Analyzing Accuracy of Phillips Relationship Using Modern Datasets

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Introduction

Standard candles like Type 1a supernovae, introduced us to ability to accurately calculate distances in space. Generally, these standard candles are able to provide distance estimations due to some form of inherent nature of the object. Type 1a supernovae (SNe 1a) are often included in the set of standard candles due to their standardized peak luminosity. Thus, with relatively simple calculations, one can calculate the distance to these kinds of supernova by comparing the apparent magnitude with the known absolute magnitude at its peak.

The reasoning behind the consistency of the peak luminosity is that SNe 1a are uniquely formed through binary star systems, where at least one of the stars is a white dwarf. As the white dwarf strips its companion star and increases in size, it edges closer to the Chandrasekhar limit of ~1.4$M_{\odot}$, or the critical mass of a white dwarf still being held through electron degeneracy pressure. When this limit is surpassed, electron degeneracy fails, and the resulting runaway reaction is a SN 1a. Since the mass and explosion process are effectively the same for most SNe 1a, it intuitively makes sense that the peak luminosity is similar.

However, upon closer inspection of SNe 1a light curves, we discover that the peak luminosity is not completely standardized and in fact fluctuates between magnitudes. We further observe that the rate of decay of luminosity is not consistent throughout all SNe 1a. This relationship is referred to as the Phillips relationship and details the luminosity decline rate relation with respect to the peak luminosity.

The Phillips relationship was named after a paper that Mark M. Phillips published. This paper, published in 1993, detailed the findings of a relationship between the peak absolute luminosity of SNe 1a and its rate of decline in luminosity. However, the sample size of thoroughly reported SNe 1a luminosity was sparse and while the paper proved that there was indeed a relationship between these two parameters – their regressions were significant – their data consisted of a mere nine supernovae for the Band V bands and only six supernovae for I band calculations. This report seeks to build off this now known relationship and ultimately test the accuracy of Phillips relationship through modern availability of supernovae sample size and refine the relationships that Phillips outlines.

Data

The data that we used is primarily extracted from the Open Supernova Catalog. In order to gather the amount of data that was required to be processed, we used the API that the website offered. Thus, it was necessary to write a script that would pull all available SNe1a in the database and their respective luminosity measurements.

In order to keep consistent with the requirements that the Phillips paper outlined for supernovae that could be included in the regression, we have only selected supernovae that satisfy the three requirements set forth in the original paper.

(1) Precise Optical Photometry

A major concern of the paper was the inaccuracies of photographic photometry and selectively only included supernovae with photometric data collected through photoelectric or

CCDs. However, photometry technology has progressed significantly since the paper's publication and modern photometry is largely concentrated in CCDs – rendering photographic photometry effectively obsolete and unused.

(2) *Well-sampled Light Curves*

The paper outlines a need for any photometry measurements to have continued for at least 20 days after the peak luminosity. In addition, Phillips arbitrarily defines that the light curve must be sampled well enough to prevent any large interpolations. In my sampling, we chose to keep the existing criteria and additionally add that there must be at least 1 light curve measurement for every 5 days in order to concretely define the large interpolation criteria.

(3) *Accurate Relative Distance*

Phillips chooses to only find supernovae that have accurate host galaxy distance measurements, specifically restricting acceptable distance measurements to Tully-Fisher (TRF) and surface brightness fluctuations (SBF). Similarly to (1), we find that distance measurement capabilities to have increased significantly since the paper has published. In addition, TRF and SBF are restricted to ~100Mpc while the SNe 1a can act as standard candles to distances of ~3000Mpc. Thus, we choose to select any supernovae that simply has any other method of distance detection, softening the original SBF and TRF restrictions.

With these criteria in mind, our script would have to eliminate supernovae that did not qualify as acceptable. As my preliminary goal was to simply verify the accuracy of the Phillips' relationship, we needed to also calculate the percentage of supernovae that passed the restrictions but also fell within the range of the Phillips' relationship.

Results

The first results that we sought were to find the accuracy of Phillips' model, specifically targeting the B (λ = 445 nm), V (λ = 551 nm), and I (λ = 806 nm) magnitudes. For each of the following bands, Phillips' regression shows that

$$M_{max} = -21.726 + 2.698\,\Delta m_{15}(B)$$

$$M_{max} = -20.883 + 1.949\,\Delta m_{15}(V)$$

$$M_{max} = -19.591 + 1.076\,\Delta m_{15}(I)$$

where $\Delta m_{15}(B)$, $\Delta m_{15}(V)$, and $\Delta m_{15}(I)$ are the change of B, V and I-magnitudes of the supernova in the span of 15 days after the peak luminosity respectively.

After processing all SNe 1a from the Open Supernova Catalog, we first eliminated any supernovae that did not satisfy Phillips' requirement (2) and (3). After that, we calculated Δm_{15} of each of these supernovae. This was not as straightforward as we had hoped. First, we had to find the apparent magnitude 15 days after the peak. Since many of the luminosity readings had irregular timestamps, we found m_{15} by taking the magnitudes of the closest two timestamps before and after t = 15 days and linearly interpolated between the two. In addition, the Phillips relationship uses absolute magnitudes for their regressions while only apparently magnitude data was available for us. Thus, we compensated by pulling the luminosity distance to each of these supernovae and calculating the absolute magnitude of the m_{peak} by using the following relationship:

$$M_{absolute} = m_{apparent} - 5(log_{10}D_L - 1)$$

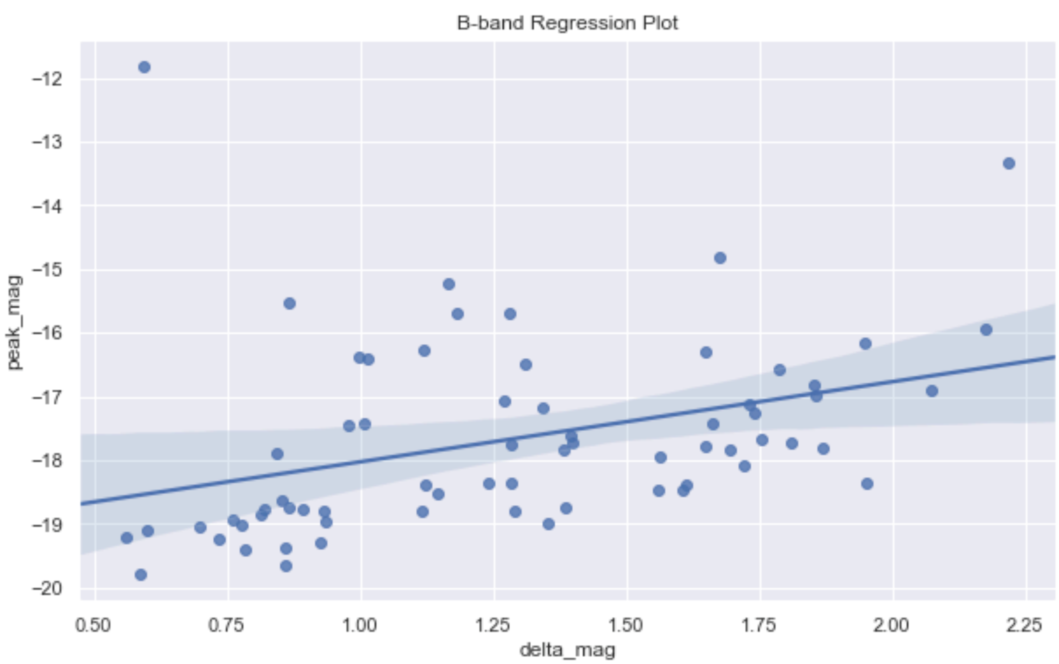
Finally, since we knew the absolute magnitude of the peak, we were able to plug in each supernovae's calculated Δm_{15} into their respective Phillips' regressions and compare the expected M_{max} with the actual m_{peak}. The Phillips paper also suggested several error margins for the regressions based on the type of band magnitude and we used these same error margins for our accuracy tests as we did see a need to change them. Our calculated percentages are displayed in Figure A.

Band	Error Margin	# of Restriction-passed SNe	% of Phillips-passed SNe
B	0.8	50	0.6800
V	0.6	54	0.2778
I	0.5	31	0.3548

Figure A. Accuracy test results using Phillips' regressions and error margins

Surprisingly, all of the Phillips regressions only moderately accounts for the relationship between the rate of decay and the peak magnitude in our data set. While the B-band regression is significantly more accurate than the other two bands, it still does not seem as strong as it should be. Moving forward from here, we decided to attempt to find a better regression that would more accurately depict the relationship.

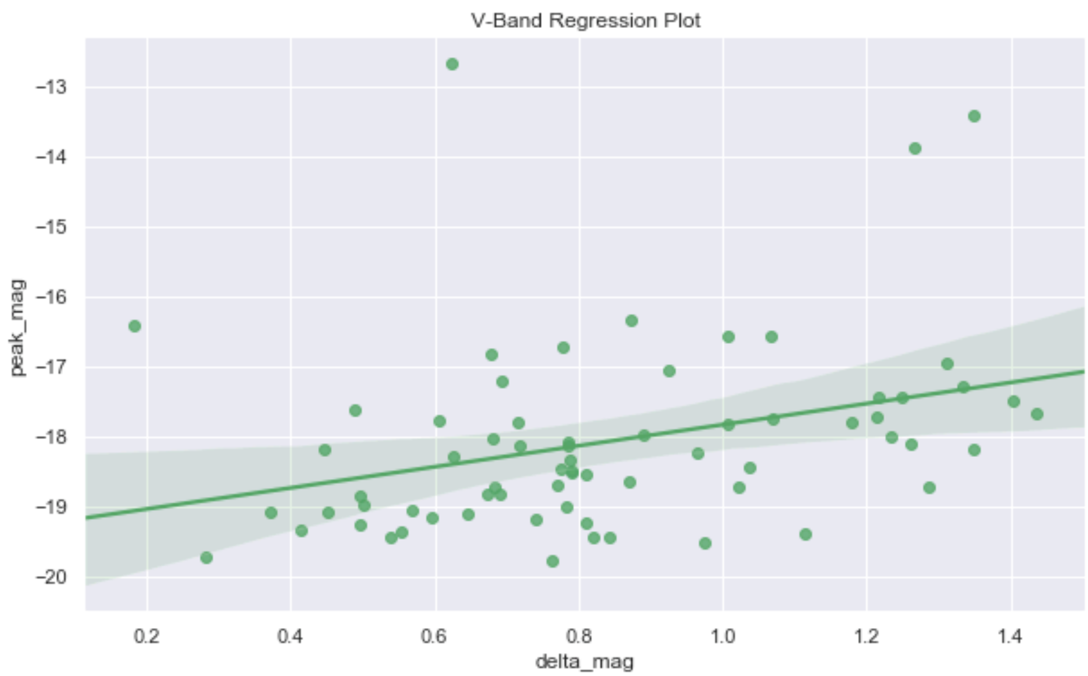
In order to calculate the regressions, we modified our script to generate three CSVs for each band where the files contained the supernova identifier, the peak magnitude, and the magnitude 15 days after the peak. We then plotted and ran ordinary least-squares on the data. Linear regression results can be seen in Figure B, C and D.



```
Residuals:
    Min      1Q  Median      3Q     Max 
-1.5368 -0.7873 -0.5476  0.5318  6.7210 

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)          -19.2861     0.5305 -36.351   <2e-16 ***
new_b_data$delta_mag   1.2573     0.3923   3.205   0.0021 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

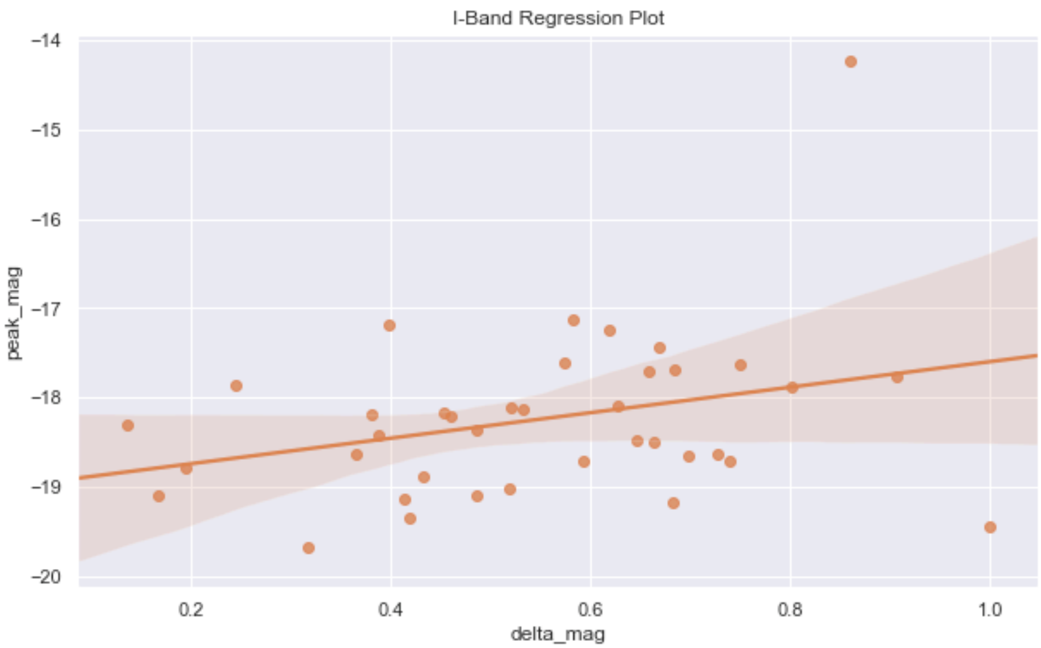
Figure B. OLS results for B-band




```
Residuals:
    Min      1Q  Median      3Q     Max 
-1.7250 -0.6704 -0.3267  0.1587  5.7363 

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)          -19.3446     0.4701  -41.15  < 2e-16 ***
new_v_data$delta_mag   1.5092     0.5276    2.86  0.00566 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure C. OLS results for V-band




```
Residuals:
    Min      1Q  Median      3Q     Max
-1.8529 -0.6019 -0.0117  0.3565  3.5727

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)          -19.0288     0.4290 -44.352   <2e-16 ***
new_i_data$delta_mag   1.4284     0.7373   1.937   0.0606 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure D. OLS results for I-band

We see that both the intercept and the 15 day magnitude delta parameters are significant in the regression – except for the I-band. With these new regressions, we propose that the following relationships will be a better fit for the Phillips relationship:

$$M_{max} = -19.286 + 1.257\,\Delta\, m_{15}(B)$$

$$M_{max} = -19.345 + 1.509\,\Delta\, m_{15}(V)$$

$$M_{max} = -19.029 + 1.428\,\Delta\, m_{15}(I)$$

To test our new regressions, we reran our script on our supernovae data to see if there was any improvement in the accuracy test.

Band	Error Margin	# of Restriction-passed SNe	% of Phillips-passed SNe
B	0.8	50	0.6600
V	0.6	54	0.4815
I	0.5	31	0.5161

Figure E. Accuracy test results using our own regressions but same error margins.

We can see that while the B-band is less accurate by a slight margin, there are significant improvements in the V and I band accuracy. The decrease in B band percentage and still subpar V and I band percentages suggest that there are perhaps other confounding unaccounted for variables when calculating the peak magnitude. Therefore, we see this as a potential opportunity to discover these unknowns in the form of a final project for this class.

Conclusion

Phillips' dataset of six to nine supernovae was incredibly small and thus we sought an improvement on his regressions based on the much more robust supernovae datasets that we have today. We discovered that the percentage of current supernovae data that fit under Phillips' relationship is relatively low. In order to refine Phillips' regressions, we then used the supernovae data that we had gathered and calculated a more holistically accurate relationship. However, the fact that the Phillips paper's regressions are still viable is a testament to the peak luminosity – rate of decline relationship and the work needed to clarify the origins and reasoning behind the relationship's existence.

Bibliography

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Observational Astronomy Midterm

April 11, 2019

In [1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns; sns.set()
from sklearn import linear_model
import math, sys
```

In [2]:
```
# CONSTANTS #
BASE = 'https://api.astrocats.space/'
B_CONDITIONAL = '/photometry/time+magnitude+e_magnitude+band?format=csv&band=B'
V_CONDITIONAL = '/photometry/time+magnitude+e_magnitude+band?format=csv&band=V'
I_CONDITIONAL = '/photometry/time+magnitude+e_magnitude+band?format=csv&band=I'
BAND_CONDITIONALS = [B_CONDITIONAL, V_CONDITIONAL, I_CONDITIONAL]

B_BAND = 0
V_BAND = 1
I_BAND = 2
BANDS = [B_BAND, V_BAND, I_BAND]
BAND_LABELS = ["B", "V", "I"]

B_BAND_ERROR_MARGIN = 0.8
V_BAND_ERROR_MARGIN = 0.6
I_BAND_ERROR_MARGIN = 0.5
BAND_ERROR_MARGINS = [B_BAND_ERROR_MARGIN, V_BAND_ERROR_MARGIN, I_BAND_ERROR_MARGIN]

INTERPOLATION_CONSTANT = 5
CONTINUITY_CONSTANT = 20
PHILLIPS_DELTA = 15

B_BAND_A = -21.726
B_BAND_B = 2.698
V_BAND_A = -20.883
V_BAND_B = 1.949
I_BAND_A = -19.591
I_BAND_B = 1.076
```

In [3]:
```
def calc_absolute_magnitude(lumdist, app_mag):
    return app_mag - (5 * (math.log10(lumdist*1000000) - 1))
```

In [4]:
```
def calc_apparent_magnitude(lumdist, abs_mag):
    return abs_mag + (5 * (math.log10(lumdist*1000000) - 1))
```

In [5]:
```
def within_margin(margin, actual, expected):
    return actual < expected + margin and actual > expected - margin
```

In [6]:
```
def calc_phillips_expected_mag(lumdist, band_mag_15, band_type):
    """
    Calculate expected b-band peak absolute magnitude BASEd on the
    band's magnitude after 15 days using the Phillips relationship

    band_mag_15 -- the band's magnitude 15 days after the peak
    """
    expected_mag = 0
    if band_type == B_BAND:
        expected_mag = B_BAND_A + B_BAND_B * band_mag_15

    elif band_type == V_BAND:
        expected_mag = V_BAND_A + V_BAND_B * band_mag_15
    elif band_type == I_BAND:
        expected_mag = I_BAND_A + I_BAND_B * band_mag_15
    return calc_apparent_magnitude(lumdist, expected_mag)
```

In [7]:
```
def interpolate(date1, mag1, date2, mag2, date15):
    """
    Calculate interpolated magnitude for 15 days after peak

    date1   -- date of first mag reading
    mag1    -- value of first mag reading
    date2   -- date of second mag reading
    mag2    -- value of second mag reading
    date15  -- exact date of 15 days after peak

    Returns magnitude at 15 day mark
    """
    slope = (mag2 - mag1) / (date2 - date1)
    days_to_15 = date15 - date1
    mag_delta = slope * days_to_15
    return mag1 + mag_delta
```

In [8]:
```
def check_criteria(sn, sn_data, band_type, f):

    """
    Checks whether a certain supernova fits the time
    and margin criteria for a specific band magnitude

    sn          -- [name, lumdist] of the supernova event
    sn_data     -- Magnitudes of the supernova
    band_type   -- Band type that correspondes to the sought after band magnitude
```

```python
    Returns an array of booleans: [time_criteria, margin_criteria]
    """

    criteria = [False, False]
    if not sn_data["magnitude"].empty:
        neg_delta_time_tracker = [-float("inf"), None]
        pos_delta_time_tracker = [float("inf"), None]

        # Find time at highest magnitude
        peak_mag = sn_data["magnitude"].min()
        peak_index = sn_data["magnitude"].idxmin()
        peak_time = np.floor(sn_data.at[peak_index, 'time'])
        curr_time = peak_time

        # Check for 1 reading every 2 days and for 20 days of readings after peak
        for reading in sn_data[["time", "magnitude"]].values:
            if reading[0] < peak_time:
                continue
            elif reading[0] - INTERPOLATION_CONSTANT > curr_time:
                break
            elif reading[0] > peak_time + CONTINUITY_CONSTANT:
                criteria[0] = True
                break
            curr_time = reading[0]

            #check for closest to PHILLIPS_DELTA value
            curr_delta = peak_time - (reading[0] - PHILLIPS_DELTA)
            neg_delta = (peak_time - (neg_delta_time_tracker[0] - PHILLIPS_DELTA))
            pos_delta = (peak_time - (pos_delta_time_tracker[0] - PHILLIPS_DELTA))
            if curr_delta > 0 and curr_delta < neg_delta:
                neg_delta_time_tracker = [reading[0], reading[1]]
            elif curr_delta < 0 and curr_delta > pos_delta:
                pos_delta_time_tracker = [reading[0], reading[1]]

        #linearly interpolate between 2 points to reach mag15
        if (
            neg_delta_time_tracker[0] is not None and
            neg_delta_time_tracker[1] is not None and
            pos_delta_time_tracker[0] is not None and
            pos_delta_time_tracker[1] is not None
        ):
            mag15 = interpolate(
                neg_delta_time_tracker[0],
                neg_delta_time_tracker[1],
                pos_delta_time_tracker[0],
                pos_delta_time_tracker[1],
                peak_time + PHILLIPS_DELTA
```

```
            )
            expected_mag = calc_phillips_expected_mag(
                sn[1],
                mag15 - peak_mag,
                band_type
            )
            criteria[1] = within_margin(
                BAND_ERROR_MARGINS[band_type],
                peak_mag,
                expected_mag
            )
            print("Currently observing: " + str(sn[0]))
            print("\tInterpolated mag: " + str(mag15))
            print("\tPeak mag: " + str(peak_mag))
            print("\tLumdist: " + str(sn[1]))
            print("\tExpected mag: " + str(expected_mag))
            sys.stdout.flush()

            f.write(str(sn[0]))
            f.write("\tInterpolated mag: " + str(mag15))
            f.write("\tPeak mag: " + str(peak_mag))
            f.write("\tLumdist: " + str(sn[1]))
            f.write("\tExpected mag: " + str(expected_mag) + "\n")

    return criteria
```

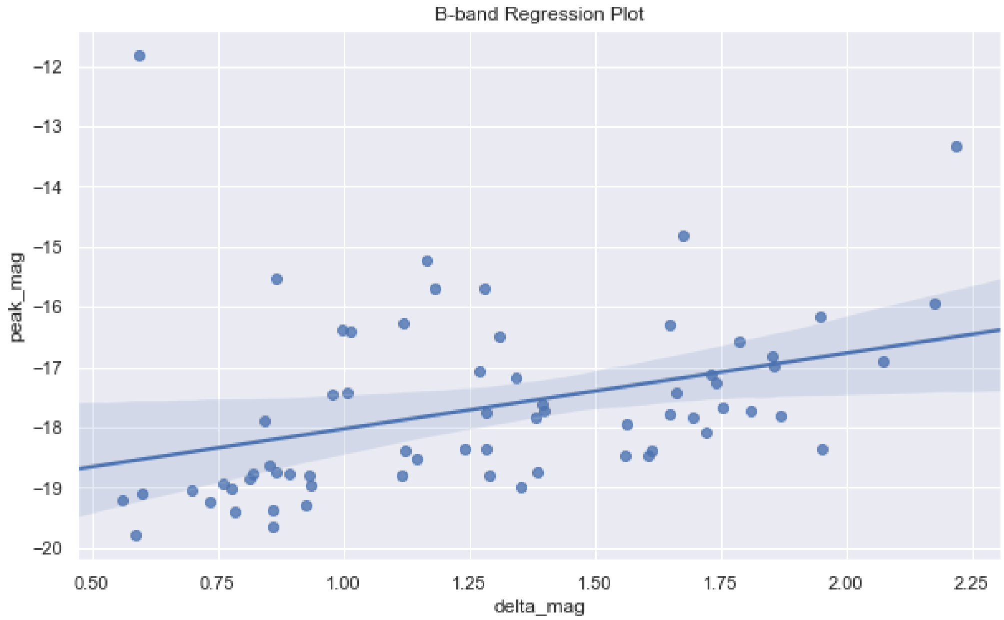
```
In [9]: def run():
    """
    Finds the list of supernova names and
     iterate through all sought after band magnitudes.
    Calculates the percentage of supernova that
     pass criteria set by Phillips' paper.
    """
    b_criteria_sne = []
    v_criteria_sne = []
    i_criteria_sne = []

    f = open("output.txt", "w+")

    for sn in DATA[["event", "lumdist"]].values:
        for band in BANDS:
            #Get data - convert columns to numerical values
            sn_data = pd.read_csv(BASE + sn[0] + BAND_CONDITIONALS[band])
            sn_data['magnitude'] = pd.to_numeric(sn_data['magnitude'])
            time_criteria, margin_criteria = check_criteria(sn, sn_data, band, f)

            #All evaluated supernovae satisfy criteria #1
            if time_criteria:
```

```
                if band == B_BAND:
                    b_criteria_sne.append([sn[0], margin_criteria])
                elif band == V_BAND:
                    v_criteria_sne.append([sn[0], margin_criteria])
                elif band == I_BAND:
                    i_criteria_sne.append([sn[0], margin_criteria])

        f.close()

        criteria_sne = [b_criteria_sne, v_criteria_sne, i_criteria_sne]
        for band in BANDS:
            within_margin_ratio = 0
            # print("List of SNe that satisfy light curves")
            for sn in criteria_sne[band]:
                if sn[1]:
                    within_margin_ratio += 1
            total_number_of_sne = len(criteria_sne[band])
            print("--------------------------------------------------------")
            print("Total SNe that satisfy light curve requirements for "
                    + BAND_LABELS[band] + "-band: " + str(total_number_of_sne))
            print("Percentage of SNe that satisfy" + BAND_LABELS[band]
                  + "-band error on expected peak mag: "
                  + str(within_margin_ratio/total_number_of_sne))
```

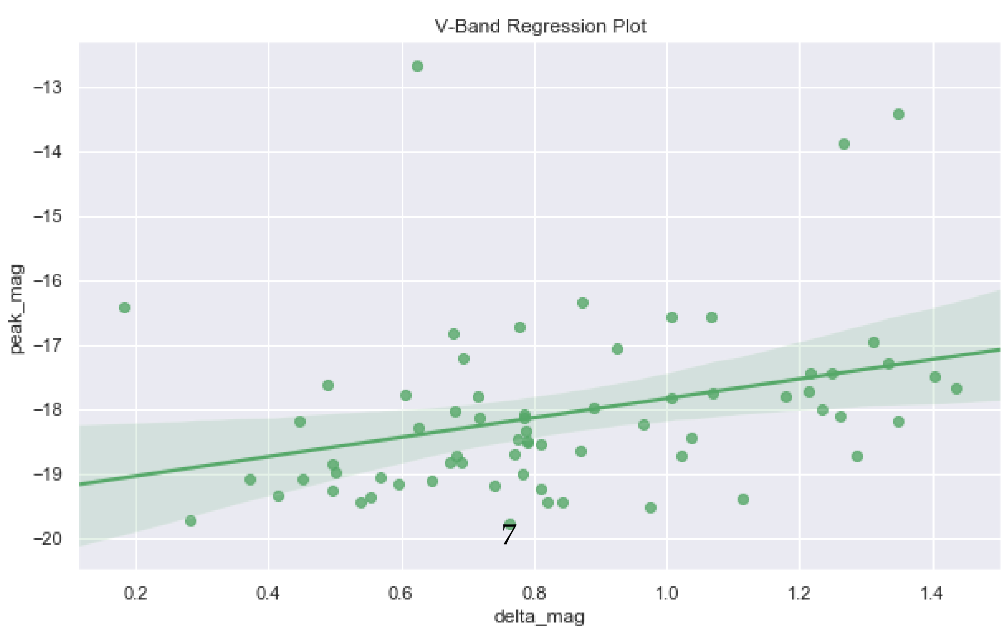
```
In [10]: def calc_delta(lst):
             for row in lst:
                 row[1] = row[1] - row[0]
             return pd.DataFrame(lst, columns=['peak_mag', 'delta_mag'])
```

```
In [11]: fig1, (ax1, ax2, ax3) = plt.subplots(nrows=3, ncols=1, figsize=(10,20))
         bands = {
             'B-band Regression Plot':
                 (pd.read_csv('./b_band.csv', usecols=['peak_mag', '15_mag']), ax1),
             'I-Band Regression Plot':
                 (pd.read_csv('./i_band.csv', usecols=['peak_mag', '15_mag']), ax2),
             'V-Band Regression Plot':
                 (pd.read_csv('./v_band.csv', usecols=['peak_mag', '15_mag']), ax3)
         }

         for band_type in bands:
             band_info = bands[band_type][0]
             delta_df = calc_delta(band_info.values)

             bands[band_type][1].set_title(band_type)
             sns.regplot(
                 x='delta_mag',
                 y='peak_mag',
                 data=delta_df,
```

```
    label=band_type,
    ax=bands[band_type][1]
)
```


B-band Regression Plot
peak_mag
delta_mag
−12
−13
−14
−15
−16
−17
−18
−19
−20
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25


I-Band Regression Plot
peak_mag
delta_mag
−14
−15
−16
−17
−18
−19
−20
0.2
0.4
0.6
0.8
1.0


V-Band Regression Plot
peak_mag
delta_mag
−13
−14
−15
−16
−17
−18
−19
−20
0.2
0.4
0.6
0.8
1.0
1.2
1.4

In [ ]: