Report of Assignment 3

First of all, we analyze sampling error when comparing non-Normal approximations to the true (out-of-sample) loss distribution. The results of VaRs and CVaRs are shown in the table below:

Non-normal Model		VaR (99.0%)		CVaR (99.0%)		VaR (99.9%)		CVaR (99.9%)	
		Value	Error (%)	Value	Error (%)	Value	Error (%)	Error	Error (%)
Portfolio 1	True Loss	$25950421.00	/	$38422607.32	/	$53059476.34	/	$60072708.47	/
	MC 1	$37044256.19	43%	$44395881.41	16%	$53366472.35	1%	$60206750.37	0%
	MC 2	$29133844.19	12%	$35630492.57	-7%	$43381316.61	-18%	$50516529.04	-16%
Portfolio 2	True Loss	$27661160.53	/	$34829745.23	/	$44856174.49	/	$50990427.81	/
	MC 1	$27359269.99	-1%	$33265704.25	-4%	$40723196.29	-9%	$46402879.69	-9%
	MC 2	$25248597.41	-9%	$30617937.43	-12%	$36871519.94	-18%	$43065650.50	-16%

**Note: Error = (simulation value-true value) / true value*

For portfolio 1, our 2 non-Normal approximations both over-estimate the VaR at quantile level 99% compared to the true loss distribution. Moreover, Monte Carlo approximation 2 under-estimates CVaR at quantile level 99%, while Monte Carlo approximation 1 over-estimates CVaR at quantile level 99% comparing to the out-of-sample loss distribution. However, at the level 99.9%, Monte Carlo approximation 1 outperforms Monte Carlo approximation 1 in estimating the true VaR and CVaR: MC1 achieves a very close estimates of VaR and CVaR with an absolute error of $53059476.34-$53366472.35 = $306996 and $60072708.47-$60206750.37 = $134041.9 respectively, compared to the true value, whereas MC2 greatly under-estimates VaR and CVaR with absolute errors both around $10000000.

For portfolio 2, our 2 non-Normal approximations both under-estimate VaR and CVaR at quantile level 99% and 99.9% compared to the true loss distribution. However, the estimation errors from MC1 are smaller than the estimation errors from MC2. Specifically, estimation errors of MC1 as percentages of true values are within 10%, while estimation errors of MC2 as percentages of true values are ranging from 9% to 18%.

Secondly, we analyze model error when comparing Normal approximations to the true (out-of-sample) loss distribution. According to the six loss distribution plots in appendix, we can see that, both true values of VaR and CVaR at any quantile level are significantly underestimated under the assumption of normal distribution. As we can see from the true distribution histogram, the loss distribution is highly skewed with fat tail, normality distributed assumption for loss is very inappropriate. In addition, we can look at the total effect of model error and sampling error together, and the results are summarized in the table below. It clearly shows that the estimation errors of VaR and CVaR due to both model error and sampling error at any quantile level become even larger, compared to those with only sampling error.

Normal Model for MC		VaR (99.0%)	CVaR (99.0%)	VaR (99.9%)	CVaR (99.9%)
Portfolio 1	True Loss	$25950421.00	$38422607.32	$53059476.34	$60072708.47
	MC 1	$26152121.18	$29035378.15	$32651660.41	$35007315.09
	MC 2	$22102590.57	$24344691.46	$27156813.24	$28988635.91
Portfolio 2	True Loss	$27661160.53	$34829745.23	$44856174.49	$50990427.81
	MC 1	$21045387.10	$23204139.79	$25911723.31	$27675449.52
	MC 2	$19545847.09	$21439555.51	$23814711.19	$25361893.13

Value at Risk (VaR) calculates the maximum loss expected (or worst-case scenario) on an investment, over a given time period and given a specified degree of confidence. Under-estimating VaR means that we under-estimate the minimum capital required in the worst-case scenario. CVaR is a coherent risk measures that calculate the average loss beyond VaR at level of α. Therefore, with model and sampling errors, if we report the in-sample VaR and CVaR to decision-makers in the bank, the bank will suffer from big loss and even go bankrupt, due to insufficient amount of capital, when bad situations actually happened. This is a mistake that make most of institutions failed during the financial crisis.

In order to minimizing impacts of sampling and model errors, the first suggestion is increasing sample size. In our case, we only have 5000 scenarios in total during the simulation, however, it is fairly small compared to the population size (100000 scenarios). Through increasing the number of scenarios in the simulation process, we could have more observations in the tail, which will achieve more robust estimates of VaR and CVaR as noise decreases. Secondly, we should analyze the historical data carefully before making any assumptions to the model. Capturing the correct characteristics of distributions from the data, for example, variance, skewness and kurtosis, to avoid making wrong model assumption in estimations. In our case, with the knowledge of skewness and kurtosis of distribution from historical data, we would avoid making normally distributed model assumption.

Appendix

1. Output from the Python code:

Portfolio 1:

Out-of-sample: VaR 99.0% = $25950421.00, CVaR 99.0% = $38422607.32
In-sample MC1: VaR 99.0% = $37044256.19, CVaR 99.0% = $44395881.41
In-sample MC2: VaR 99.0% = $29133844.19, CVaR 99.0% = $35630492.57
In-sample No: VaR 99.0% = $20850017.04, CVaR 99.0% = $23026621.57
In-sample N1: VaR 99.0% = $26152121.18, CVaR 99.0% = $29035378.15
In-sample N2: VaR 99.0% = $22102590.57, CVaR 99.0% = $24344691.46

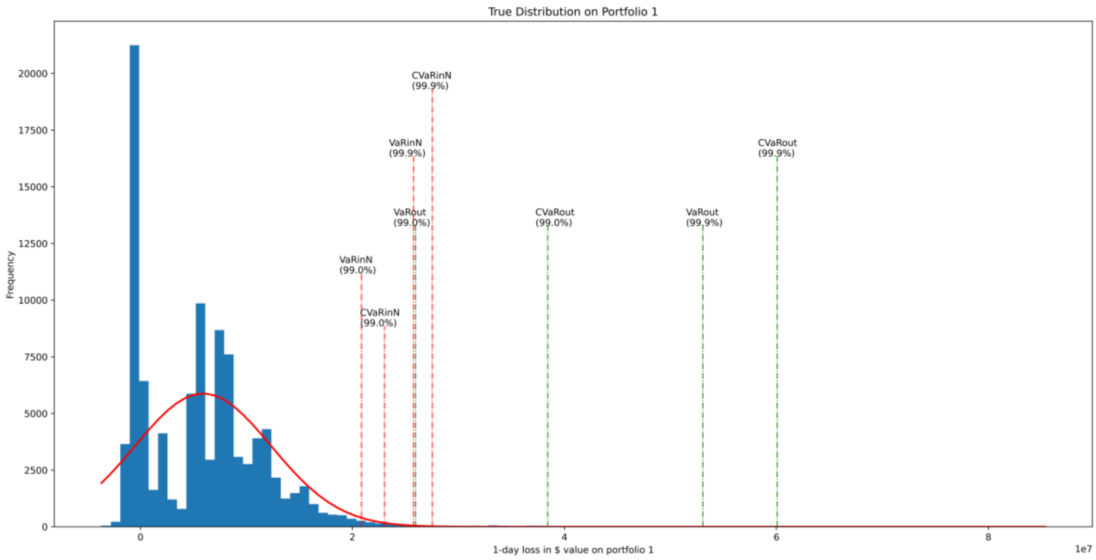
Out-of-sample: VaR 99.9% = $53059476.34, CVaR 99.9% = $60072708.47
In-sample MC1: VaR 99.9% = $53366472.35, CVaR 99.9% = $60206750.37
In-sample MC2: VaR 99.9% = $43381316.61, CVaR 99.9% = $50516529.04
In-sample No: VaR 99.9% = $25756595.51, CVaR 99.9% = $27534906.89
In-sample N1: VaR 99.9% = $32651660.41, CVaR 99.9% = $35007315.09
In-sample N2: VaR 99.9% = $27156813.24, CVaR 99.9% = $28988635.91

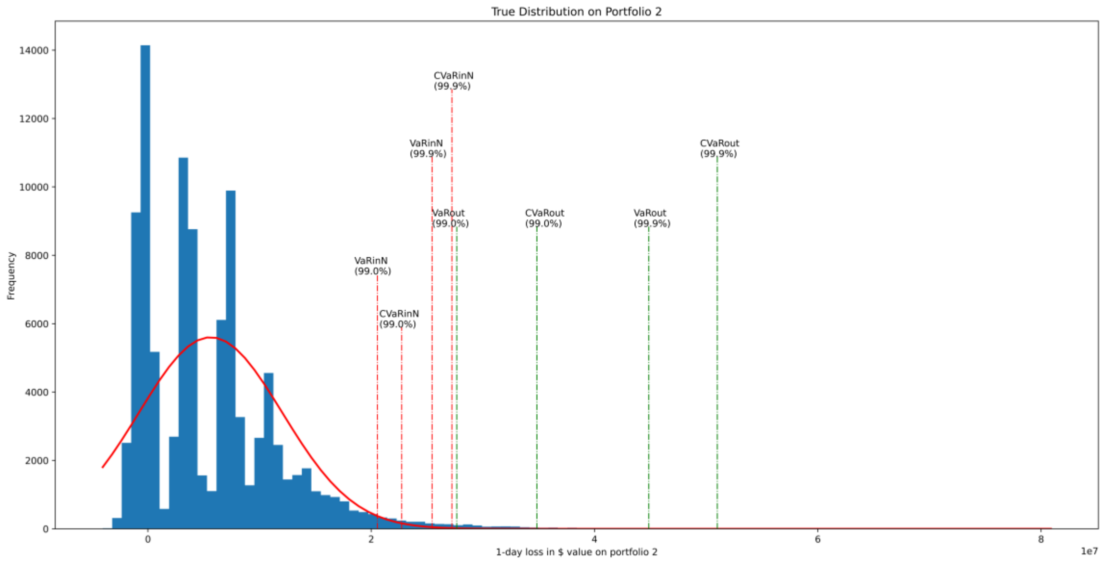
Portfolio 2:

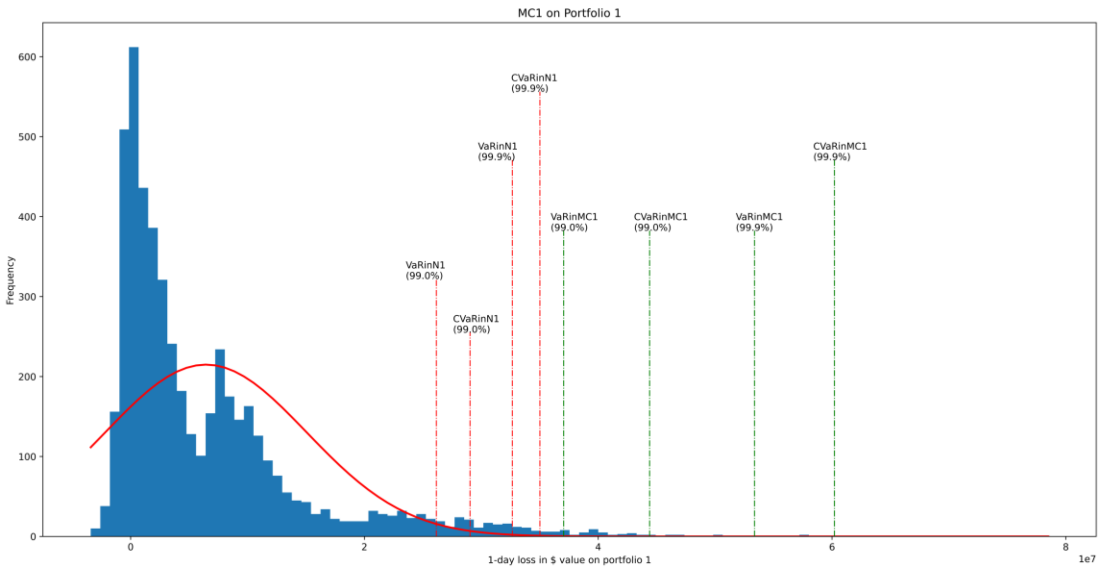
Out-of-sample: VaR 99.0% = $27661160.53, CVaR 99.0% = $34829745.23
In-sample MC1: VaR 99.0% = $27359269.99, CVaR 99.0% = $33265704.25
In-sample MC2: VaR 99.0% = $25248597.41, CVaR 99.0% = $30617937.43
In-sample No: VaR 99.0% = $20544714.86, CVaR 99.0% = $22719671.08
In-sample N1: VaR 99.0% = $21045387.10, CVaR 99.0% = $23204139.79
In-sample N2: VaR 99.0% = $19545847.09, CVaR 99.0% = $21439555.51

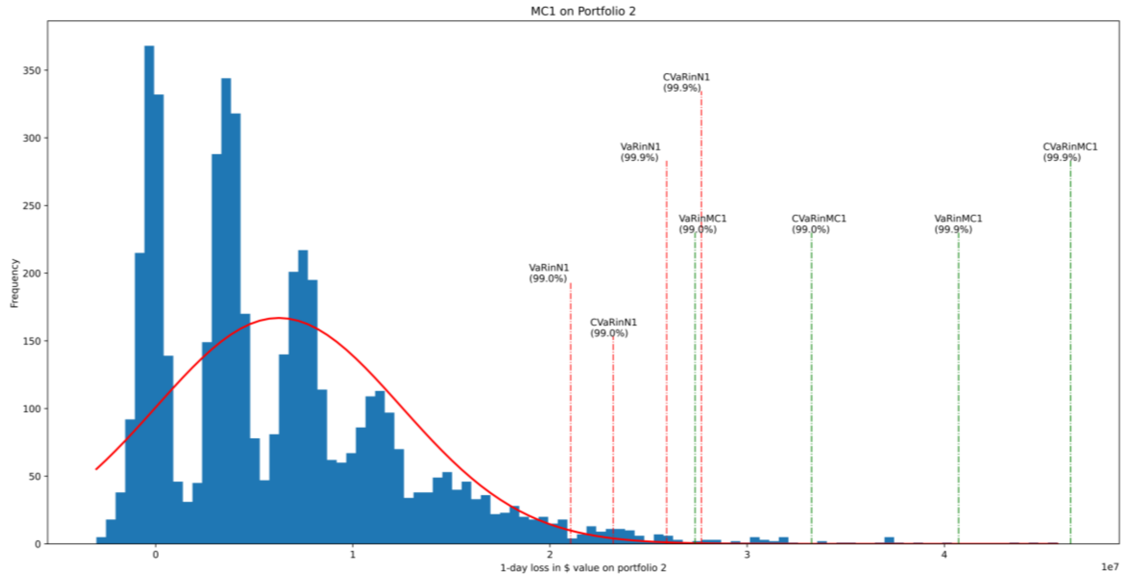
Out-of-sample: VaR 99.9% = $44856174.49, CVaR 99.9% = $50990427.81
In-sample MC1: VaR 99.9% = $40723196.29, CVaR 99.9% = $46402879.69
In-sample MC2: VaR 99.9% = $36871519.94, CVaR 99.9% = $43065650.50
In-sample No: VaR 99.9% = $25447577.64, CVaR 99.9% = $27224542.32
In-sample N1: VaR 99.9% = $25911723.31, CVaR 99.9% = $27675449.52
In-sample N2: VaR 99.9% = $23814711.19, CVaR 99.9% = $25361893.13

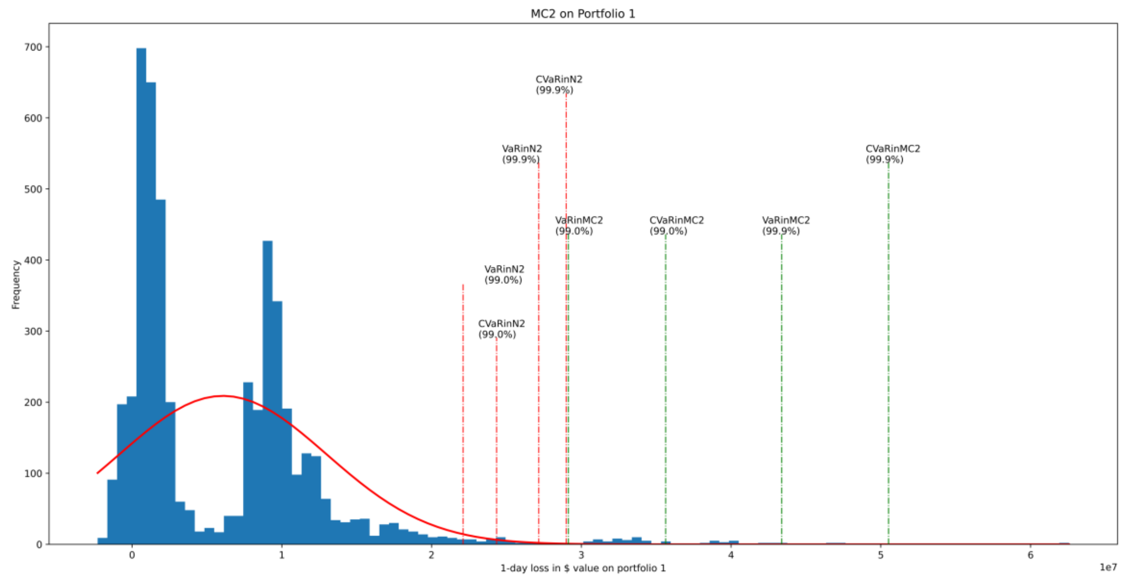
2. Loss distribution plots:

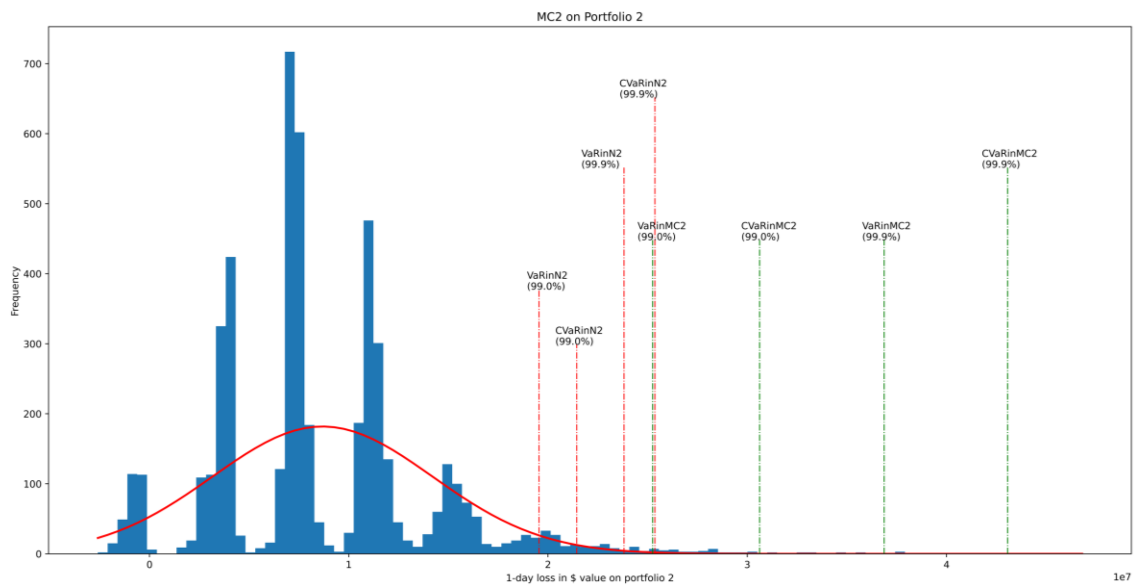

True Distribution on Portfolio 1
CVaRinN (99.9%)
VaRinN (99.9%)
VaRout (99.0%)
CVaRout (99.0%)
VaRout (99.9%)
CVaRout (99.9%)
VaRinN (99.0%)
CVaRinN (99.0%)
Frequency
1-day loss in $ value on portfolio 1
1e7


True Distribution on Portfolio 2
CVaRinN (99.9%)
VaRinN (99.9%)
CVaRout (99.9%)
VaRout (99.0%)
CVaRout (99.0%)
VaRout (99.9%)
VaRinN (99.0%)
CVaRinN (99.0%)
Frequency
1-day loss in $ value on portfolio 2
1e7


MC1 on Portfolio 1
CVaRinN1 (99.9%)
VaRinN1 (99.9%)
CVaRinMC1 (99.9%)
VaRinMC1 (99.0%)
CVaRinMC1 (99.0%)
VaRinMC1 (99.9%)
VaRinN1 (99.0%)
CVaRinN1 (99.0%)
Frequency
1-day loss in $ value on portfolio 1
1e7


MC1 on Portfolio 2
Frequency
1-day loss in $ value on portfolio 2
1e7
VaRinN1
(99.0%)
CVaRinN1
(99.0%)
VaRinN1
(99.9%)
CVaRinN1
(99.9%)
VaRinMC1
(99.0%)
CVaRinMC1
(99.0%)
VaRinMC1
(99.9%)
CVaRinMC1
(99.9%)


MC2 on Portfolio 1
Frequency
1-day loss in $ value on portfolio 1
1e7
VaRinN2
(99.0%)
CVaRinN2
(99.0%)
VaRinN2
(99.9%)
CVaRinN2
(99.9%)
VaRinMC2
(99.0%)
CVaRinMC2
(99.0%)
VaRinMC2
(99.9%)
CVaRinMC2
(99.9%)


MC2 on Portfolio 2
Frequency
1-day loss in $ value on portfolio 2
1e7
VaRinN2
(99.0%)
CVaRinN2
(99.0%)
VaRinN2
(99.9%)
CVaRinN2
(99.9%)
VaRinMC2
(99.0%)
CVaRinMC2
(99.0%)
VaRinMC2
(99.9%)
CVaRinMC2
(99.9%)