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Another Example

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Let e.value denote the value of e. Then:

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Example Let $+_3$ and $*_3$ be the 3-ary plus and times operators, and let $*_2$ and $-_2$ the binary times and minus operators.
Suppose that x has value 7 and that y has value 4.
We now show evaluation of: **x 2 3 y $+_3$ 1 x $*_2$ $-_2$ 5 $*_3$**

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STACK **(rightmost item = topmost item):**

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UNREAD **INPUT:** 3 **y $+_3$ 1 x $*_2$ $-_2$ 5 $*_3$**

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- When a k-ary operator **op** is seen, *pop* off k values, *apply* **op** to those values (with the i^{th}-last value to be popped as the i^{th} argument), and *push* the result.

After the entire expression has been processed in this way, the value of the expression will be the only thing on the stack.

Example Let $+_3$ and $*_3$ be the 3-ary plus and times operators, and let $*_2$ and $-_2$ the binary times and minus operators.
Suppose that x has value 7 and that y has value 4.
We now show evaluation of: **x 2 3 y $+_3$ 1 x $*_2$ $-_2$ 5 $*_3$**

***UNREAD* INPUT:** x $*_2$ $-_2$ 5 $*_3$

STACK (<u>rightmost</u> item = topmost item)**: 7 9 1 7**

Evaluation of Postfix Expressions Using a Stack

Postfix expressions can be evaluated as follows:

- Read the expression *from left to right*.
- When a variable or constant is seen, *push* its value.
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We now show evaluation of: **x 2 3 y $+_3$ 1 x $*_2$ $-_2$ 5 $*_3$**

***UNREAD* INPUT:** $*_2$ $-_2$ 5 $*_3$

STACK (rightmost item = topmost item)**: 7 9 1 7 7**

Evaluation of Postfix Expressions Using a Stack

Postfix expressions can be evaluated as follows:

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Suppose that x has value 7 and that y has value 4.
We now show evaluation of: **x 2 3 y $+_3$ 1 x $*_2$ $-_2$ 5 $*_3$**

UNREAD **INPUT:** $-_2$ **5 $*_3$**

STACK (**<u>rightmost</u> item = topmost item**)**: 7** 9 7 **2**

Evaluation of Postfix Expressions Using a Stack

Postfix expressions can be evaluated as follows:

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***UNREAD* INPUT:** 5 $*_3$

STACK (<u>rightmost</u> item = topmost item)**: 7 2 5**

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***UNREAD* INPUT:** $*_3$

STACK (<u>rightmost</u> item = topmost item): 7 2 5 **70**

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We now show evaluation of: **x 2 3 y $+_3$ 1 x $*_2$ $-_2$ 5 $*_3$**

value of expression

UNREAD **INPUT:**

STACK (rightmost item = topmost item): **70**

Evaluation of Prefix Expressions Using a Stack

Prefix expressions can be evaluated as follows:

Evaluation of Prefix Expressions Using a Stack

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Suppose that x has value 7 and that y has value 4.
We now show evaluation of: $\mathbf{*_3\ x\ -_2\ +_3\ 2\ 3\ y\ *_2\ 1\ x\ 5}$

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***UNREAD* INPUT:** $*_3$ **x** $-_2$ $+_3$ **2 3 y** $*_2$ **1 x 5**

STACK (leftmost item = topmost item):

Evaluation of Prefix Expressions Using a Stack

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***UNREAD* INPUT:** $*_3$ **x** $-_2$ $+_3$ **2 3 y** $*_2$ **1 x** 5

STACK (leftmost item = topmost item)**:** **5**

Evaluation of Prefix Expressions Using a Stack

Prefix expressions can be evaluated as follows:

- Read the expression *from right to left*.
- When a variable or constant is seen, *push* its value.
- When a k-ary operator **op** is seen, *pop* off k values, *apply* **op** to those values (with the i^{th} value to be popped as the i^{th} argument), and *push* the result.

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We now show evaluation of: $*_3$ **x** $-_2$ $+_3$ **2 3 y** $*_2$ **1 x 5**

***UNREAD* INPUT:** $*_3$ **x** $-_2$ $+_3$ **2 3 y** $*_2$ **1** x

STACK (leftmost item = topmost item)**:** **7 5**

Evaluation of Prefix Expressions Using a Stack

Prefix expressions can be evaluated as follows:

- Read the expression *from right to left*.
- When a variable or constant is seen, *push* its value.
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Example Let $+_3$ and $*_3$ be the 3-ary plus and times operators, and let $*_2$ and $-_2$ the binary times and minus operators. Suppose that x has value 7 and that y has value 4. We now show evaluation of: $*_3$ x $-_2$ $+_3$ 2 3 y $*_2$ 1 x 5

***UNREAD* INPUT:** $*_3$ x $-_2$ $+_3$ 2 3 y $*_2$ 1

STACK (leftmost item = topmost item)**:** **1 7 5**

Evaluation of Prefix Expressions Using a Stack

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We now show evaluation of: $*_3$ **x** $-_2$ $+_3$ **2 3 y** $*_2$ **1 x 5**

UNREAD **INPUT:** $*_3$ **x** $-_2$ $+_3$ **2 3 y** $*_2$

STACK (leftmost item = topmost item)**:** **7 1 7 5**

Evaluation of Prefix Expressions Using a Stack

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***UNREAD* INPUT:** $\mathbf{*_3\ x\ -_2\ +_3\ 2\ 3}$ y

STACK (leftmost item = topmost item): 4 7 5

Evaluation of Prefix Expressions Using a Stack

Prefix expressions can be evaluated as follows:

- Read the expression *from right to left*.
- When a variable or constant is seen, *push* its value.
- When a *k*-ary operator **op** is seen, *pop* off *k* values, *apply* **op** to those values (with the i^{th} value to be popped as the i^{th} argument), and *push* the result.

After the entire expression has been processed in this way, the value of the expression will be the only thing on the stack.

Example Let $+_3$ and $*_3$ be the 3-ary plus and times operators, and let $*_2$ and $-_2$ the binary times and minus operators. Suppose that x has value 7 and that y has value 4.
We now show evaluation of: $*_3$ **x** $-_2$ $+_3$ **2 3 y** $*_2$ **1 x 5**

***UNREAD* INPUT:** $*_3$ **x** $-_2$ $+_3$ **2** 3

STACK (leftmost item = topmost item)**:** **3 4 7 5**

Evaluation of Prefix Expressions Using a Stack

Prefix expressions can be evaluated as follows:

- Read the expression *from right to left*.
- When a variable or constant is seen, *push* its value.
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We now show evaluation of: $\mathbf{*_3\ x\ -_2\ +_3\ 2\ 3\ y\ *_2\ 1\ x\ 5}$

***UNREAD* INPUT:** $\mathbf{*_3\ x\ -_2\ +_3}$ 2

STACK (leftmost item = topmost item)**:** **2 3 4 7 5**

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UNREAD **INPUT:** $*_3$ **x** $-_2$ $+_3$

STACK (leftmost item = topmost item)**:** **9** 2 3 4 **7** **5**

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We now show evaluation of: $*_3$ **x** $-_2$ $+_3$ **2 3 y** $*_2$ **1 x 5**

UNREAD **INPUT:** $*_3$ **x** $-_2$

STACK (leftmost item = topmost item): **2** 9 7 **5**

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***UNREAD* INPUT:** $\mathbf{*_3}$ x

STACK (leftmost item = topmost item)**:** **7 2 5**

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***UNREAD* INPUT:** $*_3$

STACK (leftmost item = topmost item)**: 70** 7 2 5

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***UNREAD* INPUT:**

value of expression

STACK (leftmost item = topmost item): 70

Translating Prefix/Postfix Notations to Lisp/“rpnLisp”

Recall:

- ***Prefix*** notation = ***Lisp*** *notation* *without parentheses*.
- ***Postfix*** notation = “***rpnLisp***” *notation* *without parentheses*.

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Lisp: $(*_3\ x\ (-_2\ (+_3\ 2\ 3\ y)\ (*_2\ w\ x)\ )\ 5\ )$

rpnLisp: $(\ x\ (\ (2\ 3\ y\ +_3)\ (w\ x\ *_2)\ -_2)\ 5\ *_3)$

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Prefix notation:	$*_3\ x\ -_2\ +_3\ 2\ 3\ y\ *_2\ w\ x\ 5$
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Q. Given a prefix / postfix expression, how can we insert parentheses to produce an equivalent Lisp / rpnLisp expression?

A.

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Notation: We will write $\boxed{\textbf{op}\ e_1\ \ldots\ e_k}$ and $\boxed{e_1\ \ldots\ e_k\ \textbf{op}}$ for the Lisp and rpnLisp expressions $(\textbf{op}\ e_1\ \ldots\ e_k)$ and $(e_1\ \ldots\ e_k\ \textbf{op})$.

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The Lisp expression $(*_3\ x\ (-_2\ (+_3\ 2\ 3\ y)\ (*_2\ w\ x))\ 5)$

will be written $\boxed{*_3\ x\ \boxed{-_2\ \boxed{+_3\ 2\ 3\ y}\ \boxed{*_2\ w\ x}}\ 5}$

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The Lisp expression $(*_3\ x\ (-_2\ (+_3\ 2\ 3\ y)\ (*_2\ w\ x))\ 5)$

will be written $\boxed{*_3\ \ x\ \ \boxed{-_2\ \ \boxed{+_3\ 2\ 3\ y}\ \ \boxed{*_2\ w\ x}}\ \ 5}$

The rpnLisp expression $(x\ ((2\ 3\ y\ +_3)\ (w\ x\ *_2)\ -_2)\ 5\ *_3)$

will be written $\boxed{x\ \ \boxed{\boxed{2\ 3\ y\ +_3}\ \ \boxed{w\ x\ *_2}\ \ -_2}\ \ 5\ *_3}$

We can use a stack as follows to translate a prefix expression to Lisp:

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We can use a stack as follows to translate a prefix expression to Lisp:

- Read the expression *from right to left*.
- *Push* each variable or constant that is seen.
- Whenever a *k*-ary operator **op** is seen:
  - *Pop* off *k* expressions e_1, ..., e_k.
  - *Push* the Lisp expression $\boxed{\textbf{op}\ e_1\ \ldots\ e_k}$.

We can use a stack as follows to translate a prefix expression to Lisp:

- Read the expression *from right to left*.
- *Push* each variable or constant that is seen.
- Whenever a *k*-ary operator **op** is seen:
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After the entire expression has been processed in this way, the Lisp equivalent of the prefix expression will be the only thing on the stack.

Example

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Example Translate the following prefix expression into Lisp.

$$*_3\ \ x\ \ -_2\ \ +_3\ 2\ \ 3\ \ y\ \ *_2\ w\ \ x\ \ 5$$

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

We can use a stack as follows to translate a prefix expression to Lisp:

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$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2 \quad w \quad x \quad 5$

STACK:

We can use a stack as follows to translate a prefix expression to Lisp:

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***UNREAD* INPUT:** $*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2 \quad w \quad x \quad 5$

STACK: **5**

We can use a stack as follows to translate a prefix expression to Lisp:

- Read the expression *from right to left*.
- *Push* each variable or constant that is seen.
- Whenever a k-ary operator **op** is seen:
  - *Pop* off k expressions $e_1, \ldots, e_k$.
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Example Translate the following prefix expression into Lisp.

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$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2 \quad w \quad x$

STACK: **x 5**

We can use a stack as follows to translate a prefix expression to Lisp:

- Read the expression *from right to left*.
- *Push* each variable or constant that is seen.
- Whenever a k-ary operator **op** is seen:
  - *Pop* off k expressions $e_1, \ldots, e_k$.
  - *Push* the Lisp expression $\boxed{\textbf{op}\ e_1 \ldots e_k}$.

After the entire expression has been processed in this way, the Lisp equivalent of the prefix expression will be the only thing on the stack.

Example Translate the following prefix expression into Lisp.

$$*_3\ \ x\ \ -_2\ \ +_3\ \ 2\ \ 3\ \ y\ \ *_2\ \ w\ \ x\ \ 5$$

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3\ \ x\ \ -_2\ \ +_3\ \ 2\ \ 3\ \ y\ \ *_2\ \ w$

STACK: **w x 5**

We can use a stack as follows to translate a prefix expression to Lisp:

- Read the expression *from right to left*.
- *Push* each variable or constant that is seen.
- Whenever a k-ary operator **op** is seen:
  - *Pop* off k expressions $e_1, \ldots, e_k$.
  - *Push* the Lisp expression $\boxed{\mathbf{op}\ e_1\ \ldots\ e_k}$.

After the entire expression has been processed in this way, the Lisp equivalent of the prefix expression will be the only thing on the stack.

Example Translate the following prefix expression into Lisp.

$$*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2 \quad w \quad x \quad 5$$

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3 \quad x \quad -_2 \quad +_3 \quad 2 \quad 3 \quad y \quad *_2$

STACK: $\boxed{*_2 \quad w \quad x} \quad 5$

We can use a stack as follows to translate a prefix expression to Lisp:

- Read the expression *from right to left*.
- *Push* each variable or constant that is seen.
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***UNREAD* INPUT:** $*_3\ \ x\ \ -_2\ \ +_3\ \ 2\ \ 3\ \ y$

STACK: $y\ \ \boxed{*_2\ \ w\ \ x}\ \ 5$

We can use a stack as follows to translate a prefix expression to Lisp:

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Example Translate the following prefix expression into Lisp.

$*_3$ x $-_2$ $+_3$ 2 3 y $*_2$ w x 5

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3$ x $-_2$ $+_3$ 2 3

STACK: 3 y $\boxed{*_2\ \text{w}\ \text{x}}$ 5

We can use a stack as follows to translate a prefix expression to Lisp:

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***UNREAD* INPUT:** $*_3$ x $-_2$ $+_3$ 2

STACK: 2 3 y $\boxed{*_2\ w\ x}$ 5

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***UNREAD* INPUT:** $*_3\ \ x\ \ -_2\ \ +_3$

STACK: $\boxed{+_3\ 2\ 3\ y}\ \ \boxed{*_2\ w\ x}\ \ 5$

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$$*_3\ \ x\ \ -_2\ \ +_3\ \ 2\ \ 3\ \ y\ \ *_2\ \ w\ \ x\ \ 5$$

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3\ \ x\ \ -_2$

STACK: $\boxed{-_2\ \boxed{+_3\ 2\ \ 3\ \ y}\ \boxed{*_2\ w\ \ x}}\ \ 5$

We can use a stack as follows to translate a prefix expression to Lisp:

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$$*_3\ \ x\ \ -_2\ \ +_3\ \ 2\ \ 3\ \ y\ \ *_2\ \ w\ \ x\ \ 5$$

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3$ x

STACK: x $\boxed{-_2\ \boxed{+_3\ 2\ 3\ y}\ \boxed{*_2\ w\ x}}$ 5

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$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:** $*_3$

STACK: $\boxed{*_3 \quad x \quad \boxed{-_2 \quad \boxed{+_3 \quad 2 \quad 3 \quad y} \quad \boxed{*_2 \quad w \quad x}} \quad 5}$

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Example Translate the following prefix expression into Lisp.

$$*_3\quad x\quad -_2\quad +_3\quad 2\quad 3\quad y\quad *_2\quad w\quad x\quad 5$$

$+_3$ and $*_3$ are 3-ary operators; $*_2$ and $-_2$ are binary operators.

***UNREAD* INPUT:**

STACK: $\boxed{*_3\quad x\quad \boxed{-_2\quad \boxed{+_3\quad 2\quad 3\quad y}\quad \boxed{*_2\quad w\quad x}}\quad 5}$

Lisp equivalent of the expression

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After the entire expression has been processed in this way, the “rpnLisp” equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $\mathbf{x \ 2 \ 3 \ +_3 \ y \ -_2 \ u \ x \ 5 \ *_2 \ *_3 \ -_1}$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

We can use a stack as follows to translate a postfix expression to “rpnLisp”:

- Read the expression *from left to right*.
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Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $\mathbf{x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

STACK:

We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
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After the entire expression has been processed in this way, the "rpnLisp" equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: **x 2 3 $+_3$ y $-_2$ u x 5 $*_2$ $*_3$ $-_1$**

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** x **2 3 $+_3$ y $-_2$ u x 5 $*_2$ $*_3$ $-_1$**

STACK: **x**

We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a k-ary operator **op** is seen:
  - *Pop* off k expressions $e_k, \ldots, e_1$.
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After the entire expression has been processed in this way, the "rpnLisp" equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $\mathbf{x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $\mathbf{2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

STACK: **x 2**

We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a k-ary operator **op** is seen:
  - o *Pop* off k expressions $e_k, \ldots, e_1$.
  - o *Push* the rpnLisp expr $\boxed{e_1 \ \ldots \ e_k \ \mathbf{op}}$.

After the entire expression has been processed in this way, the "rpnLisp" equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: **x 2 3 $+_3$ y $-_2$ u x 5 $*_2$ $*_3$ $-_1$**

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** 3 **$+_3$ y $-_2$ u x 5 $*_2$ $*_3$ $-_1$**

STACK: **x 2 3**

We can use a stack as follows to translate a postfix expression to “rpnLisp”:

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a *k*-ary operator **op** is seen:
  - *Pop* off *k* expressions $e_k, \ldots, e_1$.
  - *Push* the rpnLisp expr $\boxed{e_1 \ldots e_k \textbf{ op}}$.

After the entire expression has been processed in this way, the “rpnLisp” equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $x\ 2\ 3\ +_3\ y\ -_2\ u\ x\ 5\ *_2\ *_3\ -_1$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $+_3\ y\ -_2\ u\ x\ 5\ *_2\ *_3\ -_1$

STACK: $\boxed{x\ 2\ 3\ +_3}$

We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a k-ary operator **op** is seen:
    - *Pop* off k expressions $e_k, \ldots, e_1$.
    - *Push* the rpnLisp expr $\boxed{e_1 \ \ldots \ e_k \ \mathbf{op}}$.

After the entire expression has been processed in this way, the "rpnLisp" equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1$

STACK: $\boxed{x \quad 2 \quad 3 \quad +_3} \quad y$

We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a *k*-ary operator **op** is seen:
  - o *Pop* off *k* expressions e_k, ..., e_1.
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Example Translate the following postfix expression into rpnLisp: $\mathbf{x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $\mathbf{-_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

STACK: $\boxed{\boxed{\mathbf{x \quad 2 \quad 3 \quad +_3}} \quad \mathbf{y \quad -_2}}$

We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a *k*-ary operator **op** is seen:
    - o *Pop* off *k* expressions $e_k, \ldots, e_1$.
    - o *Push* the rpnLisp expr $\boxed{e_1 \ \ldots \ e_k \ \mathbf{op}}$.

After the entire expression has been processed in this way, the "rpnLisp" equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $x \ 2 \ 3 \ +_3 \ y \ -_2 \ u \ x \ 5 \ *_2 \ *_3 \ -_1$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $u \ x \ 5 \ *_2 \ *_3 \ -_1$

STACK: $\boxed{\boxed{x \ 2 \ 3 \ +_3} \ y \ -_2} \ u$

We can use a stack as follows to translate a postfix expression to “rpnLisp”:

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a k-ary operator **op** is seen:
  - *Pop* off k expressions $e_k, \ldots, e_1$.
  - *Push* the rpnLisp expr $\boxed{e_1 \ \ldots \ e_k \ \mathbf{op}}$.

After the entire expression has been processed in this way, the “rpnLisp” equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $\mathbf{x \ 2 \ 3 \ +_3 \ y \ -_2 \ u \ x \ 5 \ *_2 \ *_3 \ -_1}$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $\mathbf{x \ 5 \ *_2 \ *_3 \ -_1}$

STACK: $\boxed{\boxed{\mathbf{x \ 2 \ 3 \ +_3}} \ \mathbf{y \ -_2}} \ \mathbf{u \ x}$

We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a *k*-ary operator **op** is seen:
  - *Pop* off *k* expressions $e_k, \ldots, e_1$.
  - *Push* the rpnLisp expr $\boxed{e_1 \ \ldots \ e_k \ \mathbf{op}}$.

After the entire expression has been processed in this way, the "rpnLisp" equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $\mathbf{x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1}$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $\mathbf{5 \quad *_2 \quad *_3 \quad -_1}$

STACK: $\boxed{\boxed{\mathbf{x \quad 2 \quad 3 \quad +_3}} \quad \mathbf{y \quad -_2}} \quad \mathbf{u \quad x \quad 5}$

We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
- *Push* each variable or constant that is seen.
- Whenever a *k*-ary operator **op** is seen:
    - o *Pop* off *k* expressions $e_k, \ldots, e_1$.
    - o *Push* the rpnLisp expr $\boxed{e_1 \ldots e_k \text{ op}}$.

After the entire expression has been processed in this way, the "rpnLisp" equivalent of the postfix expression will be the only thing on the stack.

Example Translate the following postfix expression into rpnLisp: $x \quad 2 \quad 3 \quad +_3 \quad y \quad -_2 \quad u \quad x \quad 5 \quad *_2 \quad *_3 \quad -_1$

Here $+_3$ and $*_3$ are 3-ary, $*_2$ and $-_2$ are binary, and $-_1$ is unary.

***UNREAD* INPUT:** $*_2 \quad *_3 \quad -_1$

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We can use a stack as follows to translate a postfix expression to "rpnLisp":

- Read the expression *from left to right*.
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that we solved above: Substituting $@_3$, $!_3$, $\hat{}_2$, $\#_2$, and $\sim_1$ for $+_3$, $*_3$, $*_2$, $-_2$, and $-_1$ in our solution to the latter problem gives a solution to the former problem.

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  However, a prefix or postfix expression may be ambiguous if you don't know the arities of operators.

- In prefix and postfix notations, operators are ***not*** divided into different precedence classes.

- In prefix and postfix notations, there is no concept of left- or right-associativity.

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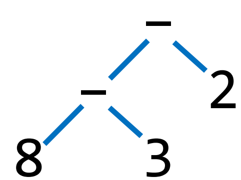

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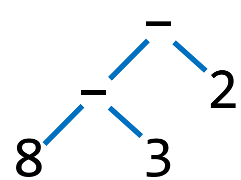

Two expressions are said to be *equivalent* if they have the same AST.

Thus the above three expressions are equivalent.

Note that ASTs do ***not*** have parentheses as nodes!

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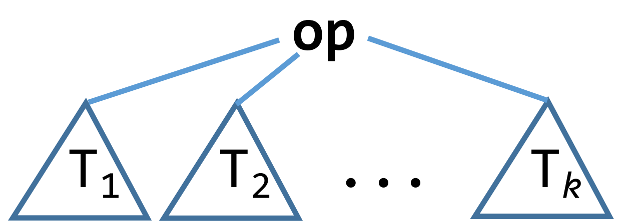
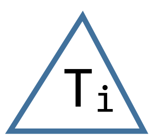


where 

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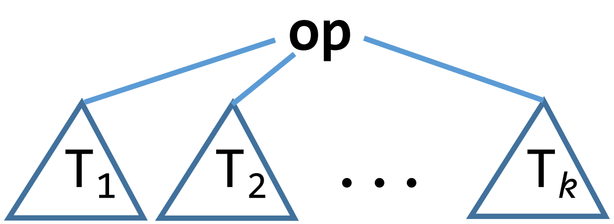
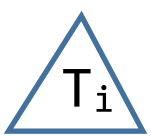


where 

is the AST of the subexpression e_i.

ASTs of *infix* expressions are binary trees, because infix notation doesn’t allow operators of arity > 2.

Example: Draw the AST of the following expression, which is written in Lisp notation:

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(& x (^ (# 2 4 z) (F u (% 3))) ($ y t) 9)
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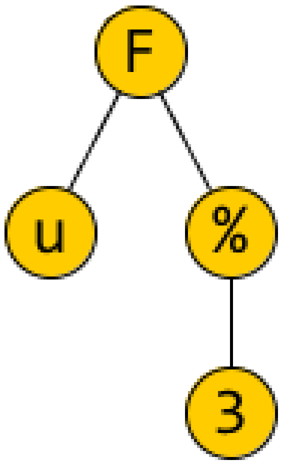

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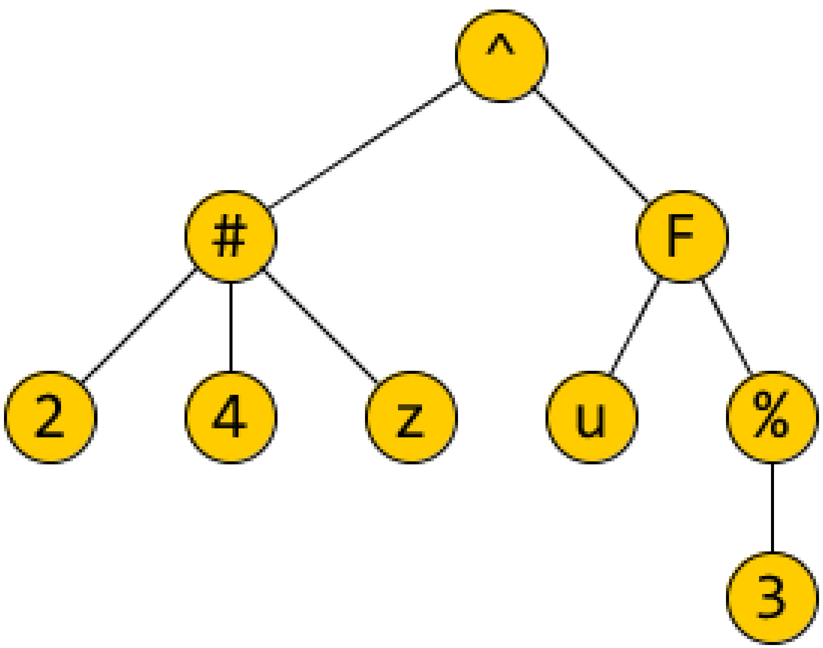

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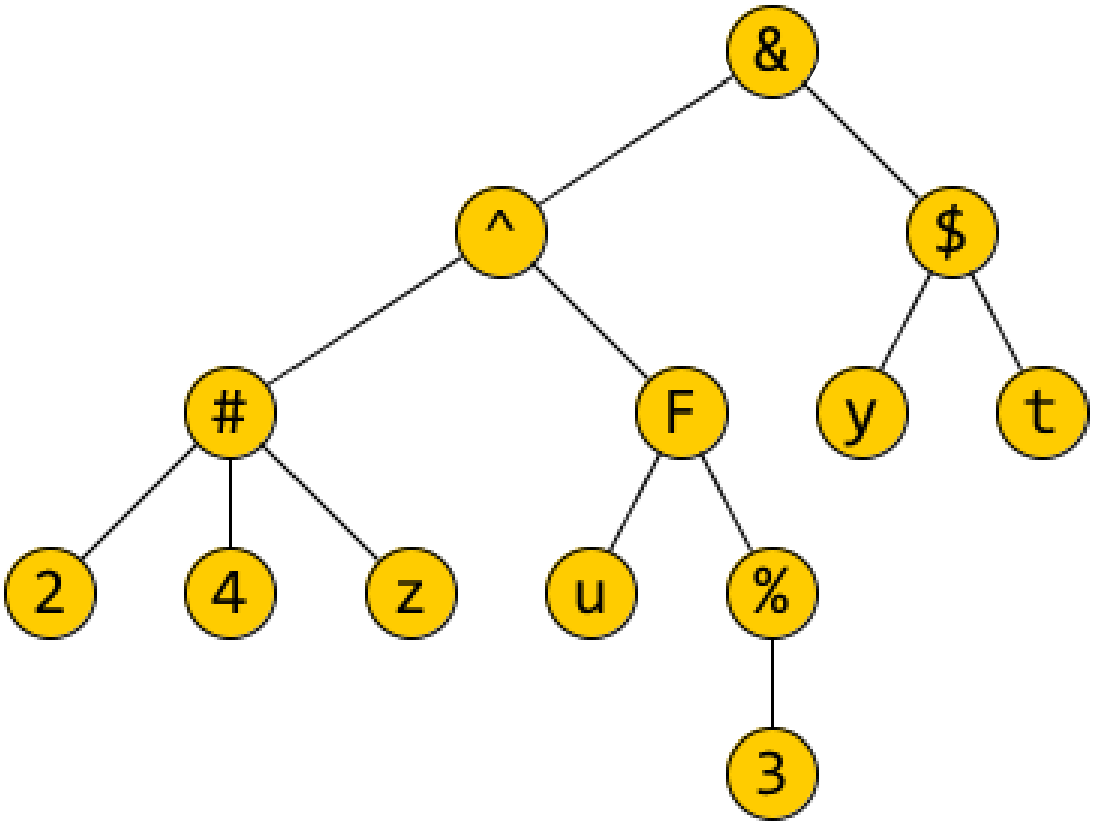

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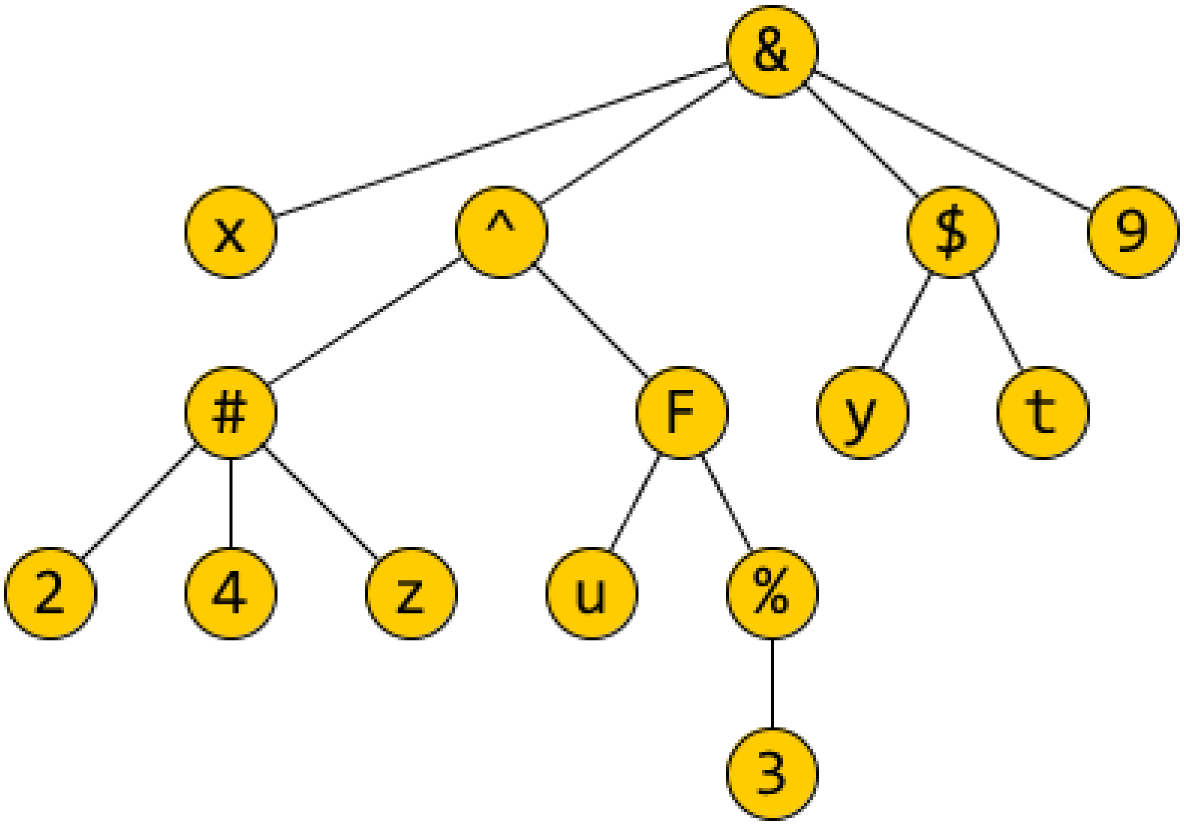

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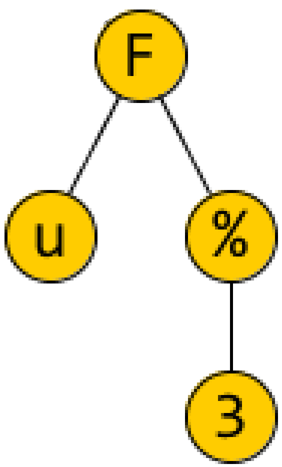

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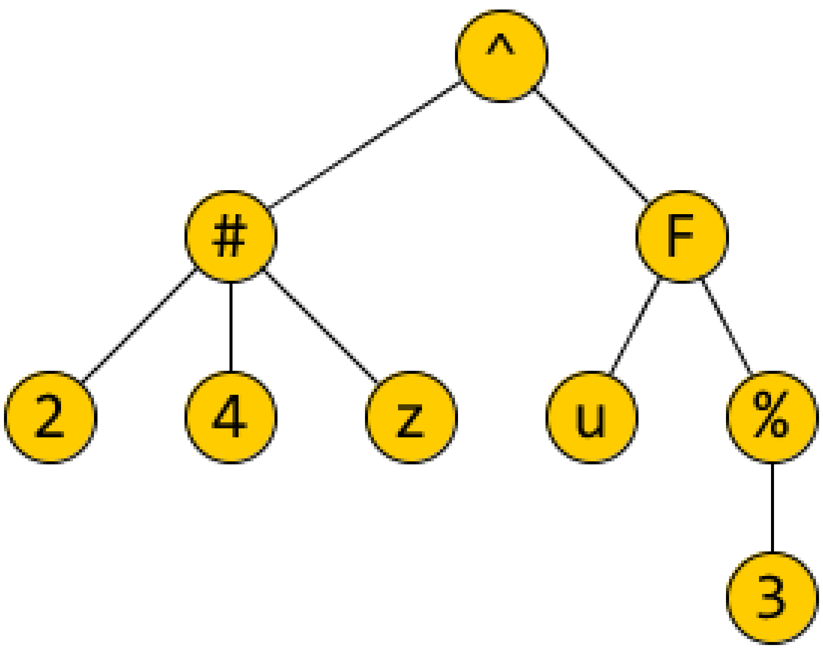

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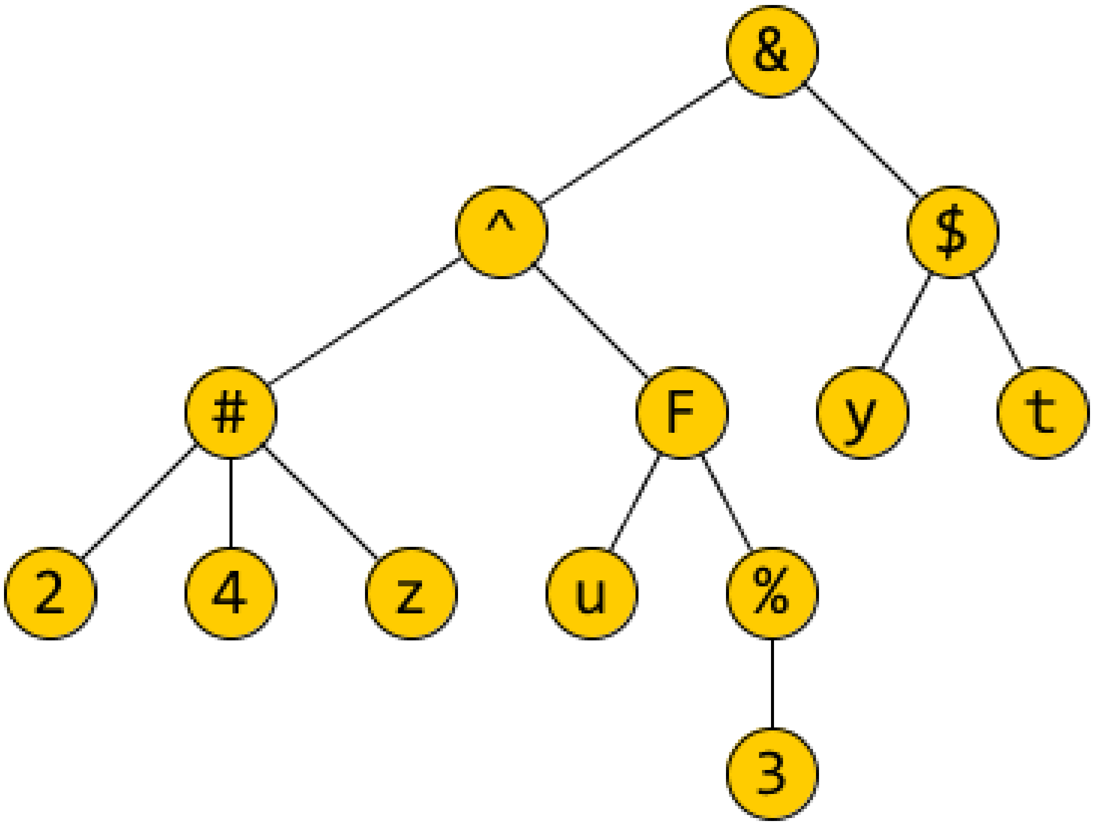

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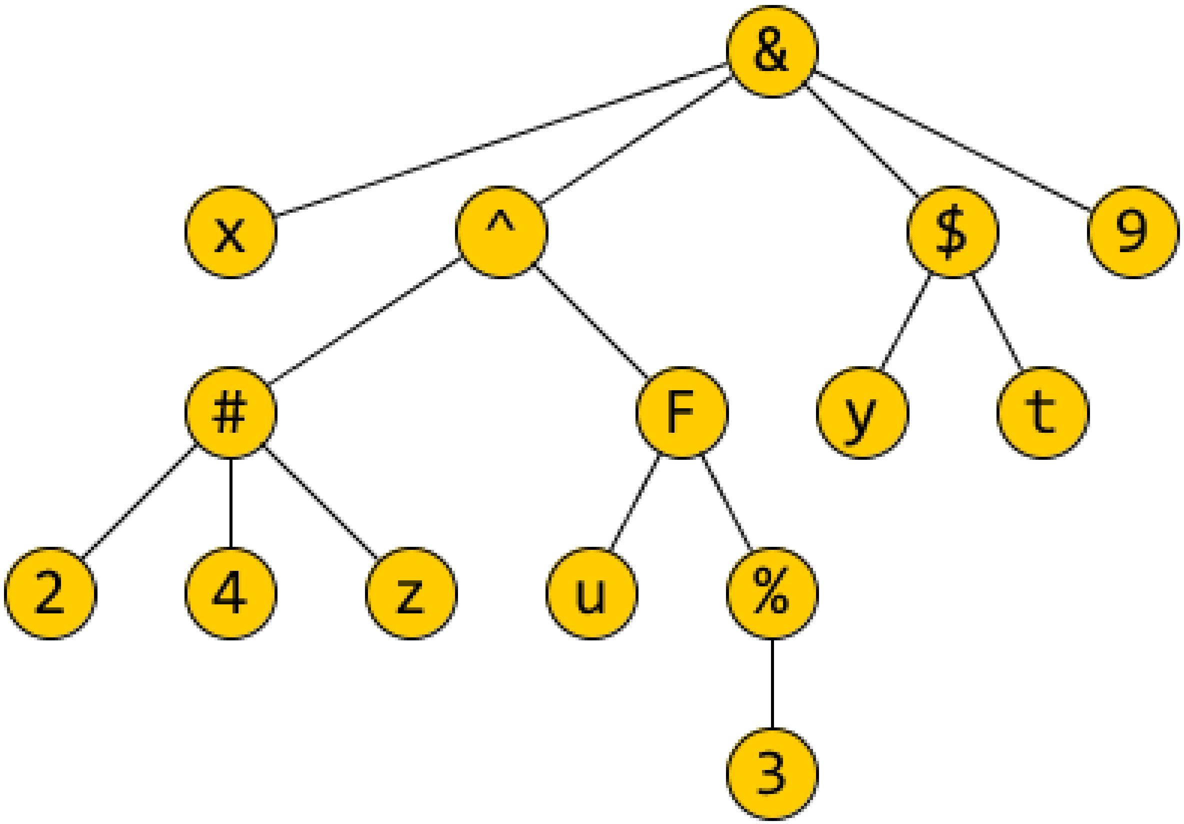

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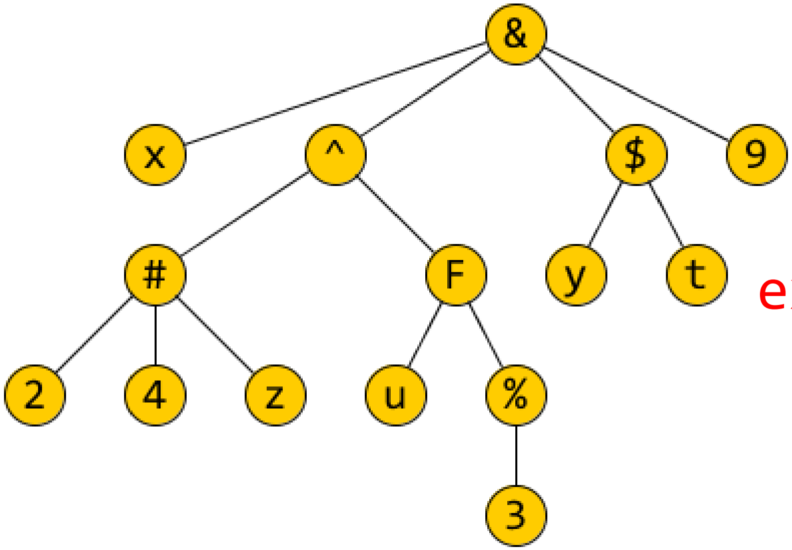

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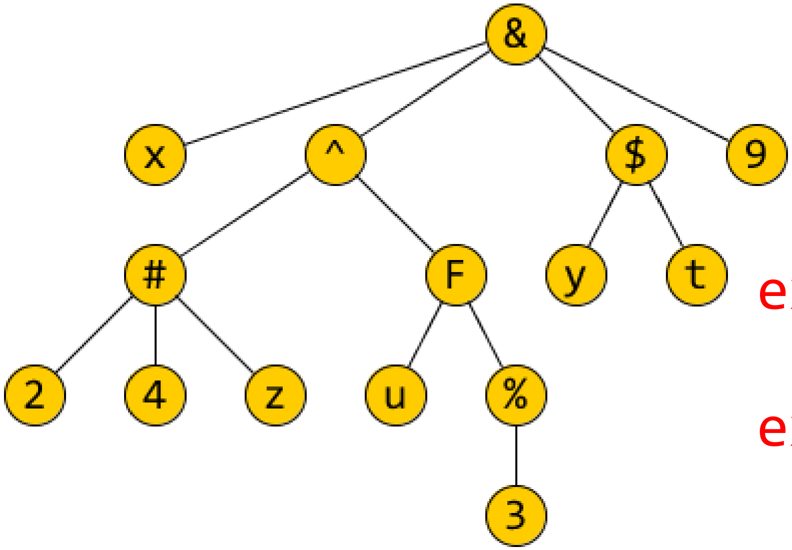

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Example: Draw the AST of the infix expression

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y − 1

assuming the operators' precedence classses are as follows:

	prefix unary ops	**binary ops**	**associativity**
Class 1	~		*right*-associative
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For $1 \le i < 4$, class *i* has ***higher*** precedence than class *i*+1.

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y - 1

Two of the subexpressions in blue boxes have more than one operator. In each case, find the operator that's applied last and its operands:

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y - 1

Example: Draw the AST of the infix expression

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y – 1

assuming the operators' precedence classses are as follows:

	prefix unary ops	**binary ops**	**associativity**
Class 1	~		*right*-associative
Class 2	+ –	+ –	*left*-associative
Class 3		& ^ @	*right*-associative
Class 4		# $	*left*-associative

For $1 \leq i < 4$, class *i* has ***higher*** precedence than class *i*+1.

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y – 1

Example: Draw the AST of the infix expression

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Class 1	~		*right*-associative
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For $1 \leq i < 4$, class *i* has ***higher*** precedence than class *i*+1.

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y - 1

One of the subexpressions in the inner red boxes has more than one operator. Find the operator of that subexpression that's applied last, and its operands:

Example: Draw the AST of the infix expression

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y - 1

assuming the operators' precedence classses are as follows:

	prefix unary ops	binary ops	associativity
Class 1	~		*right*-associative
Class 2	+ −	+ -	*left*-associative
Class 3		& ^ @	*right*-associative
Class 4		# $	*left*-associative

For $1 \leq i < 4$, class *i* has ***higher*** precedence than class *i*+1.

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y - 1

One of the subexpressions in the inner red boxes has more than one operator. Find the operator of that subexpression that's applied last, and its operands:

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Example: Draw the AST of the infix expression

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y - 1

$$\boxed{+\ \ x}\ \ @\ \ \boxed{\boxed{\boxed{z}\ \ \#\ \ \boxed{\boxed{\sim\ \ y}\ \ \hat{}\ \ \boxed{z}}}\ \ \&\ \ \boxed{\boxed{\boxed{a}\ \ @\ \ \boxed{\boxed{\sim\ \ z}\ \ \hat{}\ \ \boxed{x}}}\ \ \&\ \ \boxed{y\ \ -\ \ 1}}}$$

Example: Draw the AST of the infix expression

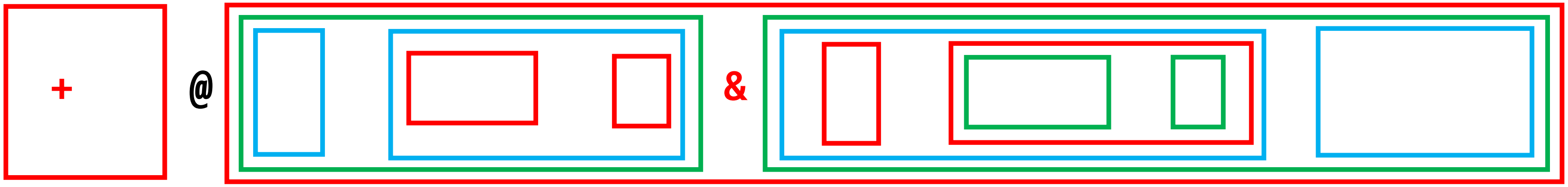
+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y - 1

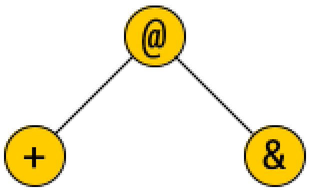
+ x @ z # ~ y ^ z & a @ ~ z ^ x & y - 1

As the subexpressions in the innermost boxes each have at most one operator, it now is straightforward to draw the AST of the entire expression:

Example: Draw the AST of the infix expression

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y – 1





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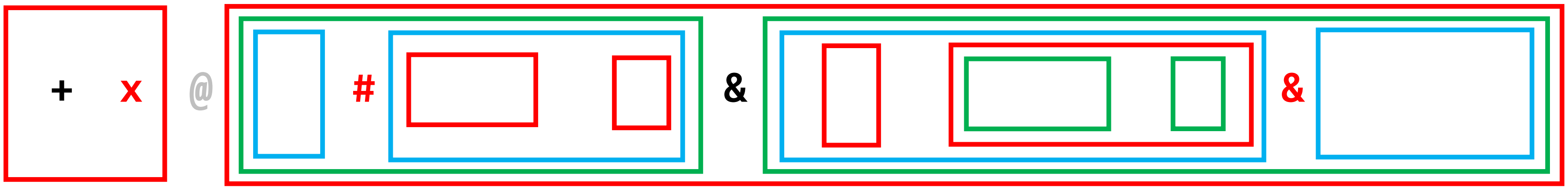

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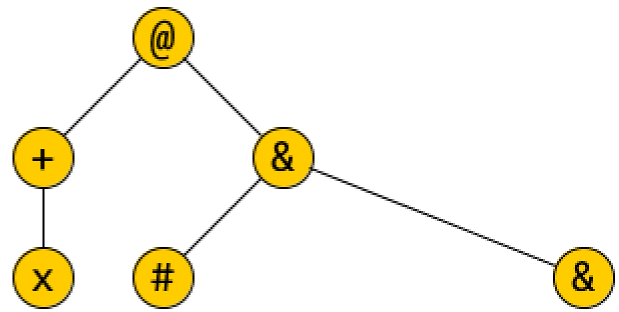

Example: Draw the AST of the infix expression

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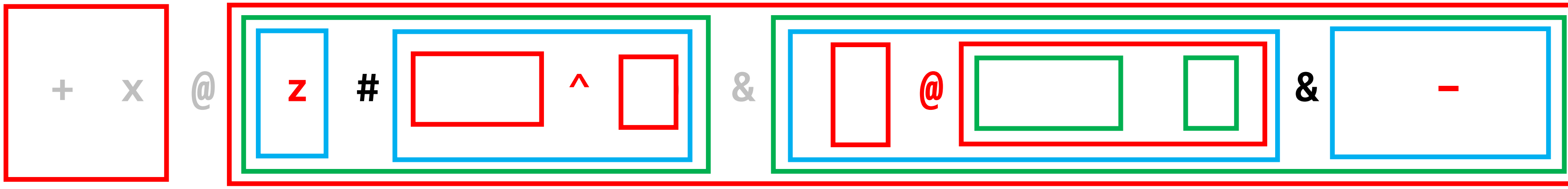
+ x @ (z # ~ y ^ z) & ( a @ ~ z ^ x) & y - 1
```



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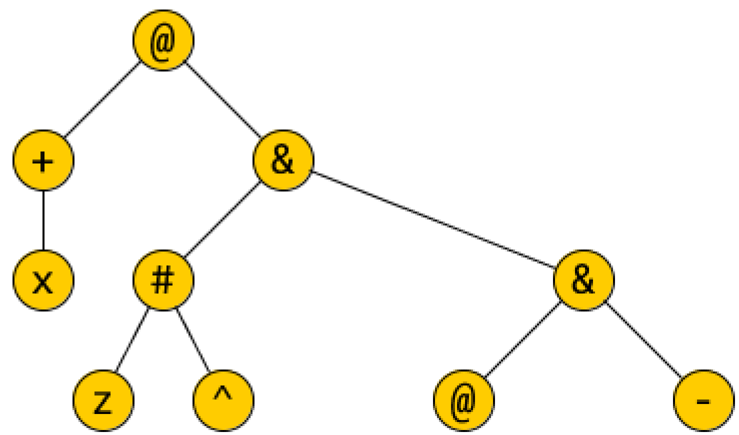

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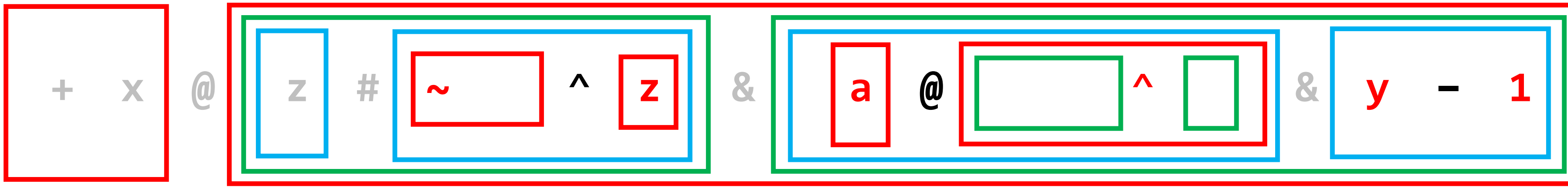

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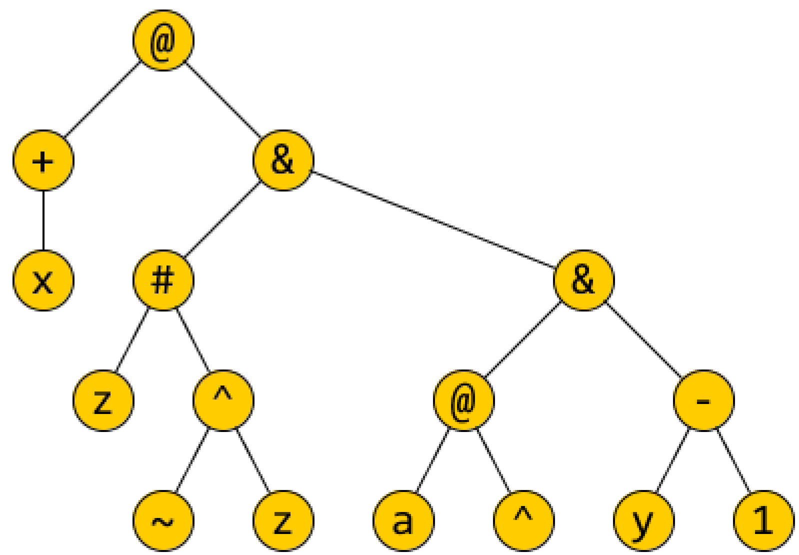

Example: Draw the AST of the infix expression

+ x @ (z # ~ y ^ z) & (a @ ~ z ^ x) & y – 1

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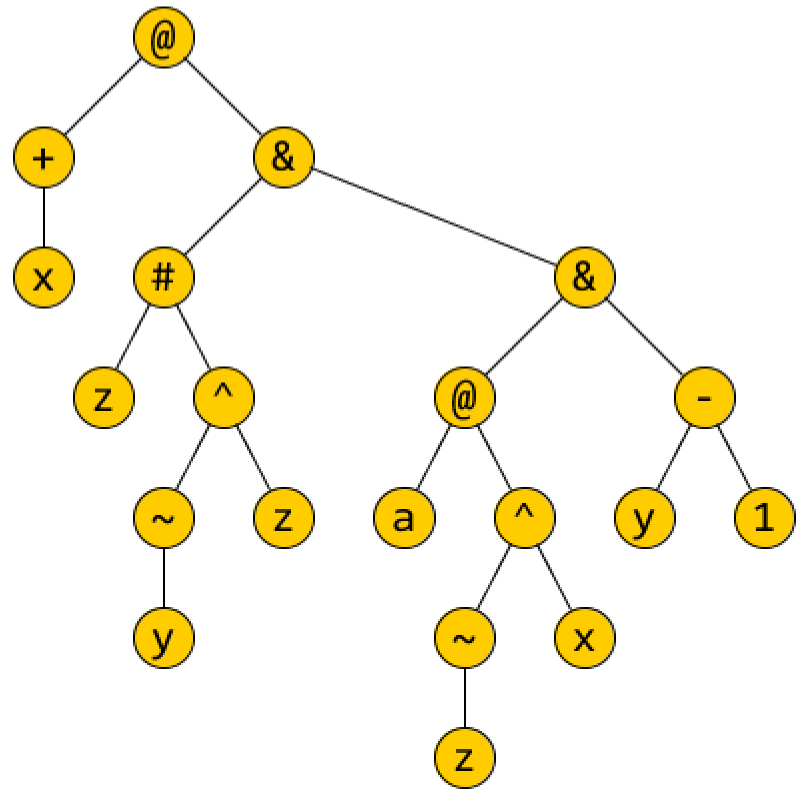

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