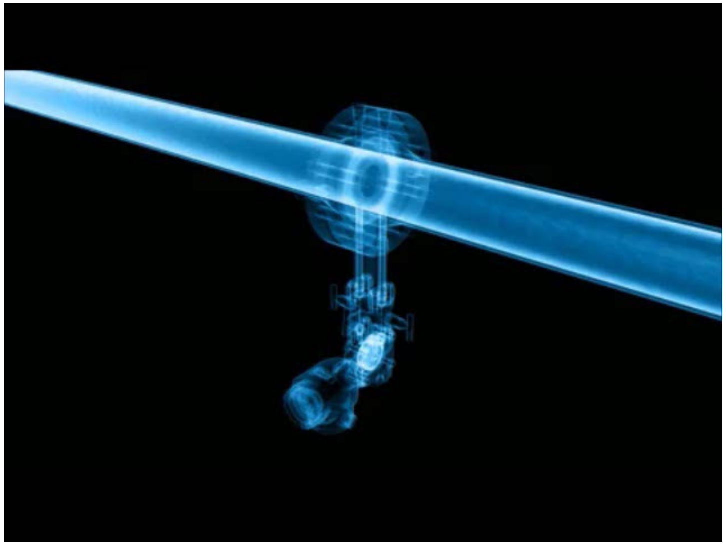
Chapter Four

Hydrodynamics

Euler

1707-1783

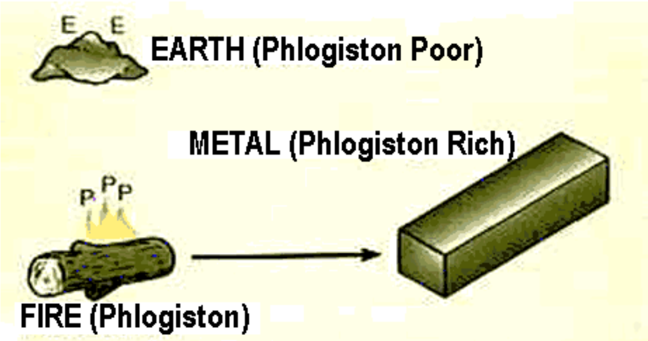
Applications



Conservation of mass

- The mass of an isolated system (closed to all matter and energy) will remain constant over time.
- The law implies that mass can neither be created nor destroyed, although it may be rearranged in space and changed into different types of particles; and that for any chemical process in an isolated system, the mass of the reactants must equal the mass of the products.

Antoine Lavoisier



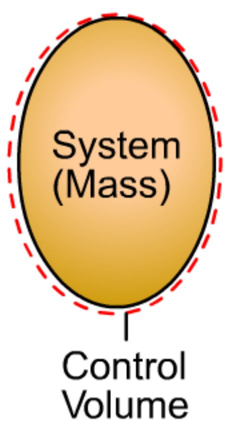
Lavoisier's quantitative experiments revealed that combustion involved oxygen rather than what was previously thought to be phlogiston

Conservation of mass

- Definitions: System and Control Volume

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}+\frac{\partial(\rho v_z)}{\partial z}=0$$

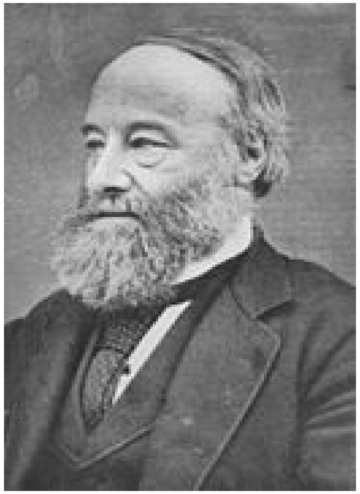
$$\begin{bmatrix}\text{time rate of change}\\ \text{of mass inside the}\\ \text{control volume}\end{bmatrix}+\begin{bmatrix}\text{net mass flow through}\\ \text{the control surface}\end{bmatrix}=0$$



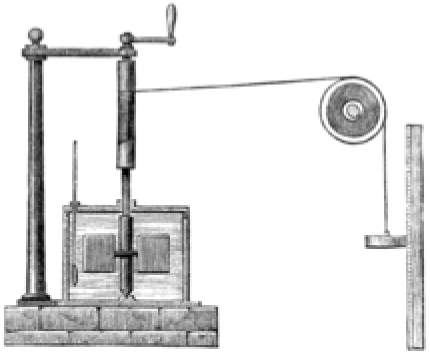
Continuous equation: $\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\vec{v})=0$

Conservation of energy

- The total amount of energy in an isolated system remains constant over time
- Energy can change its location within the system, and that it can change form within the system, for instance chemical energy can become kinetic energy, but that energy can be neither created nor destroyed.

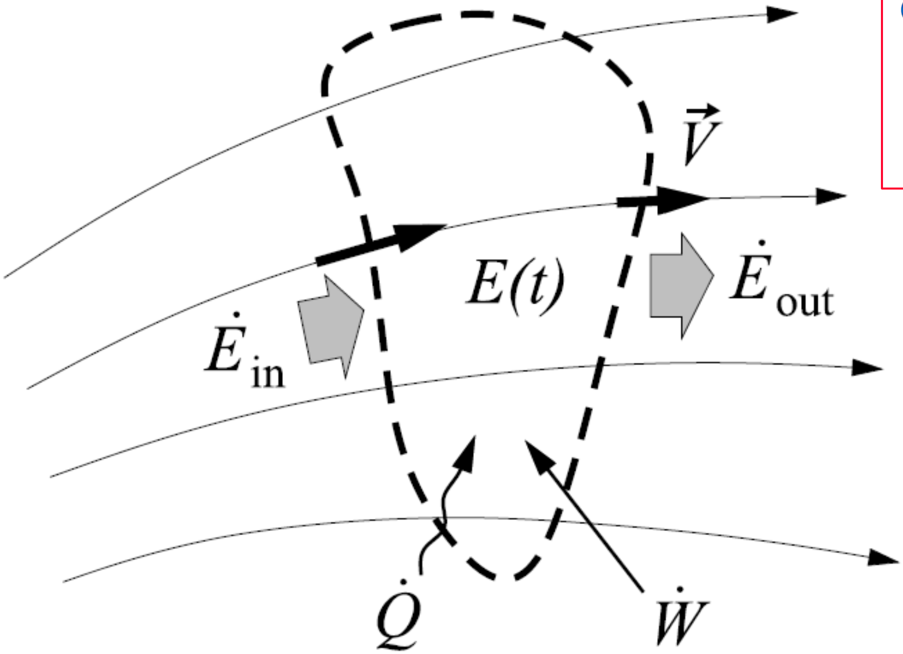


James Prescott Joule



Joule's apparatus for measuring the mechanical equivalent of heat.

Conservation of energy



- Energy can neither be created nor destroyed. It can only change forms.

$$dE = \delta Q + \delta W$$

$$\frac{dE}{dt} + \dot{E}_{out} - \dot{E}_{in} = \dot{Q} + \dot{W}$$

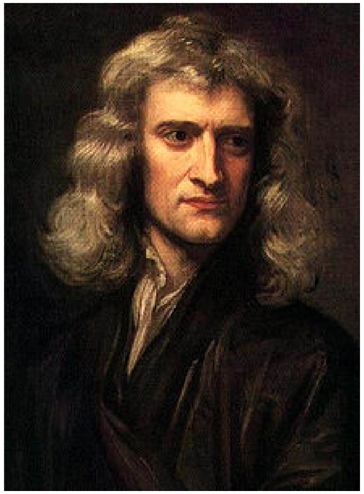
Bernoulli equation: $$\sum\left(\frac{v^2}{2} + gz + \tilde{u}\right)\dot{m} = Q_C - \sum pAV$$

$$z + \frac{p}{\rho g} + \frac{v^2}{2g} = C$$

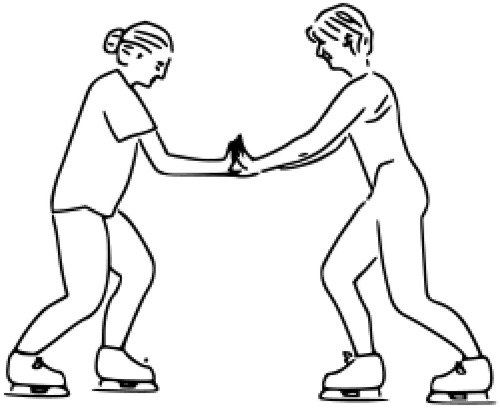
for ideal, steady, incompressible fluid

Conservation of momentum

- In a closed system (one that does not exchange any matter with the outside and is not acted on by outside forces) the total momentum is constant.

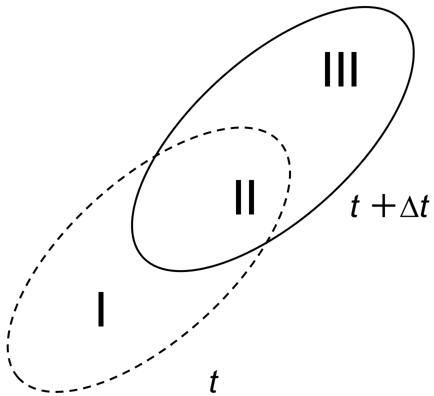


Isaac Newton



The third law states that all forces exist in pairs: if one object A exerts a force $\mathbf{F}_A$ on a second object B, then B simultaneously exerts a force $\mathbf{F}_B$ on A, and the two forces are equal and opposite: $\mathbf{F}_A = -\mathbf{F}_B$

Conservation of momentum



$$\sum \boldsymbol{F} = \frac{d(\sum m\boldsymbol{v})}{dt} = \lim_{\Delta t \to 0} \frac{\Delta(\sum m\boldsymbol{v})}{dt}$$

$$\left(m\bar{v}\right)_{t+\Delta t} = \left(m\bar{v}\right)_{II,t+\Delta t} + \left(m\bar{v}\right)_{III,t+\Delta t}$$

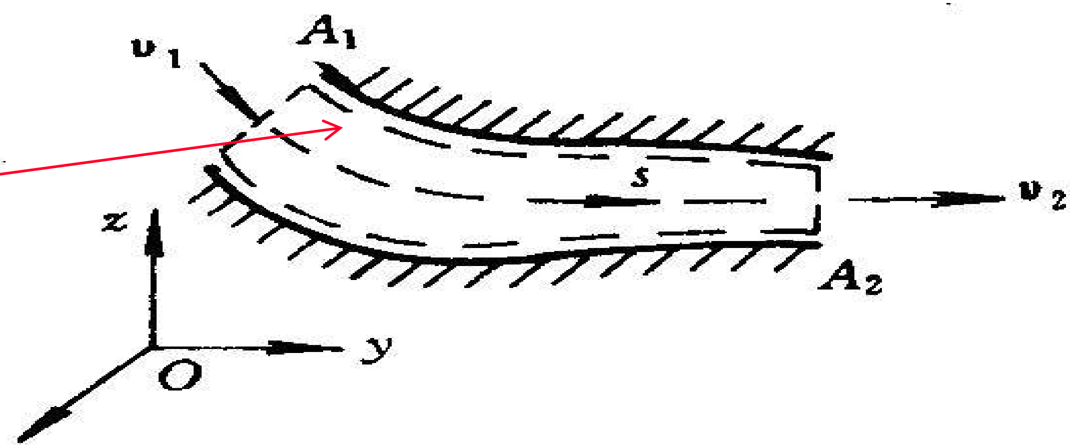
$$\left(m\bar{v}\right)_{t} = \left(m\bar{v}\right)_{I,t} + \left(m\bar{v}\right)_{II,t}$$

$$\Delta mv = \left\{\left(m\bar{v}\right)_{II,t+\Delta t} - \left(m\bar{v}\right)_{II,t}\right\} + \left\{\left(m\bar{v}\right)_{III,t+\Delta t} - \left(m\bar{v}\right)_{I,t}\right\}$$

Momentum equation: $$\sum \boldsymbol{F} = \frac{\partial}{\partial t}\iiint_V \rho \boldsymbol{v} dV + \oiint_A \rho \boldsymbol{v}(\boldsymbol{v} \cdot d\boldsymbol{A})$$

Applications of momentum

Define a control volume



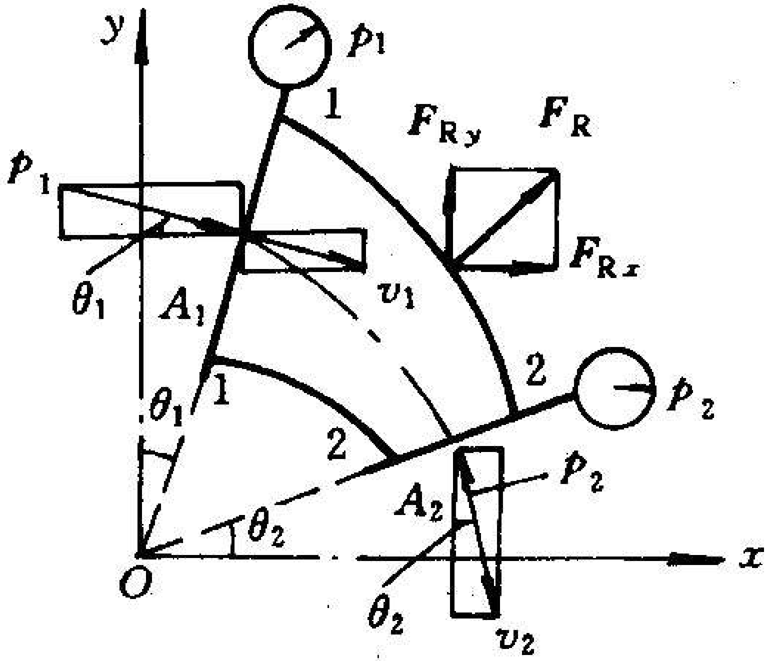
$$\Sigma \boldsymbol{F}_s = \oiint_A \rho \boldsymbol{v}(\boldsymbol{v} \cdot \mathrm{d}\boldsymbol{A}) = \int_{A_2} \rho \boldsymbol{v}_2 \, \boldsymbol{v}_2 \mathrm{d}A - \int_{A_1} \rho \boldsymbol{v}_1 \, \boldsymbol{v}_1 \mathrm{d}A$$

$$= \beta \rho q_V (\boldsymbol{v}_2 - \boldsymbol{v}_1) \approx \rho q_V (\boldsymbol{v}_2 - \boldsymbol{v}_1)$$

Projected force

$$\left.\begin{aligned} \Sigma F_x &= \beta \rho q_V (v_{2x} - v_{1x}) \approx \rho q_V (v_{2x} - v_{1x}) \\ \Sigma F_y &= \beta \rho q_V (v_{2y} - v_{1y}) \approx \rho q_V (v_{2y} - v_{1y}) \\ \Sigma F_z &= \beta \rho q_V (v_{2z} - v_{1z}) \approx \rho q_V (v_{2z} - v_{1z}) \end{aligned}\right\}$$

Applications of momentum



- *First step*:
  - Define control volume
- *Second step*:
  - Analysis of forces
- *Third step*:
  - Momentum conservation

X:

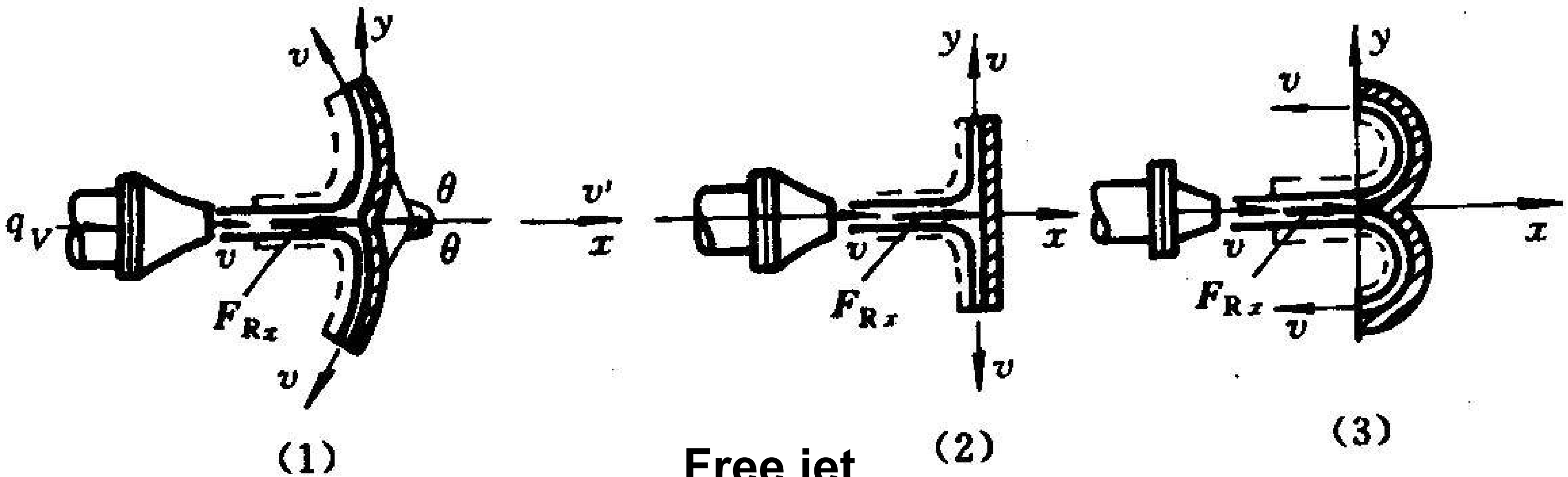
$$p_1 A_1 \cos\theta_1 - p_2 A_2 \sin\theta_2 - F_{Rx} = \rho q_V[(v_2 \sin\theta_2) - (v_1 \cos\theta_1)],$$

Y:

$$-p_1 A_1 \sin\theta_1 + p_2 A_2 \cos\theta_2 - F_{Ry} = \rho q_V[(-v_2 \cos\theta_2) - (-v_1 \sin\theta_1)]$$

$$\left.\begin{aligned} F_{Rx} &= p_1 A_1 \cos\theta_1 - p_2 A_2 \sin\theta_2 + \rho q_V(v_1 \cos\theta_1 - v_2 \sin\theta_2) \\ F_{Ry} &= p_2 A_2 \cos\theta_2 - p_1 A_1 \sin\theta_1 + \rho q_V(v_2 \cos\theta_2 - v_1 \sin\theta_1) \end{aligned}\right\}$$

Applications of momentum



Free jet

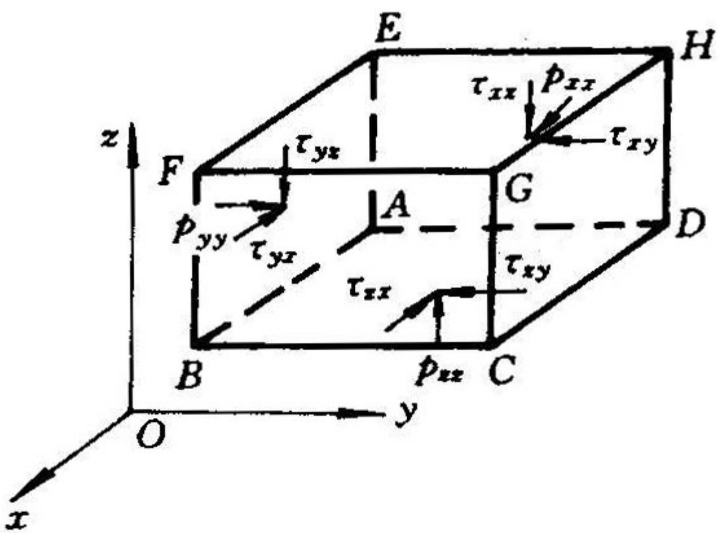
Force on fluid $$F_x = \rho\left[2\frac{q_V}{2}v\cos\theta - q_V v\right] = \rho q_V v(\cos\theta - 1)$$

Force on curve surface $$F_{Rx} = -F_x = \rho q_V v(1-\cos\theta)$$

θ=90^0 $$F_{Rx} = \rho q_V v$$

θ=180^0 $$F_{Rx} = 2\rho q_V v$$

Forces acting on a fluid element



- Body force
  - gravity $F = \rho g V$
  - electromagnetic forces etc
- surface forces
  - normal stresses
  - shear stresses $\tau = \partial v / \partial y$

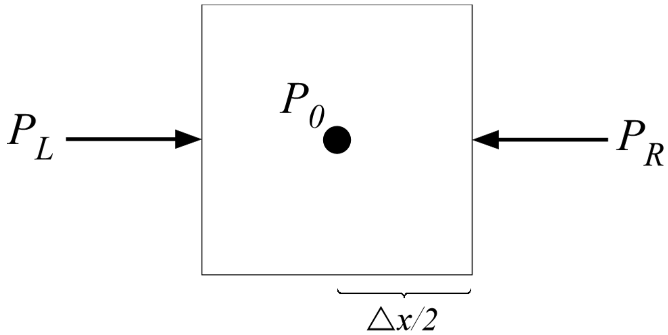
$$m\vec{a} = \vec{F}_{body} + \sum \vec{F}_{pressure\ gradient} + \sum \vec{F}_{viscous}$$

Gravity

Normal stress

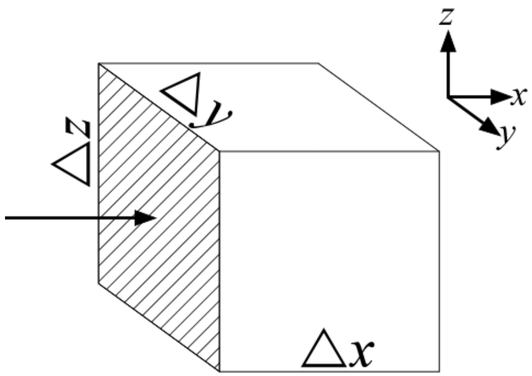
Shear stress

Pressure gradient forces



$$P_L = P_0 - \frac{\Delta x}{2}\frac{\partial P}{\partial x}$$

$$P_R = P_0 + \frac{\Delta x}{2}\frac{\partial P}{\partial x}$$



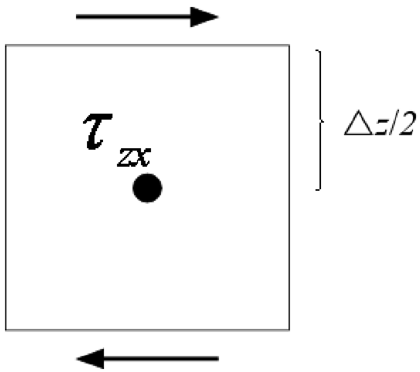
$$F_L = P_L \Delta y \Delta z = \left(P_0 - \frac{\partial P}{\partial x}\frac{\Delta x}{2}\right)\Delta y \Delta z$$

$$F_R = P_R \Delta y \Delta z = \left(P_0 + \frac{\partial P}{\partial x}\frac{\Delta x}{2}\right)\Delta y \Delta z$$

$$F_L - F_R == -\frac{\partial P}{\partial x}\Delta x \Delta y \Delta z = -\frac{\partial P}{\partial x}\Delta V$$

$$\sum \vec{F}_{pressure\ gradient} = -\frac{\partial P}{\partial x}\Delta V \vec{i} - \frac{\partial P}{\partial y}\Delta V \vec{j} - \frac{\partial P}{\partial z}\Delta V \vec{k} = -\Delta V\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right)$$

Viscous forces



$$\tau_T = \tau_0 + \frac{\Delta z}{2}\frac{\partial \tau}{\partial z}$$

$$\tau_B = \tau_0 - \frac{\Delta z}{2}\frac{\partial \tau}{\partial z}$$

$$F_T = \tau_T \Delta x \Delta y = \left(\tau_0 + \frac{\partial \tau}{\partial z}\frac{\Delta z}{2}\right)\Delta x \Delta y$$

$$F_B = \tau_B \Delta x \Delta y = \left(\tau_0 - \frac{\partial \tau}{\partial z}\frac{\Delta z}{2}\right)\Delta x \Delta y$$

$$\Rightarrow \quad F_T - F_B = \frac{\partial \tau}{\partial z}\Delta x \Delta y \Delta z = \frac{\partial \tau}{\partial z}\Delta V$$

$$\sum F_{x,\,\text{viscous stresses}} = \frac{\partial \tau_{xx}}{\partial x}\Delta V + \frac{\partial \tau_{yx}}{\partial y}\Delta V + \frac{\partial \tau_{zx}}{\partial z}\Delta V = \Delta V\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$$

$$\frac{\sum \vec{F}_{\text{viscous stresses}}}{V} = \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)\vec{i} + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)\vec{j} + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right)\vec{j}$$

Navier-Stokes Equation

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}=f_x-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}=f_y-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\mu}{\rho}\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right]$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}=f_z-\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{\mu}{\rho}\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

****The equations cannot be solved in their present form because the stresses* ... *t been recast in terms of velo*

$$\tau_{xy} = \tau \ldots \left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)$$

$$\tau_{yz} = \tau \ldots \left(\frac{\partial v_z}{\partial y}+\frac{\partial v_y}{\partial z}\right)$$

$$\tau_{zx} \ldots \left(\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial x}\right)$$

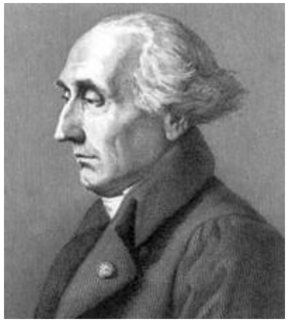
Navier(1785-1836)

Stokes(1819-1903)

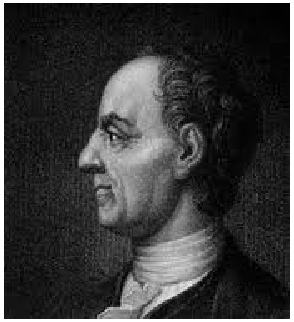
How to describe fluid flow?

- The Lagrangian description of the flow field is a way of looking at fluid motion where the observer follows an individual fluid parcel as it moves through space and time. Plotting the position of an individual parcel through time gives the pathline of the parcel. This can be visualized as sitting in a boat and drifting down a river.

- The Eulerian description of the flow field is a way of looking at fluid motion that focuses on specific locations in the space through which the fluid flows as time passes. This can be visualized by sitting on the bank of a river and watching the water pass the fixed.



Joseph-Louis Lagrange (1736 –1813)



Leonhard Euler (1707–1783)

Relation between Lag and Euler

$$\begin{cases} x = x(a,b,c,t) \\ y = y(a,b,c,t) \\ z = z(a,b,c,t) \end{cases}$$

$$\boldsymbol{V}(a,b,c,t) = \frac{\partial}{\partial t}\boldsymbol{r}(a,b,c,t).$$

$$\boldsymbol{a}(a,b,c,t) = \frac{\partial}{\partial t}\boldsymbol{V}(a,b,c,t) = \frac{\partial}{\partial t^2}\boldsymbol{r}(a,b,c,t).$$

$$\left.\begin{aligned} u(a,b,c,t) &= \frac{\partial x(a,b,c,t)}{\partial t}, \\ v(a,b,c,t) &= \frac{\partial y(a,b,c,t)}{\partial t}, \\ w(a,b,c,t) &= \frac{\partial z(a,b,c,t)}{\partial t}, \end{aligned}\right\}$$

Lagrangian system

$$\left.\begin{aligned} a_x(a,b,c,t) &= \frac{\partial u}{\partial t} = \frac{\partial^2 x}{\partial t^2}, \\ a_y(a,b,c,t) &= \frac{\partial v}{\partial t} = \frac{\partial^2 y}{\partial t^2}, \\ a_z(a,b,c,t) &= \frac{\partial w}{\partial t} = \frac{\partial^2 z}{\partial t^2}. \end{aligned}\right\}$$

***The shape and position of the system of fluid particles will change

Relation between Lag and Euler

$$B = B(x, y, z, t) = B\,(x\,(a, b, c, t), y\,(a, b, c, t), z\,(a, b, c, t),\ t)$$

$$\boldsymbol{a}(x, y, z, t) = \frac{\mathrm{D}}{\mathrm{D}t}\mathbf{V}(x, y, z, t) = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z},$$

Eulerian system

$$\boldsymbol{a} = \frac{\mathbf{DV}}{\mathbf{D}t} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}.$$

$$\left.\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}, \\ a_y &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}, \\ a_z &= \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}, \end{aligned}\right\}$$

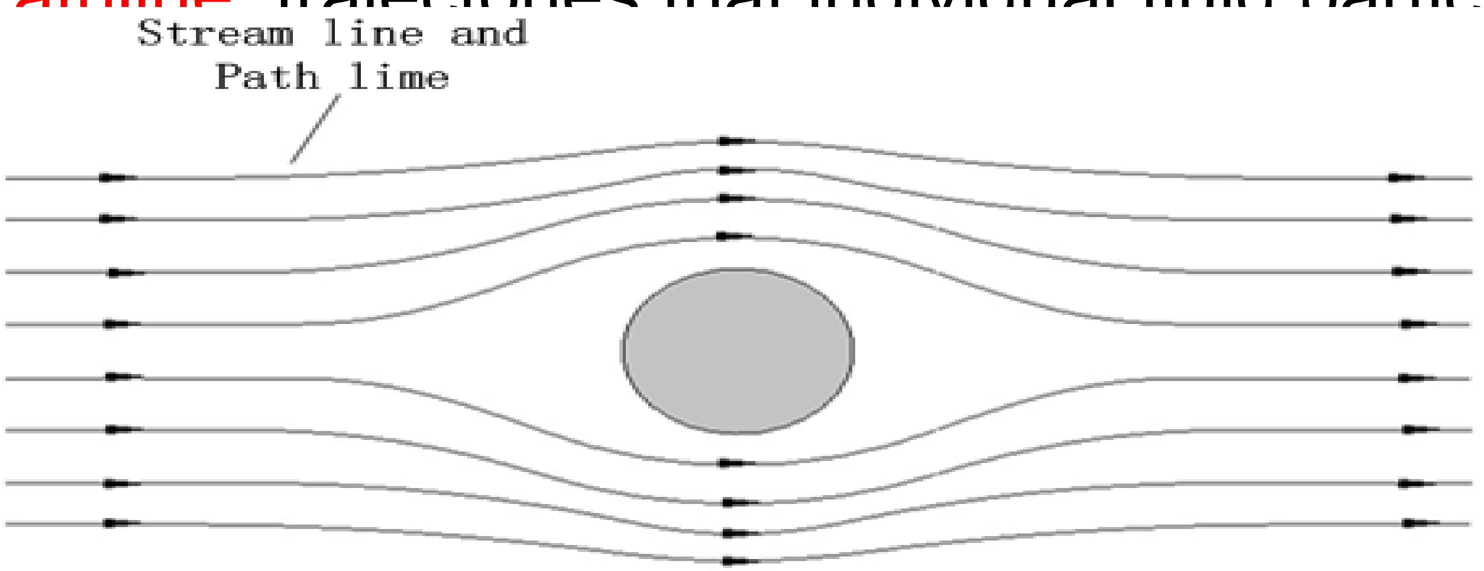
***The shape and position of control volume never change

Stream line and Path line

Streamline: curves that are instantaneously tangent to the velocity vector of the flow.

- Tells direction of velocity vector
- Does not directly indicate magnitude of velocity

Pathline: trajectories that individual fluid particles



Steady/Unsteady & Uniform/Non-uniform

Steady: Streamline patterns are not changing over time

Unsteady: Velocity at a point on a streamline changes over time

$$\frac{\partial V}{\partial t} = 0\,?$$

Uniform: Velocity is constant along a streamline (Streamlines are straight and parallel)

Non-uniform: Velocity changes along a streamline (Streamlines are curved and/or not parallel)

$$\frac{\partial V}{\partial s} = 0\,?$$

Flow Visualization

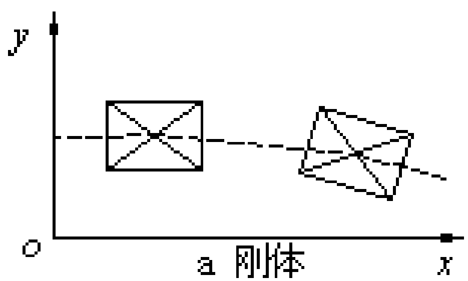
- Flow visualization is the art of making flow patterns visible. Most fluids (air, water, etc.) are transparent, thus their flow patterns are invisible to us without some special methods to make them visible.

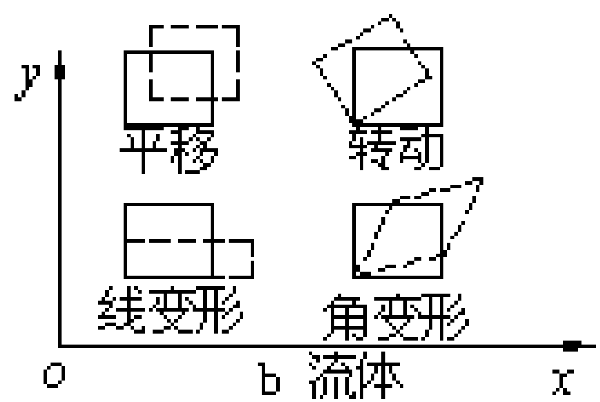
Flow Visualization

- Flow visualization is the art of making flow patterns visible. Most fluids (air, water, etc.) are transparent, thus their flow patterns are invisible to us without some special methods to make them visible.

Rigid Body Vs. Fluid Element

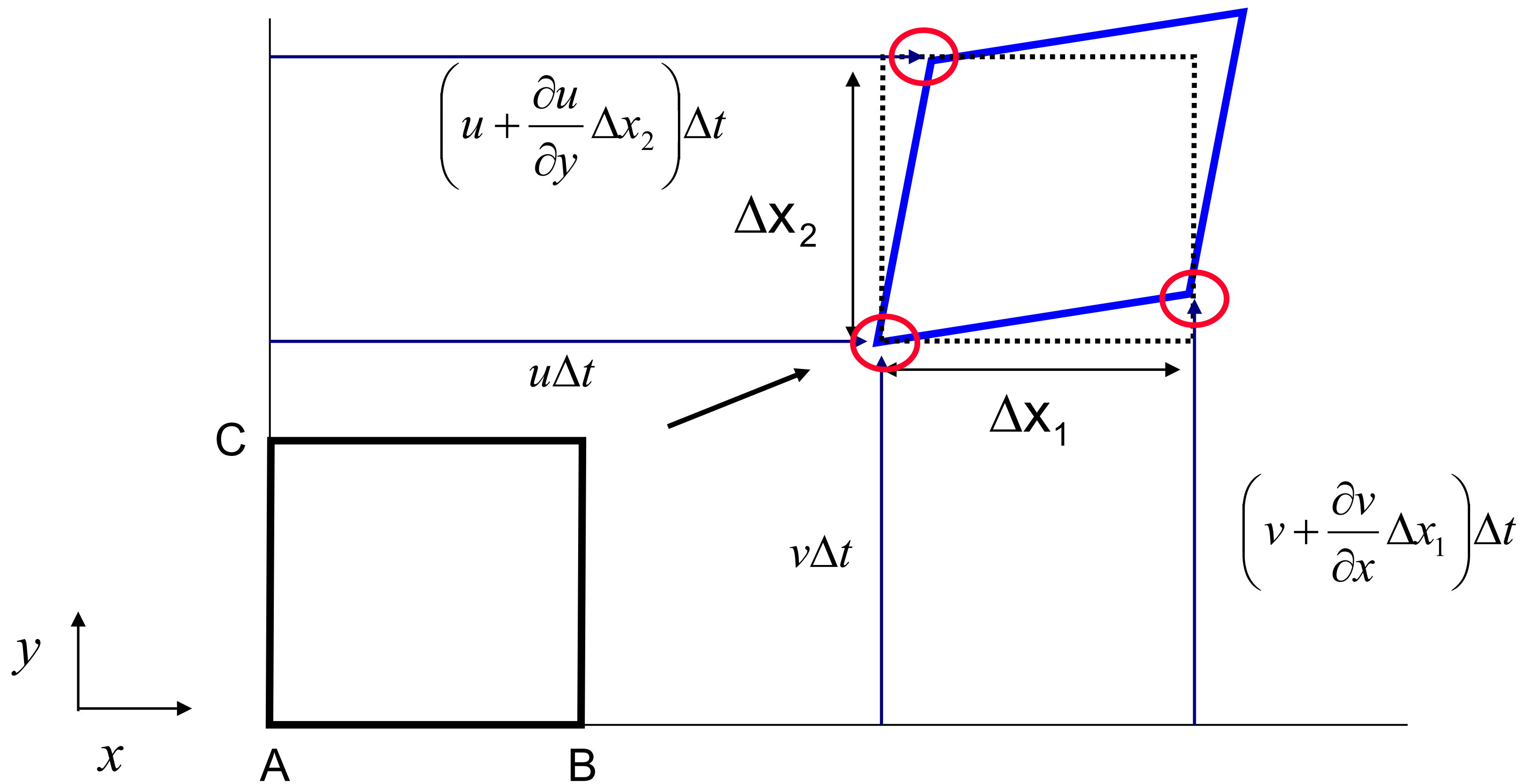
- Rigid Body may undergo two fundamental types of motion or deformation: (a) translation, and (b) rotation
- In fluid mechanics an element may undergo four fundamental types of motion or deformation: (a) translation, (b) rotation, (c) linear strain (sometimes called extensional strain), and (d) shear strain.





Movement of Fluid Element

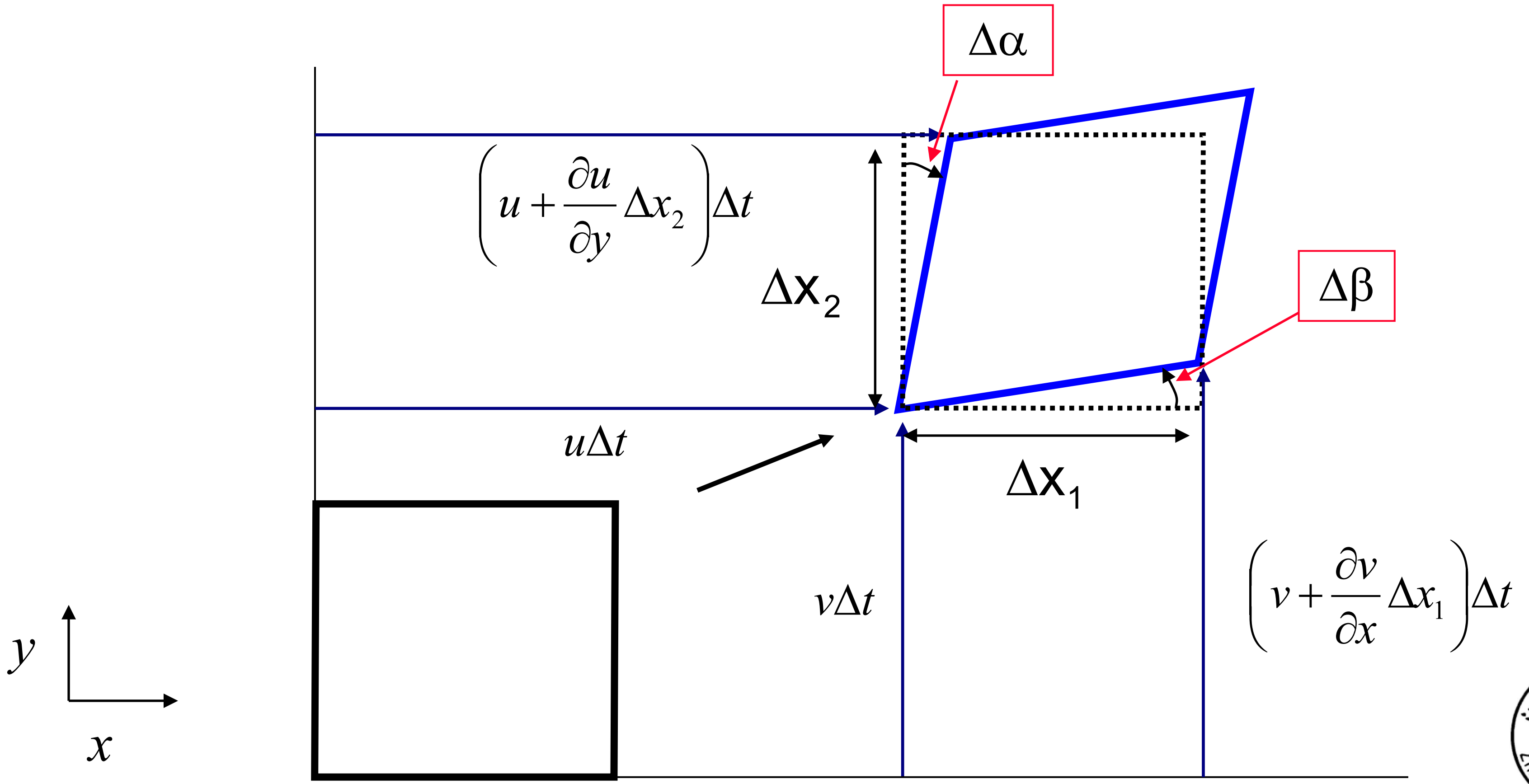
- The body undergoes shear deformation over time Δt. That is:
- Point A moves a distance uΔt in the x direction and vΔt in the y direction;
- Point B moves a distance [v + (∂v/∂x)Δx1]Δt in the y direction; and
- Point C moves a distance [u + (∂u/∂y)Δx2]Δt in the x direction.



Movement of Fluid Element

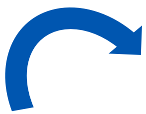
$$\Delta\alpha = \frac{\left(u + \frac{\partial u}{\partial y}\Delta x_2\right)\Delta t - u\Delta t}{\Delta x_2} = \frac{\partial u}{\partial y}\Delta t$$

$$\Delta\beta = \frac{\left(v + \frac{\partial v}{\partial x}\Delta x_1\right)\Delta t - v\Delta t}{\Delta x_1} = \frac{\partial v}{\partial x}\Delta t$$



Rotational Vs. Irrotational Flow

$$\Delta\dot{\alpha} = \frac{\partial v}{\partial x}$$



CLOCKWISE

$$\Delta\dot{\beta} = \frac{\partial u}{\partial y}$$

ANTICLOCKWISE

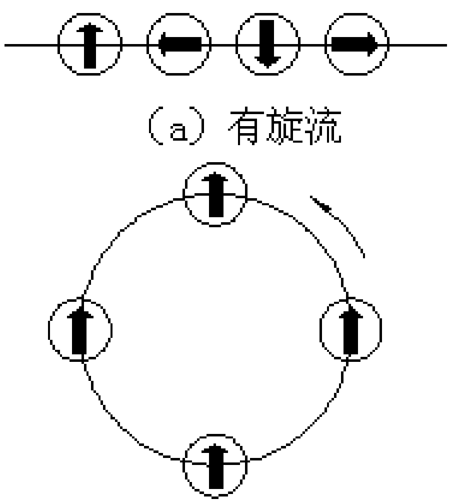
$$\omega_z = \frac{1}{2}\left(\Delta\dot{\alpha} - \Delta\dot{\beta}\right) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega = \left[\omega_x i + \omega_y j + \omega_z k\right] = 0?$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

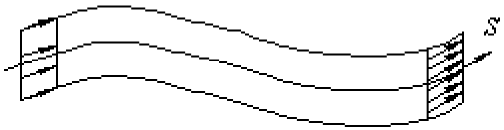


（a）有旋流

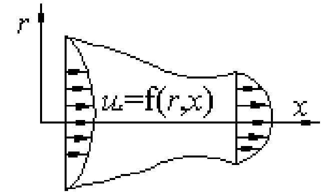
（b）无旋流

One/Two/Three Dimensional Flow

- One dimensional flow

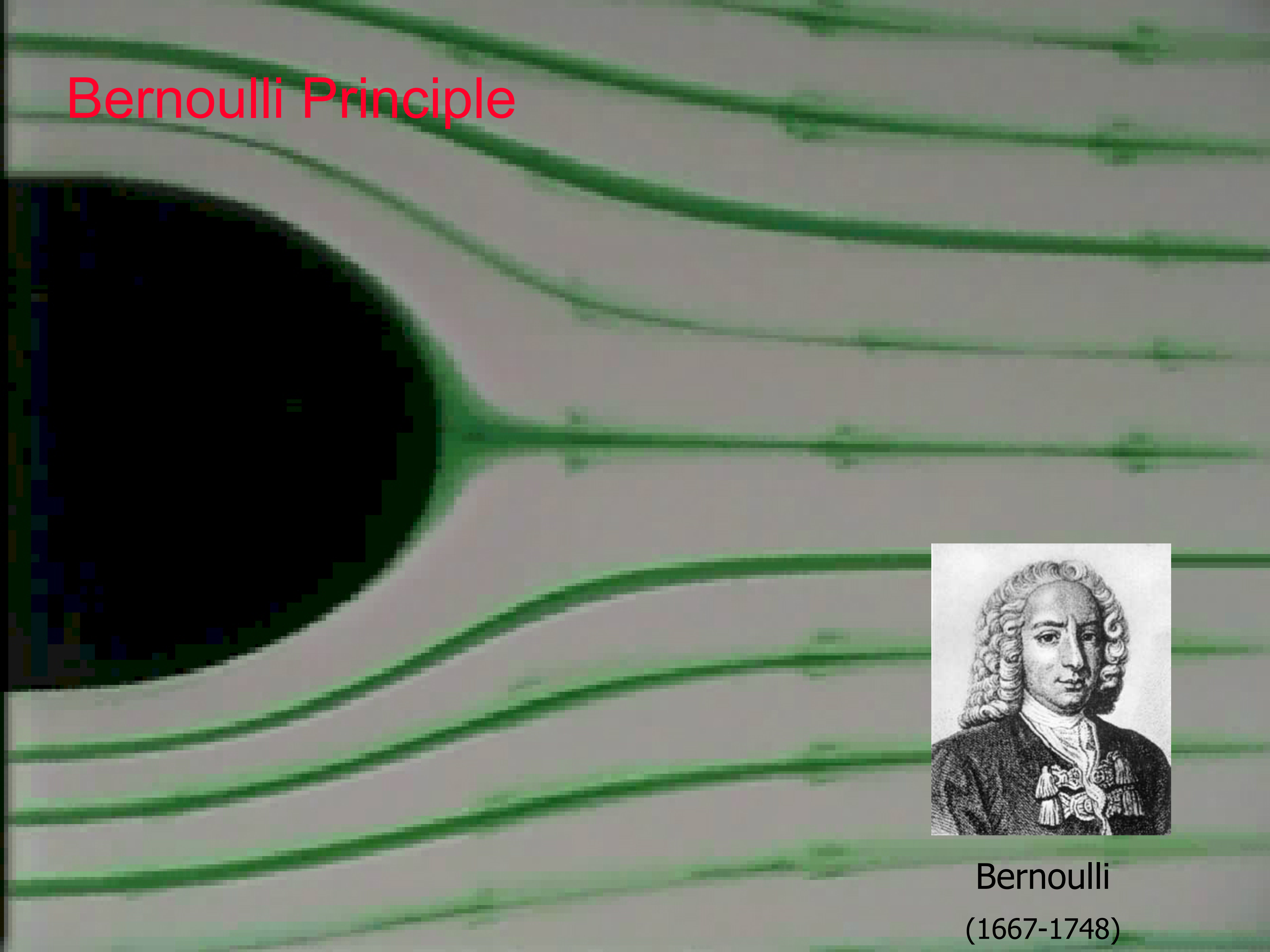


  - The fluid parameters (velocity, pressure…) remains constant throughout any cross-section normal to the flow-direction. Flow field is represented by streamlines which are essentially straight & parallel.



- Two dimensional flow
  - The flow parameters vary along two-directions. Viscosity make the velocity change from zero at the boundary to a maximum value in the flow field.
- Three dimensional flow
  - The flow properties vary in all the three directions.

Bernoulli Principle



Bernoulli

(1667-1748)

Bernoulli Equation

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}=f_x-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]\Big)+$$

The fluid particle moves a distance **s** along the stream line

The work done on the fluid particle;

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}=f_y-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\mu}{\rho}\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right]$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}=f_z-\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{\mu}{\rho}\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

Bernoulli Equation

$$-\mathrm{d}W - \frac{\mathrm{d}p}{\rho} - f\mathrm{d}s = \mathrm{d}\left(\frac{v^2}{2}\right)$$

$$\mathrm{d}\left[W + \int \frac{\mathrm{d}p}{\rho} + \frac{v^2}{2} + \int f\mathrm{d}s\right] = 0$$

$$z + \frac{p}{\rho g} + \frac{v^2}{2g} + \frac{1}{g}\int f\mathrm{d}s = C$$

——Steady、impressible flow

$$z + \frac{p}{\rho g} + \frac{v^2}{2g} = C$$

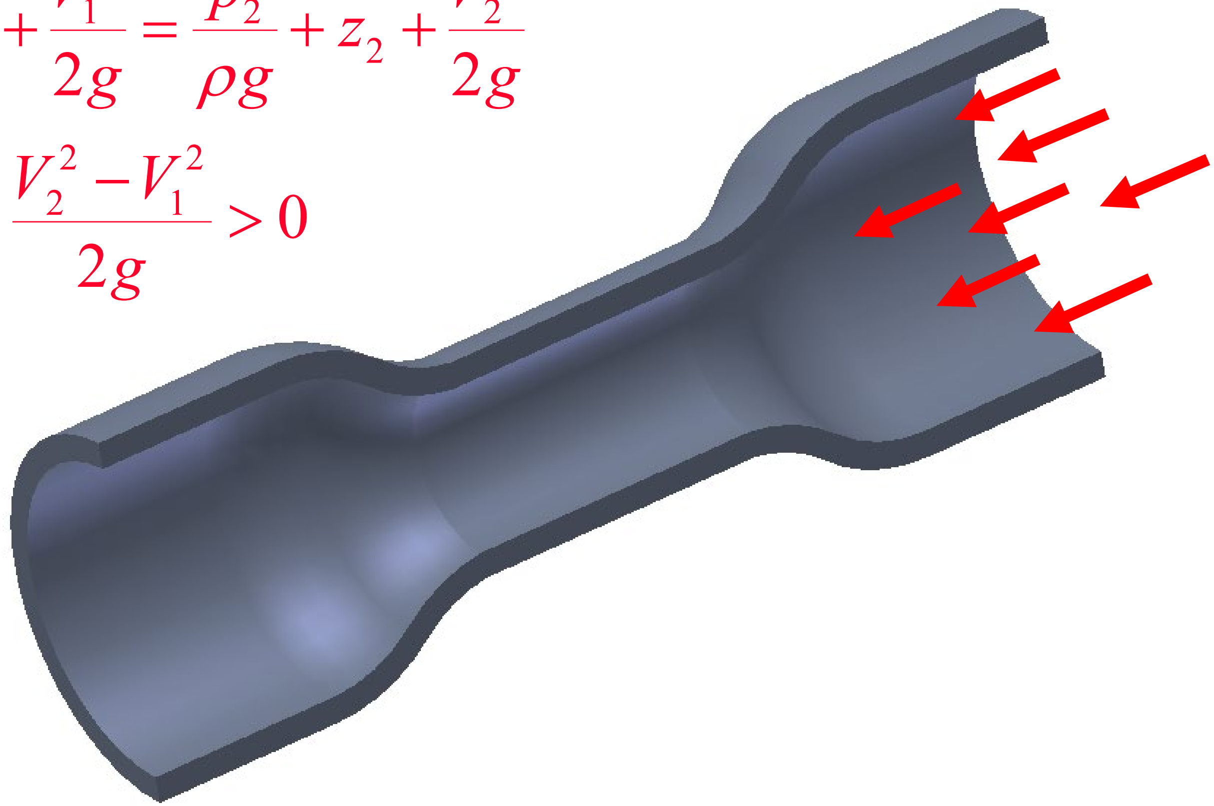
——Inviscid、steady、impressible flow

Applications of Bernoulli Principle

- An increase in velocity results in a decrease in pressure. Likewise, a decrease in velocity results in an increase in pressure.

$$\frac{p_1}{\rho g}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+z_2+\frac{V_2^2}{2g}$$

$$\frac{p_1-p_2}{\rho g}=\frac{V_2^2-V_1^2}{2g}>0$$



Applications of Bernoulli Principle

- What is an example of a fluid experiencing a change in elevation, but remaining at a constant pressure? Free jet

$$\cancel{\frac{p_1}{g}} + z_1 + \frac{V_1^2}{2g} = \cancel{\frac{p_2}{g}} + z_2 + \frac{V_2^2}{2g}$$

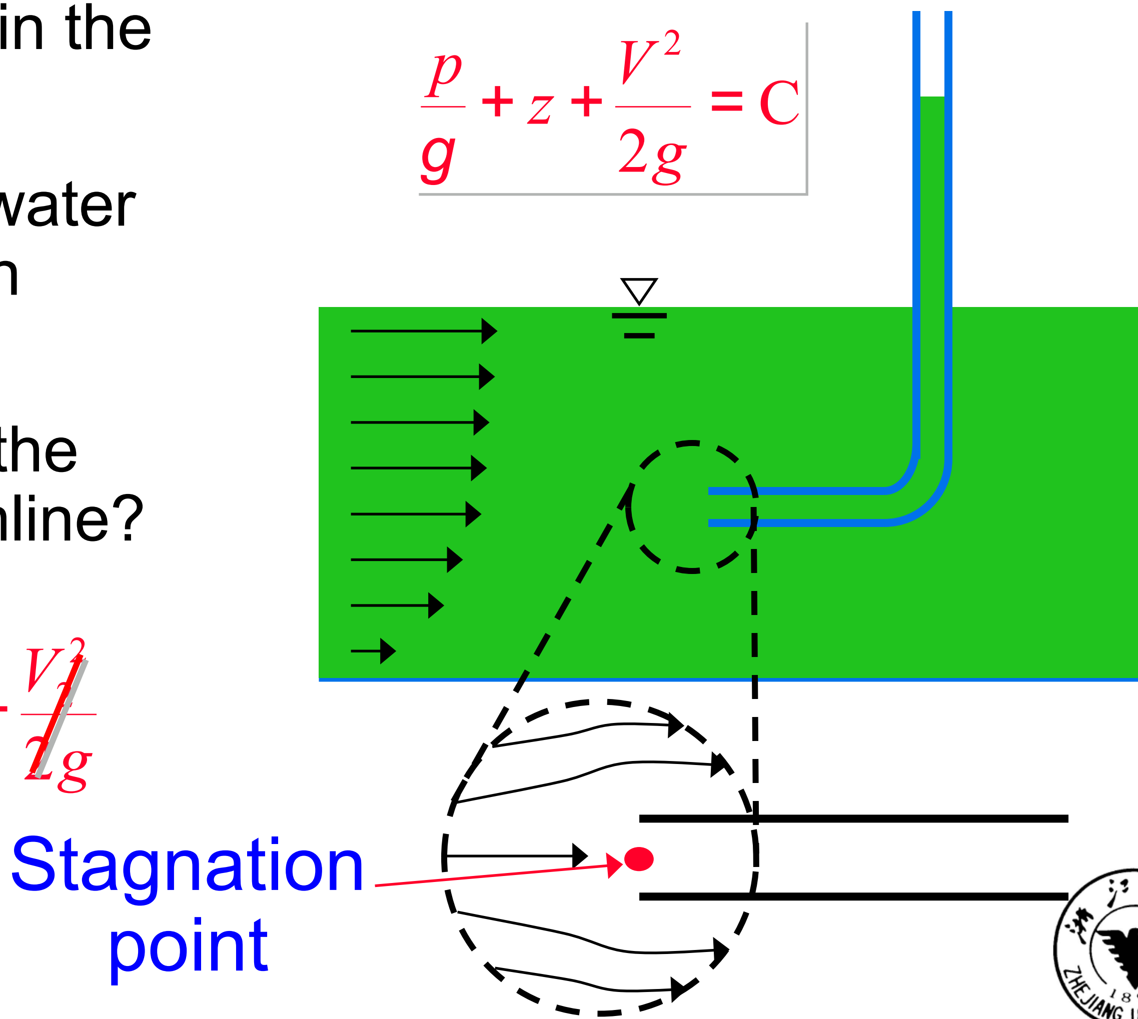
$$z_1 + \frac{V_1^2}{2g} = z_2 + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2g(z_1 - z_2) + V_1^2}$$

Applications of Bernoulli Principle

- What happens when the water starts flowing in the channel?
- How high does the water rise in the stagnation tube?
- How do we choose the points on the streamline?

$$\frac{p}{g} + z + \frac{V^2}{2g} = C$$



$$\frac{p_1}{g} + z_1 + \frac{V_1^2}{2g} = \cancel{\frac{p_2}{g}} + z_2 + \cancel{\frac{V_2^2}{2g}}$$

Pitot Tubes

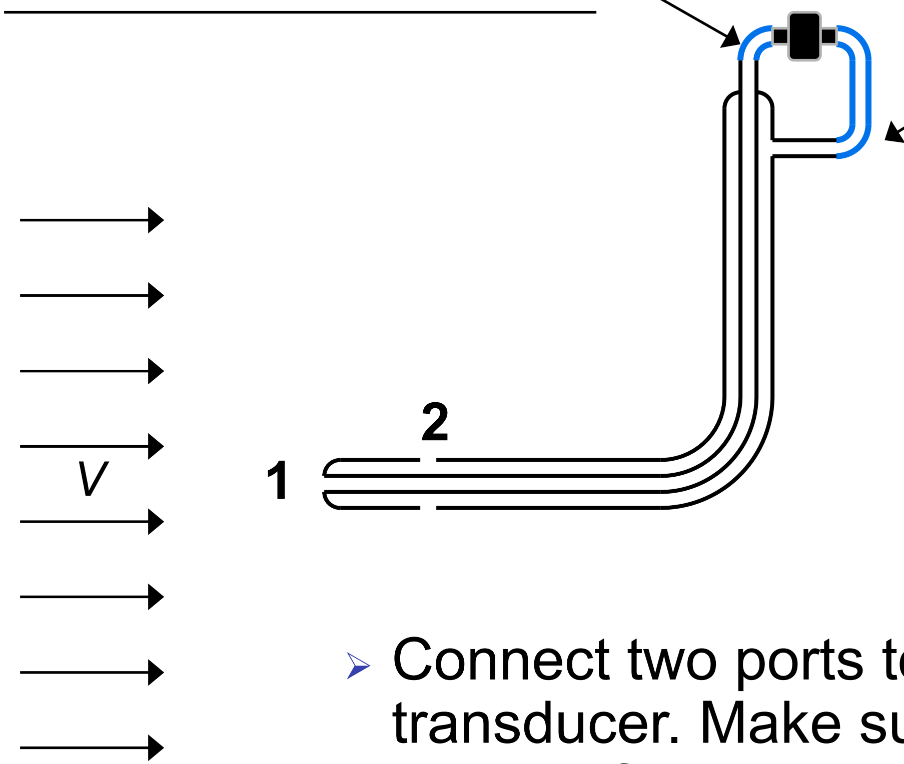
- Used to measure air speed on airplanes
- Can connect a differential pressure transducer to directly measure $V^2/2g$
- Can be used to measure the flow of water in pipelines

<u>Point measurement!</u>

Pitot Tube

Stagnation pressure tap

Static pressure tap



$$\frac{p_1}{g} + \cancel{z_1} + \frac{\cancel{V_1^2}}{2g} = \frac{p_2}{g} + \cancel{z_2} + \frac{V_2^2}{2g}$$

- $V_1 = \underline{0}$
- $z_1 = z_2$

$$V = \sqrt{\frac{2}{r}(p_1 - p_2)}$$

- Connect two ports to differential pressure transducer. Make sure Pitot tube is completely filled with the fluid that is being measured.
- Solve for velocity as function of pressure difference

Robot (Bernoulli suction)

Single driving cylinder

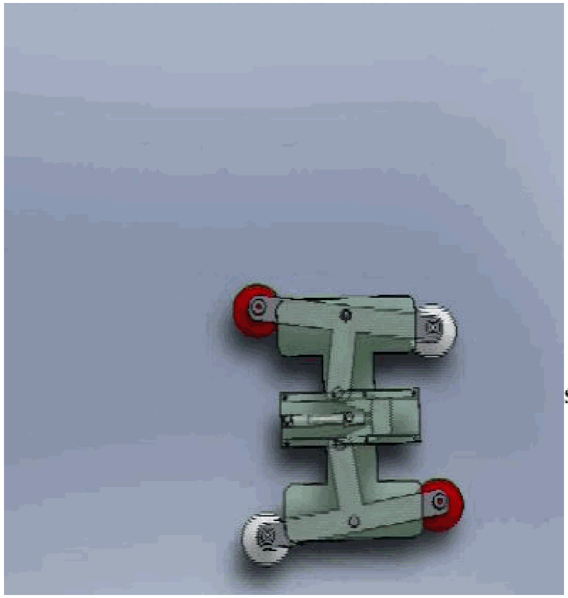
——crawling and turning with single cylinder

- **Crawling:**

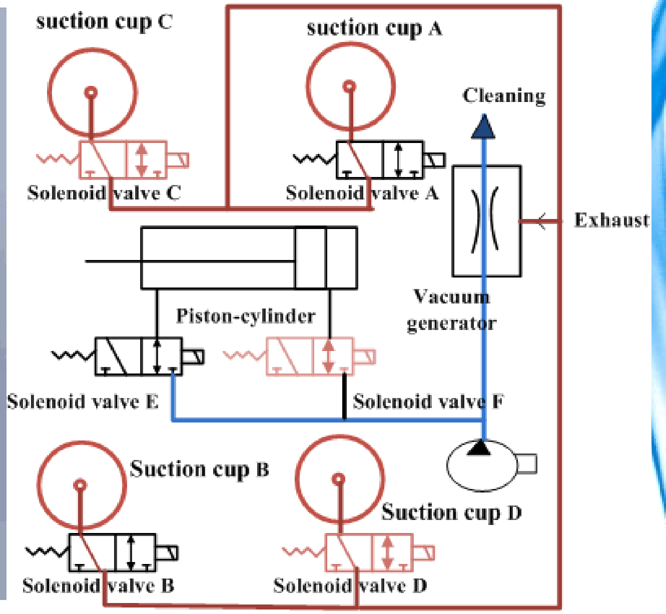
  through alternate sucking of A+B or C+D

- **Turning:**

  through alternate sucking of A+C or B+D



Mechanism schematic



Hydraulic system chart

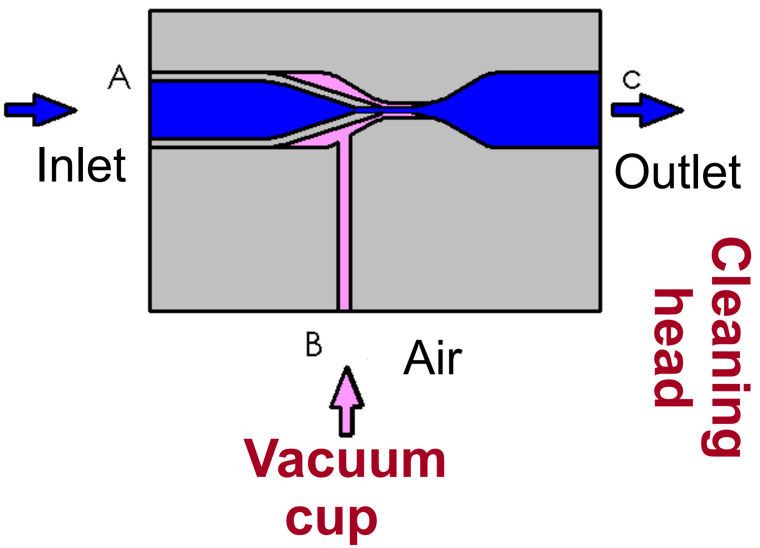
Robot (Bernoulli suction)

Multiple functions of Water

——including suction, climb, and wash, making the whole system tiny and light

- **Climbing function:** using water pressure to drive the cylinder to achieve reciprocating motion
- **Sucking function:** using the vacuum caused by the Venturi effect
- **Cleaning function:** using ultra-sound to clean

the Venturi effect



可实现在多种光滑壁面上进行爬行
和清洗工作
climb on different smooth walls

Example

- Which of the following velocity fields are kinematically possible for an incompressible flow ? Which of the following velocity fields is the irrotational flow? The acceleration of (i)?

(i) $u = x^2 + y^2$, $v = y^2 + z^2$, $w = -2(x + y)z$

(ii) $\vec{v} = x^2 y \vec{i} + (x + y + z)\vec{j} + (z^2 + x^2)\vec{k}$

(iii) $u = -\frac{kx}{y}, \quad v = k\log(xy)$

(iv) $u = \frac{k(x^2 - y^2)}{(x^2 + y^2)^2}, \quad v = \frac{2kxy}{(x^2 + y^2)^2}$

•[(i) yes, (ii) No, (iii) yes, (iv) No]

(k is a constant)

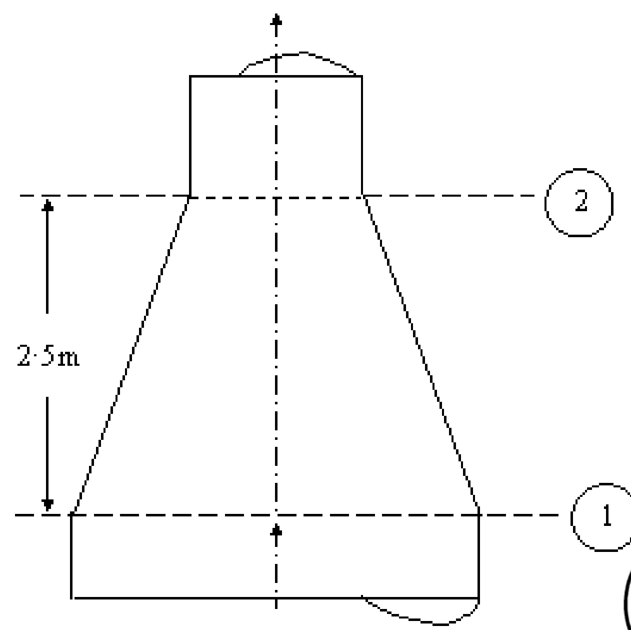
Example

- Consider a vertical nozzle of inlet and outlet diameters of 0.6 m and 0.3 m respectively as shown in Fig 13.2. The pressure at section 1 is 20 kPa (gauge), and the volume flow rate is 0.6 m^3 /s. Find

(i) the velocities at section 1 and section 2,

(ii) total force acting on the walls of the nozzle.

[Neglect frictional resistance]

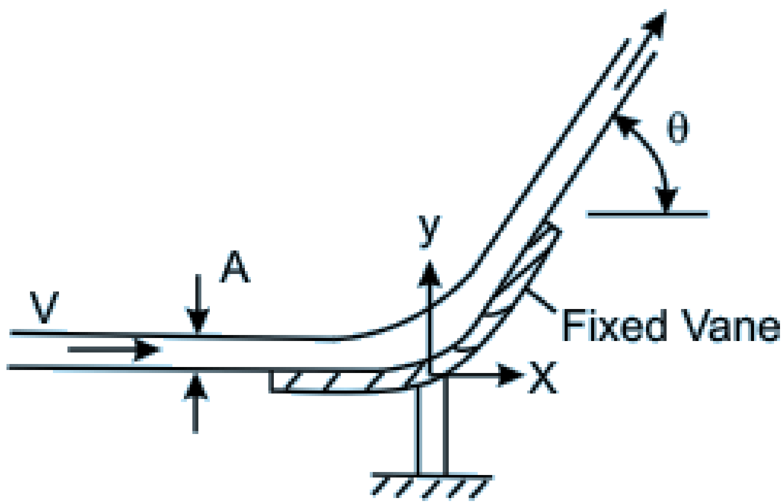


[(i) V_1 = 2.12 m/s, V_2 = 8.45 m/s

(ii) 0.517 kN (vertically upwards)]

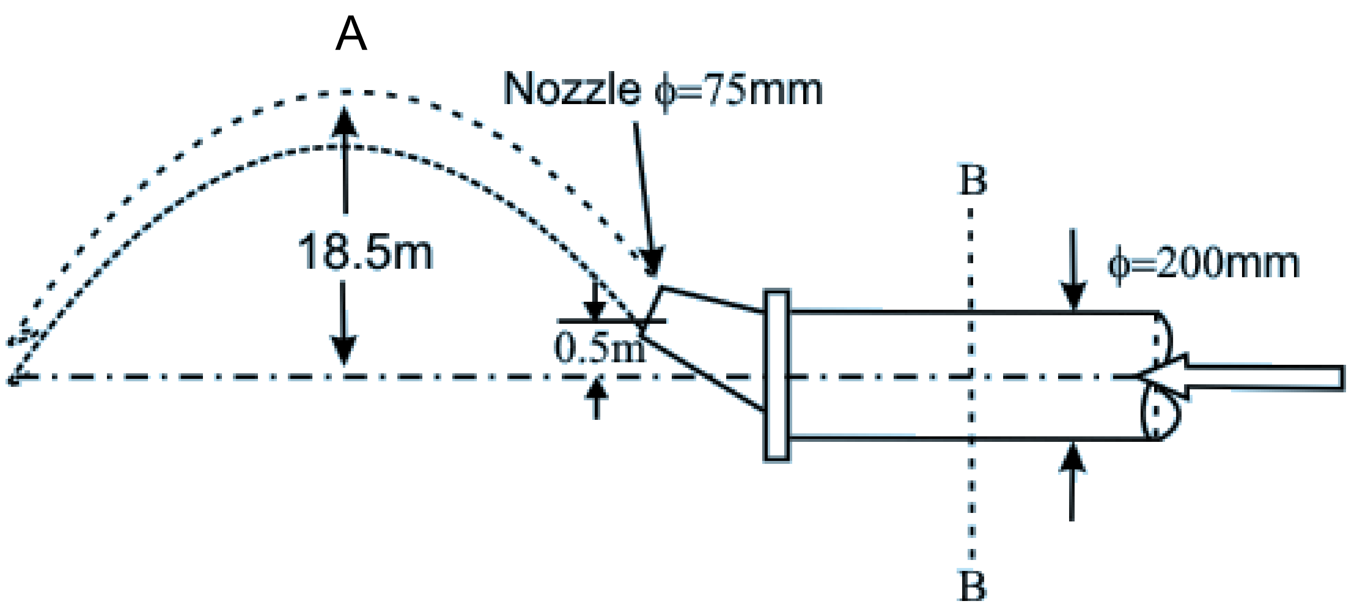
Example

- A horizontal jet of water with velocity V and cross sectional area A impinges on a stationary vane, which deflects the jet through an angle θ . Derive expressions for the horizontal and vertical force components X and Y acting on the vane. Neglect effects of gravity and friction.



Example

- A free jet of water is produced using a 75 mm diameter nozzle attached to a 200 mm diameter pipe, as shown in Figure 16.8. If the average velocity of water at plane B is 3.8 m/s, calculate the velocity of water at point A in the free jet. Neglect friction losses in the nozzle and pipe



(19.4 m/s)