Equity Option Prices and Firm Characteristics *

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Abstract

We investigate the information content of firm characteristics for the cross-section of equity option prices. We first show that 42 out of 86 characteristics significantly explain differences in the implied volatility surface (IVS) across stocks. Then, we exploit this relation to price equity options. We estimate the IVS of a given stock by using machine learning to optimally pool option information from stocks with similar characteristics, which overcomes the illiquidity of the equity option market. Our method improves upon stock-specific benchmarks in pricing options out-of-sample and allows us to uncover the nonlinear interactions between characteristics and option prices.

Keywords: Equity options, firm characteristics, implied volatility, random forest, machine learning

JEL Classification: C58, G12, G13

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1. Introduction

Prices of options across strikes and maturities reflect investors' perceptions of different types of risk in the underlying asset (e.g., volatility and downside risk). This is such that there is a large literature studying the pricing of index options and its implications for risk expectations in the aggregate stock market.[1] In contrast, relatively little is known about the structure of individual equity option prices. In this regard, two main questions arise. First, what types of stocks are perceived as riskier by investors in the option market? Second, how can this information be exploited to better price equity options? Our paper addresses these questions by investigating whether and how firm characteristics, which have been shown to be relevant for stock returns, help explain differences in option prices across stocks.[2]

We start by summarizing the cross-section of option prices for a given stock with the level, moneyness slope and term structure of the implied volatility surface (IVS). Then, we estimate end-of-month Fama and MacBeth (1973) regressions to determine how each of these features are related to 86 firm characteristics from Green et al. (2017). We simultaneously include all characteristics as explanatory variables, which allows us to identify the characteristics that provide independent information about equity option prices. To address data-snooping concerns from using many regressors and characteristics already previously identified in the literature, we deem a given characteristic to be significant if the absolute value of its Fama-MacBeth t-statistic is at least 3.0, as recommended by Harvey et al. (2016).

We find that 42 out of the 86 firm characteristics are priced in the option market, in the sense that they significantly explain cross-sectional differences in perceived sources of risk as measured by features of the IVS.[3] This is about three times as large as the number of

[1]See, for instance, Jackwerth and Rubinstein (1996), Bakshi et al. (1997), Aït-Sahalia and Lo (1998), Dumas et al. (1998), Bollerslev and Todorov (2011), Kelly et al. (2016), Andersen et al. (2015, 2017) and Almeida and Freire (2022).

[2]It is worth emphasizing that our goal of explaining how option prices (which reflect investors' expectations of risk in the underlying asset) change across stocks differs from that of explaining how equity delta-hedged option returns (which reflect investors' compensation for bearing option risk) change in the cross-section, as done by, e.g., Goyal and Saretto (2009) and Zhan et al. (2022).

[3]As in Ilhan et al. (2021), "priced" here means option prices reflect that certain stocks are perceived as riskier than others, rather than that investors are compensated with an expected return for taking risk.

characteristics that independently explain average returns across stocks (Green et al., 2017). Overall, the most important characteristics are price trends covering different momentum variables. High-momentum stocks (i.e., those that did well in the past) have lower IVS levels, but are associated with higher moneyness slopes, indicating that the cost of protection against downside risk is larger for these firms. This supports the idea that there is a crash risk component in momentum strategies (Daniel and Moskowitz, 2016).

These are followed by liquidity measures (such as bid-ask spread, market value and Amihud illiquidity). Illiquid stocks and small stocks are associated with higher IVS levels, being thus perceived as more volatile. Such a negative relation between size and volatility is consistent with the model of firm networks from Herskovic et al. (2020). Also among the most important characteristics are volatility variables (including idiosyncratic volatility, market beta and return volatility). Naturally, stocks with high return volatility are perceived as more volatile, with higher IVS levels. As for market beta, the CAPM stochastic volatility model of Christoffersen et al. (2018) predicts that firms with higher betas should have a higher level, a steeper moneyness slope and a steeper term structure of the IVS. We confirm only the latter two predictions, which suggests that the relation between market betas and IVS levels is affected by controlling for other characteristics.

We then proceed to exploit the information in firm characteristics to price equity options. The main challenge of dealing with equity options is their relative illiquidity. While there are on average more than a thousand options on the S&P 500 index each day, the average number of equity option observations per stock-day is only 21. This makes leading option pricing techniques, which are option data intensive, unfeasible for the vast majority of individual equities.[4] Motivated by our previous results, we address this challenge by employing pooled estimation of IVS features which groups stocks with similar characteristics together. The clustering and the estimation are carried out simultaneously by means of local linear forests (Athey et al., 2019; Friedberg et al., 2020), which extend the original idea of random forests

[4]For instance, Christoffersen et al. (2018) and Bakshi et al. (2021) restrict their sample to 30 and 50 firms with the most liquid cross-sections of options, respectively, to estimate stochastic volatility models.

(Breiman, 2001) by fitting linear models inside the leaves of the decision trees instead of just taking an average of the observations. The group structure of our IVS forest alleviates joint signals across stocks while reducing estimation error.[5]

As an empirical validation of our method, we conduct an out-of-sample option pricing exercise. Our main benchmark is the ad-hoc Black-Scholes (AHBS) of Dumas et al. (1998), which is stock-specific and models the IVS with a quadratic polynomial of moneyness and maturity.[6] Our approach can be seen as a pooled AHBS across stocks that are optimally grouped in terms of characteristics, such that this comparison can shed further light on the relative information content of characteristics for the IVS. On a given day of our sample, we fit the models using half of the option data for each stock, and predict the other half. We show that our local linear random forest outperforms the univariate AHBS benchmark. The outperformance is mainly driven by stocks that have relatively illiquid cross-sections of options, which are the majority in the option market. For these stocks, pooling option information from similar stocks is extremely helpful as it reduces estimation uncertainty.

The local linear random forest models how the IVS differs across stocks taking into account nonlinear interactions of the stock characteristics. This allows us to provide additional insights into the relation between characteristics and option prices. We first analyze the variable importance of the characteristics using our fitted model. While the set of most relevant characteristics overlaps with those that are significant in the Fama-MacBeth regressions, we find that the most influential variables differ when taking into account nonlinearities. For instance, characteristics that are industry-specific (such as industry momentum) are more important in the local linear decision trees as they allow the effects of other characteristics to differ across industries. Then, we examine the model-implied marginal local effects of the characteristics for features of the IVS. While the signs of the effects generally coincide with

[5]In this regard, our model is related to Kleen and Tetereva (2022), who employ local linear forests to obtain superior realized volatility forecasts in a large cross-section of stocks.

[6]The AHBS model is a popular benchmark in the option pricing literature (Dumas et al., 1998; Heston and Nandi, 2000; Christoffersen and Jacobs, 2004; Barone-Adesi et al., 2008; Almeida et al., 2023a) and is often used to summarize the IVS (Gonçalves and Guidolin, 2006).

the cross-sectional regression analysis, we show that these relations can be highly nonlinear and often concentrated on a particular region of the characteristic space.

The remainder of the paper is organized as follows. After a brief review of the related literature, Section 2 describes our data and presents the results of our cross-sectional regression analysis. In Section 3, we propose the local linear random forest for modeling the IVS as a function of characteristics. Section 4 contains the results for our prediction exercise, which provide empirical support for our approach, and analyzes the most important characteristics and their nonlinear interactions with IVS features. Section 5 concludes the paper.

1.1. Related literature

Our paper is mainly related to three strands of the literature. The first studies the pricing of equity options.[7] Bakshi et al. (2003) document that risk-neutral equity distributions are less skewed to the left than index distributions, which is related to a flatter moneyness slope of the IVS. Bollen and Whaley (2004) associate differences in the structure of index and equity option prices to differential demands for each type of option. Duan and Wei (2009) show that stocks with higher systematic risk proportion have a higher level and steeper slope of the implied volatility curve. Christoffersen et al. (2018) develop an option pricing model capturing the factor structure of equity options, which predicts that firms with higher market betas have a higher IVS level and steeper slope and term structure. While these papers are mainly interested in the relative pricing of index versus equity options and consider up to 30 stocks in their analyses, we focus on the role of firm characteristics in explaining differences in features of the IVS across the entire universe of optionable stocks. More closely related to our work, Dennis and Mayhew (2002), An et al. (2014) and Chen et al. (2023) investigate the relative importance of 5, 9 and 13 firm characteristics in explaining risk-neutral skewness, predicting changes in implied volatility levels, and explaining 30-day implied volatilities, respectively. We consider a broader set of characteristics and examine implications for the

[7]As previously mentioned, a different literature instead studies the cross-section of delta-hedged equity option returns (see, e.g., Goyal and Saretto, 2009; Zhan et al., 2022).

level, slope and term structure of the entire IVS.[8] We also analyze nonlinear interactions between characteristics and option prices and provide a new option pricing method.

The second strand of the literature proposes option pricing techniques for individual equity options. Christoffersen et al. (2018) develop a model for equity option valuation featuring stochastic volatility and a CAPM factor structure, while Bakshi et al. (2021) compare a number of traditional models with stochastic volatility and jumps for pricing equity options. These methods require stocks with relatively liquid options, while we put forward a flexible machine learning technique that models the IVS as a function of firm characteristics, exploiting information from the cross-section of stocks to overcome illiquidity. Almeida et al. (2023b) use information from the time series of stock returns coupled with risk-neutralization algorithms to price equity options. While they require a relatively long time series of returns for a given stock, our approach exploits cross-sectional information and requires only characteristics data.

The third strand explores implications of machine learning in finance. Gu et al. (2020) compare a number of machine learning methods for predicting stock returns based on a large set of predictive signals consisting mostly of firm characteristics. We use the same set of characteristics to model how the IVS differs across stocks with random forests. Bryzgalova et al. (2021) construct efficient portfolios by conditionally sorting stocks on characteristics with decision trees. Our local linear random forest builds a different option pricing model for each group of stocks that is endogenously selected using information from characteristics. In the context of options, Hutchinson et al. (1994) and Garcia and Gençay (2000) use neural networks to estimate the index option pricing function, while Almeida et al. (2023a) apply neural networks to correct parametric option pricing models. These methods require large quantities of option data and as such have been designed for index options. In contrast, our approach optimally combines option information across similar stocks using firm characteristics to overcome the illiquidity of equity options.

[8]See Vasquez (2017) for the economic importance of explaining the IVS term structure dimension.

2. Implied volatility surfaces and firm characteristics

In this section, we first describe the option, stock price and characteristics data we use. Then, we provide empirical evidence on the role of firm characteristics for explaining differences in the IVS across stocks. In particular, from a large set of characteristics, we identify those that are independently informative about perceived sources of risk implied by equity option prices and discuss the outcomes.

2.1. Data

We obtain end-of-month option data for individual equities from OptionMetrics for the sample ranging from January, 2000 to December, 2021. The data contain information about the entire U.S. equity option market and include the end-of-day bid and ask quotes, implied volatility, strike price, expiration date and volume for each option.[9] We apply a handful of standard filters to the raw data. We exclude options with: zero volume, null bid or ask, zero bid, bid greater than ask, mid-point of bid and ask below 1/8 and null implied volatility. Our source for stock price data is the Center for Research in Security Prices (CRSP). As for the firm characteristics, we use those calculated and provided by Gu et al. (2020).[10] We follow the common practice of replacing missing values in the characteristics by the cross-sectional median in that month and standardizing the characteristics on a monthly basis by mapping the cross-sectional ranks into the $[-1, 1]$ interval. In light of the results by Green et al. (2017), we exclude eight characteristics that are most strongly related to other characteristics (betasq, dolvol, lgr, maxret, mom6m, pchquick, quick, and stdacc) in order to avoid multicollinearity issues in our Fama-MacBeth regressions. After that, we are left with a total of 86 firm characteristics.[11] We use a linking table provided by Wharton Research Data Services (WRDS) to merge the option, stock price and characteristics datasets.

[9] The implied volatilities are computed by OptionMetrics using binomial trees. They properly take into account that equity options are American and their prices can be influenced by an early exercise premium.

[10] https://dachxiu.chicagobooth.edu/download/datashare.zip. We thank Dacheng Xiu for making the data available on his website. As the characteristics are pre-lagged, the month December, 2021 is missing.

[11] For the list of characteristics and their description, see Green et al. (2017) and Gu et al. (2020).

We consider options across all maturities and within the broad range of moneyness between 0.5 and 2.0, where moneyness is defined as the ratio between the stock price and the strike. Our final sample consists of 13,389,513 options, across 8,917 distinct underlying stocks. Since we work directly with implied volatilities, we do not make a distinction between calls and puts. There is an average of 21 options in the cross-section per stock-day in our sample, across on average 4 maturities. This is in stark contrast with the thousands of options observed for the S&P 500 index each day.

Throughout the paper, we focus on implied volatilities.[12] Table 1 reports summary statistics of our implied volatility data across different moneyness and time to maturity categories. Short-term equity options present a pronounced U-shaped pattern across moneyness categories popularly known as a "smile", where average implied volatilities decrease from out-of-the-money (OTM) calls (or in-the-money (ITM) puts) to at-the-money (ATM) options and then increase again for OTM puts (or ITM calls). The same pattern is observed for longer maturities, albeit with a flatter shape. Average implied volatilities are also decreasing with maturity, for each of the moneyness categories. In terms of number of observations, most of the short-term options are ATM, while medium- and long-term options mostly fall in the deep OTM categories (i.e., with moneyness between [0.5, 0.9) or [1.1, 2]).

To help visualize how observations are spread out across stocks, we plot the histogram of the average number of options per stock. Since we have an unbalanced panel of stocks, the average number of observations is calculated only across months for which a particular stock has observed options. Figure 1 depicts the results. There is a large number of stocks (about 300) with an average number of options of 1, which indicates the presence of many stocks with only one observation in the entire sample. In addition, most of the stocks have on average between 2 and 10 options on a given day of our sample. Beyond that region, the higher the number of observations the lower is the number of stocks satisfying the threshold.

[12] There is a one-to-one mapping between option prices and implied volatilities. Implied volatilities are preferable because, in contrast to option prices, they are comparable across options of different moneyness and time to maturity, across different stocks and over time. There is also a close connection between the shape of the IVS and the risk-neutral distribution of underlying returns.

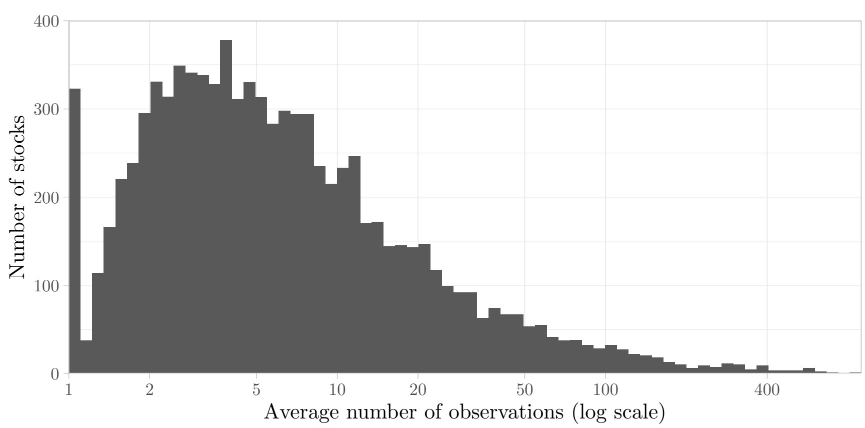
Table 1: Summary statistics of equity options

	Moneyness				
	[0.5, 0.9)	[0.9, 0.97)	[0.97, 1.03)	[1.03, 1.10)	[1.1, 2]
$\tau \leq 90$	62.05% [32.14%, 99.65%] {4640.37}	42.69% [22.95%, 67.38%] {6069.76}	39.35% [21.28%, 62.06%] {8793.85}	44.08% [25.30%, 67.65%] {5845.16}	61.03% [34.13%, 94.51%] {6054.39}
$90 < \tau \leq 365$	48.04% [25.41%, 76.08%] {4452.18}	38.20% [21.01%, 60.43%] {2389.01}	38.16% [21.46%, 59.54%] {2250.68}	40.15% [23.43%, 61.68%] {1918.21}	49.36% [29.21%, 74.38%] {4580.35}
$\tau \geq 365$	40.87% [23.43%, 63.40%] {1336.01}	37.00% [21.94%, 56.52%] {409.54}	36.76% [22.25%, 55.28%] {402.36}	37.85% [23.40%, 56.37%] {363.50}	43.26% [27.27%, 63.55%] {1405.32}

Notes: This table presents summary statistics of our equity option data. In each last day of the month, we group all options into buckets by moneyness and time-to-maturity (the definitions are indicated in row and column names). Per bucket and month, we calculate (1) the average implied volatility in %, (2) the 10% and 90% quantiles of implied volatilities (in brackets), and (3) the number of observations (in curly braces). The table reports the time-series averages of these quantities. The sample ranges from January, 2000 to December, 2021.

In particular, the 30 or 50 stocks with most liquid cross-sections of options represent the very far right tail of the histogram.[13]

Fig. 1: Histogram of average number of end-of-month observations per stock



Notes: We calculate the total number of traded options per stock and end-of-month and take the stock-specific time-series average of the number of traded options. The histogram is based on these time-series averages. Averages are only taken across months with at least one observation; that is, the average number of observations cannot be lower than 1. The sample ranges from January, 2000 to December, 2021.

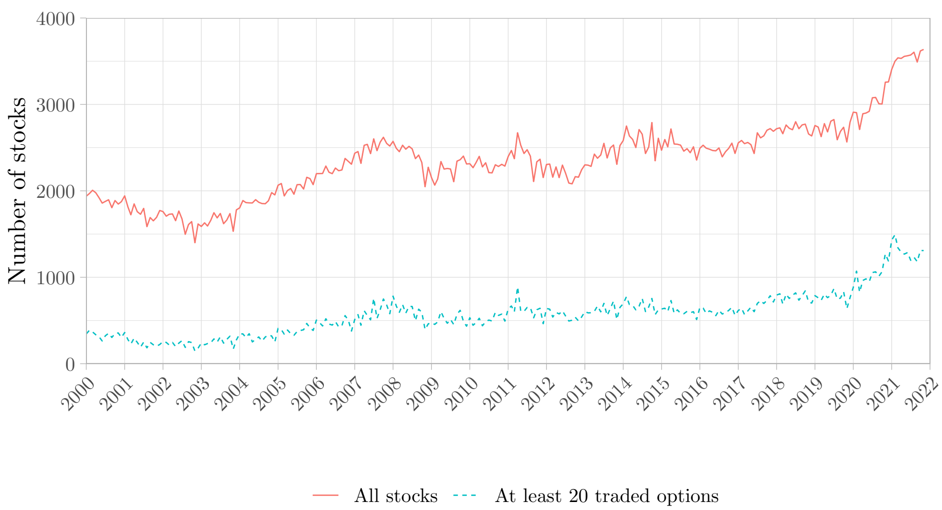
For most of our analysis, we will work with a more restricted sample that, each end-of-month, collects the stocks that have at least 20 option observations. We refer to it as the

[13]It is worth noting that stocks beyond those with relatively liquid cross-sections of options are still considerably large in terms of market capitalization, further justifying the importance of studying an extended universe of equity options. For instance, Zhan et al. (2022) show that, relative to the CRSP sample, the average size percentile of optionable stocks is 80%.

liquid sample. This sample guarantees that for each stock we have enough option data to reliably compute features of the IVS such as its level, slope and term structure. This is important for our cross-sectional analysis of the relation between the IVS and firm characteristics and for the stock-specific benchmark in the out-of-sample pricing exercise. On the other hand, this would not be necessary for our local linear random forest as it optimally pools stocks together to estimate IVS features, such that there is always enough option information for each stock. In fact, we also estimate a version of our model using all available option data as input.

Figure 2 shows the evolution of the number of unique stocks for each month, both for the entire sample and for the liquid sample. In both cases, the plot is mostly increasing over time, reflecting the increase in liquidity and size that the equity option market has experienced over the last decades. The number of unique stocks in the entire sample increases from 2000 in January, 2000 to around 3500 in December, 2021, while that in the liquid sample evolves from around 500 to 1500. Even so, the gap between the plots remains essentially the same, indicating that the fraction of stocks with very illiquid option cross-sections remained approximately constant.

Fig. 2: Number of stocks over time with traded options



Notes: This figure depicts, for each end-of-month, the number of stocks in our sample with traded options and the number of stocks with at least 20 traded options. The sample ranges from January, 2000 to December, 2021.

2.2. IVS features

For each stock $i = 1, \ldots, N$ and each end-of-month $t = 1, \ldots, T$, we have a cross-section $j = 1, \ldots, J_i$ of implied volatilities $\sigma_{i,j,t}$ across different moneyness $m_{i,j,t}$ and times to maturity $\tau_{i,j,t}$ (in calendar days). In our unbalanced panel, N and J_i vary over time but we avoid introducing a t subscript for brevity in notation. To summarize the cross-section of option prices, or, in other words, the IVS of each stock, we estimate its level, slope and term structure. More specifically, for each month and stock, we run the following regression:

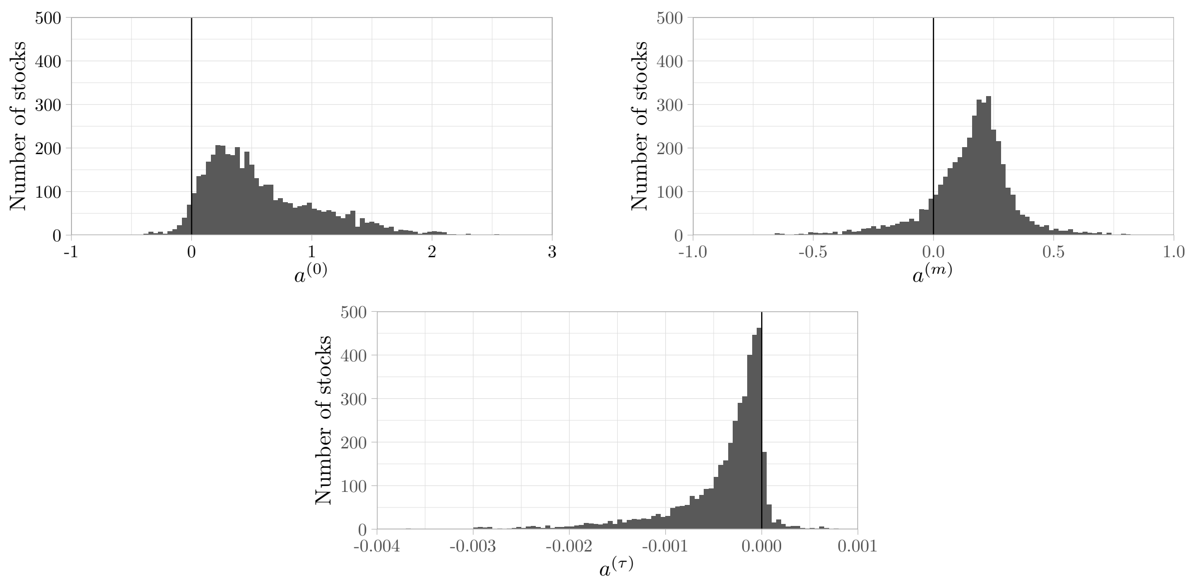
$$\sigma_{i,j,t} = a_{i,t}^{(0)} + a_{i,t}^{(m)} m_{i,j,t} + a_{i,t}^{(\tau)} \tau_{i,j,t} + \varepsilon_{i,j,t}, \qquad j = 1, \ldots, J_i. \tag{1}$$

Each of the coefficients above convey different information about perceived sources of risk for stock i. The level $a_{i,t}^{(0)}$ captures risk-neutral expectations of future volatility for the underlying asset. The slope $a_{i,t}^{(m)}$ is closely related to the risk-neutral skewness of stock returns. A higher slope indicates negative skewness and a higher cost of protection against downside risk, as OTM puts are relatively more expensive than OTM calls. The term structure slope $a_{i,t}^{(\tau)}$, on the other hand, reflects market views on how volatility is expected to change over time.

While a small number of shape features cannot fully represent the IVS, there is a general consensus that they offer a comprehensive and intuitive description of implied volatility data. Christoffersen et al. (2018) adopt a similar approach to identify common components in equity implied volatilities. Andersen et al. (2015) and Almeida et al. (2023a) consider, as a model diagnosis tool, the ability of option pricing models in reproducing the dynamics of IVS features. Aït-Sahalia et al. (2021) propose to estimate stochastic volatility models by directly fitting such observed features of the IVS and show that this method works well. They also highlight the economic importance of the risks captured by IVS features as several trading strategies have been designed to get exposure to or hedge against these risks. Examples include straddles, risk reversals and calendar spreads exposing to the level, moneyness slope and term structure of the IVS, respectively.

Figure 3 reports the histogram of the average level, slope and term structure of stocks in the liquid sample. Most of the stocks have an average level between 20% and 40%, aligned with the average implied volatilities of ATM options in Table 1. Even so, many stocks have higher levels, which can reach up to 200%. A few stocks display negative average level coefficients. This does not mean that implied volatilities fitted by regression (1) would be negative, as the other coefficients have to be taken into account as well. What matters is that stocks with higher IVS levels are perceived as more volatile.

Fig. 3: Histogram of average level, slope and term structure



Notes: Each month we estimate the regression in Eq. (1) for all stocks that have more than 20 traded options. Thereafter, we calculate the time-series averages for the level, slope and term structure coefficient per stock. The histogram is based on 4217 time-series averages per coefficient. To be consistent with the results in Figure 4, the histograms are based on the winsorized estimates. The sample ranges from January, 2000 to December, 2021.

The second plot of Figure 3 shows that the majority of stocks has a positive moneyness slope on average, meaning that protection against downside risk is costly as OTM puts are relatively more expensive in the option cross-section. The higher the slope, the higher this cost. On the other hand, there are a few stocks with a negative average slope. For these equities, OTM calls are relatively more expensive than OTM puts. The third plot depicts the histogram for the term structure coefficient. As can be seen, the vast majority of stocks has a negatively sloped term structure on average, suggesting that stocks are expected to become less volatile in the future.

2.3. Explaining IVS features with characteristics

To investigate whether firm characteristics help explain cross-sectional differences in features of the IVS, we run Fama and MacBeth (1973) regressions. More specifically, for each end-of-month and each IVS feature we run the following cross-sectional regression:

$$a_{i,t}^{(.)} = \lambda_{0,t} + \lambda_{1,t} c_{i,t} + \epsilon_{i,t}, \qquad i = 1, \ldots, N, \tag{2}$$

where $c_{i,t}$ is a vector containing the 86 firm characteristics we consider. Green et al. (2017) show that simultaneously evaluating all characteristics is feasible as the correlations across characteristics are small. By doing so, we can identify the characteristics that provide independent information about features of the IVS. The Fama-MacBeth approach is also naturally well-suited to handle unbalanced panels of stocks. The effect of a given characteristic is assessed by the time-series average of its coefficient. We deem a characteristic to be significant if the absolute Newey-West t-statistic of its average coefficient is above 3.0. Our hurdle for statistical significance is conservative and follows the recommendation of Harvey et al. (2016) to address data-snooping concerns.

Figure 4 summarizes the results of our cross-sectional analysis by plotting, for each IVS feature, the heatmap of the Fama-MacBeth t-statistic of each characteristic. Insignificant relations are omitted from the plot and characteristics are ordered according to the sum of their absolute t-statistics across all features of the IVS. We find that many of the characteristics that have been shown to be relevant for stock returns also explain cross-sectional differences in option prices. There are in total 42 characteristics that are priced in the option market, in the sense that they significantly explain at least one of the IVS features. In particular, 31 characteristics are significant for the level, 21 for the moneyness slope and 18 for the term structure. While Green et al. (2017) find that only 12 characteristics are able to explain the cross-section of stock returns when evaluated simultaneously, the number of characteristics that are independently informative about perceived sources of risk implied by options is much higher.

The most informative characteristics are price trends, including 4 of the top 10 variables: 12-month momentum (mom12m), 1-month momentum (mom1m), 36-month momentum (mom36m) and change in 6-month momentum (chmom). The signs of the estimated coefficients are the same across all these variables. Stocks that did well in the past (high momentum) have lower IVS levels, i.e., they are perceived as less volatile. However, stocks with high momentum are associated with higher moneyness slopes, indicating that the cost of protection against downside risk is larger for these firms. This is consistent with Daniel and Moskowitz (2016), who show that there is a crash risk component in momentum strategies as they experience persistent negative returns during crises. We provide option-based evidence that the exposure of momentum to downside risk is reflected in investors' expectations.

Next are liquidity measures, such as the bid-ask spread (baspread), market value (mvel1) and Amihud illiquidity (ill). Illiquid stocks (high bid-ask spread) and small stocks are associated with higher IVS levels, but flatter slopes in the moneyness and maturity dimensions. The fact that small stocks are perceived as riskier than large firms is aligned with the usual interpretation of the size effect on stock returns. In particular, the higher levels and lower slopes of small firms suggests that the size effect is related to compensation for volatility risk rather than downside risk. Such a negative relation between size and volatility is also consistent with Herskovic et al. (2020), who rationalize it with a model of firm networks.

The third most important group of characteristics are volatility variables. Naturally, stocks that are more volatile, with high idiosyncratic volatility (idiovol) and return volatility (retvol), are also perceived as more volatile, with higher IVS levels. Regarding market beta, Christoffersen et al. (2018) show that a stochastic volatility model where equity prices are driven by exposure to the market predicts that firms with higher market betas have a higher level and steeper moneyness slope and term structure of the IVS. These predictions are confirmed by our results, with the exception that stocks with higher betas have lower IVS levels. This suggests that controlling for other characteristics affects the relation between market beta and the level of the IVS.

Valuation characteristics such as leverage (lev), book-to-market (bm), dividend-to-price (dy) and earnings-to-price (ep) also significantly explain cross-sectional differences in the IVS. Firms with high leverage are associated with smaller IVS levels. Dennis and Mayhew (2002) document a similar relation and argue that low volatility firms are able to take on more debt. On the other hand, high leverage firms have higher moneyness slopes, and are thus perceived as more exposed to downside risk. This is consistent with the predictions of the option pricing model on leveraged equity by Toft and Prucyk (1997). Moreover, firms with high book-to-market are perceived as riskier with higher IVS levels, aligned with the common notion that these firms are in more financial distress. Similarly to the size characteristic, the fact that high book-to-market firms also have lower moneyness slopes suggests that the value effect is related to compensation for volatility risk rather than downside risk.

In sum, we find that a large number of firm characteristics are priced in the option market. We identify the most relevant groups of characteristics and provide interpretations for some of the significant relations we document. In the next section, we exploit the information content of characteristics for equity option prices and propose to model implied volatility surfaces as a function of characteristics.

Fig. 4: Fama-MacBeth regressions for IVS coefficients on firm characteristics

Notes: This figure reports the outcome of the two-stage Fama-Macbeth cross-sectional analysis as detailed in Section 2.2. In the first stage, we run monthly cross-sectional regressions of IVS coefficients for each stock with at least 20 traded options on firm characteristics. In the second stage, we regress the monthly slope parameters per characteristic on a constant. The magnitude of the second-stage t-statistics are highlighted if $|t| > 3$. t-statistics below -3 are depicted in blue and t-statistics above 3 are depicted in red. We employ standard errors based on Newey-West covariance estimates with 3 lags. The estimated level, slope and term structure are winsorized monthly at the 1%- and 99%-cross-sectional quantile prior to running the Fama-MacBeth regressions. The sample ranges from January, 2000 to November, 2021.

3. Modeling volatility surfaces using characteristics

Given the strong relation between firm characteristics and features of the IVS, we propose to model the IVS of a given stock as a function of its characteristics. As a first step, we note that while the specification in Eq. (1) captures the main qualitative features of the IVS shape, it may be too restrictive to provide an accurate quantitative fit of the IVS. In particular, both the moneyness and term structure slopes usually display some curvature, and the effect of moneyness on implied volatilities depends on the maturity and vice versa. For this reason, the reduced specification in Eq. (1) is often augmented to include second-order effects of moneyness, second-order effects of time to maturity, and interaction effects between these two variables:[14]

$$\sigma_{i,j} = a_i^{(0)} + a_i^{(m)} m_{i,j} + a_i^{(m^2)} m_{i,j}^2 + a_i^{(\tau)} \tau_{i,j} + a_i^{(\tau^2)} \tau_{i,j}^2 + a_i^{(m\tau)} m_{i,j} \tau_{i,j} + \varepsilon_{i,j}, \quad j = 1, \ldots, J_i. \tag{3}$$

Even though we focus on the level, slope and term structure of the IVS in our analysis of Section 2, additional results in supplementary appendix A confirm that the higher-order coefficients in Eq. (3) are also significantly related to firm characteristics.

The model in Eq. (3) is popularly known as the ad-hoc Black-Scholes (AHBS). This model was first proposed by Dumas et al. (1998) as a parsimonious but accurate model for the IVS. It has been considered as a benchmark in a number of studies and is often difficult to beat (Heston and Nandi, 2000; Christoffersen and Jacobs, 2004; Barone-Adesi et al., 2008; Almeida et al., 2023a). In previous applications, which considered hundreds or thousands of options on the S&P 500 index, estimating the AHBS model via ordinary least squares (OLS) is straightforward. However, in Figure 1 we document that even stocks in the liquid sample of our dataset have hardly more than 50 observations. This is such that the AHBS model is prone to severe estimation risk for the large majority of stocks in the option market.

One simple way of reducing estimation uncertainty is to run pooled OLS regressions in-

[14] From now on, we omit the time subscript for ease of notation.

stead of running individual regressions for each stock i. The decision to run pooled estimation even though the individual IVS features are found to be different in practice can be seen as a typical trade-off between lower estimation uncertainty and higher potential misspecification error. Even though it is reasonable to assume that the estimates of the slope coefficients in the AHBS model might improve in terms of variance, fixing the intercept coefficient in Eq. (3) is clearly too restrictive in light of the differences in average IVs across stocks.[15] Hence, for addressing the different levels of average IV with pooled OLS without explicitly estimating stock-specific fixed effects, we run one joint regression that models the deviation from the average IV of a stock instead of the IV itself:

$$\sigma_{i,j} - \bar{\sigma}_i = \widetilde{a}^{(0)} + \widetilde{a}^{(m)} m_{i,j} + \widetilde{a}^{(m^2)} m_{i,j}^2 + \widetilde{a}^{(\tau)} \tau_{i,j} + \widetilde{a}^{(\tau^2)} \tau_{i,j}^2 + \widetilde{a}^{(m\tau)} m_{i,j} \tau_{i,j} + \widetilde{\varepsilon}_{i,j}, \tag{4}$$

for $j = 1, \ldots, J_i$ and $i = 1, \ldots, N$, with stock-specific average IV $\bar{\sigma}_i$ and error term $\widetilde{\varepsilon}_{i,j}$. This leads to all non-intercept coefficients being identical across stocks. The IVSs across stocks only differ in their levels, where each stock-specific intercept is given by $\widetilde{a}^{(0)} + \bar{\sigma}_i$.

Modeling deviations from the average IV can be seen as modeling deviations from the Black and Scholes (1973) model (BS). To see that, note first that the AHBS model nests the volatility function implied by the BS model as a particular case. By setting all parameters in Eq. (3) to zero except for $a_i^{(0)}$, we obtain a constant implied volatility model. Running regression (3) only with the intercept amounts to calculating the average IV for stock i. The prediction of the BS model is then simply $\bar{\sigma}_i$ for any option on stock i. Therefore, the pooled approach in Eq. (4) essentially assumes that deviations from Black-Scholes are the same across all stocks.

Arguably, given the observed variation in IVS features across stocks in Figure 3, the pooled AHBS is too restrictive and cannot be the optimal trade-off between estimation error and misspecification error. Motivated by the strong cross-sectional relation we document

[15] In unreported results, we confirm this conjecture when including a pooled AHBS model without stock-specific fixed effects in the prediction exercise of Section 4.

between firm characteristics and IVS slope coefficients, we introduce an endogenous pooling approach via local linear forests. The idea is to optimally pool stocks that are similar in terms of characteristics and that therefore should have similar IVS shape features. In other words, we posit that deviations from Black-Scholes are the same for stocks with similar characteristics. In Section 3.1, we combine the concept of local linear trees with the pooled AHBS model of Eq. (4) and discuss how we aggregate individual local linear trees to form our proposed AHBS-Forest model.

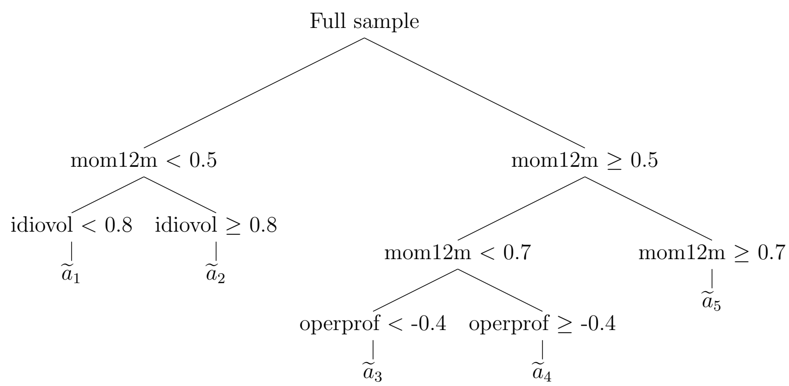
3.1. Modeling the IVS using local linear forests

We propose to model the AHBS coefficients as functions of firm characteristics by means of decision trees. The decision trees in our AHBS-Forest group stocks together based on cutoff values of our 86 firm characteristics. For instance, the estimation of a pooled AHBS model can be done separately for low-volatility stocks and for high-volatility stocks. Employing a decision tree for this partition of the firm characteristics space allows us to represent complex interactions across characteristics. To help illustrate, we provide an example of an IVS tree with local AHBS models in Figure 5. In this example, the set of splitting covariates $\mathcal{K}$ contains the standardized characteristics idiosyncratic volatility (idiovol), 12-month momentum (mom12m), and operating profitability (operprof). The final partitioning contains 5 regions with 5 different AHBS coefficient vectors $\widetilde{a}_1, \ldots, \widetilde{a}_5$.

The advantage of using trees to model AHBS coefficients instead of using trees to model the IVS directly is two-fold. First, approximating the smooth IVS via a regression tree is costly as simple regression trees are locally constant, whereas the AHBS model already captures the main features of the IVS. Second, we are able to recover the stock-specific IVS coefficients at any point in time. This leads to interpretable estimation results instead of a hard-to-interpret machine learning model. Furthermore, with the IVS coefficients, one can predict implied volatility for any moneyness and maturity.

We assume that all splitting rules in the tree $\mathcal{T}$ are based on our 86 firm characteristics.

Fig. 5: Example of a tree with AHBS models in each leaf



Notes: At each node, we decide how to go through the tree based on the value of a splitting variable; for example mom12m at the parent node. Each leaf of the tree consists of a set of coefficients that constitute the local AHBS model in this leaf.

The states of these K splitting variables for all N stocks are collected in the state matrix $Z \in \mathbb{R}^{N \times K}$. Now, $\mathcal{T}$ assigns each possible value of $Z_{i,\cdot}$, the ith row of Z,[16] one of L terminal nodes denoted by $R_1, \ldots, R_L$. The local model in terminal node R_l is given by Eq.(4) but with leaf-specific parameter vector $\widetilde{a}_l = \left(\widetilde{a}_l^{(0)}, \widetilde{a}_l^{(m)}, \widetilde{a}_l^{(m^2)}, \widetilde{a}_l^{(\tau)}, \widetilde{a}_l^{(\tau^2)}, \widetilde{a}_l^{(m\tau)}\right)^T$. Given that the splitting rules of the tree $\mathcal{T}$ are fixed and the final node assignment is determined by $Z_{i,\cdot}$ for each stock i, the tree implies a mapping $\widetilde{\mathcal{T}}(y)$, $\widetilde{\mathcal{T}} : \mathbb{R}^K \to \mathbb{R}^6$, from the vector of characteristics $Z_{i,\cdot}$ to the stock's coefficients $\widetilde{a}_l$,

$$\widetilde{\mathcal{T}}(Z_{i,\cdot}) = \left(\widetilde{\mathcal{T}}^{(0)}(Z_{i,\cdot}), \widetilde{\mathcal{T}}^{(m)}(Z_{i,\cdot}), \widetilde{\mathcal{T}}^{(m^2)}(Z_{i,\cdot}), \widetilde{\mathcal{T}}^{(\tau)}(Z_{i,\cdot}), \widetilde{\mathcal{T}}^{(\tau^2)}(Z_{i,\cdot}), \widetilde{\mathcal{T}}^{(m\tau)}(Z_{i,\cdot})\right)^T = \widetilde{a}_l,$$

if $\mathcal{T}(Z_{i,\cdot}) = R_l$. The optimal split at each node in the tree is determined via minimizing the sum of squares across all possible splits for all characteristics:

$$\min_{\tilde{c} \in \mathbb{R},\ k \in \mathcal{K}} \left(\min_{\widetilde{a}} \sum_{i | z_{i,k} < \tilde{c}} \widehat{\widetilde{\varepsilon}}_{i,j}^2 + \min_{\widetilde{a}} \sum_{i | z_{i,k} \geq \tilde{c}} \widehat{\widetilde{\varepsilon}}_{i,j}^2 \right), \tag{5}$$

where $\widehat{\widetilde{\varepsilon}}_{i,j}$ are derived from cross-sectional AHBS models with individual fixed effects (see Eq. (4)) on the subset of data at the node. Going back to our example in Figure 5, this

[16]For a given time t, the realizations $z_{i,\cdot}$ of the random vector $Z_{i,\cdot}$ are equal to $c_{i,t}$ in Section 2.

means that the coefficients of the left-most leaf are obtained from a pooled AHBS model fitted on all stocks with mom12m < 0.5 and idiovol < 0.8.

As individual decision trees are prone to overfitting, our AHBS-Forest model aggregates B individual IVS trees $\widetilde{\mathcal{T}}_1, \ldots, \widetilde{\mathcal{T}}_B$. In the fashion of random forest, each tree is constructed on a subsample of the dataset, with a random subset of splitting variables at each node. The prediction of the AHBS-Forest is then the average prediction across trees. Likewise, the vector of characteristic-dependent coefficients of the AHBS forest for stock i is given by:

$$\widetilde{\mathcal{F}}(Z_{i,\cdot}) = \left(\widetilde{\mathcal{F}}^{(0)}(Z_{i,\cdot}), \widetilde{\mathcal{F}}^{(m)}(Z_{i,\cdot}), \widetilde{\mathcal{F}}^{(m^2)}(Z_{i,\cdot}), \widetilde{\mathcal{F}}^{(\tau)}(Z_{i,\cdot}), \widetilde{\mathcal{F}}^{(\tau^2)}(Z_{i,\cdot}), \widetilde{\mathcal{F}}^{(m\tau)}(Z_{i,\cdot})\right)^T, \qquad (6)$$

with $\widetilde{\mathcal{F}}^{(\cdot)} = \frac{1}{B}\sum_{b=1}^{B} \widetilde{\mathcal{T}}_b^{(\cdot)}(Z_{i,\cdot})$. Note that taking the average coefficient across trees allows us to extract a single AHBS coefficient vector for each stock i that fully represents the stocks's model-implied IVS.

For fitting random forests, the most often used subsampling technique is bootstrap aggregation (bagging) introduced by Breiman (1996). However, we employ subsample aggregation (subagging) for two reasons. First, theoretical results regarding local linear forests (Wager and Athey, 2018) are typically derived under the assumption of subagging and honesty. Honest linear trees use one subsample of the data to choose the optimal split and the complimentary subsample to estimate the parameters of interest. Hence, our model and estimation framework are more in line with current theoretical developments for local linear random forests.[17] Second, already Bühlmann and Yu (2002) note that subagging is less computationally expensive without loss in accuracy.[18]

In our baseline analysis, we consider forests containing $B = 200$ trees per forest. We grow the tree until we reach a minimum node size of 30 or there are no more than 2 different stocks left in a node. To save computational time, we only consider the splitting variable's

[17]Given the cross-sectional structure of our data, we do not sample randomly the entire data but we sample stocks identifiers only and divide the dataset according to these identifiers. This means that we have two non-overlapping datasets that contain observations for the same number of stocks but not necessarily the same number of observations. Our argument for this is the lack of balance in observations per stock.

[18]Using bagging instead of subagging leads to negligible differences in prediction performance in Section 4.

5%-, 5.25%-, ..., 94.75%-, 95%-quantiles as potential splitting points. Cutting of the splitting's variable grid at the 5%- and 95%-quantiles leads to less extreme splits at the top of the tree, which reduces the number of trees needed to obtain good approximations of the characteristic-dependent parameters. To counterbalance sensitivity to the number of trees grown, we employ a trimmed mean aggregation across trees with trimming cutoffs at the 5%-quantile and 95%-quantile. As decorrelated trees help reduce the variance of the averaging step, we follow Goulet Coulombe (2021) and draw random subsets of features $\mathcal{K}^{-} \subset \mathcal{K}$ with subsampling rate 1/3 instead of $\mathcal{K}$ in Eq. (5). In supplementary appendix B, we report additional results demonstrating the robustness of our analysis to these modeling choices.

4. Empirical results for IVS forests

In this section, we analyze to what extent our approach provides an accurate modeling of the IVS across stocks by considering an out-of-sample prediction exercise with different benchmarks. Moreover, we examine the nonlinear interactions between characteristics and option prices implied by our local linear random forest model.

4.1. Implementation details of the prediction exercise

For each day in our sample, we use half of the option data of each stock in the estimation, and predict the other half in the same day. This tells us how well our model generalizes to unobserved implied volatilities coming from the same IVS. In this exercise, we mimic the situation where one observes only a few liquid options for each stock and uses this information to price other options on the stock on the same day. For this, we randomly split the option observations per stock in two equal-sized groups. The first group of observations is used to fit our models (i.e., the estimation sample) and the second group of observations is used to evaluate the implied volatility predictions (i.e., the test sample).[19] To be able to include

[19] For all models, we incorporate the sanity filter of Dumas et al. (1998), that takes the minimum between the model prediction and 0.01, to avoid the very rare cases that an implied volatility is predicted to be

the stock-specific univariate AHBS models as benchmarks in our prediction exercise, all evaluation is done on our liquid sample, that is, on stocks that have at least 20 traded options on each end-of-month t.

Our measure of prediction performance is the implied volatility root mean squared error $IVRMSE^j_{i,t}$, which is calculated every month t for stock i and model j. This is aligned with the loss function that the models we consider minimize in the estimation. We aggregate the performance across stocks in three complementary ways. First, we simply compute the average IVRMSE across stocks for a given model. Second, we calculate how often model j outperforms the benchmark AHBS model in the cross-section of stocks with the outperformance rate:

$$OR^j_t = \frac{1}{N}\sum_{i=1}^{N} \mathbb{1}(IVRMSE^j_{i,t} < IVRMSE^{AHBS}_{i,t}).$$

Third, we consider the median loss ratio of model j relative to the benchmark AHBS:

$$MedLR^j_t = \underset{i=1,\ldots,N}{\text{median}} \frac{IVRMSE^j_{i,t}}{IVRMSE^{AHBS}_{i,t}}.$$

To put the performance of our approach into perspective, we include several variations of the AHBS model in our prediction exercise. The main benchmark is the stock-specific AHBS model. Even under the assumption of it being the true model, the stock-specific AHBS model is prone to high estimation risk if we have few option data per stock. On the other hand, we also include the pooled AHBS model in Eq. (4) that has low estimation uncertainty but a large misspecification error by construction. Our AHBS-Forest can be seen as bridging these two models by optimizing the misspecification-estimation uncertainty trade-off using characteristics to define which stocks are pooled together. Furthermore, we consider additional stock-specific AHBS models with reduced specifications aiming at lowering estimation uncertainty (at the risk of a more severe model misspecification). We

negative. This filter has no effect on our empirical results.

include: the Black-Scholes (BS) model; the reduced AHBS model without higher-order terms from Section 2; an AHBS model without moneyness terms; and an AHBS model without time to maturity terms. These benchmarks will help assess the relative importance of each of the IVS coefficients.

Finally, we implement four variations of our AHBS-Forest, differing in the specification at the terminal nodes of the trees (reduced AHBS without higher-order terms vs. full AHBS specification) and on whether we employ data in the estimation that is not part of the liquid sample (liquid vs. all).[20] To shed light on the added value of accounting for nonlinear interactions between firm characteristics and IVS coefficients with our random forests, we also consider a final benchmark, AHBS (pooled) Interaction, that specifies the coefficients in the pooled model in Eq. (4) to be linear functions of characteristics.[21]

4.2. Prediction results

Table 2 presents the time-series averages of the cross-sectional average IVRMSE, out-performance rate (OR) and median loss ratio (MedLR) for each model, based on different subsets of stocks.[22] Panel A shows results for all stocks in the liquid sample, while Panels B and C focus on subsets with a specific number of traded options: stocks with at least 50 traded options ("$n > 50$") and with in between 20 and 50 traded options ("$50 \geq n \geq 20$"), respectively. The first row of each panel also includes the average number of stocks in these subsets over the time period of January 2000 to November 2021. Around 2/3 of the stocks in our liquid sample have less than 50 traded options per end-of-month.

Focusing first on the aggregate performance in Panel A, the BS model has the worst combination of OR and MedLR, which is not surprising given its well-known moneyness and

[20]While we require at least 20 options for fitting univariate AHBS models, a natural question is whether our AHBS-Forest allows to ease this restriction. Note that all models are evaluated on the liquid sample.

[21]That is, $\widetilde{\alpha}^{(\cdot)} = \widetilde{\alpha}^{(\cdot)}\tilde{c}_i$, where $\widetilde{\alpha}^{(\cdot)}$ is a vector of length 87 and $\tilde{c}_i = (1, c_i)$ is a vector containing the 86 characteristics of stock i. The model can thus be seen as a pooled AHBS with interaction effects for the characteristics: $\sigma_{i,j} - \bar{\sigma}_i = \widetilde{\alpha}^{(0)}\tilde{c}_i + \widetilde{\alpha}^{(m)}\tilde{c}_i m_{i,j} + \widetilde{\alpha}^{(m^2)}\tilde{c}_i m_{i,j}^2 + \widetilde{\alpha}^{(\tau)}\tilde{c}_i \tau_{i,j} + \widetilde{\alpha}^{(\tau^2)}\tilde{c}_i \tau_{i,j}^2 + \widetilde{\alpha}^{(m\tau)}\tilde{c}_i m_{i,j}\tau_{i,j} + \widetilde{\varepsilon}_{i,j}$.

[22]Our prediction results are stable over time, as can be seen with the monthly performance metrics of our main specification depicted in Figure B.1 of the supplementary appendix.

Table 2: Prediction performance in the cross-section of equity options

	No. Obs.	IVRMSE	OR	MedLR
Panel A: All	564.046			
AHBS (benchmark)		0.106	—	—
BS		0.078	0.374	1.221
AHBS (reduced)		0.064	0.541	0.967
AHBS (maturity only)		0.097	0.359	1.195
AHBS (moneyness only)		0.076	0.451	1.052
AHBS (pooled)		0.063	0.508	0.992
AHBS (pooled) Interaction		0.064	0.500	1.001
AHBS-Forest (reduced, liquid)		0.063	0.508	0.993
AHBS-Forest (liquid)		0.060	0.550	0.955
AHBS-Forest (reduced, all)		0.063	0.516	0.984
AHBS-Forest (all)		**0.060**	**0.562**	**0.942**
Panel B: $n > 50$	218.985			
AHBS (benchmark)		0.073	—	—
BS		0.083	0.212	1.426
AHBS (reduced)		0.064	0.380	1.063
AHBS (maturity only)		0.082	0.238	1.310
AHBS (moneyness only)		0.074	0.298	1.146
AHBS (pooled)		0.066	0.369	1.095
AHBS (pooled) Interaction		0.064	0.386	1.067
AHBS-Forest (reduced, liquid)		0.067	0.344	1.118
AHBS-Forest (liquid)		0.064	0.424	1.051
AHBS-Forest (reduced, all)		0.067	0.350	1.113
AHBS-Forest (all)		**0.064**	**0.427**	**1.051**
Panel C: $50 \geq n \geq 20$	345.061			
AHBS (benchmark)		0.127	—	—
BS		0.075	0.459	1.095
AHBS (reduced)		0.063	0.625	0.875
AHBS (maturity only)		0.106	0.422	1.131
AHBS (moneyness only)		0.078	0.531	0.981
AHBS (pooled)		0.061	0.582	0.895
AHBS (pooled) Interaction		0.063	0.563	0.922
AHBS-Forest (reduced, liquid)		0.060	0.595	0.881
AHBS-Forest (liquid)		0.058	0.615	0.858
AHBS-Forest (reduced, all)		0.060	0.604	0.870
AHBS-Forest (all)		**0.058**	**0.630**	**0.839**

Notes: We report (Panel A) time-series averages of different monthly performance measures: the cross-sectional average IVRMSE, the outperformance rate (OR) defined as the share of stocks for which the models achieve a lower IVRMSE than the AHBS model (i.e., higher is better), and the monthly median loss ratio (MedLR) (i.e., lower is better). Panel B/C report the same performance measures for the subsets of stocks with more than 50 trades/at most 50 trades per time period. All models are evaluated on the liquid sample; that is, the sample containing only those stocks per month for which we observe at least 20 option trades. AHBS-Forest (all) and AHBS-Forest (reduced, all) are the only models that include non-liquid observations during estimation. The monthly data spans 2000:M1 until 2021:M11. 2021:M12 is missing due to the availability of stock characteristics.

time to maturity biases. Those deviations are precisely what the other benchmarks and our approach aim to capture. The reduced AHBS model performs better than the full AHBS model for 54.1% of stocks and has a MedLR below one, reflecting the estimation uncertainty associated with the inclusion of higher-order IVS coefficients in a stock-specific model. The severe estimation risk of the AHBS is also evident from the OR of the pooled AHBS, which is greater than 50%. In fact, while the AHBS model is difficult to beat for a large part of the stocks, it has the highest average IVRMSE among all models. This highlights the lack of robustness of this model, as for the cross-sections of options that it fails to perform well, the pricing errors can be large. The two restricted models that contain either only time to maturity or moneyness terms perform consistently worse than the AHBS across stocks, suggesting that it is important to account for both moneyness and term structure effects.

Regarding the AHBS-Forests, there are three main takeaways. First, in contrast to the stock-specific models, it is not beneficial to employ the reduced AHBS as the local linear model. The endogenous pooling and the resulting larger sample sizes are enough to reduce estimation uncertainty, such that a simpler specification is not needed. Second, we observe that including additional data in the estimation from stocks with less than 20 option observations slightly improves the performance of the AHBS-Forest. Third, and most important, the AHBS-Forest (all) is the model with highest OR and smallest MedLR and IVRMSE. That is, our endogenous pooling based on characteristics offers the best misspecification-estimation error trade-off among the considered models. In particular, our approach outperforms the AHBS (pooled) Interaction model, which defines coefficients as a linear function of characteristics. This suggests that accounting for nonlinearities in the relation between characteristics and option prices is paramount for an accurate cross-sectional prediction.

Panels B and C of Table 2 provide further insights into the relative pricing performance of the models. For stocks with at least 50 observed options, the AHBS model is more difficult to beat. This is consistent with the fact that estimation uncertainty is lower given larger sample sizes. The AHBS-Forest (all), which has the highest OR and lowest MedLR, only outperforms the AHBS for 42.7% of the stocks in this subset. Even so, the IVRMSE associated with the AHBS-Forest (all) is still smaller than the benchmark, reinforcing the robustness of our approach. As for the stocks with limited option data between 50 and 20 observations, the performance of the AHBS model is substantially worse, with an average IVRMSE of 12.7%. In contrast, our method has even smaller pricing errors, outperforms the AHBS model for more than 60% of the stocks and has a MedLR of 0.839, being again the best model overall. These results indicate that the outperformance of the AHBS-Forest (all) in Panel A is mainly driven by stocks that have relatively illiquid cross-sections of options, which are the majority in the option market. For these stocks, pooling option information from stocks with similar characteristics is extremely helpful.

In sum, we provide compelling empirical evidence in support of our local linear random forest model. Our approach outperforms the stock-specific AHBS model across all aggregate performance metrics. The superior performance is mainly attributed to stocks with limited option observations, which constitute around 2/3 of the total number of stocks in the option market. For the few stocks with ample option data, a stock-specific approach can be preferred, but our method is still more robust in terms of smaller average IVRMSE. The AHBS-Forest also performs better than alternative models that aim to reduce estimation uncertainty, such as the unconditional pooled model. This further reinforces the explanatory power of firm characteristics for cross-sectional differences in the IVS. Importantly, nonlinearities captured by the random forest are fundamental for its strong performance.

4.3. The nonlinear effects of firm characteristics on IVS coefficients

To shed light on the nonlinear dependencies between characteristics and IVS features captured by the AHBS-Forest, we apply a model-agnostic technique, called Accumulated Local Effect (ALE) plots (Apley and Zhu, 2020), that is more robust to correlated regressors than partial dependence plots. The ALE can be interpreted as the main local effect of a covariate on the AHBS-Forest coefficient compared to the average coefficient of the AHBS-

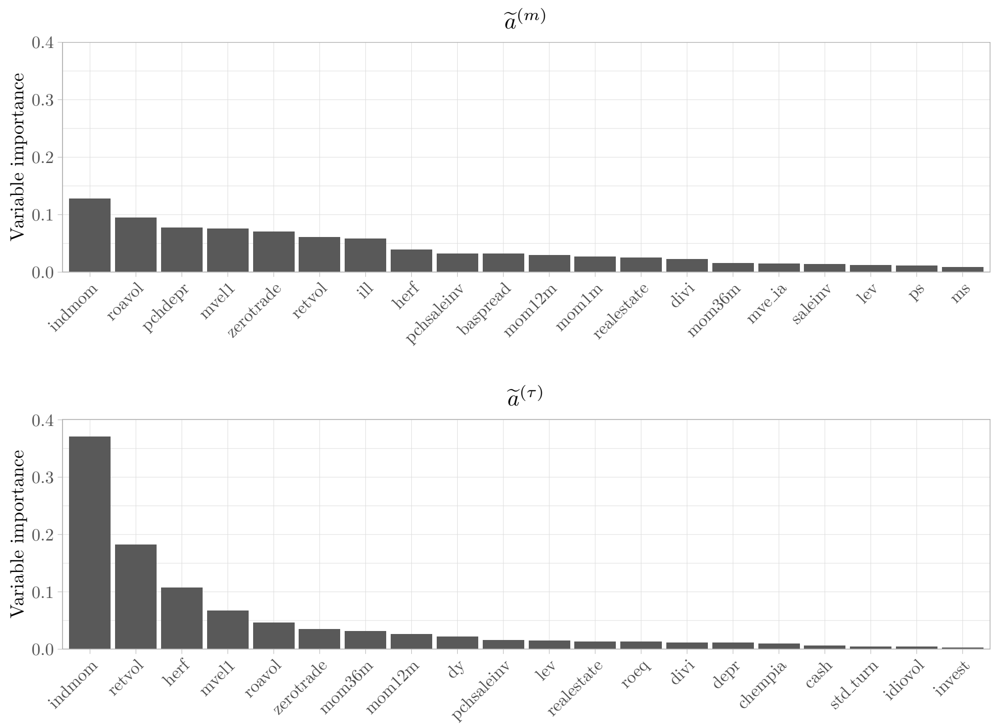
Forest. For example, an ALE value of -0.1 for idiovol = 0.5 means that—conditional on idiovol = 0.5—the coefficient is smaller by 0.1 than the average AHSB-Forest coefficient. Section C of the supplementary appendix provides further details about ALE plots.

Now, let $\widehat{ALE}_k(\xi_m)$ denote the estimated ALE function of variable $k \in \mathcal{K}$ for one of the IVS forest coefficients. As a measure of variable importance (VI) that is directly linked to the ALE plots, we calculate the standard deviation $VI_k = \sqrt{\frac{1}{M-1}\sum_{m=1}^{M}\left(\widehat{ALE}_k(\xi_m)\right)^2}$ across a grid $[\xi_0, \xi_1], (\xi_1, \xi_2], \ldots, (\xi_{M-1}, \xi_M]$, which we choose to be [-1, -0.9], (-0.9, -0.8], ..., (0.9, 1]. As we estimate a different AHBS-Forest every end-of-month, we report the average variable importance and the average ALE function across months in the following Figures 6 to 7.

We begin by analyzing which characteristics are important for the AHBS-Forest. Figure 6 displays the top 20 characteristics for the moneyness and term structure coefficients, $\widetilde{a}^{(m)}$ and $\widetilde{a}^{(\tau)}$, according to our variable importance measure. That is, we focus on how first-order deviations from Black-Scholes across stocks depend on firm characteristics. Of particular interest are the five most important variables: industry momentum (indmom; Moskowitz and Grinblatt, 1999), return volatility (retvol; Ang et al., 2006), earnings volatility (roavol; Francis et al., 2004), percentage change in depreciation (pchdepr; Holthausen and Larcker, 1992), and industry sales concentration (herf; Hou and Robinson, 2006). We interpret below the results for indmom, and the remaining ones in Section B.1 of the supplementary appendix.

Interestingly, indmom is the most influential variable in our AHBS-Forest model. This seems to contradict our Fama-MacBeth regression analysis in Section 2, in which indmom was not among the most significant characteristics for either the slope coefficient $a^{(m)}$ or the term structure coefficient $a^{(\tau)}$. However, in the local linear random forest, indmom might serve different purposes. First, as the AHBS-Forest model includes stocks with less than 20 traded options, it encompasses a wider range of relatively smaller firms compared to the Fama-MacBeth regressions. For these firms, spillover effects from their industry might act as a more reliable price trend than the momentum of the firm itself. Second, even in the absence of significant spillover effects, indmom may serve as an implicit grouping variable

Fig. 6: Variable importance based on accumulated local effects



Notes: We present the average importance of firm characteristics for the slope and the term structure as inferred by the AHBS-Forest model. These averages are calculated from the variable importance measures of each monthly AHBS-Forest model. Our method of determining the importance of variables is based on accumulated local effects, as discussed in Section 4.3. The importance measure of each variable is standardized to ensure that the sum across all variables equals 1. We truncate the histogram at the 20 most important features per coefficient. The monthly data spans 2000:M1 until 2021:M11. 2021:M12 is missing due to the availability of stock characteristics.

for industry classification if the trees run deep enough. In particular, given the partitioning feature of decision trees, indmom may play an important role by allowing to differentiate the effects of other characteristics across industries.

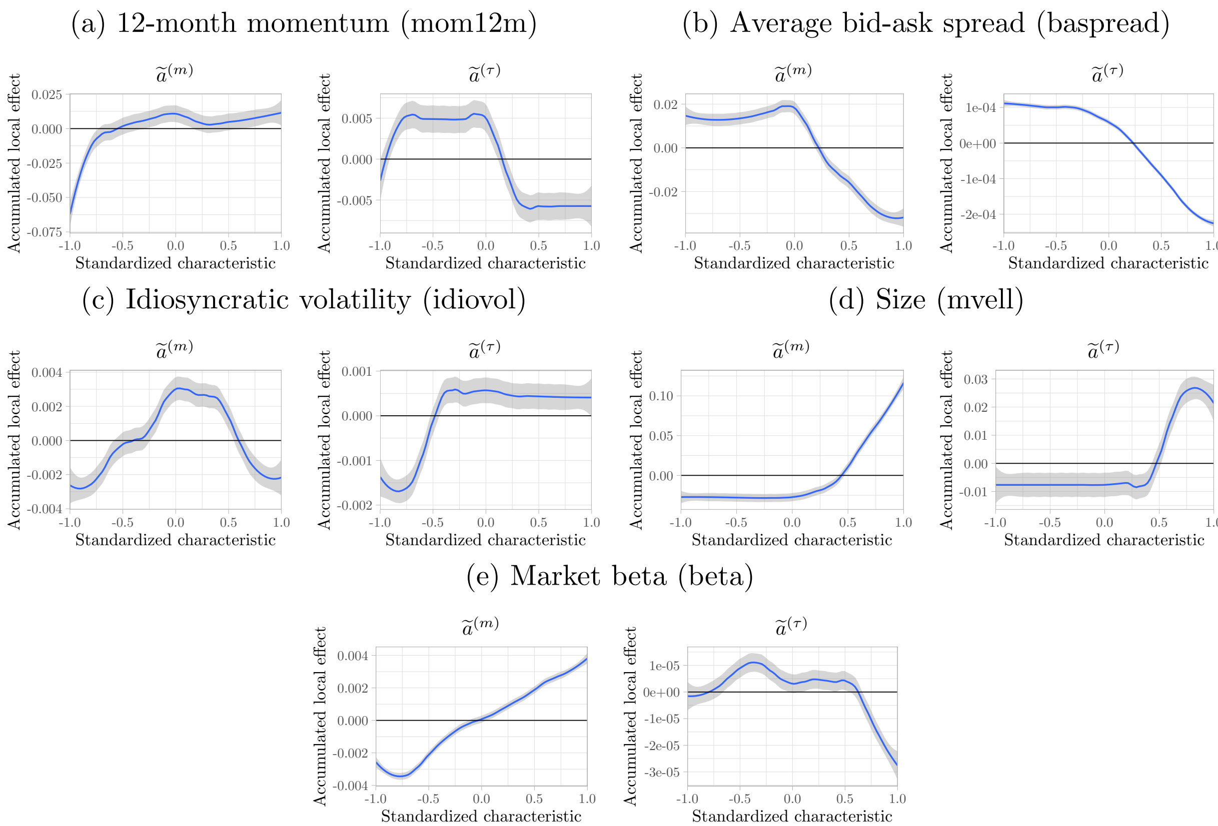
In Figure 7, we further plot the accumulated local effects of characteristics that were strongly significant in the Fama-MacBeth regressions of Section 2.[23] The ALE functions and corresponding 90% confidence intervals depicted are based on monthly ALE functions that are aggregated via local linear regression (Cleveland and Devlin, 1988).[24] Since the effects of these characteristics are of different magnitudes, the scale of the plot is different

[23]Most of these characteristics also feature prominently in Figure 6 as important variables for the AHBS-Forest model. Section B.1 of the supplementary appendix further analyzes the local effects of other top characteristics according to the variable importance measure.

[24]Our confidence intervals can only be seen as rough approximations as we do not take the functional nature of monthly ALE curves nor the measurement error in the monthly ALE curves into account. However, to the best of our knowledge, there are no rigorous statistical tests for ALE plots as of now.

for each panel. With that, we focus on the shape of the nonlinear local effects rather than their relative importance. The first panel confirms the Fama-MacBeth result that higher momentum is associated with a higher moneyness slope and thus higher cost of protection against downside risk. However, the ALE plot also shows that this effect is mostly concentrated on low-momentum stocks, i.e., investors require less downside risk protection for stocks that already performed poorly in the recent past. In contrast, the net effect of momentum on the term structure is close to zero, consistent with the fact that mom12m is insignificant for this coefficient in the cross-sectional regression approach.

Fig. 7: ALE plots for influential characteristics in Fama-MacBeth regressions



Notes: In this figure, we plot ALE functions as defined in Eq (7) for five characteristics that have been identified as influential in Fama-MacBeth regressions in Section 2. The ALE functions and the corresponding 90% confidence intervals depicted are based on monthly ALE functions from our monthly re-estimation scheme that are aggregated via local linear regression (Cleveland and Devlin, 1988). The monthly data spans 2000:M1 until 2021:M11. 2021:M12 is missing due to the availability of stock characteristics.

Panel (b) illustrates the patterns for baspread. Again, the ALE plots confirm the evidence from Fama-MacBeth regressions of a negative relation between baspread and the moneyness slope and term structure, particularly in stocks with above-average baspread. The negative effect of idiovol on $a^{(m)}$ obtained from the cross-sectional regression is also confirmed in Panel (c). However, the ALE plot reveals a highly nonlinear relation, where the local effect is actually positive until the midrange of idiovol, and then decreases for higher levels of the characteristic. For the size effect depicted in Panel (d), there is a positive relation for both IVS coefficients, again aligned with results in Section 2. The plots show that the effect is mostly flat for smaller stocks, and increasing for stocks with above-average market equity. Finally, we can see from Panel (e) that the relation between market beta and the moneyness slope is positive and approximately linear. While this supports the Fama-MacBeth evidence, the same is not true for the term structure coefficient, where the ALE plot displays an overall negative relation. Even so, the magnitude of the effect is small, indicating that beta is not relevant for $a^{(\tau)}$ in the AHBS-Forest.

5. Conclusion

We document that many of the firm characteristics that have been previously identified as explaining the cross-section of stock returns also explain cross-sectional differences in equity option prices. To further explore this strong relation, we investigate whether information from firm characteristics can be useful for pricing equity options. We model the IVS of a given stock as a function of its characteristics by using a local linear random forest, which endogenously pools option information from similar equities to estimate the IVS. This method overcomes the illiquidity of equity options and outperforms different stock-specific models in pricing options out-of-sample. By using our estimated model, we provide new evidence of the complex, nonlinear interactions between firm characteristics and option prices.

References

Aït-Sahalia, Y., Li, C., and Li, C. X. (2021). Implied stochastic volatility models. *Review of Financial Studies*, 34(1):394–450.

Aït-Sahalia, Y. and Lo, A. W. (1998). Nonparametric estimation of state-price densities implicit in financial asset prices. *Journal of Finance*, 53(2):499–547.

Almeida, C., Fan, J., Freire, G., and Tang, F. (2023a). Can a machine correct option pricing models? *Journal of Business and Economic Statistics*, 41(3):995–1009.

Almeida, C. and Freire, G. (2022). Pricing of index options in incomplete markets. *Journal of Financial Economics*, 144(1):174–205.

Almeida, C., Freire, G., Azevedo, R., and Ardison, K. (2023b). Nonparametric option pricing with generalized entropic estimators. *Journal of Business and Economic Statistics*, 41(4):1173–1187.

An, B.-J., Ang, A., Bali, T., and Cakici, N. (2014). The joint cross section of stocks and options. *Journal of Finance*, 69(5):2279–2337.

Andersen, T. G., Fusari, N., and Todorov, V. (2015). The risk premia embedded in index options. *Journal of Financial Economics*, 117(3):558–584.

Andersen, T. G., Fusari, N., and Todorov, V. (2017). Short-term market risks implied by weekly options. *Journal of Finance*, 72(3):1335–1386.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006). The cross-section of volatility and expected returns. *Journal of Finance*, 61(1):259–299.

Apley, D. W. and Zhu, J. (2020). Visualizing the effects of predictor variables in black box supervised learning models. *Journal of the Royal Statistical Society. Series B: Statistical Methodology*, 82(4):1059–1086.

Athey, S., Tibshirani, J., and Wager, S. (2019). Generalized random forests. *Annals of Statistics*, 47(2):1179–1203.

Bakshi, G., Cao, C., and Chen, Z. (1997). Empirical performance of alternative option pricing models. *Journal of Finance*, 52(5):2003–2049.

Bakshi, G., Cao, C., and Zhong, Z. (2021). Assessing models of individual equity option prices. *Review of Quantitative Finance and Accounting*, 57(1):1–28.

Bakshi, G., Kapadia, N., and Madan, D. (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *Review of Financial Studies*, 16(1):101–143.

Barone-Adesi, G., Engle, R. F., and Mancini, L. (2008). A GARCH option pricing model with filtered historical simulation. *Review of Financial Studies*, 21(3):1223–1258.

Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654.

Bollen, N. P. and Whaley, R. E. (2004). Does net buying pressure affect the shape of implied volatility functions? *Journal of Finance*, 59(2):711–753.

Bollerslev, T. and Todorov, V. (2011). Tails, fears, and risk premia. *Journal of Finance*, 46(6):2165–2211.

Breiman, L. (1996). Bagging predictors. *Machine Learning*, 24(2):123–140.

Breiman, L. (2001). Random forests. *Machine Learning*, 45(1):5–32.

Bryzgalova, S., Pelger, M., and Zhu, J. (2021). Forest through the trees: Building cross-sections of stock returns. *Available at SSRN: 3493458*.

Bühlmann, P. and Yu, B. (2002). Analyzing bagging. *Annals of Statistics*, 30(4):927–961.

Chen, D., Guo, B., and Zhou, G. (2023). Firm fundamentals and the cross-section of implied volatility shapes. *Journal of Financial Markets*, 63:100771.

Christoffersen, P., Fournier, M., and Jacobs, K. (2018). The factor structure in equity options. *Review of Financial Studies*, 31(2):595–637.

Christoffersen, P. and Jacobs, K. (2004). Which GARCH model for option valuation? *Management Science*, 50(9):1204–1221.

Cleveland, W. S. and Devlin, S. J. (1988). Locally weighted regression: An approach to regression analysis by local fitting. *Journal of the American Statistical Association*, 83(403):596–610.

Daniel, K. and Moskowitz, T. J. (2016). Momentum crashes. *Journal of Financial Economics*, 122(2):221–247.

Dennis, P. and Mayhew, S. (2002). Risk-neutral skewness: Evidence from stock options. *The Journal of Financial and Quantitative Analysis*, 37(3):471.

Duan, J. C. and Wei, J. (2009). Systematic risk and the price structure of individual equity options. *Review of Financial Studies*, 22(5):1981–2006.

Dumas, B., Fleming, J., and Whaley, R. E. (1998). Implied volatility functions : Empirical tests. *Journal of Finance*, 53(6):2059–2106.

Fama, E. F. and MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy*, 81(3):607–636.

Francis, J., LaFond, R., Olsson, P. M., and Schipper, K. (2004). Costs of equity and earnings attributes. *Accounting Review*, 79(4):967–1010.

Friedberg, R., Tibshirani, J., Athey, S., and Wager, S. (2020). Local linear forests. *Journal of Computational and Graphical Statistics*, 30(2):503–517.

Garcia, R. and Gençay, R. (2000). Pricing and hedging derivative securities with neural networks and a homogeneity hint. *Journal of Econometrics*, 94(1-2):93–115.

Gonçalves, S. and Guidolin, M. (2006). Predictable dynamics in the S&P 500 index options implied volatility surface. *Journal of Business*, 79(3):1591–1635.

Goulet Coulombe, P. (2021). The macroeconomy as a random forest. *Available at SSRN: 3633110*.

Goyal, A. and Saretto, A. (2009). Cross-section of option returns and volatility. *Journal of Financial Economics*, 94(2):310–326.

Green, J., Hand, J. R., and Zhang, X. F. (2017). The characteristics that provide independent information about average U.S. monthly stock returns. *Review of Financial Studies*, 30(12):4389–4436.

Gu, S., Kelly, B., and Xiu, D. (2020). Empirical asset pricing via machine learning. *The Review of Financial Studies*, 33(5):2223–2273.

Harvey, C. R., Liu, Y., and Zhu, H. (2016). ... and the cross-section of expected returns. *Review of Financial Studies*, 29(1):5–68.

Herskovic, B., Kelly, B., Lustig, H., and Nieuwerburgh, S. V. (2020). Firm volatility in granular networks. *Journal of Political Economy*, 128(11):4097–4162.

Heston, S. L. and Nandi, S. (2000). A closed-form GARCH option valuation model. *Review of Financial Studies*, 13(3):585–625.

Holthausen, R. W. and Larcker, D. F. (1992). The prediction of stock returns using financial statement information. *Journal of Accounting and Economics*, 15(2-3):373–411.

Hou, K. and Robinson, D. T. (2006). Industry concentration and average stock returns. *Journal of Finance*, 61(4):1927–1956.

Hutchinson, J. M., Lo, A. W., and Poggio, T. (1994). A nonparametric approach to pricing and hedging derivative securities via learning networks. *The Journal of Finance*, 49(3):851–889.

Ilhan, E., Sautner, Z., and Vilkov, G. (2021). Carbon tail risk. *Review of Financial Studies*, 34(3):1540–1571.

Jackwerth, J. C. and Rubinstein, M. (1996). Recovering probability distributions from option prices. *Journal of Finance*, 51(5):1611–1631.

Kelly, B., Pástor, Ľ., and Veronesi, P. (2016). The price of political uncertainty: Theory and evidence from the option market. *Journal of Finance*, 71(5):2417–2480.

Kleen, O. and Tetereva, A. (2022). A forest full of risk forecasts for managing volatility. *Available at SSRN: 4161957*.

Moskowitz, T. J. and Grinblatt, M. (1999). Do industries explain momentum? *Journal of Finance*, 54(4):1249–1290.

Toft, K. B. and Prucyk, B. (1997). Options on leveraged equity: Theory and empirical tests. *The Journal of Finance*, 52(3):1151.

Vasquez, A. (2017). Equity volatility term structures and the cross section of option returns. *Journal of Financial and Quantitative Analysis*, 52(6):2727–2754.

Wager, S. and Athey, S. (2018). Estimation and inference of heterogeneous treatment effects using random forests. *Journal of the American Statistical Association*, 113(523):1228–1242.

Zhan, X. E., Han, B., Cao, J., and Tong, Q. (2022). Option return predictability. *The Review of Financial Studies*, 35(3):1394–1442.

Equity Option Prices and Firm Characteristics

– Supplementary Appendix –

A. Additional results for Section 2

Fig. A.1: Fama-MacBeth regressions for AHBS coefficients on firm characteristics

Notes: Alternative version of Figure 4 which employs the full AHBS model (see Eq. (3)) instead of the reduced AHBS model (see Eq. (1)). This figure reports the outcome of the two-stage Fama-Macbeth cross-sectional analysis as detailed in Section 2.2. In the first stage, we run monthly cross-sectional regressions of IVS coefficients for each stock with at least 20 traded options on firm characteristics. In the second stage, we regress the monthly slope parameters per characteristic on a constant. The magnitude of the second-stage t-statistics are highlighted if $|t|> 3$. t-statistics below -3 are depicted in blue and t-statistics above 3 are depicted in red. We employ standard errors based on Newey-West covariance estimates with 3 lags. The IVS coefficients are winsorized monthly at the 1%- and 99%-cross-sectional quantile prior to running the Fama-MacBeth regressions. The sample ranges from January, 2000 to November, 2021. December 2021 is missing due to the availability of stock characteristics.

B. Additional results for Section 4

Table B.1: Same setup as in Table 2 but the forest uses the median instead of the trimmed mean in the prediction aggregation

	No. Obs.	IVRMSE	OR	MedLR
Panel A: All	564.046			
AHBS (benchmark)		0.106	—	—
BS		0.078	0.374	1.221
AHBS (reduced)		0.064	0.541	0.967
AHBS (maturity only)		0.097	0.359	1.195
AHBS (moneyness only)		0.076	0.451	1.052
AHBS (pooled)		0.063	0.508	0.992
AHBS (pooled) Interaction		0.064	0.500	1.001
AHBS-Forest (reduced, liquid)		0.063	0.511	0.990
AHBS-Forest (liquid)		0.060	0.558	0.948
AHBS-Forest (reduced, all)		0.062	0.519	0.981
AHBS-Forest (all)		**0.059**	**0.572**	**0.934**
Panel B: $n > 50$	218.985			
AHBS (benchmark)		0.073	—	—
BS		0.083	0.212	1.426
AHBS (reduced)		0.064	0.380	1.063
AHBS (maturity only)		0.082	0.238	1.310
AHBS (moneyness only)		0.074	0.298	1.146
AHBS (pooled)		0.066	0.369	1.095
AHBS (pooled) Interaction		0.064	0.386	1.067
AHBS-Forest (reduced, liquid)		0.067	0.347	1.115
AHBS-Forest (liquid)		0.063	0.432	1.046
AHBS-Forest (reduced, all)		0.067	0.352	1.112
AHBS-Forest (all)		**0.063**	**0.439**	**1.041**
Panel C: $50 \geq n \geq 20$	345.061			
AHBS (benchmark)		0.127	—	—
BS		0.075	0.459	1.095
AHBS (reduced)		0.063	0.625	0.875
AHBS (maturity only)		0.106	0.422	1.131
AHBS (moneyness only)		0.078	0.531	0.981
AHBS (pooled)		0.061	0.582	0.895
AHBS (pooled) Interaction		0.063	0.563	0.922
AHBS-Forest (reduced, liquid)		0.060	0.598	0.877
AHBS-Forest (liquid)		0.058	0.622	0.850
AHBS-Forest (reduced, all)		0.060	0.607	0.867
AHBS-Forest (all)		**0.057**	**0.639**	**0.832**

Notes: We report (Panel A) time-series averages of different monthly performance measures: the cross-sectional average IVRMSE, the outperformance rate (OR) defined as the share of stocks for which the models achieve a lower IVRMSE than the AHBS model (i.e., higher is better), and the monthly median loss ratio (MedLR) (i.e., lower is better). Panel B/C report the same performance measures for the subsets of stocks with more than 50 trades/at most 50 trades per time period. All models are evaluated on the liquid sample; that is, the sample containing only those stocks per month for which we observe at least 20 option trades. AHBS-Forest (all) and AHBS-Forest (reduced, all) are the only models that include non-liquid observations during estimation. The monthly data spans 2000:M1 until 2021:M11. 2021:M12 is missing due to the availability of stock characteristics.

Table B.2: Same setup as in Table 2 but the forest uses a feature subsampling rate of 1/2 instead of 1/3

	No. Obs.	IVRMSE	OR	MedLR
Panel A: All	564.046			
AHBS (benchmark)		0.106	—	—
BS		0.078	0.374	1.221
AHBS (reduced)		0.064	0.541	0.967
AHBS (maturity only)		0.097	0.359	1.195
AHBS (moneyness only)		0.076	0.451	1.052
AHBS (pooled)		0.063	0.508	0.992
AHBS (pooled) Interaction		0.064	0.500	1.001
AHBS-Forest (reduced, liquid)		0.063	0.507	0.994
AHBS-Forest (liquid)		0.061	0.547	0.957
AHBS-Forest (reduced, all)		0.063	0.515	0.985
AHBS-Forest (all)		**0.060**	**0.559**	**0.945**
Panel B: $n > 50$	218.985			
AHBS (benchmark)		0.073	—	—
BS		0.083	0.212	1.426
AHBS (reduced)		0.064	0.380	1.063
AHBS (maturity only)		0.082	0.238	1.310
AHBS (moneyness only)		0.074	0.298	1.146
AHBS (pooled)		0.066	0.369	1.095
AHBS (pooled) Interaction		0.064	0.386	1.067
AHBS-Forest (reduced, liquid)		0.067	0.342	1.117
AHBS-Forest (liquid)		**0.064**	**0.420**	**1.054**
AHBS-Forest (reduced, all)		0.067	0.349	1.114
AHBS-Forest (all)		0.064	0.422	1.055
Panel C: $50 \geq n \geq 20$	345.061			
AHBS (benchmark)		0.127	—	—
BS		0.075	0.459	1.095
AHBS (reduced)		0.063	0.625	0.875
AHBS (maturity only)		0.106	0.422	1.131
AHBS (moneyness only)		0.078	0.531	0.981
AHBS (pooled)		0.061	0.582	0.895
AHBS (pooled) Interaction		0.063	0.563	0.922
AHBS-Forest (reduced, liquid)		0.061	0.594	0.881
AHBS-Forest (liquid)		0.059	0.613	0.860
AHBS-Forest (reduced, all)		0.060	0.602	0.870
AHBS-Forest (all)		**0.058**	**0.628**	**0.842**

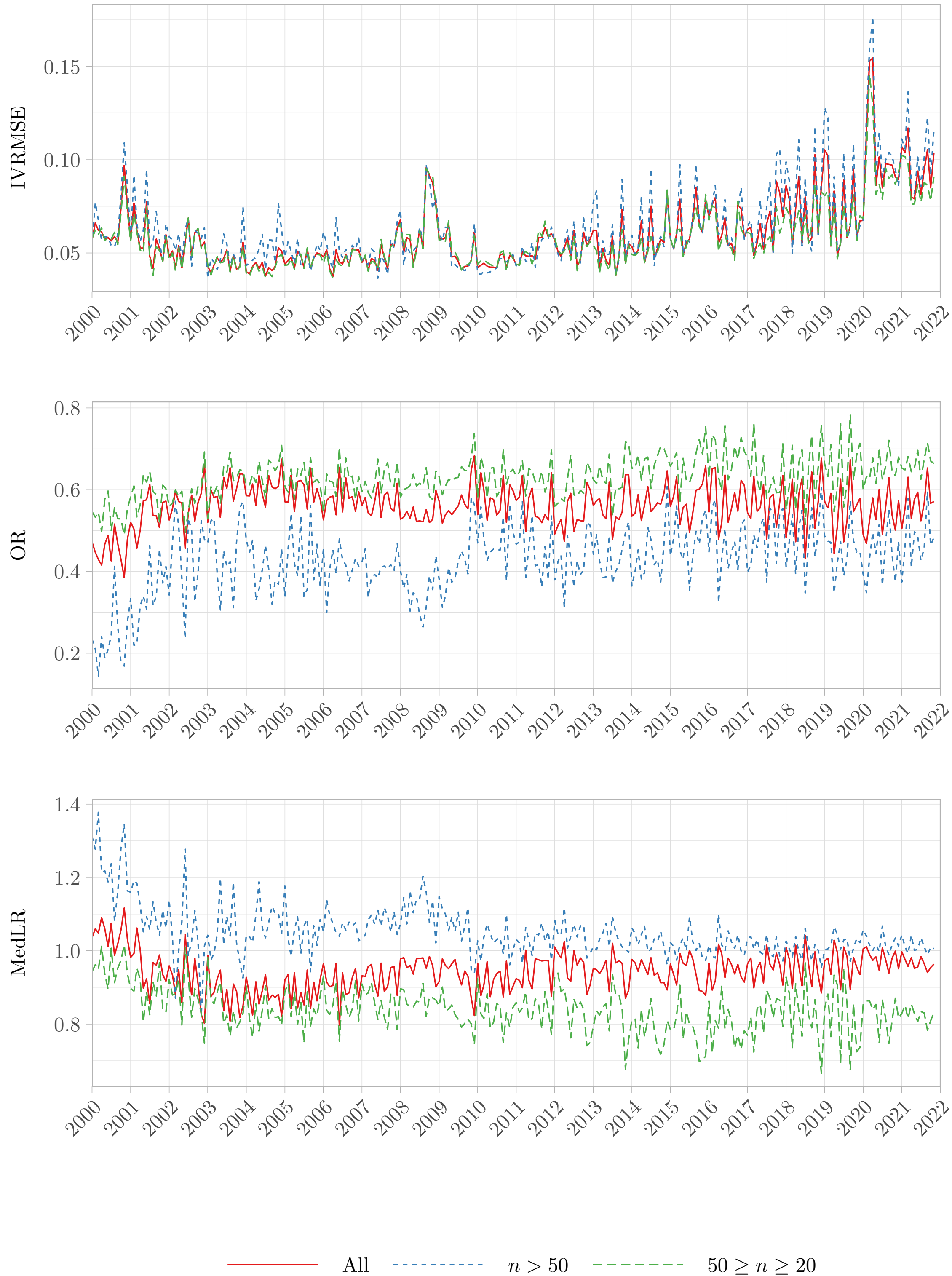
Notes: We report (Panel A) time-series averages of different monthly performance measures: the cross-sectional average IVRMSE, the outperformance rate (OR) defined as the share of stocks for which the models achieve a lower IVRMSE than the AHBS model (i.e., higher is better), and the monthly median loss ratio (MedLR) (i.e., lower is better). Panel B/C report the same performance measures for the subsets of stocks with more than 50 trades/at most 50 trades per time period. All models are evaluated on the liquid sample; that is, the sample containing only those stocks per month for which we observe at least 20 option trades. AHBS-Forest (all) and AHBS-Forest (reduced, all) are the only models that include non-liquid observations during estimation. The monthly data spans 2000:M1 until 2021:M11. 2021:M12 is missing due to the availability of stock characteristics.

Table B.3: Same setup as in Table 2 but the forest is based on 50 trees instead of 200 trees

	No. Obs.	IVRMSE	OR	MedLR
Panel A: All	564.046			
AHBS (benchmark)		0.106	—	—
BS		0.078	0.374	1.221
AHBS (reduced)		0.064	0.541	0.967
AHBS (maturity only)		0.097	0.359	1.195
AHBS (moneyness only)		0.076	0.451	1.052
AHBS (pooled)		0.063	0.508	0.992
AHBS (pooled) Interaction		0.064	0.500	1.001
AHBS-Forest (reduced, liquid)		0.063	0.508	0.993
AHBS-Forest (liquid)		0.060	0.550	0.955
AHBS-Forest (reduced, all)		0.063	0.516	0.984
AHBS-Forest (all)		**0.060**	**0.560**	**0.944**
Panel B: $n > 50$	218.985			
AHBS (benchmark)		0.073	—	—
BS		0.083	0.212	1.426
AHBS (reduced)		0.064	0.380	1.063
AHBS (maturity only)		0.082	0.238	1.310
AHBS (moneyness only)		0.074	0.298	1.146
AHBS (pooled)		0.066	0.369	1.095
AHBS (pooled) Interaction		0.064	0.386	1.067
AHBS-Forest (reduced, liquid)		0.067	0.345	1.118
AHBS-Forest (liquid)		**0.064**	**0.424**	**1.051**
AHBS-Forest (reduced, all)		0.067	0.348	1.113
AHBS-Forest (all)		0.064	0.423	1.055
Panel C: $50 \geq n \geq 20$	345.061			
AHBS (benchmark)		0.127	—	—
BS		0.075	0.459	1.095
AHBS (reduced)		0.063	0.625	0.875
AHBS (maturity only)		0.106	0.422	1.131
AHBS (moneyness only)		0.078	0.531	0.981
AHBS (pooled)		0.061	0.582	0.895
AHBS (pooled) Interaction		0.063	0.563	0.922
AHBS-Forest (reduced, liquid)		0.061	0.595	0.881
AHBS-Forest (liquid)		0.058	0.615	0.858
AHBS-Forest (reduced, all)		0.060	0.603	0.870
AHBS-Forest (all)		**0.058**	**0.629**	**0.842**

Notes: We report (Panel A) time-series averages of different monthly performance measures: the cross-sectional average IVRMSE, the outperformance rate (OR) defined as the share of stocks for which the models achieve a lower IVRMSE than the AHBS model (i.e., higher is better), and the monthly median loss ratio (MedLR) (i.e., lower is better). Panel B/C report the same performance measures for the subsets of stocks with more than 50 trades/at most 50 trades per time period. All models are evaluated on the liquid sample; that is, the sample containing only those stocks per month for which we observe at least 20 option trades. AHBS-Forest (all) and AHBS-Forest (reduced, all) are the only models that include non-liquid observations during estimation. The monthly data spans 2000:M1 until 2021:M11. 2021:M12 is missing due to the availability of stock characteristics.

Fig. B.1: Prediction performance over time of the AHBS-Forest (all)



Notes: Monthly IVRMSE, outperformance rate (OR), and median loss ratio (MedLR) over time. The sample ranges from January, 2000 to November, 2021. December 2021 is missing due to the availability of stock characteristics.

B.1. Effect of characteristics identified by variable importance

We first continue the interpretation of the most important characteristics according to the variable importance measure. As can be seen in Figure 6 of the main text, the option market values two distinct forms of volatility, retvol and roavol, differently. The term structure coefficient $\widetilde{a}^{(\tau)}$ is significantly affected by return volatility. This can be attributed to the approximate square-root-of-time rule of return volatility. On the other hand, the coefficient $\widetilde{a}^{(m)}$ is significantly impacted by earnings volatility. This shows that companies with a less steady business model, characterized by higher earnings volatility, are priced differently in terms of cost of protection against downside risk than those with lower earnings volatility.

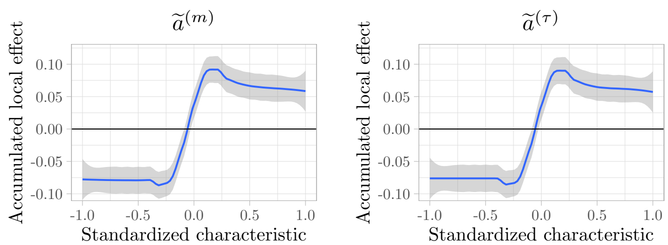
Moreover, the changes in depreciation have a significant impact on $\widetilde{a}^{(m)}$, but not on $\widetilde{a}^{(\tau)}$. This indicates that changes in depreciation related to property, plant, and equipment provide insight into a company's current state, which is reflected in the company's exposure to downside risk. Additionally, research by Hou and Robinson (2006) suggests that high industry sales concentration can lead to a high degree of undiversifiable risk, resulting in higher downside risk and higher expected future volatility. As we discuss below, the ALE plots indicate that this is indeed the direction in which herf affects the model-implied IVS coefficients.

Figure B.2 displays the ALE plots for the most important variables in the AHBS-Forest, as identified in Figure 6 of the main text. The scale is consistent across all panels, allowing for comparison of the magnitude of effects. The first panel shows a positive relation between indmom and the slope $\widetilde{a}^{(m)}$ and between indmom and the term structure $a^{(\tau)}$. This is in line with the results of other momentum-based characteristics in Section 2. However, the ALE plots provide a more nuanced view: the local effect of industry momentum is most pronounced in its mid-range, while for very low and high ranks of indmom, there is a slight reversal to the average model-implied coefficient.

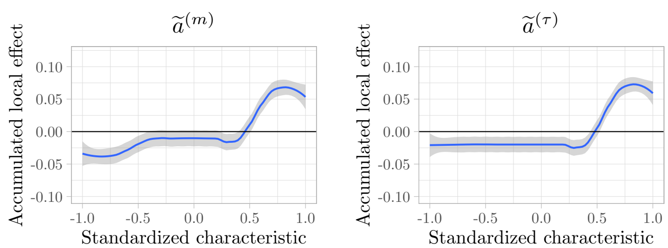
Panel (b) of Figure B.2 shows that ALE plots are nearly flat for both the slope and term structure coefficients up to the 60%-cross-sectional quantile of return volatility. However,

Fig. B.2: ALE plots for influential characteristics in variable importance analysis

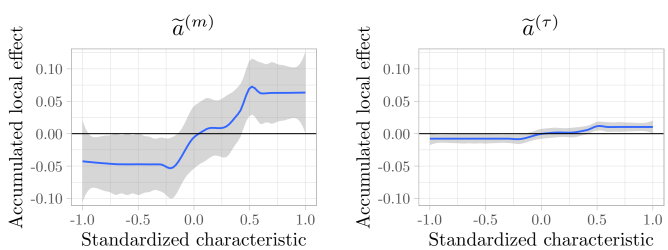
(a) Industry momentum (indmom)



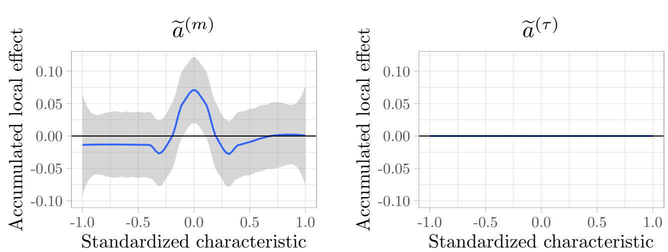
(b) Return volatility (retvol)



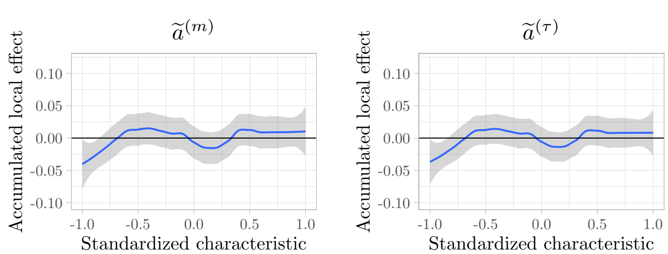
(c) Earnings volatility (roavol)



(d) Change in depreciation (pchdepr)



(e) Industry sales concentration (herf)



Notes: In this figure, we plot ALE functions as defined in Eq (7) for five characteristics that have been identified to be important by our variable importance measure. The ALE functions and the corresponding 90% confidence intervals depicted are based on monthly ALE functions from our monthly re-estimation scheme that are aggregated via local linear regression (Cleveland and Devlin, 1988). The monthly data spans 2000:M1 until 2021:M11. 2021:M12 is missing due to the availability of stock characteristics.

above this quantile, increases in volatility have a significantly larger positive effect on the model-implied coefficients, indicating that the option market prices mostly above-average levels of return volatility. In other words, higher costs of protection against downside risk and higher expected future volatility are concentrated on high-volatility stocks.

Panels (c) and (d) confirm the findings from the variable importance analysis that earnings volatility and change in depreciation have a different impact on the two coefficients. The ALE plots show that there is a greater effect on the coefficient $\widetilde{a}^{(m)}$. For instance, an increase in earnings volatility is associated with a pronounced increase in perceived downside risk. Lastly, Panel (e) illustrates a nonlinear relation between industry sales concentration and the two IVS features. A very low level of industry sales concentration leads to a lower perceived risk in both the moneyness and time to maturity dimensions.

C. ALE plots details

For defining the ALE plots, we introduce some additional notation. By $z_{i,(x;-k)}$ we denote the altered characteristics vector $z_{i,\cdot}$ in which the kth element is replaced by x. Moreover, let $[\xi_0, \xi_1], (\xi_1, \xi_2], \ldots, (\xi_{M-1}, \xi_M]$ be an equal-sized partition of the interval [-1,1]. The accumulated local effect function of variable $k \in \mathcal{K}$ on the moneyness slope coefficient $\widetilde{a}^{(m)}$ is defined as follows:

$$ALE_k^{(m)}(z) = \int_{-1}^{z} \mathbb{E}\left[\frac{\partial \widetilde{\mathcal{F}}^{(m)}}{\partial y_k}\left(Z_{i,(x;-k)}\right)\right] \mathrm{d}x + C, \tag{7}$$

where $\frac{\partial \widetilde{\mathcal{F}}^{(m)}}{\partial y_k}$ is the first partial derivative with respect to the kth characteristic and C is chosen such that $\mathbb{E}[ALE_k^{(m)}(Z_{i,k})] = 0$. We assume the function $\widetilde{\mathcal{F}}^{(m)}$ to be sufficiently smooth and differentiable in its arguments such that Eq. (7) is well defined. In Eq. (7), the term $\frac{\partial \widetilde{\mathcal{F}}^{(m)}}{\partial y_k}\left(Z_{i,(x;-k)}\right)$ captures the local effect of small changes in the kth characteristic while holding all other characteristics constant. The accumulated local effect can then be interpreted as the combined impact of the kth characteristic at the value z compared to the average model-implied coefficient.

We estimate the ALE function via its empirical analogue by approximating the local effect by finite differences. For $z \geq \xi_1$, the estimated uncentered ALE function is

$$\widehat{\widetilde{ALE}}_k^{(m)}(z) = \frac{1}{N_1} \sum_{i: z_{i,k} \in [\xi_0, \xi_1]} \left[\widetilde{\mathcal{F}}^{(m)}(z_{i,(\xi_1;-k)}) - \widetilde{\mathcal{F}}^{(m)}(z_{i,(\xi_0;-k)}) \right] + \sum_{m>1: \xi_m \leq z} \frac{1}{N_m} \sum_{i: z_{i,k} \in (\xi_{m-1}, \xi_m]} \left[\widetilde{\mathcal{F}}^{(m)}(z_{i,(\xi_m;-k)}) - \widetilde{\mathcal{F}}^{(m)}(z_{i,(\xi_{m-1};-k)}) \right],$$

where N_m, $m = 1, \ldots, M$, is the amount of observations for which $Z_{\cdot,k}$ is in the interval $(\xi_{m-1}, \xi_m]$. For $z < \xi_1$, the estimated uncentered ALE function equals zero. The (centered) ALE function is then simply given by

$$\widehat{ALE}_k^{(m)}(z) = \widehat{\widetilde{ALE}}_k^{(m)}(z) - \frac{1}{M} \sum_{m=1}^{M} \widehat{\widetilde{ALE}}_k^{(m)}(\xi_m).$$

Note that we can take the simple arithmetic average across the grid to center the ALE function because our features are rank-standardized. The ALE functions for other AHBS-Forest coefficients are defined accordingly by replacing, for example, $\widetilde{\mathcal{F}}^{(m)}$ by $\widetilde{\mathcal{F}}^{(\tau)}$.