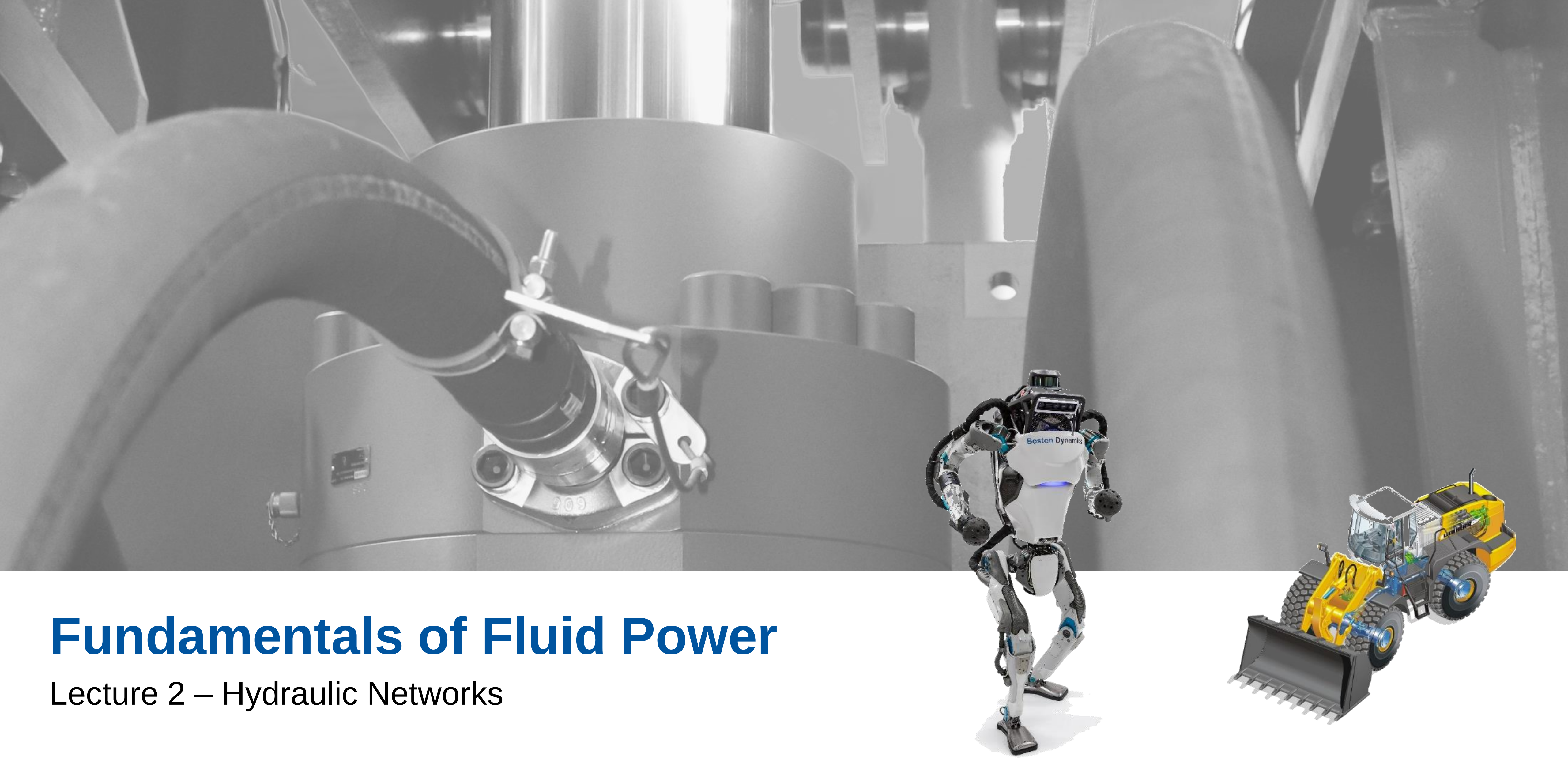
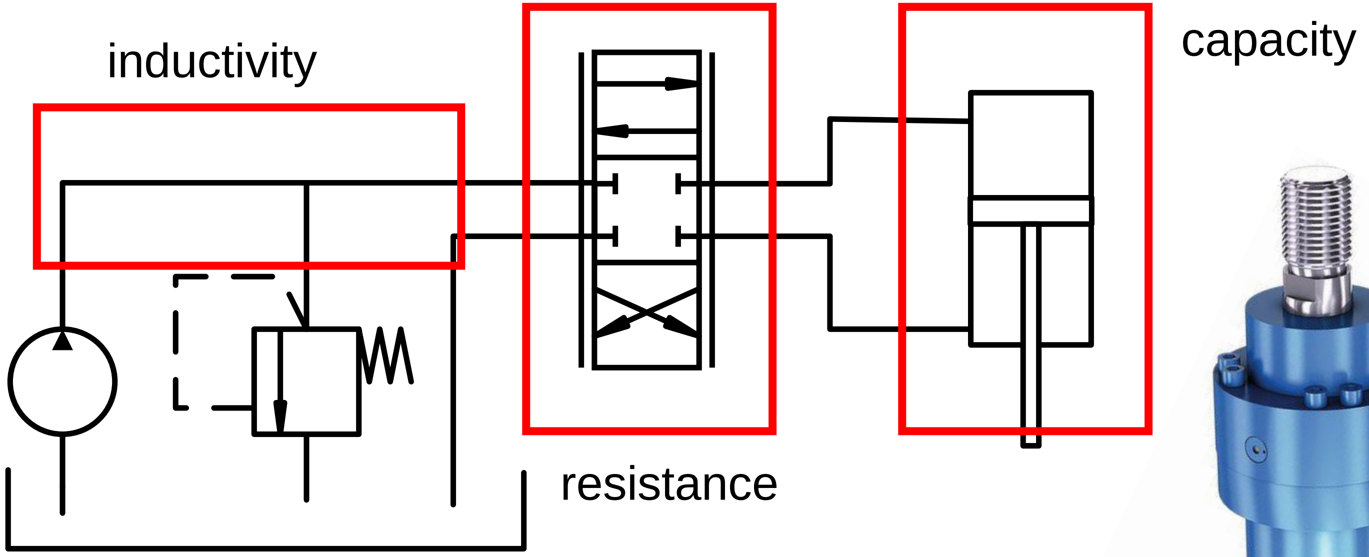


Fundamentals of Fluid Power

Lecture 2 – Hydraulic Networks

ifas Institute for Fluid Power Drives and Systems
RWTH AACHEN UNIVERSITY

Design of a hydraulic system



Source: Bosch Rexroth

Today:

Calculation of hydraulic networks

Outline of todays lecture

1 Components in a hydraulic network

1.1 Hydraulic resistance

1.2 Hydraulic capacity

1.3 Hydraulic inductivity

2 Calculation of a hydraulic network

3 Liquid collumn as homogeneouse resonator

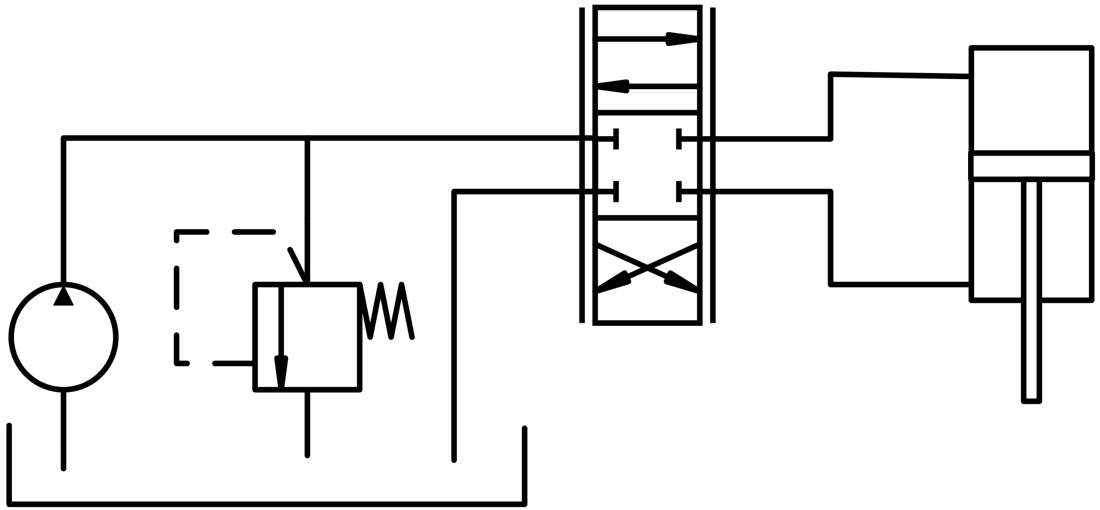
4 Summary

Hydraulic networks

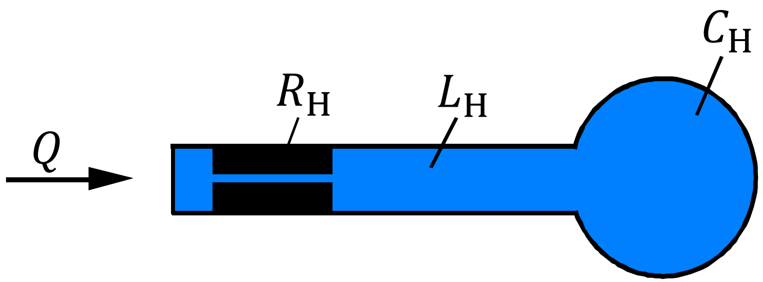
- resistance: $$R_{\mathrm{H}} = \frac{\Delta p}{Q}$$

- capacity: $$C_{\mathrm{H}} = \frac{Q}{\dot{p}}$$

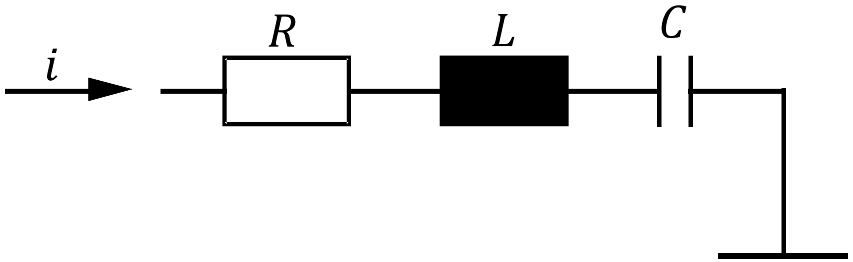
- inductivity: $$L_{\mathrm{H}} = \frac{\Delta p}{\dot{Q}}$$



Serial connection of complex resistors



analogy with electrical engineering



complex resistance:

$$p = R_H \cdot Q + L_H \cdot \frac{dQ}{dt} + \frac{1}{C_H} \cdot \int Q dt$$

analogy with electrical engineering:

$$u = R \cdot i + L \cdot \frac{di}{dt} + \frac{1}{C} \cdot \int i dt$$

$p \mathrel{\widehat{=}} u$

$Q \mathrel{\widehat{=}} i$

Outline of todays lecture

1 Components in a hydraulic network

1.1 Hydraulic resistance

1.2 Hydraulic capacity

1.3 Hydraulic inductivity

2 Calculation of a hydraulic network

3 Liquid collumn as homogeneouse resonator

4 Summary

Basic resistances: throttle / gap

throttle: Hagen-Poisouille equation:

$$Q = \frac{\pi \cdot r^4}{8 \cdot \eta \cdot l} \cdot (p_1 - p_2)$$

Hydraulic resistance:

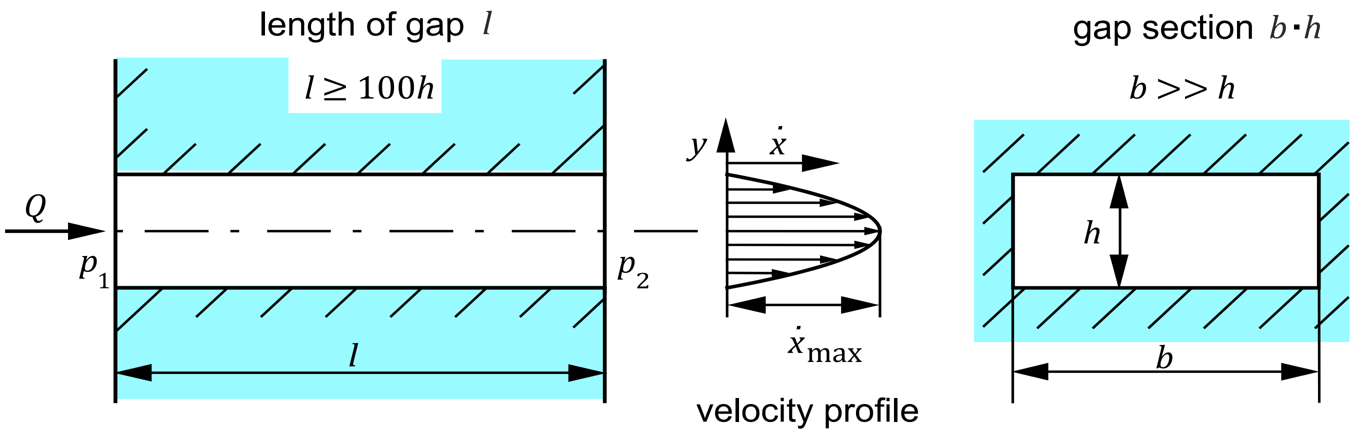
$$\Delta p = Q \cdot R_{\mathrm{H}} \qquad R_{\mathrm{H}} = \frac{8 \cdot \eta \cdot l}{\pi \cdot r^4}$$

gap: Hagen-Poisouille equation:

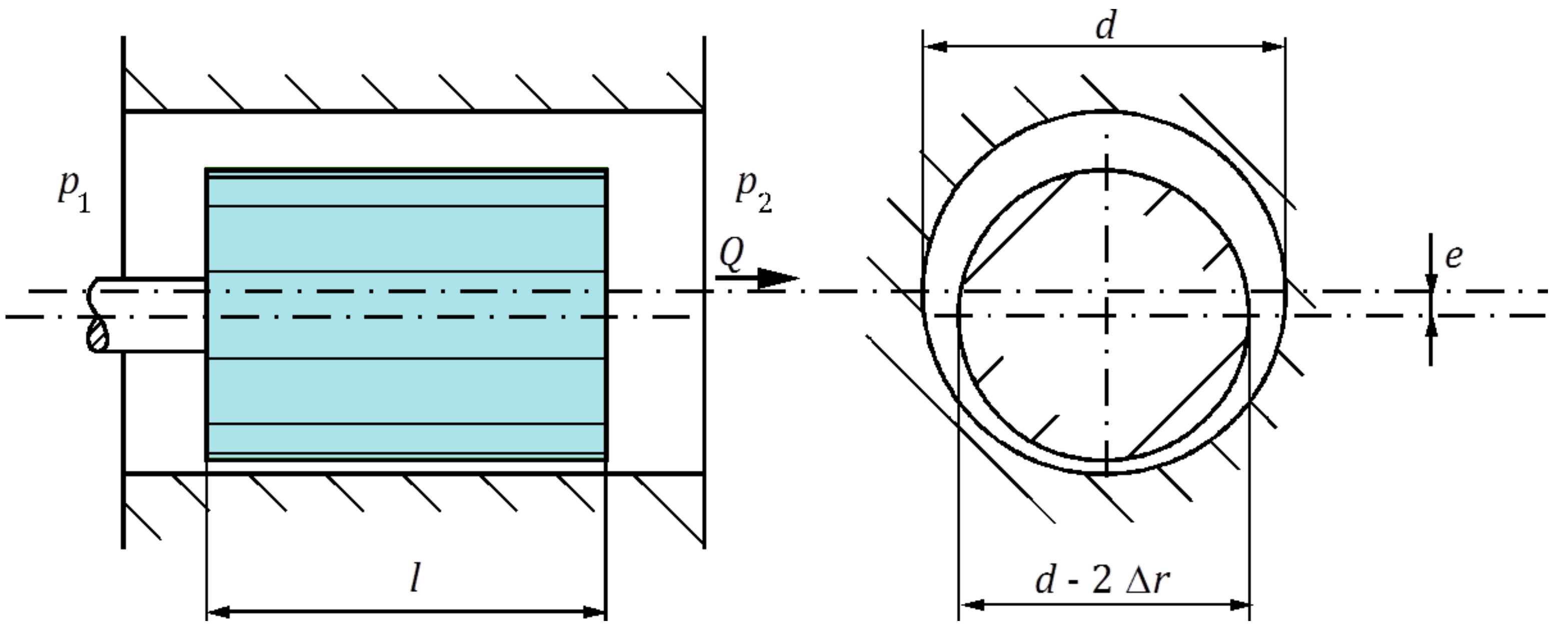
$$Q = \frac{b \cdot h^3}{12 \cdot \eta \cdot l} \cdot (p_1 - p_2)$$

Hydraulic resistance:

$$R_{\mathrm{H}} = \frac{12 \cdot \eta \cdot l}{b \cdot h^3}$$

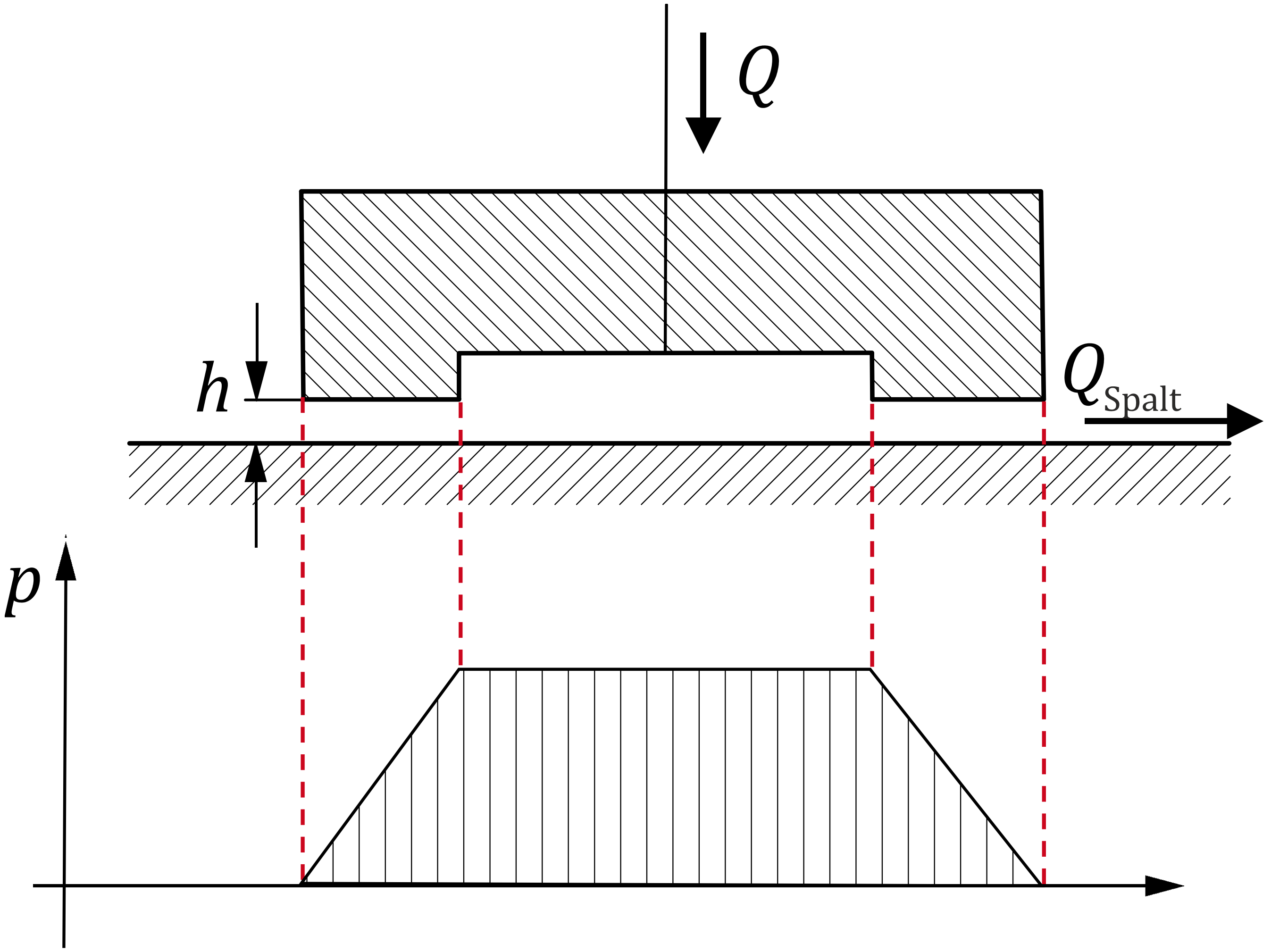


Flow through an Excentric Annular Gap

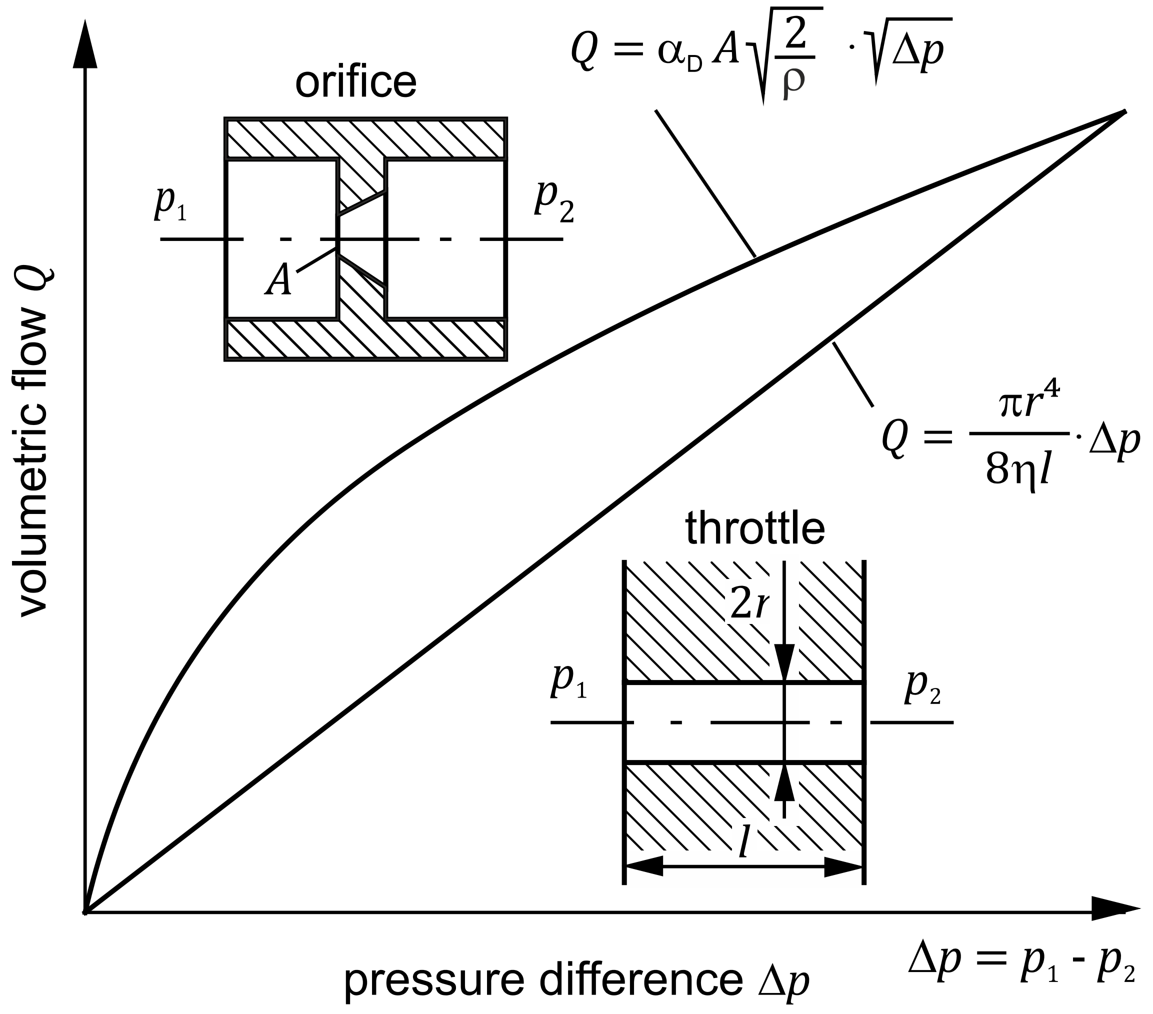


$$Q = \frac{d \cdot \pi \cdot \Delta r^3}{12 \cdot \eta \cdot l} \cdot \left[1 + 1{,}5 \cdot \left(\frac{e}{\Delta r}\right)^2\right] \cdot (p_1 - p_2)$$

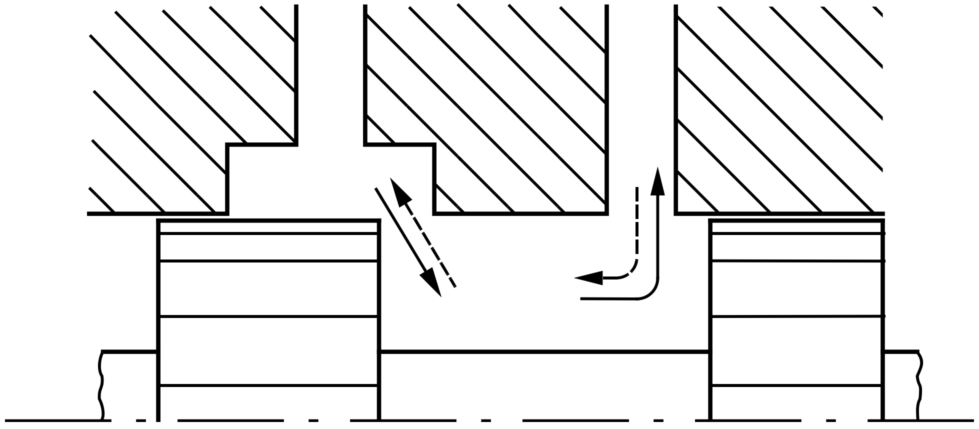
Hydrostatic Bearing



Basic Resistances: throttle - orficie



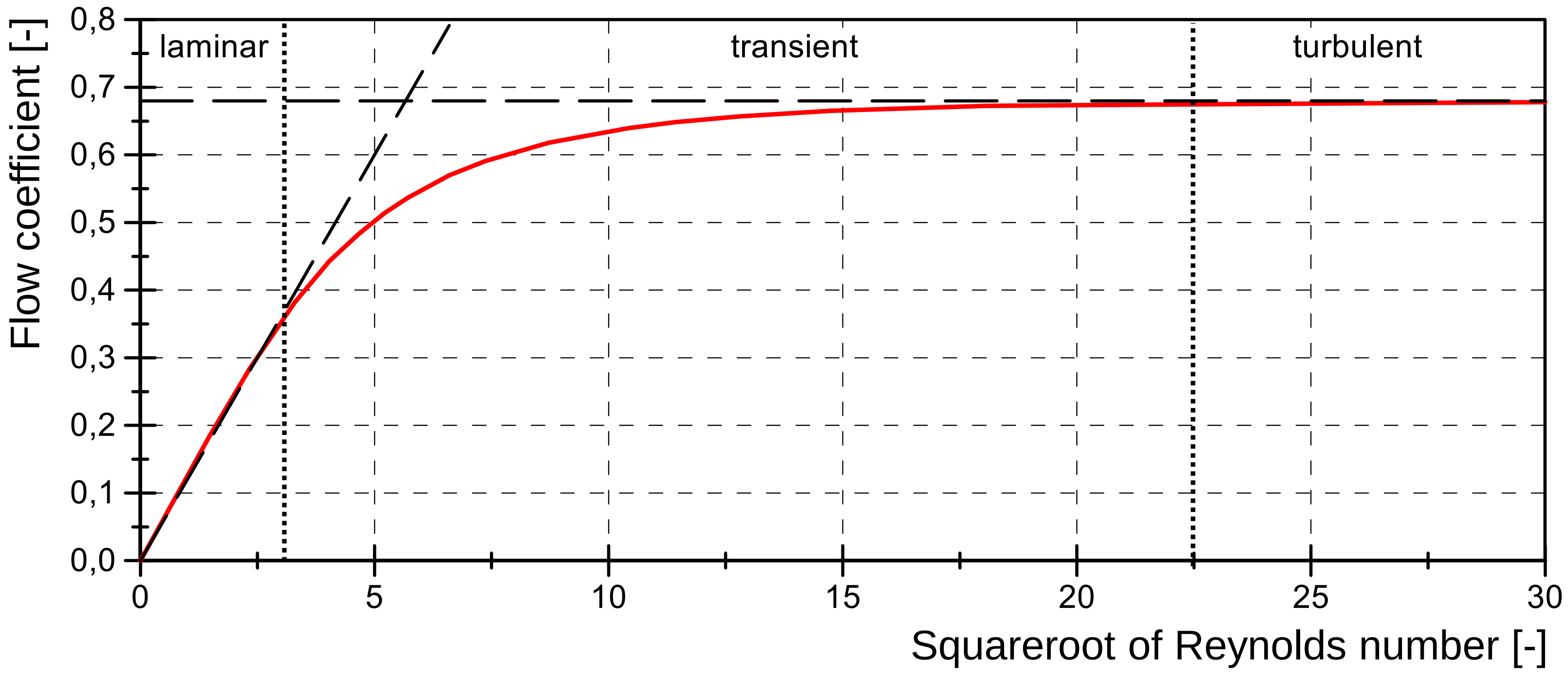
Example: Valve Spool



Orifice: turbulent resistance
temperature indepentent

Throttle: laminar resistance
temperature depentent

Flow coefficient as a function of the Reynolds number



Pressure losses in hydraulic circuits

1. Pressure loss on pipes (Blasius)

$$\Delta p_{\mathrm{R}} = \sum \lambda \cdot \frac{l}{d} \cdot \frac{\rho}{2} \cdot v^2$$

λ – pipe loss factor

2. Pressure loss in fittings (bends, intersections, etc.)

$$\Delta p_{\mathrm{F}} = \sum_i \xi_i \cdot \frac{\rho}{2} \cdot v_i^2$$

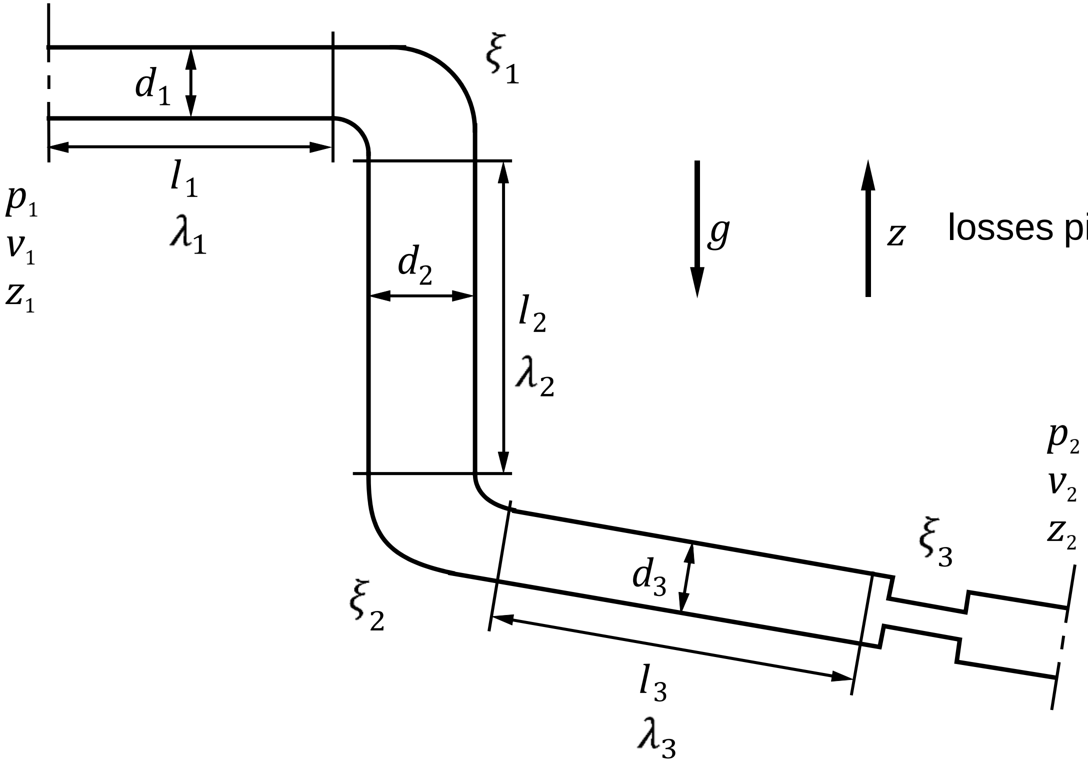
ξ – resistance coefficient

Pressure drop in a pipe assembly

Bernoulli with losses

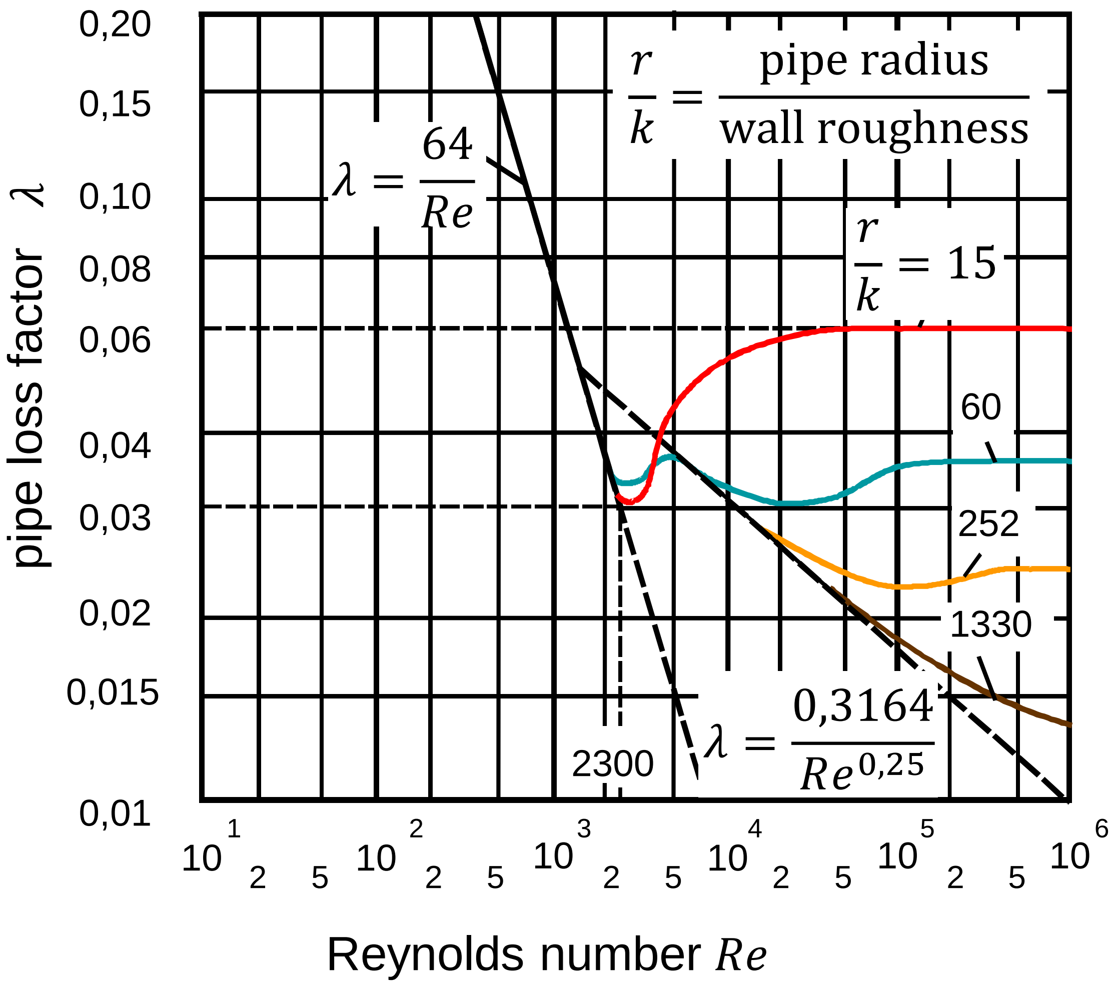
condition 1 condition 2

$$\left(p_1 + \frac{\rho}{2} \cdot v_1^2 + \rho \cdot g \cdot z_1\right) -$$



losses pipe friction losses elbow

Loss factor for smooth pipes



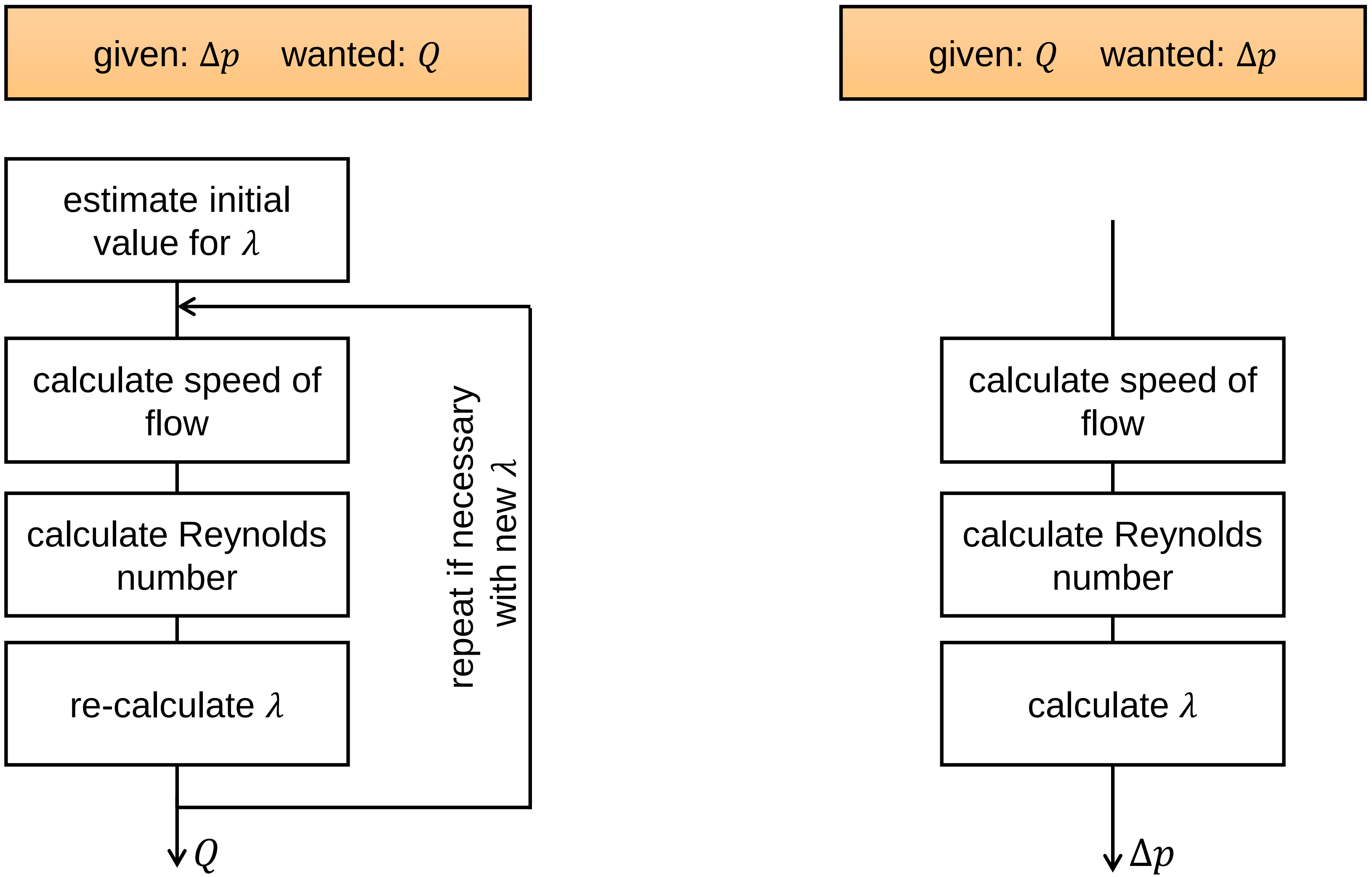
laminar flow:

$$\lambda = \frac{64}{Re}$$

turbulent flow:

$$\lambda = \frac{0{,}3164}{Re^{0{,}25}}$$

Calculation of pipe resistance according blasius



Comparison Flow Coefficient vs. Loss Factor

orifice equation:

$$Q = \alpha_\mathrm{D} \cdot A \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

$$\Rightarrow v = \alpha_\mathrm{D} \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

pressure drop:

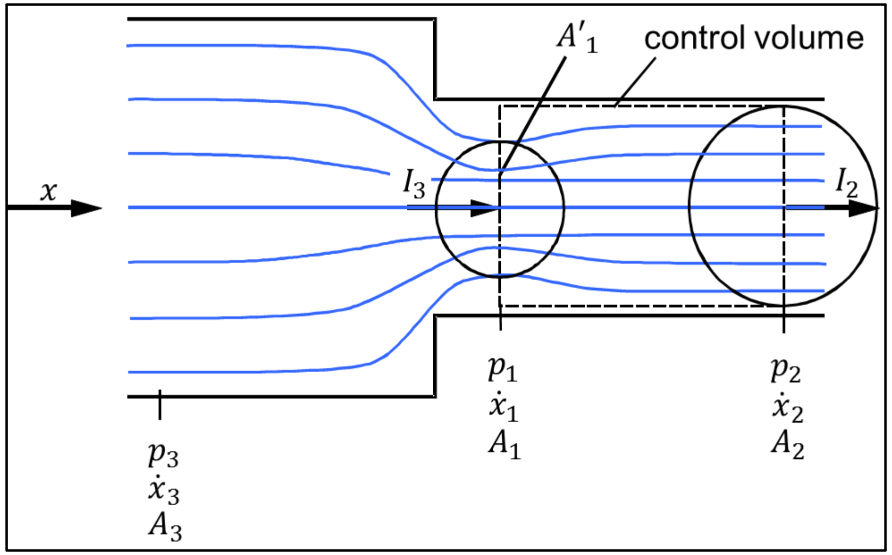
$$\Delta p = \xi \cdot \frac{\rho}{2} \cdot v^2$$

$$v = \sqrt{\frac{1}{\xi}} \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

comparison yields:

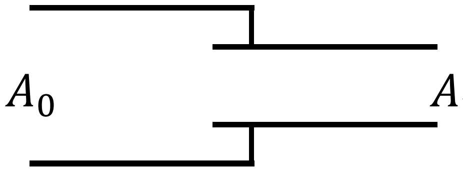
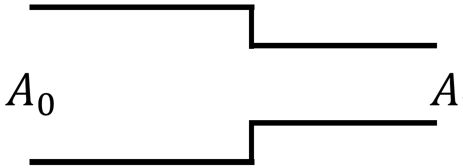
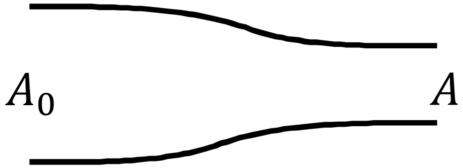
$$\alpha_\mathrm{D} = \sqrt{\frac{1}{\xi}}$$

Pressure Loss at Cross Section Changes



$$\Delta p_V = \frac{\rho}{2} \cdot \dot{x}_2{}^2 \cdot \left(\frac{1}{\alpha_K} - 1\right)^2$$

$$\xi = \frac{\Delta p_V}{\frac{\rho}{2} \cdot \dot{x}_2{}^2} = \left(\frac{1}{\alpha_K} - 1\right)^2$$

shape of reduction	α_K	ξ
A_0  A_1	0,5	1
A_0  A_1	0,61 ... 0,65	0,4 ... 0,3
A_0  A_1	0,99	0

Outline of todays lecture

1 Components in a hydraulic network

1.1 Hydraulic resistance

1.2 Hydraulic capacity

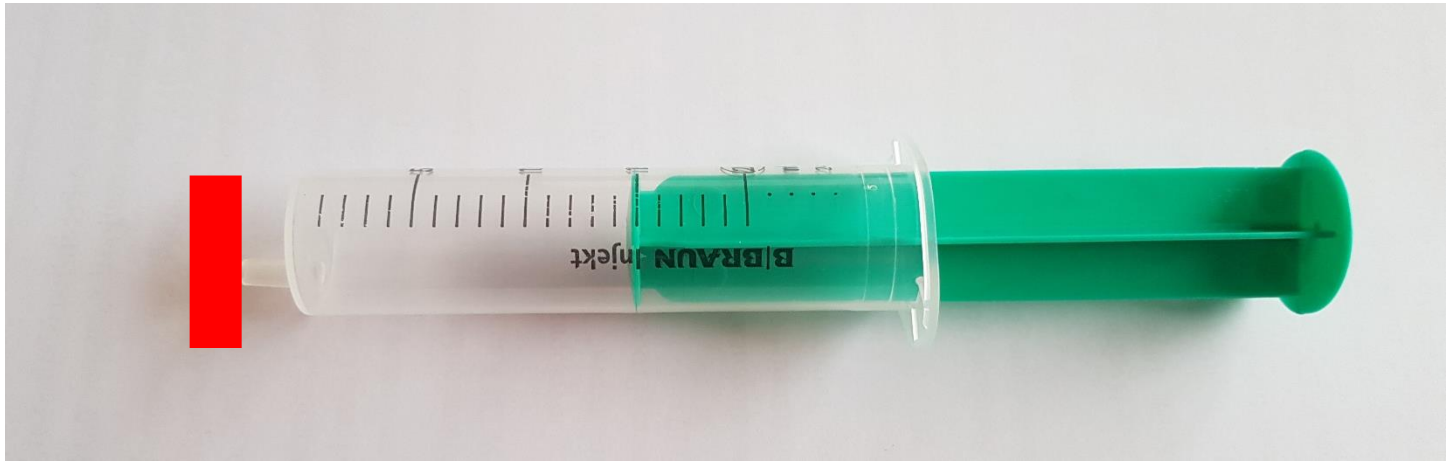
1.3 Hydraulic inductivity

2 Calculation of a hydraulic network

3 Liquid collumn as homogeneouse resonator

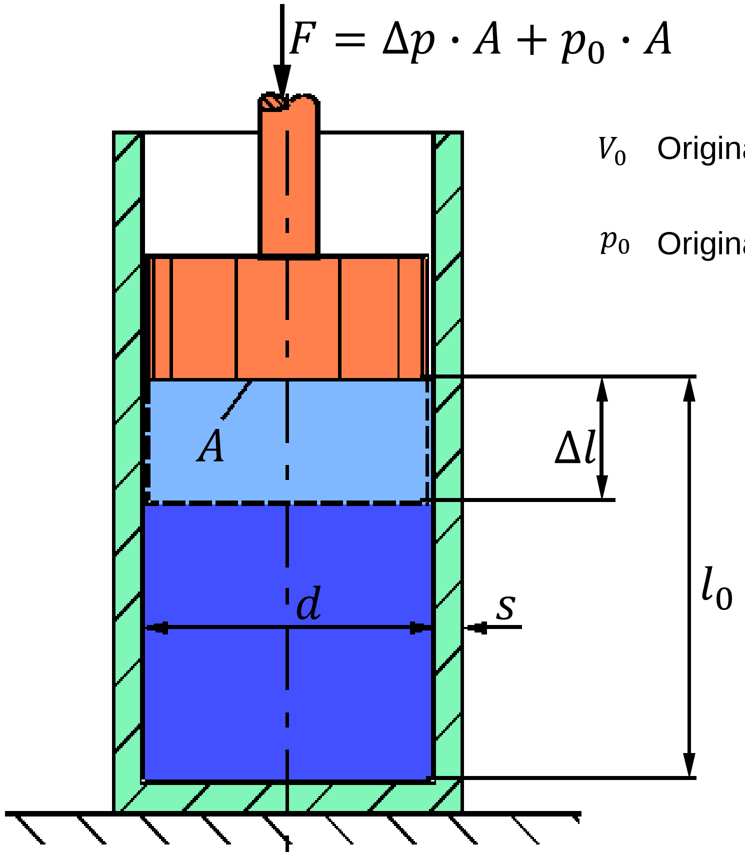
4 Summary

What is a hydraulic capacity?



In a sealed volume:
pressure build-up only because of change in volume or change in temperature
→ Compressibility of fluid influences pressure build-up and energy stored in fluid

Hydraulic Capacity



V_0 Original volumen

p_0 Original pressure

compressibility of a liquid

compressibility of the liquid

$$\Delta V_{\mathrm{Fl}} = A \cdot \Delta l = V_0 \cdot \frac{\Delta p}{E_{\mathrm{Fl}}}$$

bulk modulus

$$E_{\mathrm{Fl}} = \frac{1}{\beta} \quad \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$$

Bulk modulus is not const., better use differential writing:

$$\frac{\mathrm{d}V}{V_0} = -\frac{\mathrm{d}p}{E_{\mathrm{Fl}}}$$

Hydraulic capacity relates volumen change due to change in pressure

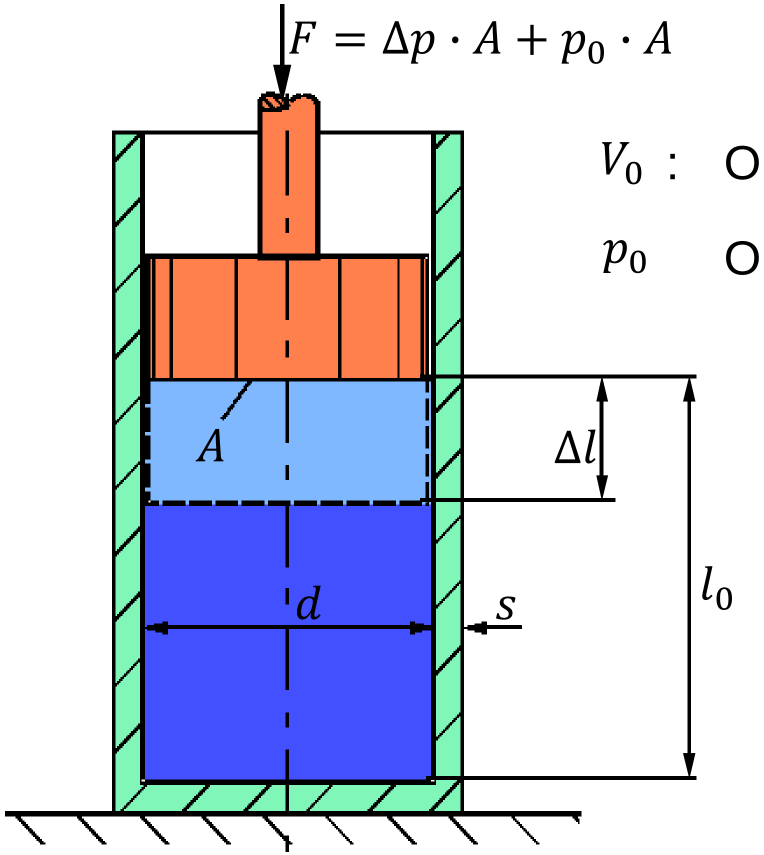
$$C_{\mathrm{H}} = -\frac{\mathrm{d}V}{\mathrm{d}p} = \frac{V_0}{E_{\mathrm{Fl}}} \qquad \left[\frac{\mathrm{m}^5}{\mathrm{N}}\right]$$

with $\quad \mathrm{d}V = -Q \cdot \mathrm{d}t \qquad \Rightarrow p = \frac{1}{C_{\mathrm{H}}} \cdot \int Q\mathrm{d}t$

Electrical engineering:

$$u = \frac{1}{C} \cdot \int i\mathrm{d}t$$

Hydraulic Capacity – Influences of compressibility



V_0 : Original volumen

p_0 Original pressure

Bulk modulus

$$E_{\mathrm{Fl}} = \frac{1}{\beta} \quad \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$$

Not only compression of liquid!

1. elasticity of the pipe wall

$$\Delta V_{\mathrm{R}} = V_0 \cdot \frac{\Delta p}{E_{\mathrm{R}}} \cdot \frac{d}{s}$$

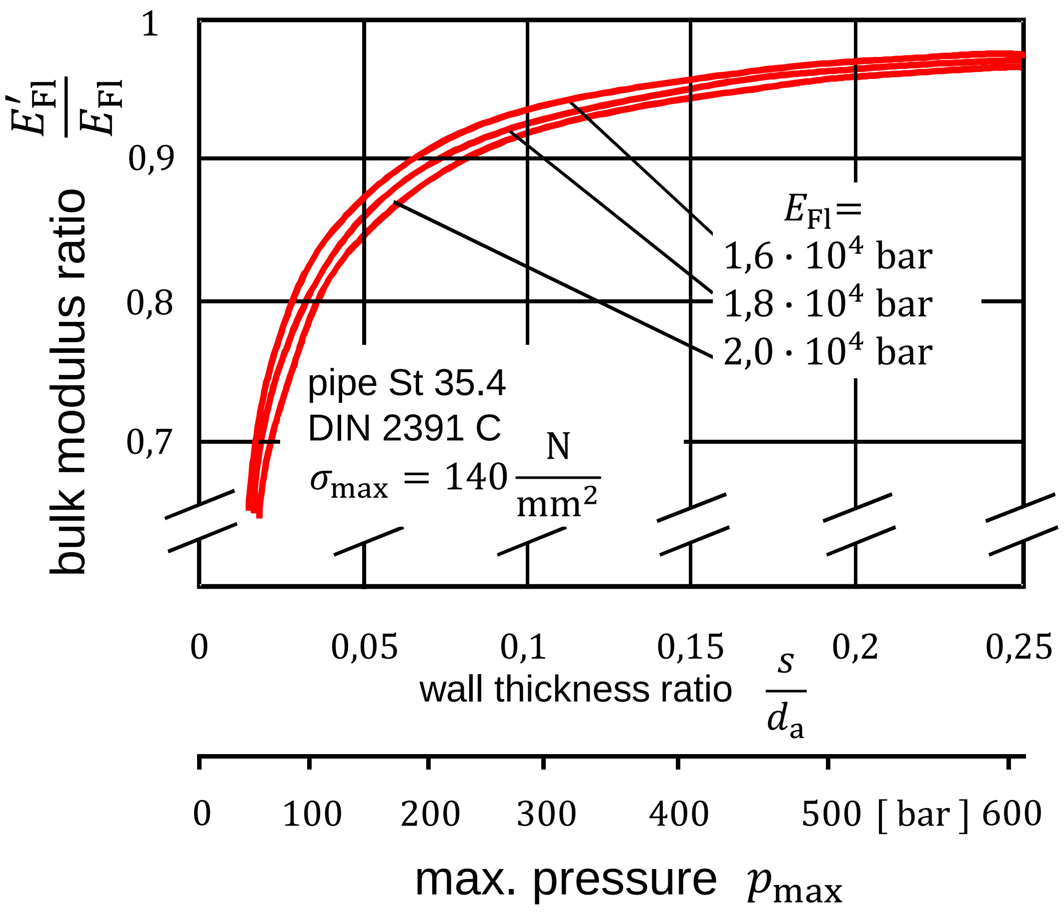
2. compressibility of air bubbles

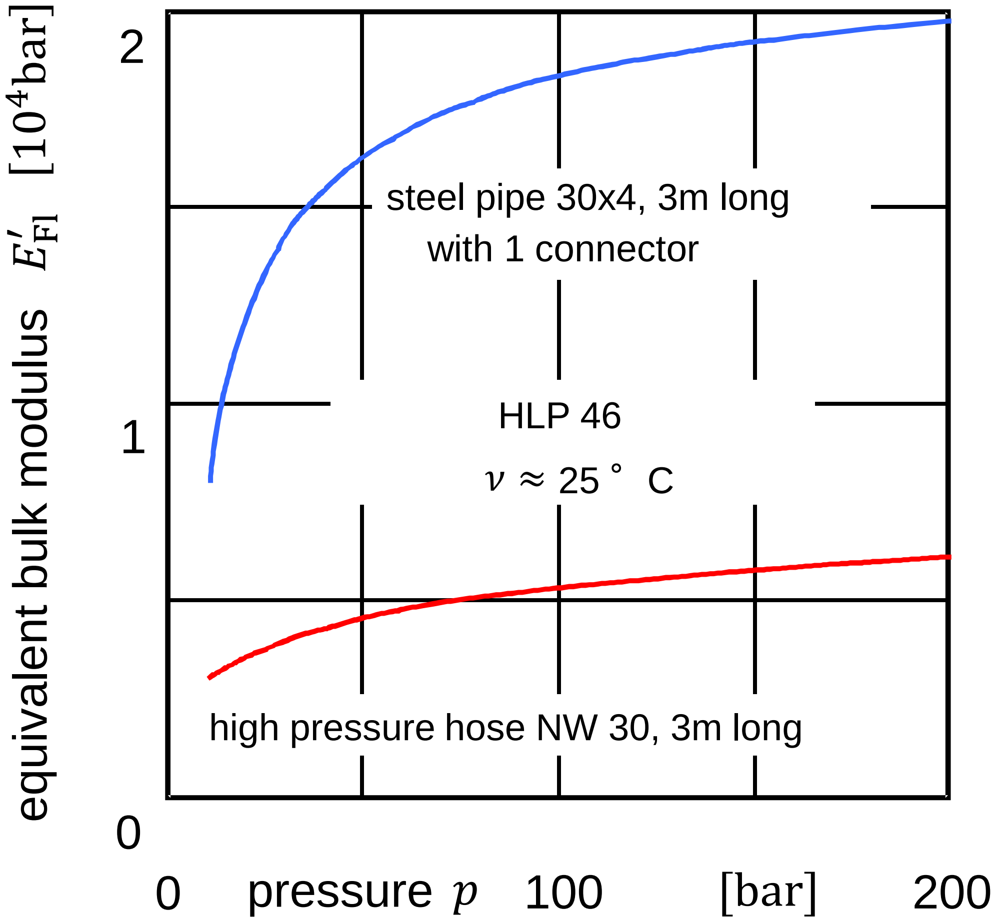
$$\Delta V_{\mathrm{L}} = \frac{V_{\mathrm{L},0}}{p_0} \cdot \frac{1}{\kappa} \cdot \Delta p$$

→ equivalent bulk modulus

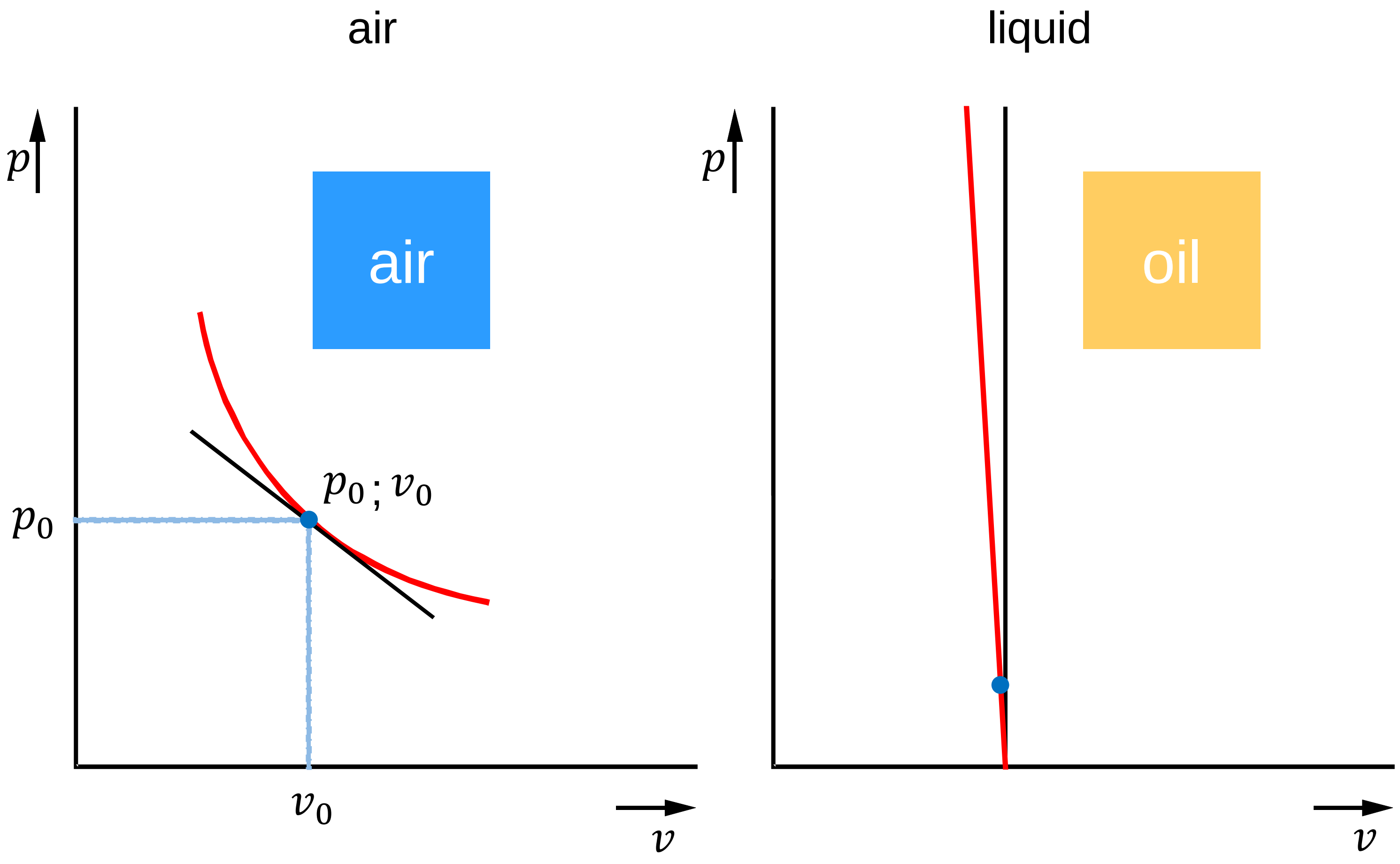
$$E'_{\mathrm{Fl}} = \frac{V_0 \cdot \Delta p}{\Delta V_{\mathrm{Fl}} + \Delta V_{\mathrm{R}} + \Delta V_{\mathrm{L}}} = \frac{1}{\frac{1}{E_{\mathrm{Fl}}} \cdot \left(1 + \frac{E_{\mathrm{Fl}}}{E_{\mathrm{R}}} \cdot \frac{d}{s}\right) + \frac{\Delta V_{\mathrm{L}}}{V_0 \cdot \Delta p}}$$

Influence of Pipe Elasticity on (Equivalent) Bulk Modulus

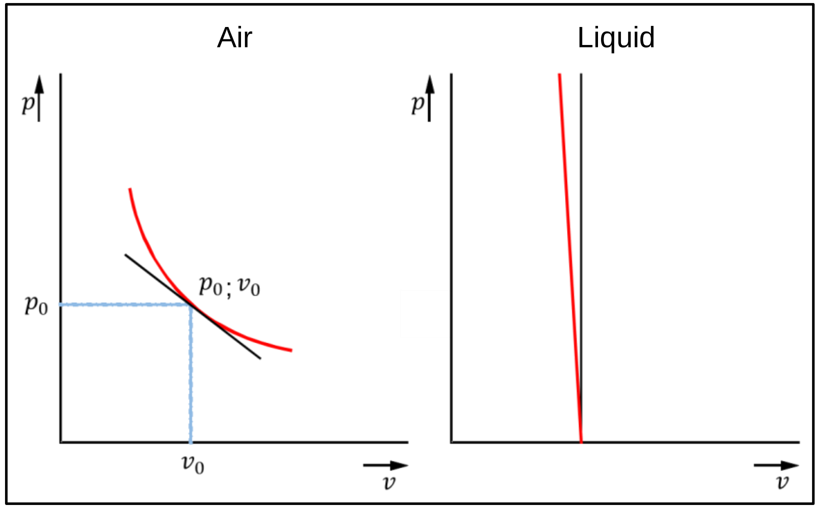




Comparison of Stiffness of Air and Liquid Volumes



Comparison of Stiffness of Air and Liquid Volumes



compression of an air volume:

$$\frac{\partial V}{\partial p} = -\frac{V_0}{p_0 \cdot \kappa}$$

compression of a liquid volume:

$$\frac{\partial V}{\partial p} = -\frac{V_0}{E_{\mathrm{Fl}}}$$

same stiffness if

$$p_{\mathrm{air}} = \frac{E_{\mathrm{Fl}}}{\kappa} \approx 10^4 \mathrm{bar}$$

Outline of todays lecture

1 Components in a hydraulic network

1.1 Hydraulic resistance

1.2 Hydraulic capacity

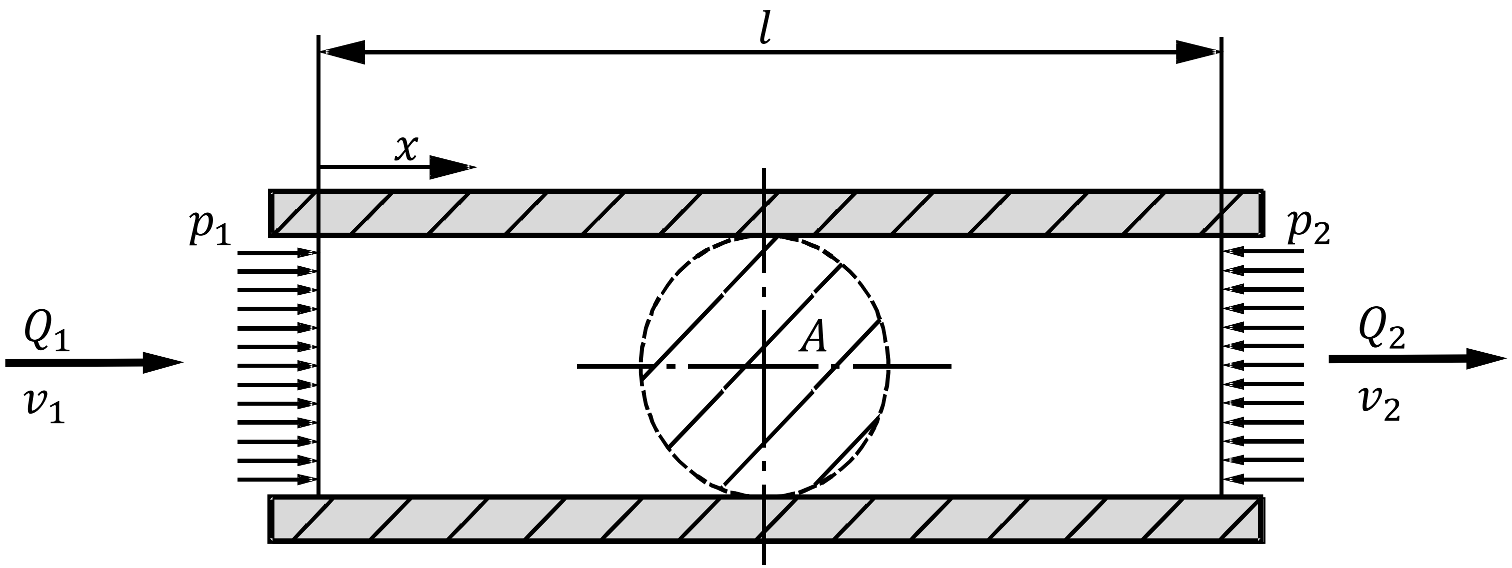
1.3 Hydraulic inductivity

2 Calculation of a hydraulic network

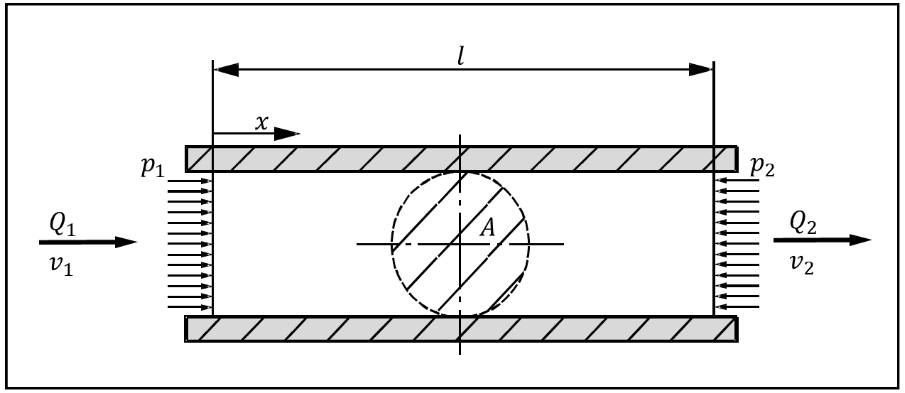
3 Liquid collumn as homogeneouse resonator

4 Summary

Hydraulic Inductivity of a Short Section of Pipe



Hydraulic Inductivity of a Short Section of Pipe



liquid with mass $\Rightarrow$ hydraulic inductivity L_H

$$F = m \cdot a = A \cdot l \cdot \rho \cdot a$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{A} \cdot \frac{\mathrm{d}Q}{\mathrm{d}t}$$

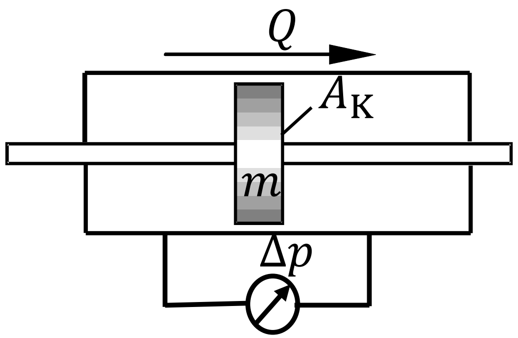
$$\Rightarrow \Delta p = \frac{l \cdot \rho}{A} \cdot \frac{\mathrm{d}Q}{\mathrm{d}t}$$

hydraulic inductivity L_H

compare with electrical engineering:

$$u = L \cdot \frac{\mathrm{d}i}{\mathrm{d}t}$$

Hydraulic Inductivity of a Symmetric Cylinder



inductivity: $$L_{\mathrm{H}} = \frac{\Delta p}{\dot{Q}} \qquad (1)$$

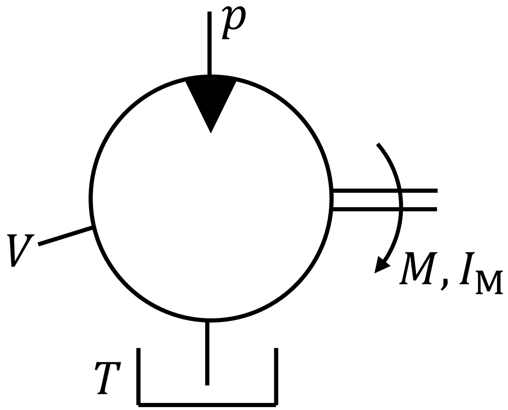
flow „through“ the cylinder: $$Q = A_{\mathrm{K}} \cdot \dot{x}$$ $$\Rightarrow \dot{Q} = A_{\mathrm{K}} \cdot \ddot{x} \qquad (2)$$

Newton: $$F = m \cdot \ddot{x} = \Delta p \cdot A_{\mathrm{K}}$$ $$\Rightarrow \ddot{x} = \frac{\Delta p \cdot A_{\mathrm{K}}}{m} \qquad (3)$$

(3) and (2) in (1): $$L_{\mathrm{H}} = \frac{\Delta p}{\frac{A_{\mathrm{K}}^2}{m} \cdot \Delta p} = \frac{m}{A_{\mathrm{K}}^2}$$

Hydraulic Inductivity of a Rotating Motor

hydro motor



$$M = I_M \cdot \dot{\omega} = \frac{V}{2\pi} \cdot \Delta p$$

$$Q = \frac{V}{2\pi} \cdot \omega$$

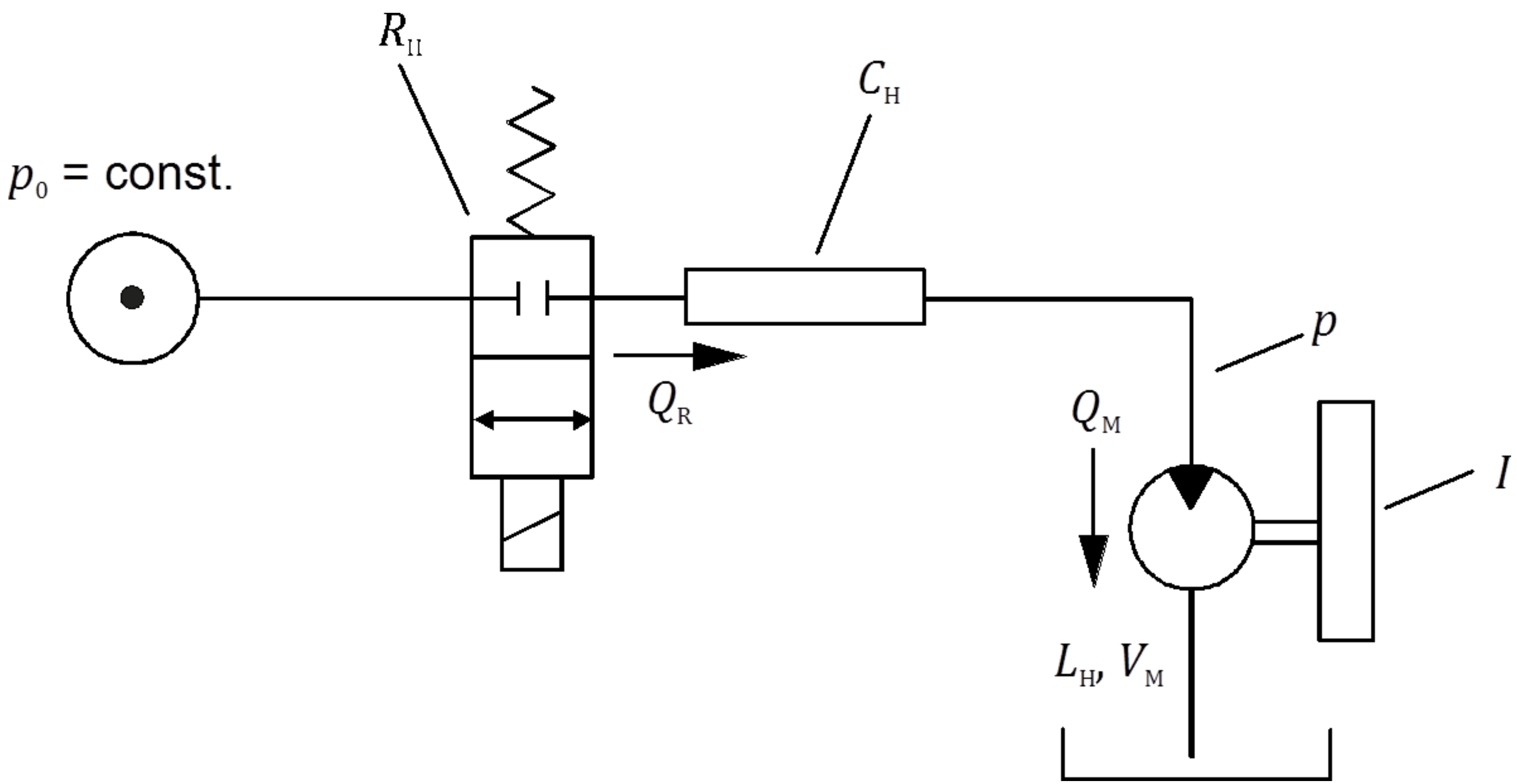
$$\frac{V}{2\pi} \cdot \Delta p = I_M \cdot \frac{2\pi \cdot \dot{Q}}{V}$$

$$L_H = \frac{\Delta p}{\dot{Q}} \quad \text{yields} \quad L_M = \frac{I_M}{\left(\frac{V}{2\pi}\right)^2}$$

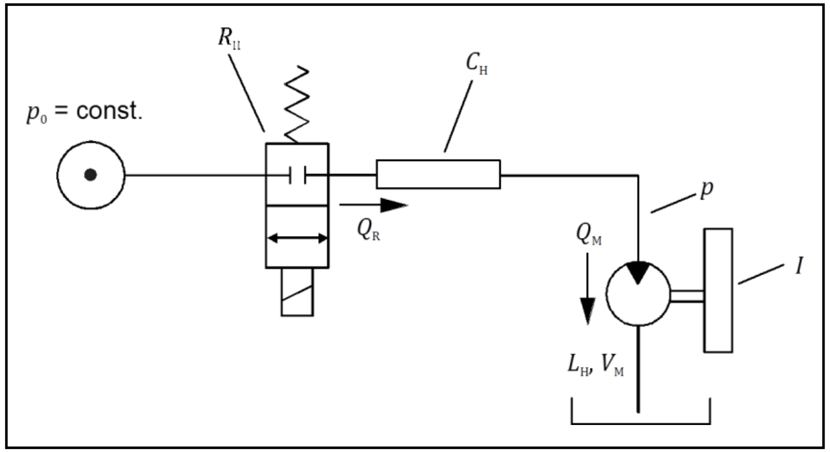
Outline of todays lecture

1	Components in a hydraulic network
1.1	Hydraulic resistance
1.2	Hydraulic capacity
1.3	Hydraulic inductivity
2	Calculation of a hydraulic network
3	Liquid collumn as homogeneouse resonator
4	Summary

Calculation Example for a Hydraulic Network



Hydraulic Network



pressure rise in the capacity

$$\dot{p} = \frac{1}{C_\mathrm{H}} \cdot \sum_i Q_i = \frac{1}{C_\mathrm{H}} \cdot (Q_\mathrm{R} - Q_\mathrm{M})$$

$$\Rightarrow \dot{p} = \frac{1}{C_\mathrm{H}} \cdot (Q_\mathrm{R} - Q_\mathrm{M})$$

inflow over the valve

$$Q_\mathrm{R} = \frac{1}{R_\mathrm{H}} \cdot (p_0 - p)$$

$$\Rightarrow Q_\mathrm{R} = -\frac{1}{R_\mathrm{H}} \cdot p$$

acceleration of the motor

$$\dot{Q}_\mathrm{M} = \frac{p}{L_\mathrm{H}}$$

Insertion yields pressure rise equation:

$$\ddot{p} + \frac{1}{R_\mathrm{H} \cdot C_\mathrm{H}} \cdot \dot{p} + \frac{1}{C_\mathrm{H} \cdot L_\mathrm{H}} \cdot p = 0$$

eigen angular frequency:

$$\omega_0 = \sqrt{\frac{1}{L_\mathrm{H} \cdot C_\mathrm{H}}}$$

damping:

$$D_\mathrm{H} = \frac{1}{2R_\mathrm{H}} \cdot \sqrt{\frac{L_\mathrm{H}}{C_\mathrm{H}}}$$

Outline of todays lecture

1 Components in a hydraulic network

1.1 Hydraulic resistance

1.2 Hydraulic capacity

1.3 Hydraulic inductivity

2 Calculation of a hydraulic network

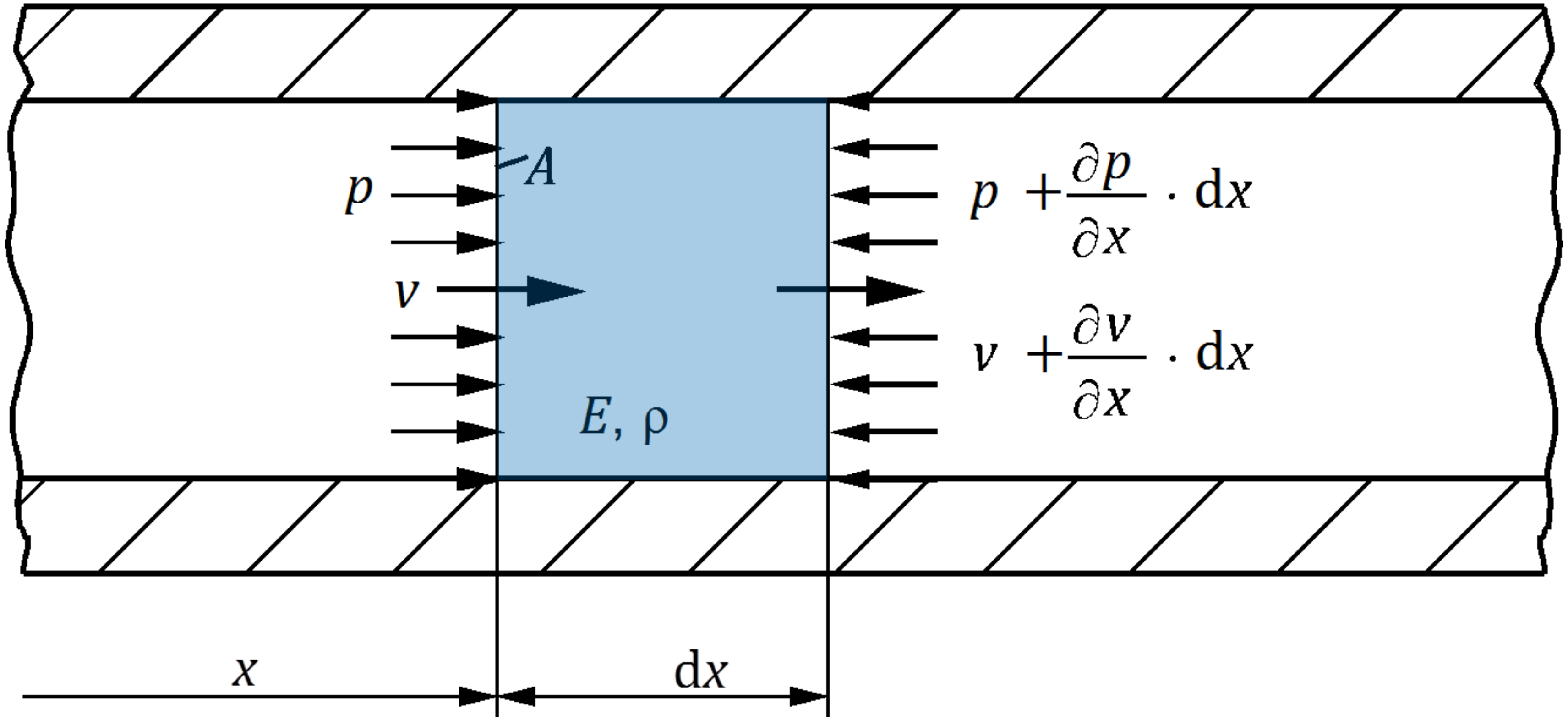
3 Liquid collumn as homogeneouse resonator

4 Summary

Transition from Lumped to Distributed Parameters

Distributed parameters: pressure & flow rate depending on time & location

Due to compression / expansion
Due to accelleration / decelleration
→ resonances in the system



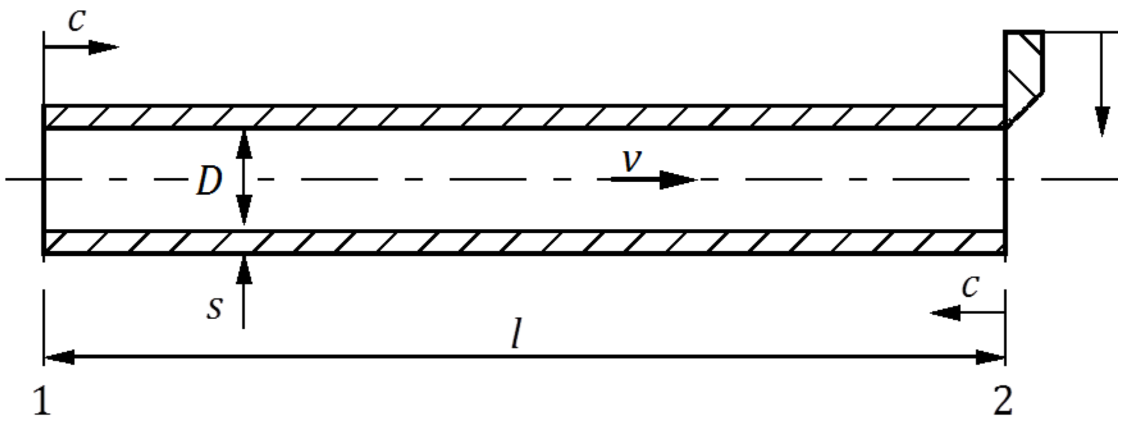
Basics for liquid collumn

- Velocity of pressure transfer in a pipe → speed of sound in oil

$$c = \sqrt{\frac{E'_{\mathrm{Fl}}}{\rho}}$$

- Pressure waves travel with speed of sound through pipes and systems

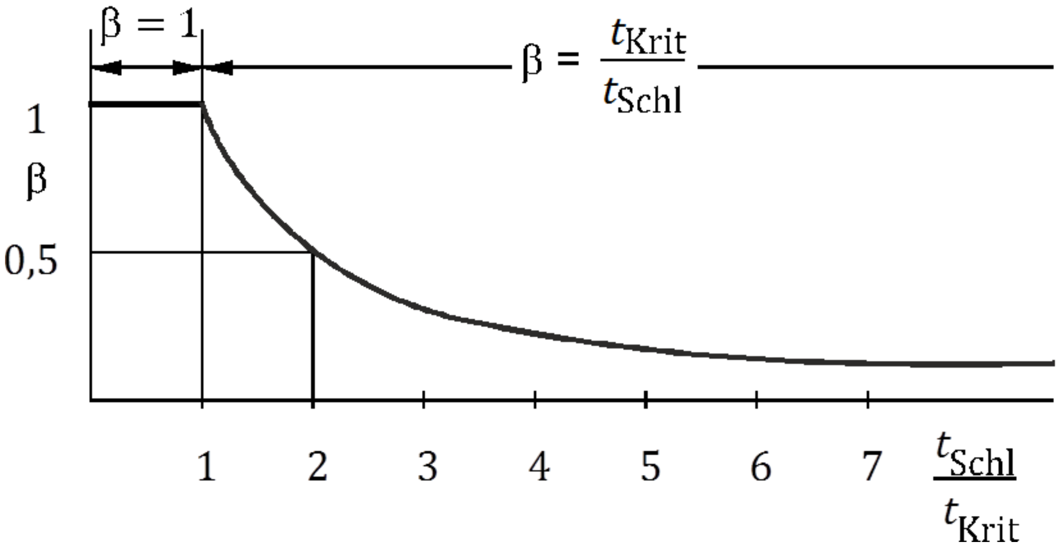
Pressure Surge in a Pipe when closing valve



Fast closing of a valve results in pressure surge (Joukowsky-surge)

$$t_{\mathrm{Schl}} \leq t_{\mathrm{Krit}} \qquad t_{\mathrm{Krit}} = \frac{2 \cdot l}{c} \qquad \text{(2 x pipe passage)}$$

$$\Delta p = \rho \cdot c \cdot \Delta v$$



Slower closing results in a reduced reflected wave reducing pressure surge

$$\Delta p = \frac{t_{\mathrm{Krit}}}{t_{\mathrm{Schl}}} \cdot \rho \cdot c \cdot \Delta v$$

Example for pressure surge

- Given.: steel pipe 25x3, 4m length
  Liquid: $E'_{Fl} = 1{,}6 \cdot 10^4$ bar, ρ = 850 kg/m³, c = 1370 m/s

- Critical closing time of the valve is:

$$t_{\text{Krit}} = \frac{2 \cdot l}{c} = 5{,}8\ ms$$

- When closing the valve in faster than 5,8 ms, the pressure surge with a change in velocity of Δv = 3 m/s has a value

$$\Delta p = \rho \cdot c \cdot \Delta v = 35\ bar$$

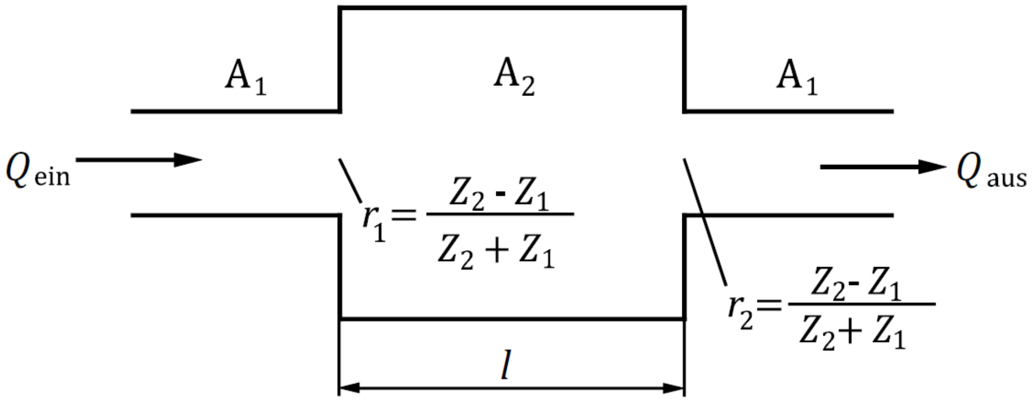
- When closing the valve slower, e.g. with a closing time of 20 ms, the pressure surge reduces

$$\Delta p = \frac{5{,}8\ ms}{20\ ms} \cdot 35\ bar = 10\ bar$$

Dynamic consideration often necessary

- Wave propagation in systems:
  - impair the control behavior
  - Lead to noise
  - Impair the life of the components
  - Lead to "shaking loose" of fittings

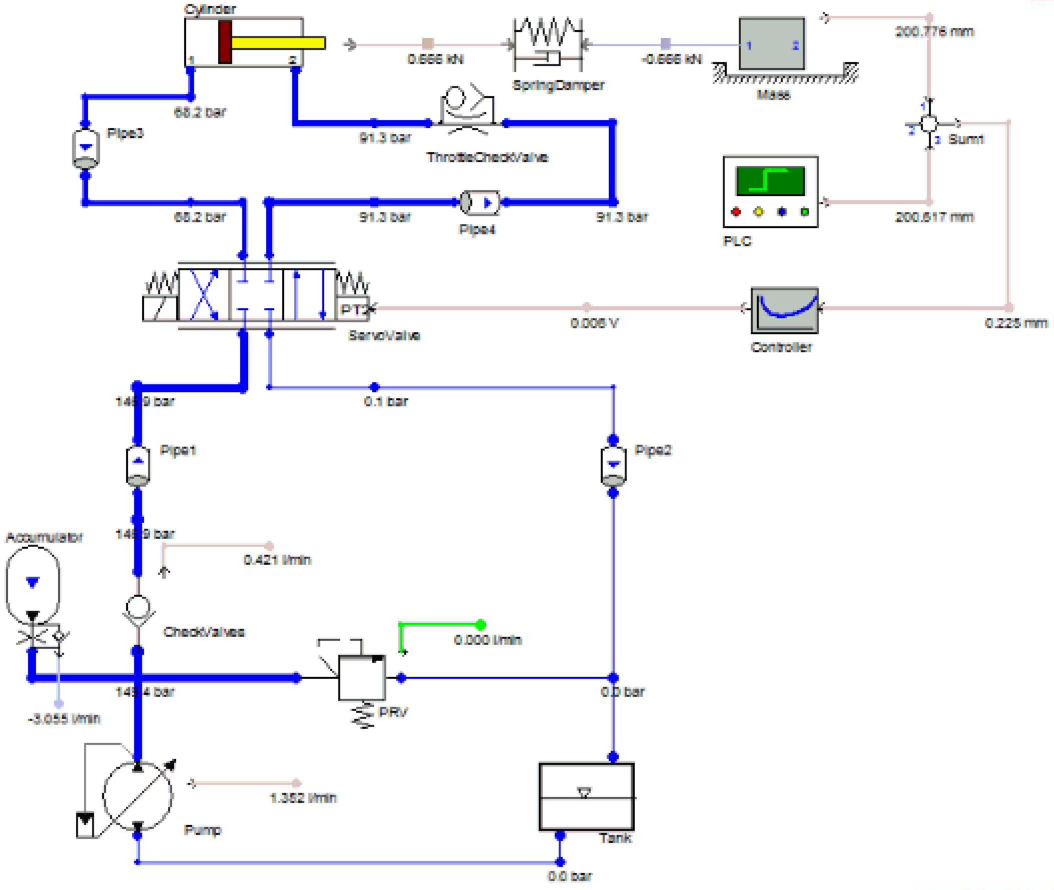
- Remedies:
  - Meaningful dimensioning of piping / system (natural frequency of pipes / liquid column and connected components must not be too close to each other)
  - Reflection Muffler (Silincer)



Simulation of hydraulic systems

- System simulation consists of
  - capacities
  - resistances
  - inductances

- Visualisation of dynamic effects in system



Outline of todays lecture

1 Components in a hydraulic network

1.1 Hydraulic resistance

1.2 Hydraulic capacity

1.3 Hydraulic inductivity

2 Calculation of a hydraulic network

3 Liquid collumn as homogeneouse resonator

4 Summary

Zusammenfassung

- What is a hydraulic resistance?
  - Throtteling of the flow that leads to pressure loss $\Delta p = Q \cdot R_{\mathrm{H}}$

- Which basic resistances exist and what are the differences?
  - Orifice: turbulent resistance, temperature independent; Throttle: laminar resistance, temperature dependent

- What is a hydraulic capacity?
  - Ratio of volumen change with corresponding pressure change

$$C_{\mathrm{H}} = -\frac{\mathrm{d}V}{\mathrm{d}p} = \frac{V_0}{E_{\mathrm{Fl}}} \qquad \left[\frac{\mathrm{m}^5}{\mathrm{N}}\right] \qquad \Rightarrow p = \frac{1}{C_{\mathrm{H}}} \cdot \int Q \mathrm{d}t$$

- What is a hydraulic inductivity?
  - Resistance against acceleration

$$L_H = \frac{l \cdot \rho}{A} \qquad \Rightarrow \Delta p = L_H \cdot \frac{\mathrm{d}Q}{\mathrm{d}t}$$

- What is the Joukowski surge? $\Delta p = \rho \cdot c \cdot \Delta v$

Thank you for your attention.