Discussion

Based on the results we get for 12 rebalance periods in appendix 1, the strategy 4 (Maximum Sharpe Ratio) outperforms the other 3 strategy in every periods. It shows robust growth since the first period, surpassing the initial portfolio value $ 1000012.93 at the end of period 2. Even though its return experiences some mild fluctuations between period 5 and period 9 such that portfolio values at the end time are lower than values at the start time within some periods, strategy 4 demonstrates significant increase in portfolio values since period 10 and reach a peak at $ 2289768.42 portfolio value at the end of period 12, which is more than twice as much as the initial portfolio value.

Meanwhile, the portfolio allocation plot for strategy 4 in appendix 3 illustrates interesting pattern. In each rebalance period, the Sharpe ration of a specific stock strictly dominant the other stocks, leading to all weights are put on that stock and our portfolio consisting only single stock. For instance, the stock NVDA's Sharpe ration dominant other stocks in period 2, 4 and 10, such that its weight is 1 in the portfolio.

On the other hand, among strategy 1, 2, and 3, the strategy 2 (equally weighted) has the best performance since period 3. According to appendix 2, the daily portfolio value plot, the strength of strategy 2 is brought to the fore as the gaps of portfolio values becomes larger and larger compared to strategy 1 and 3 since August 2020: its portfolio value increases to over $1.1 million and rises steadily reaching more than $1.6 million at the end of period 12.

Whereas, strategy 1 and 3 illustrate weak performance with daily portfolio value fluctuating around the initial point $1 million for 2 years. Although the portfolio value of strategy 3 appears to be a bit higher than that of strategy 1 in 2021, the buy and hold strategy are much easier and more efficient to manipulate in the real world without conducting too much computations. Therefore, that little gain from strategy 3 couldn't compensate much effort that was spent on it in compare to strategy 1.

Therefore, as the results shown, I would choose strategy 4, maximum Sharpe ratio, for managing my own portfolio. In addition, the strategy I would definitely not choose is strategy 3, minimum variance based on its performance. However, it makes sense that strategy 3 turns out to be the worst strategy when evaluating the return. As its objective is to minimize the variance while putting return aside, stocks with lower risk have lower return in general, this would lead to low portfolio value. However, for a risk-averse investor, this strategy might be attractive because it is much less risky while still have some gain on assets over the passive investment (buy and hold).

Lastly, I test my Python program for different variations of your strategies, e.g., select "1/n" portfolio at the beginning of period 1 and hold it till the end of period 12 (as if the rebalancing strategy required large transaction costs). As the daily portfolio value plot shown in appendix 4, the strategy 5 appears to have similar results as the equally weighted strategy (strategy 2), while yielding slightly higher results from period 9 to

period 12. Therefore, there are some improvements through implementing this new strategy in my program.

In summary, the strategy 4 (maximum Sharpe ratio) performs the best in my Python program. However, there can be improvement on the algorithm of this strategy. In my program, I first compute efficient frontier, and find 50 portfolios on the efficient frontier computing their Sharpe ratios. Then find the highest ratio as my optimal portfolio and do the validation process. In fact, the result I get is just an estimate which is close to the highest Sharpe ratio on the efficient frontier. It is possible that the real optimal one is somewhere in between my chosen portfolios. Therefore, I could find the exact maximum Sharpe ratio by solving the optimization problem as shown below. The results by using this algorithm might help me to achieve a better performance.

A portfolio w^* with the maximum Sharpe ratio can be found by solving:

$$\begin{aligned} \min_{y \in \mathbb{R}^n, \kappa \in \mathbb{R}} \quad & y^T Q y \\ \text{s.t.} \quad & \textstyle\sum_i (\mu_i - r_f) y_i = 1 \\ & (y, \kappa) \in \{\kappa > 0, \tfrac{y}{\kappa} \in \mathcal{F}\} \cup (0, 0) \end{aligned}$$

and computing $w^* = \frac{y}{\kappa}$

Appendix

1. Results for 12 periods:

Initial portfolio value = $ 1000012.93

Period 1: start date 01/02/2020, end date 02/28/2020

Strategy "Buy and Hold", value begin = $ 1000012.93, value end = $ 893956.75

Strategy "Equally Weighted Portfolio", value begin = $ 990811.32, value end = $ 892822.09

Strategy "Minimum Variance Portfolio", value begin = $ 992706.83, value end = $ 916195.86

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990062.84, value end = $ 917072.81

Period 2: start date 03/02/2020, end date 04/30/2020

Strategy "Buy and Hold", value begin = $ 945076.08, value end = $ 949228.39

Strategy "Equally Weighted Portfolio", value begin = $ 930982.28, value end = $ 861993.24

Strategy "Minimum Variance Portfolio", value begin = $ 955928.63, value end = $ 850157.53

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 947474.02, value end = $ 1001794.19

Period 3: start date 05/01/2020, end date 06/30/2020

Strategy "Buy and Hold", value begin = $ 937916.81, value end = $ 913415.30

Strategy "Equally Weighted Portfolio", value begin = $ 830758.03, value end = $ 933762.21

Strategy "Minimum Variance Portfolio", value begin = $ 825743.83, value end = $ 853349.45

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 959601.54, value end = $ 1157696.37

Period 4: start date 07/01/2020, end date 08/31/2020

Strategy "Buy and Hold", value begin = $ 905419.63, value end = $ 994693.42

Strategy "Equally Weighted Portfolio", value begin = $ 927361.20, value end = $ 1060298.21

Strategy "Minimum Variance Portfolio", value begin = $ 855822.55, value end = $ 980963.44

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1195916.24, value end = $ 1678307.43

Period 5: start date 09/01/2020, end date 10/30/2020

Strategy "Buy and Hold", value begin = $ 993194.54, value end = $ 971914.18

Strategy "Equally Weighted Portfolio", value begin = $ 1067904.54, value end = $ 998791.02

Strategy "Minimum Variance Portfolio", value begin = $ 982710.65, value end =

$ 942059.35
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1717590.47, value end = $ 1402896.00

Period 6: start date 11/02/2020, end date 12/31/2020
Strategy "Buy and Hold", value begin = $ 983801.02, value end = $ 1004435.67
Strategy "Equally Weighted Portfolio", value begin = $ 1007621.18, value end = $ 1193698.37
Strategy "Minimum Variance Portfolio", value begin = $ 950477.21, value end = $ 1005285.11
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1378054.16, value end = $ 1531469.65

Period 7: start date 01/04/2021, end date 02/26/2021
Strategy "Buy and Hold", value begin = $ 1005601.39, value end = $ 956244.15
Strategy "Equally Weighted Portfolio", value begin = $ 1180172.25, value end = $ 1266561.95
Strategy "Minimum Variance Portfolio", value begin = $ 1003284.62, value end = $ 974449.28
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1493510.69, value end = $ 1649989.68

Period 8: start date 03/01/2021, end date 04/30/2021
Strategy "Buy and Hold", value begin = $ 957791.42, value end = $ 1019731.31
Strategy "Equally Weighted Portfolio", value begin = $ 1296914.88, value end = $ 1398169.13
Strategy "Minimum Variance Portfolio", value begin = $ 974823.74, value end = $ 1087901.70
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1724319.68, value end = $ 1660992.54

Period 9: start date 05/03/2021, end date 06/30/2021
Strategy "Buy and Hold", value begin = $ 1022204.61, value end = $ 987842.85
Strategy "Equally Weighted Portfolio", value begin = $ 1397046.90, value end = $ 1458592.23
Strategy "Minimum Variance Portfolio", value begin = $ 1087630.43, value end = $ 1076570.68
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1657290.59, value end = $ 1564819.63

Period 10: start date 07/01/2021, end date 08/31/2021
Strategy "Buy and Hold", value begin = $ 993283.49, value end = $ 975250.12
Strategy "Equally Weighted Portfolio", value begin = $ 1465990.00, value end = $ 1517002.40

Strategy "Minimum Variance Portfolio", value begin = $ 1076589.93, value end = $ 1086503.39

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1588808.07, value end = $ 1759908.66

Period 11: start date 09/01/2021, end date 10/29/2021

Strategy "Buy and Hold", value begin = $ 974520.08, value end = $ 949068.41

Strategy "Equally Weighted Portfolio", value begin = $ 1512774.80, value end = $ 1562673.35

Strategy "Minimum Variance Portfolio", value begin = $ 1080947.99, value end = $ 1057065.34

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1746757.08, value end = $ 1909368.37

Period 12: start date 11/01/2021, end date 12/31/2021

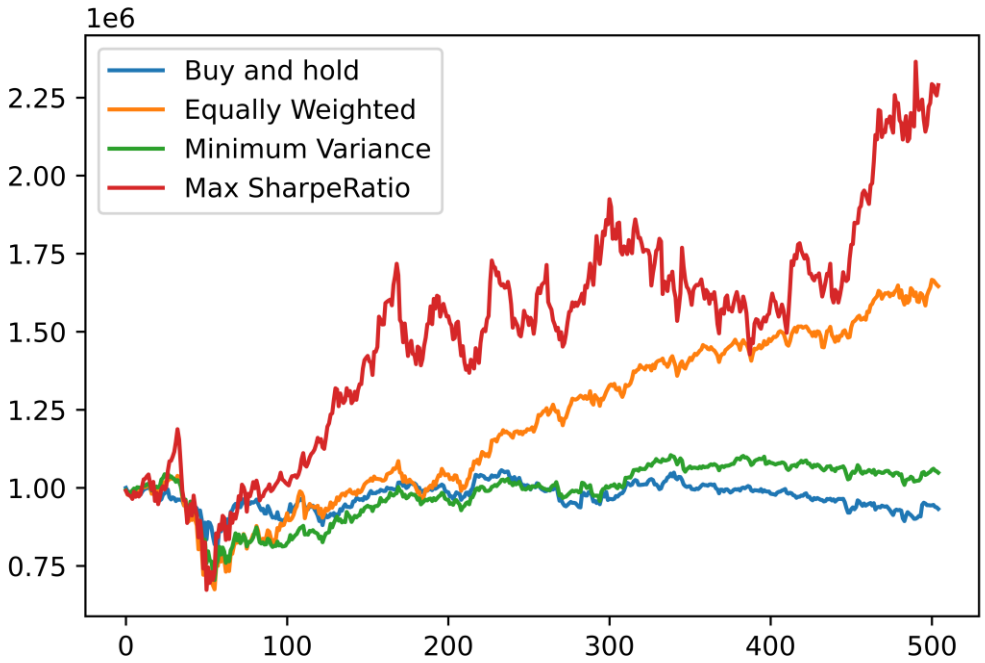
Strategy "Buy and Hold", value begin = $ 951350.41, value end = $ 932471.35

Strategy "Equally Weighted Portfolio", value begin = $ 1584031.18, value end = $ 1645811.90

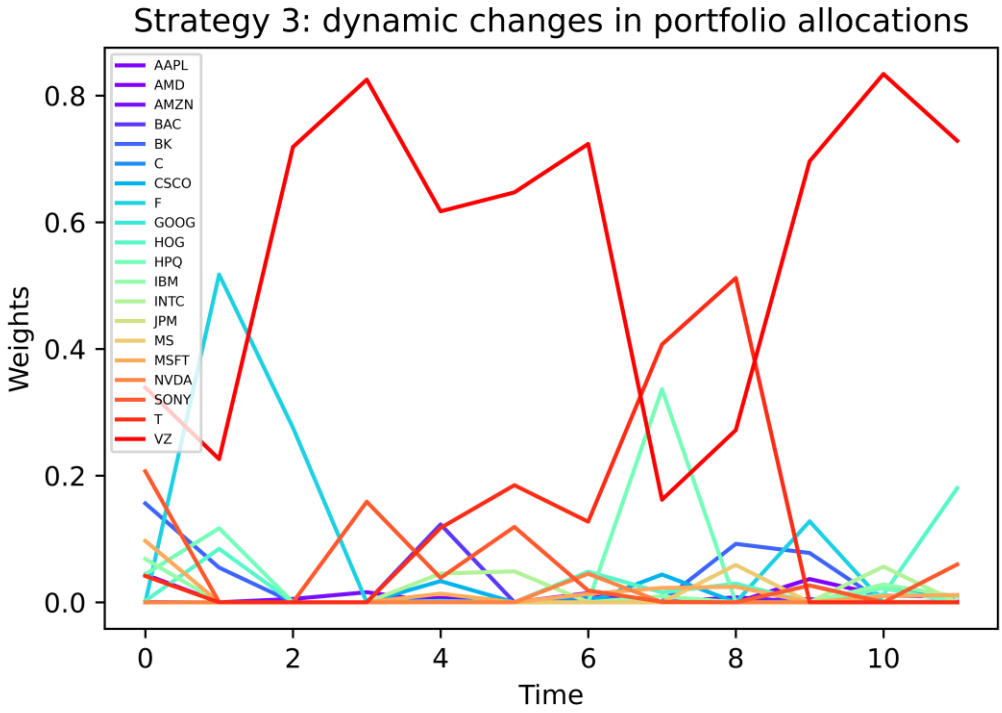
Strategy "Minimum Variance Portfolio", value begin = $ 1054320.20, value end = $ 1048451.03

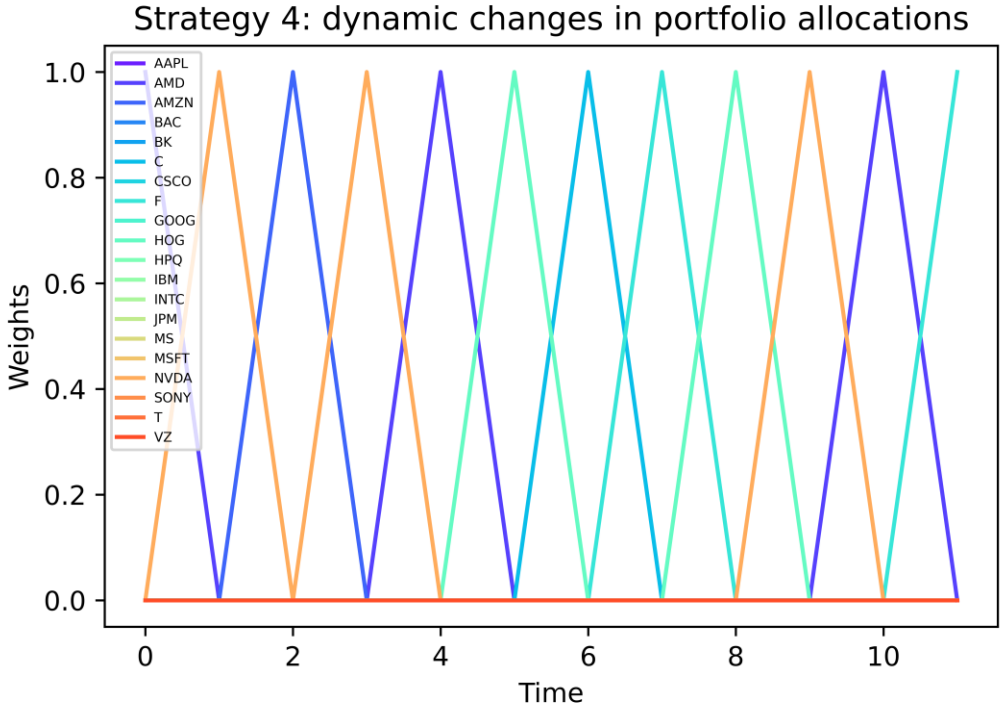
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1968980.72, value end = $ 2289768.42

2. **Daily value of portfolio (for each trading strategy) over the years 2020-2021:**

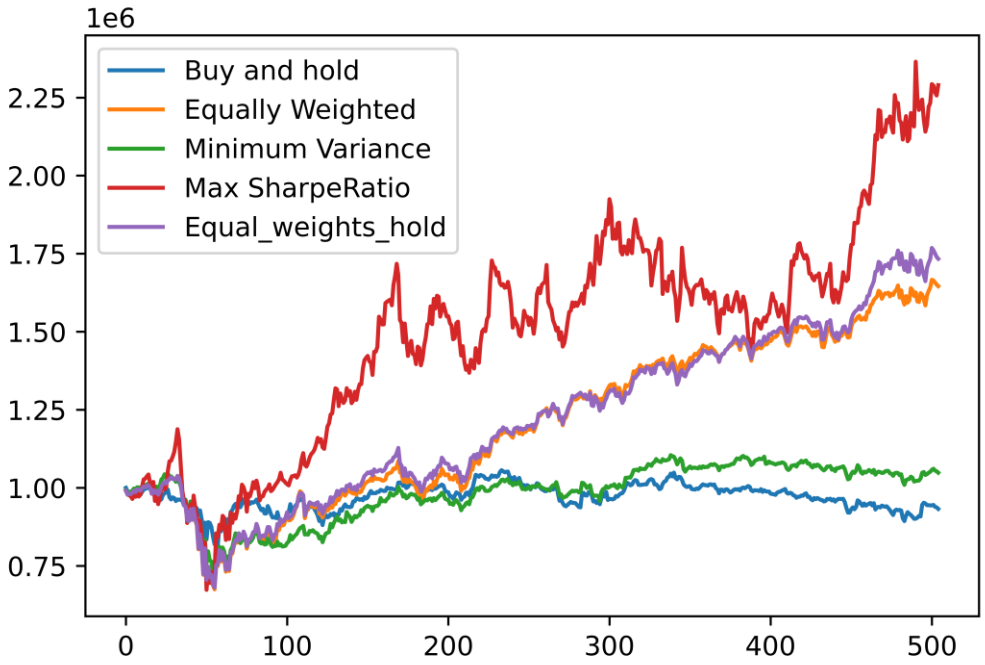


3. **Dynamic changes in portfolio allocations for strategy 3 and 4：**





4. **Test another Strategy:**



Period 1: start date 01/02/2020, end date 02/28/2020

Strategy "Buy and Hold", value begin = $ 1000012.93, value end = $ 893956.75

Strategy "Equally Weighted Portfolio", value begin = $ 990811.32, value end = $ 892822.09

Strategy "Minimum Variance Portfolio", value begin = $ 992706.83, value end = $ 916195.86

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990062.84, value end = $ 917072.81

Strategy "1/n weight and hold", value begin = $ 990811.32, value end = $ 892822.09

Period 2: start date 03/02/2020, end date 04/30/2020

Strategy "Buy and Hold", value begin = $ 945076.08, value end = $ 949228.39

Strategy "Equally Weighted Portfolio", value begin = $ 930982.28, value end = $ 861993.24

Strategy "Minimum Variance Portfolio", value begin = $ 955928.63, value end = $ 850157.53

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 947474.02, value end = $ 1001794.19

Strategy "1/n weight and hold", value begin = $ 931431.86, value end = $ 869697.72

Period 3: start date 05/01/2020, end date 06/30/2020

Strategy "Buy and Hold", value begin = $ 937916.81, value end = $ 913415.30

Strategy "Equally Weighted Portfolio", value begin = $ 830758.03, value end = $ 933762.21

Strategy "Minimum Variance Portfolio", value begin = $ 825743.83, value end =

$ 853349.45

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 959601.54, value end = $ 1157696.37

Strategy "1/n weight and hold", value begin = $ 838544.93, value end = $ 946637.59

Period 4: start date 07/01/2020, end date 08/31/2020

Strategy "Buy and Hold", value begin = $ 905419.63, value end = $ 994693.42

Strategy "Equally Weighted Portfolio", value begin = $ 927361.20, value end = $ 1060298.21

Strategy "Minimum Variance Portfolio", value begin = $ 855822.55, value end = $ 980963.44

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1195916.24, value end = $ 1678307.43

Strategy "1/n weight and hold", value begin = $ 943358.63, value end = $ 1100144.95

Period 5: start date 09/01/2020, end date 10/30/2020

Strategy "Buy and Hold", value begin = $ 993194.54, value end = $ 971914.18

Strategy "Equally Weighted Portfolio", value begin = $ 1067904.54, value end = $ 998791.02

Strategy "Minimum Variance Portfolio", value begin = $ 982710.65, value end = $ 942059.35

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1717590.47, value end = $ 1402896.00

Strategy "1/n weight and hold", value begin = $ 1111609.97, value end = $ 1021938.96

Period 6: start date 11/02/2020, end date 12/31/2020

Strategy "Buy and Hold", value begin = $ 983801.02, value end = $ 1004435.67

Strategy "Equally Weighted Portfolio", value begin = $ 1007621.18, value end = $ 1193698.37

Strategy "Minimum Variance Portfolio", value begin = $ 950477.21, value end = $ 1005285.11

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1378054.16, value end = $ 1531469.65

Strategy "1/n weight and hold", value begin = $ 1028951.18, value end = $ 1202324.39

Period 7: start date 01/04/2021, end date 02/26/2021

Strategy "Buy and Hold", value begin = $ 1005601.39, value end = $ 956244.15

Strategy "Equally Weighted Portfolio", value begin = $ 1180172.25, value end = $ 1266561.95

Strategy "Minimum Variance Portfolio", value begin = $ 1003284.62, value end =

$ 974449.28

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1493510.69, value end = $ 1649989.68

Strategy "1/n weight and hold", value begin = $ 1189791.15, value end = $ 1262081.45

Period 8: start date 03/01/2021, end date 04/30/2021

Strategy "Buy and Hold", value begin = $ 957791.42, value end = $ 1019731.31

Strategy "Equally Weighted Portfolio", value begin = $ 1296914.88, value end = $ 1398169.13

Strategy "Minimum Variance Portfolio", value begin = $ 974823.74, value end = $ 1087901.70

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1724319.68, value end = $ 1660992.54

Strategy "1/n weight and hold", value begin = $ 1292297.23, value end = $ 1382847.52

Period 9: start date 05/03/2021, end date 06/30/2021

Strategy "Buy and Hold", value begin = $ 1022204.61, value end = $ 987842.85

Strategy "Equally Weighted Portfolio", value begin = $ 1397046.90, value end = $ 1458592.23

Strategy "Minimum Variance Portfolio", value begin = $ 1087630.43, value end = $ 1076570.68

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1657290.59, value end = $ 1564819.63

Strategy "1/n weight and hold", value begin = $ 1379386.64, value end = $ 1463825.14

Period 10: start date 07/01/2021, end date 08/31/2021

Strategy "Buy and Hold", value begin = $ 993283.49, value end = $ 975250.12

Strategy "Equally Weighted Portfolio", value begin = $ 1465990.00, value end = $ 1517002.40

Strategy "Minimum Variance Portfolio", value begin = $ 1076589.93, value end = $ 1086503.39

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1588808.07, value end = $ 1759908.66

Strategy "1/n weight and hold", value begin = $ 1471827.47, value end = $ 1544995.31

Period 11: start date 09/01/2021, end date 10/29/2021

Strategy "Buy and Hold", value begin = $ 974520.08, value end = $ 949068.41

Strategy "Equally Weighted Portfolio", value begin = $ 1512774.80, value end = $ 1562673.35

Strategy "Minimum Variance Portfolio", value begin = $ 1080947.99, value end =

$ 1057065.34

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1746757.08, value end = $ 1909368.37

Strategy "1/n weight and hold", value begin = $ 1542806.45, value end = $ 1615992.76

Period 12: start date 11/01/2021, end date 12/31/2021

Strategy "Buy and Hold", value begin = $ 951350.41, value end = $ 932471.35

Strategy "Equally Weighted Portfolio", value begin = $ 1584031.18, value end = $ 1645811.90

Strategy "Minimum Variance Portfolio", value begin = $ 1054320.20, value end = $ 1048451.03

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1968980.72, value end = $ 2289768.42

Strategy "1/n weight and hold", value begin = $ 1636560.31, value end = $ 1733525.11