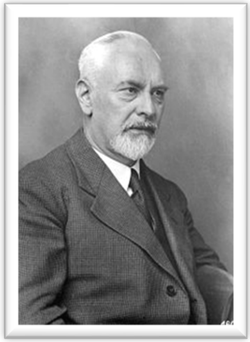
Chapter Nine

Boundary Layer

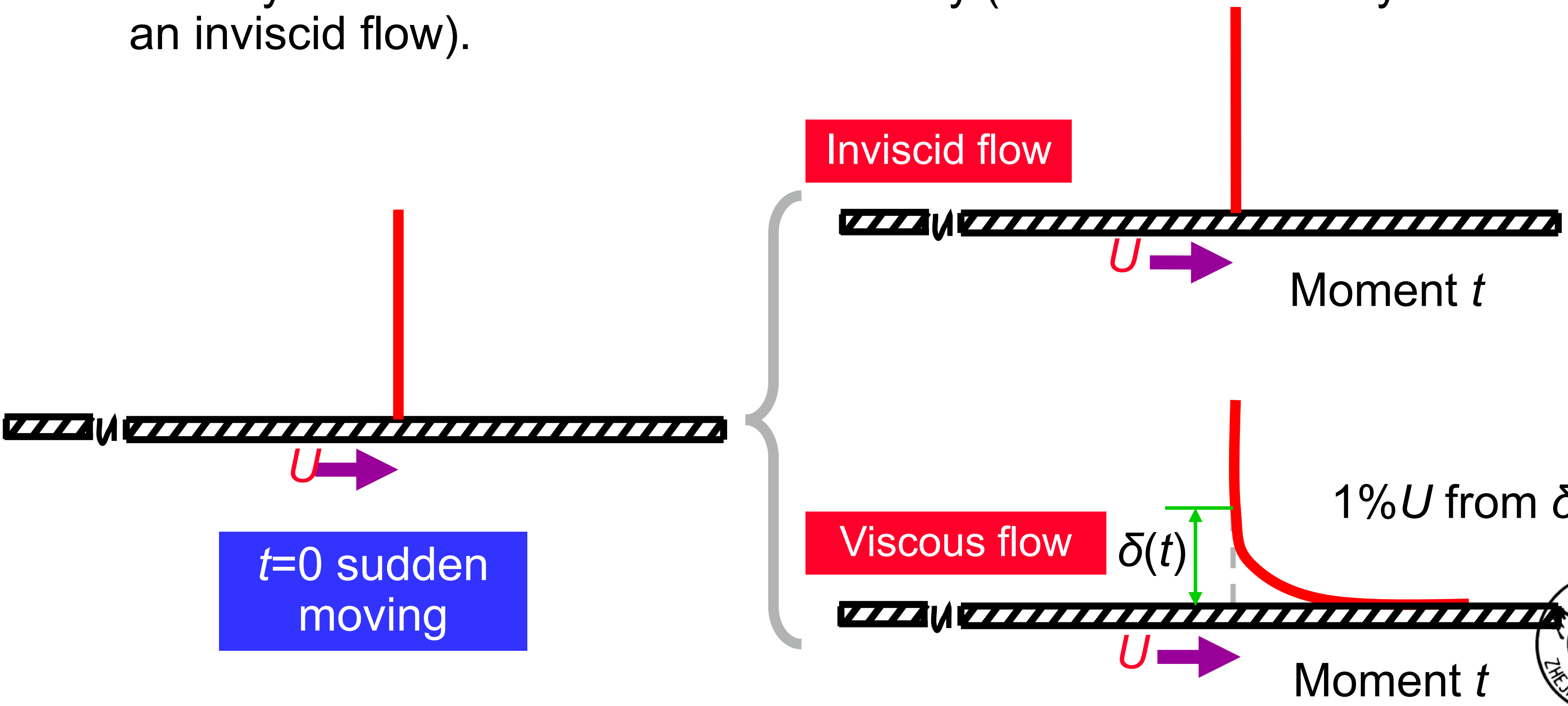
Prandtl

1875 –1953

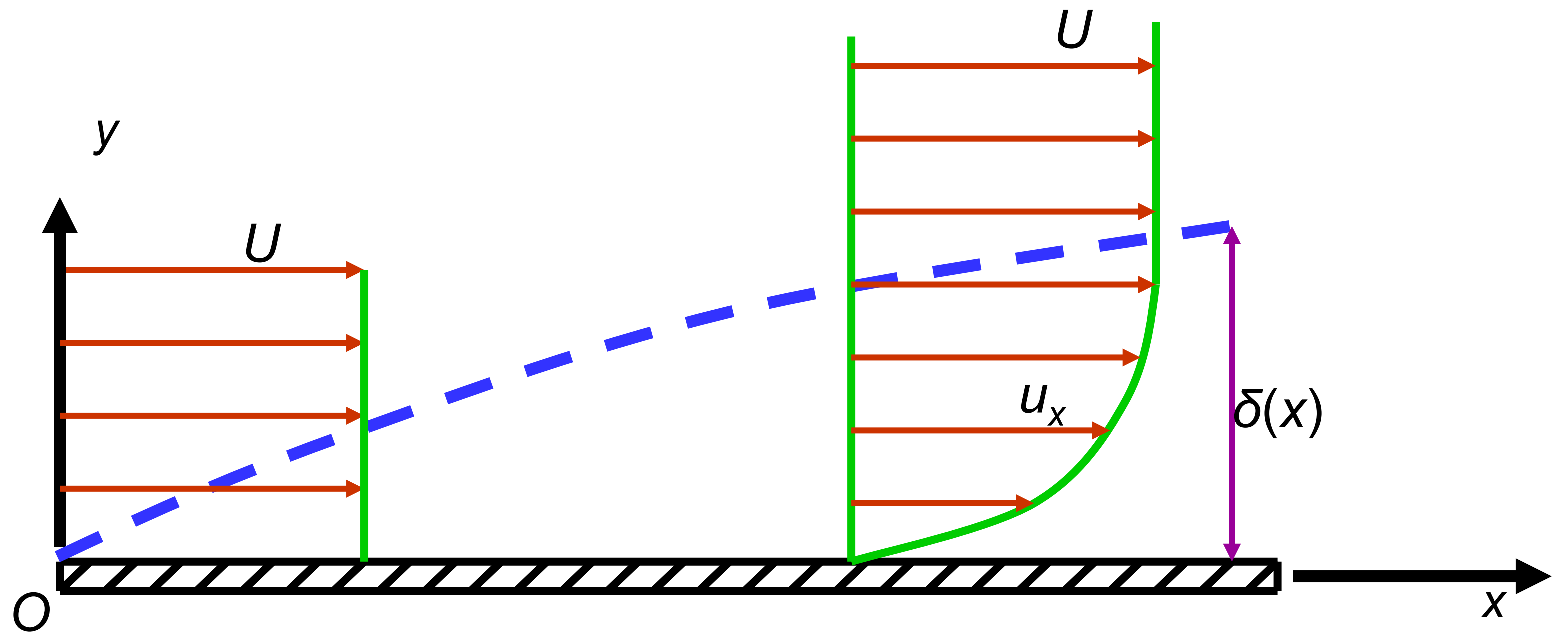


Definition

- The boundary layer is the layer of fluid in the immediate vicinity of a bounding surface where the effects of viscosity are significant.
- The thickness of the velocity boundary layer is normally defined as the distance from the solid body at which the viscous flow velocity is 99% of the freestream velocity (the surface velocity of an inviscid flow).



Height of boundary layer

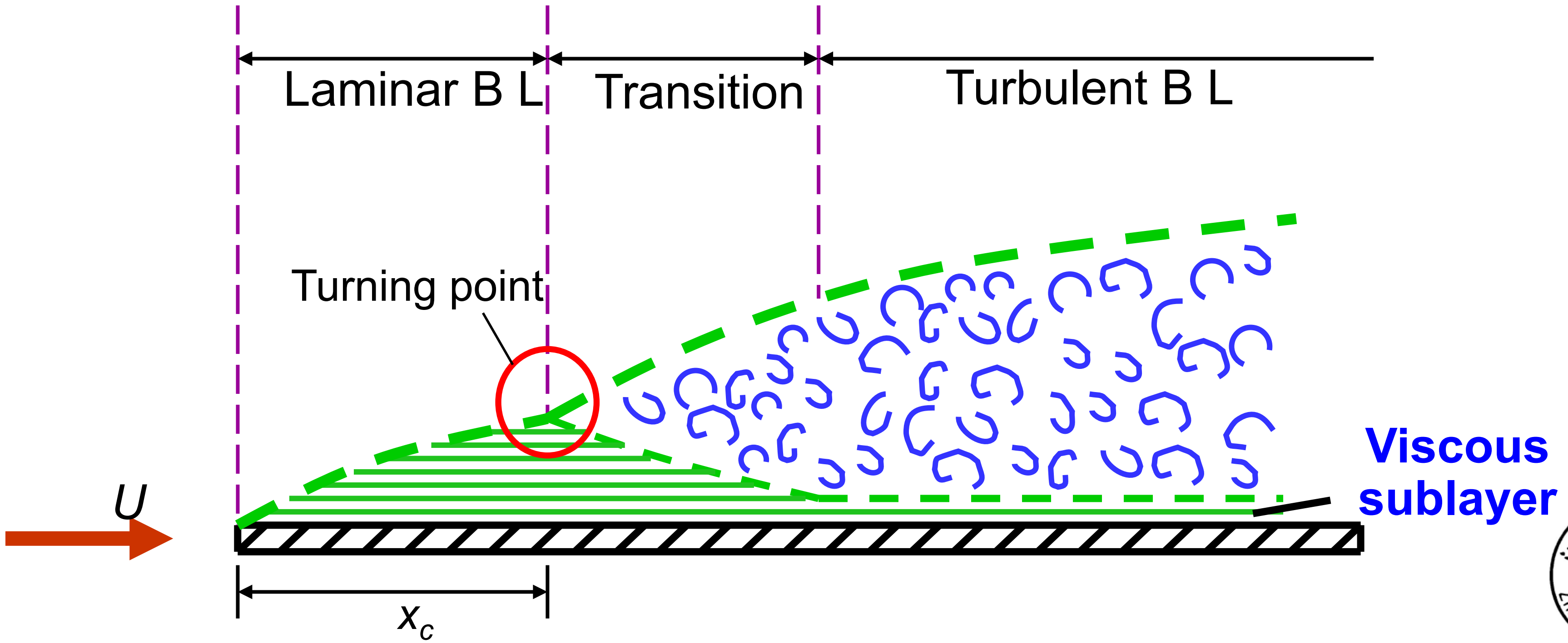


- The fluid particle moves from the head of the plate. The interval is x/U, and the height of boundary is increasing, which can be calculated by

$$\delta(x) \propto \sqrt{\frac{\nu x}{U}} = \frac{x}{(R_{ex})^{1/2}}$$

Laminar and turbulent boundary layer

- As the boundary layer increases in thickness, the viscous shear stresses become small. they are no longer able to hold the flow in layers and the fluid starts to rotate
- At points very close to the boundary the velocity gradients become very large and the velocity gradients become very large. This region is known as the laminar sub-layer.



Laminar Boundary Layer

- The laminar boundary layer is a very smooth flow, while the turbulent boundary layer contains swirls or “eddies.”
- The laminar flow creates less skin friction drag than the turbulent flow, but is less stable.
- Boundary layer flow over a wing surface begins as a smooth laminar flow. As the flow continues back from the leading edge, the laminar boundary layer increases in thickness.

Turbulent Boundary Layer

- At some distance back from the leading edge, the smooth laminar flow breaks down and transitions to a turbulent flow.
- From a drag standpoint, it is advisable to have the transition from laminar to turbulent flow as far aft on the wing as possible, or have a large amount of the wing surface within the laminar portion of the boundary layer.
- The low energy laminar flow, however, tends to break down more suddenly than the turbulent layer.

Governing equation of boundary layer

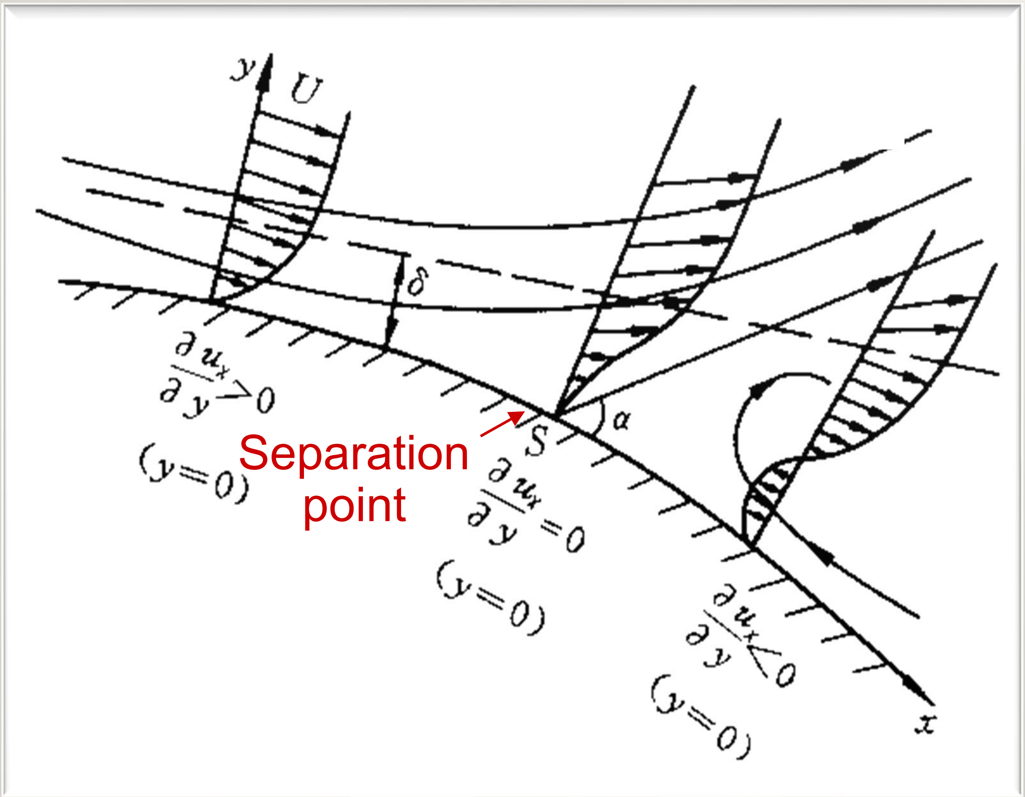
$$
\begin{cases}
u_x \dfrac{\partial u_x}{\partial x} + u_y \dfrac{\partial u_x}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \dfrac{1}{\rho}\dfrac{\partial}{\partial x}[\mu \cancel{\dfrac{\partial u_x}{\partial x}} - \overline{\rho u'_x u'_x}] + \dfrac{1}{\rho}\dfrac{\partial}{\partial y}[\mu \dfrac{\partial u_x}{\partial y} - \overline{\rho u'_x u'_y}] \\
u_x \cancel{\dfrac{\partial u_y}{\partial x}} + u_y \dfrac{\partial u_y}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} + \dfrac{1}{\rho}\dfrac{\partial}{\partial x}[\mu \cancel{\dfrac{\partial u_y}{\partial x}} - \overline{\rho u'_y u'_x}] + \dfrac{1}{\rho}\dfrac{\partial}{\partial y}[\mu \cancel{\dfrac{\partial u_y}{\partial y}} - \overline{\rho u'_y u'_y}] \\
\dfrac{\partial u_x}{\partial x} + \dfrac{\partial u_y}{\partial y} = 0
\end{cases}
$$

Simplifying

$$
\begin{cases}
u_x \dfrac{\partial u_x}{\partial x} + u_y \dfrac{\partial u_x}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \dfrac{1}{\rho}\dfrac{\partial}{\partial y}[\mu \dfrac{\partial u_x}{\partial y} - \overline{\rho u'_x u'_y}] \\
\dfrac{\partial p}{\partial y} = 0 \\
\dfrac{\partial u_x}{\partial x} + \dfrac{\partial u_y}{\partial y} = 0
\end{cases}
$$

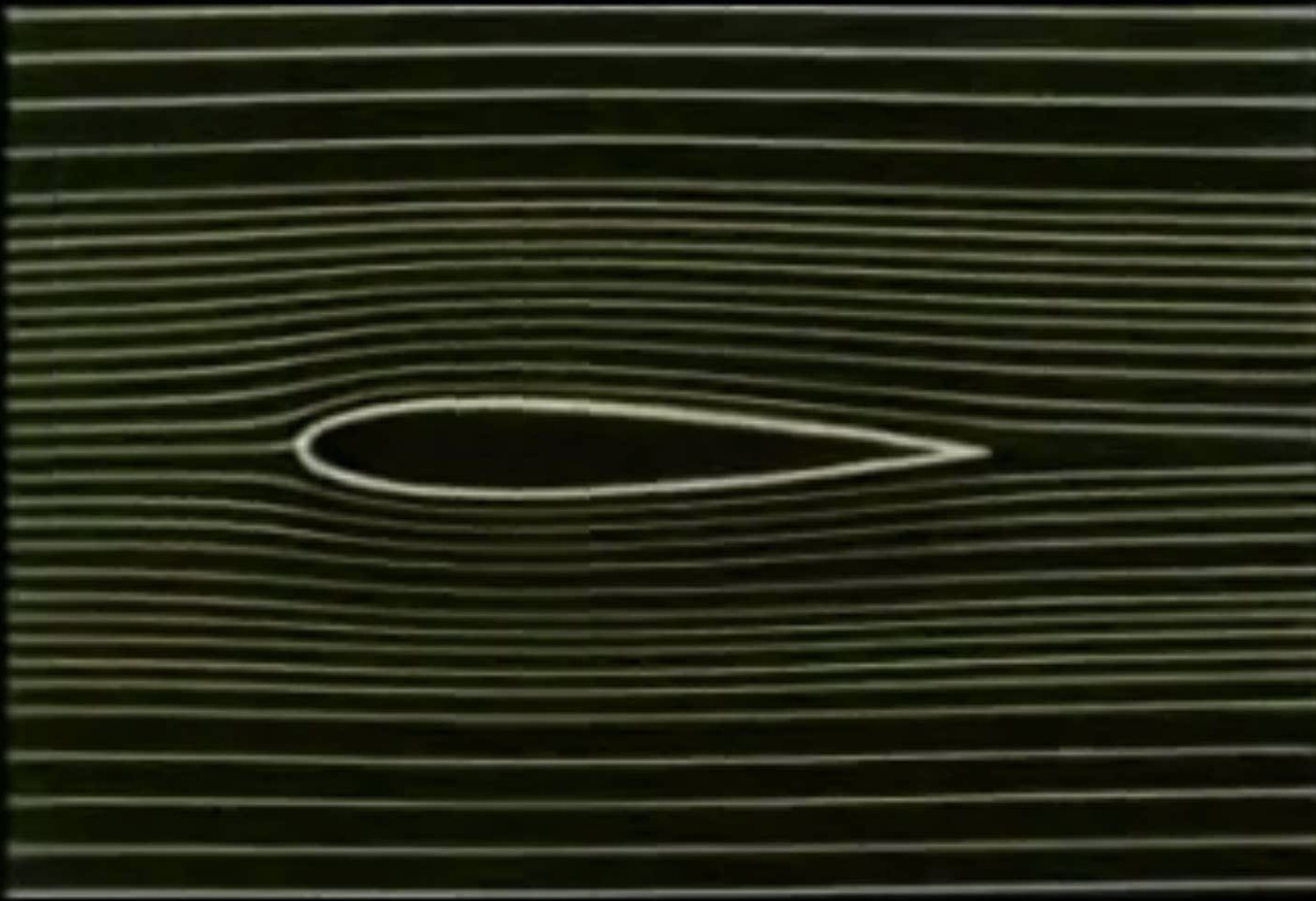
Differential equation of boundary layer

Boundary layer separation

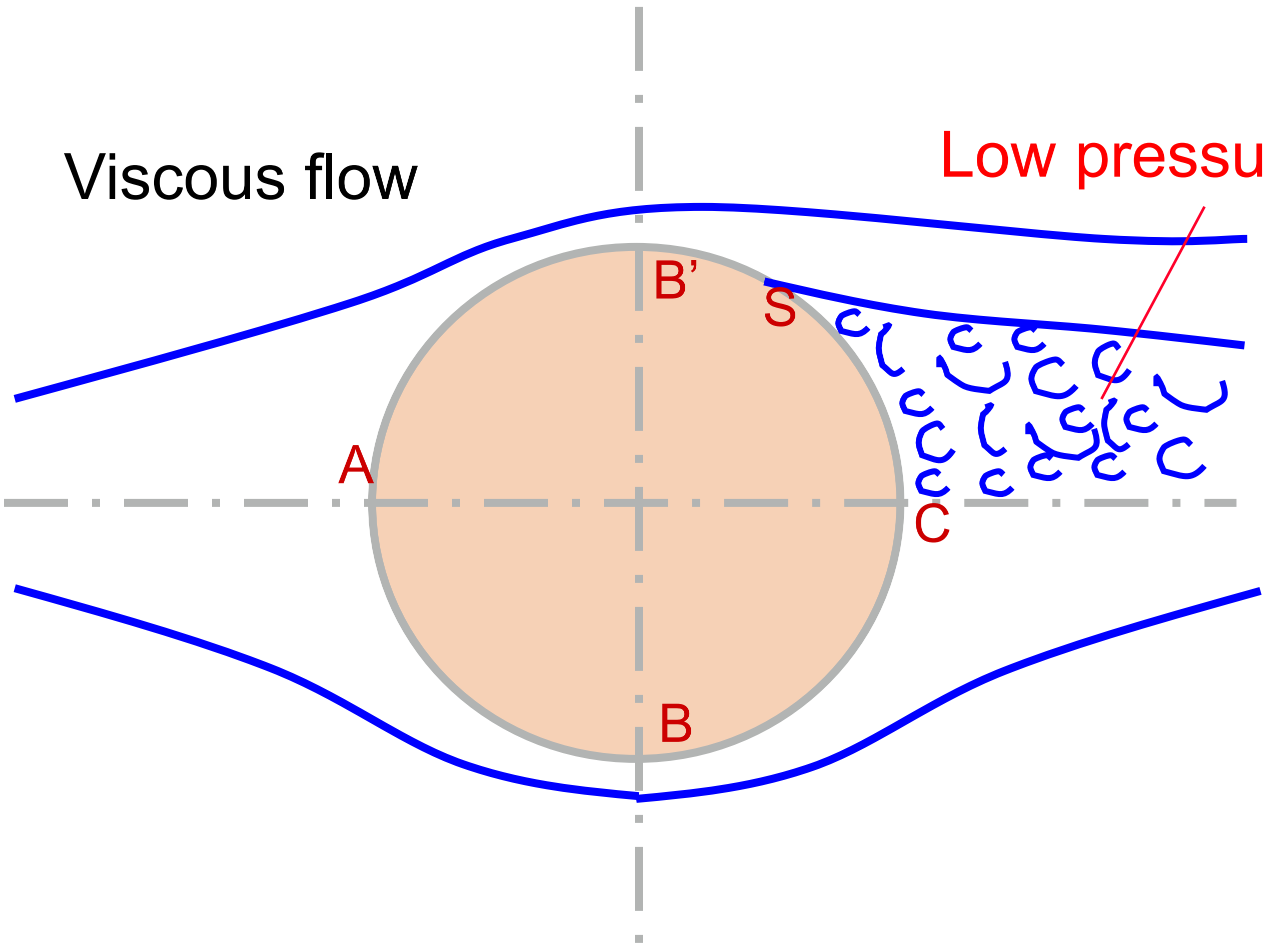


- When the pressure increases in the direction of flow, fluid outside the boundary layer has enough momentum to overcome this pressure. The fluid within the boundary layer has so little momentum that it will very quickly be brought to rest, and possibly reversed in direction.

- If this reversal occurs, it lifts the boundary layer away from the surface. That is called boundary layer separation

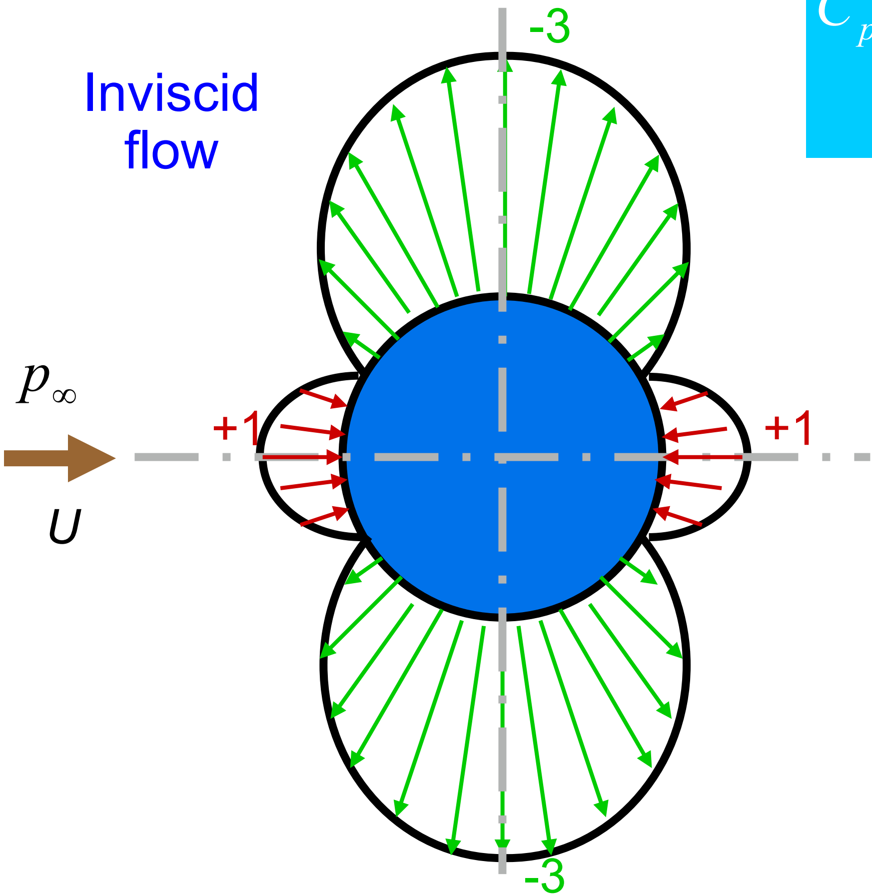


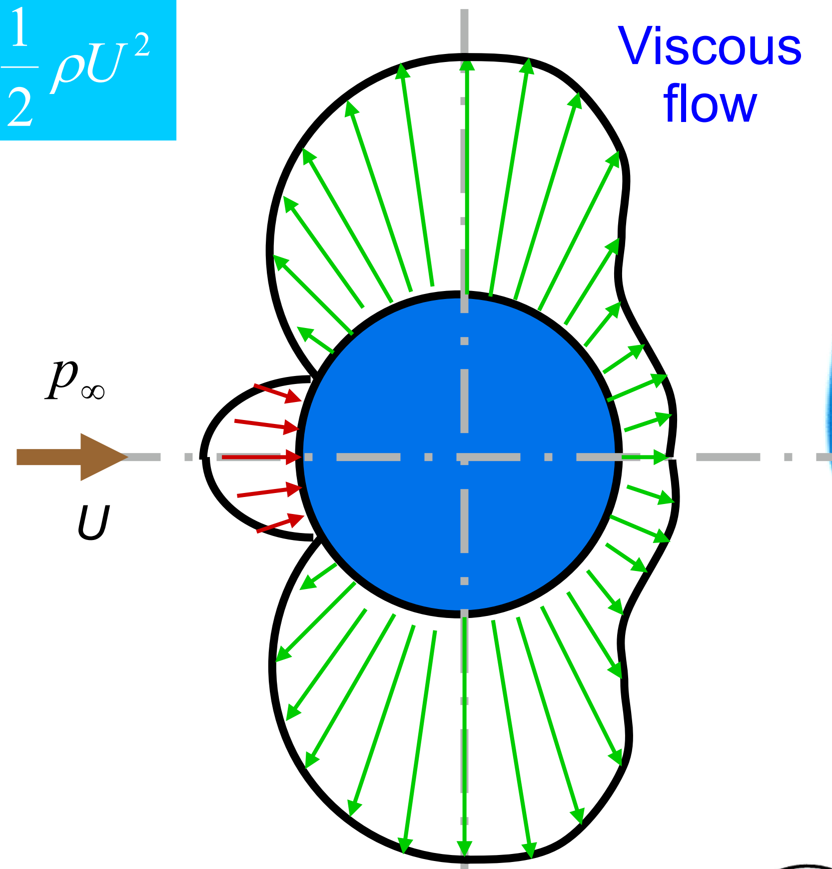
Boundary layer separation



Distribution of pressure coefficient

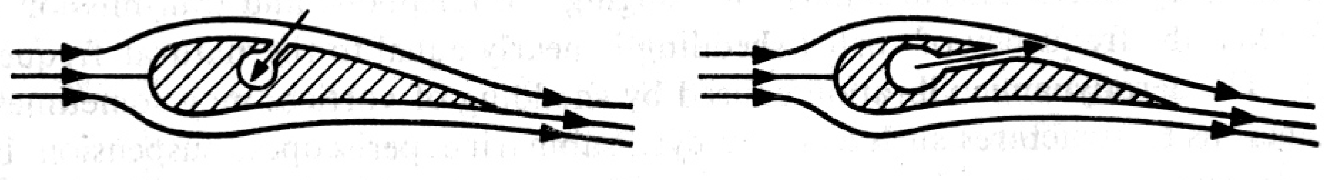
$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$$





Control of separation

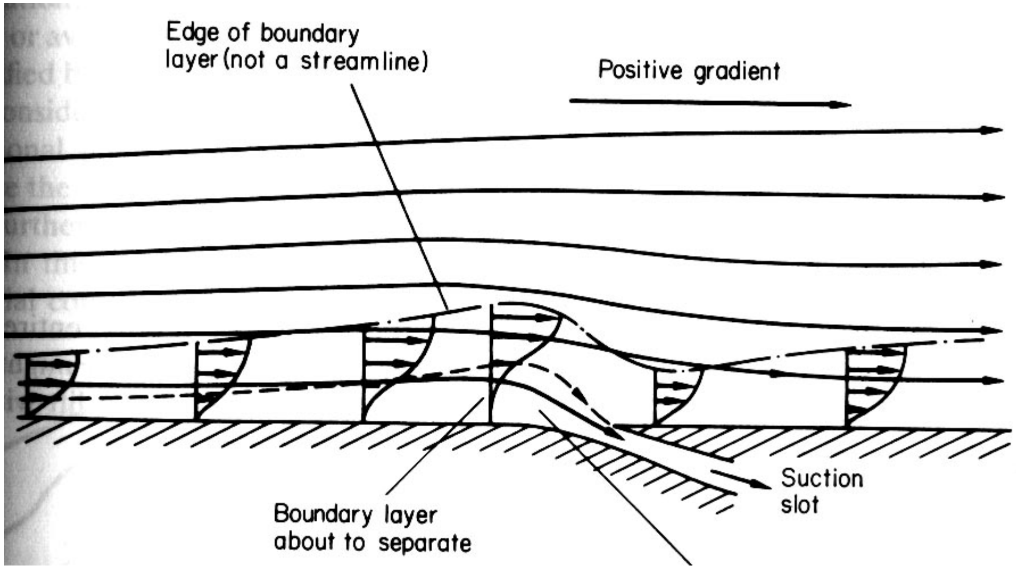
- Streamlining reduces adverse pressure gradient beyond the maximum thickness and delays separation
- Fluid particles lose kinetic energy near separation point. So these are either removed by suction or higher energy

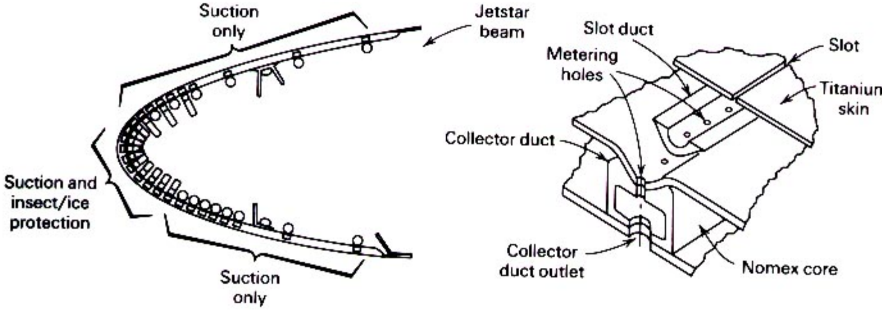


(a) Suction (b) Injection

- High energy fluid is blown near separation point
- Roughening surface to force early transition to turbulent boundary layer

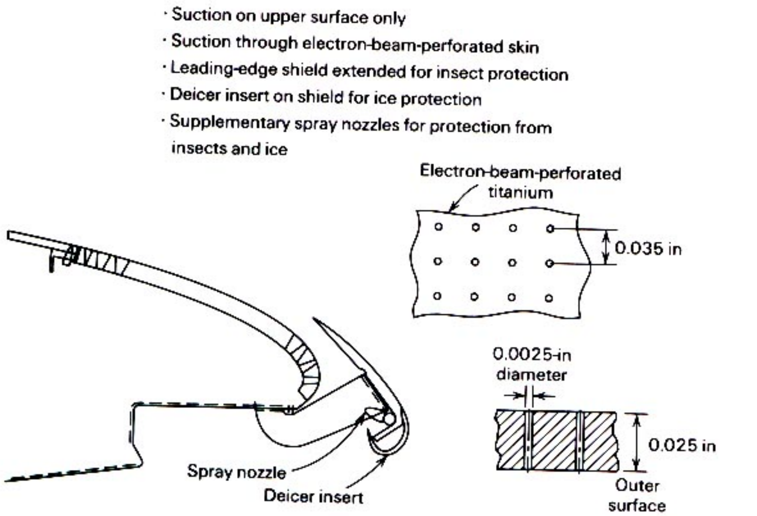
Separation delays by suction





Lockheed-Georgia LFC System for the Jetstar

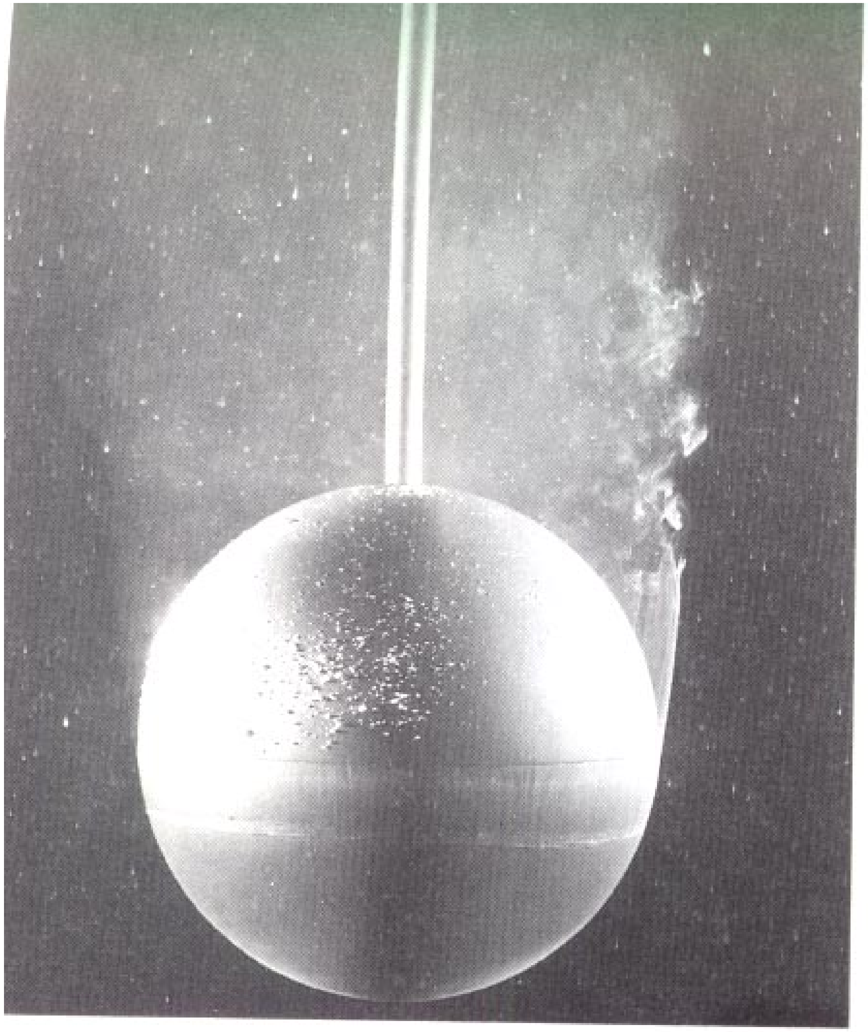
(a)



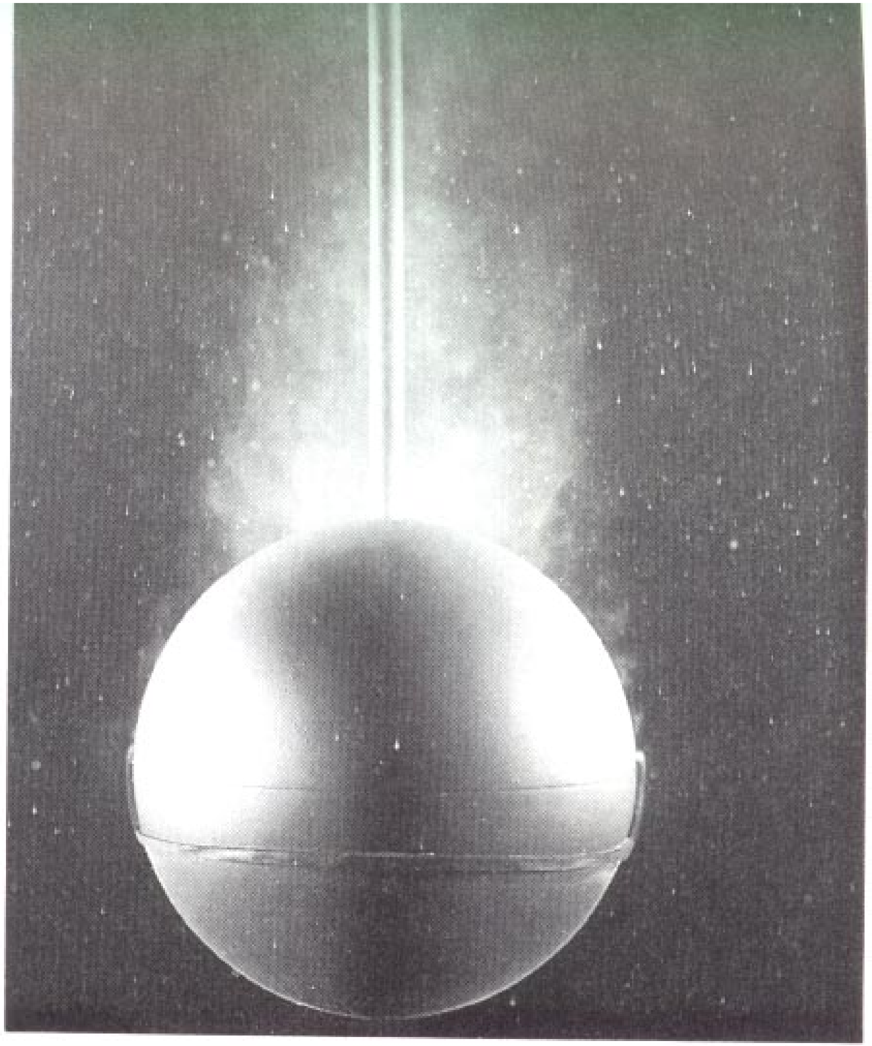
McDonnell-Douglas LFC System for the Jetstar

(b)

A wire ring on separation



No wire



With wire

Roughness on separation

Smooth ball

Rough ball