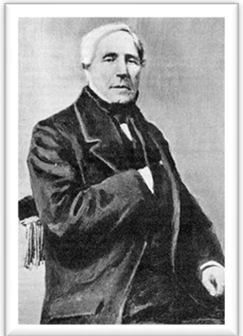
Chapter Eight

Gap Flow

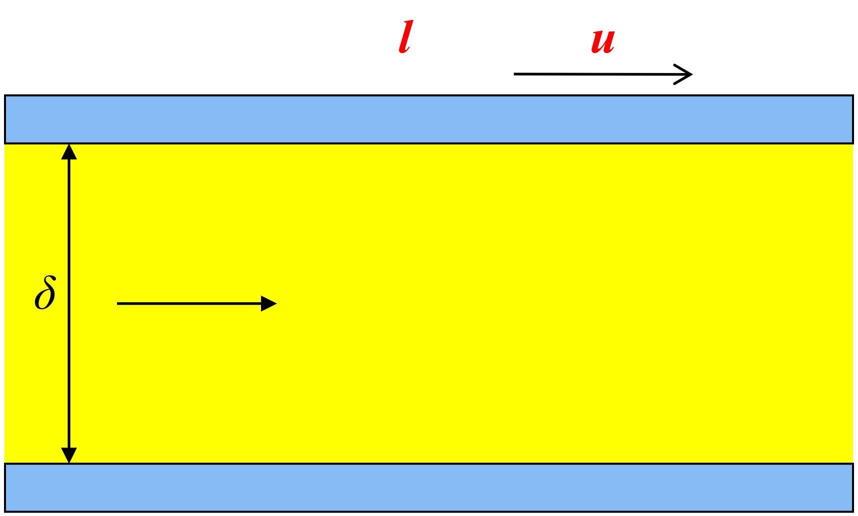
Poiseuille

1797 –1869

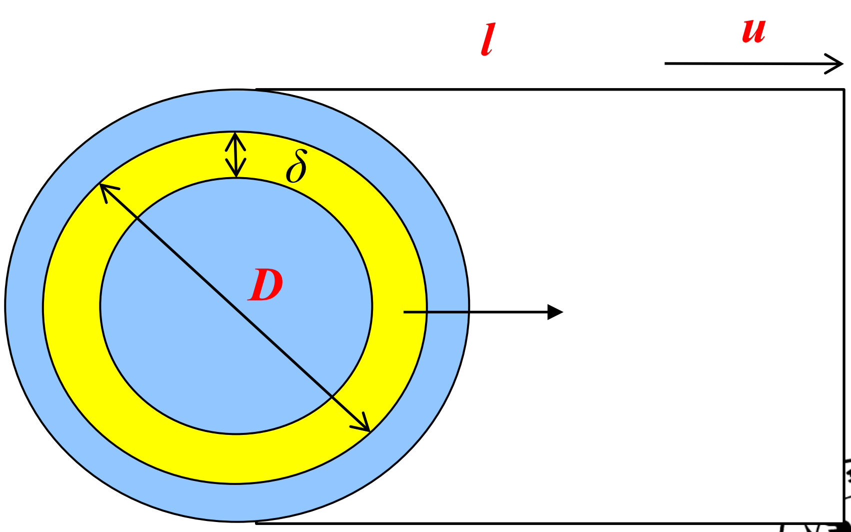


Characteristics

- *l*>>δ, *b* or *D*>>δ
  - Length *l* and width *b* are greatly larger than gap height *h*
- Re<=2300
  - The flow state is laminar
- *u*≠0, in most cases
  - The wall perhaps is moving

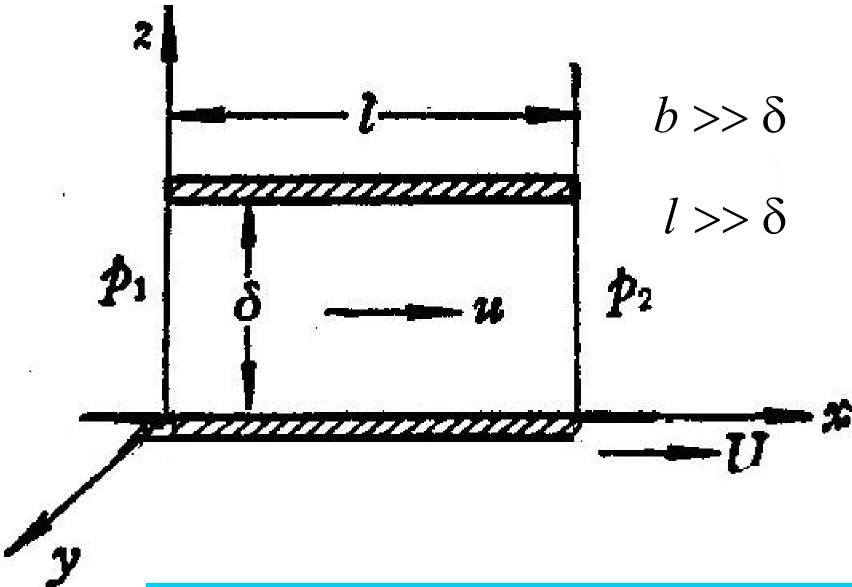


Parallel plate



Annular clearance

Parallel plate



$b >> \delta$

$l >> \delta$

Impressible flow

$$u_x = u$$

$$\begin{cases} -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \nu\dfrac{\partial^2 u}{\partial z^2} = 0 \\ -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} = 0 \\ -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} = 0 \end{cases}$$

$$\cancel{\frac{\partial v_x}{\partial t}} + v_x\cancel{\frac{\partial v_x}{\partial x}} + v_y\cancel{\frac{\partial v_x}{\partial y}} + v_z\cancel{\frac{\partial v_x}{\partial z}} = \cancel{f_x} - \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\left[\cancel{\frac{\partial^2 v_x}{\partial x^2}} + \cancel{\frac{\partial^2 v_x}{\partial y^2}} + \frac{\partial^2 v_x}{\partial z^2}\right]$$

$$\cancel{\frac{\partial v_y}{\partial t}} + \ldots v_z \ldots \cancel{\frac{\partial^2 v_y}{\partial z^2}}$$

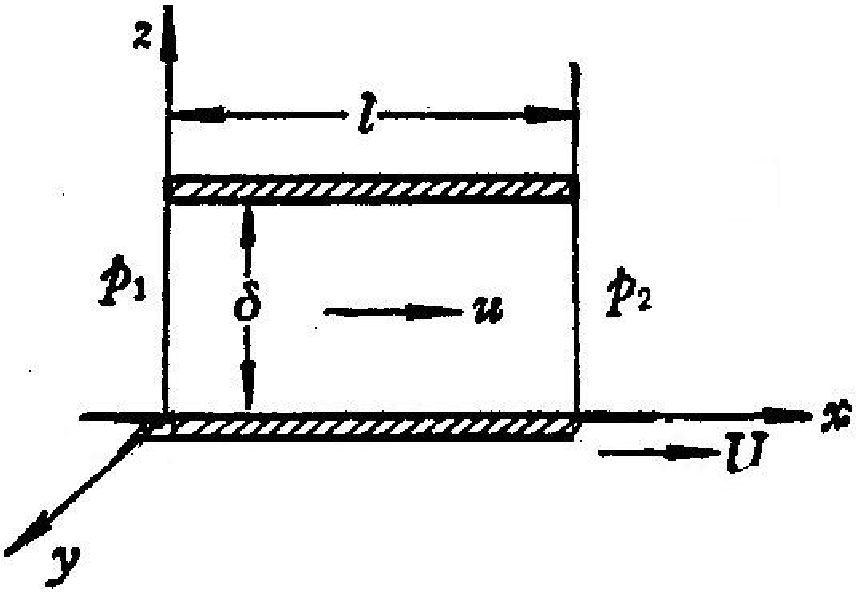
$$\cancel{\frac{\partial v_z}{\partial t}} + v_x\cancel{\frac{\partial v_z}{\partial x}} + v_y\cancel{\frac{\partial v_z}{\partial y}} + v_z \ldots + \frac{\mu}{\rho}\left[\cancel{\frac{\partial^2 v_z}{\partial x^2}} + \cancel{\frac{\partial^2 v_z}{\partial y^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right]$$

p decease uniformly

$$\frac{\partial p}{\partial x} = \frac{dp}{dx} = -\frac{p_1 - p_2}{l} = -\frac{\Delta p}{l}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{d^2 u}{dz^2}$$

Parallel plate



$$\frac{d^2u}{dz^2}=\frac{1}{\mu}\frac{dp}{dx}=-\frac{\Delta p}{\mu l}$$

Integral

$$u=-\frac{\Delta p}{2\mu l}z^2+C_1 z+C_2$$

Boundary condition

$$\begin{cases} z=0, & u=U \\ z=\delta, & u=0 \end{cases}$$

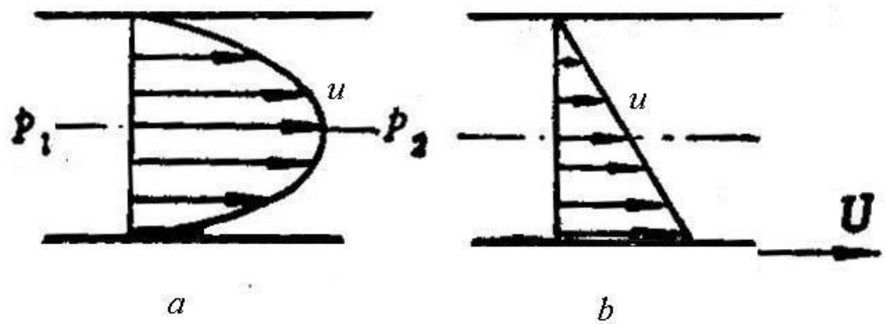
$$C_1=\frac{\Delta p}{2\mu l}\delta-\frac{U}{\delta},\quad C_2=U$$

$$u=\frac{\Delta p}{2\mu l}(\delta-z)z+U(1-\frac{z}{\delta})$$

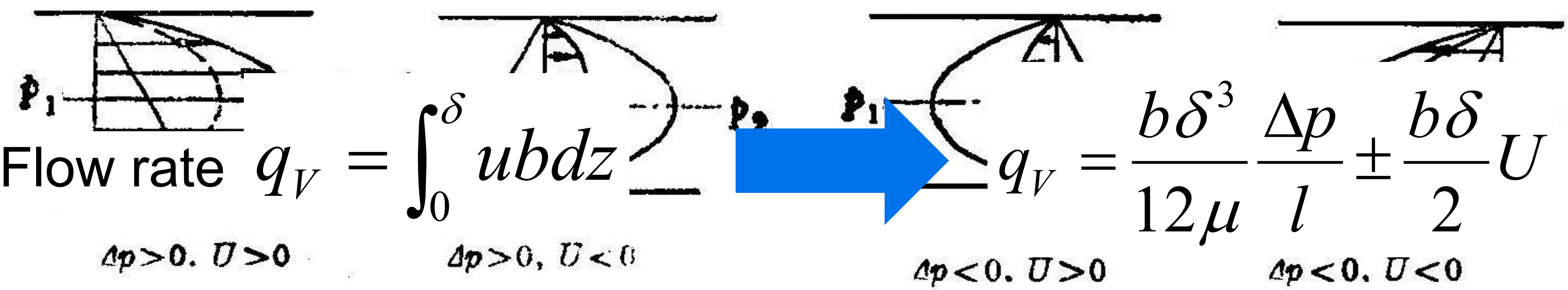
Parallel plate

$$u = \frac{\Delta p}{2\mu l}(\delta - z)z \pm U\left(1 - \frac{z}{\delta}\right)$$

- One is caused by differential pressure
- The other is caused by shear of moving plate



Flow rate $q_V = \int_0^{\delta} ubdz$ $\Rightarrow$ $q_V = \frac{b\delta^3}{12\mu}\frac{\Delta p}{l} \pm \frac{b\delta}{2}U$



Parallel plate

Flow rate loss:

$$N_Q = \Delta p q_V = \Delta p\left(\frac{b\delta^3}{12\mu}\frac{\Delta p}{l} + \frac{b\delta}{2}U\right) = \Delta p b\left(\frac{\delta^3}{12\mu}\frac{\Delta p}{l} + \frac{\delta}{2}U\right)$$

Shear friction loss:

$$F = \tau b l = -\mu b l \left.\frac{du}{dz}\right|_{z=0} \qquad F = b\left(\frac{\mu U l}{\delta} - \frac{\Delta p \delta}{2}\right)$$

$$N_F = FU = bU\left(\frac{\mu U l}{\delta} - \frac{\Delta p \delta}{2}\right)$$

Total power loss:

$$N = N_Q + N_F = b\left(\frac{\Delta p^2 \delta^3}{12\mu l} + \frac{\mu l U^2}{\delta}\right)$$

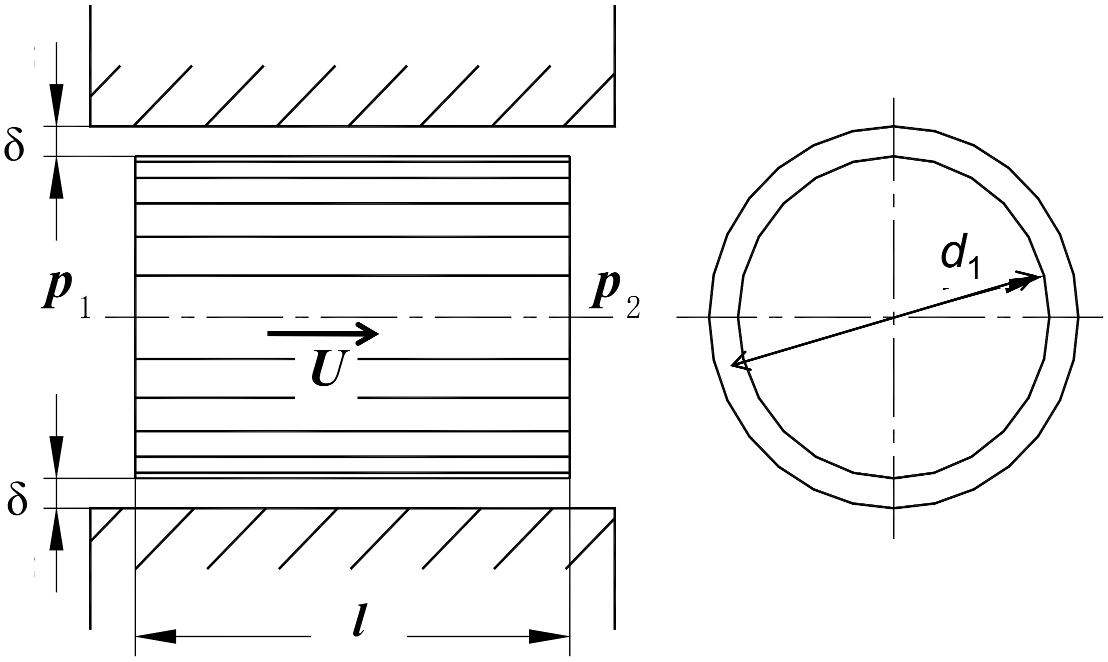
Minimum power loss exist

***Whatever *U* direction

Annular gap

$\Delta p = p_1 - p_2$

$\delta / d_1 << 1$



Parallel plate flow rate

$$q_V = \frac{b\delta^3}{12\mu}\frac{\Delta p}{l} \pm \frac{b\delta}{2}U$$

Let $b = \pi d_1$

$$q_V = \frac{\pi d_1 \delta^3}{12\mu}\frac{\Delta p}{l} \pm \frac{\pi d_1 \delta}{2}U$$

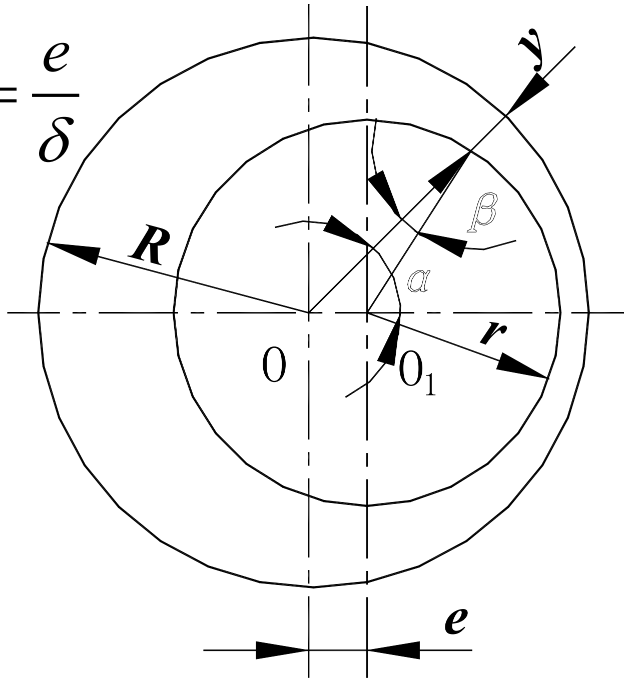
Eccentric annular gap

$$y = R - r\cos\beta - e\cos\alpha$$

$$\approx R - r - e\cos\alpha = \delta - e\cos\alpha$$

$$= \delta(1 - \varepsilon\cos\alpha)$$

$$\varepsilon = \frac{e}{\delta}$$



$$dq = \frac{\Delta p}{12\mu l} y^3 r d\alpha = \frac{\Delta p}{12\mu l} h^3 (1 - \varepsilon\cos\alpha)^3 r d\alpha$$

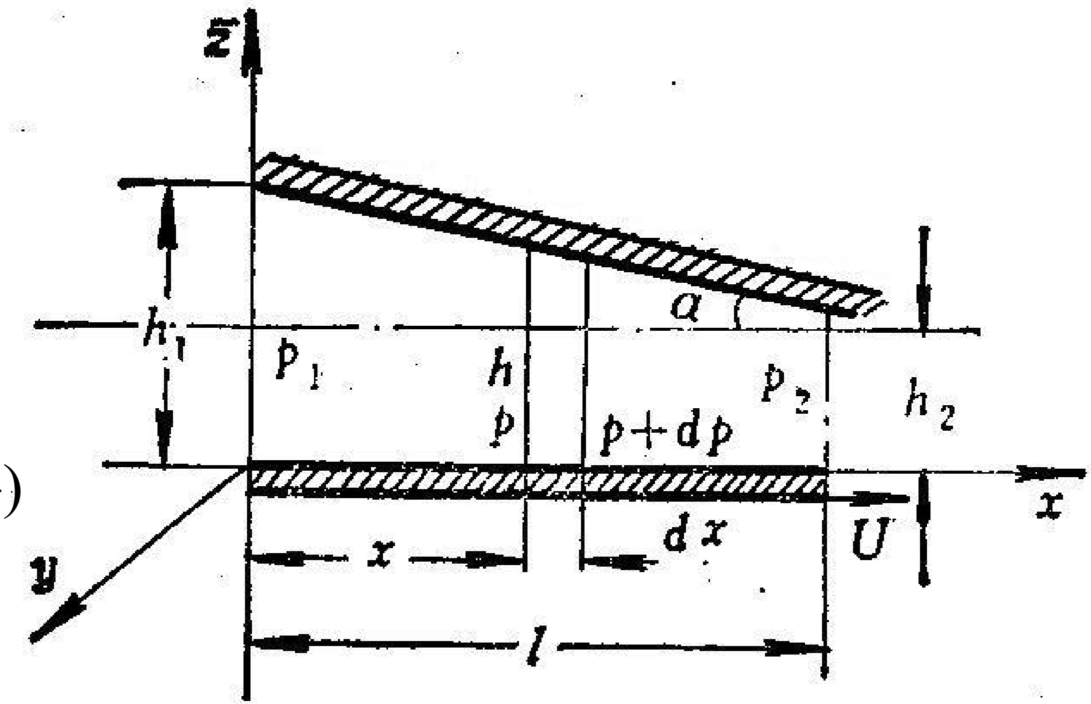
Integrating from 0 to 2π

$$\boxed{q = \frac{\pi d h^3}{12\mu l}\Delta p(1 + 1.5\varepsilon^2)}$$

- When components are assembled utterly eccentrically the leakage flow rate is 2.5 times that when utterly concentrically.

Gap between nonparallel plates

- The gap may be considered as a gap between parallel plates due to a infinitesimal length



$$u = \frac{1}{2\mu}(z^2 - hz)\frac{dp}{dx} + U(1 - \frac{z}{h})$$

When width =*b*,
the flow rate is as

$$q_V = -\frac{bh^3}{12\mu}\frac{dp}{dx} + \frac{bhU}{2} \quad \Rightarrow \quad \frac{dp}{dx} = \frac{6\mu U}{h^2} - \frac{12\mu}{bh^3}q_V$$

$$p = p_1 + \frac{6\mu q_V}{b\,tg\alpha}(\frac{1}{h_1^2} - \frac{1}{h^2}) - \frac{6\mu U}{tg\alpha}(\frac{1}{h_1} - \frac{1}{h})$$

Gap between nonparallel plates

Differential pressure P1-P2

$$\Delta p = \frac{6\mu q_V}{btg\alpha}\left(\frac{h_1^2 - h_2^2}{h_1^2 h_2^2}\right) - \frac{6\mu U}{tg\alpha}\left(\frac{h_2 - h_1}{h_1 h_2}\right)$$

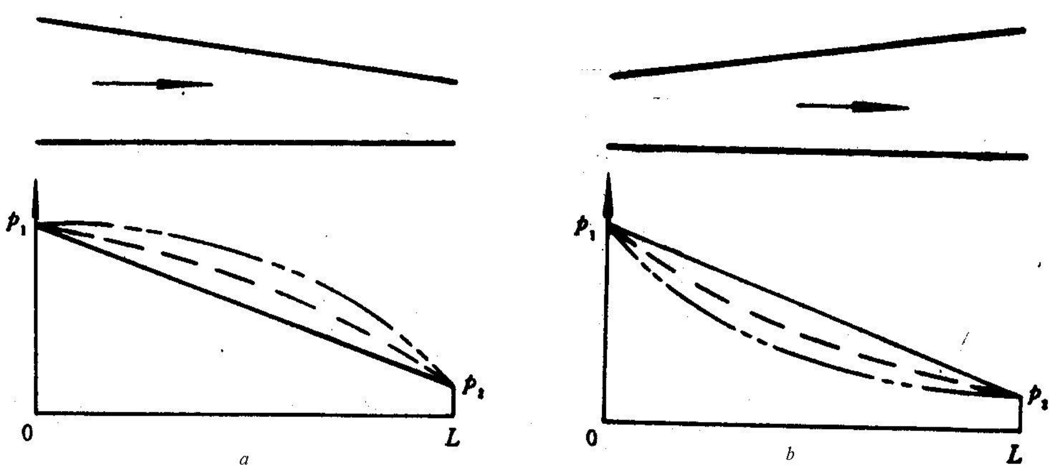
Flow rate

$$q_V = \frac{bh_1h_2}{h_1 + h_2}\left(\frac{h_1h_2}{6\mu l}\Delta p + U\right)$$

When *U*=0 Pressure

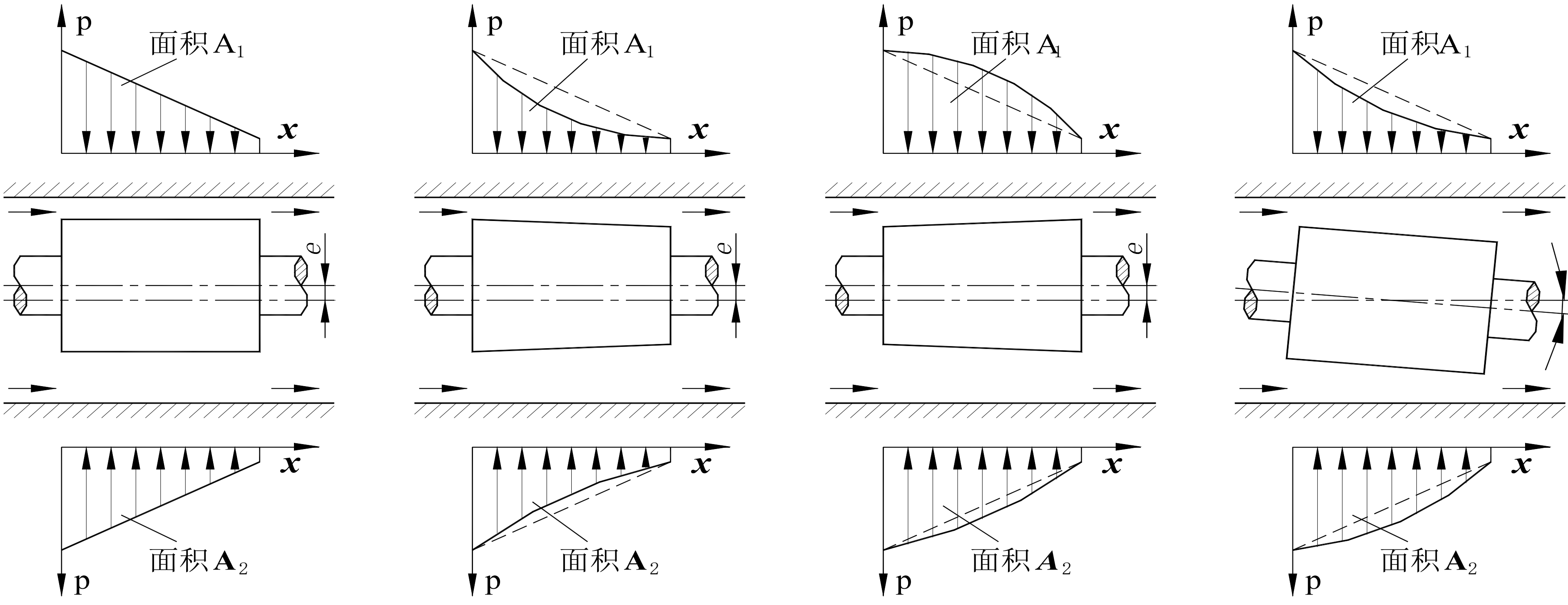
$$p = p_1 - \Delta p \frac{\left(\frac{h_1}{h}\right)^2 - 1}{\left(\frac{h_1}{h_2}\right)^2 - 1}$$

- When $h_1 > h_2$, p(x) is convex
- When $h_1 < h_2$, p(x) is concave



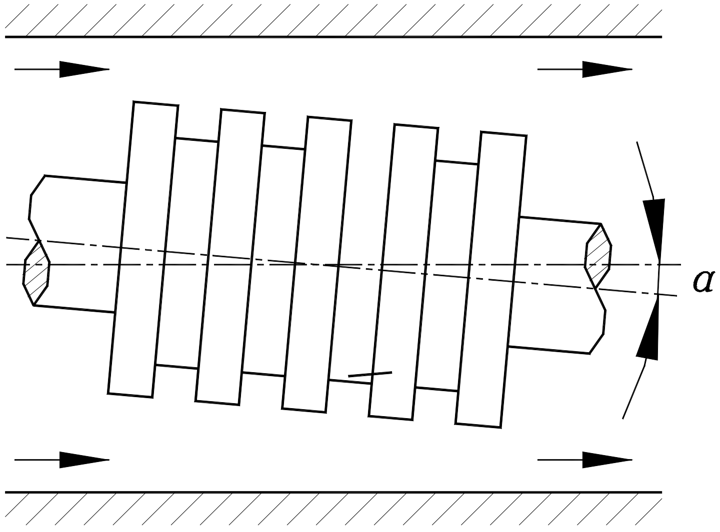
Sticking force

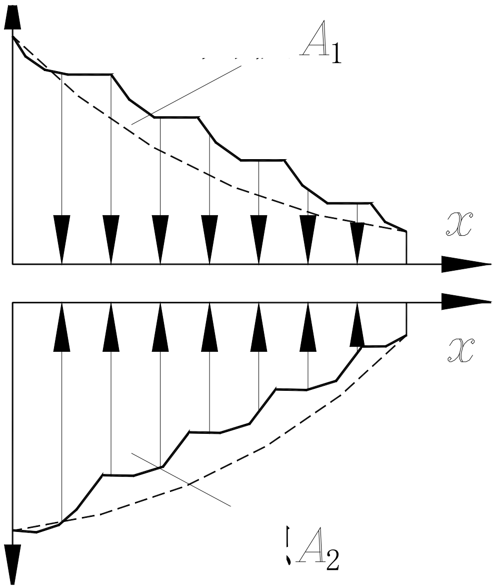
- Shape deviation and position deviation result in nonparallel clearances
- According to the geometric significance of integration, the magnitude of the force is equal to the area under the curve



Sticking force

- Solution
  - Raising the accuracy degree of components(dimension accuracy degree, shape accuracy degree and position accuracy degree)
  - Machining balancing grooves

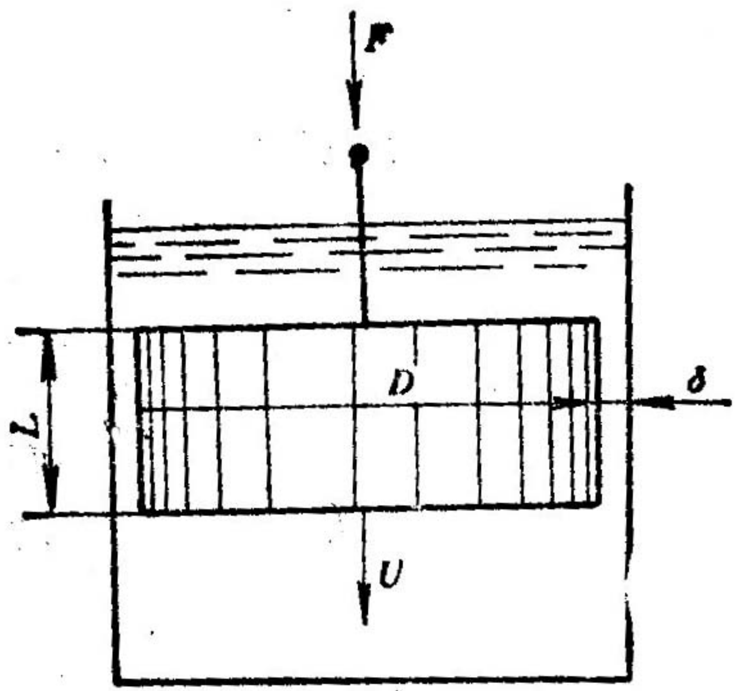




Example

- Viscous Damper shown in fig1, piston diameter is D， length is L, gap is δ, acting force F, moving speed U downward, viscosity is μ, no eccentric

$$Q_1 = \pi D\left(\frac{\Delta p \delta^3}{12\mu L} - \frac{U\delta}{2}\right)$$



$$\pi D^2$$

The first equation Q1=Q2

$$\Delta p = \frac{6\mu UL}{\delta^3}\left(\frac{D}{2} + \delta\right)$$

Example

F_p is force caused by pressure

$$F_p = \frac{\pi D^2}{4}\Delta p = \pi\mu UL\left[\frac{3}{4}\left(\frac{D}{\delta}\right)^3 + \frac{3}{2}\left(\frac{D}{\delta}\right)^2\right]$$

Shear force on piston τ

$$u = \frac{\Delta p}{2\mu L}(\delta - z)z - U\left(1 - \frac{z}{\delta}\right)$$

$$\tau_0 = \mu\left.\frac{du}{dz}\right|_{z=0} = \frac{\Delta p}{2L}(\delta - 2z) + \left.\mu\frac{U}{\delta}\right|_{z=0} = \frac{\Delta p}{2L}\delta + \mu\frac{U}{\delta}$$

Shear force F_τ

The second equation $F=F_p+F_\tau$

Sum force F

$$F = F_p + F_\tau = \pi\mu UL\left[\frac{3}{4}\left(\frac{D}{\delta}\right)^3 + 3\left(\frac{D}{\delta}\right)^2 + 4\left(\frac{D}{\delta}\right)\right]$$