Decentralised Finance

Networks of automated market makers
arbitrage, order routing ...

Lecture 11

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Outline

1. what is DeFi = decentralised finance
2. what is an automated market maker (aMM) = state machine to exchange assets
    1. Uniswap example
    2. Variants
3. why aMMs are so successful
4. networks of aMMs -> global price consistency -> arbitrage and order routing

1.

what is DeFi = decentralised finance

Decentralised Finance (aka open finance)

assets being exchanged = ERC20 tokens

- users (=accounts) stay in control of their assets
- ownership = secret keys = no custody, ie no delegation of access rights, no IOUs
- financial transactions are mediated by smart contracts ~ state machines with controlled access
- smart contracts run on a neutral computational platform; confidence in code replaces trust in intermediaries
- financial functionalities are code hence are composable (unclear how much this is used; akropolis' hack)
- not clear which other substrates would be fit for DeFi

What is being traded?

assets = tokens

- A, B, C … represent tokens (ERC20) which people exchange/swap
- ERC20 can be freely created
- … and hooked to any contract
- some of them are utility tokens (BAT), some security tokens (wBTC) <- "fundamental value" problem …

ERC20 building block

Interface

```
interface IERC20 {
  function totalSupply() external view returns (uint256);

  function balanceOf(address who) external view returns (uint256);

  function allowance(address owner, address spender)
    external view returns (uint256);

  function transfer(address to, uint256 value) external returns (bool);

  function approve(address spender, uint256 value)
    external returns (bool);

  function transferFrom(address from, address to, uint256 value)
    external returns (bool);

  event Transfer(
    address indexed from,
    address indexed to,
    uint256 value
  );

  event Approval(
    address indexed owner,
    address indexed spender,
    uint256 value
  );
}
```

getters

transitions

events

2.

what is an automated market maker (aMM)

What is a market maker

an **MM** is a special kind of actor in a market always willing to trade

- MMs are liquidity providers posting offers eg Bs-for-As
- traders take offers
- MMs provide liquidity, traders consume it
- <u>w/o MMs</u> there is **less liquidity**

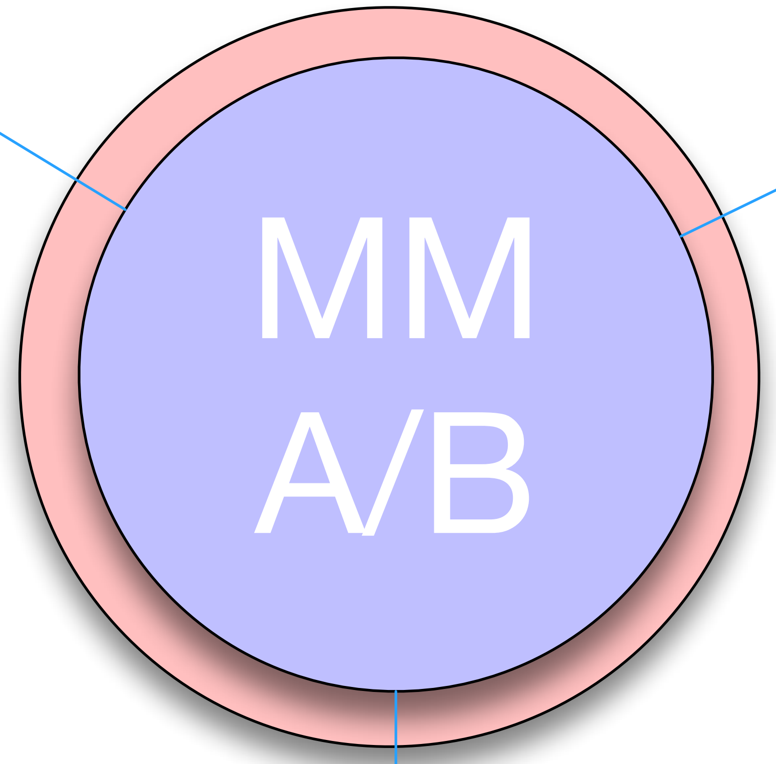
  -here liquidity means: 1) possibility to always trade, 2) trade at low "price impact"

Interacting with an MM

3 roles

Traders
A's-for-B's
move price
off equilibrium

Arbitrageurs
correct price
back to equilibrium

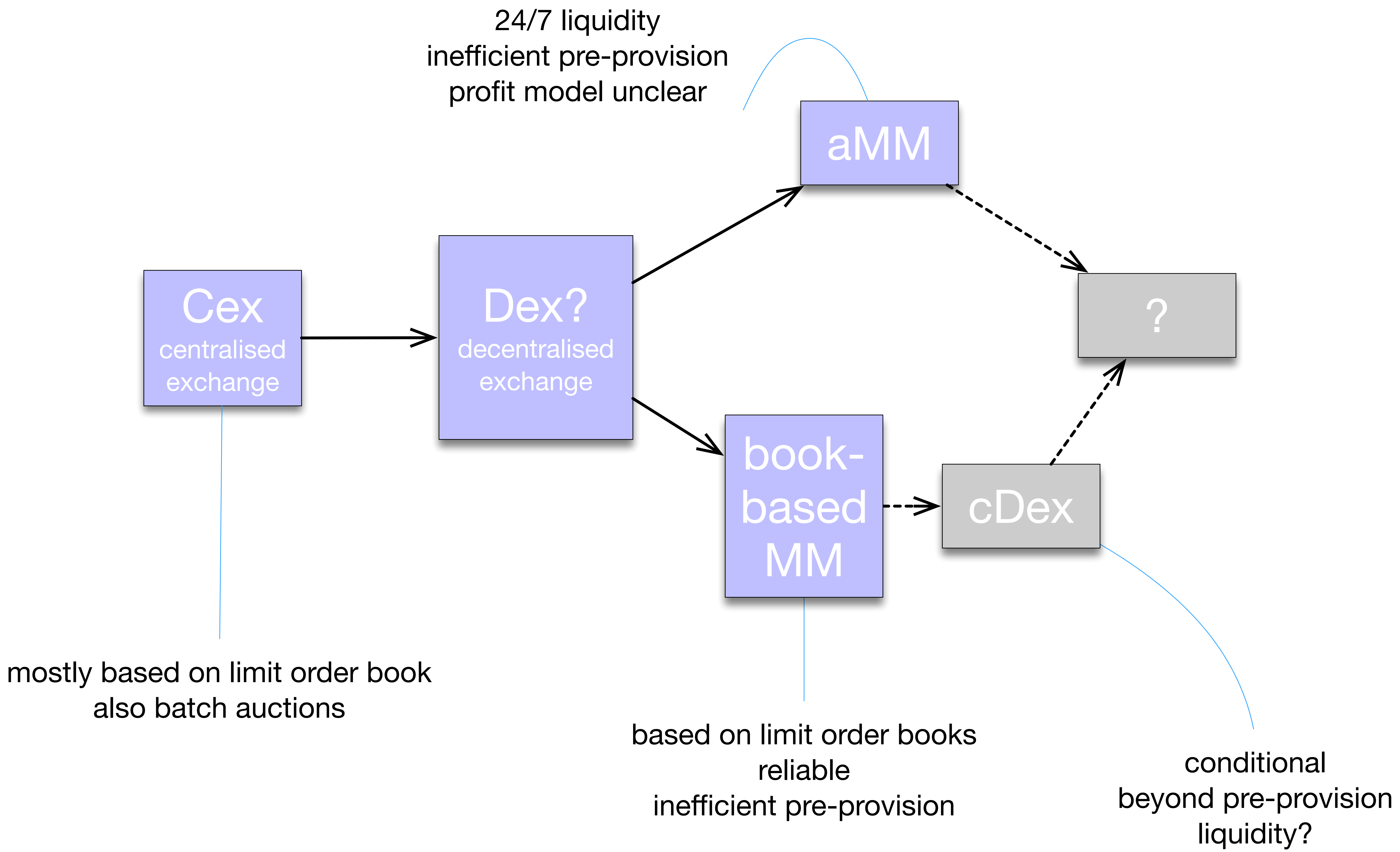


Liquidity providers
± liquidity
changes stay on equilibrium

<u>equilibrium</u> in this context means
no-arbitrage
(to be defined later)

Types of markets

evolution of market structures



A/B Order book

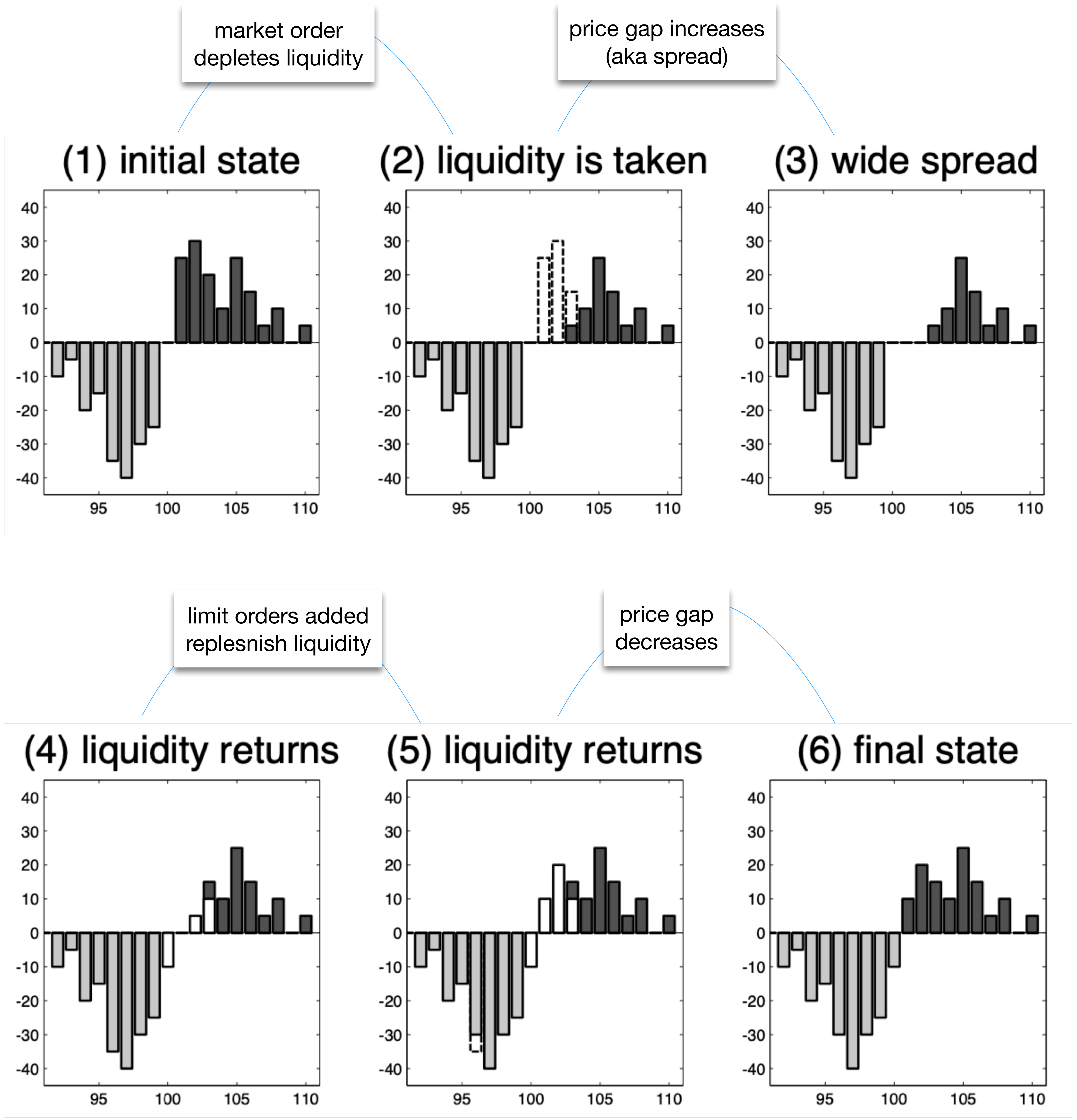
market order vs limit order

can think of the OB as a continuous auction mechanism:

1. **Liquidity providers (LPs)** place and cancel offers (aka limit orders): price in **A** and amount of **B**; <u>provide</u> liquidity

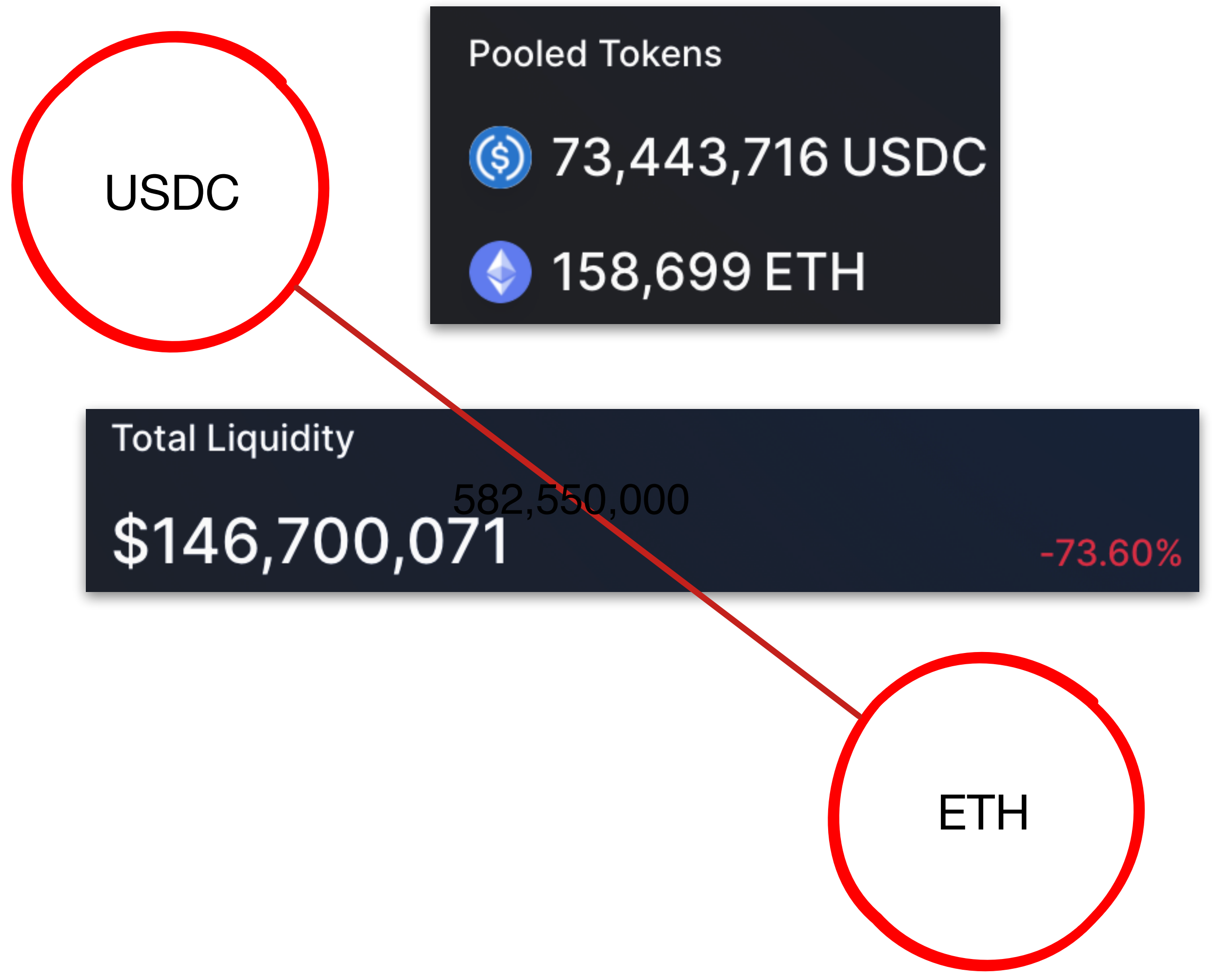
2. **Traders** issue market orders: amount of **B**; <u>consume</u> liquidity

3. LPs are competing



aMMs

comparison with an order book



- no order book or other type of auction, no price oracle
- instead:
  - Liquidity providers are not competing = they are passively providing liquidity and share fees
  - liquidity provided by LPs feed reserves (aka "pools")
  - price is computed by a **price function** from current <u>reserves</u> = state machine (described in the next slides)
- in Cex'es books liquidity is "promised", here it is captive

2.1
The Uniswap example

Uniswap state machine 1

Trader action

State of an edge $\theta = ([A],[B],\gamma)$ in $\mathbb{R}^2_+ \times [0,1]$ with $[A]$ the amount of A in the pool, $[B]$ the amount of B, and $0 \leq 1-\gamma \ll 1$ the fee.

Trader action 1 The price function gives the amount y of B paid-out for an amount x of A paid-in:

$$f_\theta(x) = \gamma[B]\frac{x}{\gamma x + [A]} = ([B]/[A])\frac{\gamma(x/[A])}{\gamma(x/[A])+1}$$

NB1 - f_θ factorises through $\phi(x) = x/(1+x)$, meaning in works in relative sizes of trades.

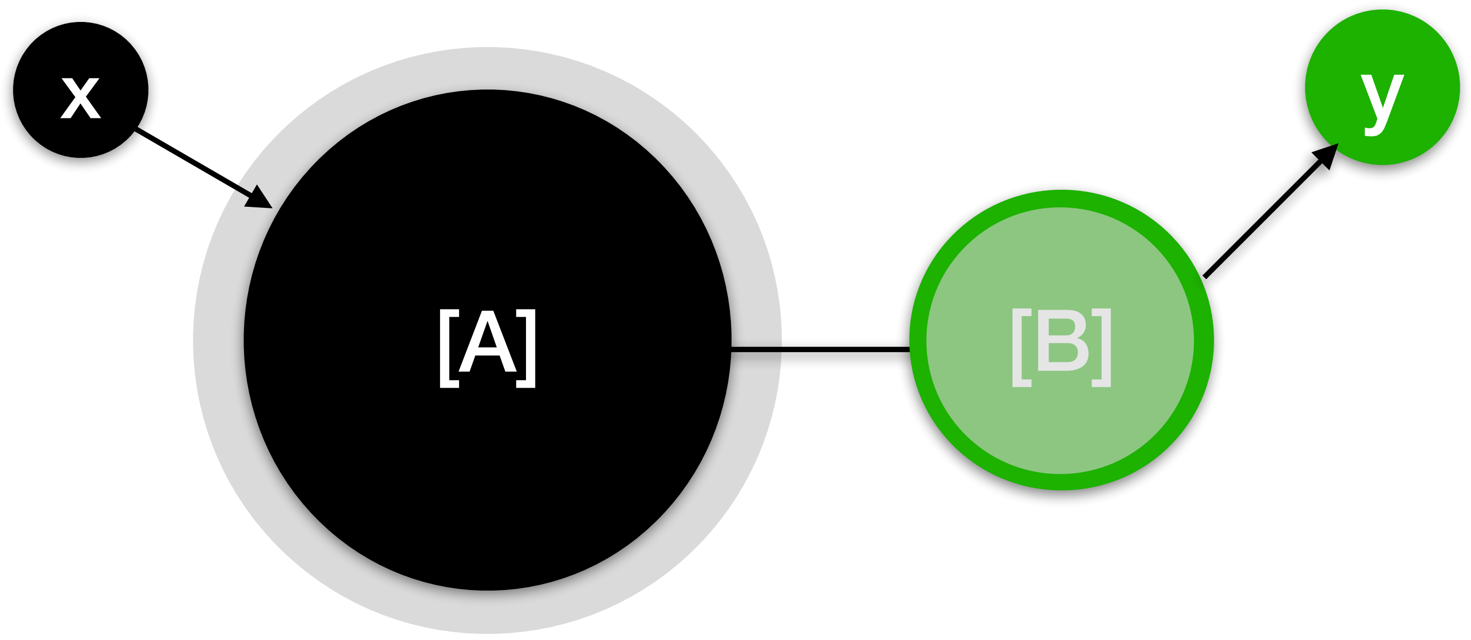
NB2 - We see $f_\theta(0) = 0$, f_θ is non-decreasing, and strictly concave:

$$f'_\theta(x) = \gamma[A][B]\frac{1}{(\gamma x + [A])^2} > 0$$

$$f''_\theta(x) = -2\gamma^2[A][B]\frac{1}{(\gamma x + [A])^3} < 0$$

NB3 - Marginal (or linear) price $f'_\theta(0) = \gamma([B]/[A])$:

$$f_\theta(x) = f'_\theta(0)x + o(x) = \gamma([B]/[A])x$$



Abstract description 1

qualitative properties of the price function

Reasonably:

$$f_\theta(0) = 0 \qquad \text{(no money for nothing)}$$
$$f_\theta < [B] \qquad \text{(never dry)}$$
$$f_\theta \ \textit{non-decreasing}$$
$$f_\theta \ \textit{concave}$$

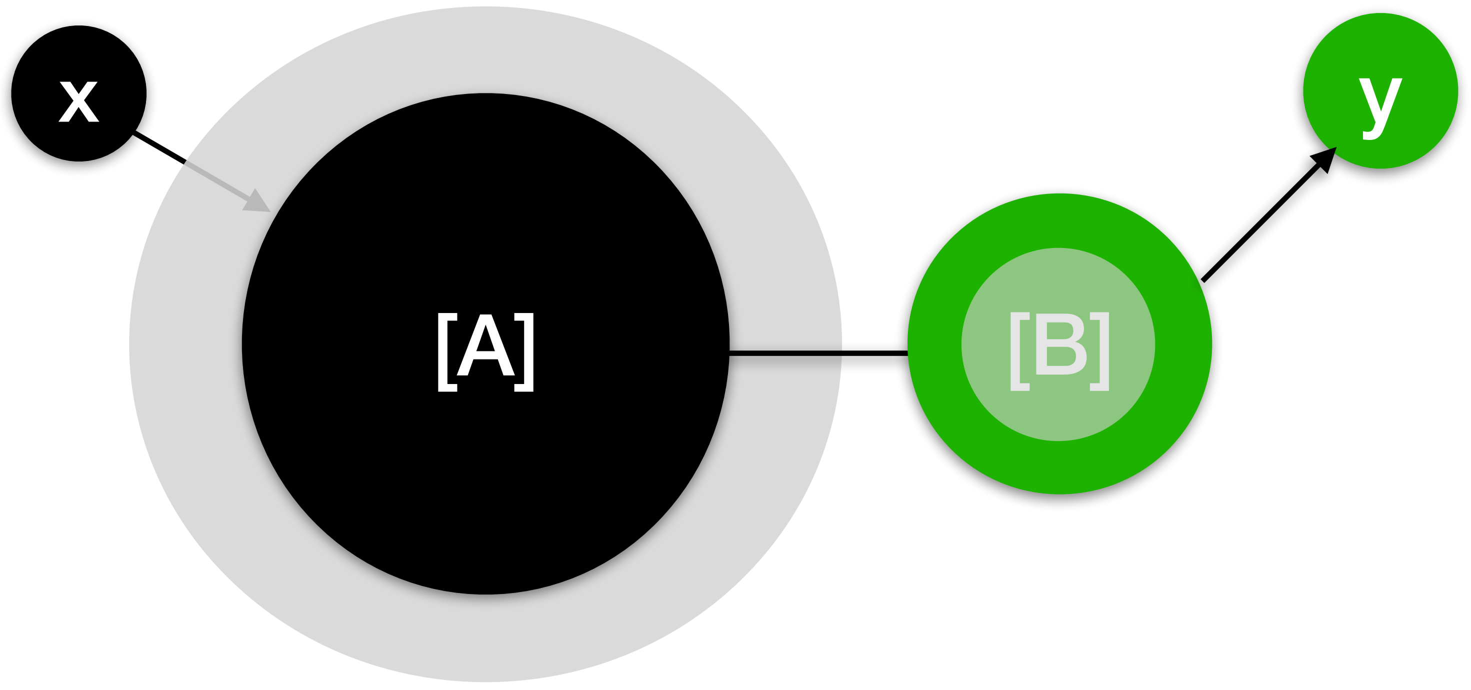
Uniswap state machine 2

Trader action 2

Trader action 2 Trader action updates state as follows:

$$x \cdot ([A], [B], \gamma) \;=\; [A] + x, [B]\frac{[A]}{[A] + \gamma x}, \gamma$$

Has impact on price (scarcer token appreciates: $f_{x \cdot \theta} \leq f_\theta$).



Example Suppose the pool is $100A + 100B$, hence $\rho = [B]/[A] = 1$;

[linear regime] suppose $a = 1$, then $\alpha = a/[A] = \frac{1}{100}$, $\beta = b/[B] = \frac{1}{101}$, so $b = \frac{100}{101}$, ie very nearly a mean price of 1;

[mid regime] suppose $a = 100$, then $\alpha = 1$, $\beta = \frac{1}{2}$, hence the actual mean price is $p(B|A) = 1/2$, to compare with the linear regime of $1/1$

[saturated regime] suppose $a = 10000$, then $\alpha = 100$, $\beta = \frac{1}{1.01}$, $b = \frac{100}{1.01}$, with terrible mean price is $p(B|A) = \frac{1}{101}$.

Uniswap state machine 3

LP action

LP action The LP action is defined for $x \in [\max(-[A], -[B]), \infty)$ and updates as follows:

$$x : ([A], [B], \gamma) \;=\; [A] + x, [B] + x([B]/[A]), \gamma$$

No impact on marginal price:

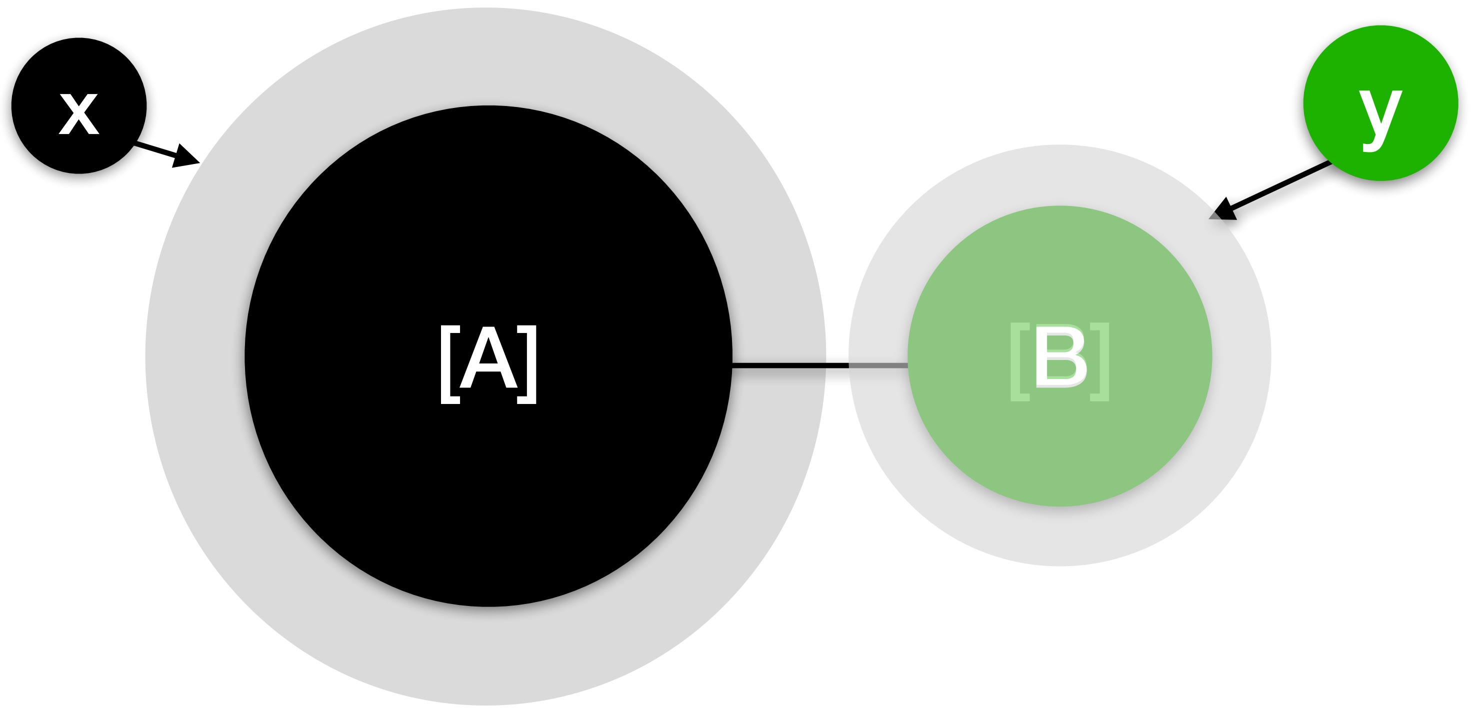
$$f'_{x:\theta}(0) \;=\; \gamma([B] + x[B]/[A])/([A] + x) \;=\; \gamma([B]/[A]) \;=\; f'_\theta(0)$$

One can show *no slicing*

$$f_\theta(x) + f_{x\cdot\theta}(y) \;\leq\; f_\theta(x+y) \;\leq\; f_\theta(x) + f_\theta(y)$$

The second inequality is by sub-additivity, which follows from concavity and $f(0) = 0$. So we also get for free the weaker:

$$f_{x\cdot\theta} \;\leq\; f_\theta$$



Peek at the code

code is open, transitions are logged as events

```
contract UniswapExchange {
    using SafeMath for uint256;

    /// EVENTS
    // transitions
    event EthToTokenPurchase(address indexed buyer, uint256 indexed ethIn, uint256 indexed tokensOut);
    event TokenToEthPurchase(address indexed buyer, uint256 indexed tokensIn, uint256 indexed ethOut);
    event Investment(address indexed liquidityProvider, uint256 indexed sharesPurchased);
    event Divestment(address indexed liquidityProvider, uint256 indexed sharesBurned);
```

```
contract UniswapFactory is FactoryInterface {
    event ExchangeLaunch(address indexed exchange, address indexed token);
```

2.2
Uniswap variants

Abstract description 2

details of the price function do not matter qualitatively

- unclear what exact class is the right one but:
  - any map which is (strictly) **concave**, **0@0**, and **non-decreasing** will "work"
  - **no slicing** seems natural to ask and entails "price sensing"
- of course details matter for finding closed forms, Uniswap is algebraically simple
- changing the price function is the source of many variants (next slide)

$$f_\theta(x) + f_{x\cdot\theta}(y) \quad \leq \quad f_\theta(x+y) \quad \leq \quad f_\theta(x) + f_\theta(y)$$

$$f_{x\cdot\theta} \quad \leq \quad f_\theta$$

A convenient representation

implicit description of the price function via invariant

Supposing $\gamma = 1$ (no fee): y the amount of B paid out is related to x the amount of A received by the *constant product rule*. That is to say x and y have to be such that:

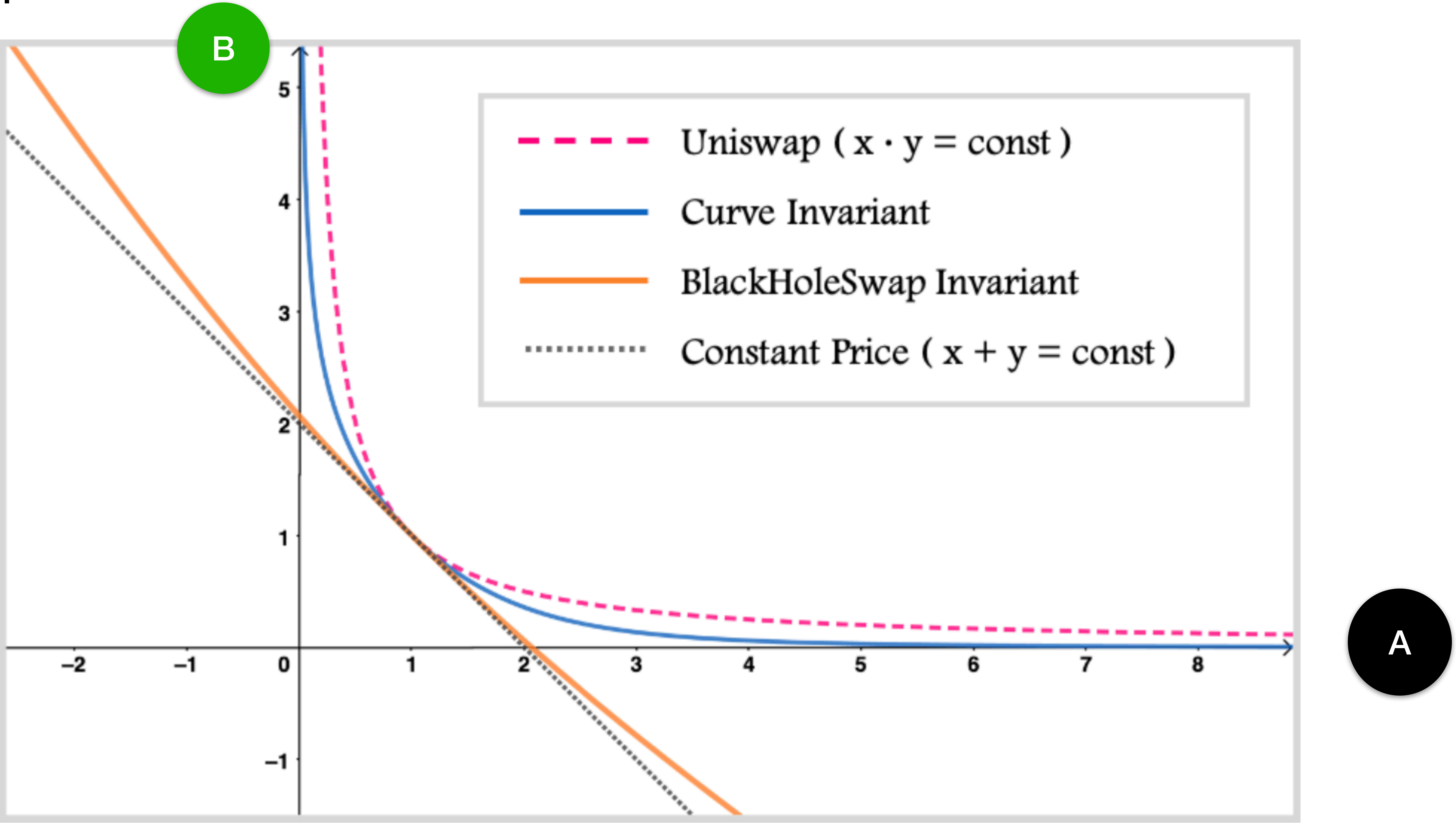
$$([A]+x)([B]-y) = [A][B]$$

If we introduce relative changes $\alpha = x/[A]$, $\beta = y/[B]$ we get the intensive form of the invariant:

$$(1+\alpha)(1-\beta) = 1$$

Make your own curve!

implicit description

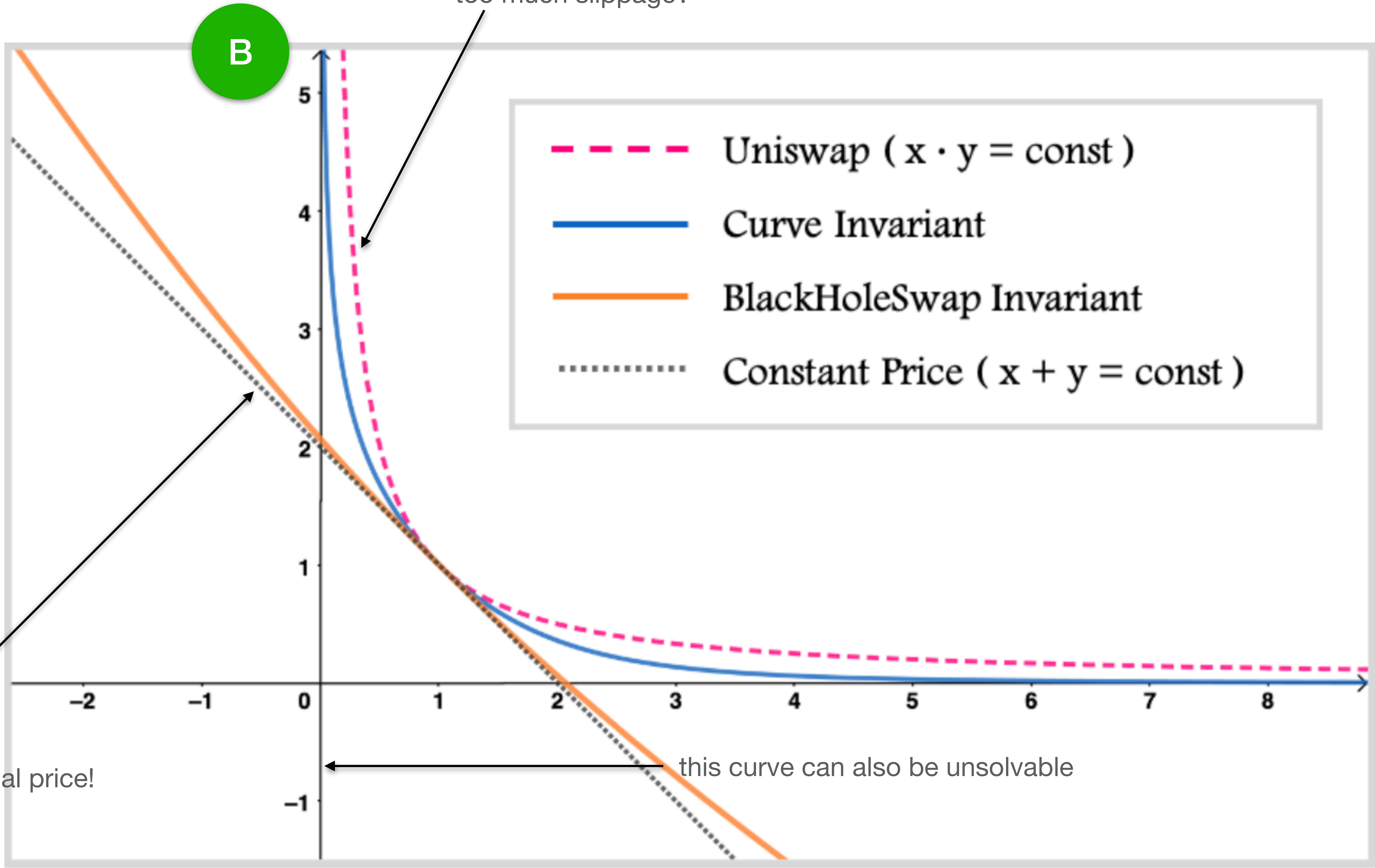


The slippage vs arb trade-off

make your own curve!

caveat: implicit description

too much slippage?



A

no price impact
aka slippage

infinite liquidity
but you never learn the external price!

3.
why aMM are so successful and whether it will last …

Reasons for aMM dominance

may not last …

- low resources <- good for gas price
- simple code <- good for confidence
- programmable lego brick - can create a token and hook it up somewhere to form a new pair
- TKRs are happy because it is 24/7 liquid and the price is arb'ed -so good
- LPs are happy because they get fees on each swap
- ARBs are happy, who doesn't like a free lunch

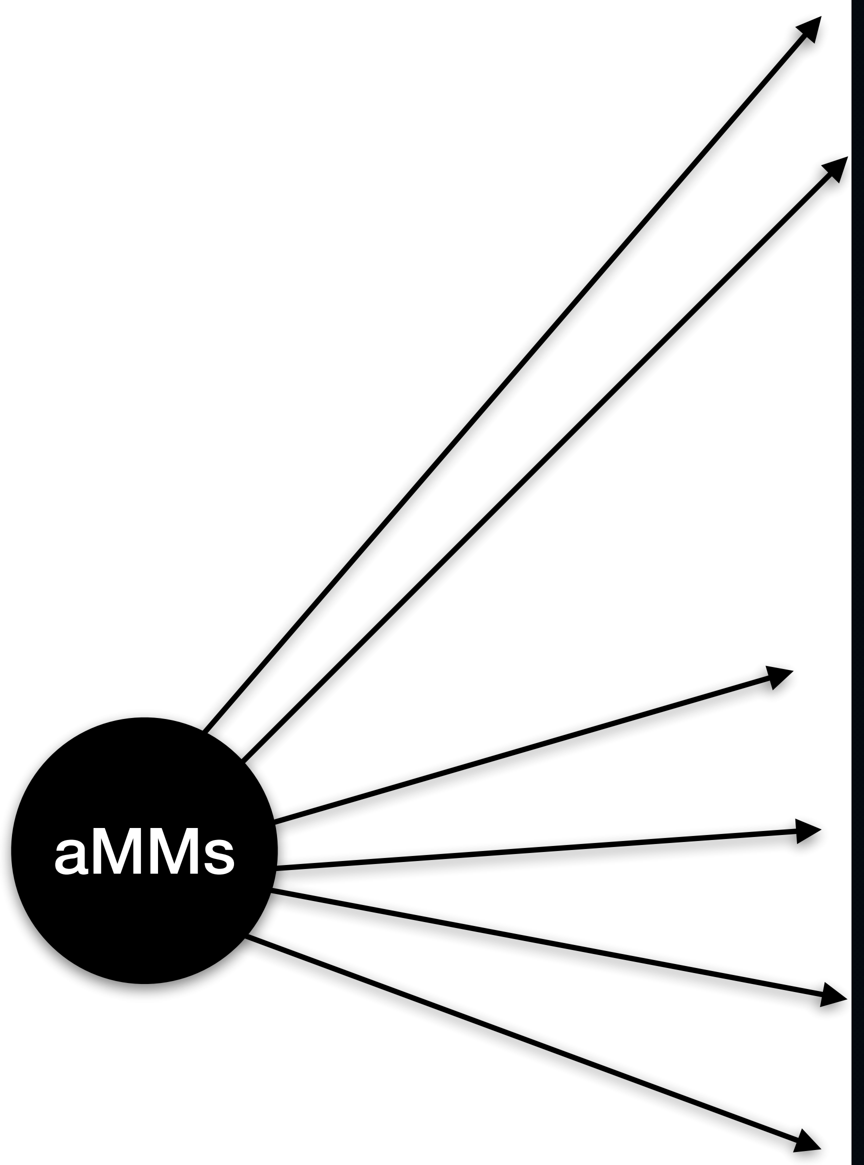
Confidence in the code v1 (300 lines) - eg **LP** action

event

```
function investLiquidity(uint256 _minShares) external payable exchangeInitialized
{
    require(msg.value > 0 && _minShares > 0);
    // user injects x = msg.value ETH to buy a bunch of shares Sp
    // why bother to check if x > 0 and min_share > 0?
    uint256 ethPerShare = ethPool.div(totalShares);
    // price of share in ETH: [ETH]/[S]
    // what do you do with the remainder -> it goes in the pocket of the pool!
    // division-by-zero! div must contain some test
    // should price of share not be in the permanent state of the contract
    // rather than be a temporary variable?
    // what if ethPool = 0? ethPerShare = 0 and then revert?
    require(msg.value >= ethPerShare);
    // this "require" is not needed nor desirable
    // it costs gas for every user not just idiots who do not provide enough money
    // besides if you have not enough money for 1 share you will get zero which is perfect
    uint256 sharesPurchased = msg.value.div(ethPerShare);
    // Sp = x / ([ETH]/[S]) = x [S]/[ETH]
    // solve Sp.{x = [ETH]/[S] * Sp}
    // division-by-zero again; remainder is pocketed (no change!)
    // would return zero for idiots in the absence of the "require" right before
    require(sharesPurchased >= _minShares);
    // this is a limit order, it is OK to put it
    // suppose require-2 is not there, and msg.value < ethPerShare
    // then sharesPurchased = 0 (.div means Euclidean quotient) as intended
    uint256 tokensPerShare = tokenPool.div(totalShares);
    // price of share in DAI: [DAI]/[S]
    // same remark - should be a state variable
    uint256 tokensRequired = sharesPurchased.mul(tokensPerShare); // Sp * [DAI]/[S]
    // series of updates
    shares[msg.sender] = shares[msg.sender].add(sharesPurchased); // add Sp to user account
    totalShares = totalShares.add(sharesPurchased);               // update total Shares
    ethPool = ethPool.add(msg.value);                             // update ETH pool
    tokenPool = tokenPool.add(tokensRequired);                    // update DAI pool
    invariant = ethPool.mul(tokenPool);                           // update invariant
    Investment(msg.sender, sharesPurchased);                      // event
    // execute a request for tokens on the sender's behalf
    require(token.transferFrom(msg.sender, address(this), tokensRequired));
    // token.trasnferFrom returns false if order does not succeed and "require" fails then and everything reverts
    // we rely on contract to contract communication with no authentication necessary
    // should be earlier in the sequence, right after computing: "tokensRequired"
    // if idiot user has not provisioned, updates will be paid for and then undone
}
```

Many clones

an example of price performance for a txn: ETH 100 -> x DAI

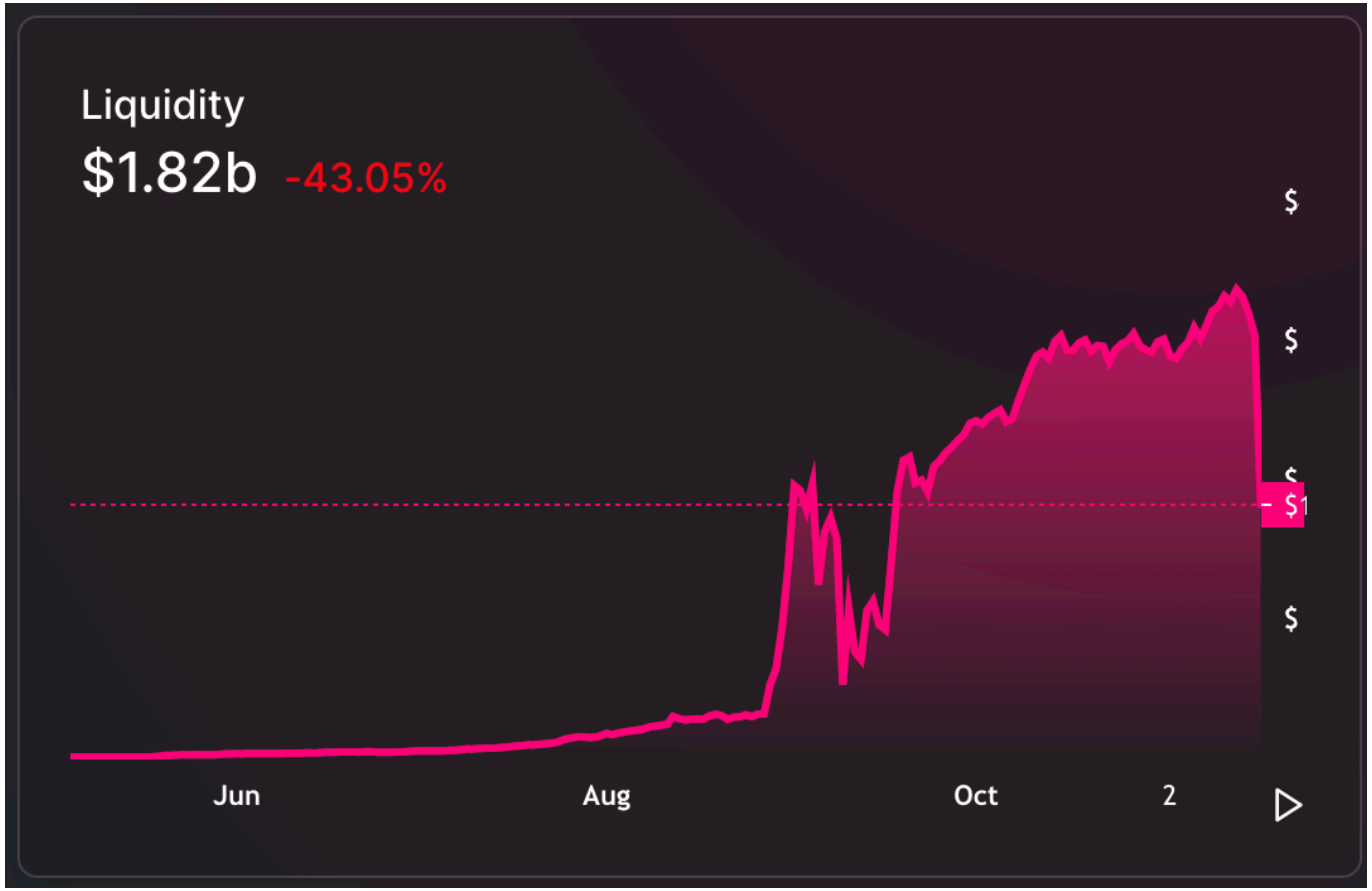


1inch	0.0021653124020748885	BEST
Kyber	0.0021743879862040913	-0.4 %
Uniswap V2	0.002176483429305303	-0.47 %
Sushiswap	0.0021768407202040598	-0.49 %
Balancer	0.002184541380176417	-0.84 %
Oasis	0.0022605410939401474	-4.19 %
Mooniswap	0.0023061155110256325	-6.07 %
DeFi Swap	0.0023426968986100553	-7.53 %
Bancor	0.0026232674180975066	-17.43 %
Uniswap V1	0.00263236581034658	-17.71 %

differ by price curve

Many clones -> competition

vampire attacks - another macro-variable to monitor

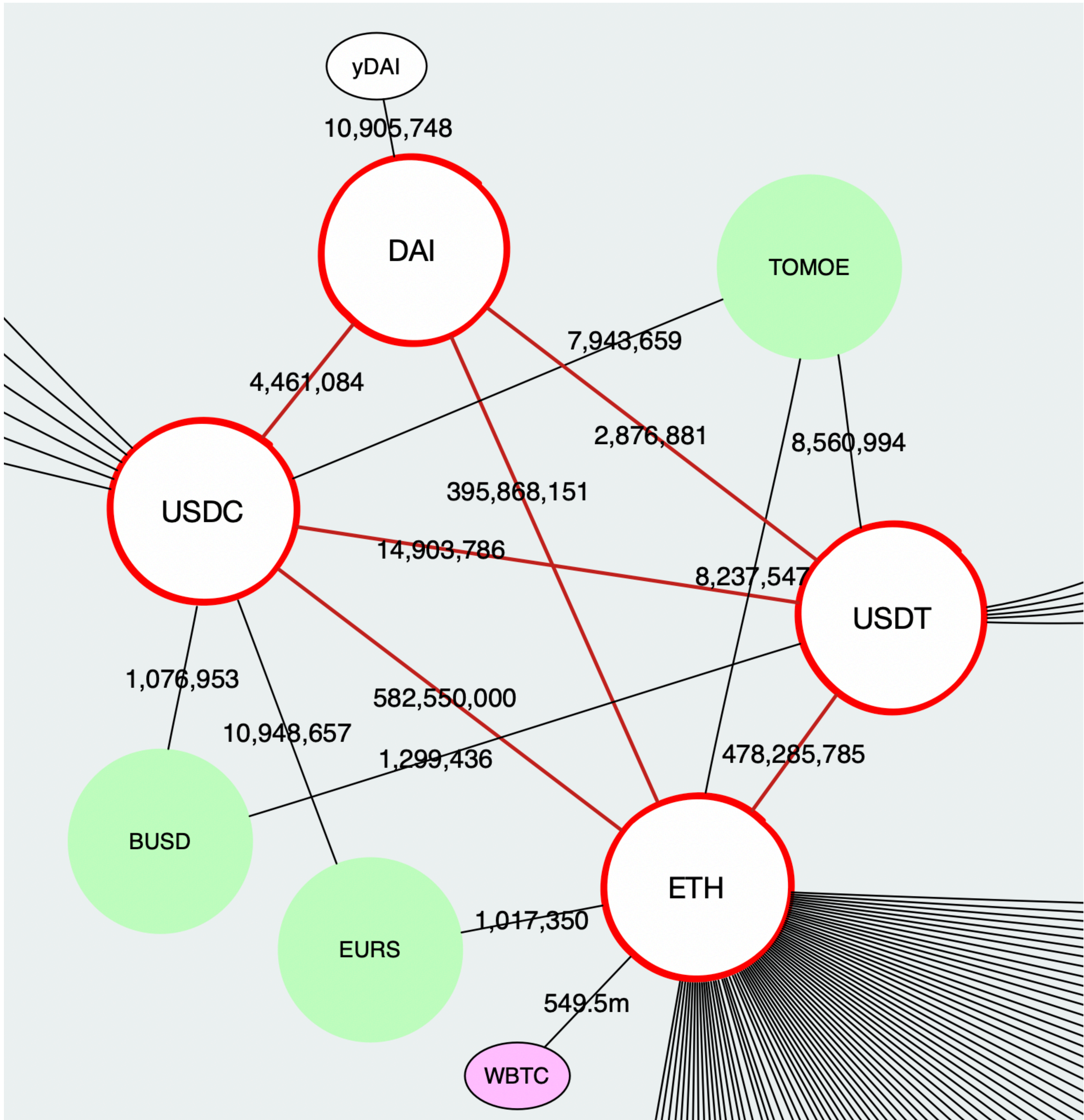


4.
networks of aMMs

The aMM network

superimpose all Uniswap edges

- real-time adjustement at freq = 1/15s
- notice the cycles
- should superimpose also: all the other swap machines (including Cexes)
- aggregators do that (1inch, paraswap, etc)



Network specific questions

- **arbitrage**: looking for cyclic sequences of swaps which guarantee risk-less profi t
- **order routing**: best combination of paths from **A** to **B** for a given amount **a**
- **liquidity migration**: LPs move from a pool to another

Arbitrage

abstract approach

NB: non-decreasing concave 0@0 functions compose; eg the **price function** of a **cycle**

cycle -> arbitrage condition f(x) ≥ x

include all swap contracts (not just aMMs) indeed OBs are also non-dec concave and 0@0!

There are closed formulas for 1) no-arb, and 2) max profit arb for uniswap

Corollary 1 *Arbitrage zones are downward closed, ie $f(x_0) \geq x_0$ implies $f(x) \geq x$ for all $x \leq x_0$.*

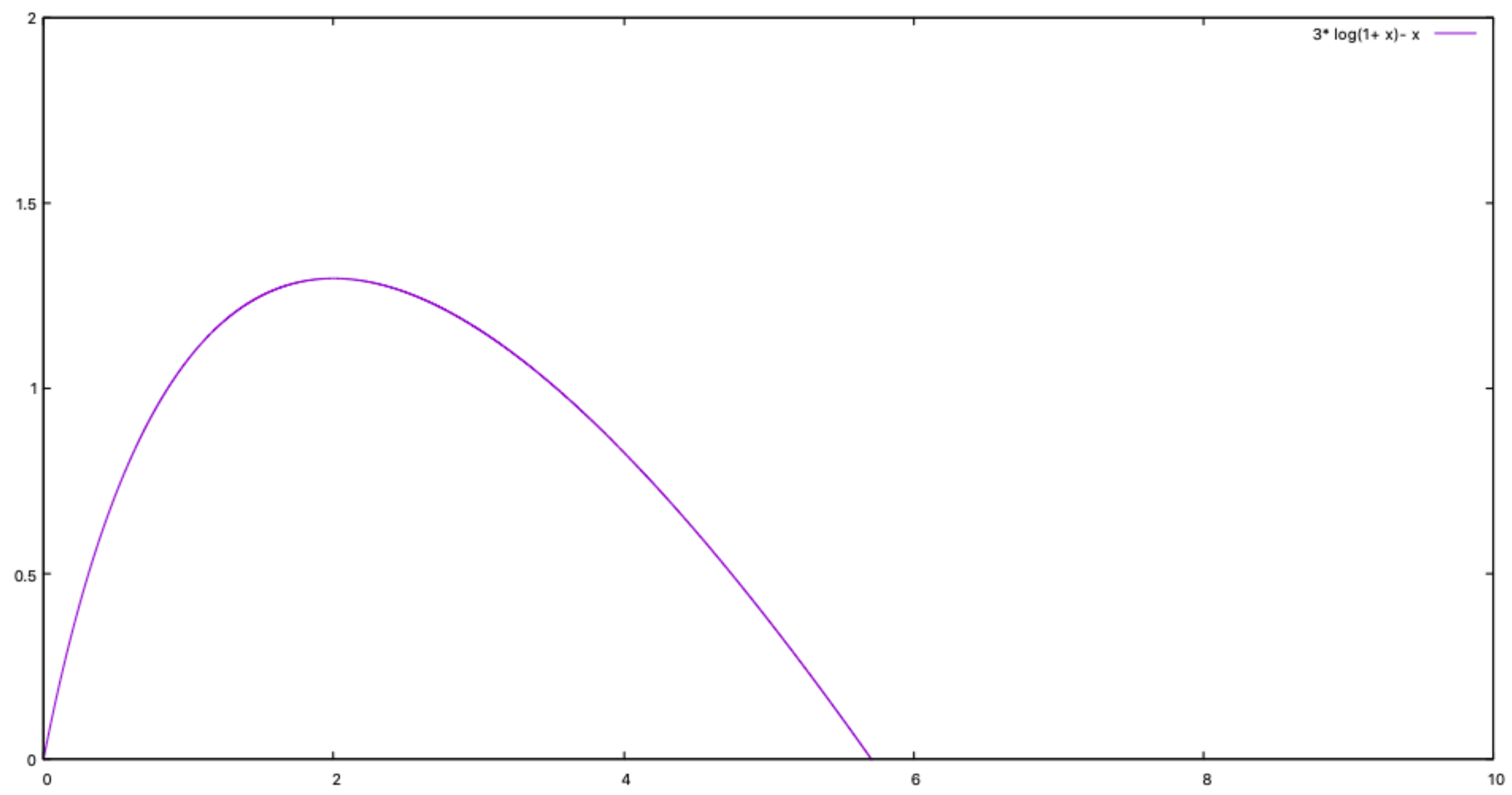
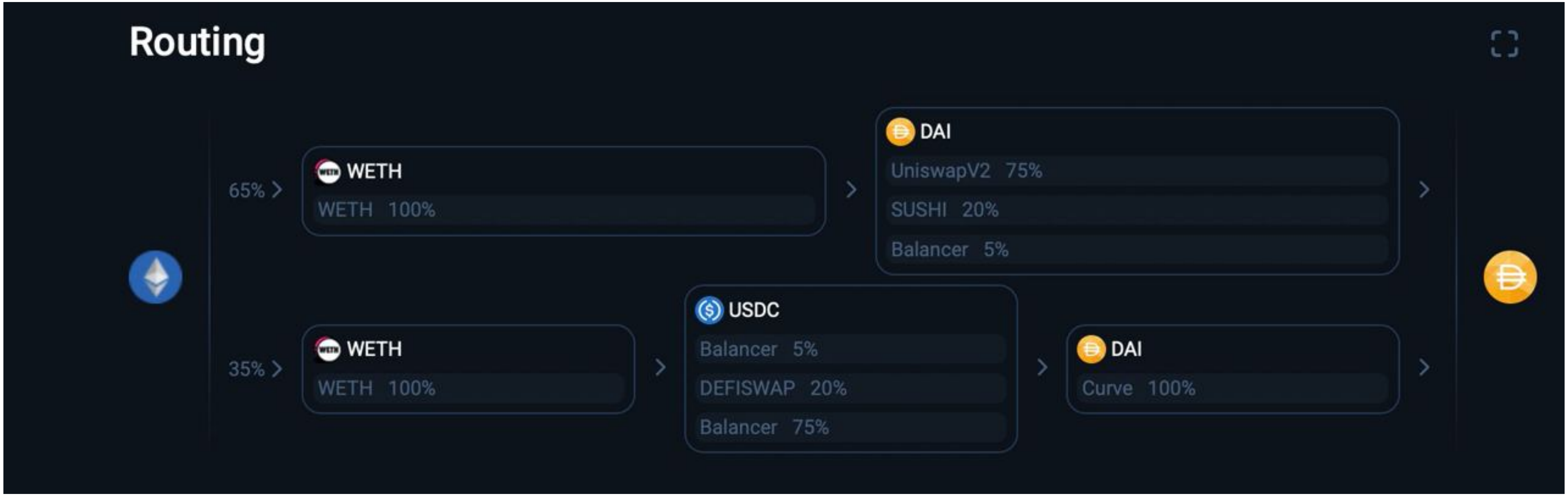


Figure 1: Plotting $f(x) - x$ with $f(x) = 3\log(1+x)$ a zz, ndec, concave function: profit is when $f(x) - x \geq 0$, the profit zone is a closed interval; max profit is obtained somewhere left of the middle of the profit interval.

Corollary 2 *If $f \not\geq I$ (eg f is bounded), either $K = (f - I)^{-1}[0, +\infty)$ is empty, or it is a compact interval and $f - I$ has a unique maximum which belongs to K.*

Routing orders

convex combination of 6 paths: ETH -> DAI



caveat: noise = asynchrony + front-running

noise and malicious noise …

reasons for hope:

secure multiparty computation and cryptographic techniques such as zero-knowledge

(the topics of the previous two lectures)

can be used to resolve front-running in theory …

but it is still an open question how