Assignment 3 Report

Firstly, based on the expected returns of all 6 portfolios on the out-of-sample data for a $1000 investment, we can conclude that RE optimization achieves higher returns from all 3 (minimum variance, maximize return and maximize Sharpe ratio portfolio) strategies compared to Classical Mean-Variance Optimization. The reason RE outperforms MV optimization on the test data is that RE optimization does not maximize in-sample parameters, rather it accounts for the possibility these parameters are wrong. Specifically, since MV optimization ignore the uncertainty in historical data (input), it failed to making profit (with a return of $-584.6) on investment in 2021 under the maximum return strategy. In contrast, RE technology could earn as high as $327.3 from $1000 investment under the maximum Sharpe ration strategy in 2021.

Secondly, the in-sample classical and resampled efficient frontiers appear to have different behaviors in the plot. The REF plots below the classical efficient frontier, and it does not extend as far as, the classical efficient frontier. It shows that with an information level constraint, resampled efficient frontier expects less return on a given level of risk and restricts risk to a narrower range. "It is noteworthy that this result is consistent with much professional practice; few recommended asset allocations involve portfolios near the top of the MV efficient frontier. RE optimization quantifies this important investment intuition" (Michaud, R. O., & Michaud, R. ,2007). Since resampled efficient frontier portfolios are an average of properly associated MV optimal portfolios, they appear to be safer and less extreme investments.

Thirdly, portfolio compositions for classical and resampled frontiers demonstrate different patterns as shown in the plots below in appendix. For example, the maximum return MV optimal portfolio represents a 100% bet in MULN stock. In contrast, the maximum return REF portfolio is more diversified, with significant weights putting on stock AMD, MULN, TSLA, and AAPL. Thus, REF optimized portfolios tend to be less risky which reflect investment sense. RE optimality. Moreover, in the classical case no more than half of 25 stocks are included in the MV efficient frontier portfolios, and many of them are composed of only 5 stocks: AMD, MULN, TSLA, AAPL, and GLD. Whereas, resampled efficient frontier portfolios include more stocks, and there is a smooth transition from one risk level to another. It never put all weights on a single asset.

In summary, RE technology allows investors to include the level of certainty in risk-return estimates in the optimization, which in turn improve the investment performance on the out-of-sample data. Comparing to the classical Mean-Variance Optimization, Resampled Efficient Frontier Optimization provides a safer, more useful and reliable solutions to the investors.

Appendix

1. The expected return on the 2021 (out-of-sample) data for a $1000 investment:

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On the Classical Efficient Frontier, the minimum variance portfolio returns $ [28.22424688]
On the Classical Efficient Frontier, the maximum return portfolio returns $ [-584.59094047]
On the Classical Efficient Frontier, the optimal Sharpe ratio portfolio returns $ [279.5883226]

On the Resampled Efficient Frontier, the minimum variance portfolio returns $ [59.06580056]
On the Resampled Efficient Frontier, the maximum return portfolio returns $ [37.33403957]
On the Resampled Efficient Frontier, the optimal Sharpe ratio portfolio returns $ [327.28394219]
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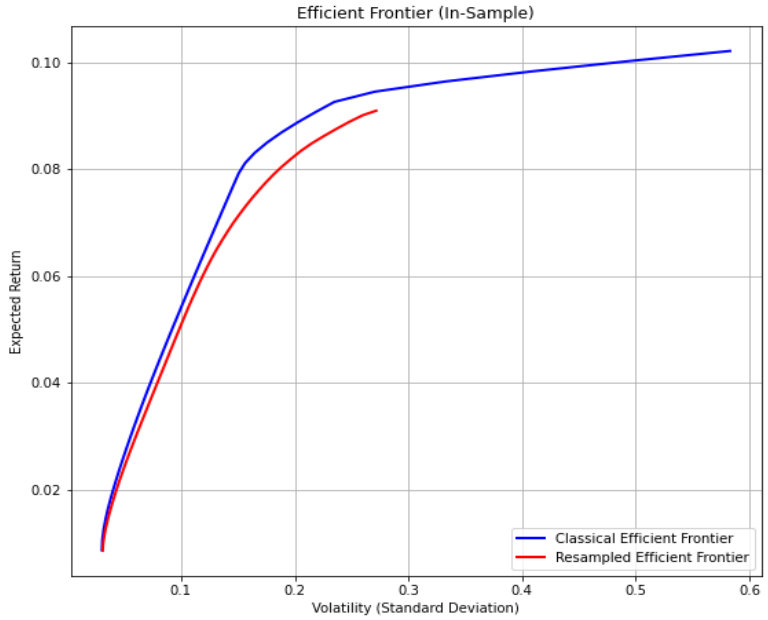
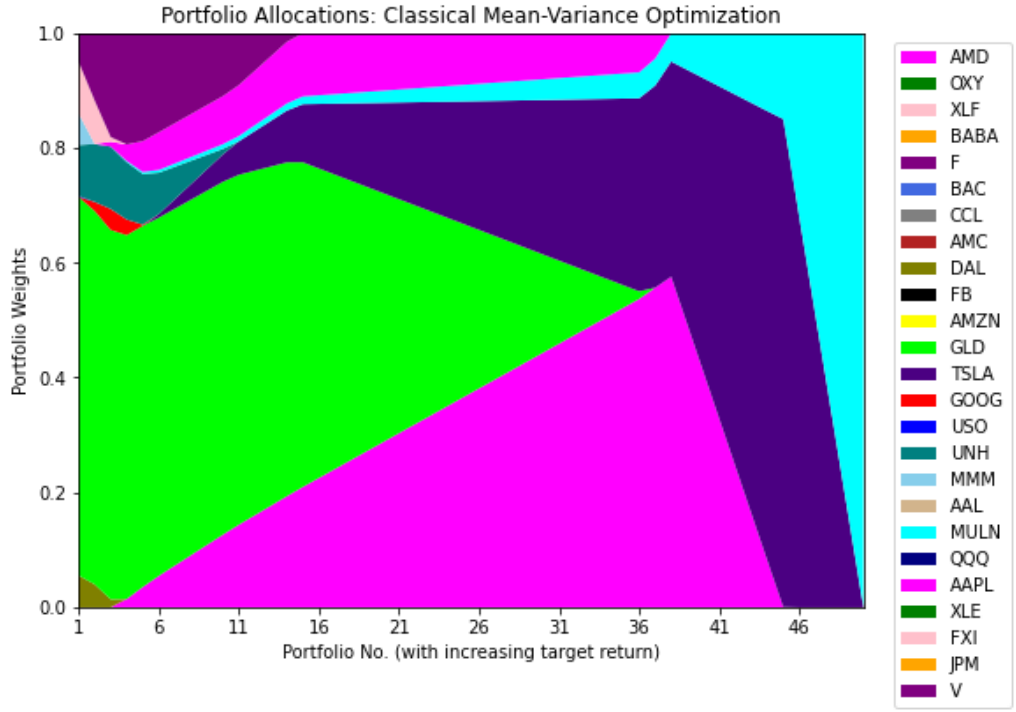
2. The classical and resampled efficient frontiers:



3. Portfolio compositions for classical frontiers:



4. Portfolio compositions for resampled frontiers: