An analysis of Shenzhen car plates allocation mechanism and price forecasting

Abstract

This paper examines the participants' choices, the allocation results, and the determination of the lowest transaction price in auctions under the car plate allocation mechanism in Shenzhen. Various statistical methods are used to predict the lowest transaction price. The paper presents the estimation of the equality and efficiency of car plate allocation in Shenzhen. The average efficiency of each period is 971.1 million yuan, with a ratio of consumer surplus to government revenue of about 5:1. The average Gini coefficient of car plate accessibility is 0.5075. The study suggests that distributing additional car plate quotas through auction would improve efficiency but worsen equality. (3) This paper presents multivariate linear regression models, proposing that the lowest transaction price this month is a linear function of the lowest price in the past or the second broadcast average price, modified by the current supply and demand. An increase in the auction quota will cause the price to adjust downward, and an increase in the number of bidders will cause the price to adjust upward. (4) The study establishes a VAR model to analyze the dynamic relationship between the lowest price and the number of bidders. The results show a two-way Granger causality between the two variables. (5) In addition, a seasonal ARIMA model and a random forest algorithm are employed to predict the lowest transaction price in the future. The random forest algorithm exhibits the highest prediction accuracy and can serve as a reliable reference for bidding.

Keywords: Car Plate Allocation; Hybrid Allocation Mechanism; Bayesian Nash Equilibrium; Random Forest

Chapter 1 Introduction

Due to population growth and urbanization, cars have become a household necessity. However, their proliferation has resulted in negative externalities such as air pollution and road congestion. Since it is important to address these issues to ensure sustainable development, several cities in China have implemented policies to restrict the growth of automobiles. These policies include a quota system for new automobile licenses and restrictions on non-local licensed vehicles. Beijing and Shanghai have adopted lottery and auction as allocation mechanisms, respectively, while Guangzhou has implemented an auction-lottery hybrid mechanism.

Shenzhen, another first-tier city, faces a similar challenge. According to the Shenzhen Municipal People's Government's Circular on the Implementation of Small Vehicle Incremental Regulation and Management, by December 20, 2014, the number of registered motor vehicles in the city had exceeded 3.14 million, with an average annual growth rate of 16%. In January 2015, Shenzhen implemented a car plate allocation mechanism, similar to Guangzhou's hybrid mechanism, to restrict vehicle growth.

The aim of this paper is to examine how participants choose between auction and lottery under the allocation mechanism of Shenzhen's car plates, the equality and efficiency, and how the minimum transaction price is determined. Additionally, this paper aims to predict the minimum transaction price using various methods.

Chapter 2 Model and Equilibrium Analysis

In this chapter, I introduce a theoretical framework to analyze lottery participants' and bidders' strategies under the mixed allocation mechanism, derive a symmetric Bayesian Nash Equilibrium and estimate measures of equality and efficiency. The model's BNE indicates that participants adopt a 'threshold strategy',meaning that there exists a willingness-to-pay threshold above which those who choose to participate in the auction and the rest in the lottery. The comparative static analysis shows that the threshold value is positively correlated with the number of participants and negatively correlated with the quota allocated to auction.

2.1 Mechanism Description

Each eligible participant must choose between participating in an auction or a lottery each period. Allocation typically occurs monthly. At the start of each month, the Shenzhen Small Vehicle Incremental Regulation Management Information System announces the total number of car plates to be allocated, with the auction taking place on the 25th, and lottery on the 26th.

The auction is conducted online, with bidders submitting sealed offers between 9:00 and 15:00. The bidder's offer must be valid and meet the following criteria: it must be greater than or equal to the reserve price, less than or equal to twice the average transaction price of the previous period, and an integer multiple of ¥100. A broadcast of the average price will be made once at 11:00 and once at 13:00. The average broadcast price is calculated by excluding the highest and lowest 10% of bids at the announcement, and rounding the result to the nearest whole number.

The system will only accept the last bid to determine the winning bidders, the highest bidders or the earliest bidders if the bids are the same will win the plates.At the end of each auction, the bidding system publishes key information, including the lowest transaction price, average transaction price, number of bidders at the lowest winning bid as recorded by the bidding system.

Figure 2.1 shows that the total number of participants, particularly those in the lottery, has continued to increase over time, resulting in a steady decline in the winning rate in lottery, which is the number of car plates for lottery divided by the number of participants in the lottery. Additionally, it reveals that the number of car plates in each period is almost equally distributed between the lottery and the auction. However, the government prefers to allocate more to the auction when there are additional licenses available. Finally, the winning rate in auction is calculated as car plates for auction divided by the number of auction participants.

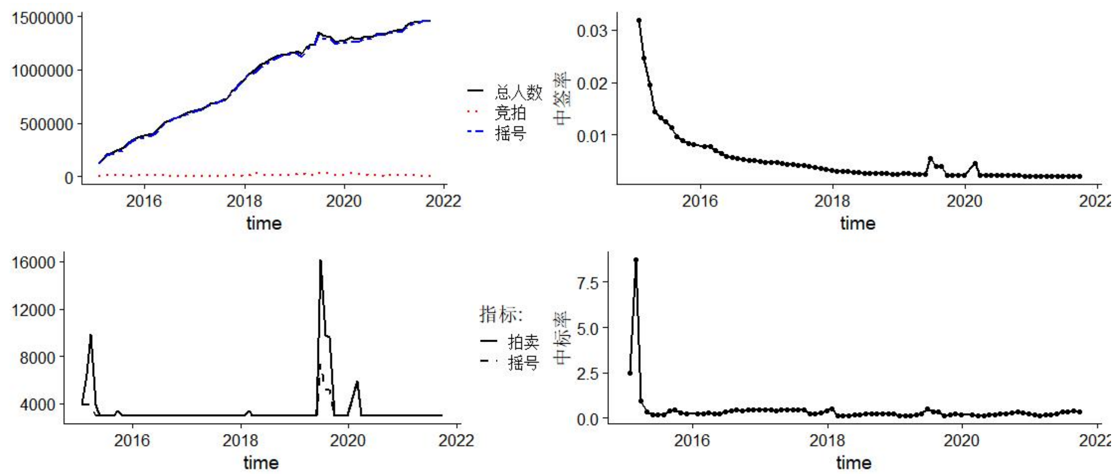

Figure 2.1 Time series of information on the allocation of car plates in Shenzhen

2.2 Model

In each period, $n+m$ plates are allocated, of which n units are allocated through a multi-unit discriminatory auction (DA) with a reserve price of $r \geq 0$ and m units are allocated through a lottery. Assuming that the car plates are homogeneous, there are N participants in the allocation ($N > n+m$), each of whom is risk-neutral and maximizes his or her expected payoff, and has demand for only one unit. The specific value of v_i that a participant i values for the car plate is his or her private information and is not affected by others' valuation; v_i is independently and homogeneously distributed and obeys a distribution function $F(v_i)$, $v_i \in [0, \bar{v}]$, $F(v_i)$ is strictly monotonically increasing and differentiable, with its probability density function $f(v_i)$. Information other than the specific value of v_i is the common knowledge of N participants.

First, N participants independently and simultaneously decide to choose either the lottery or the auction, and once chosen, have no opportunity to participate in the other. If they participate in the auction, bidders are required to submit offers b_i greater than or equal to the reserve price r, the highest n bidders wins the licence with a payoff $v_i - b_i$; the payoff for losers is set to 0. Assume that a bidder's offer strategy, $b_i = \beta(v_i)$, is a strictly monotonous function of his willingness to pay v_i, $\beta(\cdot)$ is monotonically increasing and differentiable over the domain and $b_i \leq v_i$. If there are fewer bidders than plates for the auction, all bidders will be awarded plates, but the remaining will not be reallocated. If they participate in the lottery, the participants will be awarded with a certain probability that is less than 1 if there are more participants than car plates for lottery, while all lottery participants will be awarded if the number of lottery participants is less than the number of plates, with the remaining likewise not be reallocated. (as in Figure 2.2).

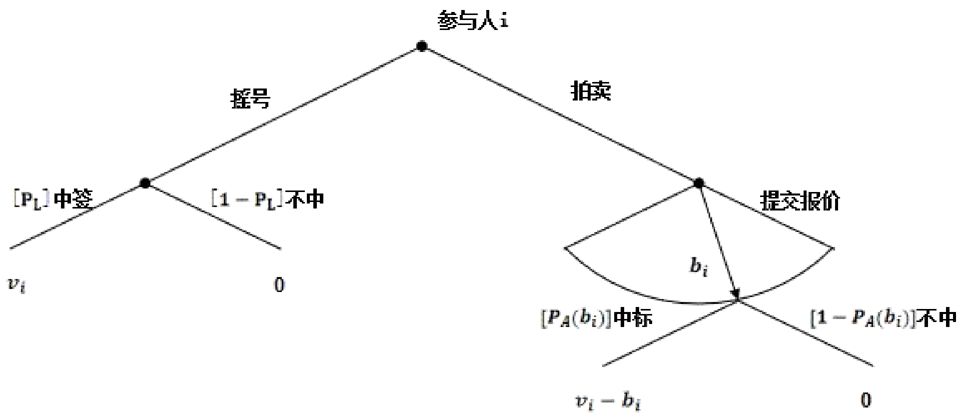

Figure 2.2 illustration on the process of allocating car plates

The article by Huang and Wen (2019) proves that there exists a symmetric Bayesian Nash Equilibrium for this game, i.e., all participants follow a threshold strategy: they participate in the auction when their willingness-to-pay v_i is not less than the threshold $\tilde{v}$, and otherwise they participate in the lottery, $\tilde{v} \in [0, \overline{v}]$. For a participant whose willingness to pay is exactly equal to the threshold $\tilde{v}$, the lottery or auction should be non-differentiable for him/her, i.e., the expected payoffs from the lottery or the auction should be equal; and if he/she participates in the auction, his/her willingness to pay must be the lowest among all bidders, and he/she submits the lowest possible offer (i.e., the reserve price), which is represented by the following equation

$$\tilde{v}P_L = (\tilde{v} - r)P_A(r) \tag{2.1}$$

$$\beta(\tilde{v}) = r \tag{2.2}$$

Given $\tilde{v}$, the expected payoff from the lottery $\Pi_L(v_i)$ can be derived as in Equation (2.4). Assume that for participant i, the probability that there are $k-1$ other participants whose willingness to pay is not less than the threshold $\tilde{v}$ is p_k. If i chooses lottery, then there are $N-k+1$ lottery participants, and the winning probability for i is $\frac{m}{N-k+1}$ when $k \le N-m$ and 1 at $N-m+1 \le k \le N$.

$$p_k = \binom{N-1}{k-1} F\ \tilde{v}^{\ N-k}[1 - F(\tilde{v})]^{k-1},\ k = 1,2,\ \ldots\ldots, N \tag{2.3}$$

$$\Pi_L(v_i) = v_i \times P_L = v_i \times \left[\sum_{k=1}^{N-m} p_k \frac{m}{N-k+1} + \sum_{k=N-m+1}^{N} p_k \right] \tag{2.4}$$

If participant i chooses the auction, the winning probability depends on the bidding strategy as in Equation (2.5). The probability of winning when $k \le n$ is 1. However, when more than n individuals bid, the winning probability is related to his or her bid, which means i can win the target only if b_i is among the top n bids.

$$Q_k(b_i) = \sum_{h=1}^{n} \binom{k-1}{h-1} \left[\frac{F(\beta^{-1}(b_i)) - F(\tilde{v})}{1-F(\tilde{v})} \right]^{k-h} \left[\frac{1 - F(\beta^{-1}(b_i))}{1-F(\tilde{v})} \right]^{h-1} \tag{2.5}$$

And expected payoff for i in auction to be:

$$\Pi_A(b_i,\ v_i) = (v_i - b_i) \times P_A(b_i) = (v_i - b_i) \times \left[\sum_{k=1}^{n} p_k + \sum_{k=n+1}^{N} p_k Q_k(b_i) \right] \tag{2.6}$$

In equilibrium $b_i = \beta(v_i)$ will maximize the expected payoff $\Pi_A(b_i,\ v_i)$.

2.3 Comparative static analysis

Theoretically for a game given the number of participants as well as car plates, the reservation price, and the distribution function of willingness to pay, the threshold value in BNE can be derived according to Equation (2.1). In this paper, by setting $N = 100$, $n = 10$, $m = 10$, $r = 0.1$, willingness to pay $V \sim U(0,1)$, I find an approximate numerical solution as Fig. 2.3 shows.

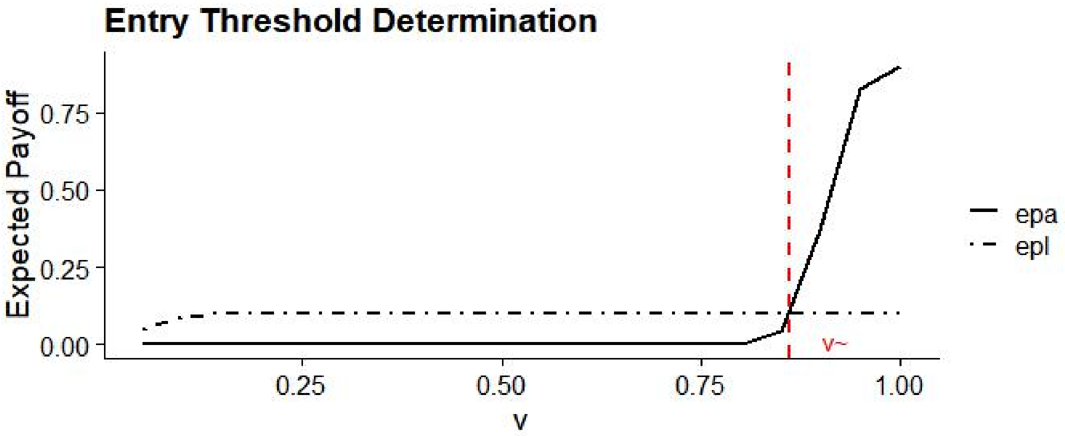

Figure 2.3 Method of determining willingness-to-pay thresholds

From equation (2.1), the parameters affecting the threshold value include the auction quota n , the number of participants N , and the reserve price r .This paper examines how changes in two exogenous variables, n and N , affect the threshold value in BNE, while keeping the total number of allocated car plates constant. This paper conjectures that (1) other things like the number of participants being equal, the higher the percentage of car plates allocated to the auction than to the lottery, the higher the probability of winning the auction than winning the lottery, thus expected return from participation in auction increases while that in lottery decreases, leading to a decline in the threshold value $\tilde{v}$; (2) other things being equal, if the number of participants increases, the threshold value $\tilde{v}$ will also rise. Although increased competition for car plates will decrease the probability of winning both the auction and the lottery, a participant that's at original threshold will now have a lower winning rate in the auction than in the lottery due to exposure to more dominant competitors. In contrast, each participant in the lottery has an equal chance of winning. Therefore, considering the change in expected returns, the original threshold participant will choose lottery instead.

The results of simulation with specific numerical values are shown in Fig. 2.3: (1)

reflecting the case of $N = 100$, $r = 0.1$, and $n + m = 20$, $\tilde{v}$ decreases with the increase of n; (2) reflecting the case of $r = 0.1$, $n = 10$, and $m = 10$, $\tilde{v}$ rises with the increase of N.

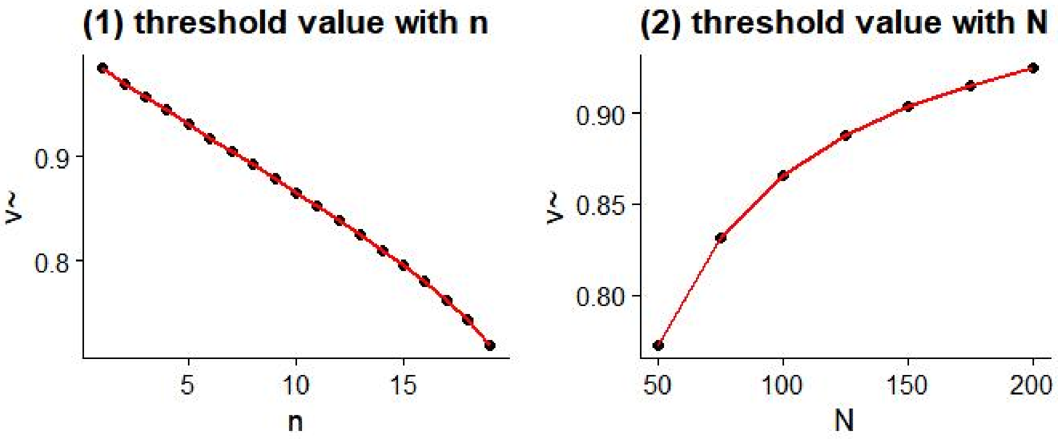

Figure 2.4 Comparitive static analysis results

The results of the simulation support the previous conjectures and align with the law in economics that supply and demand affect the price - when the supply of car plates for auction increases, the transaction price decreases; when the demand for car plates increases, the transaction price rises.

2.4 Equality and efficiency

In order to objectively assess the efficacy of Shenzhen's car plate allocation mechanism in allocating resources, this paper makes estimation of equality and efficiency based on a total of 79 periods of allocation information from January 2015 to September 2021 in Shenzhen. However, this paper is limited in its ability to collect micro-data, such as individual bids in auctions. As a result, it only has access to publicly available aggregated statistical information on the official website. This information is insufficient to accurately estimate the distribution of participants' willingness to pay. Therefore, the paper chooses to refer to Nie's (2022) method, which uses aggregated statistical data to approximate the estimation of equality and efficiency after modifying the model's assumptions.

This paper relax the assumption that v_i is a private value, assuming that the willingness to pay of all participants is common knowledge and that there is a Nash equilibrium with $v_1 > v_2 > \ldots > v_n > v_{n+1} \ldots > v_N$. The n individuals with the highest valuation participate in the auction, the individual rank $n+1$ chooses the auction with a certain probability, and the rest participate in the lottery; the highest bid in the auction is determined by the $n+1$ highest willingness to pay for the car plate $p_{max} = (1 - \frac{m}{N-n})v_{n+1}$.

In this paper, the efficiency is measured by the expected social surplus, and the willingness to pay of the participants for the allocation is calculated, which consists of the expected surplus in the auction and the expected surplus in the lottery. In order to estimate the efficiency, it is first assumed that the willingness to pay of auction participants follows a Pareto distribution, and the parameters are estimated using $P(v > v_{n+1}) = kv_{n+1}^{-\alpha}$ and collected data. The probability of participating in the auction $P(v > v_{n+1})$ is measured by the ratio of the number of bidders to the total number of participants in each period $p = \frac{n}{N}$, while v_{n+1} is estimated using $v_{n+1} = \frac{2p_a - p_{min}}{1 - \frac{m}{N-n}}$, where $2p_a - p_{min}$ is an estimate of the highest bid in the auction. In addition, after the outbreak of corona virus pandemic, urban citizens may show stronger demand for cars for safety concerns, resulting in a significant rise in number of participants, thus negatively affecting p ; therefore, when establishing the regression equation, the variable $corona$ (0: pre-pandemic; 1: pandemic), is included in the equation (2.7), and the estimation shows $\alpha = 1.2281$ (Fig. 2.5). The expected surplus in the auction is $E_a = n \times E\left(v \mid v > v_{n+1}\right) = n\frac{\alpha}{\alpha - 1}v_{n+1}$. Assuming that the willingness to pay of the lottery participants obeys a uniform distribution from 0 to the auction reserve price r , the efficiency of the lottery is estimated $E_l = m\frac{r}{2}$. Eventually the total efficiency of car plate allocation can be obtained $E = E_a + E_l$.

$$log\left(p\right) = log\left(k\right) - \alpha log\left(v_{n+1}\right) + corona + \varepsilon \quad (2.7)$$

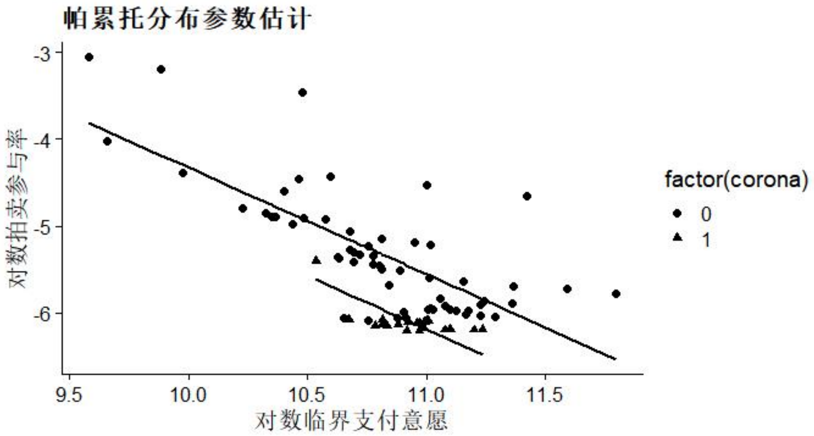

Figure 2.5 Estimated parameters of the Pareto distribution

In addition, the government, as the supplier of car plates, receives a certain amount of revenue R (equivalent to the producer's surplus), which is equal to the product of the average price of the auction and the quota of the auctioned car plates $R = np_a$; while the participants who obtain car plates enjoy a certain amount of consumer surplus as consumers, of which the consumer surplus in auction is their willingness to pay minus the bids they paid, i.e. $CS_a = E_a - R$, while the consumer surplus of the lottery participants, who do not have to pay, is equal to the efficiency of the lottery $CS_l = E_l$. The total consumer surplus is the sum of the consumer surplus in both auction and lottery $CS = CS_a + CS_l$.

This paper adopts the Gini coefficient, a widely accepted indicator, to measure the equality of the mechanism. The Gini coefficient was originally used as a measure of equality in the distribution of income or wealth, but in the case of the car plate allocation mechanism it can be applied as a measure of the difference in the winning probability, reflecting the equality of the mechanism; the lower the Gini coefficient, the fairer the mechanism. Since the participant with $n+1$ highest willingness to pay randomly chooses between auction and lottery with a certain probability, there are two measures of the availability of car plates: if this participant participates in the lottery, all the n participants in the auction get a plate, and all the other participants in the lottery win at a probability of $\frac{m}{N-n}$, the Gini coefficient is $Gini_1 = \frac{n}{m+n} - \frac{n}{N}$; if this participant chooses auction and the participant with the highest willingness to pay fails to win, the Gini coefficient is $Gini_2 = \frac{n}{m+n} - \frac{n}{N} - \frac{1}{N}\left(1 + \frac{n}{n+m}\right)$. The Gini coefficient of the distribution of car plate availability in BNE should be somewhere in between, i.e. $Gini_2 \le Gini \le Gini_1$.

Table 2.1 Statistics on Allocation of Car Plates in Shenzhen, January 2015-September 2021

	observation	mean	median	minimum	maximum
efficiency	79	9.711	8.914	3.410	35.17
consumer surplus	79	8.131	7.554	2.878	30.00
revenue	79	1.58	1.507	0.532	5.17
Gini1	79	0.5075	0.4984	0.4685	0.6763
Gini2	79	0.5075	0.4984	0.4685	0.6763
number of participants	79	929132	1068677	124152	1467288
lottery winning rate	79	0.004971	0.003077	0.002003	0.032535
quota for lottery	79	3119	2933	2933	7333
quota for auction	79	3488	2945	2934	16188

Table 2.1 reports the descriptive statistics of Shenzhen's car plate allocation from January 2015 to September 2021 for 79 periods. It can be seen that in the history data, an average of 3,119 plates were allocated by lottery and 3,448 were allocated by auction per period, which is close to the ratio of 1:1; an average of 929,132 people participated in the allocation per period, with the highest number of participants at 14,672,888, and the average winning rate in lottery was 0.004971. The average efficiency in each period was RMB971.1 million, the revenue from auctions was 158 million, consumer surplus

is RMB813.1 million, and the ratio of consumer surplus to auction revenue is about 5:1; the two estimates of the Gini coefficient for the opportunity distribution of car plate availability are close to each other, with an average value of 0.5075.

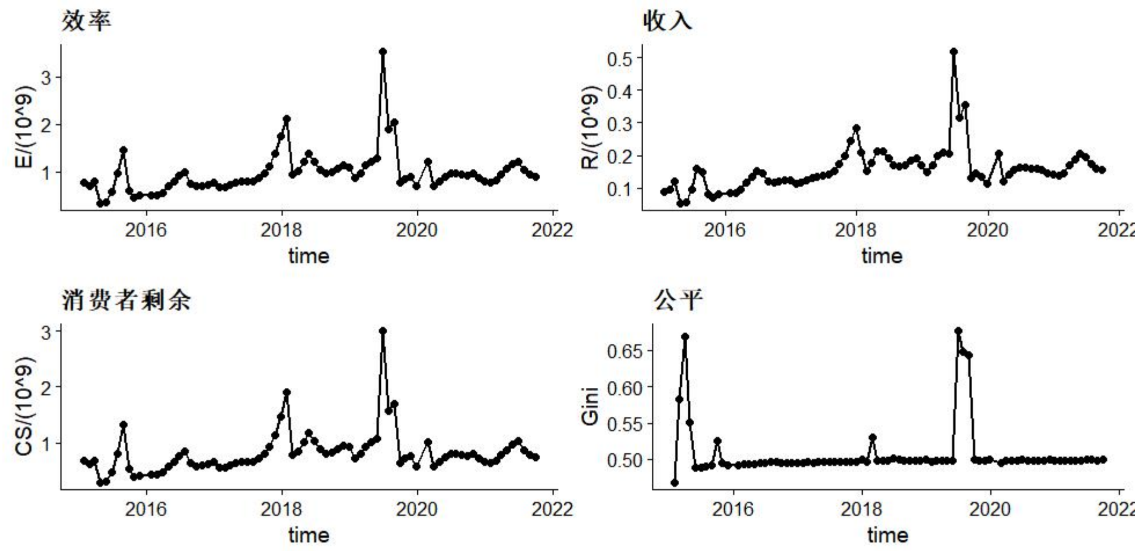

Figure 2.6 Equality and Efficiency Time Series Plot of Car Plate Allocation

Figure 2.6 reflects that efficiency, revenue, and consumer surplus have fluctuated above and below the mean in most periods in history, and the Gini coefficient has been around 0.5, reflecting the fact that car plates were allocated between auction and lottery close to 1:1. However, in June to August 2019, additional car plates were allocated, with a corresponding surge in efficiency, revenue and consumer surplus, and since the additional plates were not equally distributed , it resulted in a 2:1 allocation ratio between auction and lottery and a sharp deterioration in equality. Therefore, if total car plates increase in the future with the implementation of the policy of encouraging consumption, although allocating more quotas to auctions can significantly improve efficiency and government revenue, Shenzhen Government should carefully consider about the negative impact of the sharp deterioration in equality brought about by such a move, and weigh efficiency and equality to reasonably allocate plates between auction and lottery.

It should be admitted that efficiency and consumer surplus in this paper may be overestimated. The reason is that under the assumption of perfect information, all participants whose willingness to pay is ranked below $n+1$ will go to the lottery, but in reality, the number of bidders k tends to be larger than the number of car plates in auction n. So for losers in auction (i.e., participants whose willingness to pay is ranked between $n+1$ and k), their willingness to pay have never been realized, resulting in efficiency loss. The figure below illustrates the difference in efficiency between the model and actual situation, with the grey bars representing participants' willingness to pay and the blue shaded area representing efficiency.

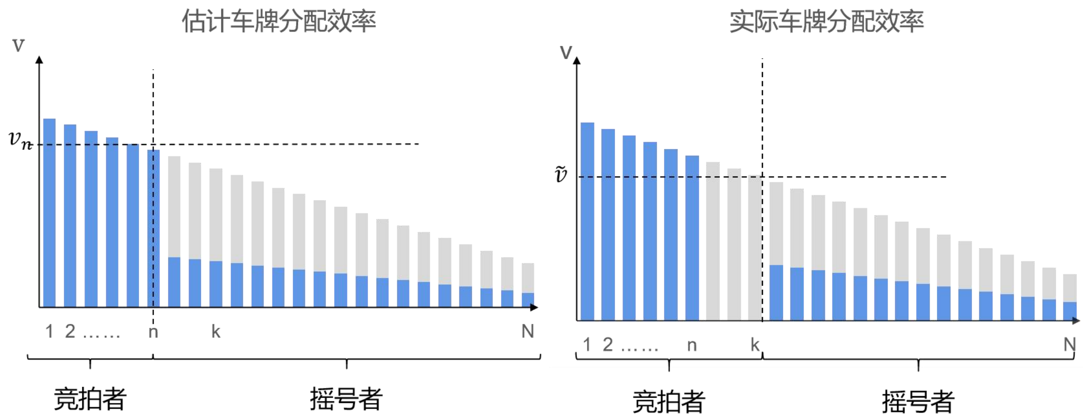

Figure 2.7 Difference between estimated and actual efficiency

Chapter 3 Empirical Analysis

The previous chapter's theoretical modeling simplified actual allocation rules to a mixed system of single-period discriminatory auctions plus lotteries. However, it did not consider the time series formed by historical allocation information and the phasing rules in the auction process. Additionally, it assumed that all participating subjects are rational persons. The conclusion was that participants' choice in equilibrium and the bidders' bid are mainly affected by three exogenous variables: the number of car plates, the number of participants, and the reservation price. However, this chapter explores additional factors that affect the minimum transaction price and the interaction between the minimum transaction price and the number of bidders through empirical analyses. It also summarizes the determination mechanism of the minimum transaction price, which is a linear function based on the minimum transaction price of the previous periods or the average price of the second broadcast in the current period, adjusted by quota for auction and the number of bidders in the current period. This chapter also demonstrates the mutual Granger causality between the minimum price and the number of bidders, with changes in one side having a lagged effect on the other. Finally, this chapter explores other methods for predicting the minimum selling price, and finds that the Random Forest model outperforms the Multiple Linear Regression model in terms of prediction accuracy, and can provide a more accurate bidding reference for bidders.

3.1 Minimum transaction price determination: multiple linear regression approach

From the theoretical analysis above, it can be seen that in the Bayesian Nash equilibrium, the lowest transaction price of the auction depends on the bidding strategy of the person with n highest valuation , i.e., $P_{\min} = \beta(v_n)$; whereas the comparative static analysis shows that the threshold of willingness-to-pay decreases with the increase of quota for auction and increases with the increase of the number of participants, so it is conjectured that v_n , which is greater than or equal to the threshold and obeys the same distribution, and the corresponding $P_{\min}$, will also decrease due to the increase of the quota for auction and rise due to the increase of the number of bidders.

However, some of the assumptions in the model do not hold in the real-life process of allocation; bidders' bidding strategies are influenced by more factors other than the supply and demand for plates for auction .

First, the information on the allocation in each period of history constitutes a time series. Therefore, time-series related properties such as trend and seasonality should be taken into account. Figure 3.1 shows an overall increase in the minimum transaction price over time, which may be due to the fact that the number of bidders is larger than the auction quota in almost every period, and the accumulation of losers from previous periods who have a higher willingness to pay drives up the price. Additionally, the allocations are held once a month, which also has a seasonal effect. Thus, this chapter introduces the trend term time t and the seasonal term month $month$ in the empirical analysis.

Secondly, the participants in reality may not always meet the rational hypothesis, the imperfect information and the disparate ability in collecting effective information make the participants insufficient to form rational expectations consistent with the equilibrium in the theoretical model. Bidders may refer to the lowest transaction price in the previous period $P_{\min}(t-1)$, the average price of the first and the second broadcasts of the current period (P_{fir} , $P_{\sec}$), the number of bidders in the previous period $Nauc(t-1)$ to form their expectations. Fig. 3.2 shows that the lowest transaction price is generally higher than the average price of the first and second broadcasts, which may indicate that the average price of both broadcasts has a positive impact on the lowest transaction price.

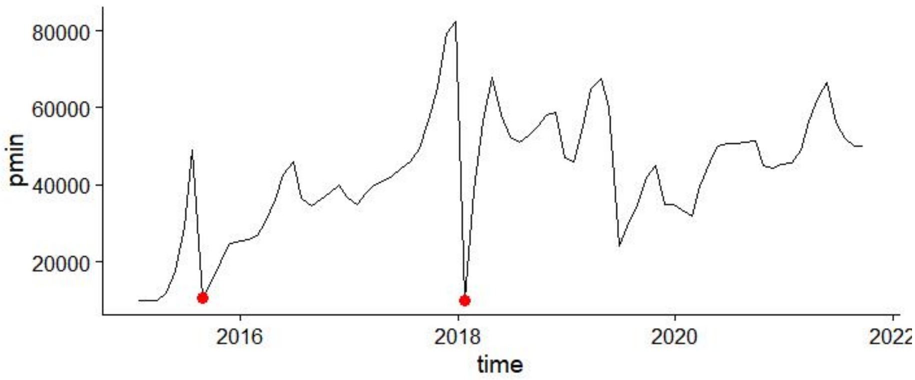

Figure 3.1 Time series of lowest transaction price

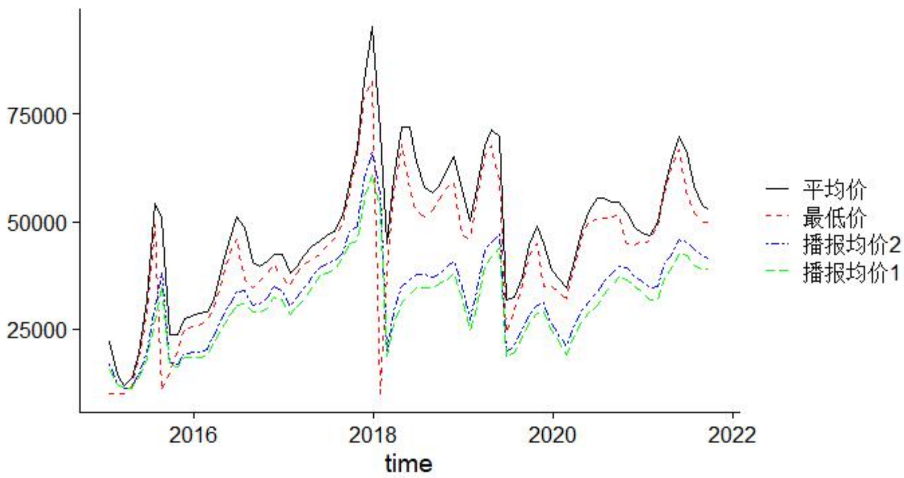

Figure 3.2 Time series of other prices

3.1.1 Modelling and analysis of results

Four initial models are developed in this paper.

$$P_{min}(t) = \beta_1 + \beta_2 t + \beta_3 n + \beta_4 Nauc + \beta_5 month + \epsilon_t \tag{3.1}$$

$$P_{min}(t) = \beta_1 + \beta_2 t + \beta_3 P_{min}(t-1) + \beta_4 n + \beta_5 Nauc + \beta_6 month + \epsilon_t \tag{3.2}$$

$$P_{min}(t) = \beta_1 + \beta_2 P_{sec} + \beta_3 n + \beta_4 Nauc + \beta_5 month + \epsilon_t \tag{3.3}$$

$$P_{min}(t) = \beta_1 + \beta_2 P_{fir} + \beta_3 P_{sec} + \beta_4 n + \beta_5 Nauc + \beta_6 month + \epsilon_t \tag{3.4}$$

The static model without lagged variables was firstly built, i.e., equation (3.1). Even though the result (as in model(1)) shows that the coefficients of quota for auction and the number of bidders are significant, the D.W. test indicates that there is positive first-order auto correlation in the residuals of the series that t-tests of the coefficients fail and that the adjusted R-squared is 40.05%, which is not a very good fit, suggesting that important explanatory variables may be omitted or a dynamic model should be built considering history information. Therefore, two approaches were taken to correct the static model: adding a lagged dependent variable to the explanatory variables or adding the average price of broadcasts.

The inclusion of lagged dependent variables in the regression is based on the assumption that participants in the current period remember the history allocation information, and refer to the number of bidders and quota for auction in all previous periods to guess the number of participants and other information in the current period, so as to form their bidding strategies; assuming that impact of lagged variables is geometrically decreasing, the model including the lagged dependent variable is established, as in equation (3.5).

$$\begin{aligned} Y_t &= \alpha + \lambda^0 \beta_0 X_t + \lambda^1 \beta_1 X_{t-1} + \lambda^2 \beta_2 X_{t-2} + \lambda^3 \beta_3 X_{t-3} + \ldots + \varepsilon_t \\ &= \alpha + \lambda Y_{t-1} + \beta_0 X_t + \varepsilon_t \end{aligned} \tag{3.5}$$

The results of model(2) are obtained from equation(3.2), which show that after introducing one-period lagged dependent variable, the quota for auction is still negatively correlated with the minimum transaction price in current period, and the number of bidders is positively correlated with it. In addition, the quota for auction and the number of bidders in the current period will have an impact on the future minimum transaction price at $\frac{\beta_4}{1-\beta_3}, \frac{\beta_5}{1-\beta_3}$ respectively.

The results of model (3) are obtained from equation (3.3), except for the coefficients of time t and month $month$, the coefficients of other variables reject the original hypothesis at least at 99% confidence interval, all independent variables can explain 61.51% of the change of the minimum transaction price, and there is no residual autocorrelation. Models (3) and (4) involving average price of broadcasts have stronger explanatory effects compared to model (2) with the inclusion of the previous lowest transaction price, with an adjusted R-squared of more than 70% and no residual autocorrelation; however, model (4) is multicollinear, and the coefficient of average price of first broadcast is negative, which is also counter-intuitive. The multicollinear problem is solved by applying ridge regression method, the significance of the coefficients is increased by reducing the overfitting and the coefficients of average price of both broadcasts are adjusted to be positive, and the results are shown in model (5). The ridge regression method means adding a regularization term to the least squares, i.e., the second term of equation (3.6), where $\widehat{P_{min}}^{(i)}$ is the fitted value in model (5), $\boldsymbol{\beta}$ refers to the vector of coefficients in the regression, and λ is automatically selected as 0.0250332. The significantly positive coefficient on average price of broadcast could be due either to the fact both the average price of broadcast and lowest transaction price belong to an almost identical distribution, or to the fact that participants raised their bids with reference to average price of broadcast in the last stage of the auction (i.e.snipping bidding).

$$\min_{\beta} \frac{1}{2} \sum_{i=1}^{79} \left(\widehat{P_{min}}^{(i)} - P_{min}^{(i)} \right)^2 + \lambda \|\boldsymbol{\beta}\|^2 \tag{3.6}$$

However, models (1)-(5) are not easy to put into practice from a prediction point of view: the data on the number of bidders in each period are actually obtained after the auction is over, and the number of bidders is not available through public channels before or during the current auction. Therefore, this paper establishes models (6) and (7) to replace the number of bidders in the current period with the number of bidders in the previous period. In model (6) (7), the coefficient of the number of bidders in the last period is significantly positive, which indicates that when the number of bidders in the last period has a tendency to rise, more participants choose auction or bidders form an expectation of a rise in price and raise their bids correspondingly; but the goodness of fit of the model is rather low, indicating that it is not conducive for bidders to form rational expectations under imperfect information, and efficacy of the allocation mechanism will be undermined as well.

Table 3.1 Multiple linear regression models

	(1) Pmin(t)	(2) Pmin(t)	(3) Pmin(t)	(4) Pmin(t)	(5) Pmin(t)	(6) Pmin(t)	(7) Pmin(t)	(8) Nauc(t)
t	299.51 (66.48***)	65.62 (64.42)				107.30 (63.60*)	105.39 (54.53*)	504.87 (113.5***)
t^2								-4.83 (1.23***)
Pmin(t-1)		0.55 (0.086***)				0.51 (0.086***)		-0.19 (0.04***)
n	-3.13 (0.75***)	-2.73 (0.60***)	-1.74 (0.54***)	-1.50 (0.545***)	-1.7***	-2.08 (0.575***)	-0.96 (0.55*)	
Nauc	0.456 (0.235*)	0.67 (0.19***)	1.003 (0.143***)	0.85 (0.16***)	0.98***			
Nauc (t-1)						0.435 (0.17**)	0.61 (0.15***)	0.56 (0.075***)
Month	816.02 (444.95*)	662.5 (357.3*)	506.6 (300.2*)	622.24 (302.55**)	480.8*			-442.86 (146.9***)
Pfir				-4.1412 (2.2876*)	0.4234***			
Psec			1.107 (0.099***)	4.9098 (2.1029**)	0.7001***		0.9265 (0.117***)	
β_1	30648.61 (5023***)	12990 (4866***)	-4895 (4777)	-2975.20 (4823.04)	-4070	18350 (4315***)	2885.81 (4991.71)	6928.74 (1803***)
sample size	78	78	79	79	79	78	78	78
D.W test	0.84328	1.8173	1.7611	1.8162	1.8162	2.0423	2.1045	1.6551
Adjusted R^2	0.4005	0.6151	0.7279	0.7361		0.5908	0.6739	0.6761

3.1.2 Outlier detection and result adjustment

Figure 3.1 shows that there have been two occasions when the lowest transaction price close to or equal to the reserve price, clearly deviating from the overall trend, and the residual results of the regression (Figure 3.3) also indicate that these two ultra-low prices are outliers in the sample. The appearance of these two ultra-low prices is not representative, because with more and more people participating in the allocation in the future, the probability of winning a plate at reserve price will only be smaller under the general trend of a widening gap between supply and demand. Therefore, this paper directly deletes these two outliers and re-estimates Eq. 3.3 and Eq. 3.4 based on the samples with the outliers removed. Compared with the results of model (3)(4), the significance of the coefficients is basically unchanged, but the adjusted R-square of the model improved. The results of the regression after removing the outliers are as follows:

$$\widehat{P_{min}(t)} = 13100 - 43.56t + 0.802P_{min}(t-1) - 2.431n + 0.493Nauc + 93.07month$$
$$(2983^{***})\ (40.89)\ (0.0573^{***})\quad (0.3696^{***})\ (0.1189^{***})\ (229.1)$$
$$Adjusted\ R^2 = 0.8399 \tag{3.8}$$

$$\widehat{P_{min}(t)} = -6473 + 1.342P_{sec} - 1.235n + 0.7861Nauc - 101month$$
$$(2106^{***})\ (0.0460^{***})\ (0.2377^{***})\ (0.0641^{***})\ (138.8)$$
$$Adjusted\ R^2 = 0.9423 \tag{3.9}$$

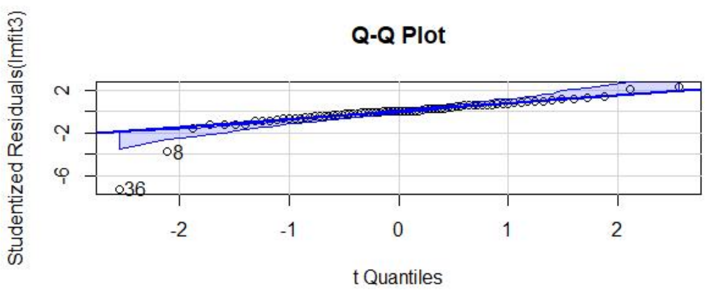

Figure 3.3 Q-Q plot for outlier detection

Based on equations (3.8) and (3.9), this paper summarizes the mechanism for determining the minimum transaction price: the minimum transaction price in current period is basically a linear function of the previous minimum price or the average price of the second broadcast in current period, and is adjusted by the supply and demand in current period (the auction quota and the number of bidders), with an increase in supply adjusting the price downwards and an increase in demand adjusting the price upwards.

Finally, predictions of the minimum transaction price for the four periods from October 2021 to January 2022 are made using equation (3.9), with the results shown in Table 3.2.

Table 3.2 Multiple linear regression predictions

time	actual value	prediction	absolute error	relative error
2021.10	47100	50024.38	2924.38	0.06209
2021.11	45100	47144.99	2044.99	0.04534
2021.12	45100	46388.51	1288.51	0.02857
2022.01	45300	45947.83	647.83	0.01430

3.2 Minimum transaction price and the number of bidders

In section 3.1 of this chapter, when using equation (3.2) to explore the factors affecting the minimum transaction price, some assumptions are implicit: the number of bidders and the quota for auction are exogenous variables independent of each other; and there is no feedback between the minimum transaction price and the number of bidders. Quota for auction can be identified as a variable that is only affected by exogenous policie, but result in model (8) of equation (3.10) shows that the number of bidders may not be fully exogenous. Specifically, the number of bidders has a tendency to rise and then fall over time; the higher the minimum transaction price in the previous period, the lower the number of bidders in the current period; the number of bidders in the current period is significantly positively correlated with the number of bidders in the previous period. This also reflects the law in economics that supply and demand determine price, and price signals in turn regulate supply and demand, reflecting the dynamic relationship between price and supply and demand. Thus, this paper employs a VAR(2) model, as shown in equation (3.11), using first-order difference data. The original time series of both the minimum transaction price and the number of bidders were found to be unstable after seasonal adjustment, but became stable after the first-order difference (Table 3.3).

$$Nauc(t) = \beta_1 + \beta_2 t + \beta_3 t^2 + \beta_4 P_{min}(t-1) + \beta_5 Nauc(t-1) + \beta_6 month + \varepsilon_t \tag{3.10}$$

$$\begin{pmatrix} \Delta P_{\min t} \\ \Delta Nauc_t \end{pmatrix} = \begin{pmatrix} c_{10} \\ c_{20} \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} \Delta P_{\min t-1} \\ \Delta Nauc_{t-1} \end{pmatrix} + \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix} \begin{pmatrix} \Delta P_{\min t-2} \\ \Delta Nauc_{t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{P_{\min},t} \\ \varepsilon_{Nauc,t} \end{pmatrix}$$
$$\Delta P_{\min t} = P_{\min t} - P_{\min t-1} \tag{3.11}$$

Table 3.3 ADF test results for time series

Variable (sa: seasonally adjusted)	ADF test p-value	result
Pminsa	0.3483	not stationary
Naucsa	0.8522	not stationary
dPminsa	<0.01	stationary
dNaucsa	<0.01	stationary

This paper puts forward two hypotheses: (1) the minimum transaction price and the number of bidders share a mutual Granger causality, i.e., changes in the minimum transaction price help to predict the changes in the number of bidders and vice versa; (2) the change in the minimum transaction price affects the number of bidders negatively, while the change in the number of bidders affects the minimum transaction price positively, and both of them have a time lag.

The fitting result of VAR (2) (Table 3.4) not only proves hypothesis (2), but also the coefficients of $\Delta P_{\min t-1}$ and $\Delta P_{\min t-2}$ on $\Delta P_{\min t}$ are significantly negative, reflecting that when the fluctuation of the minimum transaction price becomes larger in the history, it has a negative impact on the fluctuation in the future, which reflects its nature of stabilization. In addition, the results of the Granger causality test proved hypothesis (1), and there is a greater probability that the change in the minimum transaction price is the Granger cause of the change in the number of bidders (Table 3.5).

Thus, if the minimum price and the number of bidders are considered as two endogenous variables in a system, the VAR model describes their interaction: they share mutual Granger causality and a rise in the increase of minimum transaction price will dampen the increase in the number of bidders in future periods, whereas a rise in the increase of the number of bidders will expand increase in minimum transaction price.

Table 3.4 VAR(2) fitting results

	Dependent variable.	
	(1) $\Delta P_{\min t}$	(2) $\Delta Nauc_t$
$\Delta P_{\min t-1}$	-0.377 (0.111***)	-0.181 (0.037***)
$\Delta Nauc_{t-1}$	0.545 (0.303*)	-0.148 (0.102)
$\Delta P_{\min t-2}$	-0.235 (0.117**)	-0.154 (0.039***)
$\Delta Nauc_{t-2}$	0.675 (0.290**)	0.010 (0.098)
const	657.062 (1197.624)	237.494 (403.187)
Observations	78	78
R^2	0.215	0.318
Adjusted R^2	0.172	0.281
Residual Std. Error (df = 73)	10532.770	3545.920
F Statistic (df = 4; 73)	5.006***	8.511***

Table 3.5 Results of Granger causality test

Null hypothesis H0	statistic	p-value
ΔNauc is not the Granger cause of ΔPmin	F=4.2349	0.0163**
ΔPmin is not the Granger cause of ΔNau	F=15.67	< 0.001***
ΔPmin and ΔNauc have no immediate Granger causility	$\chi^2 = 0.21967$	0.6393

3.3 Other forecasting methods

The multiple linear regression model in Section 3.1 provides a method to predict the minimum transaction price, but in order to find algorithms with higher prediction accuracy than the multiple linear regression, so as to better provide bidding references, this paper tries to apply other methods to make predictions, including time-series prediction methods and machine learning prediction methods.

3.3.1 Seasonal ARIMA Model and Extensions

Since January 2015, Shenzhen has been allocating car plates almost every month through both auctions and lotteries, and the minimum transaction price in auctions is also a time series with periodicity. Using the seasonal ARIMA model to fit the history information of the minimum transaction price may be a better predictor of the future trend in the short term. The ARIMA model, i.e. auto regressive integrated moving average, is denoted as ARIMA(p,d,q); and the seasonal ARIMA model is denoted as ARIMA(p,d,q)(P,D,Q)[m], where p, d, and q are the non-seasonal terms in the the number of autoregression terms,differences, and moving average terms, P, D, and Q are the parameters in the seasonal term, and m is the seasonality.

In order to determine the optimal parameters of the seasonal ARIMA, attempts were made to perform one difference and one seasonal difference on the original series, and after observing the time series plot, autocorrelation and partial autocorrelation plots (Fig. 3.4), the ARIMA (0,1,1)(1,1,1) [12] and models (10)-(13) were initially established. Test results of models above indicate that the residuals are white noise, except for model (9) whose Q-statistic corresponds to a p-value of less than 0.05, which fails the Ljung-Box test; then the model with significant coefficients and smallest AIC is selected as the best model, and the winning model is ARIMA (2,1,1) (0,1,1) [12].

Then the ARIMA (2,1,1) (0,1,1) [12] is applied to predict minimum transaction price for a total of four months from October 2021 to January 2022, and the relative error between the predicted value and the true value in Table (3.7) is large, which indicates that the seasonal ARIMA model predicts poorly, and that the use of history information alone does not accurately predict the current and future information, because the changes generated by external variables over time are not taken into account.

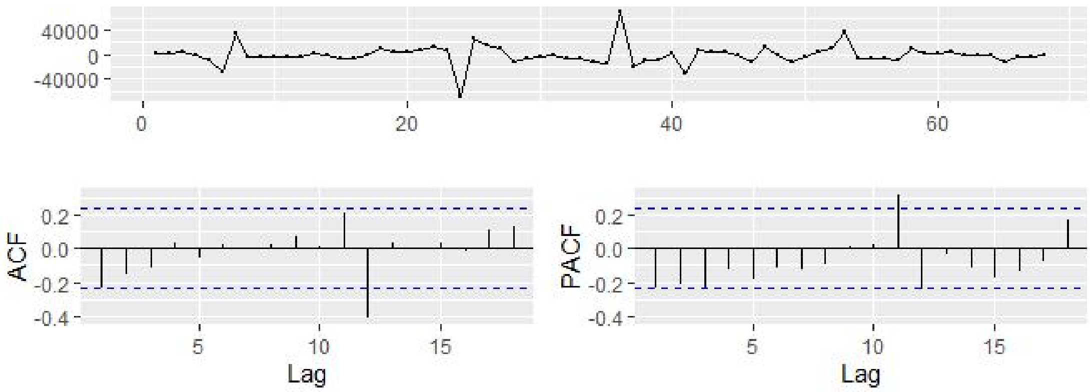

Figure 3.4 Time series, ACF, PACF after two differences

Table 3.6 ARIMA models

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Ar1		0.4124 (0.1485***)	0.1431 (0.2698)	0.4015 (0.1529**)	0.4016 (0.1530**)			
Ar2				-0.2233 (0.1285*)	-0.2152 (0.1328)			
Ma1	-0.6077 (0.1685***)	-0.8393 (0.0842***)	-0.5112 (0.2512**)	-0.7546 (0.1147***)	-0.7597 (0.1161***)	-0.8608 (0.0708***)	-0.8510 (0.0738***)	-0.8619 (0.0676***)
Ma2			-0.2472 (0.1693)					
Sar1	0.1047 (0.1480)				0.0378 (0.1488)			
Sma1	-0.9997 (0.3571***)	-1.0000 (0.6799)	-0.9997 (0.4138**)	-1.000 (0.334***)	-1.0000 (0.294***)	-1.0000 (0.3297***)	-1.0000 (0.3518***)	-0.9999 (0.2474***)
Pfir							0.8577 (0.1667***)	-8.5636 (2.3220***)
Psec						0.8254 (0.1494***)		8.6841 (2.1261***)
AIC	1492.15	1485.48	1485.82	1484.69	1486.62	1471.18	1473.61	1460.63
Q	22.647	13.385	9.6172	6.7301	6.3242	13.067	14.563	7.174

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
P-value	0.04612	0.4185	0.6495	0.8749	0.8509	0.4426	0.3354	0.8459

Table 3.7 ARIMA (2,1,1) (0,1,1) [12] predictions

time	actual value	prediction	absolute error	relative error
2021.10	47100	55023.60	7923.60	0.16823
2021.11	45100	58877.61	13777.61	0.30549
2021.12	45100	57673.94	12573.94	0.27880
2022.01	45300	45013.64	286.36	0.006321

Therefore, in this paper, we consider extending the ARIMA model by adding external regression variables and including average price of broadcasts into the model to form a dynamic regression model. The results of models (14)-(16) in Table 3.6 show that compared with the pure seasonal ARIMA, the AIC of the model is improved and the residuals passed the Ljung-Box test. The optimal model is model (16) and the fitting results are shown in equation (3.12), as determined by both AIC and Q-statistics.

$$
\begin{aligned}
&P_{mint} = P_{min0} - 8.5636P_{firt} + 8.6841P_{sect} + \eta_t \\
&(1 - L)(1 - L^{12})\eta_t = (1 - 0.8619L)(1 - 0.9999L^{12})\varepsilon_t \\
&\varepsilon_t \sim NID(0,\ 79947493)
\end{aligned}
\tag{3.12}
$$

The model (16) is used to forecast the four periods from September 2021 to January 2022. The relative error between the forecast and the true value is improved compared to the seasonal ARIMA model alone, but it is still above 5%, which cannot be considered a valid prediction. This reflects that even though the newly added broadcast price plays an important role in the prediction and improves the prediction performance, it is still not able to reflect all the external information.

Table 3.8 Predictions from dynamic regression model (16)

time	actual value	prediction	absolute error	relative error
2021.10	47100	49578.98	2478.98	0.05263
2021.11	45100	51639.76	6539.76	0.14500
2021.12	45100	48486.55	3386.55	0.07509
2022.01	45300	41687.62	3612.38	0.07974

Both the seasonal ARIMA model and the extended dynamic regression model have shown to be inaccurate in their predictions, and the ARIMA model may not be suitable for predicting the minimum transaction price. Therefore, this paper will explore alternative prediction methods.

3.3.2 Random Forest Models

From the prediction point of view, a single algorithm or a simple combination of several algorithms such as multiple linear regression, ARIMA or dynamic regression models may cause large prediction errors due to their own shortcomings, whereas integrated algorithms tend to combine multiple learners and take the mean of multiple results, which not only improves the accuracy of the model but also resists overfitting much better; Random Forests is a kind of integrated algorithms that builds multiple CART decision trees and integrates their results using an improved bagging method. The advantage of random forest includes not only the use of integrated algorithm to improve the prediction accuracy, but also insensitivity to multivariate covariance as well as the ability to incorporate as much relevant information as possible into the model; therefore, this paper uses random forest to predict the minimum transaction price in the hope of improving the prediction performance.

The base learner for Random Forest is the CART decision tree. CART, which stands for Classification and Regression Decision Tree Model, is an algorithm that classifies data using a series of rules. If the output variable is continuous, the regression tree is applied. Specifically, the regression tree determines the optimal features and the optimal sample division point by minimizing the sum of squared residuals, and continues to iteratively divide the two parts of the sample after the division until the stopping condition is met. Figure 4.2 is a simple illustration of a CART regression tree (illustration from Crawley 2007 The R Book p691).

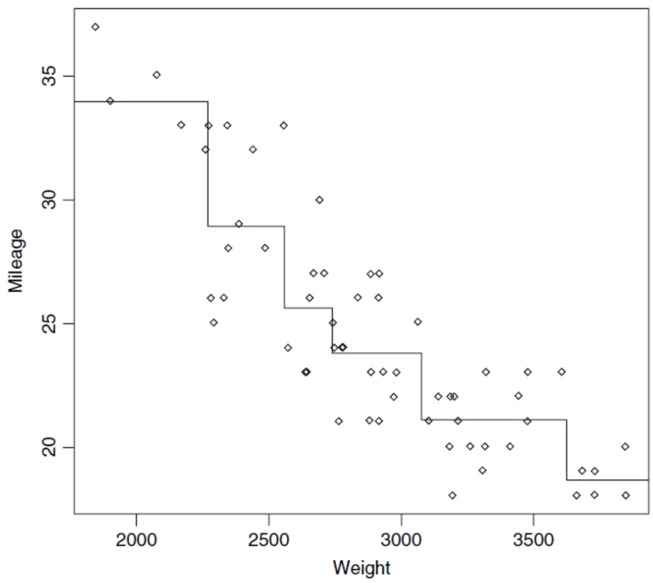

Figure 3.5 Example of CART

The random forest algorithm employs regression tree CART integration to create a 'forest' process. This involves randomly sampling the sample set $D = \{(x_1, y_1), (x_2, y_2), \ldots (x_m, y_m)\}$ T times, each time collecting m samples to obtain T sample sets $D_t, t = 1, 2 \ldots T$. T weak learners $G_t(x)$ are trained on each set, with a part of the variables randomly selected during each round of decision tree training (hence the 'random' in 'random forest'). The final output is the arithmetic mean of the T base learners, which reflects the accuracy of the algorithm. Additionally, for a given sample in the sample set D_t, the probability of being included in one random sample is 1/m, and the probability of not being included in m samples will converge to a certain value $\lim_{m \to \infty} \left(1 - \frac{1}{m}\right)^m = e^{-1} \approx 0.368$. In other words, if frequency is used to estimate probability, approximately 36.8% of the data in the sample set is not used for training, which is referred to as ‘out-of-bag’. These data will be used in T rounds of iterations to estimate the generalization error, thus reducing overfitting of the model.

In this paper, we use the R language function randomForest to make predictions of the lowest transaction price from October 2021 to January 2022, and the independent variables include time, quota for auction, the number of bidders, the average price of the first and the second broadcasts, month, and corona; Table 3.9 reports that the relative errors between the predicted values of randomForest and the true values are relatively small, basically less than 5%.

Table 3.9 Predictions of the Random Forest

time	actual value	prediction	absolute error	relative error
2021.10	47100	47097.63	2.37	0.0000502
2021.11	45100	45245.44	145.44	0.0032249
2021.12	45100	49332.56	4232.56	0.093848
2022.01	45300	47478.89	2178.89	0.048099

Although the Random Forest algorithm for prediction is often considered a black box, this paper analyzes the prediction process and results by scoring the importance of each variable in the training and test sets.

The variables in the training set were ranked based on their importance using the measure of increase in node purity, which is determined by the sum of squared residuals. This measure represents the effect of each variable on the heterogeneity of the nodes that divide the decision tree. The results indicate that the average price of the two broadcasts plays the most significant role in building the random forest model. Although the average price of the two broadcasts is highly correlated, the importance ranking still concludes that the average price of the second broadcast is the most important factor (see Figure 3.6). Rational bidders should choose to submit their final valid bid at the end of the auction because the average price of second broadcast is crucial in predicting the lowest transaction price.

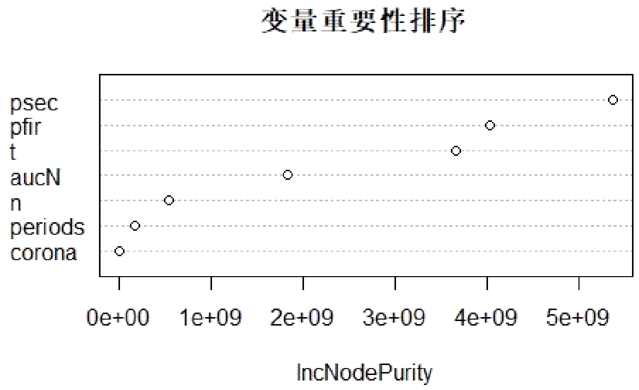

Figure 3.6 Importance of Variables in Random Forest Training Set

This paper explores the contributions of variables that affect the predicted value of the lowest transaction price in October 2021. The Random Forest algorithm gives its prediction of 7,097.634 yuan, which is slightly higher than the actual price of 43,141.827 yuan. The number of bidders *Nauc* had the greatest negative contribution on the lowest transaction price. The second most influential variable was the average price of second broadcast P_{sec}, which had a positive contribution. (Figure 3.7)

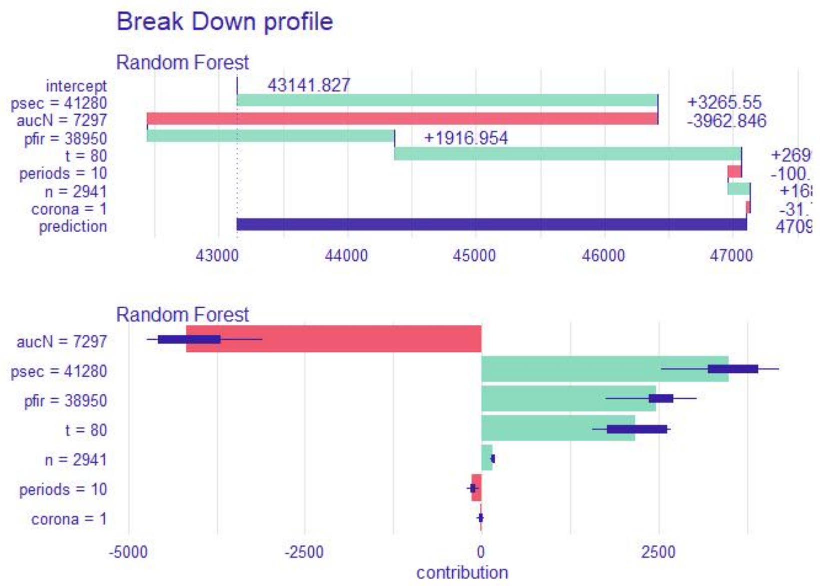

Figure 3.7 Importance of Random Forest Training Set Variables

3.3.3 Comparison of predicted effects

Table 3.10 demonstrates the prediction accuracy of the multiple linear regression, dynamic regression model (i.e., extended ARIMA), and random forest model, measured by the mean absolute percentage error (MAPE); the results show that the prediction accuracy of the random forest is the best among all the models, which demonstrates the advantages of the integrated algorithms; however, on the other hand, the multiple linear regression model is not much less accurate but simpler, easier to understand and operate, thus provides a suitable bidding reference.

Table 3.10 Comparison of prediction accuracy

models	MLR	dynamic regression	random forest
MAPE	3.757582%	8.81178%	3.63057%

Chapter 4 Conclusion

This paper establishes a game theory model based on private value to represent the allocation mechanism, and finds that in the Bayesian Nash equilibrium, participants adopt a "threshold strategy", those whose willingness to pay is higher than the threshold participate in the auction, and those whose willingness to pay is lower than the threshold participate in the lottery, which leads to the formation of a diversion. The threshold is affected by the supply and demand of car plates, and the higher the quota for auction, the lower the threshold, and the more people participated, the higher the threshold. Next, we estimate the equality and efficiency of Shenzhen's car plate allocation, with an average allocation efficiency of 971.1 million yuan per period, in which the ratio of consumers' surplus to auction revenue is about 5:1, and the average Gini coefficient of the opportunity distribution of car plate availability is 0.5075. Historical data shows that the equality of Shenzhen's car plate allocation worsened while the efficiency improveed significantly, which often happens when the government allocates more of the additional plates to the auction.

In order to test the conclusions of the comparative static analysis in the theoretical model and explore the determination mechanism of the minimum transaction price, this paper establishes an empirical multiple linear regression model using the historical data, and finds that the minimum transaction price of the current period is a linear function of the minimum transaction price in the past or the average price of the second broadcast in the current period, and is corrected by the supply and demand of the current period, and the increase of the auction quota causes the price to be adjusted downward, while the increase of the number of bidders causes the price to be adjusted upward. This paper also establishes a VAR model to study the dynamic relationship between the minimum transaction price and the number of bidders, and proves that the two are Granger causality, and that changes in the number of bidders have a time-lagged positive effect on the minimum transaction price, while changes in the minimum transaction price have a time-lagged negative effect on the number of bidders. Finally, the seasonal ARIMA model and the random forest algorithm are used to predict the future minimum transaction price, and the random forest algorithm has the highest prediction accuracy and can provide a good bidding reference.

Based on the above findings, this paper makes recommendations for individuals and Shenzhen government involved in the car plate allocation. For the bidders, it is more rational to wait until the final stage of the bidding and get the information of the average price of the second broadcast before submitting the final bid, and the fact that the lowest price is higher than the average price of the second broadcast and the crucial predictive role of the average price of the second broadcast in the multiple linear regression and random forest model are strong evidence. As for the government, it should strive to make the allocation mechanism more open and transparent, such as announcing the number of bidders in real time and clarifying the relationship between supply and demand, which is conducive to the formation of rational expectations of bidders; and it should also appropriately shorten the auction time, because if more and more bidders choose to submit the final bid in the last stage, most of the time before that is spent waiting, and in terms of the opportunity cost caused by waiting, the auction time is too long is a waste of social resources.

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