The 5th Report of Assignments of Advanced Mathematics II

Cheng-Chao Huang

Department of Computer Science and Technology,
East China Normal University, Shanghai, China
ecnucchuang@126.com

1 Exercises of §8.1

1-(2) $y > x \geq 0 \wedge x^2 + y^2 < 1$.
1-(4) $x \neq 0 \wedge y \neq 0 \wedge z^2 \leq x^2 + y^2$.
2-(1) $\ln(2)$.
2-(2)

$$\lim_{(x,y)\to(0,0)} \frac{2-\sqrt{xy+4}}{xy} = \lim_{(x,y)\to(0,0)} \frac{4-(xy+4)}{xy(2+\sqrt{xy+4})} = -\frac{1}{4} \ .$$

2-(4)

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{(x^2+y^2)e^{x^2y^2}} = \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)^2}{2} \frac{1}{(x^2+y^2)e^{x^2y^2}} = 0 \ .$$

4-(1) $f(x,y)$ is continuous at $(0,0)$, since

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)^{-1}}{-(x^2+y^2)^{-2}} = 0 = f(0,0) \ .$$

4-(2) $f(x,y)$ is continuous at $(0,0)$, since when $\lim_{(x,y)\to(0,0)}(x+y) = 0$ and $\cos(1/x)$ is bounded s.t.

$$\lim_{(x,y)\to(0,0)} (x+y)\cos\frac{1}{x} = 0 = f(0,0) \ .$$

2 Exercises of §8.2

1 & 2 See the solutions in the course book.
3 The included angle between X-axis and the tangent line of the curve at $(2,4,5)$ is $\pi/4$, since $\partial z/\partial x\big|_{(x,y,z)=(2,4,5)} = 1$.
5 & 6 See the solutions in the course book.

3 Exercises of §8.3

1 & 2 See the solutions in the course book.
3

Proof. $f(x, y)$ is continuous at $(0, 0)$, since $\lim_{(x,y)\to(0,0)} = f(0, 0)$. Furthermore, we compute the first-order partial derivative of $f(x, y)$ at $(0, 0)$ as $f_x(0, 0) = f_y(0, 0) = 0$. Then we have

$$\begin{aligned}
&\Delta f - f_x(0,0)\Delta x - f_y(0,0)\Delta y \\
=&(\Delta x^2 + \Delta y^2)\sin\frac{1}{\Delta x^2 + \Delta y^2} \\
=&o(\sqrt{\Delta x^2 + \Delta y^2})\ .
\end{aligned}$$

Thus, $f(x, y)$ is differentiable at $(0, 0)$. However, its partial derivatives are not continuous at $(0, 0)$ because they are not defined at $(0, 0)$.

4-(1)
Let $g(x, y) = \sqrt{x^3 + y^3}$, $(\hat{x}, \hat{y}) = (1, 2)$ and $\Delta x, \Delta y) = (0.02, -0.03)$. It's easy to prove that $g(x, y)$ is differentiable at $(\hat{x}, \hat{y})$. Thus,

$$\begin{aligned}
&g(\hat{x} + \Delta x, \hat{y} + \Delta y) \\
\approx&g(\hat{x}, \hat{y}) + f_x(\hat{x}, \hat{y})\Delta x + f_y(\hat{x}, \hat{y})\Delta y \\
\approx&3 + 0.5 \cdot 0.02 + 2 \cdot (-0.03). \\
\approx&2.95\ .
\end{aligned}$$

4 Exercises of §8.4

1 & 2 See the solutions in the course book.
3
Let $\Psi(t) = \int e^{-t^2} dt$. Then we have

$$\begin{aligned}
&\frac{d\int_{2u}^{v^2+u} e^{-t^2} dt}{dx} \\
=&\frac{d\Psi(v^2+u)}{dx} - \frac{d\Psi(2u)}{dx} \\
=&\frac{d\Psi(v^2+u)}{d(v^2+u)}\frac{d(v^2+u)}{dx} - \frac{d\Psi(2u)}{d(2u)}\frac{d(2u)}{dx} \\
=&e^{-(v^2+u)^2}\left(\frac{dv^2}{dx} + \frac{du}{dx}\right) + e^{-(2u)^2}\frac{d(2u)}{dx} \\
=&(2e^{2x} + \cos x)e^{-(e^{2x}+\sin x)^2} - 2\cos x e^{-4\sin^2 x}\ .
\end{aligned}$$

6 & 7 See the solutions in the course book.