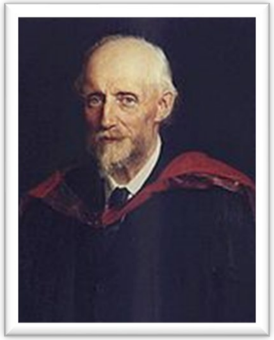
Chapter Six

Pipe Flow

Reynolds

1842-1912



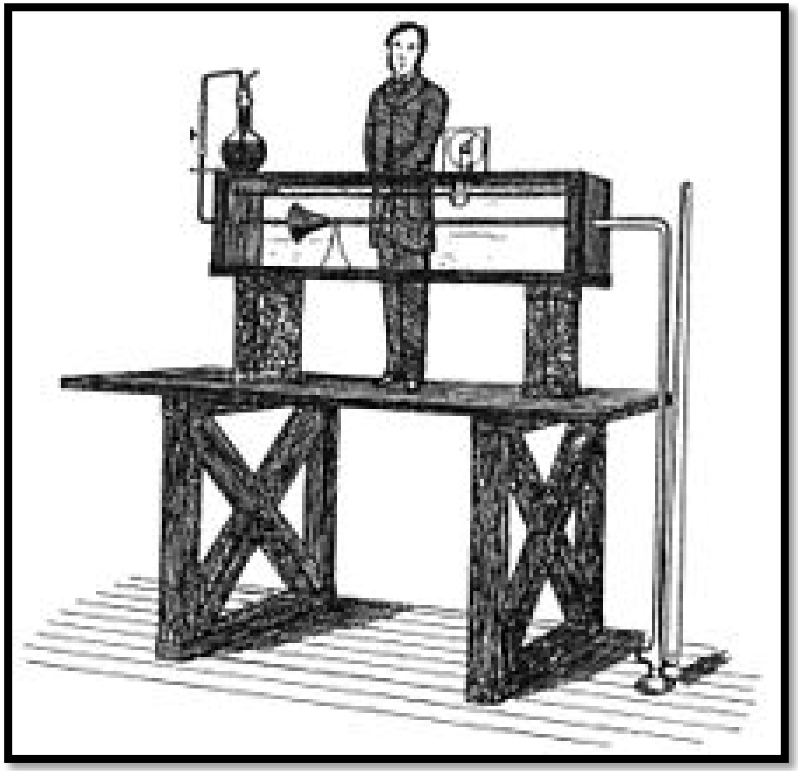
Objectives

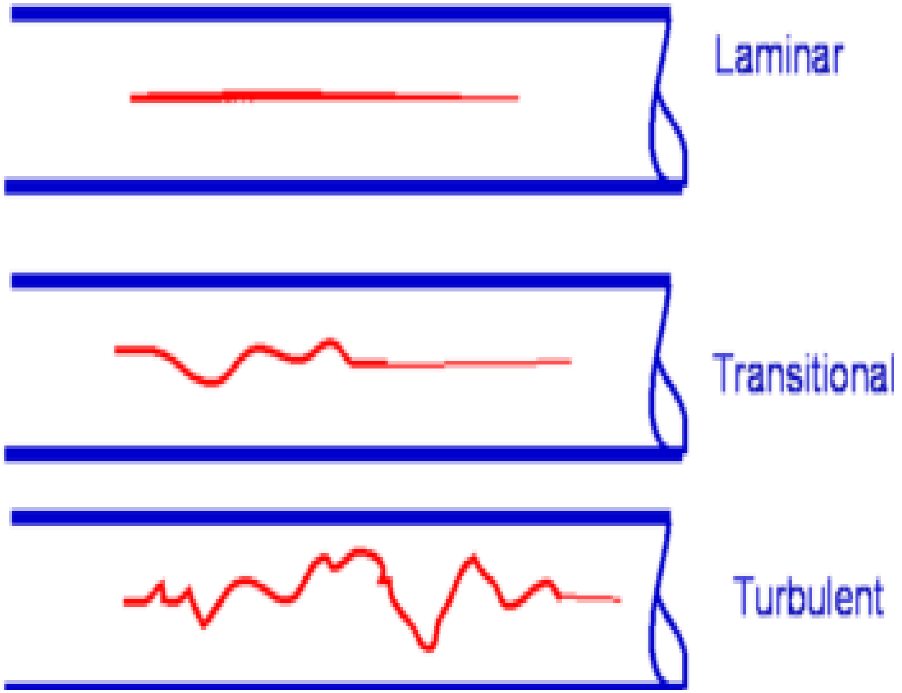
- Differentiate between laminar and turbulent flows in pipelines.
- Describe the velocity profile for laminar and turbulent flows.
- Compute Reynolds number for flow in pipes.
- Define the friction factor, and compute the friction losses in pipelines.
- Recognize the source of minor losses, and compute minor losses in pipelines.
- Analyze simple pipelines, pipelines in series, parallel, and simple pipe networks.

Laminar and turbulent flow

- Laminar flow
  - Known as streamline flow, occurs when a fluid flows in parallel layers, with no disruption between the layers
- Turbulent flow
  - A flow regime characterized by chaotic and stochastic property changes including low momentum diffusion, high momentum convection, and rapid variation of pressure and velocity in space and time
- The process of a laminar flow becoming turbulent is known as laminar-turbulent transition, which at present is not fully understood

Reynolds' experiment





When the water was slow, the filament remained distinct through the entire length of the tube. When the speed was increased, the filament broke up at a given point and diffused throughout the cross-section.

Critical Reynolds Number

- Laminar flow occurs when *Re*<2300 (Lower critical number)
- Turbulent flow occurs when Re>13800 (Upper critical number)
- Transition flows occurs When 2300 <Re<13800, laminar and turbulent flows are possible depending on other factors, such as pipe roughness and flow uniformity.
- Upper critical *Re* (Laminar flow -> Turbulent flow), sensitive to small disturbance
- Lower critical *Re* (Turbulent flow -> Laminar flow), used to delimit the flow state

$$Re = \frac{\rho v d_H}{\mu}, \qquad d_H = \frac{4A}{S}$$

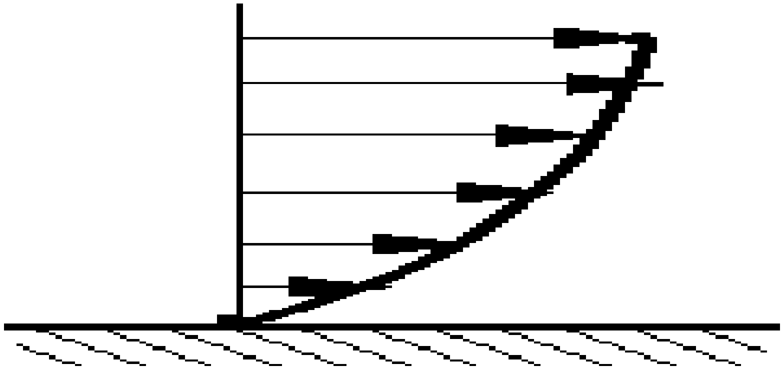
**d_H is hydraulic diameter, *A* i the cross sectional area and *S* is the wetted perimeter of the cross-section

Laminar flow in pipe

- Viscous shears dominate in laminar flow. The shear stress is governed by Newton's law of viscosity:

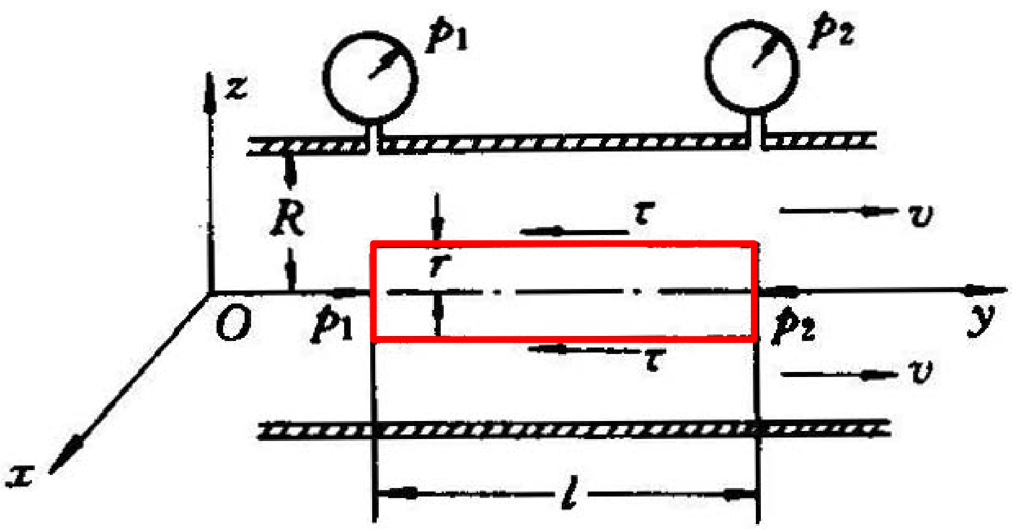
$$\tau = \mu \frac{dv}{dy}$$

- The effect of friction shows itself as a pressure loss.
- The shear stress at the wall retard the flow.



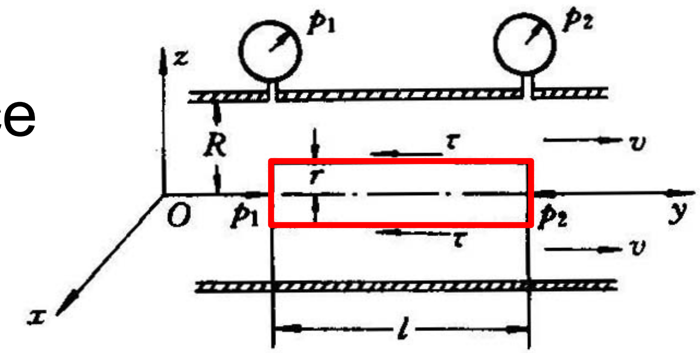
Laminar flow in pipe

- In laminar flow the flow particle flows along its layer, does not cross other layers, We suppose that flow in pipe is composed of concentric cylinder layers.
- Lets consider a cylinder of fluid with a length l, radius r, flowing steadily in the center of pipe.



Laminar flow in pipe

- The fluid is in equilibrium, shearing forces equal the pressure forces.
- Shearing force = Pressure force



$$\tau 2\pi r l = \Delta P A = \Delta P \pi r^2$$

$$\tau = \frac{\Delta P}{l}\frac{r}{2}$$

Taking: $$\tau = -\mu \frac{dv_r}{dr}$$

$$\frac{dv_r}{dr} = -\frac{p_1 - p_2}{2\mu l} r = -\frac{\Delta p}{2\mu l} r$$

ΔP = change in pressure

l = pipe length

R = pipe radius

r =distance from the pipe center

Laminar flow in pipe

- In an integral form this gives an expression for velocity, with the values of *r* = 0 (at the pipe center) to *r* = R (at the pipe wall)

$$v_r = -\frac{\Delta P}{l}\frac{1}{2\mu}\int_0^r r dr = -\frac{\Delta p}{4\mu l} r^2 + C$$

When *r* =*R* $v_y = -\frac{\Delta p}{4\mu l} R^2 + C = 0 \Rightarrow C = \frac{\Delta p}{4\mu l} R^2$

- The maximum velocity is at the center of the pipe, i.e. when *r* = 0.

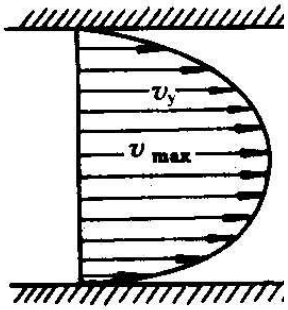
$$v_{r=0} = \frac{\Delta p}{4\mu l} R^2$$

- The minimum velocity is at the wall of the pipe, i.e. when *r* = R.

$$v_{r=R} = 0$$

Laminar flow in pipe

- The distribution of velocity



$$v_r = \frac{\Delta p}{4\mu l}(R^2 - r^2)$$

- The flow rate of the pipe

$$q_V = \int dq_V = \int_0^R \frac{\Delta p}{4\mu l}(R^2 - r^2)2\pi r dr = \frac{\pi R^4 \Delta p}{8\mu l} = \frac{\pi d^4 \Delta p}{128\mu l}$$

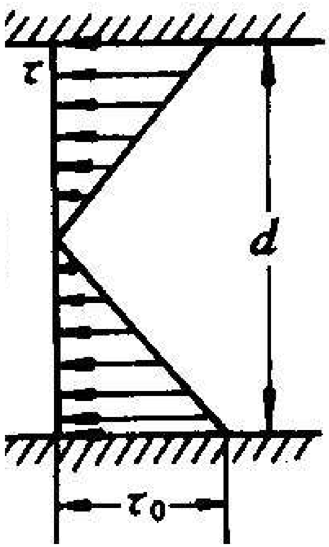
——Hagen–Poiseuille law

- The average velocity

$$v_{average} = \frac{\Delta p}{8\mu l}R^2 = \frac{1}{2}v_{\max}$$

Laminar flow in pipe

- The distribution of shear stress



$$\tau = -\mu \frac{dv_r}{dr} = \frac{\Delta p r}{2l}$$

- The maximum shear stress is at the wall

$$\tau = \frac{\Delta p R}{2l}$$

- The minimum shear stress is at the center

$$\tau = 0$$

Laminar flow in pipe

- Pressure loss

$$\Delta p = \frac{128\mu l q_V}{\pi d^4} = \frac{32\mu l v_{average}}{d^2}$$

- Hydraulic head loss

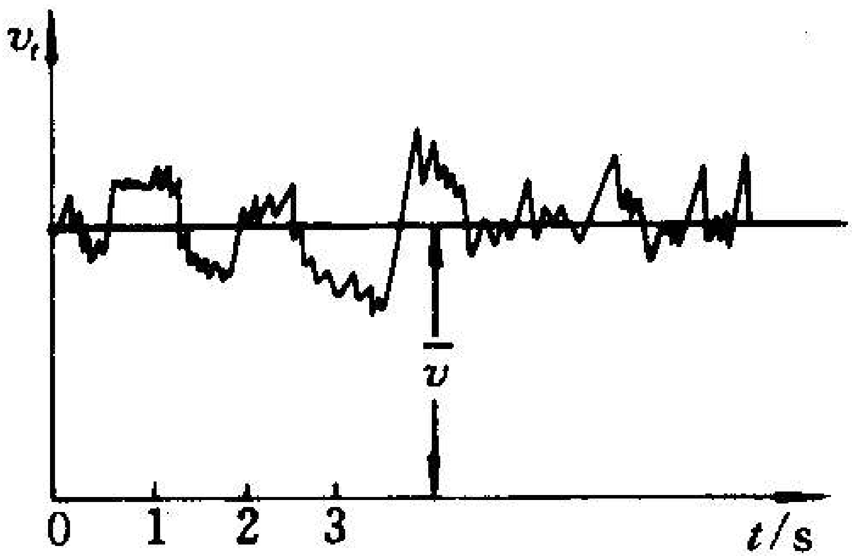
$$h_f = \frac{\Delta p}{\rho g} = \frac{32\mu l v_{average}}{\rho g d^2}$$

- Power loss

$$P = \Delta p q_V = \frac{128\mu l q_V^2}{\pi d^4}$$

Turbulent flow in pipe

- For a homogeneous fluid and an incompressible flow, the flow velocities are split into a mean part and a fluctuating part using Reynolds decomposition:



$$v = \bar{v} + v'$$

$$\bar{v} = \frac{1}{T}\int_0^T v_t dt$$

$$\frac{1}{T}\int_0^T v' dt = 0$$

v is the flow velocity, $\bar{v}$ is the mean velocity, *v'* is the fluctuating (turbulence) part of the velocity

Turbulent flow in pipe

- These velocity fluctuations *v'* cause additional shear stresses
- The total shear stresses include both viscous and turbulent shear stresses

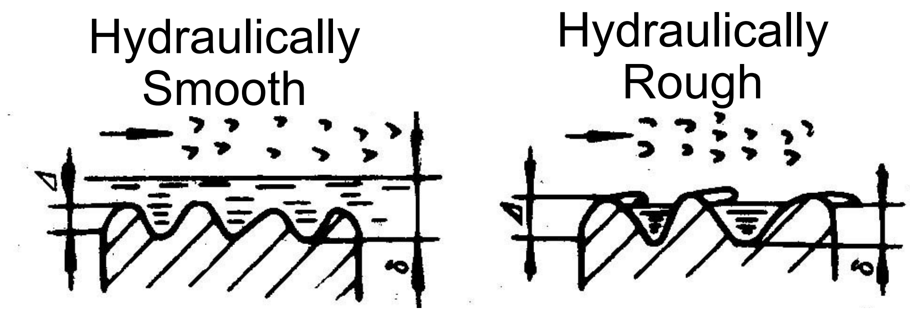
$$\tau = \mu \frac{d\bar{v}}{dy} - \rho \overline{v'_x v'_y}$$

******Viscous shear stress is caused by random motions of fluid molecules, however turbulent shear stress is induced by the fluctuating velocity of fluid particle***

Turbulent flow in pipe

- In turbulent flow, a thin layer near the surface is found to be laminar. The thickness δ of the layer is defined as

$$\delta \approx 30 \frac{d}{Re\sqrt{\lambda}}$$

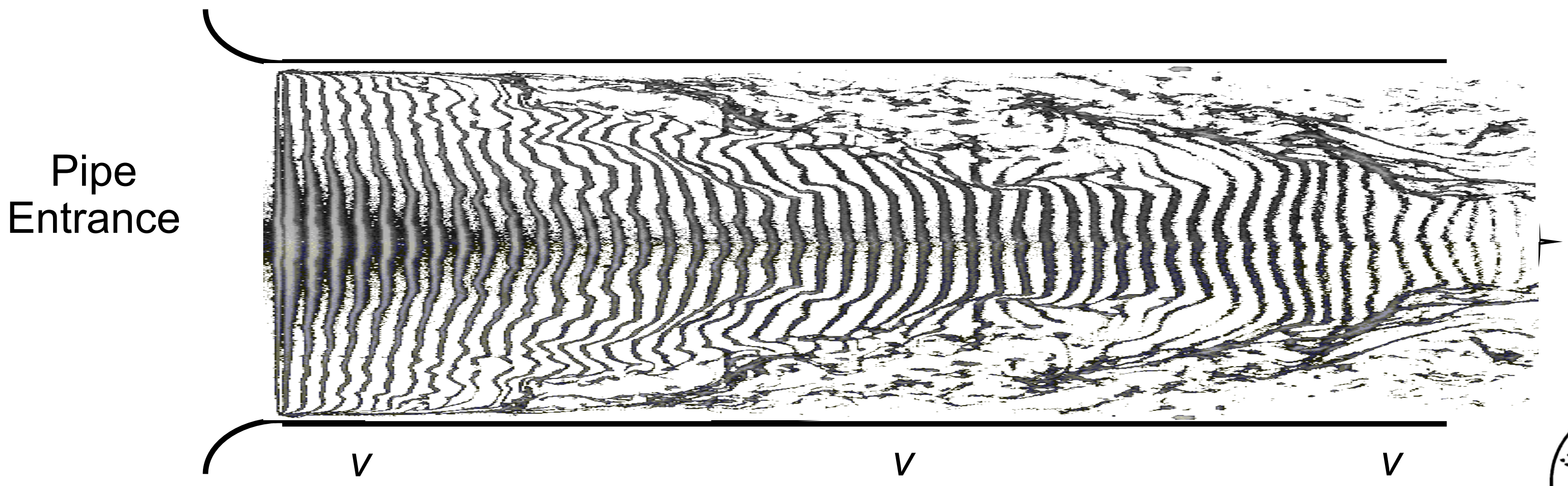


- If roughness height $\Delta < \delta$ the pipe is considered as hydraulically smooth, otherwise as hydraulically rough

******Hydraulically smooth or hydraulically rough pipe is not only determined by its geometric parameters but also by flow parameters***

Boundary layer buildup in a pipe

- Because of the shear force near the pipe wall, a boundary layer forms on the inside surface and occupies a large portion of the flow area as the distance downstream from the pipe entrance increase. At some value of this distance the boundary layer fills the flow area. The velocity profile becomes independent of the axis in the direction of flow, and the flow is said to be fully developed.



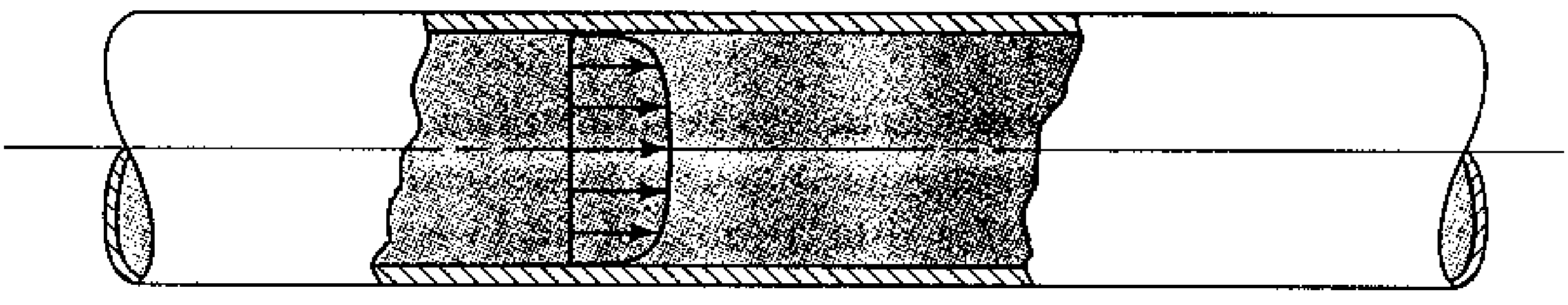
Velocity distribution of turbulent flow

- When Re<1.1 × 10^5, the velocity at any point in the cross-section will be proportional to the one-seventh power of the distance from the wall, which can be expressed as:

$$\frac{v_y}{v_{\max}} = \left(\frac{y}{R}\right)^{1/7}$$

***y* -- the distance from the wall**

***v_{max}* -- the maximum velocity in pipe (at the pipe center)**



Major losses in pipe flow

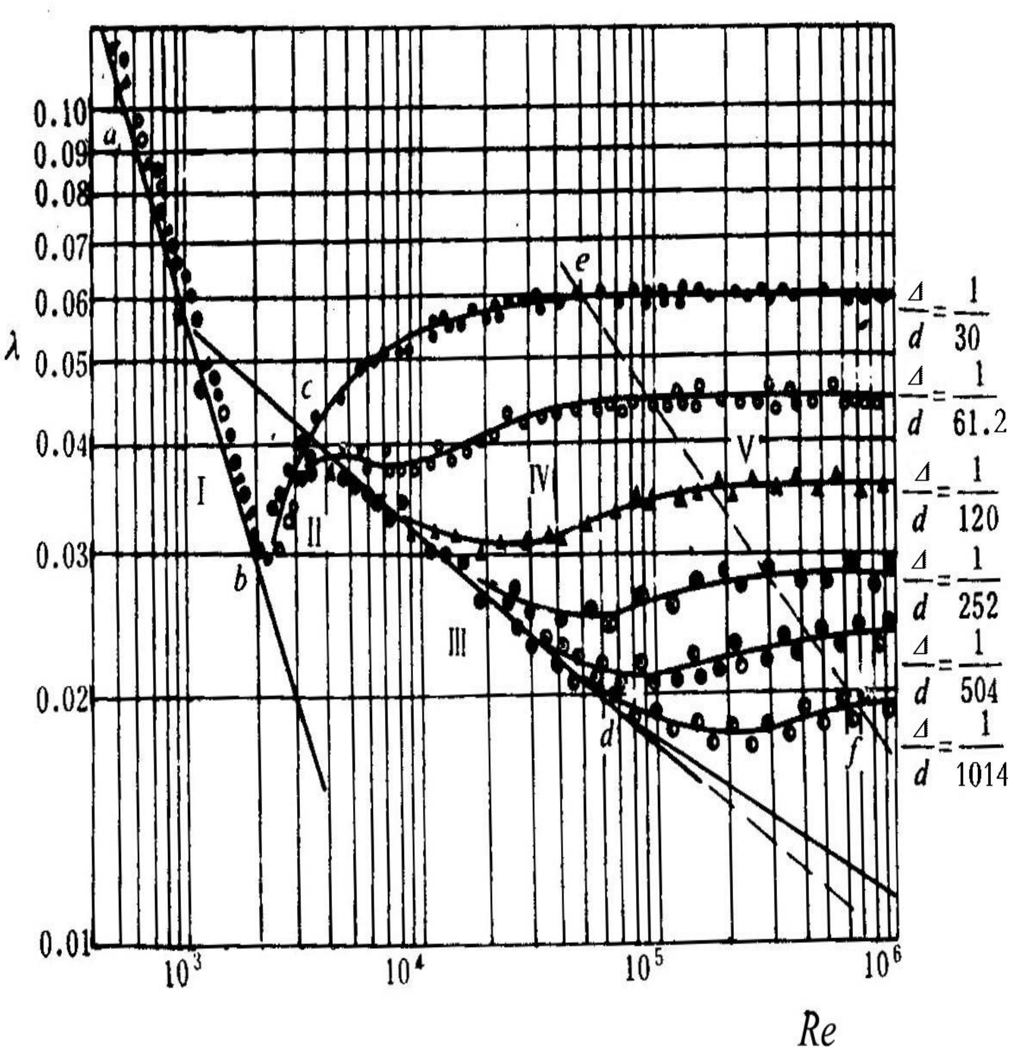
- Due to friction, significant head loss is associated with the straight portions of pipe flow, which can be calculated by considering the pressure losses along the pipelines.

$$h_f = \lambda \frac{l}{d} \frac{V^2}{2g}$$

- d pipe diameter
- l pipe length
- V flow velocity (mean velocity)
- λ is the friction coefficient

- λ is the friction coefficient. It should be noted that λ is dimensionless, and the value is not constant, and dependant on flow parameters and geometric parameters

Nikuradse's Experiments



I. Laminar flow

II. Transitional flow : $\lambda = f(Re)$

III. Smooth turbulence : $\lambda = f(Re)$

IV. Transitional turbulence

V. Rough turbulence : λ is constant for a given roughness and independent on Re

Example

Oil (ρ = 900 kg/m3, μ=46 x10^{-3} N.s/m^2) flows with a mean velocity of 5 m/s in a 10-mm diameter 1-m length pipe. Determine whether the flow is laminar or turbulent, pressure loss, and friction coefficient.

Solution:

$$Re = \frac{\rho V D}{\mu} \qquad \Delta p = \frac{128 \mu l q_V}{\pi d^4} = \frac{32 \mu l v_{average}}{d^2}$$

$$Re = \frac{900 \times 5 \times 0.01}{46 \times 10^{-3}} = 978 < 2300$$

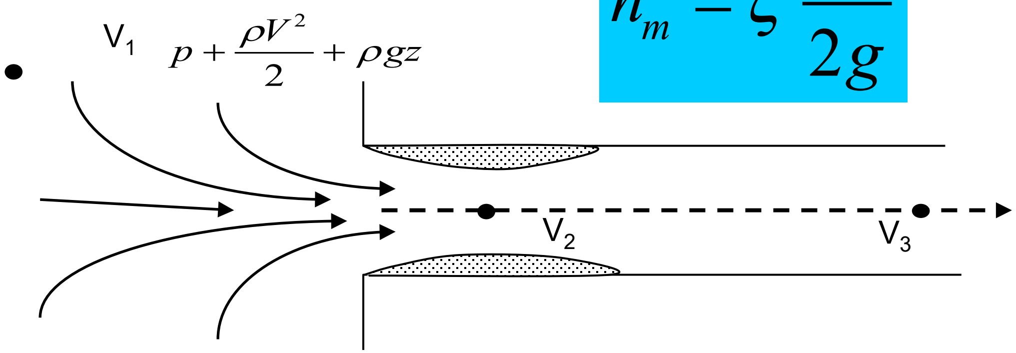
$$\Delta p = \frac{32 \times 46 \times 10^{-3} \times 1 \times 5}{0.01^2} = 7.36 \times 10^4$$

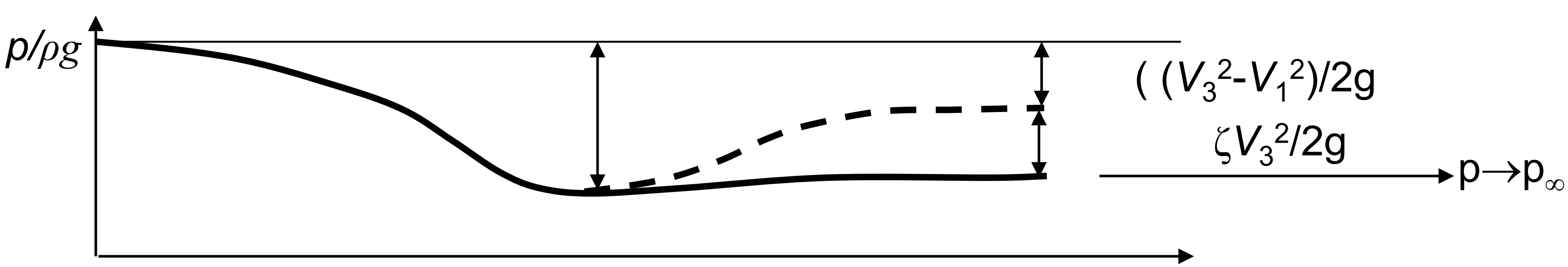
$$\lambda = \frac{64}{Re} = 0.0654$$

Minor losses in pipe flow

- Additional components (valves, bends, tees, contractions, etc) in pipe flows also contribute to the total head loss of the system. Their contributions are generally termed minor losses.
- The head losses can be characterized by using the loss coefficient ζ

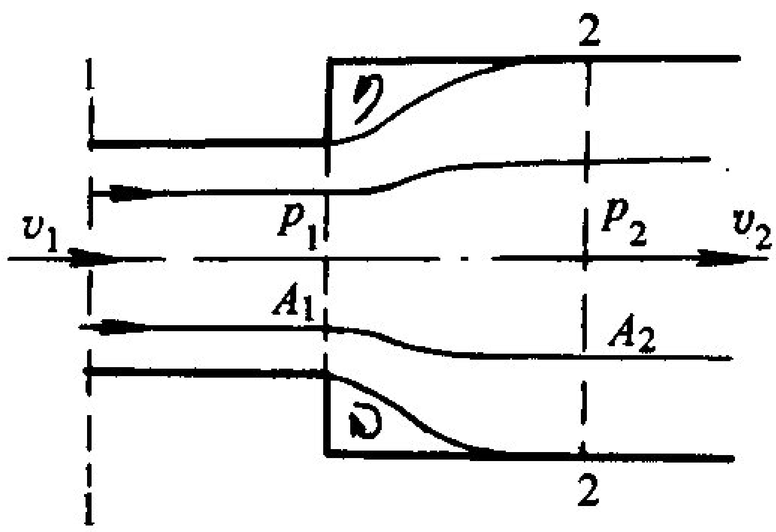
$$h_m = \zeta \frac{v^2}{2g}$$





Minor losses in pipe flow

- Sudden Enlargement
  - As fluid flows from a smaller pipe into a larger pipe through sudden enlargement, causing turbulence that generates an energy loss.
  - The minor loss (h_m)is calculated as:



$$h_m = \frac{(v_1 - v_2)^2}{2g} = \zeta \frac{v_1^2}{2g}$$

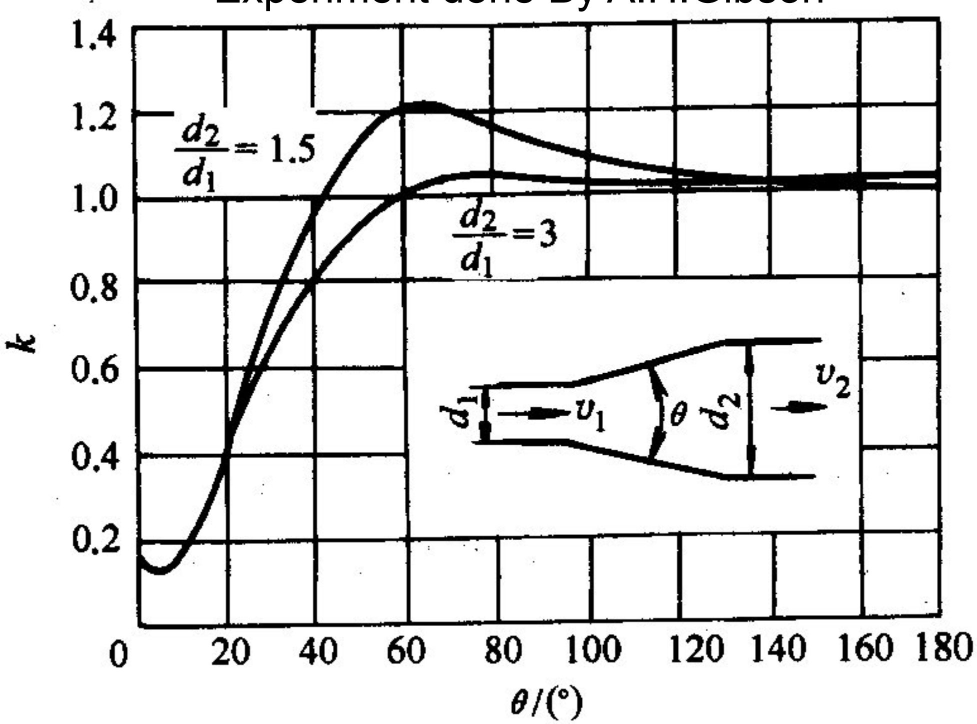
$$\zeta = (1 - \frac{A_1}{A_2})^2$$

D_1/D_2	0.0	0.2	0.4	0.6	0.8
ζ	1.00	0.92	0.70	0.41	0.13

Minor losses in pipe flow

- Gradual Expansion
  - As fluid flows from a smaller pipe into a gradually expanding pipe, generating an energy loss also.
  - The minor loss (h_m)is calculated as:

Experiment done By A.H.Gibson

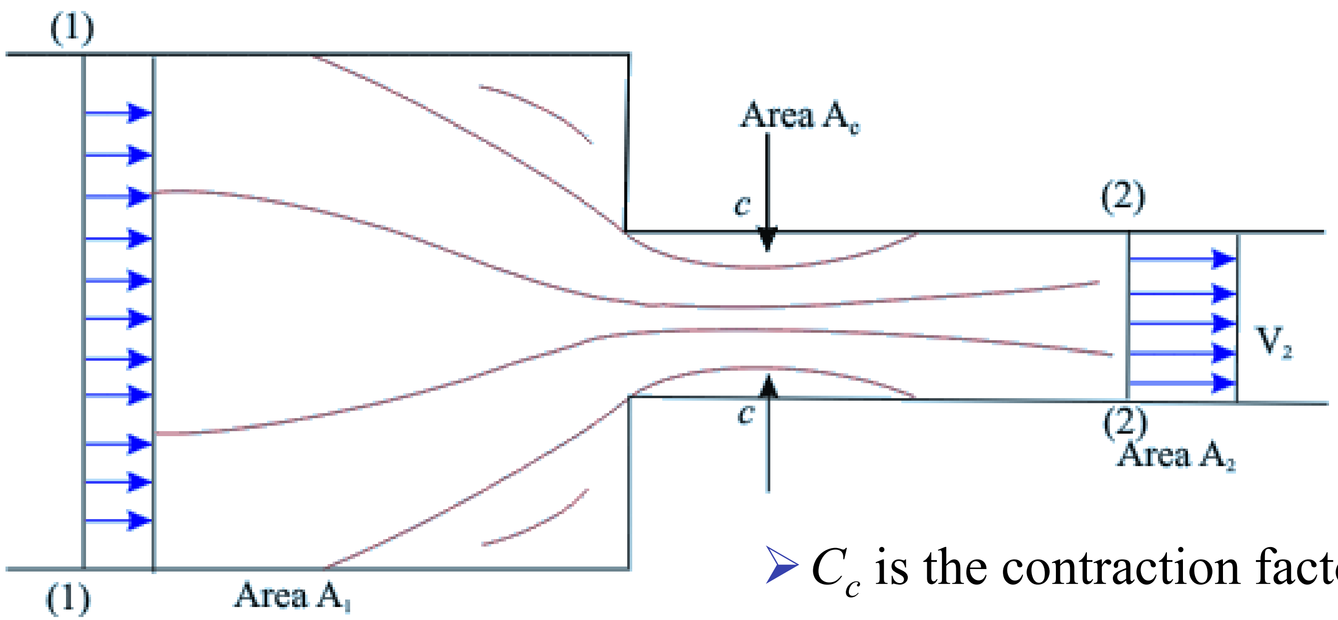


$$h_m = \zeta \frac{(v_1 - v_2)^2}{2g}$$

- The loss coefficient ζ is minimum with the angle θ within 5~7 degree
- At the peak with θ within 55~80 degree

Minor losses in pipe flow

- Sudden Contraction
  - As fluid flows from a larger pipe into a smaller pipe through sudden contraction, the minor loss (h_m)is calculated as:



$$h_m = \zeta \frac{v_2^2}{2g}$$

$$\zeta = \left(\frac{1}{C_c} - 1\right)^2$$

- C_c is the contraction factor=A_c/A_2

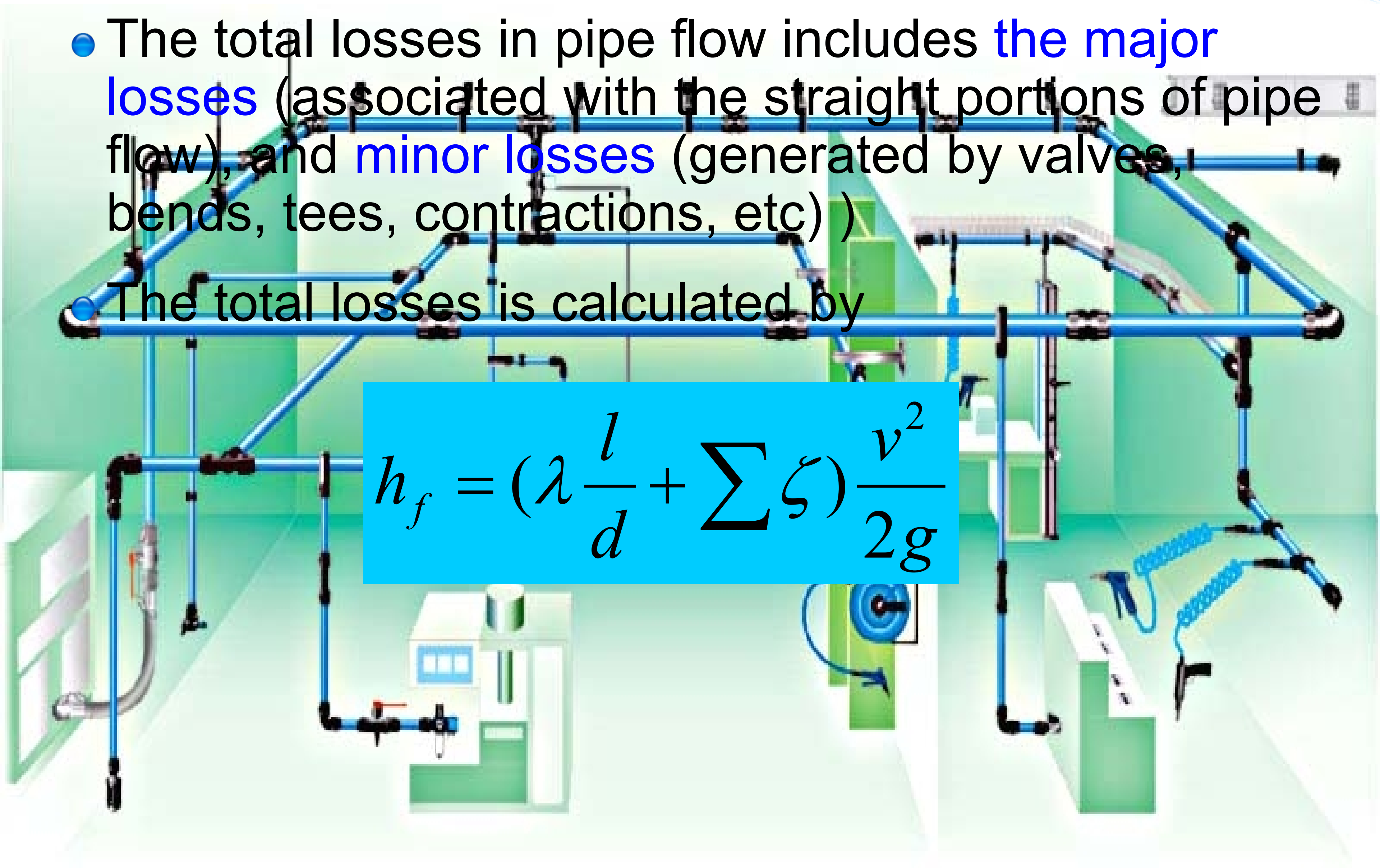
A_2/A_1	0	0.04	0.16	0.36	0.64	1.0
ζ	0.5	0.45	0.38	0.28	0.14	0

Total losses in pipe flow

- The total losses in pipe flow includes the major losses (associated with the straight portions of pipe flow), and minor losses (generated by valves, bends, tees, contractions, etc))

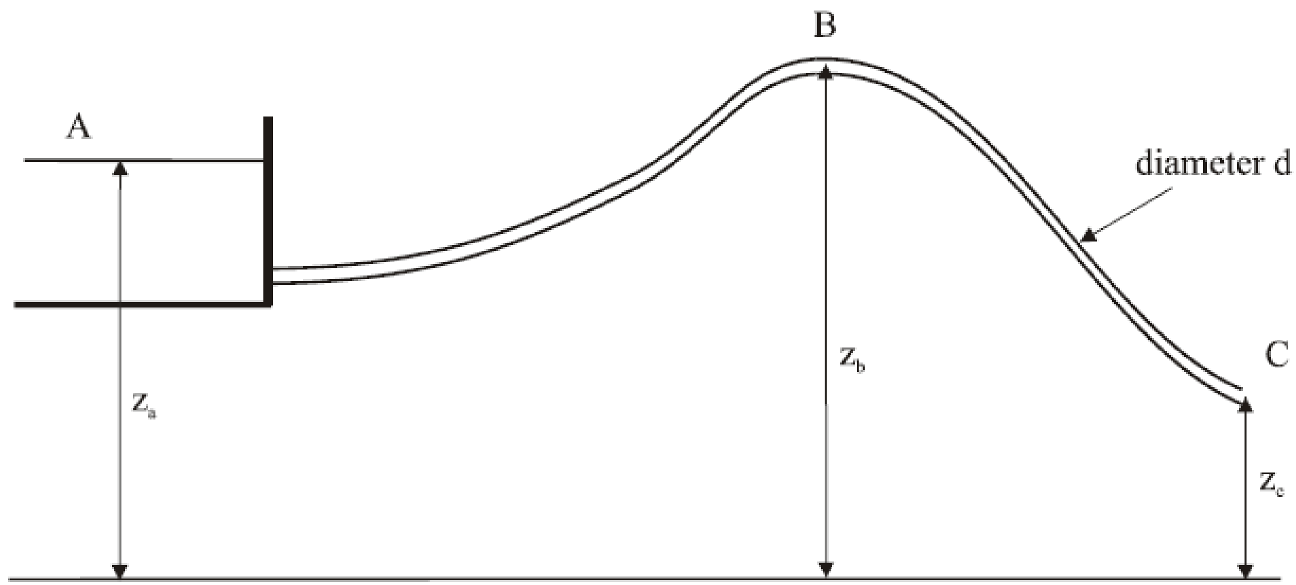
- The total losses is calculated by

$$h_f = (\lambda \frac{l}{d} + \sum \zeta) \frac{v^2}{2g}$$



Example

The pipe diameter is 100mm and has length 15m and feeds directly into the atmosphere at point C 4m below the surface of the reservoir (i.e. $z_a - z_c = 4.0$m). The highest point on the pipe is a B which is 1.5m above the surface of the reservoir (i.e. $z_b - z_a = 1.5$m) and 5 m along the pipe measured from the reservoir. Assume the entrance and exit to the pipe to be sharp and the value of friction factor f to be 0.08. Calculate a) velocity of water leaving the pipe at point C, b) pressure in the pipe at point B.



Example

For a sharp entry, ζ=0.5

$$h_L = 0.5\frac{u^2}{2g}$$

$$h_f = \frac{4fLu^2}{2gd}$$

$$z_A = \frac{u^2}{2g} + z_C + \frac{4fLu^2}{2gd} + 0.5\frac{u^2}{2g}$$

$$z_A - z_C = \frac{u^2}{2g}\left(1 + 0.5 + \frac{4fL}{d}\right)$$

****we should calculate Re to be sure of the flow state*

$$u = 1.26m/s$$

To find the pressure at B apply Bernoulli from point A to B using the velocity calculated above.

$$z_A = \frac{p_B}{\rho g} + \frac{u^2}{2g} + z_B + \frac{4fL_1u^2}{2gd} + 0.5\frac{u^2}{2g}$$

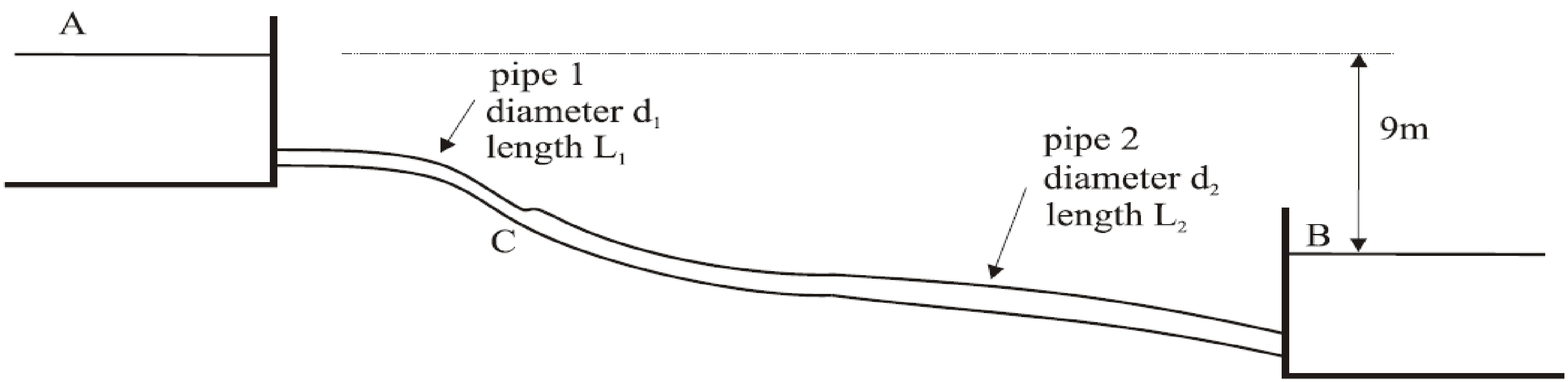
$$z_A - z_B = \frac{p_B}{\rho g} + \frac{u^2}{2g}\left(1 + 0.5 + \frac{4fL_1}{d}\right)$$

$$-1.5 = \frac{p_B}{1000 \times 9.81} + \frac{1.26^2}{2 \times 9.81}\left(1.5 + \frac{4 \times 0.08 \times 5.0}{0.1}\right)$$

$$p_B = -28.58 \times 10^3 \ N/m^2$$

Pipes in Series

- When two or more pipes of different diameters or roughness are connected in series
- The flow follows a single flow path, and have the same rate $$q_{V1} = q_{V2} = \cdots = q_V$$
- In a series pipeline the total energy loss is the sum of the individual minor losses and all pipe friction losses $$H = \sum h_{li} = h_{l1} + h_{l2} + \cdots$$



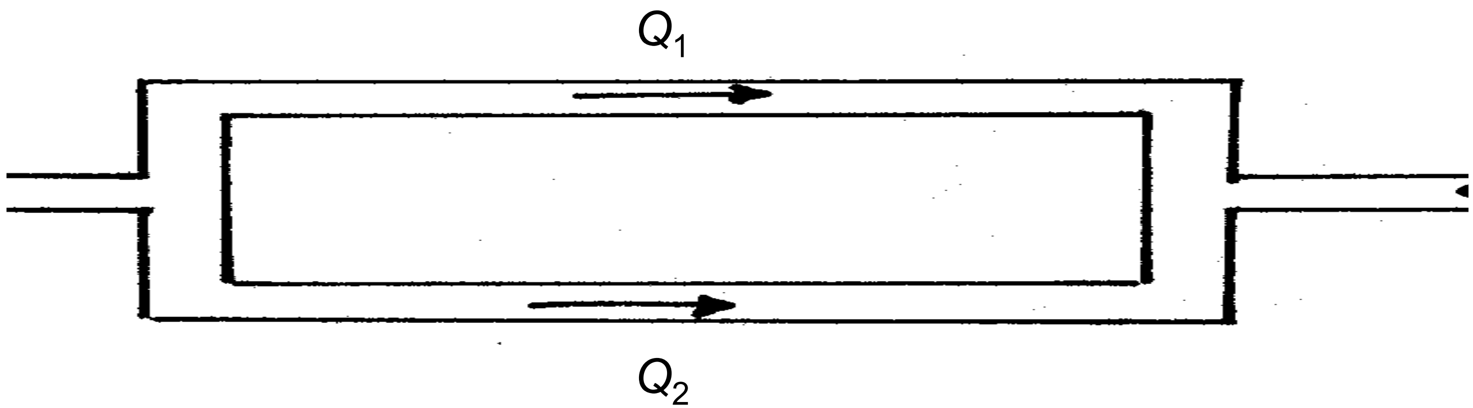
Pipes in Parallel

- A combination of two or more pipes connected between two points as in parallels
- The discharge divides at the first junction and rejoins at the next is known

$$q_V = q_{V1} + q_{V2} + \ldots$$

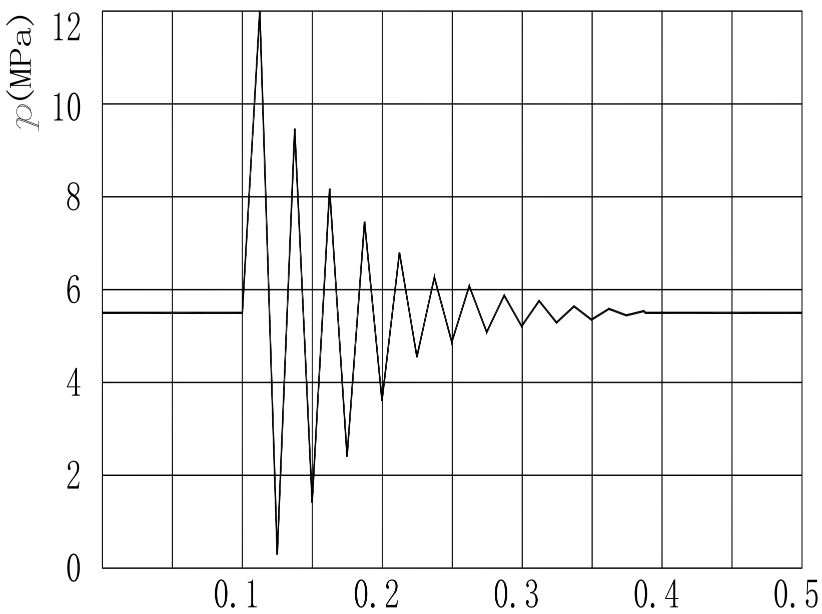
- The head loss between the two junctions is the same for all pipes

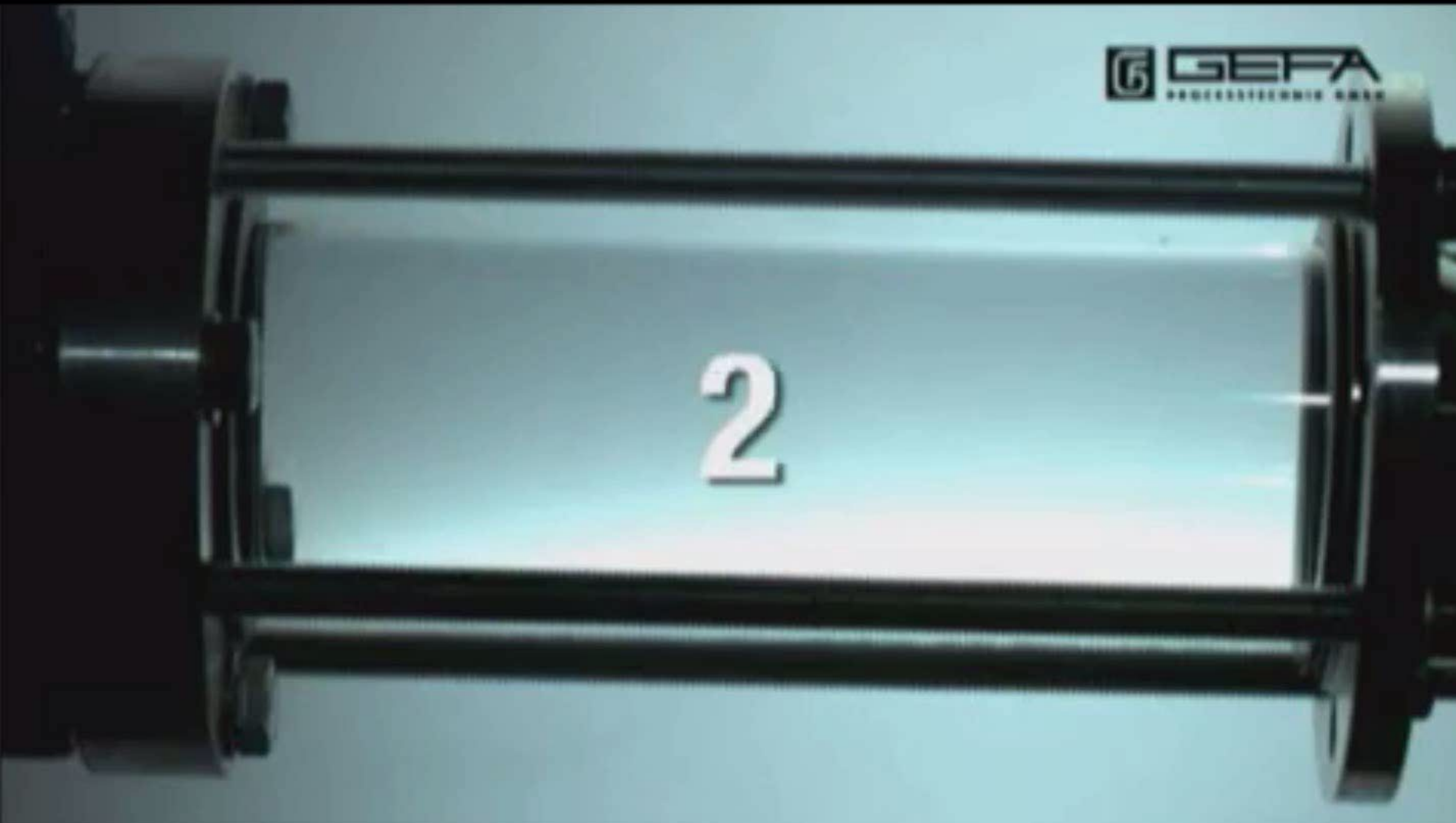
$$h_l = h_{l1} = h_{l2} = \ldots$$



Hydraulic shock

- Hydraulic shock
  - used to describe the momentary pressure rise in a piping system which results when the liquid is started or stopped quickly
- Pressure rise is caused by the momentum of the fluid
  - Rise with the increase of velocity of the liquid
  - Rise with the length of the system from the fluid source
  - Rise with an increase in the speed with which it is started or stopped

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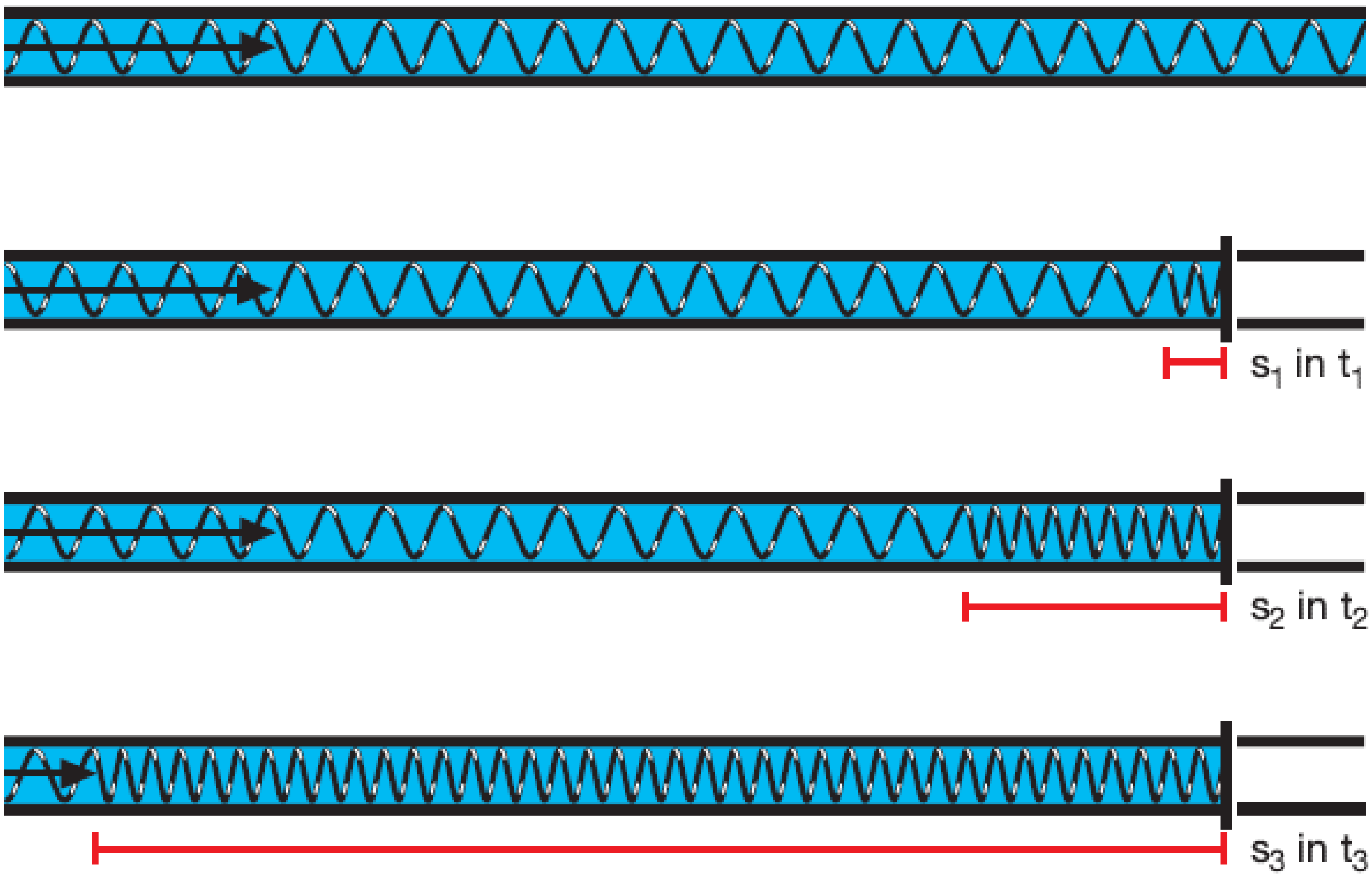
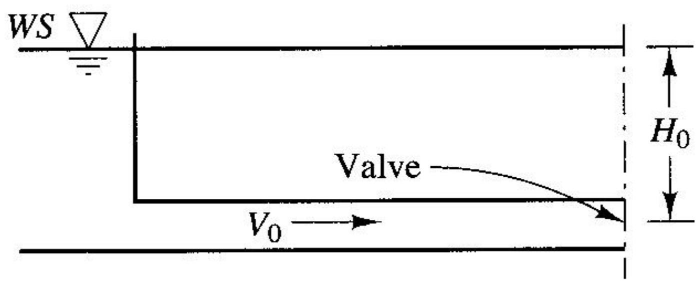
Hydraulic shock



Sudden closure of gate valve, visualized by a heavy steel spring

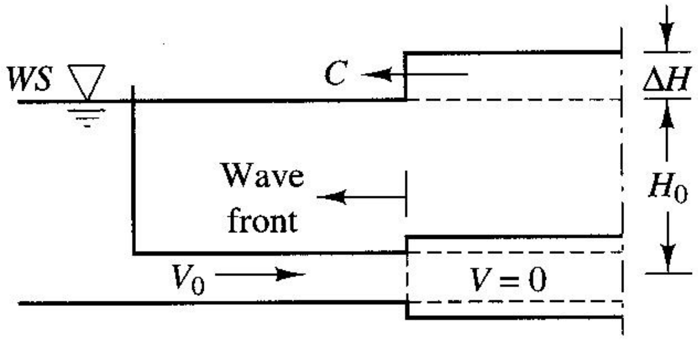
Propagation of pressure wave

Consider a pipe length L with inside diameter D, wall thickness e, and the modulus of elasticity E_p.



(a)

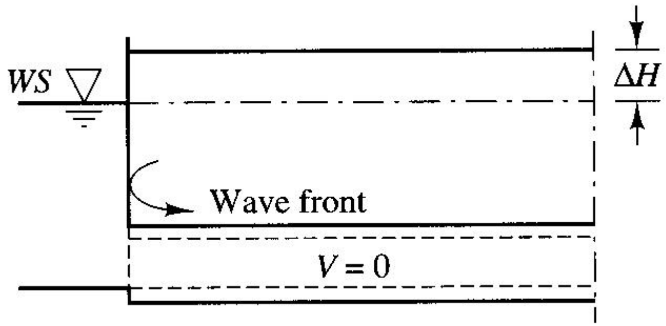
Steady state condition



(b)

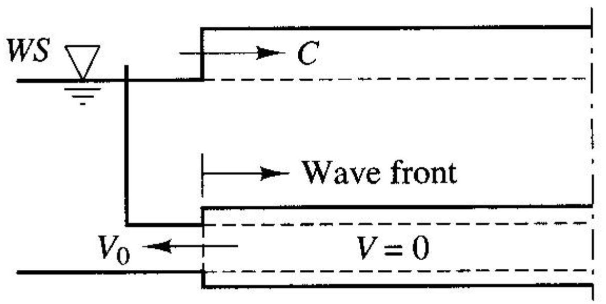
Transient condition $t < L/C$

Propagation of pressure wave



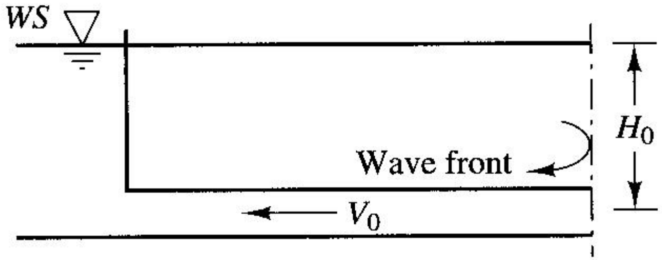
(c)

Transient condition t = L/C



(d)

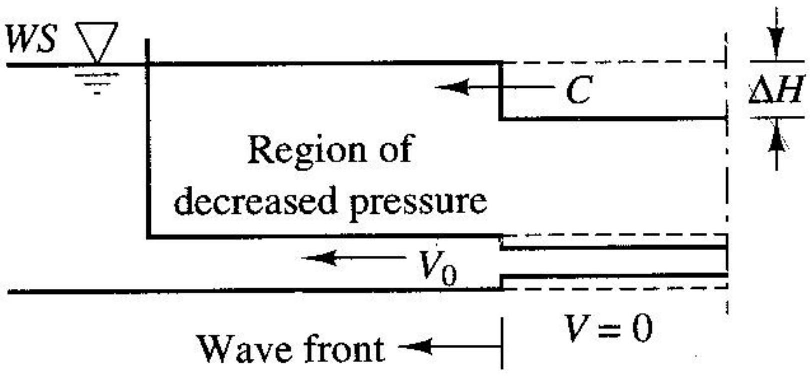
Transient condition L/C >t >2L/C



(e)

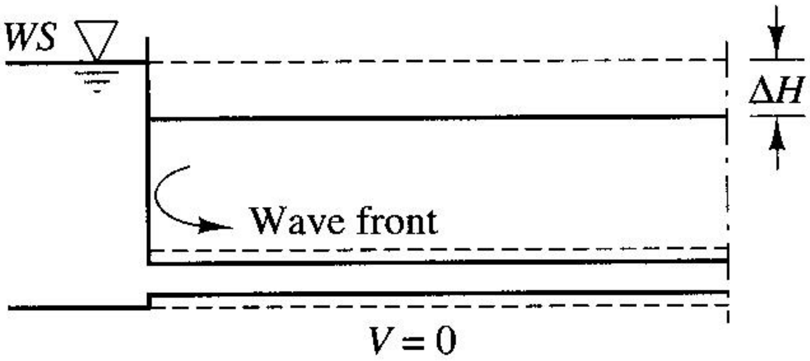
Transient condition t =2L/C

Propagation of pressure wave



(f)

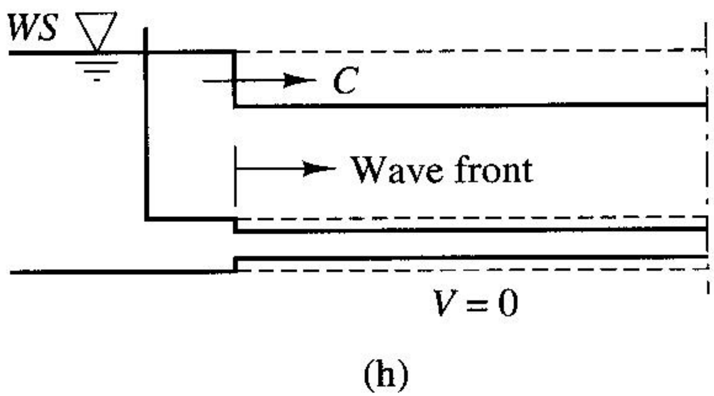
Transient condition 2L/C >t >3L/C



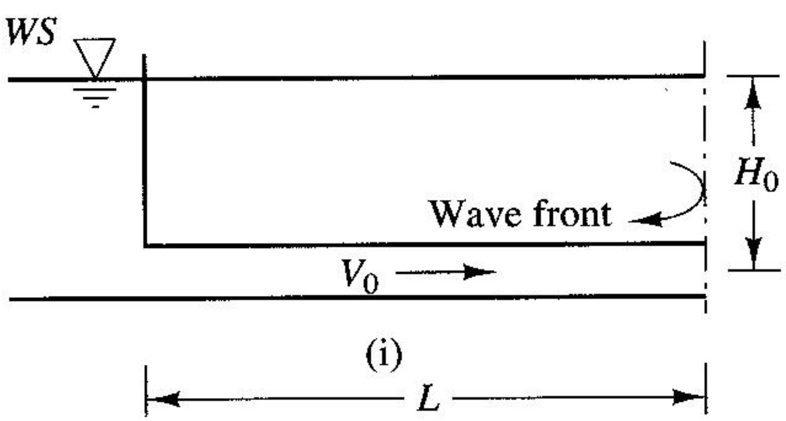
(g)

Transient condition t = 3L/C

Propagation of pressure wave



Transient condition 3L/C >t >4L/C



Transient condition t = 4L/C

Case 1: Gradual closure of valve

- If the time of closure $\boxed{t_c > \frac{2L}{C}}$, then the closure is said to be gradual and the increased pressure is

$$\boxed{\Delta P = \frac{\rho L V_0}{t}}$$

where,

- V_0 = initial velocity of water flowing in the pipe before pipe closure
- t = time of closure.
- L = length of pipe.
- ρ = water density.

- The pressure head caused by the water hammer is

$$\boxed{\Delta H = \frac{\Delta P}{\gamma} = \frac{\rho L V_0}{\rho g t} = \frac{L V_0}{g t}}$$

Case 2: Sudden closure and pipe rigid

- If the time of closure $t_c \le \frac{2L}{C}$, then the closure is said to be Sudden.
- The pressure head due caused by the water hammer is

$$\Delta P = \rho C V_0 \qquad \Delta H = \frac{C V_0}{g}$$

$$C = \sqrt{\frac{E_b}{\rho}} \qquad \Delta H = \frac{V_0}{g}\sqrt{\frac{E_b}{\rho}}$$

where E_b is the elasticity of oil

- But for rigid pipe so

$$\Delta P = V_0 \sqrt{E_b \rho}$$

Case 3: Sudden closure and pipe elastic

- If the time of closure $\boxed{t_c \le \frac{2L}{C}}$, then the closure is said to be Sudden.
- The pressure head caused by the water hammer is $\boxed{\Delta H = \frac{C V_0}{g}}$

$$\Delta P = \rho C V_0$$

- But for elastic pipe $C = \sqrt{\frac{E_c}{\rho}}$ so: $\boxed{\Delta H = \frac{V_0}{g}\sqrt{\frac{1}{\rho(\frac{1}{E_b} + \frac{DK}{E_p e})}}}$

where E_c is the composed elasticity of elastic wall and fluid within

$$\boxed{\Delta P = V_0 \sqrt{\frac{\rho}{(\frac{1}{E_b} + \frac{DK}{E_p e})}}}$$

Example

- Calculate the force F required on the piston to discharge 500 mm^3/s of water through a syringe (see Fig. 37.4), taking into account the frictional loss in the syringe needle only. Assume fully developed laminar flow in the syringe needle. Take the dynamic viscosity of water 10^{-3} Ns/m^2.