Context-Free Grammars

Grammars were invented by Chomsky in the mid-1950s for describing natural languages. In the late 1950s, one of Chomsky’s types of grammar (Type 2 or ***context-free*** grammar) was reinvented by Backus and proposed as a way to specify the syntax of the new language Algol.

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This proposal was adopted in the **Algol 60 Report** (edited by Naur), an influential document considered to have done an excellent job of specifying Algol 60.

The grammar notation used in the Algol 60 Report is now called "Backus Naur Form" or **BNF**.

Like many authors (but unlike Sethi) we use the term ***BNF*** more loosely, to simply mean "*a commonly used notation for writing context-free grammars*", and we refer to grammars written in such a notation as ***BNF specifications***.

$$
\begin{array}{rcl}
\langle expression \rangle & ::= & \langle expression \rangle + \langle term \rangle \\
& | & \langle expression \rangle - \langle term \rangle \\
& | & \langle term \rangle \\
\langle term \rangle & ::= & \langle term \rangle * \langle factor \rangle \\
& | & \langle term \rangle \, / \, \langle factor \rangle \\
& | & \langle factor \rangle \\
\langle factor \rangle & ::= & \textbf{number} \\
& | & \textbf{name} \\
& | & (\, \langle expression \rangle \,)
\end{array}
$$

A grammar written in BNF notation on p. 46 of Sethi (p. 47 in the course reader).

Figure 2.10 BNF syntactic rules for arithmetic expressions.

On p. 42, Sethi gives this equivalent grammar that is written in a similar notation. ***We will consider this notation to be BNF*,**

$$
\begin{array}{lcl}
E & ::= & E + T \mid E - T \mid T \\
T & ::= & T * F \mid T \, / \, F \mid F \\
F & ::= & \textbf{number} \mid \textbf{name} \mid (\, E \,)
\end{array}
$$

Figure 2.6 A grammar for arithmetic expressions.

$$
\begin{aligned}
\langle expression \rangle &::= \langle expression \rangle + \langle term \rangle \\
&\quad|\ \langle expression \rangle - \langle term \rangle \\
&\quad|\ \langle term \rangle \\
\langle term \rangle &::= \langle term \rangle * \langle factor \rangle \\
&\quad|\ \langle term \rangle \,/\, \langle factor \rangle \\
&\quad|\ \langle factor \rangle \\
\langle factor \rangle &::= \textbf{number} \\
&\quad|\ \textbf{name} \\
&\quad|\ (\ \langle expression \rangle\ )
\end{aligned}
$$

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Figure 2.10 BNF syntactic rules for arithmetic expressions.

On p. 42, Sethi gives this equivalent grammar that is written in a similar notation. ***We will consider this notation to be BNF***, even though it isn't exactly the same as the notation used in the Algol 60 Report and Sethi does not call it BNF.

$$
\begin{aligned}
E &::= E + T \mid E - T \mid T \\
T &::= T * F \mid T / F \mid F \\
F &::= \textbf{number} \mid \textbf{name} \mid (\ E\ )
\end{aligned}
$$

Figure 2.6 A grammar for arithmetic expressions.

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- A grammar is relatively concise way to precisely specify certain (possibly infinite) *sets of finite sequences of symbols*; those symbols are referred to as ***terminals*** of the grammar.
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- A grammar is relatively concise way to precisely specify certain (possibly infinite) *sets of finite sequences of symbols*; those symbols are referred to as ***terminals*** of the grammar.
- Each of the specified sets of finite sequences of terminals is denoted by a ***nonterminal*** of the grammar.
- One of the nonterminals is regarded as the “most important”: It is called the ***starting nonterminal*** (or *start symbol* or *sentence symbol*); the set of sequences of terminals it denotes is called the ***language*** *generated by* (or *language of*) the grammar.
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- Each of the specified sets of finite sequences of terminals is denoted by a ***nonterminal*** of the grammar.
- One of the nonterminals is regarded as the “most important”: It is called the ***starting nonterminal*** (or *start symbol* or *sentence symbol*); the set of sequences of terminals it denotes is called the ***language*** *generated by* (or *language of*) the grammar.
- We commonly think of the other nonterminals as being defined in order that they may be used in defining the starting nonterminal.

$$\langle \textit{real-number} \rangle ::= \langle \textit{integer-part} \rangle \; . \; \langle \textit{fraction} \rangle$$

$$\langle \textit{integer-part} \rangle ::= \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \langle \textit{digit} \rangle$$

$$\langle \textit{fraction} \rangle ::= \langle \textit{digit} \rangle \mid \langle \textit{digit} \rangle \langle \textit{fraction} \rangle$$

$$\langle \textit{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

Figure 2.3 BNF rules for real numbers.

A grammar given by Sethi to specify unsigned floating point literals in a simple language.

In the above grammar:

$$
\begin{array}{rcl}
\langle \textit{real-number} \rangle & ::= & \langle \textit{integer-part} \rangle \; . \; \langle \textit{fraction} \rangle \\
\langle \textit{integer-part} \rangle & ::= & \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \langle \textit{digit} \rangle \\
\langle \textit{fraction} \rangle & ::= & \langle \textit{digit} \rangle \mid \langle \textit{digit} \rangle \langle \textit{fraction} \rangle \\
\langle \textit{digit} \rangle & ::= & 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
\end{array}
$$

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In the above grammar:

The following characters are the 11 ***terminals***:

. 0 1 2 3 4 5 6 7 8 9

A ***terminal*** of a grammar is a constant symbol that is ***not*** defined by the grammar.

$$
\begin{aligned}
\langle \textit{real-number} \rangle &::= \langle \textit{integer-part} \rangle \;.\; \langle \textit{fraction} \rangle \\
\langle \textit{integer-part} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \langle \textit{digit} \rangle \\
\langle \textit{fraction} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{digit} \rangle \langle \textit{fraction} \rangle \\
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The following are the 4 ***nonterminals***:

<real-number> *<integer-part>* *<fraction>* *<digit>*

A ***nonterminal*** of a grammar is a variable that denotes a set of finite sequences of terminals. For example,

⟨*real-number*⟩	::=	⟨*integer-part*⟩ . ⟨*fraction*⟩
⟨*integer-part*⟩	::=	⟨*digit*⟩ | ⟨*integer-part*⟩ ⟨*digit*⟩
⟨*fraction*⟩	::=	⟨*digit*⟩ | ⟨*digit*⟩ ⟨*fraction*⟩
⟨*digit*⟩	::=	0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

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. 0 1 2 3 4 5 6 7 8 9

A ***terminal*** of a grammar is a constant symbol that is ***not*** defined by the grammar.

The following are the 4 ***nonterminals***:

<real-number> *<integer-part>* *<fraction>* *<digit>*

A ***nonterminal*** of a grammar is a variable that denotes a set of finite sequences of terminals. For example, *<digit>* denotes the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

$$
\begin{aligned}
\langle \textit{real-number} \rangle &::= \langle \textit{integer-part} \rangle \;.\; \langle \textit{fraction} \rangle \\
\langle \textit{integer-part} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \langle \textit{digit} \rangle \\
\langle \textit{fraction} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{digit} \rangle \langle \textit{fraction} \rangle \\
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A grammar given by Sethi to specify unsigned floating point literals in a simple language.

In the above grammar:

There are 15 rules called ***productions***. Each production:

- has a left side that is a *single nonterminal*, and
- has a right side that is a *sequence of 0 or more terminals and/or nonterminals*.

The “vertical bar” symbol | means:

The left side of this production is the same as the left side of the ***previous*** *production.*

Example: The **3rd** production of the above grammar is

$$
\begin{aligned}
\langle \textit{real-number} \rangle &::= \langle \textit{integer-part} \rangle \; . \; \langle \textit{fraction} \rangle \\
\langle \textit{integer-part} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \, \langle \textit{digit} \rangle \\
\langle \textit{fraction} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{digit} \rangle \, \langle \textit{fraction} \rangle \\
\langle \textit{digit} \rangle &::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
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Example: The **3**rd production of the above grammar is

<integer-part> ::= <integer-part> <digit>

$$\begin{aligned}
\langle \textit{real-number} \rangle &::= \langle \textit{integer-part} \rangle \,.\, \langle \textit{fraction} \rangle \\
\langle \textit{integer-part} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \langle \textit{digit} \rangle \\
\langle \textit{fraction} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{digit} \rangle \langle \textit{fraction} \rangle \\
\langle \textit{digit} \rangle &::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
\end{aligned}$$

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A grammar given by Sethi to specify unsigned floating point literals in a simple language.

⟨*real-number*⟩ ::= ⟨*integer-part*⟩ . ⟨*fraction*⟩
⟨*integer-part*⟩ ::= ⟨*digit*⟩ | ⟨*integer-part*⟩ ⟨*digit*⟩
⟨*fraction*⟩ ::= ⟨*digit*⟩ | ⟨*digit*⟩ ⟨*fraction*⟩
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A grammar given by Sethi to specify unsigned floating point literals in a simple language.

Grammar notation is "free format": We can insert whitespace characters, ***including newlines***, between symbols without changing the specified grammar!

For example, the 2nd and 3rd productions

<integer-part> ::= *<digit>* | *<integer-part>* *<digit>*

of the above grammar could be ***rewritten*** as:

⟨*real-number*⟩ ::= ⟨*integer-part*⟩ . ⟨*fraction*⟩
⟨*integer-part*⟩ ::= ⟨*digit*⟩ | ⟨*integer-part*⟩ ⟨*digit*⟩
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of the above grammar could be ***rewritten*** as:

<integer-part> ::= <digit>
| <integer-part> <digit>

$$
\begin{aligned}
\langle \textit{real-number} \rangle &::= \langle \textit{integer-part} \rangle \;.\; \langle \textit{fraction} \rangle \\
\langle \textit{integer-part} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \langle \textit{digit} \rangle \\
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    <integer-part> ::= *<digit>* | *<integer-part>* *<digit>*

of the above grammar could be ***rewritten*** as:

    <integer-part> ::= *<digit>*
                      | *<integer-part>* *<digit>*

Intuitively, a production *N* ::= ... means “any ... is an *N*”.

$$
\begin{aligned}
\langle \textit{real-number} \rangle &::= \langle \textit{integer-part} \rangle \;.\; \langle \textit{fraction} \rangle \\
\langle \textit{integer-part} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \langle \textit{digit} \rangle \\
\langle \textit{fraction} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{digit} \rangle \langle \textit{fraction} \rangle \\
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<real-number> is the ***starting nonterminal*** of the above grammar.

In this course, we use the convention that ***unless otherwise indicated***, the starting nonterminal of a grammar is the nonterminal on the left side of the ***first*** production:

⟨*real-number*⟩ ::= ⟨*integer-part*⟩ . ⟨*fraction*⟩
⟨*integer-part*⟩ ::= ⟨*digit*⟩ | ⟨*integer-part*⟩ ⟨*digit*⟩
⟨*fraction*⟩ ::= ⟨*digit*⟩ | ⟨*digit*⟩ ⟨*fraction*⟩
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In this course, we use the convention that ***unless otherwise indicated***, the starting nonterminal of a grammar is the nonterminal on the left side of the ***first*** production:

If you write a grammar and want *some other* nonterminal to be its starting nonterminal, you must ***explicitly indicate*** which nonterminal is the starting nonterminal!

$$
\begin{aligned}
\langle \textit{real-number} \rangle &::= \langle \textit{integer-part} \rangle \,.\, \langle \textit{fraction} \rangle \\
\langle \textit{integer-part} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{integer-part} \rangle \langle \textit{digit} \rangle \\
\langle \textit{fraction} \rangle &::= \langle \textit{digit} \rangle \mid \langle \textit{digit} \rangle \langle \textit{fraction} \rangle \\
\langle \textit{digit} \rangle &::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
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A grammar given by Sethi to specify unsigned floating point literals in a simple language.

⟨*real-number*⟩ ::= ⟨*integer-part*⟩ . ⟨*fraction*⟩
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⟨*digit*⟩ ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Figure 2.3 BNF rules for real numbers.

A grammar given by Sethi to specify unsigned floating point literals in a simple language.

<*empty*> denotes the empty string; other people write ϵ or λ to denote the empty string.

Example: Changing the 2nd production above from *<integer-part> ::= <digit>* to *<integer-part> ::= <empty>* will allow a number with ***no digits before the point*** (e.g., .213) to belong to the language of the grammar.

Note that <*empty*> is ***neither*** a terminal ***nor*** a nonterminal!

Parse Trees

Q. Exactly which sequences of terminals belong to the set of sequences of terminals that is denoted by a given nonterminal ***N*** of a grammar?

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A. A sequence of terminals t_1 ... t_k belongs to the set of sequences denoted by a nonterminal ***N*** *if and only if* there is a **parse tree with root *N* that generates** t_1 ... t_k.

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A. A sequence of terminals $t_1 \dots t_k$ belongs to the set of sequences denoted by a nonterminal ***N*** *if and only if* there is a **parse tree with root *N* that generates** $t_1 \dots t_k$.

Unless otherwise indicated, the term ***parse tree*** means *parse tree whose root is the starting nonterminal*.

So we can say that sequence of terminals $t_1 \dots t_k$ belongs to the language of a grammar *if and only if* there is a **parse tree that generates** $t_1 \dots t_k$.

Comment:

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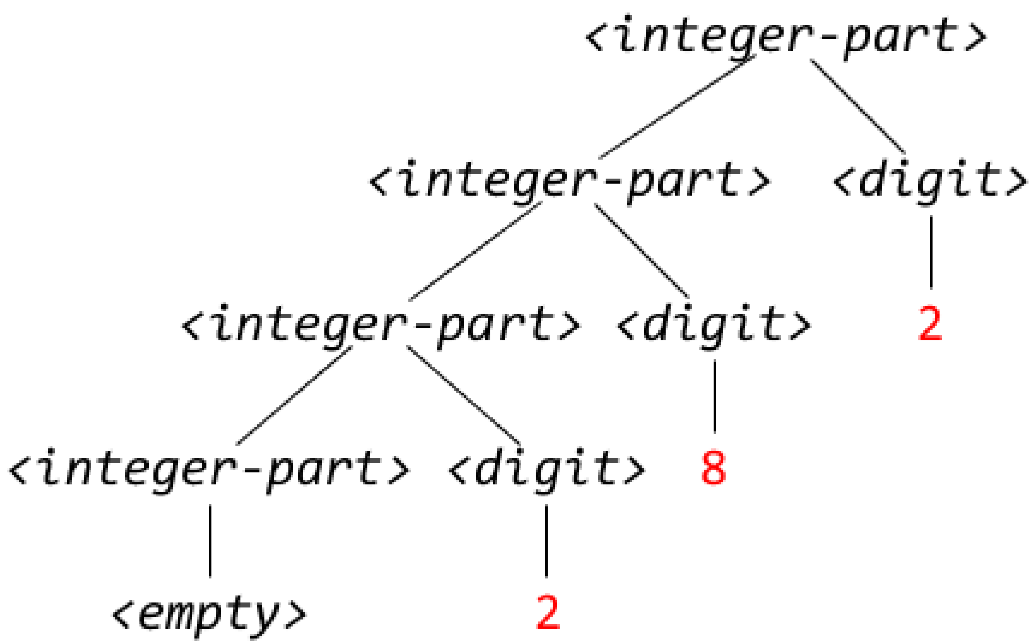
Unless otherwise indicated, the term ***parse tree*** means *parse tree* <u>*whose root is the starting nonterminal*</u>.

So we can say that sequence of terminals t_1 ... t_k belongs to the language of a grammar *if and only if* there is a **parse tree that generates** t_1 ... t_k.

Comment: Instead of using parse trees, we can also answer the above question using the concept of a <u>***derivation***</u> that is introduced on pp. 40 – 41 of Sethi.

Below is a parse tree, whose root is *<integer-part>*, that shows 282 belongs to the set of sequences denoted by *<integer-part>* in the following grammar:

<real-number> ::= *<integer-part>* . *<fraction>*
<integer-part> ::= *<empty>* | *<integer-part>* *<digit>*
<fraction> ::= *<digit>* | *<digit>* *<fraction>*
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9



Below is a parse tree that shows 282.83 belongs to the language of the same grammar:

<real-number> ::= *<integer-part>* . *<fraction>*
<integer-part> ::= *<empty>* | *<integer-part>* *<digit>*
<fraction> ::= *<digit>* | *<digit>* *<fraction>*
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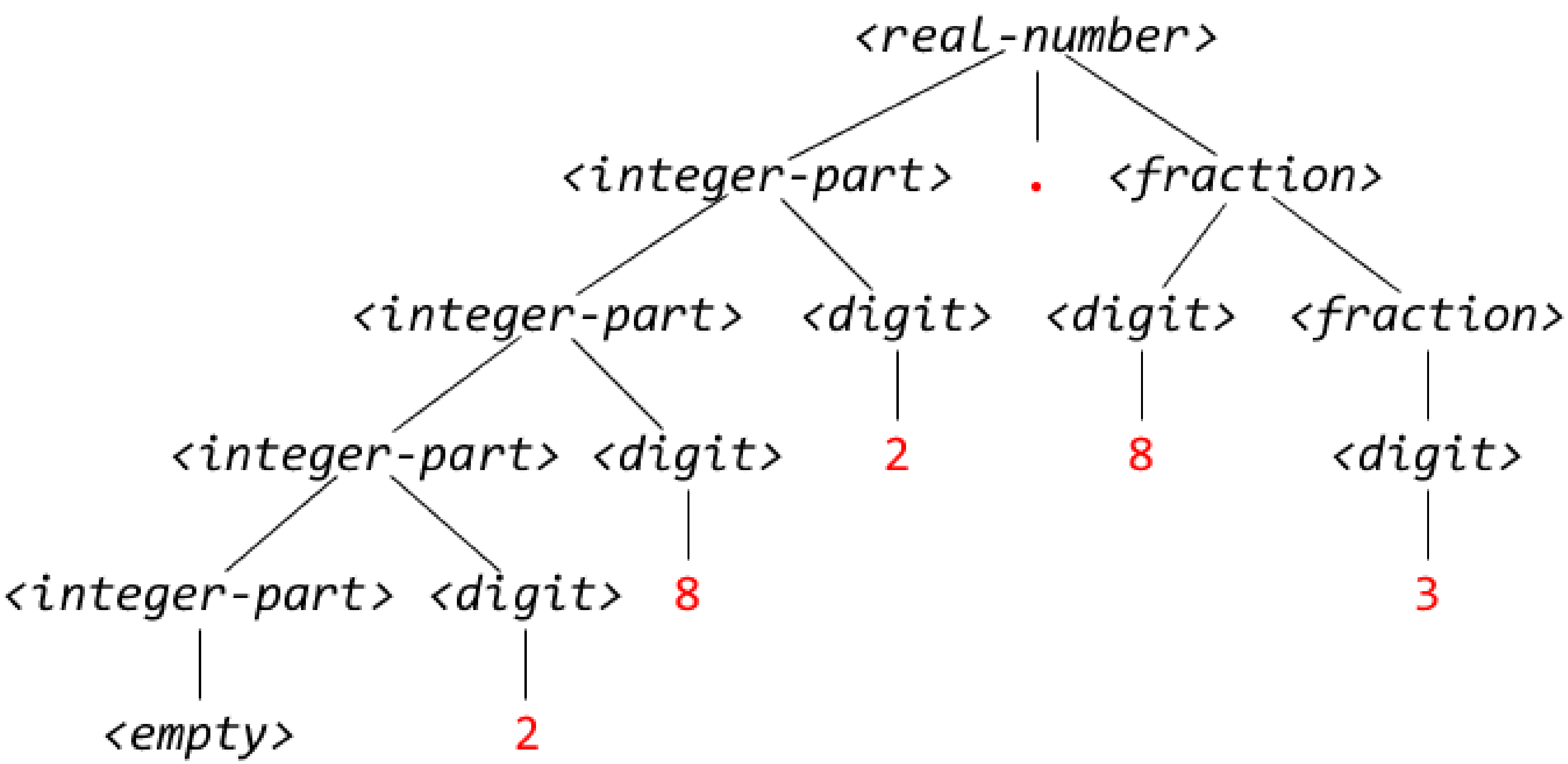

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Given terminals t_1, ..., t_k, a ***parse tree with root N that generates*** t_1 ... t_k (or ***parse tree with root N of*** t_1 ... t_k) is

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Given terminals t_1, ..., t_k, a ***parse tree with root N that generates*** t_1 ... t_k (or ***parse tree with root N of*** t_1 ... t_k) is a parse tree with root ***N*** for which the left-to-right sequence of leaves that are not <*empty*> is t_1, ..., t_k.

RECALL:

Q. Exactly which sequences of terminals belong to the set of sequences of terminals that is denoted by a given nonterminal ***N*** of a grammar?

A. A sequence of terminals t_1 ... t_k belongs to the set of sequences denoted by a nonterminal ***N*** *if and only if* there is a **parse tree with root *N* that generates** t_1 ... t_k.

Unless otherwise indicated, the term ***parse tree*** means *parse tree whose root is the starting nonterminal.*

So we can say that sequence of terminals t_1 ... t_k belongs to the language of a grammar *if and only if* there is a **parse tree that generates** t_1 ... t_k.

Comment: Instead of using parse trees, we can also answer the above question using the concept of a ***derivation*** that is introduced on pp. 40 – 41 of Sethi.

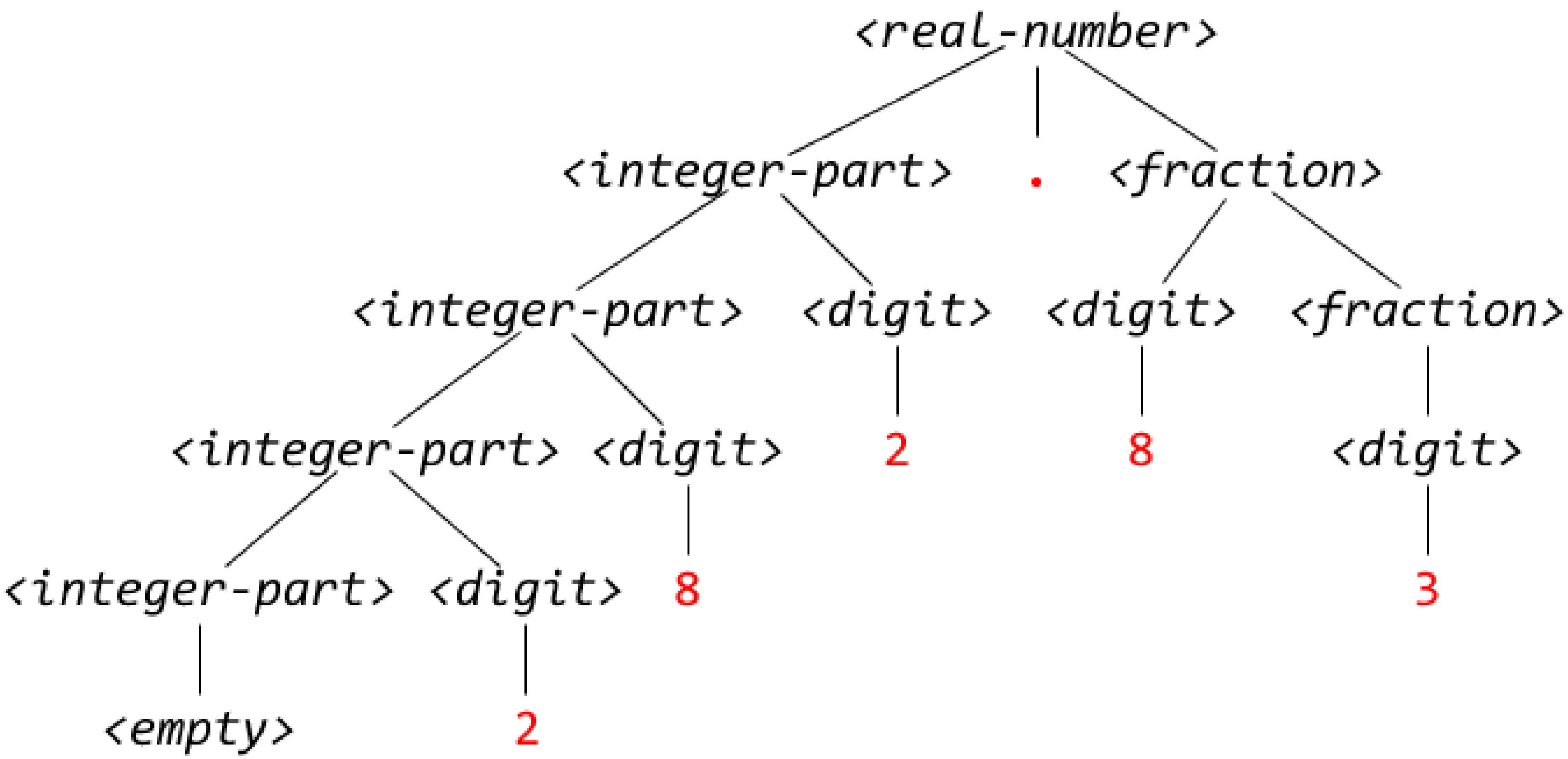
Below is a parse tree that shows 282.83 belongs to the language of the same grammar:

<real-number> ::= *<integer-part>* . *<fraction>*

<integer-part> ::= *<empty>* | *<integer-part>* *<digit>*

<fraction> ::= *<digit>* | *<digit>* *<fraction>*

<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9



Use of Grammars to Define *Syntactically* Valid Code

An important part of the work of a compiler or interpreter is ***lexical analysis*** or ***lexical scanning***.

Lexical analysis decomposes the source program into **token instances** (i.e., **instances of *tokens***).
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Each token *T* is a set of strings of characters; each member of that set is called an ***instance*** of *T*.

For Java:
3 instances of **IDENTIFIER** are: **x** **prevVal** **pi_2**
2 instances of **UNSIGNED-INT-LITERAL** are: **23** **0x1A1D**

Note:

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An important part of the work of a compiler or interpreter is ***lexical analysis*** or ***lexical scanning***.

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Note: In sec. 2.3 of Sethi, the tokens **IDENTIFIER** and **UNSIGNED-INT-LITERAL** are called **name** and **number**, and a token instance is called a ***spelling***.

If a piece of source code should be decomposed by a compiler into a sequence of token instances t_1 ... t_n in which each t_i is an instance of token T_i, we say T_1 ... T_n is the ***sequence of tokens*** of that source code.

Java Example: **IDENTIFIER = UNSIGNED-INT-LITERAL ;**
is the sequence of tokens of **x23 = 4;**

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T_1 ... T_n belongs to the language generated by G if (and, roughly speaking, only if) T_1 ... T_n is the sequence of tokens of a possibly valid L source file.

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We can then say a particular L source file is ***syntactically valid*** if its sequence of tokens belongs to the language generated by the grammar *G*.

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EBNF notation supplements BNF notation with (...), [...], and { ... } to allow simpler specifications.

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Expr ::=    + Term + Term | – Term + Term | Term + Term
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Expr ::= Term (+ | -) Term
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is equivalent to the following 2 BNF productions:

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Expr ::=    Term + Term
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```
Expr ::= [+ | -] Term (+ | -) Term
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is equivalent to

```
Expr ::= (+ | - | <empty>) Term (+ | -) Term
```

which is equivalent to these 6 BNF productions:

```
Expr ::=    + Term + Term | - Term + Term | Term + Term
          | + Term - Term | - Term - Term | Term - Term
```

```
Expr ::= Term {(+ | -) Term}
```

is equivalent to an ***infinite*** collection of BNF productions, including productions such as

```
Expr ::= Term + Term + Term - Term + Term - Term - Term
```

A grammar can only have ***finitely*** many productions. However, any EBNF rule can be translated into an equivalent ***finite*** set of BNF productions as follows.

Working from the inside outwards, eliminate all occurrences of (...), [...], and { ... }:

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- Replace each [x_1 | ... | x_k] with a new nonterminal (D, say) that is defined by these k+1 productions:

  D ::= *<empty>* | x_1 | ... | x_k
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Here k may be 1. Thus { *Digit* } can be replaced with a new nonterminal (*DigitSeq*, say) that is defined by:

DigitSeq ::= *<empty>* | *DigitSeq Digit*

Example: We now use the above method to translate

```
      Expr ::= [+ | -] Term {(+ | -) Term}        (*)
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3. Finally, replace [+ | -] with a nonterminal OptSign
   defined by OptSign ::= <*empty*> | + | -
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The result is the following set of 8 BNF productions:

Expr ::= OptSign Term Rest
OptSign ::= <*empty*> | + | -
Rest ::= <*empty*> | Rest Op Term
Op ::= + | -

While the above method is general, it will often ***not*** find a simplest BNF equivalent of the given EBNF rule.

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EBNF can be used just like BNF to define what it means for a source code file to be "***syntactically*** valid":

For many programming languages L, we can construct ~~a grammar~~ an EBNF specification *G* (whose terminals are L's tokens) such that:

T_1 ... T_n belongs to the language generated by *G*

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We can then say a particular L source file is ***syntactically valid*** if its sequence of tokens belongs to the language generated by the ~~grammar~~ EBNF specification *G*.

Note: Replacing one identifier with another and replacing a literal constant with another of the same type (e.g., changing 9/x to 3/y) will ***not*** affect the syntactic validity of a source file, as it won't change its sequence of tokens!

A Rule to Follow When Writing EBNF Specifications

In EBNF, when any of the characters | ( ) [ ] { } is a terminal, that terminal should be ***put in single quotes*** to make it clear that the character is ***not*** being used with its EBNF meaning!

Sethi says the following about this on p. 47 of his book (p. 48 of the course reader):

Symbols such as { and }, which have a special status in a language description, are called *metasymbols*.

EBNF has many more metasymbols than BNF. Furthermore, these same symbols can also appear in the syntax of a language—the index i in $A[i]$ is not optional—so care is needed to distinguish tokens from metasymbols. Confusion between tokens and metasymbols will be avoided by enclosing tokens within single quotes if needed, as in '('.

An EBNF version of the grammar in Fig. 2.6 is

$$
\begin{aligned}
\langle \textit{expression} \rangle \quad &::= \quad \langle \textit{term} \rangle \ \{\ (+ \mid -)\ \langle \textit{term} \rangle\ \} \\
\langle \textit{term} \rangle \quad &::= \quad \langle \textit{factor} \rangle \ \{\ (* \mid /)\ \langle \textit{factor} \rangle\ \} \\
\langle \textit{factor} \rangle \quad &::= \quad \text{'('}\ \langle \textit{expression} \rangle\ \text{')'} \mid \textbf{name} \mid \textbf{number}
\end{aligned}
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1. γ is a terminal, or a nonterminal, or <empty>.
2. γ is (e) for some EBNF form e.
3. γ is e_1 ... e_n for some EBNF forms e_1, ..., e_n.
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- A ::= A (α) | β is equivalent to A ::= (β) {α}
- A ::= (α) A | β is equivalent to A ::= {α} (β)

Example: Translate `Expr ::= [+ | -] Term {(+ | -) Term}` into BNF *without introducing new nonterminals*.

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`A ::=` $(\beta)\,\{\alpha\}$
if `A` is `Expr`, β is `[+ | -] Term`, and α is `(+ | -) Term`.

Example: Translate `Expr ::= [+ | -] Term {(+ | -) Term}` into BNF *without introducing new nonterminals*.

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So, since
A ::= (β) {α}   is equivalent to   A ::= A (α)
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```
Expr ::= Expr + Term | Expr - Term | + Term | - Term | Term
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2nd Example: Translate the five BNF productions

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Expr ::= Term | + Term | – Term | Expr + Term | Expr – Term
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into a non-recursive EBNF rule.

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