

# Blockchains & Distributed Ledgers

Lecture 06

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# Lecture 06 Overview

1. The Permissioned and Permissionless setting
2. Digital signatures and certificates
3. BFT = Byzantine fault tolerance. A classical BFT protocol for a permissioned ledger
  - a. Graded Consensus
  - b. Binary Byzantine Agreement
4. Proof of Stake Protocols

# Permissionless Protocols

- Bitcoin and similar PoW-based blockchain protocols provide a **permissionless** setting:
- Anyone can participate in the protocol and receive BTC as rewards by performing the PoW-based mining operation.
- The mechanism of **pouring** currency in the system via proof of work, makes it feasible for anyone (possessing sufficient hashing power) to participate.
- The ledger itself is public, readable and writeable by anyone (the latter assuming one **possesses** bitcoin)

# Permissioned Protocols

- Participation is restricted:
- Producing transactions and/or blocks can only be performed after being authorized by the other nodes.
- In their simplest form the set of nodes is **static**: the set of nodes implementing the protocol is fixed and determined at the onset of protocol execution.

# Permissioning How-To

- Most straight approach: employ a PKI (=public-key infrastructure).
- Based on digital signatures / authentication protocols.
- **Certificate authorities** can authorize other entities.
- authorization includes a signature from the CA on the entity's public-key, identity information etc.
- Sharing certificate authority information is necessary; (how, where?)

# X.509 Certificates

- Internet standard since 1988.
- Hierarchical.
- <http://www.ietf.org/rfc/rfc3280.txt>

# Structure of x.509

Version
Serial Number
Algorithm / Parameters
Issuer
Period of Validity: not before date not after date
Subject
Algorithm/ Parameters/ Key
x509v3 extensions
...
<i>Signature</i>

**X.509**  
**does not**  
**specify**  
**cryptographic**  
**algorithms**

# Digital Signatures and Certificates

- A certificate contains a digital signature.
- Recall that cryptographic design of digital signatures involves typically:
  - A cryptographic signing operation that acts on a fixed input of a specific type and has a public-verifiability feature.
  - A cryptographic hash function that takes arbitrary strings and maps them to the data type suitable for the signing operation.
  - Common setting today: SHA2 with RSA or DSA.



# Certification considerations

- All computer systems come with preloaded certificates from certificate authorities. This provide a **setup assumption**.
- Certificates need to be revoked in case the corresponding secret keys become exposed or the algorithms used are not safe anymore.
- In a blockchain system, certificate information can be provided as part of the genesis block.

# Secure channels and certificates

- Possession of mutually acceptable certificates not only permits authenticated communication (exchanging signed mechanism between two entities) but also allows building a secure channel
- Protocol **TLS 1.3** is used to build such secure channel.
  - It relies on cryptographic protocols such as the Diffie Hellman key exchange. It can ensure the confidentiality of the data exchanged.

## Static Permissioned Blockchain

- All participants are identified by self-signed certificates in the genesis block.
- The set of participants remains the same throughout the execution.
- This is the simplest form of a PKI / public-key directory.

# Permissioning

- Prior to system operation the nodes register their certificates that are included in the genesis block.
- Using such certificates, all the nodes are capable of authenticating each participant and allowing interaction with the shared state in a way that is prescribed by the participants' credentials.

# A Centralised Permissioned Ledger

(let's focus on just a “LOG” of Transactions)

- One of participants acts as a server and maintains the LOG.
- Readers and writers to the LOG authenticate with the server and can perform read and write operations.
- Consistency of the LOG is guaranteed assuming the server is trusted.
- Liveness of the LOG is guaranteed assuming the server is trusted and functional.

# Bitcoin Permissionless Ledger

- The genesis block contains no certificate information.
- Reading from the LOG is open (anyone can do it without credentials).
- Writing to the LOG can only be done in specific ways (issuing transactions).
- Nodes can obtain valid credentials (accounts) by generating a public and secret-key and either mining a block (which will reward their account with BTC) or buy BTC from another node.
- Once the LOG records their account credit, they can issue transactions.
- In essence: crediting a bitcoin account is creating a certificate that imparts the account holder with certain permissions w.r.t. the ledger.

# Distributed Permissioned Ledger

- A number of servers maintain the ledger LOG individually.
- Each share the same genesis block that identifies all participants.
- Assuming a synchronous operation, at each round, Readers and Writers authenticate with the servers and interact with the LOG in a prescribed fashion.

## Distributed Permissioned Ledger, II

- Readers authenticate to each server and obtain Read access.
- Writers authenticate to each server and provide their inputs.
- Servers run a **consensus protocol** to agree what inputs should be included in the LOG.



# Reader/Writer Management

- Readers and Writers can authenticate to each server referring to the information in the genesis block.
- It is possible to introduce additional readers and writers by suitably issuing certificates to other users.
- Note that each participant would then need to show a valid certificate chain that establishes her privileges for the specific read or write access that is requested.

# Read Requests

- Is it possible to restrict read requests as in the centralized setting?
- Nodes can maintain blocks of transactions private and issue them only to users that are authenticated.
- The TLS protocol can be used to build a secure channel between the reader and the responding node.
- Note that the above would require that all servers remain honest (as they all share the LOG).

## “Classical” BFT Consensus

- Focus on write requests next. We want to ensure LOG liveness and consistency.
- We will build a “byzantine fault tolerant” (BFT) agreement protocol that uses two important tools:
  - a graded broadcast.
  - a binary consensus protocol.

# Graded Consensus

- Parties involved : a single sender and several receivers.
  - The  $i$ -th receiver outputs  $(M_i, G_i)$ .
  - The value  $G_i$  is in  $\{0,1,2\}$ .
  - If the sender is honest then  $M_i=M_j$  for all  $i,j$  and  $G_i=2$ .
  - If the sender is malicious and one receiver outputs  $(M,2)$  then other honest receivers output  $(M, G_i)$  with  $G_i \geq 1$ .

# Graded Broadcast Protocol, 1

- Round 1.** The sender sends the message  $M$  to all receivers.
- Round 2.** The  $i$ -th receiver obtains  $M1i$  from round 1 and sends it to all receivers.
- Round 3.** The  $i$ -th receiver obtains  $M2ji$  from the  $j$ -th receiver in round 2 and performs the following:
  - if there is a single message that was sent by at least  $2n/3$  receivers then send it to all receivers. Else do nothing.

## Graded Broadcast Protocol, 2

**Output Generation.** The  $i$ -th receiver obtains  $M_{3ji}$  from the  $j$ -th receiver in round 3.

- If there is a single message that was sent by at least  $2n/3$  receivers output that message as  $M_i$  and set  $G_i=2$ .
- If there is a single message that was sent by at least  $n/3$  receivers output that message as  $M_i$  and set  $G_i=1$ .
- In any other case output *fail* and  $G_i=0$ .

# Graded Broadcast Protocol, 3

Analysis. Assume that malicious parties are  $t < n/3$ .

## **Observation #1**

If the sender is honest, then each receiver will receive the same message  $\geq 2n/3$  times in rounds 2 and 3. As a result all honest receivers will output  $G_i=2$  and that message in the output generation stage.

## Graded Broadcast Protocol, 4

**Observation #2.** If two honest receivers send a message in round 3 it **must be** the *same*.

**Proof.** Suppose an **honest party**  $P$  sends message  $M$  in round 3. Then  $P$  has received  $M$  by at least  $2n/3$  parties. Given this, observe that  $2n/3 - t > n/3$  honest parties have sent  $M$  in round 2.

Thus  $< n - n/3 = 2n/3$  parties are capable of sending a different message than  $M$  in round 2. Hence another honest party  $P'$  in round 3 will send either  $M$  or nothing.



# Graded Broadcast Protocol, 5

## Observation #3

Suppose the  $i$ -th receiver returns  $G_i=2$  and let  $M_i$  be the message it generates. For the  $j$ -th receiver's output  $(M_j, G_j)$  it holds  $M_i=M_j, G_j>0$ .

The  $i$ -th receiver has received the message  $M_i$  from at least  $2n/3$  receivers in round 3.

=> More than  $n/3$  honest receivers have sent  $M_i$  in round 3. Thus it cannot be that  $M_j=\text{fail}$ .

Now suppose  $M_j$  is not equal to  $M_i$ .

Then we deduce  $M_j$  was sent by at least  $n/3$  receivers in round 3. => at least one of them is honest. Thus by Observation #2 it holds  $M_i=M_j$ .

# From Graded Broadcast to a BFT-Ledger

A simplistic approach: execute  $n/3$  phases to guarantee an honest sender will be encountered. In each phase perform:

- A designated sender organizes all valid transactions it collected as **M** and performs a graded broadcast.
- A binary consensus protocol determines whether everyone's grade is 2 or not. If that is true each node signs the output to generate a public endorsement and appends **M** on their LOG (together with the signatures). Otherwise LOG remains the same.

# Byzantine Binary Consensus

(RECALL)  $n$  parties  $(1, 2, \dots, n)$ ,  $t$  adversarial.

Let  $v_i \in \{0, 1\}$  be the input of party  $i$ .

Honest parties should *decide* on values  $u_i \in \{0, 1\}$  satisfying the following properties.

- **Agreement:** if parties  $i$  and  $j$  are honest, then  $u_i = u_j$ .
- **Validity:** if there exists  $v \in \{0, 1\}$  such that  $v_i = v$  for each honest party  $i$ , then  $u_i = v$  for each honest party  $i$ .
- **Termination:** values  $u_i$  are well defined for all honest parties.

Note: We will examine the **synchronous** setting

# Exponential Information Gathering Algorithm (EIG)

## Algorithm Sketch.

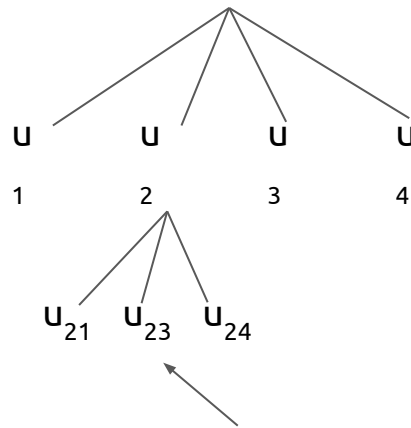
- At round 1, send to everyone your input.
- At round  $r+1$ , send to everyone all messages you received at round  $r$  (avoiding redundant messages).

Each party arranges the messages in its own EIG tree.

- Let  $u_1, \dots, u_n$  be the messages received in the first round (including itself)
- Subsequently,  $u_{xj}$  is the value received from  $j$  as the value  $u_x$  in  $j$ 's tree.

Note: there need be no repetitions in the label of a node (e.g.,  $x$  in  $u_x$  should contain distinct identifiers).

What is the size of the tree?



The value party 3 told me that party 2 send him in the previous round.

# EIG Termination

The EIG algorithm terminates after  $t+1$  rounds. The output value of each party is defined as follows.

- For each leaf  $v$  in the EIG tree, set  $z_v = u_v$ .
- For an internal node  $v$ , set  $z_v$  equal to the majority of the  $z$ -values of its children. If the majority is not defined, set  $z_v = 0$  (without loss of generality).
- Define the output as  $z_{\text{root}}$ .

# (Im)possibility results - synchronous setting

**Theorem[LSP1982]** Impossible for  $n < 3t + 1$ .

**Theorem[FL1982]** Impossible in  $t$  rounds.

**Example** The EIG algorithm with  $t=1$  needs at least 2 rounds.

1. If a party received a single 1, its output should be 0. (Because the 1 could be coming from the adversary.)
2. If a party received two 1s, its output should be 0. (Because one of them could have been sent from the adversary, while another party could have received a single 1 and will decide 0 according to the previous statement.)
3. And so on... (by induction, the output will always be 0, contradicting validity)

**Theorem[GM1998]** Doable for  $n > 3t$  in  $t+1$  rounds.

**Theorem [DS83]** Doable for  $n > 2t$  assuming a PKI.

# Impossibility results - asynchronous setting

**Theorem[BT1985]** Asynchronous Byzantine Consensus is impossible with  $n < 3t + 1$ , even if the parties have agreed on a PKI.

**Proof** Partition parties into sets A, B, C of size at most  $t$ . Consider 3 scenarios.

- A. A malicious, B and C honest with inputs 0. The adversary sends no messages. The honest parties should decide on 0 until some time  $T_A$ .
- B. B malicious, A and C honest with inputs 1. The adversary sends no messages. The honest parties should decide on 1 until some time  $T_B$ .
- C. C malicious, B and A honest with inputs 0 and 1 respectively. The adversary communicates with B as the honest C in scenario A and with A as the honest C in scenario B. At the same time every communication between A and B is delayed for time at least  $\max\{T_A, T_B\}$ .

The crux is that A has the same view in scenarios B and C. Similarly for B, in scenarios A and C. Agreement in scenario C is impossible, if validity is achieved in scenarios A and B.

# A blockchain related to proof-of-stake

Servers  $S_1, \dots, S_n$  with shared verification keys  $pk_1, \dots, pk_n$  and private signing keys  $sk_1, \dots, sk_n$ .

$B_0$ : Genesis block containing the public info.

$B_i = (k, d, sl, \sigma_{sl}, \sigma_{block})$ , where

$k$ : hash of previous block

$d$ : data

$sl$ : slot number

$\sigma_{sl}$ : signature on  $sl$  by  $S_{sl \bmod n}$

$\sigma_{block}$ : signature on whole block by  $S_{sl \bmod n}$

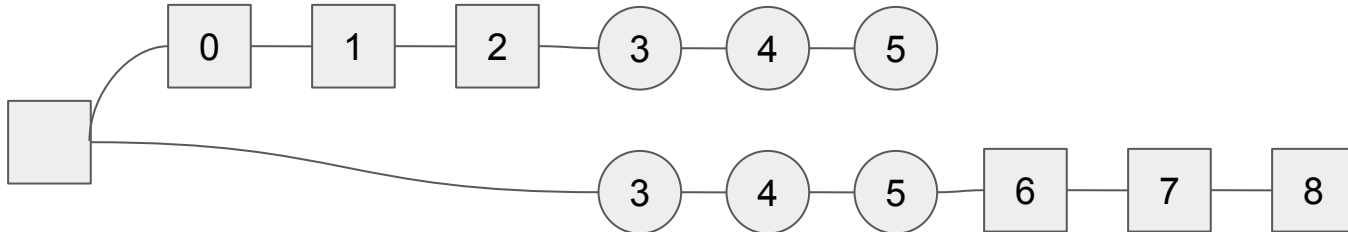
At any step, each server,  
extends the longest  
known blockchain



# Characteristic sequences and executions

The characteristic sequence of an execution with  $L$  slots  $0, 1, \dots, L-1$  is a binary string  $w$  in  $\{0, 1\}^L$  such that  $w_i = 1$  iff  $S_{i \bmod n}$  is adversarial.

**Example**  $w = 000111000$ . Squares denote honest slots (including the genesis) and circles the adversarial ones. The adversary keeps the chain ending with slot 5 on top hidden so that  $S_6$  extends the bottom chain. This results in a **fork**, two disjoint chains (except for the genesis block) of maximum length. The corresponding characteristic sequence  $w$  is called **forkable**.



# Forkable sequences

**Fact** If  $\text{weight}(w) < \text{length}(w)/3$ , then  $w$  is not forkable.

**Proof** Let  $n = \text{length}(w)$  and  $t = \text{weight}(w)$  and assume  $w$  is forkable. The two chains must have length at least  $n-t$  (prove formally by induction). Thus,  $2(n-t)$  is a lower bound on the sum of their lengths. On the other hand, the  $n-t$  honest parties have contributed at most  $n-t$ , while the adversarial parties have contributed at most  $2t$  (because a given chain contains a slot at most once). Thus,  $(n-t) + 2t$  is an upper bound on the sum of their lengths. We have

$$2(n-t) \leq (n-t) + 2t \implies n \leq 3t.$$

# Consensus inspired from proof-of-stake [KR2018]

- Fix an arbitrary ordering of the servers:  $S_1, \dots, S_n$ .
- Construct a blockchain for  $5t+2$  rounds, recording your own input bit as data in any block you create.
- Upon termination output the majority of the first  $2t+1$  blocks of your chain.

**Theorem** If  $n > 3t$ , the protocol satisfies agreement and validity.

**Proof [KR2018]** It can be shown that the first  $2t+1$  blocks are common to all honest parties; this implies agreement. Validity follows from the fact that among the first  $2t+1$  at most  $t$  are adversarial and so the majority of them belong to honest parties.

# Bitcoin Consensus

- Miners run the Bitcoin protocol recording their own input bit as data in any block they compute.
- When their chain has at least  $2k$  blocks (for some security parameter  $k$ ), the broadcast it and stop.
- Output is the majority of the bits recorded in the first  $k$  blocks.

**Theorem [GKL15]** If  $t < n/3$ , the above protocol satisfies Agreement and Validity with probability  $1 - e^{-\Omega(k)}$ .

**Remark** For Agreement,  $t < n/2$  suffices.

# Common-Prefix Property and Agreement

**Common-Prefix Property** For any pair of honest parties adopting the chains  $C_1$  and  $C_2$  at rounds  $r_1$  and  $r_2$  respectively. If  $r_1 \leq r_2$ , then  $C_1[-k]$  is a prefix of  $C_2$ , where  $C_1[-k]$  is  $C_1$  without its last  $k$  blocks.

Common-Prefix Property implies Agreement. This is because the parties are pruning at least  $k$  blocks from their chains when keeping only their initial  $k$  blocks. Thus, the initial  $k$  blocks are common to all honest parties.

In [GKL2015] it is shown that in Bitcoin the Common-Prefix fails with probability exponentially small in  $k$ , if the adversary's hashing power is sufficiently bounded below  $\frac{1}{2}$ . It follows that Agreement is satisfied with probability  $1 - e^{-\Omega(k)}$ , when  $t$  is sufficiently less than  $n/2$ .

# Chain-Quality Property and Validity

**Chain-Quality Property (informal)** Among any sufficiently large number of consecutive blocks in an honest party's chain, a fraction of at least  $(n-2t)/(n-t)$  have been computed by honest parties.

Chain-Quality implies Validity, when  $t$  is sufficiently less than  $n/3$ . This is because, in that case, the majority of the first  $k$  blocks have been computed by honest parties. Thus, if all honest parties have input  $v$ , the majority of values recorded in the first  $k$  blocks will be  $v$ .

In [GKL2015] it is shown that in Bitcoin Chain-Quality fails with probability exponentially small in  $k$ , if the adversary's hashing power is sufficiently bounded below  $1/2$ . It follows that Validity is satisfied with probability  $1 - e^{-\Omega(k)}$ , when  $t$  is sufficiently less than  $n/3$ .

# Dynamic Availability

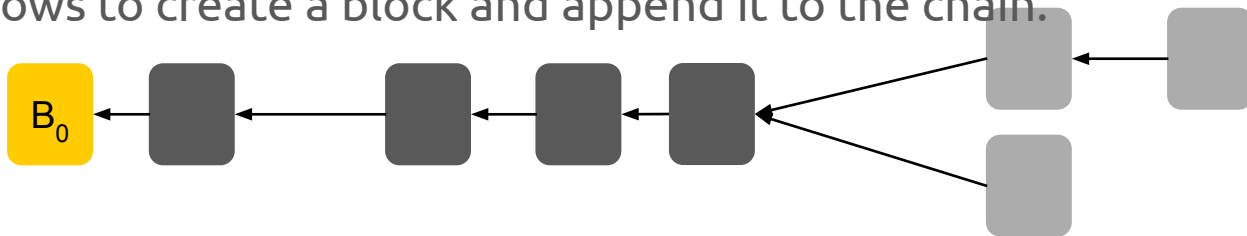
Recall: **Dynamic availability (DA).**

- Parties join and leave at will. They need to bootstrap a chain when (re-) joining.
    - Easy in Bitcoin: “**longest-chain rule**” (general: most difficult chain).
  - Number of online/offline parties changes over time
    - Analysis must account for that -
  - There is no *a priori* knowledge of participation levels is required by the protocol.
  - Unannounced disappearance.
- Classical BFT protocols do not operate under general **dynamic availability**.

# Bitcoin Energy Disadvantage

- **Bitcoin Proof of Work Mechanism:**

- Parties repeatedly try to solve cryptographic puzzles. A solution allows to create a block and append it to the chain.



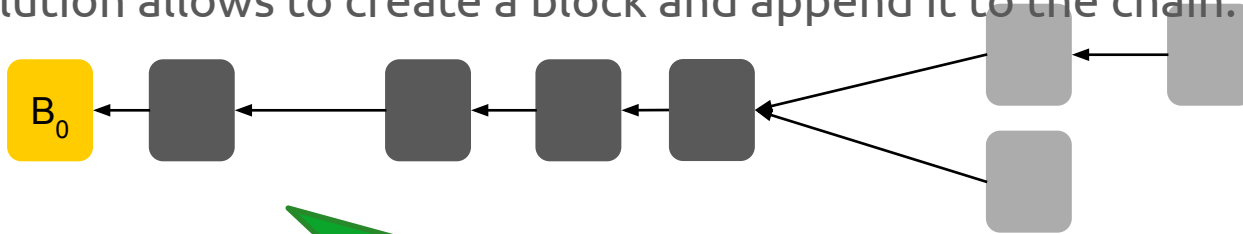
- **Bitcoin is not energy efficient** as the hash-based lottery consumes a lot of energy to ensure the protocol's security.
- Classical BFT protocols are much more energy efficient.



# Proof of Stake

- **The case of Bitcoin:**

- Parties repeatedly try to solve cryptographic puzzles. A solution allows to create a block and append it to the chain.



**Proof-of-Stake to the rescue!**

# Proof of Stake

## Proof-of-Stake Blockchains:

- Use **stake** (a virtual resource) instead of hashing power.
- Miners = **Stakeholders**.
- Next **stakeholder** to produce block **elected with probability proportional to stake**.

# Proof of Stake

## Proof-of-Stake Blockchains:

- Use Stake (a virtual resource) instead of hashing power.
- Miners = Stakeholders.
- Next stakeholder to produce block elected with probability proportional to stake.

## Two categories:

Nakamoto-style consensus (e.g., the Ouroboros protocol - KRDO16)

BFT-style consensus (e.g., Algorand - CM16)

# Realizing the Ledger

## Complications of PoS vs. PoW:



### **PoS has costless simulation:**

No physical resources to create blocks: several transaction histories could be generated “in the adversaries head”.



### **Long-Range attacks in the threat model:**

Adversary tries to deceive (new) participants into believing the “wrong” history (which are cheap to generate).

# References

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**[LSP1982]** Lamport, Shostak, Pease. The Byzantine generals problem.

**[CM2016]** Chen, Micali, Algorand a secure and efficient distributed ledger.