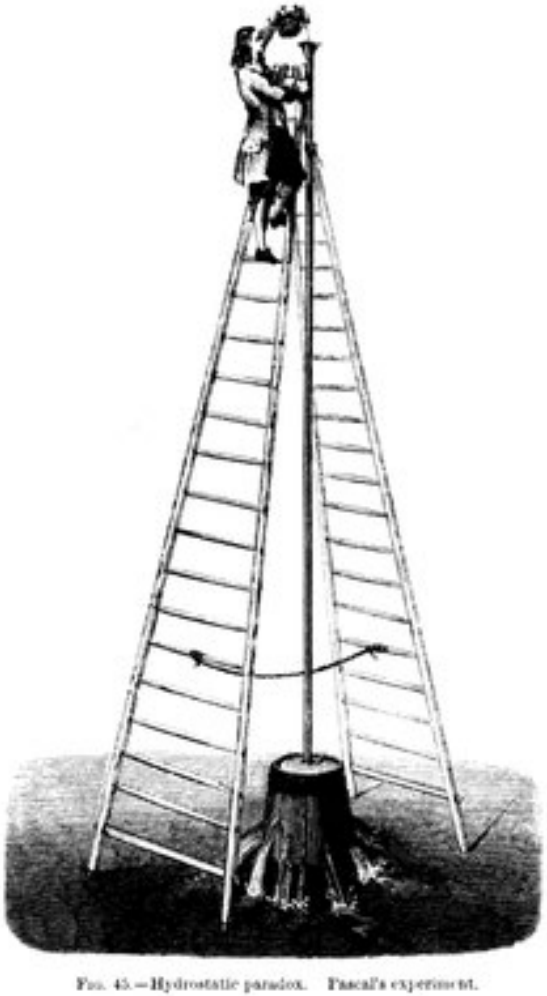
Chapter Three

Hydrostatics

Pascal

1623-1662

Hydrostatics



- The word “statics” is derived from Greek word “statikos”= motionless
- For a fluid at rest or moving in such a manner that there is no relative motion between particles there are no shearing forces present
- The only stress in fluid statics is the normal stress, which is the pressure, and the variation of pressure is due only to the weight of the fluid

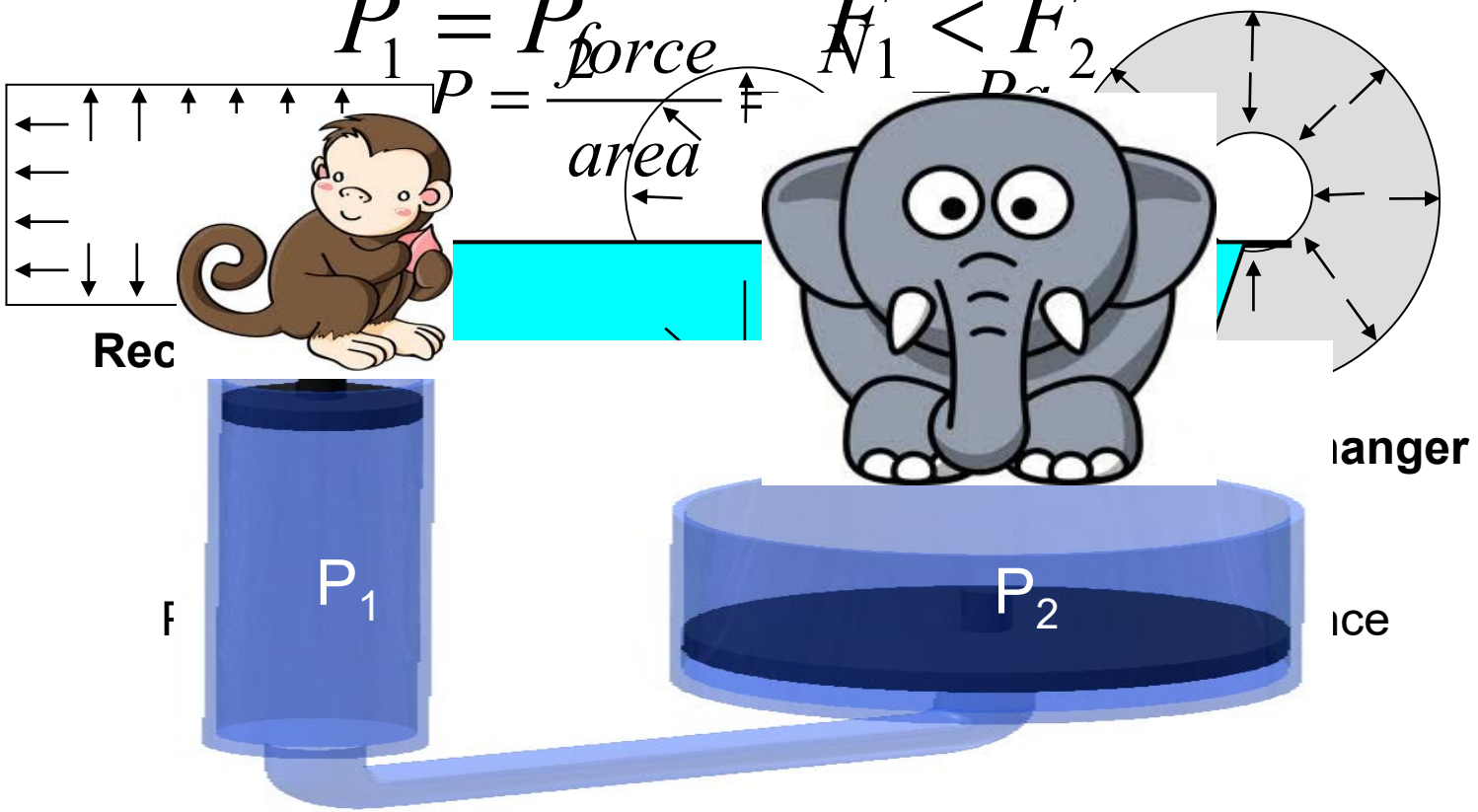
Pascal's barrel experiment in1646

Principle of statics

- The pressure exerted by a confined incompressible fluid is transmitted undiminished throughout the fluid (*Pascal's Law*)

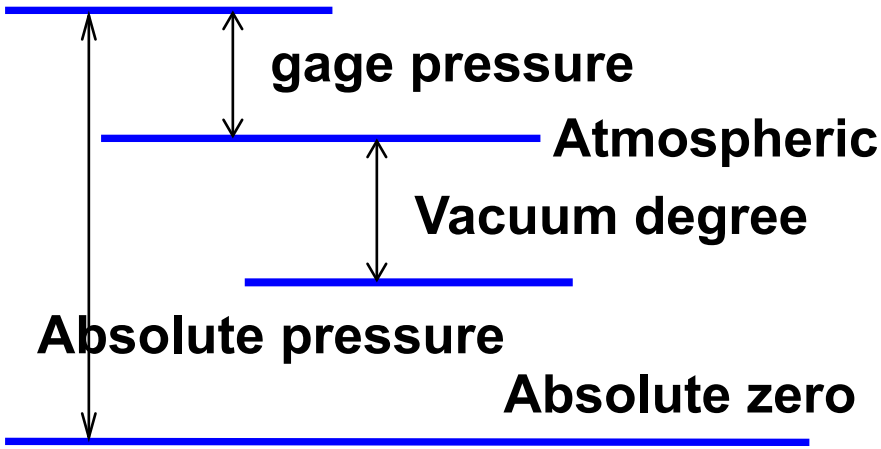
$$P = \frac{force}{area}$$

$$P_1 = P_2 \qquad F_1 < F_2$$



Definition of Pressure Terminology

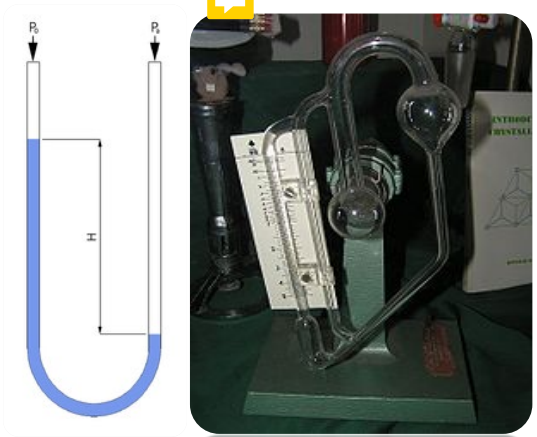
- Absolute pressure: The pressure is measured above that of vacuum
- Gage pressure: Pressure expressed as the difference between the pressure of the fluid and that of the surrounding atmosphere
- Vaccum degree: Pressure expressed as the difference between the surrounding atmospheric pressure and the absolute pressure of gases



$$p_{abs} = p_{atm} + p_{gage}$$

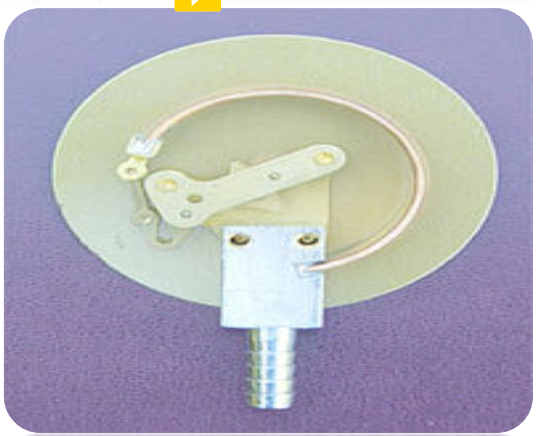
$$p_{vac} = p_{atm} - p_{abs}$$

How to measure pressure?



Liquid column

- The liquid column will rise or fall until its weight is in equilibrium with the pressure differential between the two ends of the tube
- A few 100 Pa to a few atmospheres



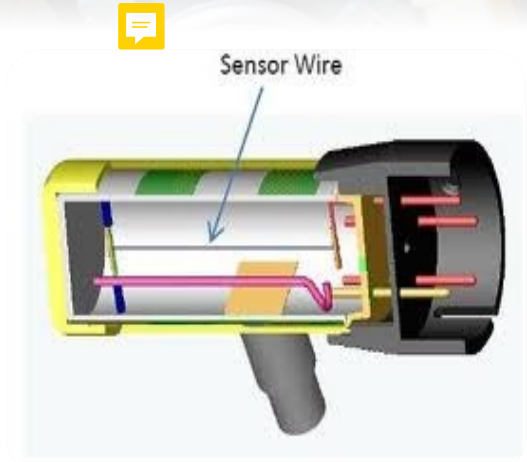
Bourdon tube

- A flattened tube tends to change to be straightened or larger circular cross-section when pressurized.
- Measuring gauge pressure

How to measure pressure?

Diaphragm

- The deformation of a thin diaphragm is dependent on the difference in pressure between its two faces.
- Measuring gauge pressure, differential pressure, or absolute pressure.



Thermal conductivity

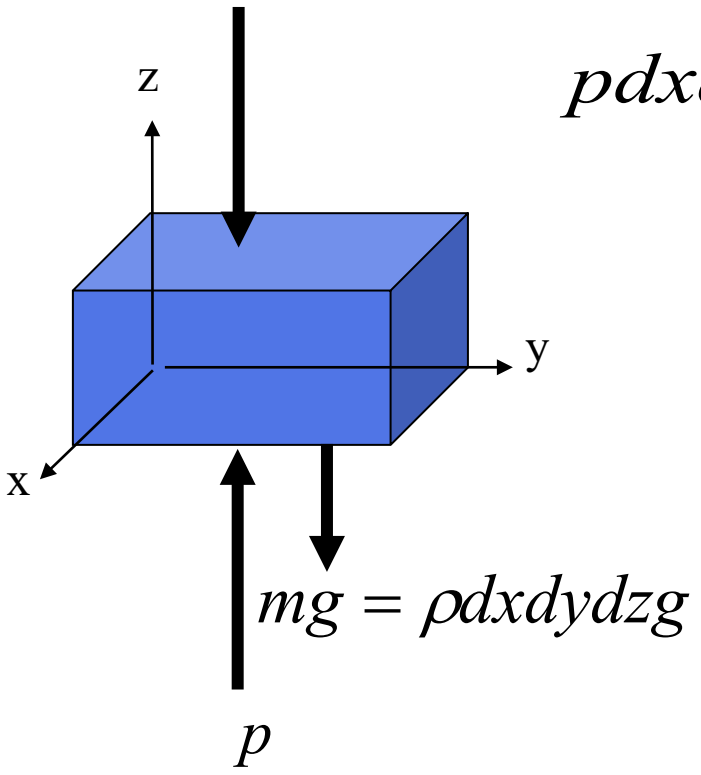
- A real gas increases in density -which may indicate an increase in pressure- its ability to conduct heat increases.
- Sensitive to the chemical composition of the gases being measured

Definition of Pressure Terminology

Unit	Definition or Relationship
1 pascal (Pa)	1 kg m^{-1} s^{-2}
1 bar	1 x 10^5 Pa
1 atm	101,325 Pa
760 mm Hg	1 atm
1 psi	6894.76 Pa
1 MPa	10^6 Pa

Pressure (the symbol: *p*) is the force per unit area applied in a direction perpendicular to the surface of an object

Basic equation of Z-direction forces



Z-direction forces

Assumption: Liquids are incompressible i.e. their density is assumed to be constant:

$$pdxdy=(p+\frac{\partial p}{\partial z}dz)dxdy+\rho gdxdydz$$

$$\Rightarrow -\frac{\partial p}{\partial z}=\rho g$$

$$\Rightarrow p_2-p_1=-\rho g(z_2-z_1)$$

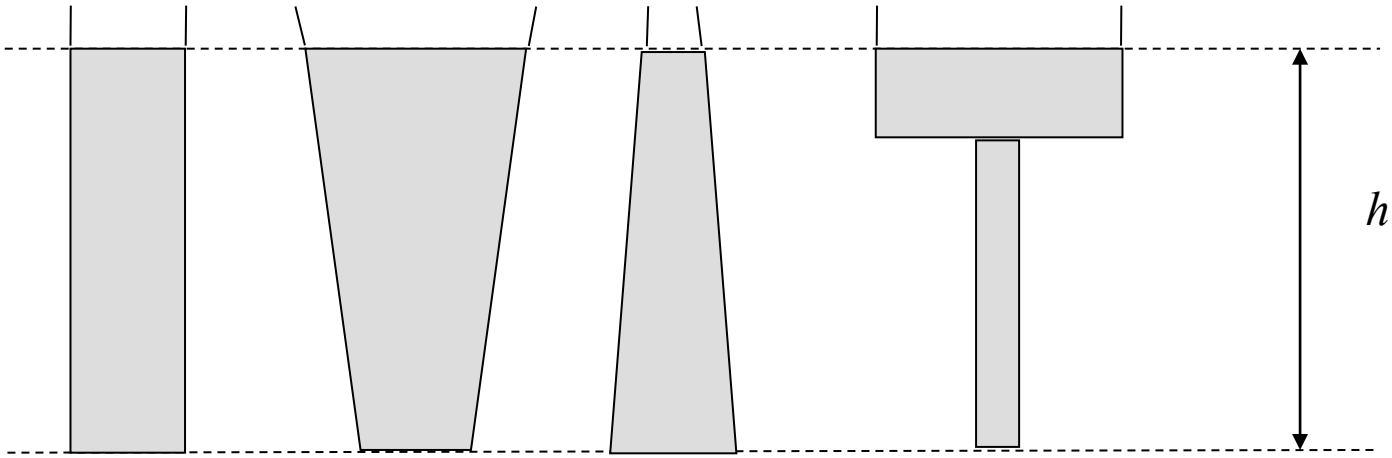
$$\Rightarrow \boxed{p=\rho gh+p_0}$$

p_o is the pressure at the free surface (p_o=p_{atm}), h is the fluid depth

The hydrostatic paradox

- The pressure in a homogeneous, incompressible fluid at rest depends on the depth of the fluid relative to some reference plane, and it is not influenced by the size or shape of the tank or container

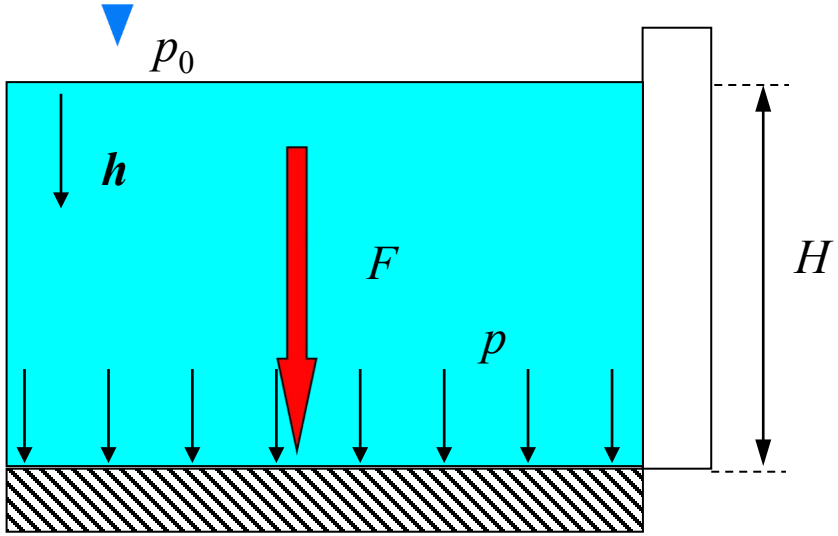
Fluid is the same in all containers



Pressure is the same at the bottom of all containers

Horizontal plane surface

- A canal is rectangular, 10 m wide, 12 m long and 15 m high. What is the net force on the bottom?



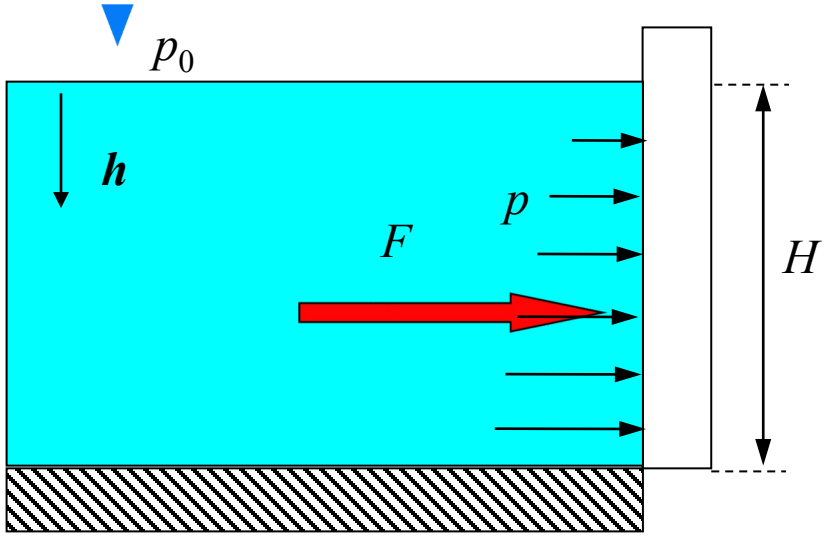
Horizontal plane surface

$$F = \int_A p dA = \int_A (p_0 + \rho g H) dA$$

$$F_{net} = \int_A (p_0 + \rho g H) dA = (10^5 + 10^3 * 9.8 * 15) * 12 * 10 \ \text{N}$$

Vertical plane surfaces

- The lock gate of a canal is rectangular, 20 m long and 10 m high. One side is exposed to the atmosphere and the other side to the water. What is the net force on the lock gate?



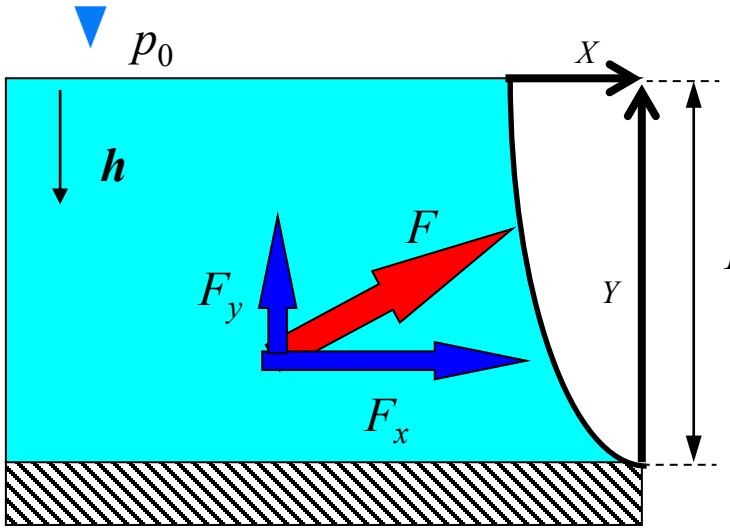
Horizontal plane surface

$$F = \int_A p dA = \int_A (p_0 + \rho g h) dA$$

$$F_{net} = \int_A \rho g h dA = 10^3 * 9.8 * 10 / 2 * 20 * 10 \ \text{N}$$

Curve surface

- The lock gate of a canal is rectangular, 20 m long and 10 m high. One side is exposed to the atmosphere and the other side to the water. What is the net force on the lock gate?



$$\vec{F} = \vec{F}_x + \vec{F}_y \Rightarrow F^2 = F_x^2 + F_y^2$$

$$F_x = \int_A p dA_y = \int_A (p_0 + pgh) dA_y$$

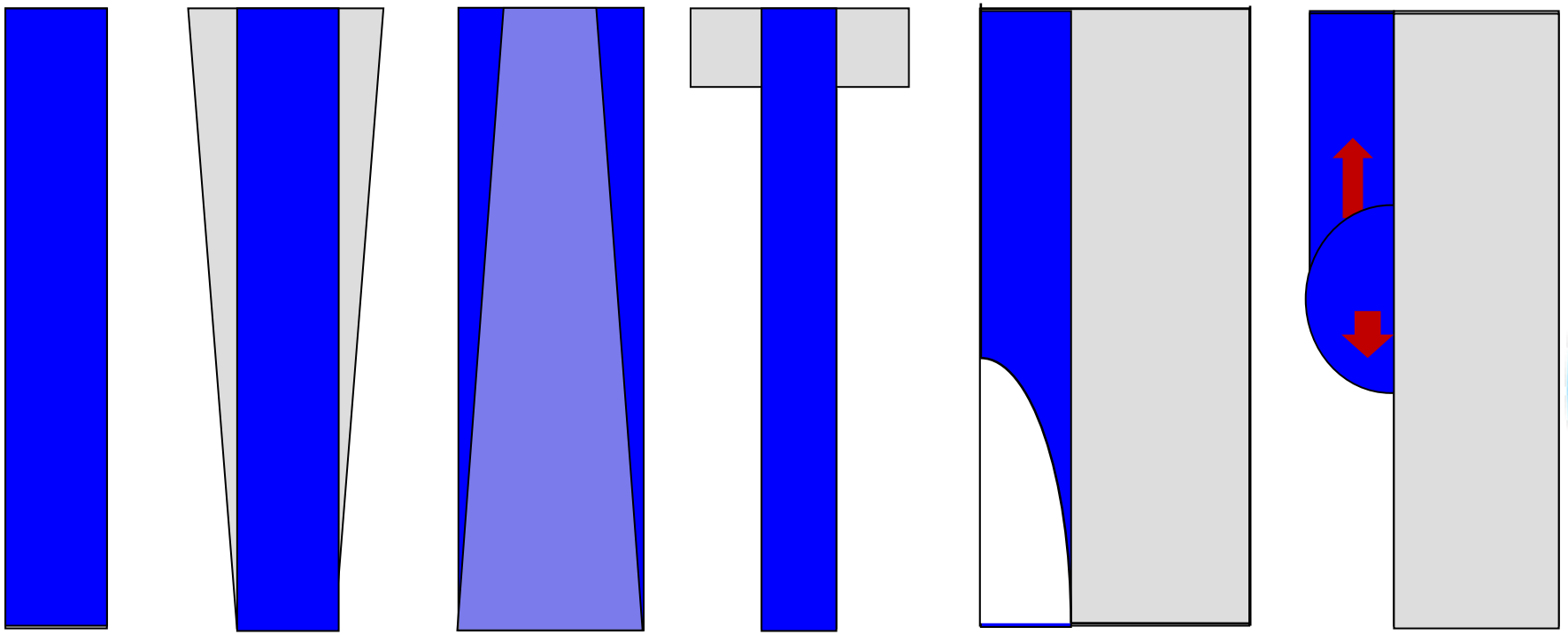
$$F_y = \int_A p dA_x = \int_A (p_0 + pgh) dA_x$$

$$F_y = \int_A p dA_x = \int_A (p_0 + \rho gh) dA_x = \int_A p_0 dA_x + \int_A \rho h dA_x g = p_0 A_x + \rho V g$$

Note: The net force on the gate in y direction equals the sum of the weight of liquid above the gate

How to estimate the volume

- The volume above the gate includes imaginary volume and objective volume of liquid.

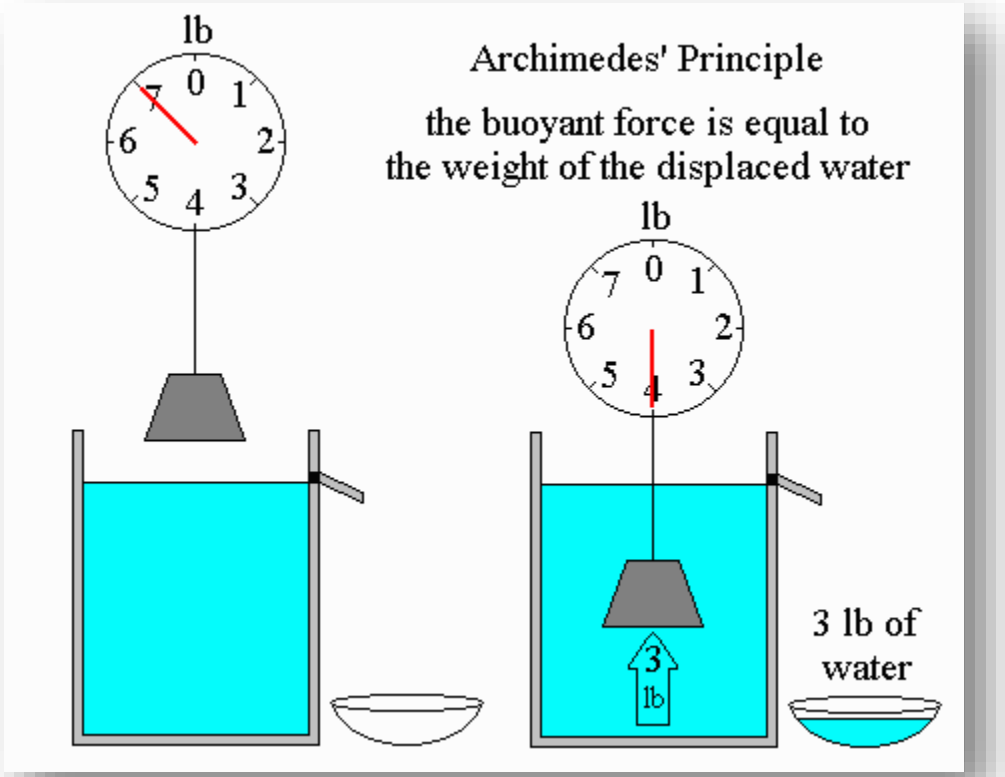


➢ ***If the volume between the curved surface and the free surface has no liquid, the volume is called imaginary volume. Force direction is upward.***

➢ ***If the volume between the curved surface and the free surface has liquid, the volume is called objective volume. Force direction is downward.***

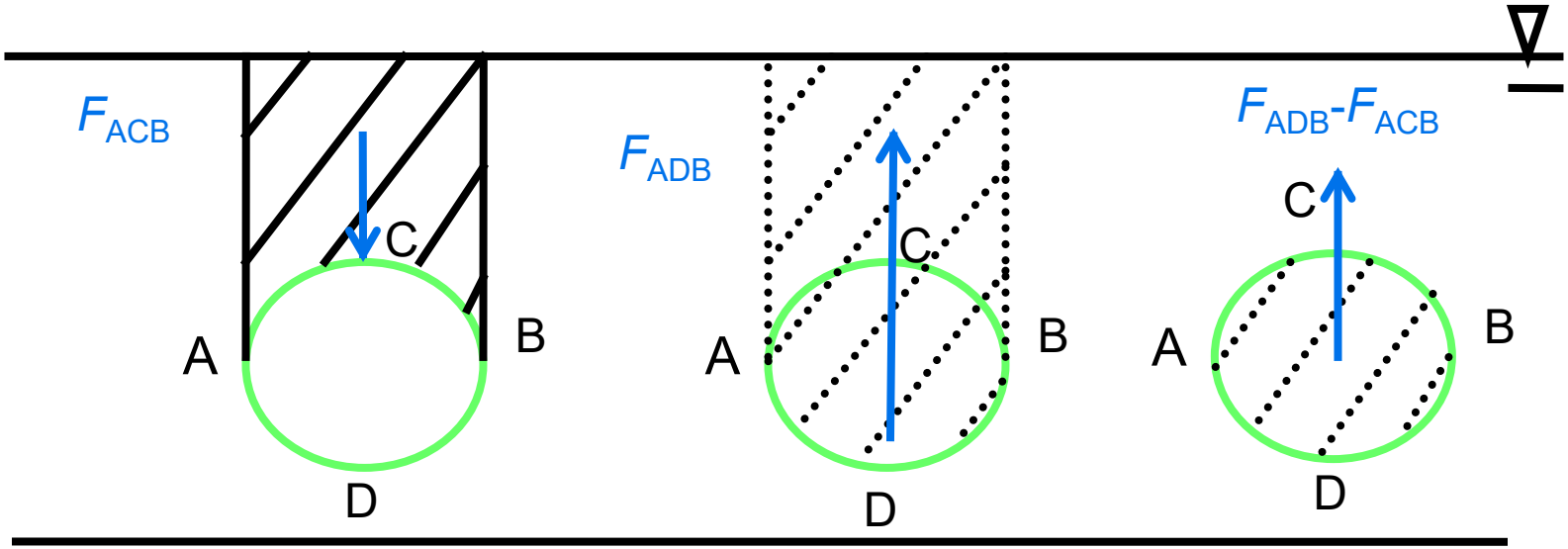
Archimedes's principle

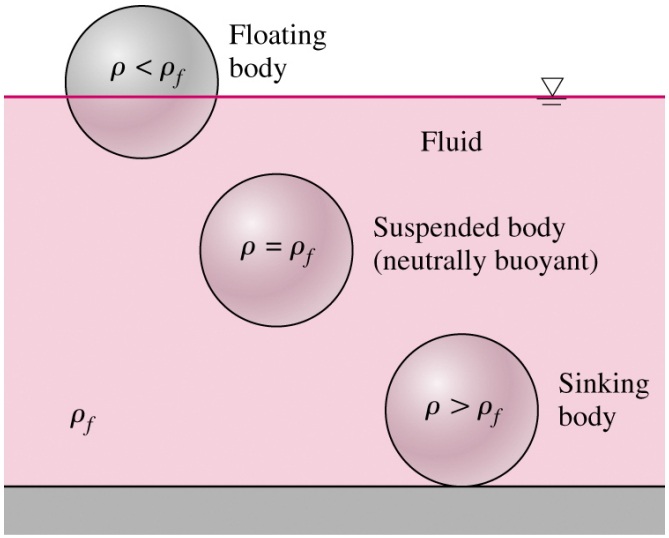
" An object is buoyed up by a force equal to the weight of the fluid displaced."



In the figure, we see that the difference between the weight in AIR and the weight in WATER is 3 lbs. This is the buoyant force that acts upward to cancel out part of the force. If you were to weight the water displaced it also would weigh 3 lbs.

Buoyant force

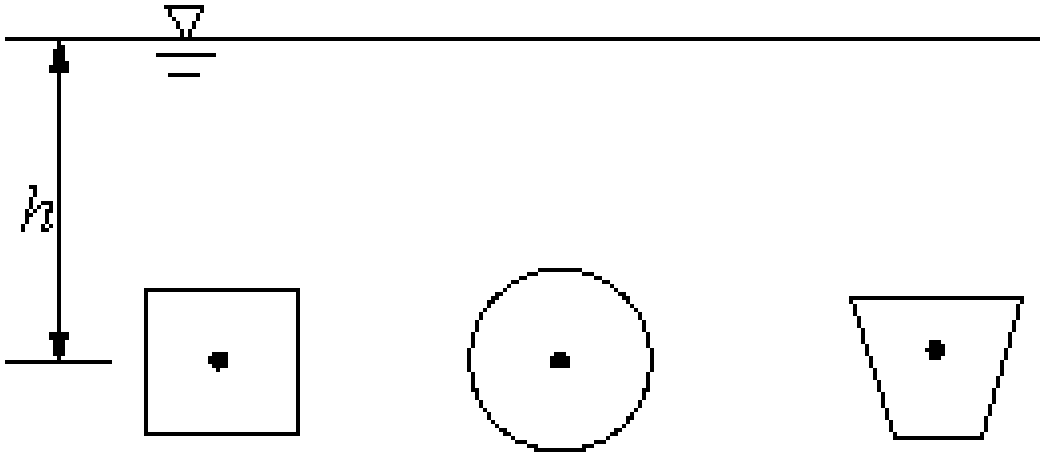




- A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its density relative to the density of the fluid.

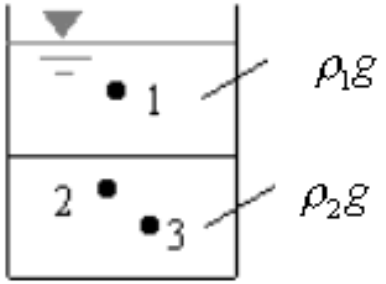
Example

- Three types of plate have the same area. Which has the maximum static pressure?



Example

- If $\rho_1 g \neq \rho_2 g$ which equation is correct?

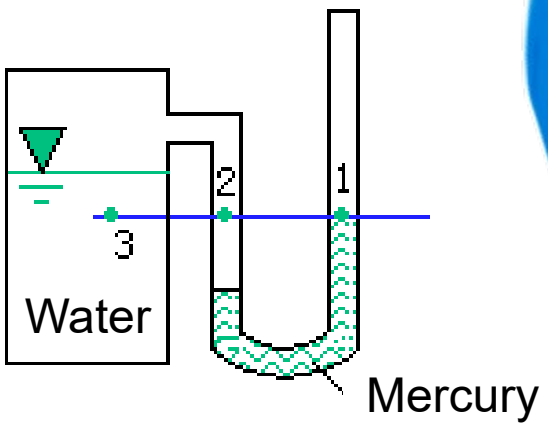


$$z_3 + \frac{p_3}{\rho g} = z_2 + \frac{p_2}{\rho g} \qquad z_1 + \frac{p_1}{\rho g} = z_2 + \frac{p_2}{\rho g}$$

- Point 1、2、3 at the same level, the following equation is correct?

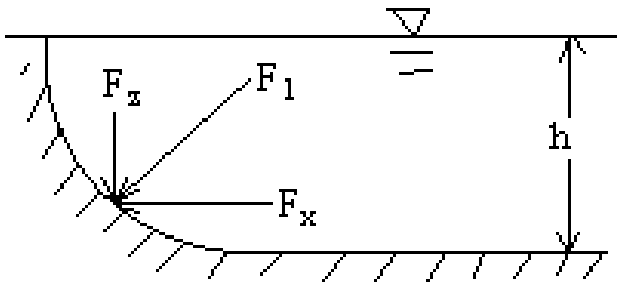
  A. $p_1=p_2=p_3$; B. $p_1>p_2>p_3$;

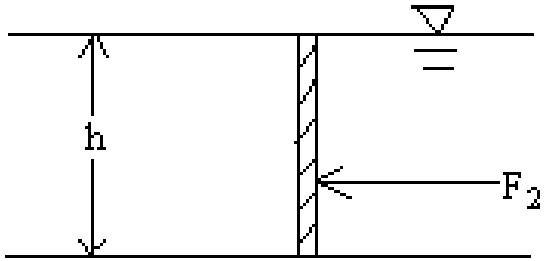
  C. $p_1<p_2<p_3$; D. $p_2<p_1<p_3$。



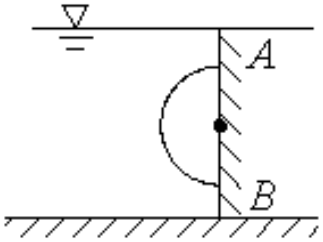
Example

- Which is right? $F_x > F_2$ $F_x = F_2$

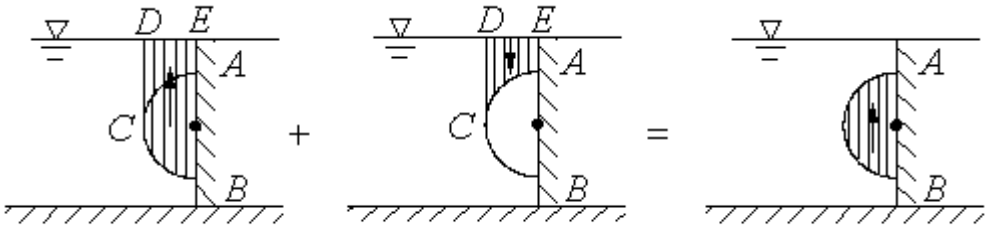




- Draw the volume

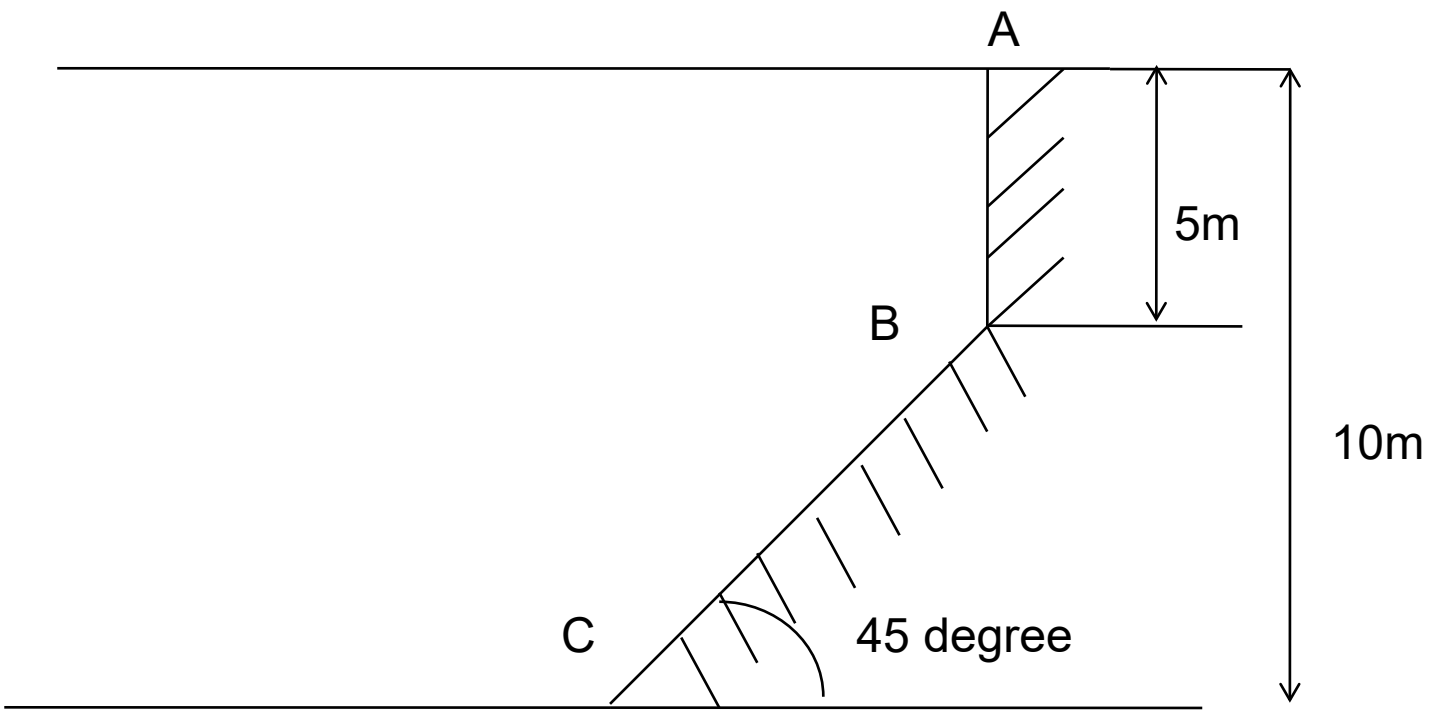


(a)



(b)

Example



Width is 1m