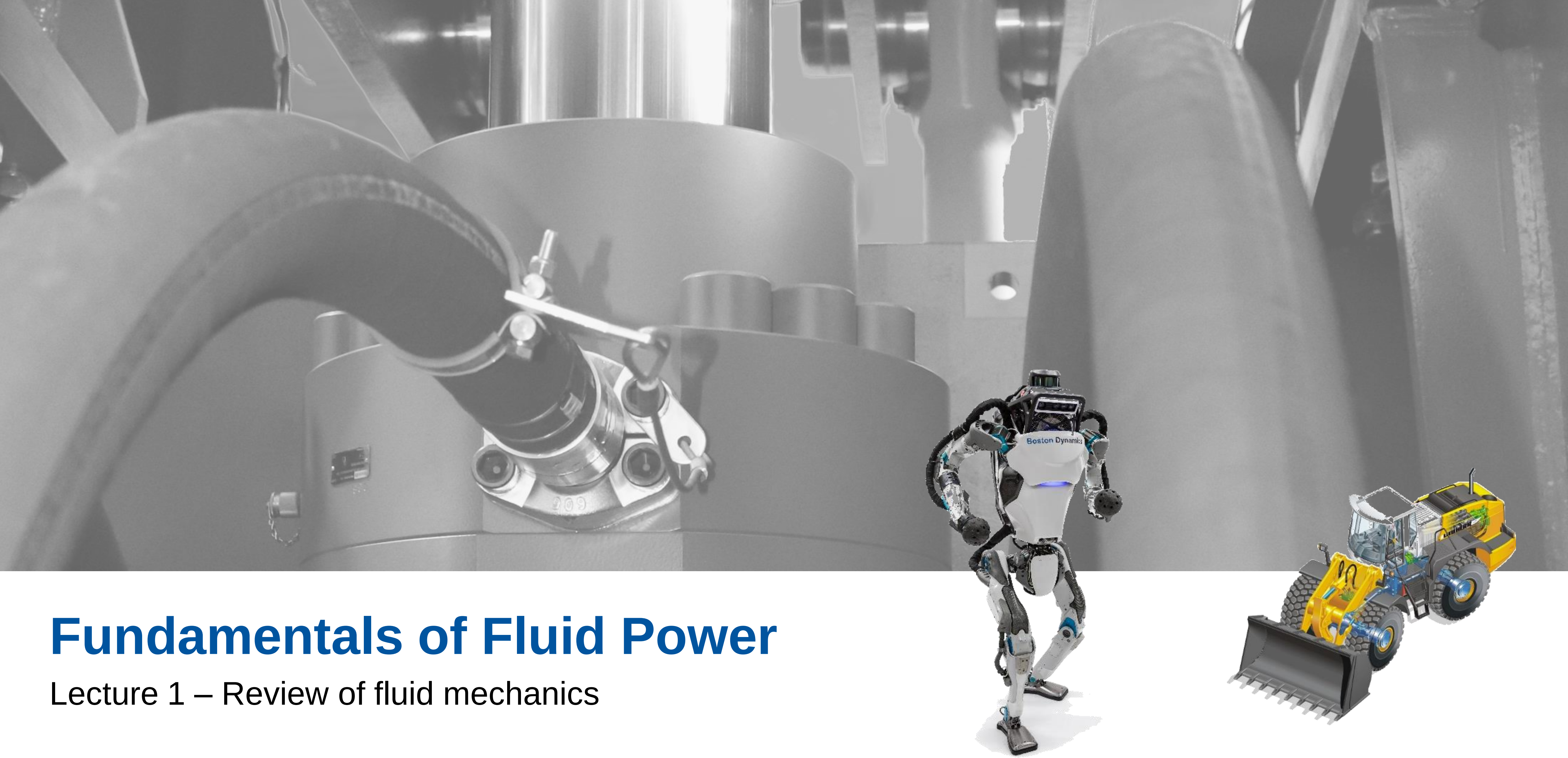

Fundamentals of Fluid Power

Lecture 1 – Review of fluid mechanics

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Definition of the subject area

- Fluid Engineering
  - Liquid media
    - Hydro static, Hydraulic
    - Hydro dynamic
  - Gaseous media
    - Pneumatic

Typical fields of application for hydraulics

Source: Hyundai

Source: Kleemann Aufzüge

Source: Fuchs-Hydraulik

Source: Zeppelin Baumaschinen

Content of today's lecture

- Review of fluid mechanics

  1. Hydrodynamics

  2. Flow through resistors

Hydrodynamics – Law of Mass Conservation

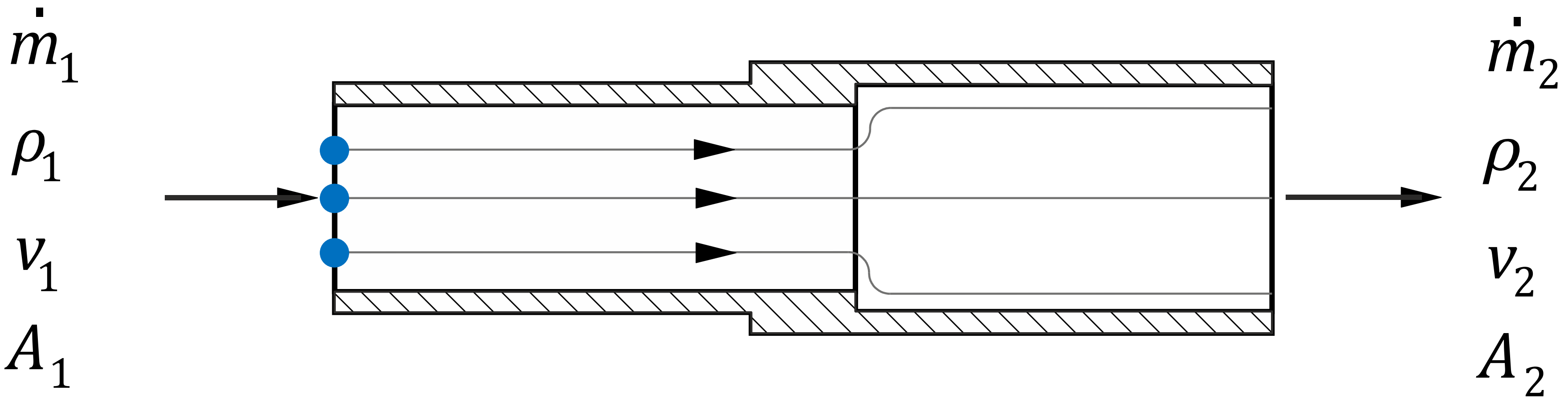
Requirement:

flow free of sources and sinks

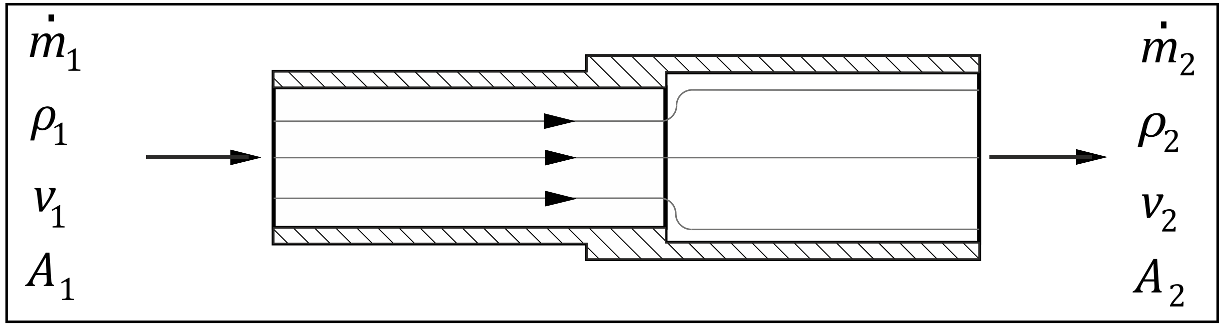
The mass entering a determined volume minus the mass leaving is equal to the mass accumulated in the volume.

$$\int_A \rho \cdot \vec{v}_\mathrm{n} \cdot \mathrm{d}A + \frac{\mathrm{d}}{\mathrm{d}t} \int_V \rho \cdot \mathrm{d}V = 0$$

Steady Flow through a Pipe



Stationary flow through a pipe



entering mass : $\dot{m}_1 = \rho_1 \cdot Q_1 = \rho_1 \cdot v_1 \cdot \mathrm{A}_1$

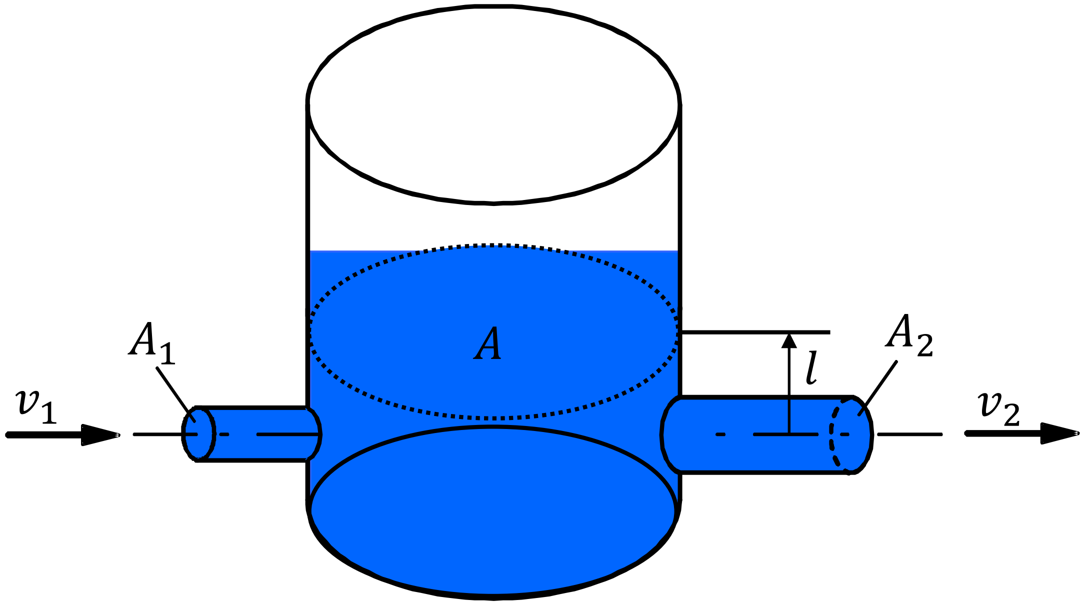
leaving mass : $\dot{m}_2 = \rho_2 \cdot Q_2 = \rho_2 \cdot v_2 \cdot \mathrm{A}_2$

steady flow: $\dot{m}_1 = \dot{m}_2$

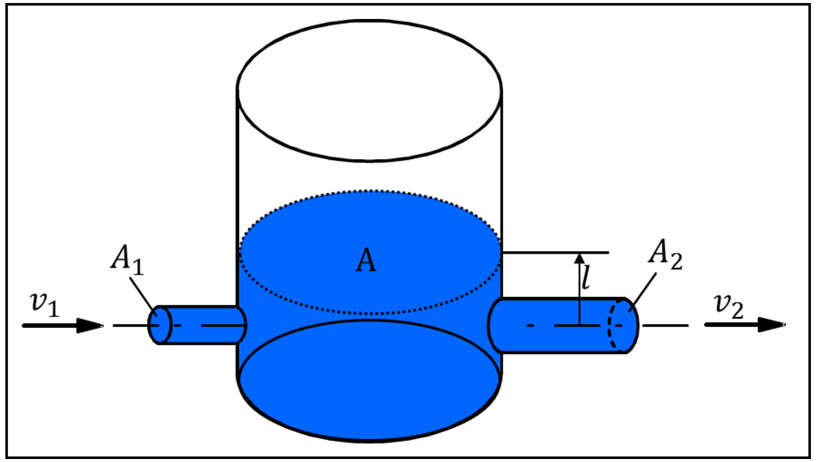
with $\rho_1 = \rho_2$

$\Rightarrow v_1 \cdot A_1 = v_2 \cdot A_2$ (continuity law)

Unsteady Flow through a Reservoir



Unsteady Flow through a Reservoir



liquid level l: $l = f(t)$

change of liquid level in time : $\dfrac{\mathrm{d}l}{\mathrm{d}t}$

change of mass in reservoir : $\dot{m} = \rho \cdot A \cdot \dfrac{\mathrm{d}l}{\mathrm{d}t}$

mass flow balance : $\rho \cdot v_1 \cdot A_1 - \rho \cdot v_2 \cdot A_2 = \rho \cdot A \cdot \dfrac{\mathrm{d}l}{\mathrm{d}t}$

with $\rho = \mathrm{const.}$: $\Rightarrow v_1 \cdot A_1 - v_2 \cdot A_2 = A \cdot \dfrac{\mathrm{d}l}{\mathrm{d}t}$

Law of conservation of the momentum

momentum: $I = \sum m_i \cdot v_i$

The change of momentum in time of a system is equal to the sum of externally acting forces.

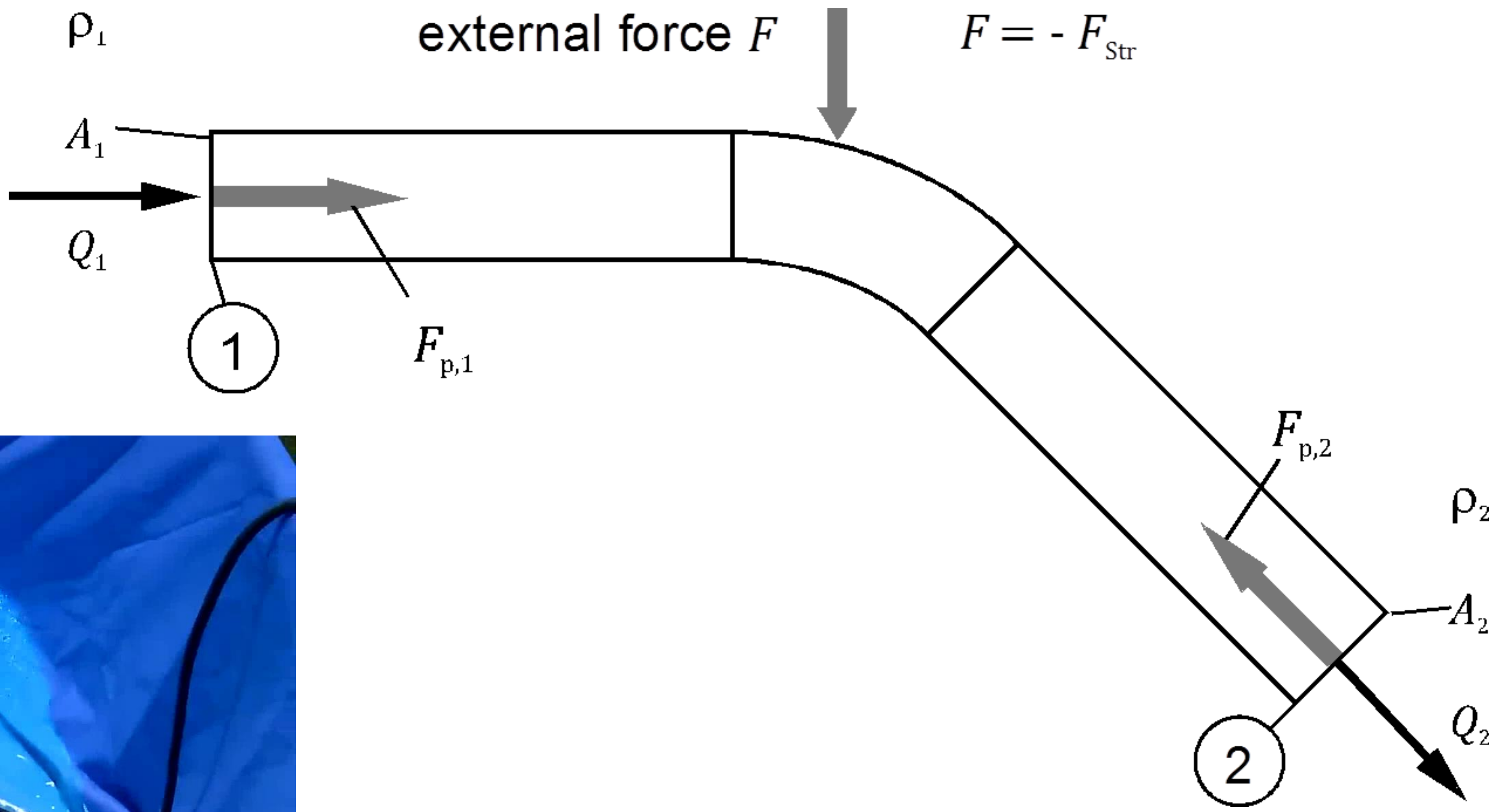
$$\sum \vec{F}_{sys} = \frac{d}{dt}\vec{I} \qquad \text{resp.} \qquad \sum \vec{F}_{sys} = \frac{d}{dt}\int_{sys} \vec{v} \cdot \rho \cdot dV$$

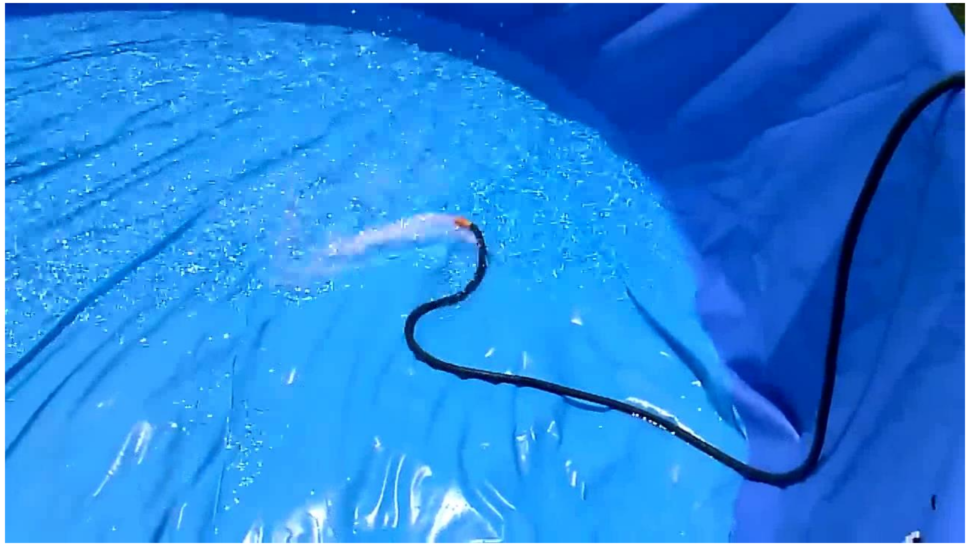
Applied to control volume KV :

$$\sum \vec{F}_{KV} = \underbrace{\frac{\partial}{\partial t}\int_{KV} \vec{v} \cdot \rho \cdot dV}_{\text{Internal change}} + \underbrace{\int_{KF} \vec{v} \cdot \rho \cdot (\vec{v} \cdot \vec{n}) dA}_{\text{external change}}$$

$$\sum \vec{F}_{KV} = \vec{F}_{Druck} + \vec{F}_{Reib} + \vec{F}_{Gew} + \vec{F}_{ext}$$

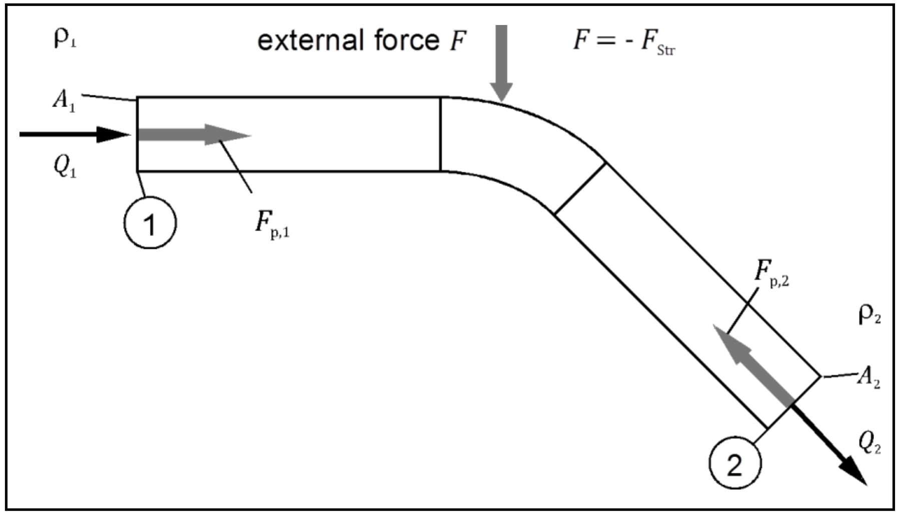
External Force at Pipe Elbow during Steady Flow





[Quelle: youtube.com]

External Force at Pipe Elbow during Steady Flow



Application of the momentum law on steady flow through an elbow:

$$\sum F_a = \int_A (\rho \cdot v) \cdot v_n \mathrm{d}A \;\Leftrightarrow\; \sum F_a = -\rho_1 \cdot v_1 \cdot A_1 \cdot v_1 + \rho_2 \cdot v_2 \cdot A_2 \cdot v_2$$

$$\sum F_a = \underbrace{F}_{\text{external forces}} + \underbrace{F_{p,1} + F_{p,2}}_{\text{pressure forces}}$$

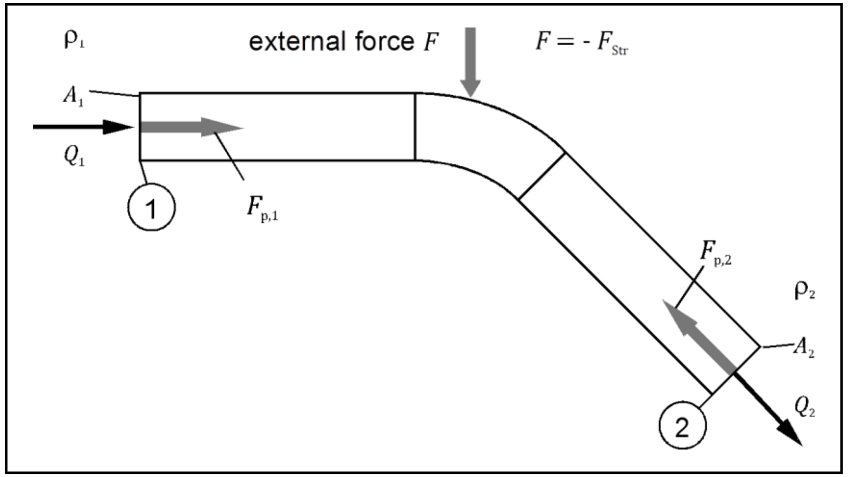
$$F = \rho_2 \cdot v_2 \cdot A_2 \cdot v_2 - \rho_1 \cdot v_1 \cdot A_1 \cdot v_1 - F_{p,1} - F_{p,2}$$

with $\rho_1 = \rho_2$ for an incompressible flow and the continuity equation

$Q = v_2 \cdot A_2 = v_1 \cdot A_1$ yields:

$$F = \rho \cdot Q \cdot (v_2 - v_1) - (F_{p,1} + F_{p,2})$$

External Force at Pipe Elbow during Steady Flow



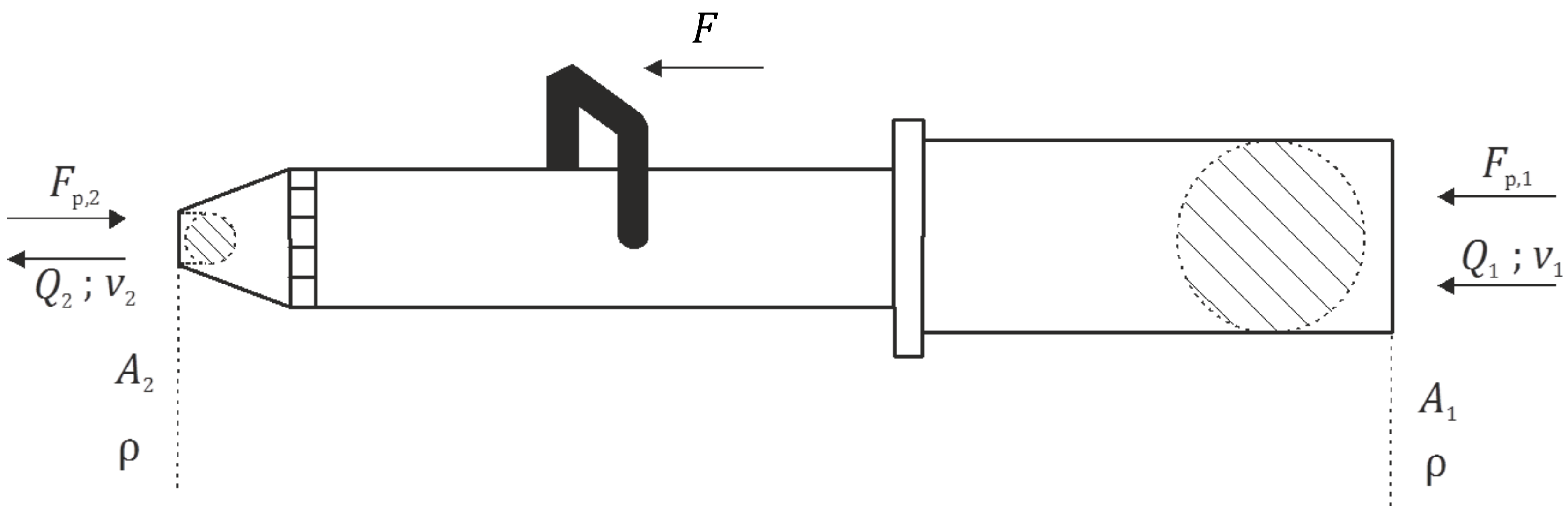
From the previous slide:

$$F = \rho \cdot Q \cdot (v_2 - v_1) - (F_{p,1} + F_{p,2})$$

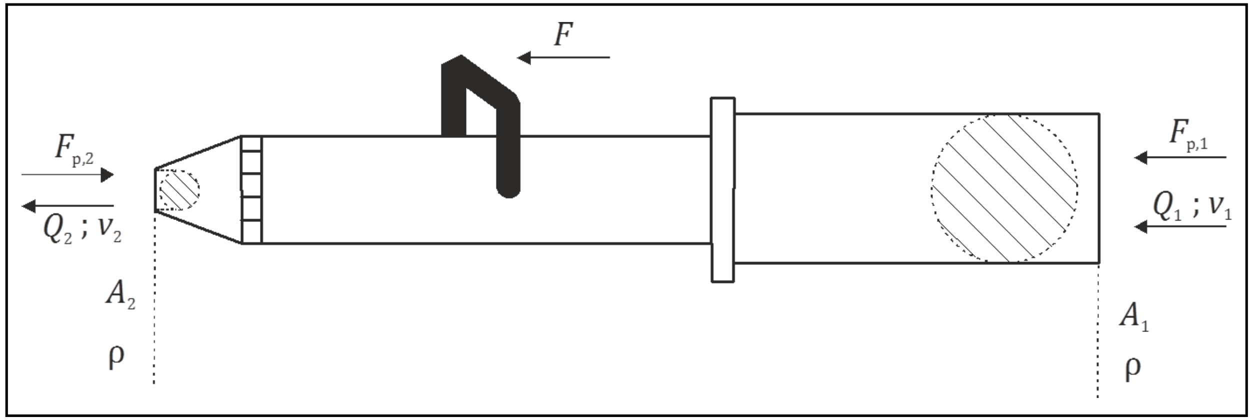
case 1: $A = \text{const.}$, no curvation $F = 0$

case 2: $A = \text{const.}$, $180°$ curvation $F = 2 \cdot A \cdot (\rho \cdot v_1^2 + p_1)$

External force acting on a fire hose



External force acting on a fire hose



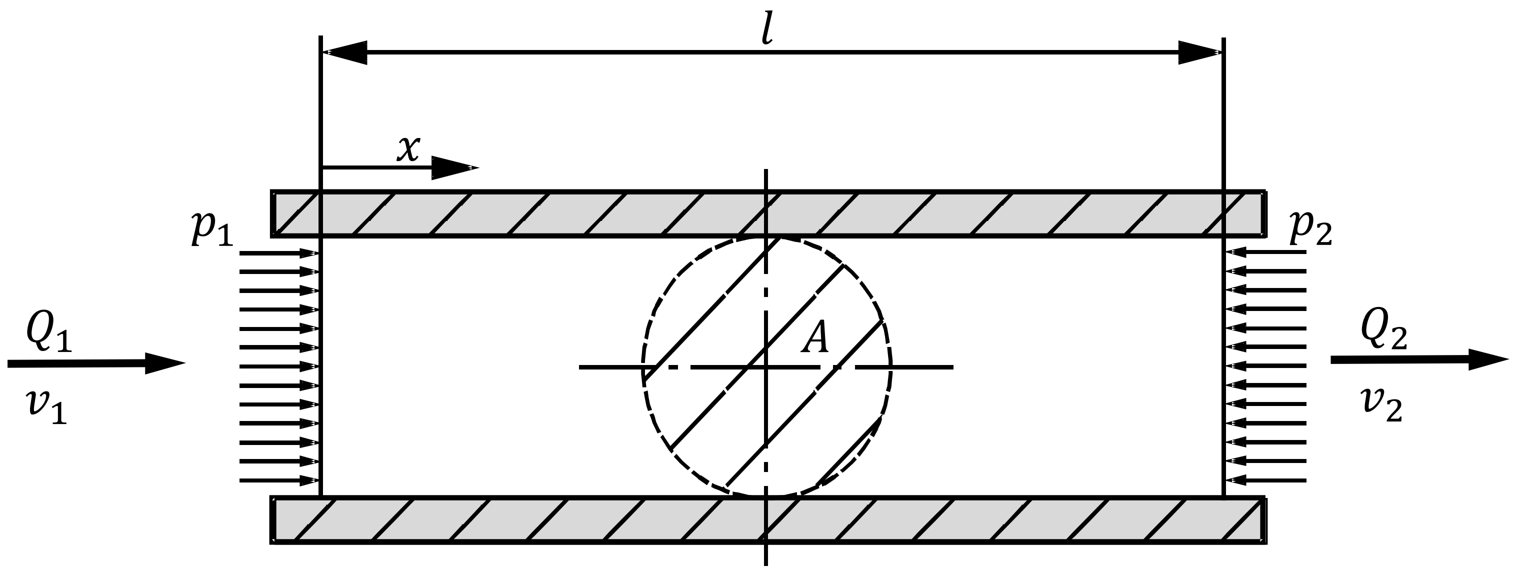
momentum of inflow: $F_1 = \rho \cdot v_1 \cdot Q_1 = \rho \cdot A_1 \cdot v_1^2$

momentum of outflow: $F_2 = \rho \cdot v_2 \cdot Q_2 = \rho \cdot A_2 \cdot v_2^2$

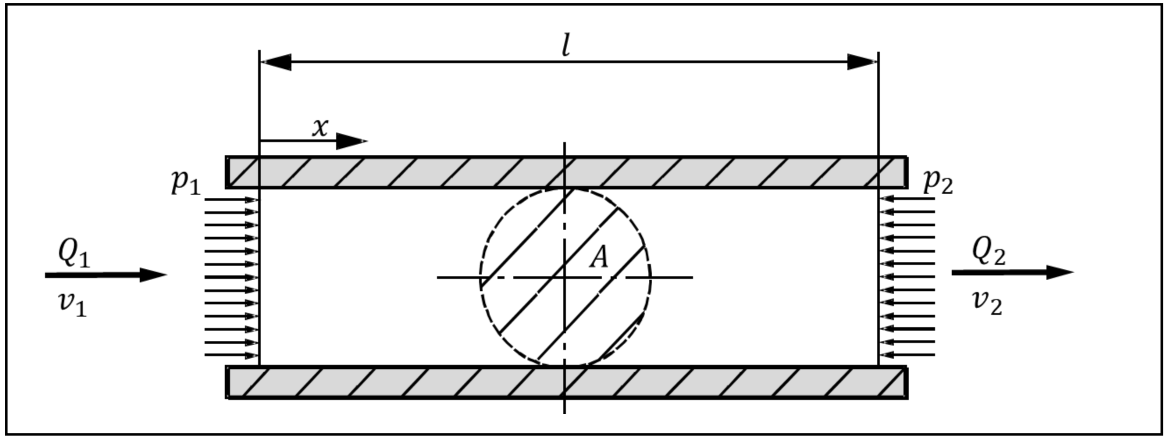
continuity equation: $\rho \cdot Q_1 - \rho \cdot Q_2 = 0$ $\Rightarrow v_2 = v_1 \cdot \frac{A_1}{A_2}$

resulting momentum force: $F = F_2 - F_1$ $\Rightarrow F = \rho \cdot v_1^2 \cdot A_1 \cdot \left(\frac{A_1}{A_2} - 1\right)$

Momentum Law for Unsteady Flow



Momentum Law for Unsteady Flow



continuity equation: $\rho \cdot Q_1 - \rho \cdot Q_2 = 0$

incompressible flow: $\Rightarrow Q_1 = Q_2 = Q$

$\Rightarrow v_1 = v_2$

momentum law: $F_\mathrm{a} = \frac{\mathrm{d}}{\mathrm{d}t}(m \cdot x) = (p_1 - p_2) \cdot A$

$$\frac{\mathrm{d}}{\mathrm{d}t}(m \cdot x) = \rho \cdot v_1 \cdot Q_1 - \rho \cdot v_2 \cdot Q_2 + \rho \cdot \int_v \frac{\mathrm{d}v}{\mathrm{d}t} \mathrm{d}V \qquad \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}(m \cdot x) = \rho \cdot \int_V \frac{\mathrm{d}v}{\mathrm{d}t} \cdot \mathrm{d}V$$

$\mathrm{d}V = A \cdot \mathrm{d}l$

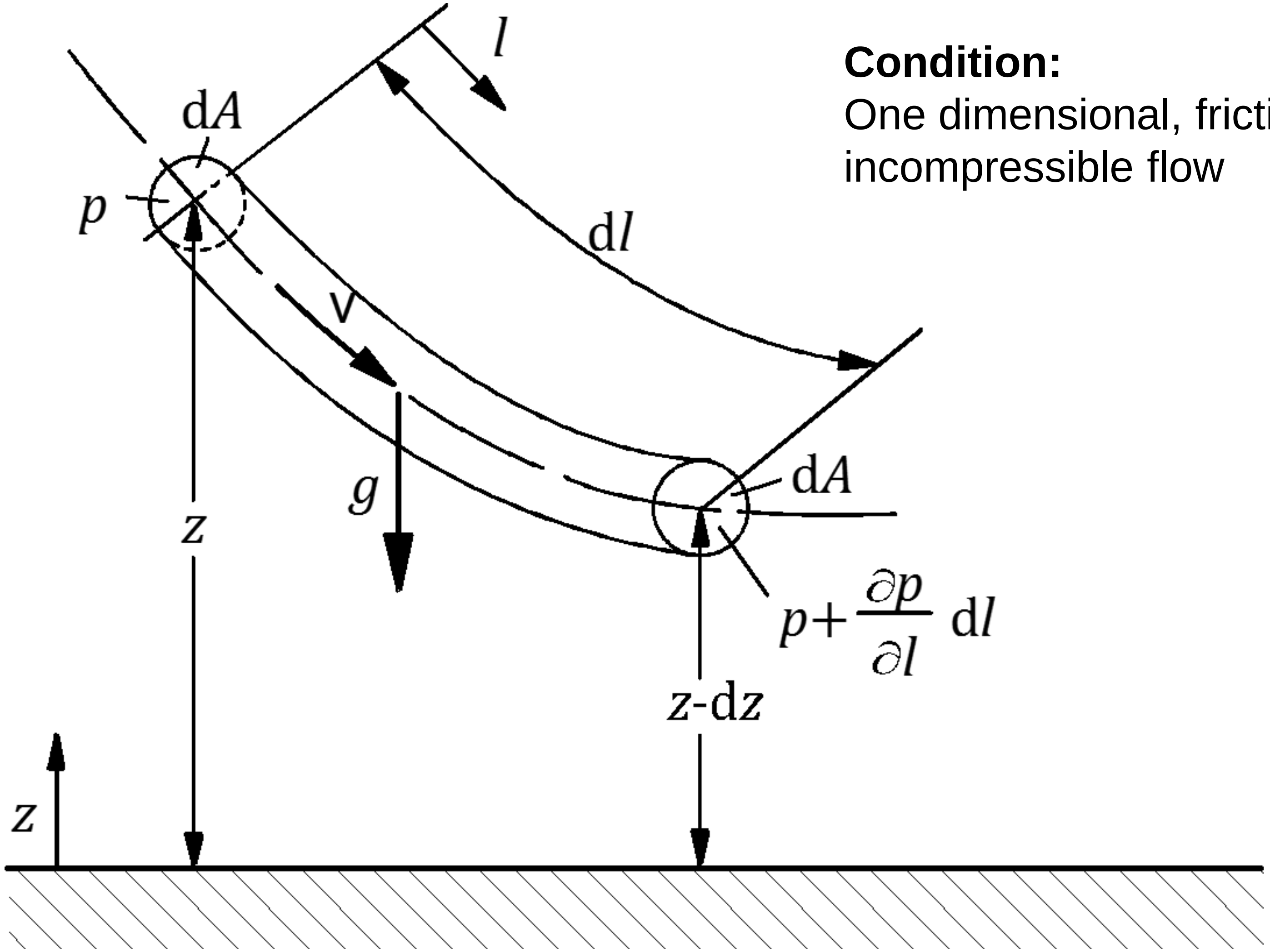
$$\Rightarrow F_\mathrm{a} = \frac{\mathrm{d}}{\mathrm{d}t}(m \cdot x) = \rho \cdot A \cdot l \cdot \frac{\mathrm{d}v}{\mathrm{d}t}$$

$$= \rho \cdot l \cdot \frac{\mathrm{d}Q}{\mathrm{d}t}$$

hydraulic inductivity L_H

$$\Rightarrow \frac{F_\mathrm{a}}{A} = p_1 - p_2 = \frac{\rho \cdot l}{A} \cdot \frac{\mathrm{d}Q}{\mathrm{d}t}$$

Law of Energy Conservation



Condition:
One dimensional, frictionless, incompressible flow

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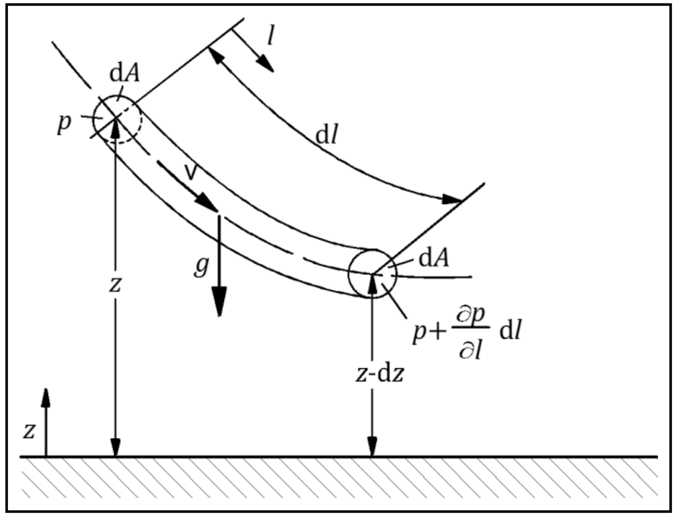
Derivation of Bernoulli‘s Equation



pressure force

$$\mathrm{d}F_\mathrm{p} = -p \cdot \mathrm{d}A + \left[p + \frac{\partial p}{\partial l} \cdot \mathrm{d}l\right] \cdot \mathrm{d}A$$

$$= \frac{\partial p}{\partial l} \cdot \mathrm{d}l \cdot \mathrm{d}A$$

gravitational force

$$G = \rho \cdot \mathrm{d}l \cdot \mathrm{d}A \cdot g \cdot \frac{\partial z}{\partial l}$$

Total velocity change :

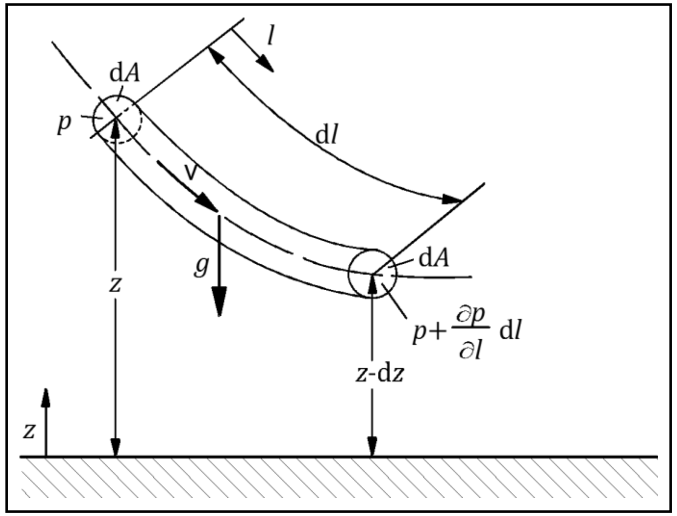
$$\mathrm{d}v = \frac{\partial v}{\partial l} \cdot \mathrm{d}l + \frac{\partial v}{\partial t} \cdot \mathrm{d}t$$

acceleration

$$\Rightarrow \frac{\mathrm{d}v}{\mathrm{d}t} = \underbrace{\frac{\partial v}{\partial l} \cdot v}_{\frac{\partial}{\partial l}\left(\frac{v^2}{2}\right)} + \frac{\partial v}{\partial t}$$

and $v = \frac{\mathrm{d}l}{\mathrm{d}t}$

Derivation of Bernoulli‘s Equation



acceleration force:

$$F = m \cdot \frac{\mathrm{d}v}{\mathrm{d}t} = \rho \cdot \mathrm{d}l \cdot \mathrm{d}A \cdot \left[\frac{\partial}{\partial l}\left(\frac{v^2}{2}\right) + \frac{\partial v}{\partial t}\right]$$

=0 for steady flow

$$\Rightarrow \mathrm{d}F_\mathrm{p} + \mathrm{d}G + \mathrm{d}F_{\ddot{x}} = 0$$

$$\frac{1}{\rho} \cdot \frac{\partial p}{\partial l} + g \cdot \frac{\partial z}{\partial l} + \frac{\partial}{\partial l}\left(\frac{v^2}{2}\right) = 0$$

Euler‘s equation (stationary case)

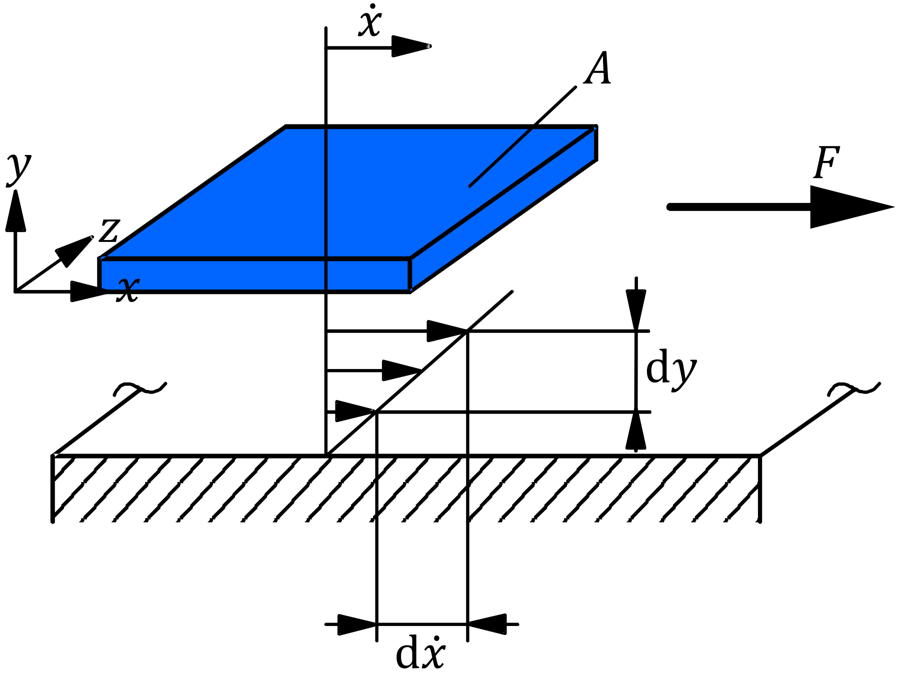
integration over $\mathrm{d}l$ yields

$$p + \rho \cdot g \cdot z + \rho \cdot \frac{v^2}{2} = \text{const.}$$

Bernoulli‘s equation

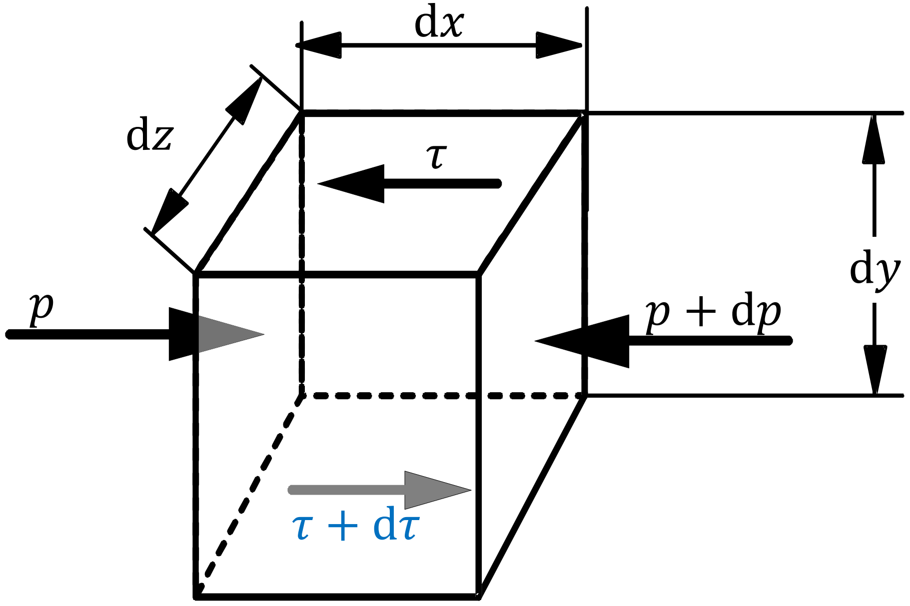
Viscosity

Couette flow in a parallel gap



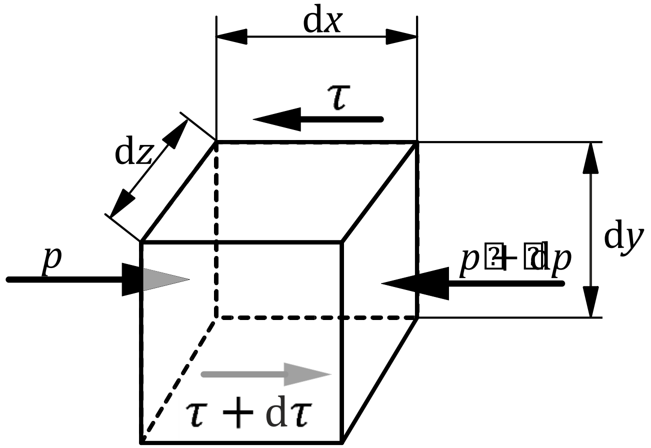
Shear stress: $\tau = \dfrac{F}{A}$

Equilibrium at a Fluid Particle



Equilibrium between shear and pressure forces

Equilibrium at a Fluid Particle



Equilibrium at a Fluid Particle:

$$[(p + \mathrm{d}p) \cdot \mathrm{d}y + \tau \cdot \mathrm{d}x] \cdot \mathrm{d}z = [p \cdot \mathrm{d}y + (\tau + \mathrm{d}\tau) \cdot \mathrm{d}x] \cdot \mathrm{d}z$$

$$\Leftrightarrow \frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\mathrm{d}\tau}{\mathrm{d}y}$$

$$\Leftrightarrow \tau = \eta \frac{\mathrm{d}\dot{x}}{\mathrm{d}y}$$

Units of viscosity

dynamic viscosity $\eta = \rho \cdot \nu$ $\left[\frac{\mathrm{Ns}}{\mathrm{m}^2}\right]$

kinematic viscosity: ν $\left[\frac{\mathrm{m}^2}{\mathrm{s}}\right]$

Dynamic viscosity η	
$1\ \mathrm{Pa\ s}$	$1\ \frac{\mathrm{N\ s}}{\mathrm{m}^2}$
$1\ \mathrm{mPa\ s}$	$10^{-3}\ \frac{\mathrm{N\ s}}{\mathrm{m}^2}$
$1\ \mathrm{P\ (Poise)}$	$1\ \frac{\mathrm{g}}{\mathrm{cm\ s}} = 0{,}1\ \frac{\mathrm{N\ s}}{\mathrm{m}^2}$
$1\ \mathrm{cP}$	$10^{-3}\ \frac{\mathrm{N\ s}}{\mathrm{m}^2}$

kinematic viscosity ν	
$1\ \frac{\mathrm{m}^2}{\mathrm{s}}$	
$1\ \frac{\mathrm{mm}^2}{\mathrm{s}}$	$10^{-6}\ \frac{\mathrm{m}^2}{\mathrm{s}}$
$1\ \mathrm{St\ (Stokes)}$	$1\ \frac{\mathrm{cm}^2}{\mathrm{s}} = 10^{-4}\ \frac{\mathrm{m}^2}{\mathrm{s}}$
$1\ \mathrm{cSt}$	$10^{-6}\ \frac{\mathrm{m}^2}{\mathrm{s}}$

Reynolds Number

Reynoldszahl (Re):

$$Re = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho \cdot \dot{x}^2}{\eta \cdot \frac{\dot{x}}{l}}$$

$$\Rightarrow Re = \frac{\rho \cdot \dot{x} \cdot l}{\eta} = \frac{\dot{x} \cdot D_\mathrm{H}}{\nu}$$

D_H : hydraulic diameter

$$D_\mathrm{H} = \frac{4 \cdot A}{U}$$

with A : flow cross section

U : wetted circumference

Reynolds Number

Examples:

1. circular cross section $\emptyset d$

$$D_{\mathrm{H}} = \frac{4 \cdot \frac{\pi \cdot d^2}{4}}{\pi \cdot d} = d$$

2. narrow gap with $\text{height } h \ll \text{width } b$

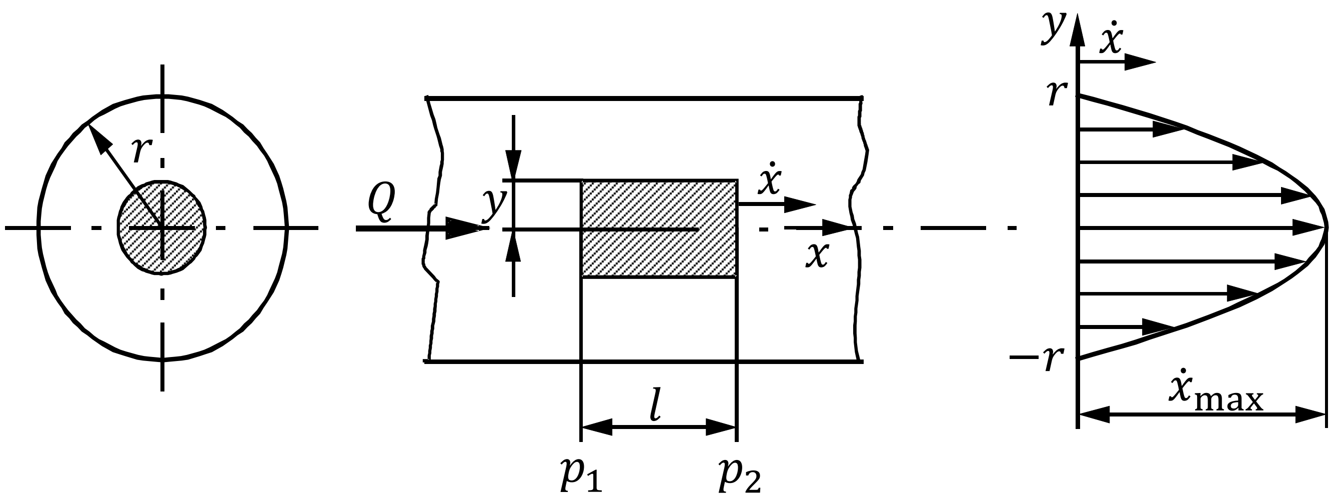
$$D_{\mathrm{H}} = \frac{4 \cdot b \cdot h}{2 \cdot (b + h)} \approx 2 \cdot h$$

Content of today's lecture

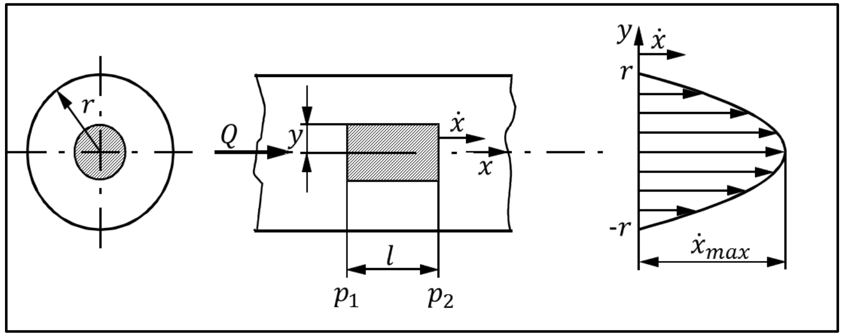
- Review of fluid mechanics
  - hydrodynamics
  - Flow through resistors

Resistance of Pipings

laminar flow in a smooth pipe



Resistance of Pipings



equilibrium at a fluid particle:

$$\tau \cdot 2\pi \cdot y \cdot l = (p_1 - p_2) \cdot \pi \cdot y^2$$

shear stress at a cylinder jacket:

$$\tau = -\eta \cdot \frac{\mathrm{d}\dot{x}}{\mathrm{d}y}$$

distribution of speed:

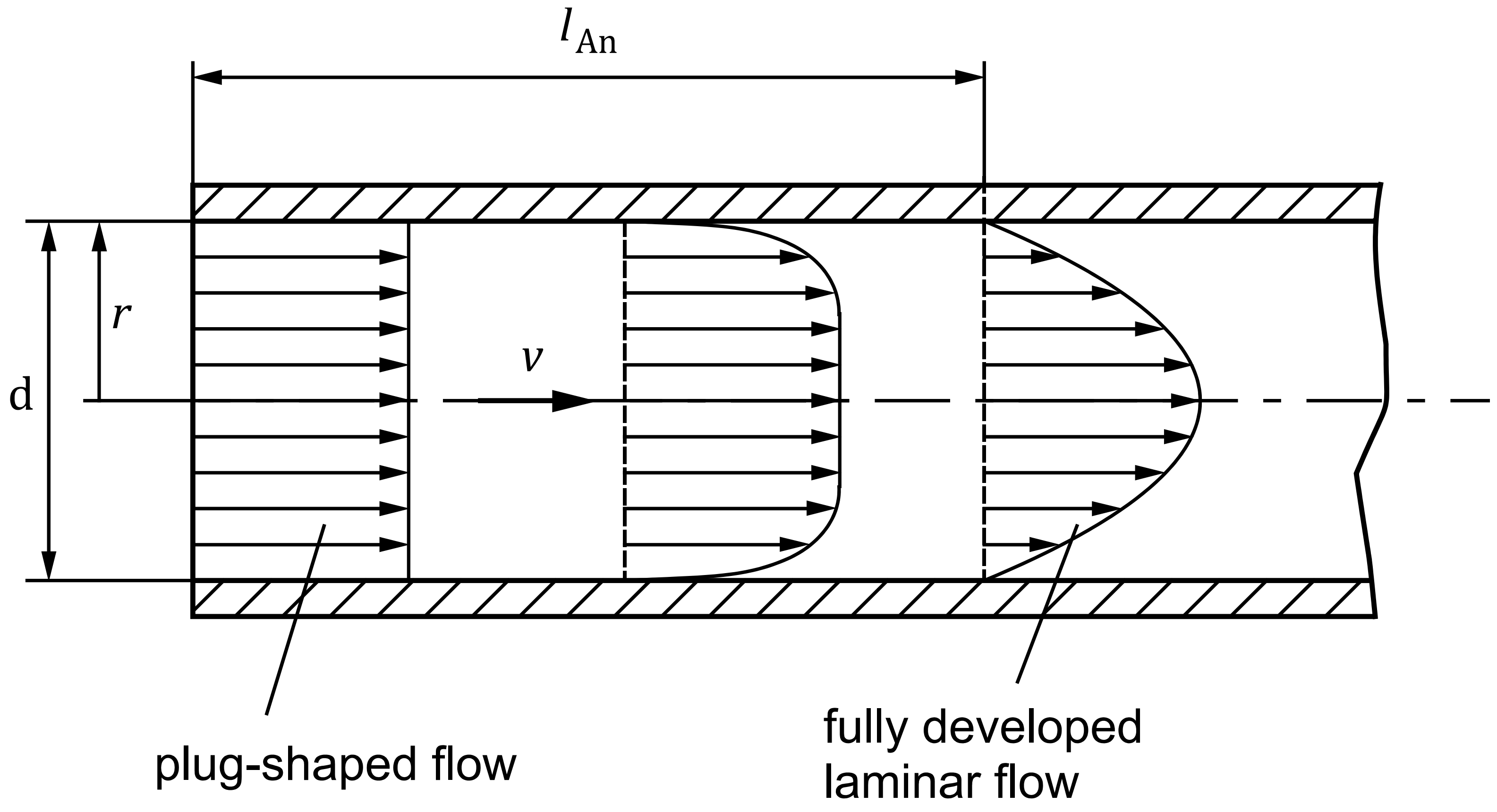
$$\dot{x} = \frac{p_1 - p_2}{4 \cdot \eta \cdot l} \cdot (r^2 - y^2) \qquad \dot{x}_{\max} = \frac{p_1 - p_2}{4 \cdot \eta \cdot l} \cdot r^2$$

flow rate:

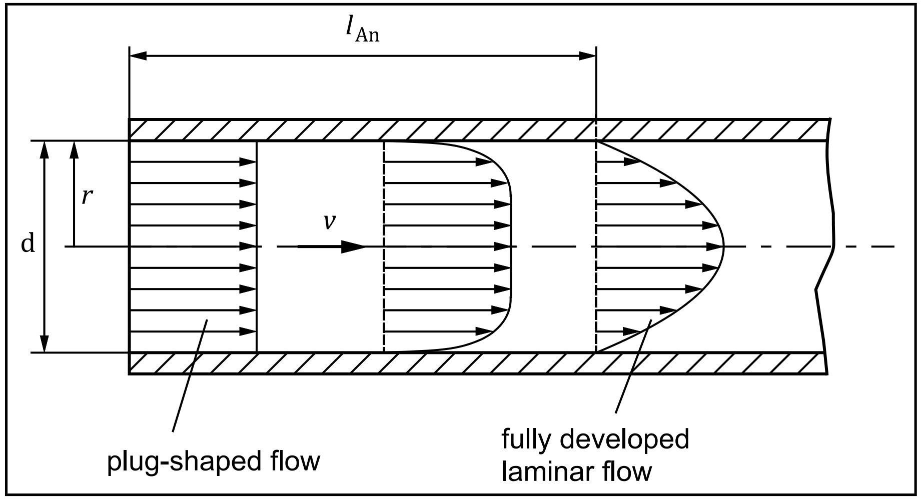
$$Q = \int_{y=0}^{y=r} \dot{x}\mathrm{d}A = \int \frac{p_1 - p_2}{4 \cdot \eta \cdot l} \cdot (r^2 - y^2) \cdot 2\pi \cdot y \cdot \mathrm{d}y$$

$$Q = \frac{\pi \cdot r^4}{8 \cdot \eta \cdot l} \cdot (p_1 - p_2)$$

Formation of the Laminar Flow State



Formation of the Laminar Flow State



plug flow: $Q = v \cdot \mathrm{A}$

fully developed laminar flow: $$Q = \int_0^{\mathrm{r}} v(r)\mathrm{d}A$$

transition length: $l_{\mathrm{An}} \approx \mathrm{d} \cdot 0{,}058 \cdot Re$

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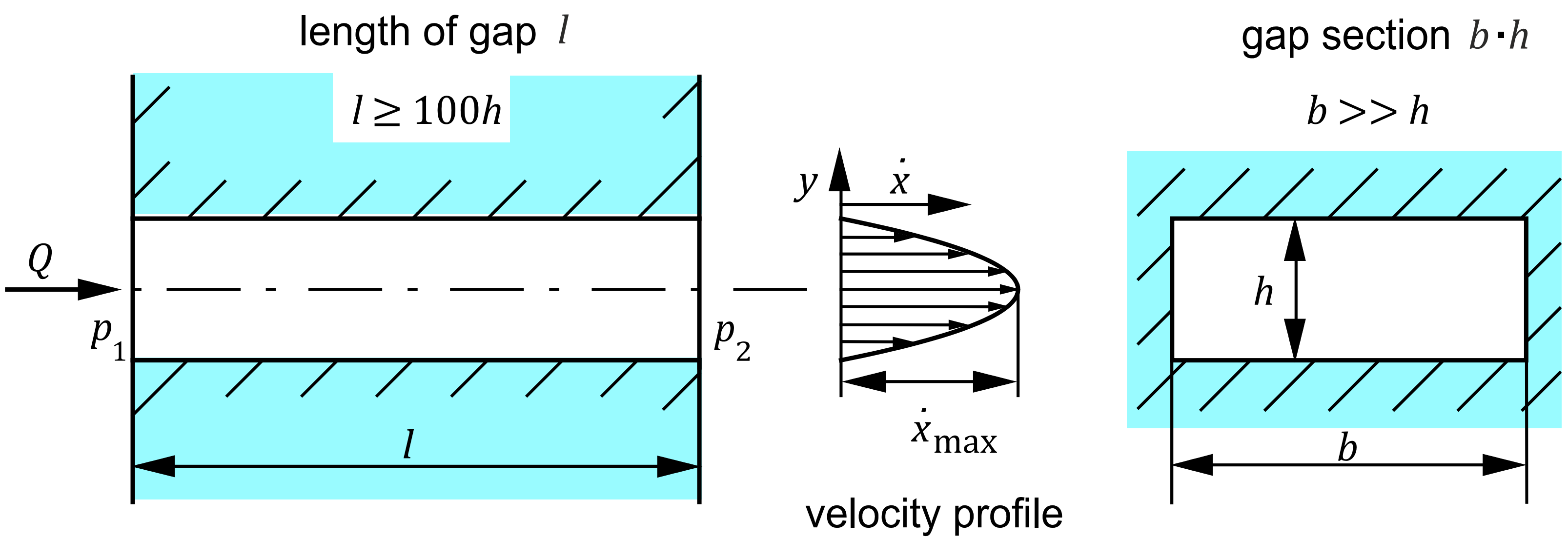
Laminar Flow in a Gap



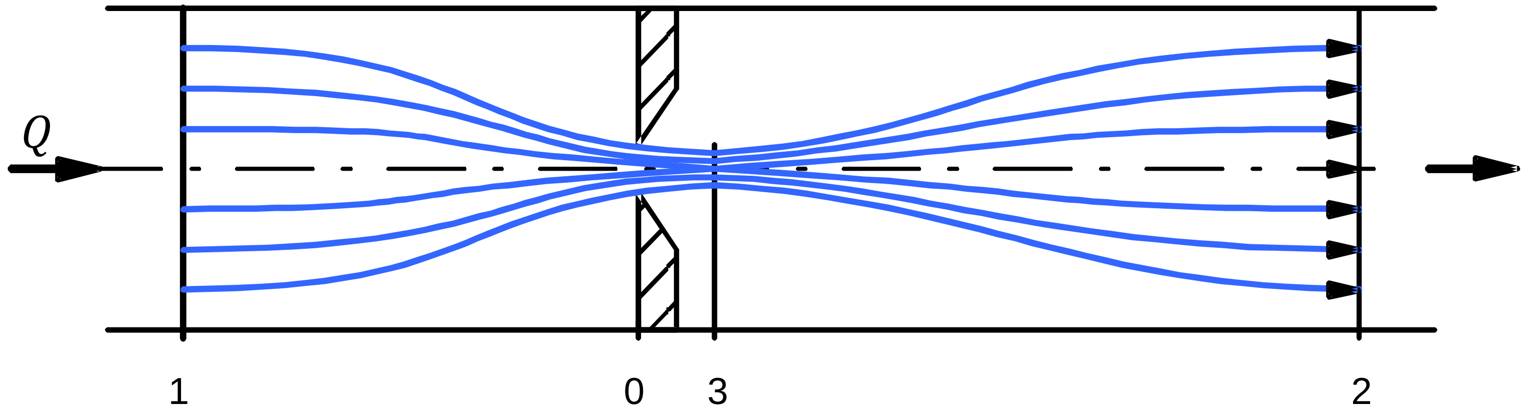
conditions: $l > 100\,h$; $b >> h$
Q: volumetric flow through gap
$\Delta p = p_1 - p_2$: pressure difference

Flow rate:

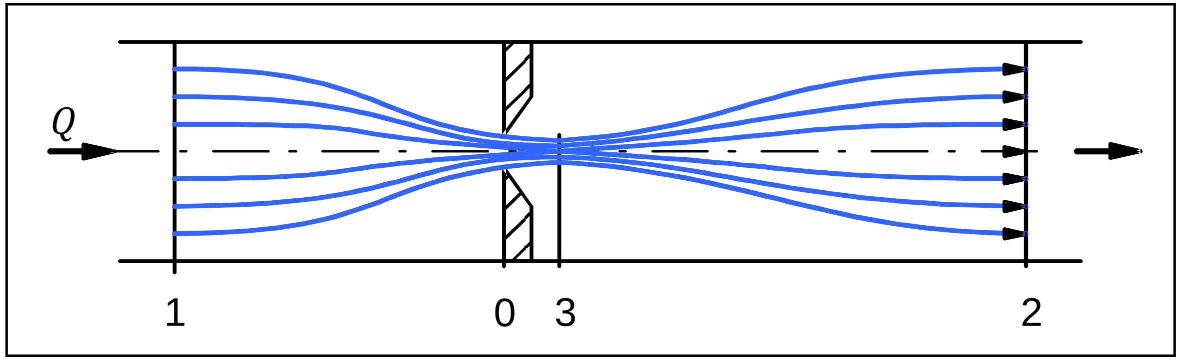
$$Q = 2 \cdot \int_{y=0}^{y=\frac{h}{2}} \dot{x} \mathrm{d}A \qquad \text{with} \quad \mathrm{d}A = b \cdot \mathrm{d}y$$

$$Q = \frac{b \cdot h^3}{12 \cdot \eta \cdot l} \cdot (p_1 - p_2)$$

Flow through an Orifice



Steady Flow through an Orifice



requirement: $A_3 \ll A_1 \Rightarrow v_1 \ll v_3$

$\rho = \text{const.}$

statement following Bernoulli: $p_1 = p_3 + \dfrac{\rho \cdot v_3^2}{2}$

velocity $v_3 = \sqrt{\dfrac{2 \cdot \Delta p'}{\rho}}$ with $\Delta p' = p_1 - p_3$

flow rate $Q = A_3 \cdot \sqrt{\dfrac{2 \cdot \Delta p'}{\rho}}$

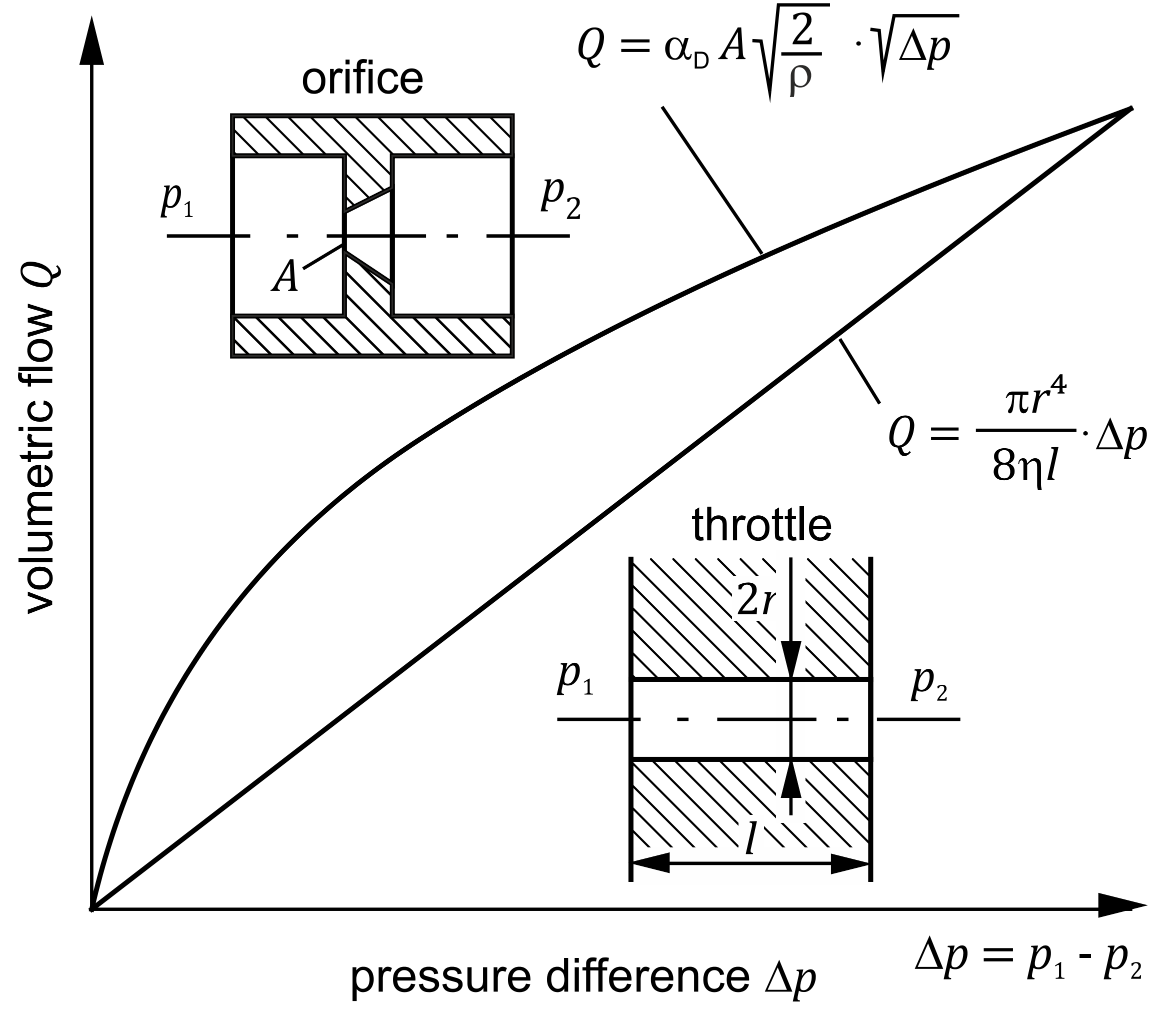
constriction factor α_K $A_3 = \alpha_K \cdot A_0$

$\Rightarrow Q = \alpha_K \cdot A_0 \cdot \sqrt{\dfrac{2 \cdot \Delta p'}{\rho}}$

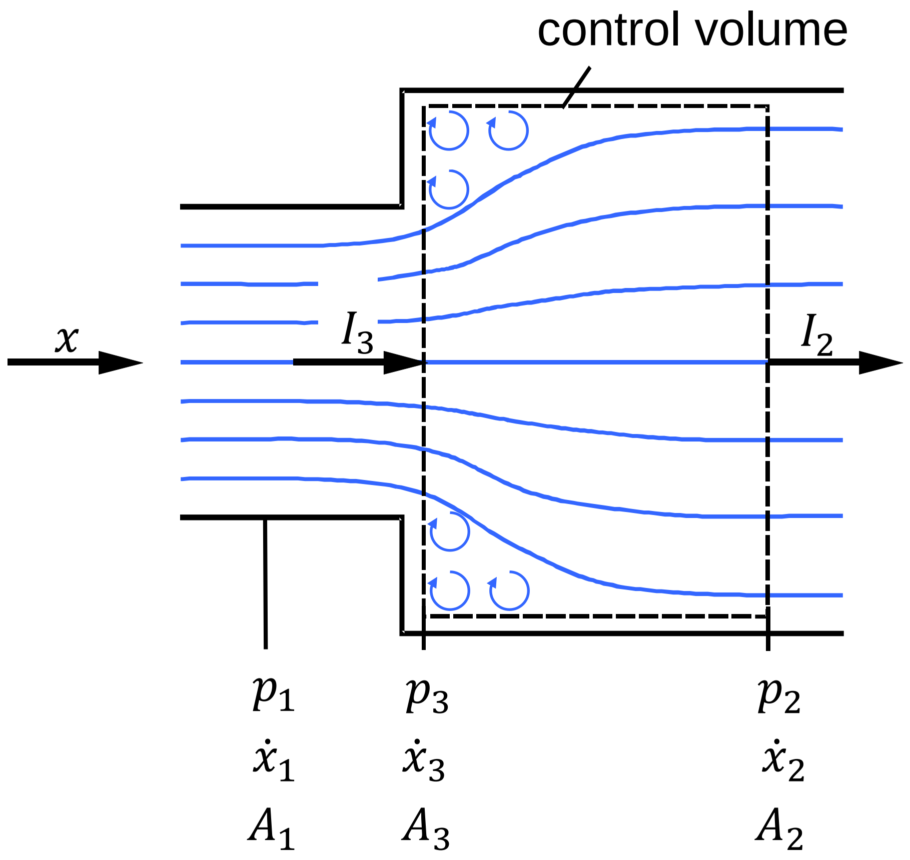
$Q = \alpha_D \cdot A_0 \cdot \sqrt{\dfrac{2 \cdot \Delta p}{\rho}}$ with $\Delta p = p_1 - p_2$

flow coefficient α_D

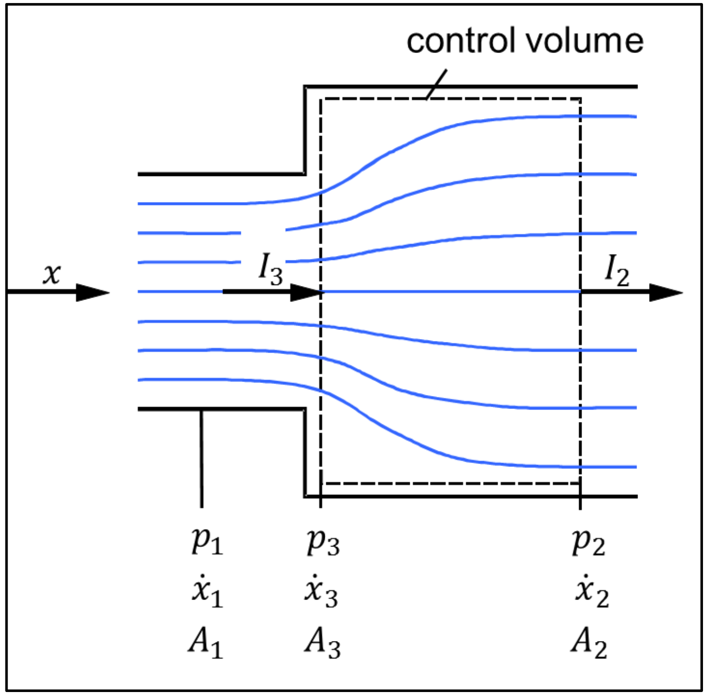
Flow Law for Orifice and Throttle



Pressure Loss at Cross Section Changes



Pressure Loss at Cross Section Enlargement



momentum law:

$$p_3 \cdot A_2 - p_2 \cdot A_2 = \rho \cdot Q_2 \cdot \dot{x}_2 - \rho \cdot Q_3 \cdot \dot{x}_3$$

$$p_3 = p_1 \qquad \dot{x}_3 = \dot{x}_1$$

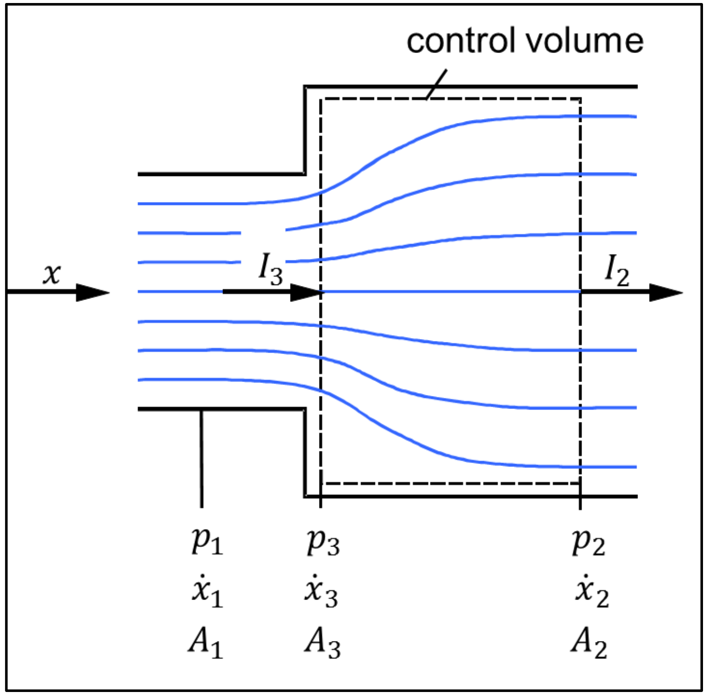
$$\Rightarrow A_2 \cdot (p_2 - p_1) = \rho \cdot Q_3 \cdot \dot{x}_1 - \rho \cdot Q_2 \cdot \dot{x}_2$$

continuity condition:

$$\rho \cdot Q_1 = \rho \cdot Q_3 = \rho \cdot Q_2 = \rho \cdot A_2 \cdot \dot{x}_2$$

$$\Rightarrow A_2 \cdot (p_2 - p_1) = \rho \cdot A_2 \cdot \dot{x}_2(\dot{x}_1 - \dot{x}_2)$$

Pressure Loss at Cross Section Enlargement



pressure difference with losses:

$$\Rightarrow p_2 - p_1 = \rho \cdot \dot{x}_2 \cdot (\dot{x}_1 - \dot{x}_2) = \Delta p_{\mathrm{mV}}$$

pressure difference without losses:

$$p_2^* - p_1 = \frac{\rho}{2} \cdot (\dot{x}_1^2 - \dot{x}_2^2) = \Delta p_{\mathrm{oV}}$$

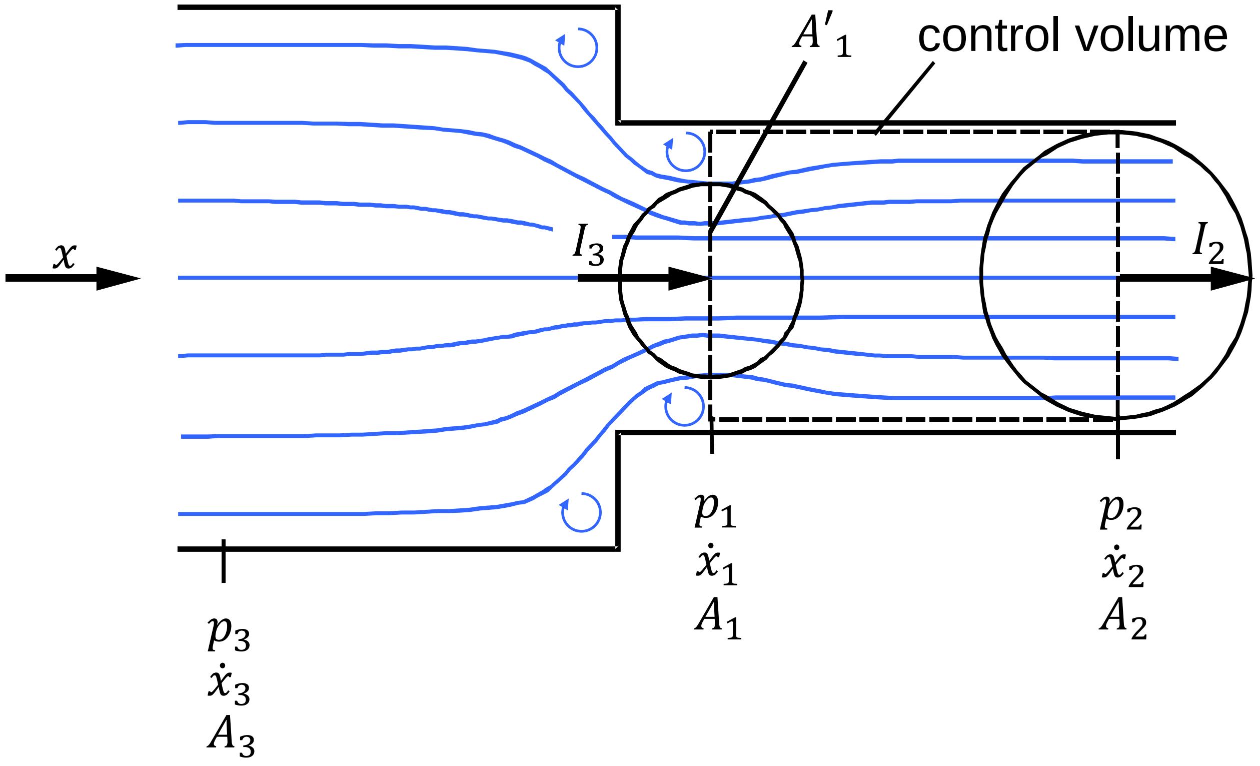
momentum loss through turbulence:

$$\Delta p_{\mathrm{SV}} = \Delta p_{\mathrm{oV}} - \Delta p_{\mathrm{mV}} = p_2^* - p_2$$

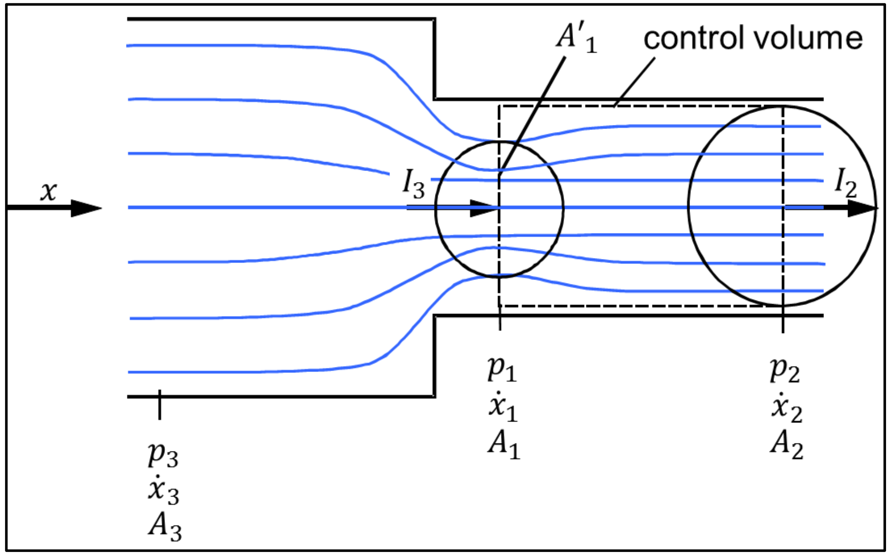
$$\Delta p_{\mathrm{SV}} = \frac{\rho}{2} \cdot (\dot{x}_1^2 - \dot{x}_2^2 - 2\dot{x}_1\dot{x}_2 + 2\dot{x}_2^2)$$

$$\Delta p_{\mathrm{SV}} = \frac{\rho}{2} \cdot (\dot{x}_1 - \dot{x}_2)^2$$

Pressure Loss at Cross Section Reduction



Pressure Loss at Cross Section Reduction



momentum law:

$$p_2 \cdot A_2 - p_1 \cdot A_1 = \rho \cdot Q_1 \cdot \dot{x}_1 - \rho \cdot Q_2 \cdot \dot{x}_2$$

$$A_1 = A_2$$

continuity condition:

$$\rho \cdot Q_1 = \rho \cdot Q_2 = \rho \cdot A_2 \cdot \dot{x}_2$$

$$\Rightarrow A_2 \cdot (p_2 - p_1) = \rho \cdot A_2 \cdot \dot{x}_2 (\dot{x}_1 - \dot{x}_2)$$

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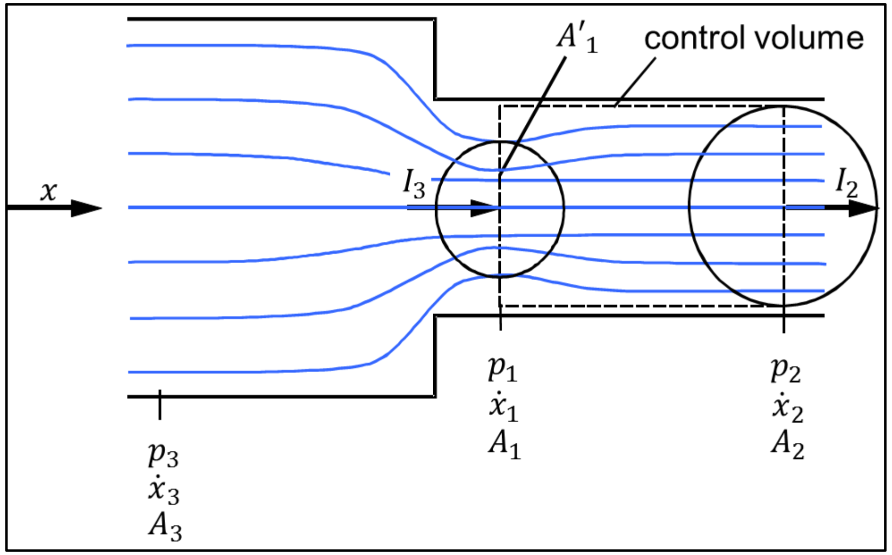
Pressure Loss at Cross Section Reduction



pressure difference with losses:

$$p_2 - p_1 = \rho \cdot \dot{x}_2 \cdot (\dot{x}_1 - \dot{x}_2) = \Delta p_{\mathrm{mV}}$$

pressure difference without losses:

$$p_2^* - p_1 = \frac{\rho}{2} \cdot (\dot{x}_1^2 - \dot{x}_2^2) = \Delta p_{\mathrm{oV}}$$

pressure loss:

$$\Delta p_{\mathrm{V}} = \Delta p_{\mathrm{oV}} - \Delta p_{\mathrm{mV}} = \frac{\rho}{2} \cdot (\dot{x}_1 - \dot{x}_2)^2$$

$$\Delta p_{\mathrm{V}} = \frac{\rho}{2} \cdot \dot{x}_2^2 \cdot \left(\frac{1}{\alpha_{\mathrm{K}}} - 1\right)^2 \qquad \alpha_{\mathrm{K}} = \frac{A_1'}{A_2} = \frac{\dot{x}_2}{\dot{x}_1}$$

Thank you for your attention!