

高等数学微积分公式大全

一、基本导数公式

$$\begin{aligned}(1) (c)' &= 0 & (2) x^\mu &= \mu x^{\mu-1} & (3) (\sin x)' &= \cos x \\(4) (\cos x)' &= -\sin x & (5) (\tan x)' &= \sec^2 x & (6) (\cot x)' &= -\csc^2 x \\(7) (\sec x)' &= \sec x \cdot \tan x & (8) (\csc x)' &= -\csc x \cdot \cot x \\(9) (e^x)' &= e^x & (10) (a^x)' &= a^x \ln a & (11) (\ln x)' &= \frac{1}{x} \\(12) (\log_a x)' &= \frac{1}{x \ln a} & (13) (\arcsin x)' &= \frac{1}{\sqrt{1-x^2}} & (14) (\arccos x)' &= -\frac{1}{\sqrt{1-x^2}} \\(15) (\arctan x)' &= \frac{1}{1+x^2} & (16) (\operatorname{arccot} x)' &= -\frac{1}{1+x^2} & (17) (x)' &= 1 & (18) (\sqrt{x})' &= \frac{1}{2\sqrt{x}}\end{aligned}$$

二、导数的四则运算法则

$$(u \pm v)' = u' \pm v' \quad (uv)' = u'v + uv' \quad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

三、高阶导数的运算法则

$$\begin{aligned}(1) [u(x) \pm v(x)]^{(n)} &= u^{(n)}(x) \pm v^{(n)}(x) & (2) [cu(x)]^{(n)} &= cu^{(n)}(x) \\(3) [u(ax+b)]^{(n)} &= a^n u^{(n)}(ax+b) & (4) [u(x) \cdot v(x)]^{(n)} &= \sum_{k=0}^n C_n^k u^{(n-k)}(x) v^{(k)}(x)\end{aligned}$$

四、基本初等函数的 n 阶导数公式

$$\begin{aligned}(1) (x^n)^{(n)} &= n! & (2) (e^{ax+b})^{(n)} &= a^n \cdot e^{ax+b} & (3) (a^x)^{(n)} &= a^x \ln^n a \\(4) [\sin(ax+b)]^{(n)} &= a^n \sin\left(ax+b+n \cdot \frac{\pi}{2}\right) & (5) [\cos(ax+b)]^{(n)} &= a^n \cos\left(ax+b+n \cdot \frac{\pi}{2}\right) \\(6) \left(\frac{1}{ax+b}\right)^{(n)} &= (-1)^n \frac{a^n \cdot n!}{(ax+b)^{n+1}} & (7) [\ln(ax+b)]^{(n)} &= (-1)^{n-1} \frac{a^n \cdot (n-1)!}{(ax+b)^n}\end{aligned}$$

五、微分公式与微分运算法则

$$\begin{aligned}(1) d(c) &= 0 & (2) d(x^\mu) &= \mu x^{\mu-1} dx & (3) d(\sin x) &= \cos x dx \\(4) d(\cos x) &= -\sin x dx & (5) d(\tan x) &= \sec^2 x dx & (6) d(\cot x) &= -\csc^2 x dx \\(7) d(\sec x) &= \sec x \cdot \tan x dx & (8) d(\csc x) &= -\csc x \cdot \cot x dx \\(9) d(e^x) &= e^x dx & (10) d(a^x) &= a^x \ln a dx & (11) d(\ln x) &= \frac{1}{x} dx\end{aligned}$$

$$(12) d(\log_a x) = \frac{1}{x \ln a} dx \quad (13) d(\arcsin x) = \frac{1}{\sqrt{1-x^2}} dx \quad (14) d(\arccos x) = -\frac{1}{\sqrt{1-x^2}} dx$$

$$(15) d(\arctan x) = \frac{1}{1+x^2} dx \quad (16) d(\operatorname{arccot} x) = -\frac{1}{1+x^2} dx$$

六、微分运算法则

$$(1) d(u \pm v) = du \pm dv \quad (2) d(cu) = cdu$$

$$(3) d(uv) = vdu + u dv \quad (4) d\left(\frac{u}{v}\right) = \frac{vdu - u dv}{v^2}$$

七、基本积分公式

$$(1) \int k dx = kx + c \quad (2) \int x^\mu dx = \frac{x^{\mu+1}}{\mu+1} + c \quad (3) \int \frac{dx}{x} = \ln|x| + c$$

$$(4) \int a^x dx = \frac{a^x}{\ln a} + c \quad (5) \int e^x dx = e^x + c \quad (6) \int \cos x dx = \sin x + c$$

$$(7) \int \sin x dx = -\cos x + c \quad (8) \int \frac{1}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + c$$

$$(9) \int \frac{1}{\sin^2 x} dx = \int \csc^2 x dx = -\cot x + c \quad (10) \int \frac{1}{1+x^2} dx = \arctan x + c$$

$$(11) \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

八、补充积分公式

$$\int \tan x dx = -\ln|\cos x| + c \quad \int \cot x dx = \ln|\sin x| + c$$

$$\int \sec x dx = \ln|\sec x + \tan x| + c \quad \int \csc x dx = \ln|\csc x - \cot x| + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c \quad \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + c \quad \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + c$$

九、下列常用凑微分公式

积分型	换元公式
$\int f(ax+b) dx = \frac{1}{a} \int f(ax+b) d(ax+b)$	$u = ax+b$
$\int f(x^\mu) x^{\mu-1} dx = \frac{1}{\mu} \int f(x^\mu) d(x^\mu)$	$u = x^\mu$
$\int f(\ln x) \cdot \frac{1}{x} dx = \int f(\ln x) d(\ln x)$	$u = \ln x$

$\int f(e^x) \cdot e^x dx = \int f(e^x) d(e^x)$	$u = e^x$
$\int f(a^x) \cdot a^x dx = \frac{1}{\ln a} \int f(a^x) d(a^x)$	$u = a^x$
$\int f(\sin x) \cdot \cos x dx = \int f(\sin x) d(\sin x)$	$u = \sin x$
$\int f(\cos x) \cdot \sin x dx = -\int f(\cos x) d(\cos x)$	$u = \cos x$
$\int f(\tan x) \cdot \sec^2 x dx = \int f(\tan x) d(\tan x)$	$u = \tan x$
$\int f(\cot x) \cdot \csc^2 x dx = \int f(\cot x) d(\cot x)$	$u = \cot x$
$\int f(\arctan x) \cdot \frac{1}{1+x^2} dx = \int f(\arctan x) d(\arctan x)$	$u = \arctan x$
$\int f(\arcsin x) \cdot \frac{1}{\sqrt{1-x^2}} dx = \int f(\arcsin x) d(\arcsin x)$	$u = \arcsin x$

十、分部积分法公式

(1) 形如 $\int x^n e^{ax} dx$, 令 $u = x^n$, $dv = e^{ax} dx$

形如 $\int x^n \sin x dx$ 令 $u = x^n$, $dv = \sin x dx$

形如 $\int x^n \cos x dx$ 令 $u = x^n$, $dv = \cos x dx$

(2) 形如 $\int x^n \arctan x dx$, 令 $u = \arctan x$, $dv = x^n dx$

形如 $\int x^n \ln x dx$, 令 $u = \ln x$, $dv = x^n dx$

(3) 形如 $\int e^{ax} \sin x dx$, $\int e^{ax} \cos x dx$ 令 $u = e^{ax}$, $\sin x, \cos x$ 均可。

十一、第二换元积分法中的三角换元公式

(1) $\sqrt{a^2 - x^2}$ $x = a \sin t$ (2) $\sqrt{a^2 + x^2}$ $x = a \tan t$ (3) $\sqrt{x^2 - a^2}$ $x = a \sec t$

【特殊角的三角函数值】

(1) $\sin 0 = 0$ (2) $\sin \frac{\pi}{6} = \frac{1}{2}$ (3) $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ (4) $\sin \frac{\pi}{2} = 1$ (5) $\sin \pi = 0$

(1) $\cos 0 = 1$ (2) $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ (3) $\cos \frac{\pi}{3} = \frac{1}{2}$ (4) $\cos \frac{\pi}{2} = 0$ (5) $\cos \pi = -1$

(1) $\tan 0 = 0$ (2) $\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$ (3) $\tan \frac{\pi}{3} = \sqrt{3}$ (4) $\tan \frac{\pi}{2}$ 不存在 (5) $\tan \pi = 0$

$$(1) \cot 0 \text{ 不存在} \quad (2) \cot \frac{\pi}{6} = \sqrt{3} \quad (3) \cot \frac{\pi}{3} = \frac{\sqrt{3}}{3} \quad (4) \cot \frac{\pi}{2} = 0 \quad (5) \cot \pi \text{ 不存在}$$

十二、重要公式

$$\begin{aligned} (1) \lim_{x \rightarrow 0} \frac{\sin x}{x} &= 1 & (2) \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} &= e & (3) \lim_{n \rightarrow \infty} \sqrt[n]{a} (a > 0) &= 1 \\ (4) \lim_{n \rightarrow \infty} \sqrt[n]{n} &= 1 & (5) \lim_{x \rightarrow \infty} \arctan x &= \frac{\pi}{2} & (6) \lim_{x \rightarrow -\infty} \arctan x &= -\frac{\pi}{2} \\ (7) \lim_{x \rightarrow \infty} \operatorname{arc cot} x &= 0 & (8) \lim_{x \rightarrow -\infty} \operatorname{arc cot} x &= \pi & (9) \lim_{x \rightarrow -\infty} e^x &= 0 \\ (10) \lim_{x \rightarrow +\infty} e^x &= \infty & (11) \lim_{x \rightarrow 0^+} x^x &= 1 \\ (12) \lim_{x \rightarrow \infty} \frac{a_0 x^n + a_1 x^{n-1} + \cdots + a_n}{b_0 x^m + b_1 x^{m-1} + \cdots + b_m} &= \begin{cases} \frac{a_0}{b_0} & n = m \\ 0 & n < m \\ \infty & n > m \end{cases} & & (\text{系数不为 } 0 \text{ 的情况}) \end{aligned}$$

十三、下列常用等价无穷小关系 ($x \rightarrow 0$)

$$\begin{aligned} \sin x &\sim x & \tan x &\sim x & \arcsin x &\sim x & \arctan x &\sim x & 1 - \cos x &\sim \frac{1}{2} x^2 \\ \ln(1+x) &\sim x & e^x - 1 &\sim x & a^x - 1 &\sim x \ln a & (1+x)^\alpha - 1 &\sim \alpha x \end{aligned}$$

十四、三角函数公式

1. 两角和公式

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \quad \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cot(A+B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A} \quad \cot(A-B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$$

2. 二倍角公式

$$\sin 2A = 2 \sin A \cos A \quad \cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

3. 半角公式

$$\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}} \quad \cos \frac{A}{2} = \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A} \quad \cot \frac{A}{2} = \sqrt{\frac{1 + \cos A}{1 - \cos A}} = \frac{\sin A}{1 - \cos A}$$

4.和差化积公式

$$\begin{aligned}\sin a + \sin b &= 2 \sin \frac{a+b}{2} \cdot \cos \frac{a-b}{2} & \sin a - \sin b &= 2 \cos \frac{a+b}{2} \cdot \sin \frac{a-b}{2} \\ \cos a + \cos b &= 2 \cos \frac{a+b}{2} \cdot \cos \frac{a-b}{2} & \cos a - \cos b &= -2 \sin \frac{a+b}{2} \cdot \sin \frac{a-b}{2} \\ \tan a + \tan b &= \frac{\sin(a+b)}{\cos a \cdot \cos b}\end{aligned}$$

5.积化和差公式

$$\begin{aligned}\sin a \sin b &= -\frac{1}{2} [\cos(a+b) - \cos(a-b)] & \cos a \cos b &= \frac{1}{2} [\cos(a+b) + \cos(a-b)] \\ \sin a \cos b &= \frac{1}{2} [\sin(a+b) + \sin(a-b)] & \cos a \sin b &= \frac{1}{2} [\sin(a+b) - \sin(a-b)]\end{aligned}$$

6.万能公式

$$\begin{aligned}\sin a &= \frac{2 \tan \frac{a}{2}}{1 + \tan^2 \frac{a}{2}} & \cos a &= \frac{1 - \tan^2 \frac{a}{2}}{1 + \tan^2 \frac{a}{2}} & \tan a &= \frac{2 \tan \frac{a}{2}}{1 - \tan^2 \frac{a}{2}}\end{aligned}$$

7.平方关系

$$\sin^2 x + \cos^2 x = 1 \quad \sec^2 x - \tan^2 x = 1 \quad \csc^2 x - \cot^2 x = 1$$

8.倒数关系

$$\tan x \cdot \cot x = 1 \quad \sec x \cdot \cos x = 1 \quad \csc x \cdot \sin x = 1$$

9.商数关系

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

十五、几种常见的微分方程

1.可分离变量的微分方程: $\frac{dy}{dx} = f(x)g(y)$, $f_1(x)g_1(y)dx + f_2(x)g_2(y)dy = 0$

2.齐次微分方程: $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

3.一阶线性非齐次微分方程: $\frac{dy}{dx} + p(x)y = Q(x)$ 解为:

$$y = e^{-\int p(x)dx} \left[\int Q(x) e^{\int p(x)dx} dx + c \right]$$