

The posterior distribution of $(\beta, \gamma, \sigma, c)$ is given as

$$\begin{aligned} p(\beta, \gamma, \sigma, c | Y) &\propto L(Y | \beta, \gamma, \sigma, c) \pi_\beta(\beta) \pi_\gamma(\gamma) \pi_\sigma(\sigma) \pi_c(c) \\ &= \frac{1}{\prod_{i=1}^n (x_i' \gamma)} p(\epsilon_1, \dots, \epsilon_n | \beta, \gamma, \sigma, c) \pi_\beta(\beta) \pi_\gamma(\gamma) \pi_\sigma(\sigma) \pi_c(c) \end{aligned}$$

where $\epsilon_i = (y_i - x_i' \beta) / (x_i' \gamma)$.

And

$$\begin{aligned} p(\epsilon_1, \dots, \epsilon_n | \beta, \gamma, \sigma, c) &= \prod_{i=1}^n \left(\left(\prod_{j=2}^M \frac{c j^2 + n_{\epsilon_1 \dots \epsilon_j(\epsilon_i)}(\epsilon_1, \dots, \epsilon_{1:i-1})}{2c j^2 + n_{\epsilon_1 \dots \epsilon_{j-1}(\epsilon_i)}(\epsilon_1, \dots, \epsilon_{1:i-1})} \right) 2^{M-1} g_\sigma(\epsilon_i) \right) \\ &\leq 2^{n(M-1)} \prod_{i=1}^n g_\sigma(\epsilon_i) \end{aligned}$$

Plug in the previous equation, it can be shown that the posterior is proper as long as it is proper under the parametric model with centering distribution G_θ when using a flat prior $p(\beta) \propto 1$.