1 T error

We want to simulate t errors.

Remark 1: If $X|W \sim N(0, W)$, $W \sim IG(\nu/2, \nu/2)$, then $X \sim t_\nu$.

Now we introduce a latent variable W, such that

$$Y_1|R = 1, W, x \sim N(\beta_{10}^{(1)} + \beta_{11}^{(1)} x, W) \tag{1}$$

$$Y_2|R = 1, W, x, Y_1 \sim N(\beta_{20}^{(1)} + \beta_{21}^{(1)} x + \beta_{22}^{(1)} Y_1, 3W/4) \tag{2}$$

where R stands for the missingness indicator.

Integrate $Y_1|R = 1, W, x$ out of (2):

$$Y_2|R = 1, W, x \sim N((\beta_{20}^{(1)} + \beta_{22}^{(1)}\beta_{10}^{(1)}) + (\beta_{21}^{(1)} + \beta_{22}^{(1)}\beta_{11}^{(1)})x, ((\beta_{22}^{(1)}))^2 + 3/4)W) \tag{3}$$

Let $\beta_{22}^{(1)} = 1/2$, thus

$$Y_2|R = 1, W, x \sim N((\beta_{20}^{(1)} + \beta_{22}^{(1)}\beta_{10}^{(1)}) + (\beta_{21}^{(1)} + \beta_{22}^{(1)}\beta_{11}^{(1)})x, W) \tag{4}$$

Now use remark 1, we have

$$Y_1|R = 1, x \sim \beta_{10}^{(1)} + \beta_{11}^{(1)} x + \epsilon_1 \tag{5}$$

$$Y_2|R = 1, x \sim (\beta_{20}^{(1)} + \beta_{22}^{(1)}\beta_{10}^{(1)}) + (\beta_{21}^{(1)} + \beta_{22}^{(1)}\beta_{11}^{(1)})x + \epsilon_2 \tag{6}$$

where $\epsilon_1, \epsilon_2 \sim t_\nu$

By similar approach, we can define distribution for $R = 0$:

$$Y_1|R = 0, x, W \sim \beta_{10}^{(0)} + \beta_{11}^{(0)} x + N(0, W) \tag{7}$$

and if we further assume $p(Y_2|R = 1, W, x) = p(Y_2|R = 0, W, x)$, we will have $p(Y_2|R = 1, x) = p(Y_2|R = 0, x)$, which yields MAR.

Therefore, the complete sampling plan for t error with MAR can be :

1. $W \sim IG(\nu/2, \nu/2)$
2. $R \sim Bernoulli(\pi)$
3. draw Y_1 based on R and (1) or (7)
4. draw Y_2 based on (2) , regardless R due to MAR

As we mentioned in 0429.pdf, we have the linear form for marginal quantile lines for the mixture of t distributions.

2 ALD error

Kuzobowski and Podgorski (2000) point out if

$$\xi \sim Gamma(1, \tau)(\text{rate})$$
$$\epsilon_p|\xi \sim N\left(\frac{1-2p}{p(1-p)}\xi, \frac{2\xi}{\tau p(1-p)}\right)$$

then marginally $\epsilon_p \sim ALD(p, 0, \tau)$, where τ is a scale parameter and $Pr(\epsilon_p < 0) = p$.

Therefore, we still introduce a latent variable ξ. Let

$$Y_1|R = 1, \xi \sim x\beta_1^{(1)} + N\left(\frac{1-2p}{p(1-p)}\xi, \frac{2\xi}{\tau p(1-p)}\right) \tag{8}$$
$$Y_2|R = 1, \xi, Y_1 \sim x\beta_2^{(1)} + \frac{1}{2}Y_1 + N\left(\frac{1}{2}\frac{1-2p}{p(1-p)}\xi, \frac{2\xi}{\tau p(1-p)} * \frac{3}{4}\right) \tag{9}$$

In (9), integrate $Y_1|R = 1, \xi$ out:

$$Y_2|R = 1, \xi \sim x(\beta_2^{(1)} + \frac{1}{2}\beta_1^{(1)}) + N\left(\frac{1-2p}{p(1-p)}\xi, \frac{2\xi}{\tau p(1-p)}\right)$$

Thus by Kuzobowski and Podgorski (2000),

$$Y_1|R = 1 \sim ALD(p, x\beta_1^{(1)}, \tau)$$
$$Y_2|R = 1 \sim ALD(p, x(\beta_2^{(1)} + \frac{1}{2}\beta_1^{(1)}), \tau)$$

Using the similar techniques in previous section, we propose a sufficient condition to yield MAR. Let $p(Y_2|R = 1, \xi, Y_1) = p(Y_2|R = 0, \xi, Y_1$, then $p(Y_2|R = 1, Y_1) = p(Y_2|R = 0, Y_1)$.

Propose distribution for $Y_1|R = 0$:

$$Y_1|R = 0, \xi \sim x\beta_1^{(0)} + N\left(\frac{1-2p}{p(1-p)}\xi, \frac{2\xi}{\tau p(1-p)}\right) \tag{10}$$

Then the complete sampling procedure for Asymmetric laplace distribution with MAR could be:

1. draw $\xi \sim Gamma(1, \tau)$
2. draw $R \sim Bernoulli(\pi)$
3. draw $Y_1|R, \xi$ from (8) or (10)
4. draw $Y_2|Y_1, R, \xi$ from (9)

Thus within each pattern (R) , Y_1 and Y_2 are ALD, and by method proposed in 0429.pdf, we can get the linear form for marginal quantile regression line for mixture of ALD distribution.