Did Retail Traders Take Over Wall Street?

A Tick-by-Tick Analysis of GameStop's Price Surge*

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Abstract

In January 2021, GameStop experienced an extraordinary surge in its stock price, soaring from \$17.25 on January 4 to a pre-market value of \$514.50 on January 28. In contrast to previous studies, we use tick-by-tick data of stock and option trading to demonstrate that this remarkable surge comes mainly from overnight trading, driven mainly by institutional orders rather than by those from retail investors. Moreover, the daily GME holdings of retail investors are trending down, rather up, right before the surge. Although sophisticated option traders typically maintain a positive gamma position, option market makers regulate their gamma exposure by participating in retail option trades. An "after-hours gamma squeeze" was triggered by a twitter catalyst, causing eventually the well-known GME short squeeze. We also provide an extended model of Brunnermeier and Pedersen (2005) that explains some of our major findings.

Keywords: GameStop, short squeeze, gamma squeeze, institutional trading, short seller.

JEL Classification: G12, G13, G14, G18.

"Markets can stay irrational longer than you can stay solvent."

— John Maynard Keynes

1 Introduction

The performance of GameStop stock between late January and early February 2021 was an extraordinary event in the stock market history. In a mere three weeks, the stock price of the video game retailer skyrocketed from \$17.25 per share to an all-time high of \$514.50,[1] an unprecedented increase of almost 3,000%. This rapid and substantial surge caused considerable financial losses to well-known short sellers. Although this accomplishes the objective of numerous online retail traders who assert this as they defeat Wall Street in their own game, there are a number of important unanswered questions. Are they really the major force behind the surge? As a group, are they making profits? From a regulatory point of view, is there really a short squeeze?

Who is responsible for the GameStop short squeeze? The news media and recent studies by Allen et al. (2023), Bradley et al. (2023), and Hu et al. (2021) suggest that the short squeeze is mainly caused by retail investors and social media platforms. Although the media, the SEC, and congressional hearings have mainly focused on these retail investors, the institutional side of the short squeeze has received little consideration. We use tick-by-tick data of both stock and option trading to identify retail and institutional trades, and find that it is the institutional investors who play a greater role in exploiting the apparent media hype that drives the meme stocks for consecutive daily rallies, and it is a subsequent twitter catalyst that triggers both short and gamma squeezes, resulting the GameStop price climax. Interestingly, prior to the surge, the daily GME holdings of retail investors are trending down instead of trending up, suggesting that both the timing of the surge and its magnitude are beyond their expectations.

Our paper reveals that the price rises of the GameStop stock are mainly attributable to after-hours sessions rather than the regular trading sessions. We investigate institutional and retail order

[1] On January 28 at 6:03 AM EST, two transactions were completed at this price.

flows over different time frames, including daily, regular and after-hour trading sessions, and 1-minute intervals. Our findings consistently show that institutional trading has a dominating positive effect on GameStop's price dynamics, particularly during periods of the price surge. Contrary to popular belief, retail trading does not have nearly the same level of influence.

Although it is difficult to definitively tell whether the overnight institutional order flow is mainly from institutional short sellers or institutional speculators, we have uncovered their large trades that have enormous impact on prices. While short sellers usually cover their short positions in a liquid market to reduce further losses, speculators who want to trigger the short squeeze prefer to buy in an illiquid market to cause price surges with limited trades. Consisient with their interest to drive up the price up, it is these institutional speculators who are active players during the day sessions of trading, primary players in the after-hour sessions, and are main drivers of the price surge. Our detailed tick by tick analysis provides a better understanding of the role of institutional traders in the GameStop short squeeze.

Do institutional speculators, as sophisticated traders, take advantage of options during a short squeeze? Mitts et al. (2022) highlight the presence of a gamma squeeze in GameStop options. Our research uses tick-by-tick option trade data to demonstrate that both retail and sophisticated option traders tend to write call and put options rather than buying them during the surge period from January 1, 2021 to March 31, 2021. Additionally, we propose a method to calculate daily net open interest delta. Its value implies that option market makers hold larger underlying positions, leading to a decrease in the supply of tradable shares. Therefore, a larger net delta predicts a higher after-hours return, while also having a negative effect on the following day's market quality. We also compute a net open interest gamma measure. Although sophisticated option traders only slightly mitigate their gamma positions during the surge period, options market makers strategically reduce their gamma risk by engaging with retail option traders. This strategy allows them to effectively manage their gamma exposure, resulting in their gamma risk being at its lowest level during the sample period prior to the onset of the short squeeze. This finding is in line with the October 2021 SEC report, which finds limited evidence of intraday gamma squeeze.

However, our paper reveals a unique type of the gamma squeeze, which we refer to as the "after-hours gamma squeeze" that is caused by a sudden increase in the after-hours stock price. This price surge turns a large quantity of out-of-the-money (OTM) call options into a large quantity of at-the-money (ATM) call options, significantly increasing gamma exposure. As a result, option market makers, who may have a minor gamma exposure at the end of the day, experience an incredible rapid increase in delta exposure due to the amplified gamma exposure, thus necessitating them to hedge their positions in the thinly traded stock market during the after-hours session. This is the only choice for them to hedge since the options market is closed during the after-hours session. For example, on the night of January 26, the position of 6,506 net long OTM call options, with strike prices ranging from \$150 to \$230, suddenly becomes deep in-the-money after the price surges over \$300, a price jump due to Elon Musk's surprising tweet that also has a link to the well-known Reddit WallStreetBets. This creates the perfect environment for an "after-hours gamma squeeze", which helps to understand the actions of the various types of option traders involved in the GameStop short squeeze.

Due to anonymity of individual players in our data, we are unable to definitively find out how and which institutional speculators use stocks and options to contribute mainly to the short squeeze. Nevertheless, to provide some insights on that, we extend the model of Brunnermeier and Pedersen (2005) to incorporate options trading and explain the actions of the speculators who target short sellers. Our model suggests that the predators, in order to maximize their profits, are likely to initiate a short squeeze in a market with limited liquidity while maintaining a positive gamma stance. This prediction is consistent with our empirical results.

Furthermore, our model describes that during the early stages of the short squeeze, both institutional speculators and institutional short sellers engage in buying stocks. As the price reaches its zenith, the institutional speculators divest their holdings to the short sellers. This dynamic, as posited by our model, engenders a balanced institutional order flow and contributes to minor price fluctuations. Remarkably, both of these phenomena were observed on January 27, 2021, when Melvin Capital closed their short position during the afternoon of the same day,[2] leading to substantial losses in the billions of dollars.

Overall, our evidence suggests that it is the sophisticated institutional investors who take advantage of the situation and go along with the retail investors, and then make the most out of it in the end. Consistent with our tick-by-tick trading analysis, anecdotal evidence shows that, although the retail investors won the war with the short sellers, they lost their money in the process.

Our study contributes to several areas of literature. Our primary focus is on the literature that examines short squeezes. A related contribution is Brunnermeier and Pedersen (2005), who develop a theoretical model for predatory trading. This concept involves intentional trading in conjunction with large traders being forced to liquidate a position. They argue that such trading would reduce market liquidity and increase costs for large traders. Allen et al. (2021) investigate the Porsche-Volkswagen (VW) short squeeze of 2008. Through a unique dataset obtained from legal proceedings against Porsche, the authors reconstruct Porsche's stock and options holdings in VW. Their analysis reveals how Porsche's management orchestrates one of the most remarkable short squeezes in history. Our research builds on this literature by conducting an empirical and theoretical analysis of the GameStop short squeeze. We use tick by tick data to analyze the price impact during regular and after-hours trading, and the role of institutions and individual traders. Our extended model takes into account the role of the options market, a key factor in short squeezes, and provides insights into the optimal strategy of predatory traders, which are consistent with our empirical findings. While the October 2021 SEC report finds limited signs of a short squeeze,[3] recent studies by Allen et al. (2023) and Mitts et al. (2022) suggest otherwise. Our study provides detailed and trading level evidence that there are short squeezes in both stocks and options hedging.

Our study adds to the literature on retail investors. Barber and Odean (2000) provide the first

[2] https://www.cnbc.com/video/2021/01/27/melvin-capital-sells-out-of-gamestop.html

[3] See the SEC's "Staff Report on Equity and Options Market Structure Conditions in Early 2021," at https://www.sec.gov/files/staff-report-equity-options-market-struction-conditions-early-2021.pdf

empirical evidence that trading is hazardous to the wealth of retail investors. Consistent with their message, we find that in the era of zero commissions, retail investors still lose in trading. Among the large body of the literature, Boehmer et al. (2021) provide an algorithm to identify retail trades, McLean et al. (2023a,b) analyze how retail investors respond to analysts' revisions, and what their return predictive ability is, and Pedersen (2022) develop a model that sheds light on how social network spillovers influence retail traders' behavior in response to influencer activity. Recently, Barber et al. (2023) refine the algorithm for identifying retail trades. In contrast to these studies, our paper is the first to study the role of retail investors in a short squeeze.

Our study also adds to the growing body of literature on intraday trading behaviors among different participants in the options market. Bryzgalova et al. (2022) develop a method to distinguish retail option trades, finding that retail investors usually opt for cheaper weekly options and lose money. Moussawi et al. (2024) discover the advantage of market makers who provide liquidity in both ETF and options markets. These market makers take advantage of their dual roles by engaging in intraday arbitrage following ETF order flow shocks, profiting from the liquidity provided by sophisticated option traders. Bryzgalova et al. (2023) reveal sub-optimal non-exercise of retail call options, with market makers profiting from these mistakes. Our contribution to this field is our proposed approach to calculate the net open interest Greeks, which serves as a proxy for the option market makers risk exposure. Additionally, we demonstrate that option market makers strategically reduce their gamma exposure before short squeezes through retail options trades, despite sophisticated option traders' tendency to keep their gamma positions.

The remainder of this paper is organized as follows. Section 2 summarizes the background of GameStop 2021 event. Section 3 describes the underlying data. Section 4 analyzes stock market order flows. Section 5 provides analyses related to the options market. Section 6 takes a close look at the days of the finals. Section 7 develops a model to explain the empirical observations. Section 8 concludes.

2 Background

Due to the Covid-19 pandemic and the advent of commission-free trading platforms such as Robinhood, there is a large increase of retail investors' involvement in the stock market prior to January 1, 2021. During this time, the stocks that piqued the interest of these retail investors were primarily those widely recognized by consumers, the so-called "meme stocks" including GME. The discussions focused on the GameStop in particular have escalated on social media platforms, and some of the postings, such as those from Keith Gill, who is a well-known writer and an influential trader, call explicitly the followers to buy GME, which, along with other meme stocks, starts to rise quickly in early January. But the trading volume and short interest are much not as extreme as at this time and the attention just starts to accumulate and explode later. As Figure 1 shows, the daily closing prices and trading volumes of GameStop have seen a dramatic increase since the beginning of 2021, with the peak occurring in late January. It should be noted that there have been no significant changes to the company during this time.

[Insert Figure 1 here.]

In countering the large, fundamental-baseless price moves, hedge funds and other institutions, such as Melvin Capital and Citron Research, enter short positions to profit from mispricing from the fundamental value. On the other hand, some other institutions and famous investors, such as RC Ventures LLC, have long large positions and take a ride in the GME price movement.[4] Hence, an ensuring battle of price between the long and the short has started.

In fact, even before the huge publicity and intensified social media attention, Melvin Capital, for example, had already bet GME to go down, which reported holding 60,000 long put options in its Q4 2020 Form 13-F (short positions are unreported as required by the SEC[5]). In contrast, RC

[4] Some speculators may go long, as the famous George Soros reported said, "When I see a bubble forming, I rush in to buy, ... this is not irrational." (see. e.g., https://seekingalpha.com/article/4085934-when-i-see-bubble-forming-i-rush-in-to-buy(2017).

[5] See for more details from "FAQ about form 13F", https://www.sec.gov/divisions/investment/13ffaq.

Ventures LLC continues to increase its stake in GameStop, reaching ownership of 9 million shares according to their December 17 Form 13-D filing.

By January 22, the GameStop stock enjoys a remarkable change in its stock price to \$65.01. On January 25, Melvin Capital was given \$2.75 billion from Citadel and Point72 to help mitigate margin call risk from its losing positions.[6]

The performance of the stock in after-hours trading was remarkable. On January 25, it opened at \$96.73, a significant jump from the previous close of \$65.01. Then it skyrocketed to a peak of \$159.18, before settling at \$76.79 at the end of the day. On January 26, after another significant surge in after-hours trading, the stock opened at \$88.56 and ended at \$147.98.

At 4:08 PM EST on January 26, Elon Musk sent out a tweet that caused an unprecedented market reaction: "Gamestonk!!". This resulted in a 140% surge in the stock price overnight, with the opening price on January 27 being \$354.83. Later that day, Melvin Capital closed its GameStop short position,[7] which had little effect on the price. The closing price at the end of the day was \$347.51, almost the same as the opening price.

On January 27, the National Securities Clearing Corporation (NSCC) experienced a record-breaking transaction volume.[8] This was followed by the release of daily margin statements shortly after 5 AM EST on January 28. As a result, many clearing members saw a significant increase in VaR charges, leading to additional capital premium charges. Robinhood, in particular, was required to provide a \$1.4 billion deposit to the NSCC pre-market on January 28, which was an increase from the \$690 million by the end of the day on January 27.[9] At 9 AM EST, the NSCC waived excess capital premium charges for clearing members.[10] Subsequently, the GameStop stock price dropped from the all-time high of \$514.50 at 6:03 AM to an opening price of \$265, before rising

[6] https://www.wsj.com/articles/citadel-point72-to-invest-2-75-billion-into-melvin-capital-management-

[7] https://www.cnbc.com/video/2021/01/27/melvin-capital-sells-out-of-gamestop.html

[8] https://www.dtcc.com/-/media/Files/PDFs/Testimony-of-Michael-Bodson-050621.pdf

[9] https://www.congress.gov/117/meeting/house/111207/witnesses/HHRG-117-BA00-Wstate-TenevV-20210218.pdf

[10] In accordance with the "T+2" settlement rule in the US equity market, a two-day "settlement period" is observed. Within this time frame, clearing members, such as Robinhood, are obligated to provide a deposit from their own funds (not customer funds) to the NSCC. This deposit serves to mitigate risk until the trade is successfully "settled."

to the intraday peak of $483. It eventually closed at $193.60 on January 28.

The price and volatility of GameStop stock remained high after its peak in late January. On February 24, a surge in short-dated call option activity was observed, with call option volumes reaching their highest point in almost three weeks.[11] The stock price rose twofold in the last 90 minutes of regular trading, closing at $91.71. This momentum continued into after-hours trading, resulting in an 85% increase to the opening price of $169.56 on February 25. The price stayed around $200 for another month. As of March 24, Reuters reported that short interest in GameStop had dropped to 15%, a significant decrease from the peak of 141% in the first week of 2021.[12] This decline was further confirmed by semi-monthly equity short interest data from the Financial Industry Regulatory Authority (FINRA) which shows a continuous downward trend from January 1, 2021 to March 31, 2021.[13] Thus, we can conclude that the price surge period has ended. After March 31, 2021, the price remained above $150, but volatility and volume have decreased significantly.

The GameStop episode in January 2021 is often described as a modern-day battle between small and large, with retail investors battling against major financial institutions. Is this really the case? Are they the winners or are they eventually the victims too? After the price surge in late March, many Reddit users reported considerable losses. Reddit CEO Steve Huffman commented that most users end up losing money.[14] In what follows, we will focus on whether retail investors are responsible for the short squeeze in general, and the surge and subsequently fall on January 27. The squeeze has reportedly caused significant financial losses, amounting to billions of dollars, for some well-known short sellers.[15]

[11] https://www.ft.com/content/50eaa1b5-d244-4b3e-b460-736828c049cd

[12] https://www.reuters.com/article/idUSKBN2BG28H

[13] https://www.finra.org/finra-data/browse-catalog/equity-short-interest/data

[14] https://www.washingtonpost.com/technology/2021/02/02/gamestop-stock-plunge-losers/

[15] By the end of January 2021, media reports indicated that Melvin Capital lost $3.75 billion and D1 Capital Partners lost $4 billion. (https://www.theguardian.com/business/2021/jan/27/gamestop-stocks-us-hedge-fund-pulls-out-after-heavy-losses and https://www.bloomberg.com/news/articles/2021-01-28/dan-sundheim-s-20-billion-d1-capital-loses-about-20-this-month#xj4y7vzkg)

3 Data and Descriptive Statistics

Using intraday trade-level information from January 1, 2020 to December 31, 2021, our paper draws from the Daily Trades and Quotes (DTAQ) and CBOE LiveVol datasets. These sources provide a detailed view of tick-by-tick GME stock and options trades. The CBOE trade data provide insights into transaction prices, implied volatility, delta, trading volume, best bid and ask quotes for options, and underlying prices. We follow the methodology of Holden and Jacobsen (2014) and Andersen et al. (2021), respectively, filter and clean the intraday stock and options data. We improve our analysis by incorporating intraday datasets with additional data sources. We obtain daily return and stock price information from CRSP, daily options volume and options' Greek information from OptionMetrics, daily stock market quality data from the Intraday Indicators database by WRDS, the daily securities lending market data from IHS Markit and daily short sale volume from FINRA.

[Insert Table 1 here.]

We present a comprehensive set of descriptive statistics regarding GameStop's performance in Table 1. The surge period, which is the time frame in which we observe a monotonic reduction in short interest, is from January 1, 2021, to March 31, 2021. The pre-surge phase is from January 1, 2020, to December 31, 2020, and the post-surge period is from April 1, 2021, to December 31, 2021.

Table 1 Panel A reveals that the all-day return (close-to-close) is significantly higher during the surge period. When broken down into its components - overnight return (close-to-open) and intraday return (open-to-close) - the substantial divergence is mainly due to the overnight return. On the other hand, the intraday return during the surge period does not differ significantly from the pre- and post-surge periods. Additionally, both overnight and intraday volumes experience a dramatic increase during the surge period. In particular, the primary cause of this substantial increase is the intraday volume, which increases from 5.98 million shares prior to the surge, to

41.27 million shares during the surge. Furthermore, using the Lee and Ready (1991) algorithm, we classify trades as buyer (seller) initiated trades and compute the signed volume as the difference between buy volume and sell volume. The signed volume indicates that only the overnight signed volume has a significant change during surge period. This is evidenced by more overnight buy trades and more intraday sell trades being executed during surge period. Furthermore, the average overnight signed volume during surge period is 100.53 thousand shares, which is close to the average all-day signed volume of 101.98 thousand shares during pre-surge period. These metrics demonstrate a distinct divergence in the trading pattern between the intraday and overnight sessions.

Table 1 Panel B provides an overview of the metrics of the options market. As expected, both realized and implied volatilities show a significant increase during the surge period. Additionally, the trading volume of options experiences a considerable surge, with a shift in the preferences of options traders. Specifically, there is an increase in the trading of put options during the surge period, while call options are predominant before and after the surge. Moreover, we classify buyer-initiated option trades as those with trade prices above the midpoint, and seller-initiated option trades as those with trade prices below the midpoint, excluding midpoint trades, as per the method outlined in Savickas and Wilson (2003).[16] The statistics demonstrate a nearly equal buying and selling pattern in both call and put options.

4 Is There a Short Squeeze?

The GameStop short squeeze has been investigated by both the Securities and Exchange Commission (SEC) and academic researchers. In October 2021, the SEC released a report that

[16] Savickas and Wilson (2003) demonstrate that such a method has the highest success rate among four popular option trade classification rules. Furthermore, Bryzgalova et al. (2022) use a recent dataset (November 2019 to June 2021) to confirm that the option trade imbalances calculated based on the quote rule we used are highly correlated (94%) with the ones with the Muravyev (2016) method.

suggests limited evidence of a short squeeze.[17] Allen et al. (2023) and Mitts et al. (2022), however, provide evidence contrary to the SEC conclusion of no short squeeze. Based on considerable increases in short interest and average borrowing fees, as well as a notable rise in available short quantity during the period of the price surge, Allen et al. (2023) conclude that short squeezes have taken place among the meme stocks, including GameStop. Mitts et al. (2022) use primarily the volume of shares returned to lenders as a metric, and they find that the associated trading volume accounts for around 80% from January 27 to 29. This pattern of high trading volume returned to lenders during the most severe price surge period provide clear evidence of short squeeze and liquidation on both sides. The SEC's investigation mainly focuses on intraday trading activities, where they find that buying by those with short positions was only a small fraction of the total buy volume.[18]

We aim to provide more direct and arguably more substantially evidence on short squeeze. First, we make a clear distinction of regular-hours trading and after-hours trading and find that the major price moves come from after-hours trading. Second, we identify retail and institutional trades and find that it is institutional traders who drive the market, especially in after-hours trading. Third, we obtain multiple evidence, from signed trading volume, weighted price contribution and intraday high-frequency analysis, to confirm further the main results. Moreover, option trading analysis, provided in the next Section, provides additional evidence on the short squeeze and on who are the main driver of the squeeze.

4.1 Short Interest and Short Volume

Allen et al. (2023) propose that a short-squeeze event is characterized by a sudden rise in stock

[17]SEC published a "Staff Report on Equity and Options Market Structure Conditions in Early 2021" that can be downloaded from
https://www.sec.gov/files/staff-report-equity-options-market-struction-conditions-early-2021.pdf

[18]Their intraday Volume Weighted Average Price (VWAP) chart, depicted as Figure 6 in the report, falls short in capturing the intricate details of after-hour price movements, particularly the overnight activity that occurred on January 26th.

price, a high short interest before the surge, and a decrease in short interest during and after the surge. They employ the securities lending market data to verify that GameStop exhibited a short-squeeze pattern during the beginning of 2021, thus conclude that there is a short squeeze on GameStop. How do short sellers behave during the surge period? Do the short-selling market run well during surge period? To answer these questions, we explore further by incorporating the daily short volume data obtained from FINRA.

[Insert Figure 2 here.]

Figure 2 displays the daily short interest and the daily short volume over time. The short interest is measured by the 'quantity on loan' in IHS Markit data, while the short volume is the total share amount of short sale trades executed on different exchanges. The amount of short interest decreases significantly from 53 million shares on January 4, 2021 to 7.7 million shares on March 31, 2021. However, daily short sales trading is highly active in the same period, with an average daily short volume of 9.5 million shares, which is six times higher than the average daily short volume in 2020 (1.5 million shares). This observation yields two insights. First, the decrease in short interest during the surge period is likely due to short sellers closing their pre-existing positions, as new short selling positions are continuously being created. The high short volume also indicates that the short-selling market is functioning properly during the surge period, which may have prevented an even more extreme price surge on GameStop. Second, active short-selling activities during the surge period make it easier for short sellers to cover their positions, as they can buy from investors who are initiating fresh short-selling positions when other investors keep holding the underlying stock. In conclusion, there is a potential short squeeze event occurring in GameStop, however, due to the highly developed short-selling market in the US, the effect of this squeeze is mild.

4.2 Large Price Moves in After-Hours Trading

In addition to regular trading hours (RTH) between 9:30 AM and 4 PM Eastern Time (EST) in the United States, both NASDAQ and NYSE offer after-hours trading sessions that include pre-market (from 4 AM to 9:30 AM EST) and after-market (from 4 PM to 8 PM EST) trading. Barclay and Hendershott (2003) demonstrate that after-hours trading has distinct characteristics compared to trading during regular market hours. During after-hours sessions, the trading volume is usually lower, market makers are less likely to provide quotes, and trading costs are usually higher than during regular trading hours. Although some popular brokers allow their customers to submit and execute orders outside of regular trading hours,[19] retail customers are often discouraged from participating due to warnings of increased risk levels. Therefore, Barclay and Hendershott (2003) suggest that the after-hours trading session is mainly dominated by professional or quasi-professional traders who have strong incentives to trade.

[Insert Figure 3 here.]

Figure 3 illustrates the cumulative overnight and intraday returns for GameStop (GME) and a benchmark stock BestBuy (BBY) which is in the same industry and of similar size but is not targeted by social media. The overnight returns on day t are calculated as the difference between the closing price at 4:00 PM on day $t-1$ and the opening price at 9:30 AM on day t. GME's overnight returns appear to have a greater impact on the stock price than its intraday returns, which are the returns from the market open to close. The high correlation between GME's overnight cumulative returns and its buy-and-hold returns (ρ=0.99) suggests that overnight trading activities significantly affect the price of GME stock. In addition, 79.19% of the GME buy-and-hold return from January 2020 to December 2021 is attributed to the overnight period. This is in agreement with the significant increase in overnight return, as seen in Table 1. If the findings of Barclay

[19] As of May 2023, several prominent brokerage firms, including Charles Schwab, E*Trade, Interactive Brokers' IBKR Lite, Robinhood, and TD Ameritrade, execute their customers' after-hours orders during two time periods: 7 AM to 9:30 AM EST and 4 PM to 8 PM EST. Additionally, Interactive Brokers' IBKR Pro offers its clients early access to the markets starting from 4AM EST.

and Hendershott (2003) still hold in recent years, these substantial price movements during surge period may be mainly driven by institutional traders.

4.3 Retail and Institutional Order Flows

The importance of retail traders in GameStop short squeezes has been highlighted in various studies, such as Allen et al. (2023) and Hu et al. (2021). However, the institutional side has not been given much consideration. In this section, we investigate the order flow patterns of both retail and institutional traders during surge period.

We employ the approach proposed by Boehmer et al. (2021) to distinguish retail trades. They observe that most of internalized retail orders usually receive a price improvement of less than one cent per share. Boehmer et al. (2021) report that this price improvement is seen in between 60% and 99% of the trades in 'retail venues', allowing them to be classified as retail trades.[20] To measure institutional trading, we use intermarket sweep orders (ISO) trades. These trades enable traders to quickly access liquidity at multiple price levels and across multiple markets to execute large block trades through parallel order submissions. Generally, institutions submit ISO orders,[21] which can be identified in the TAQ database with the sale condition code F. Chakravarty et al. (2012) provided empirical evidence that informed institutions are the main users of ISO orders.

It is essential to be aware that the methods mentioned above only account for a part of retail and institutional trades. However, for our research, we assume that retail and institutional traders maintain a steady pattern of order submission during the sample period. By taking advantage of these two proxies, internalized retail trades and ISO trades, we can analyze the order flow of both retail and institutional traders.

[20] Two critiques (Battalio et al. (2022), Barber et al. (2023)) point out the limited accuracy of the method proposed by Boehmer et al. (2021), with an approximate rate of 30% to 35%.

[21] The ISO order is an exception case of Rule 611(order protection rule) under regulation NMS. SEC requires that "when an order is designated as an ISO, the broker-dealer routing the order must assume the responsibility for transmitting additional orders, as necessary, to execute against any better-priced protected quotations". Therefore, broker-dealers have no incentive to submit ISOs for retail traders. More details refer to https://www.sec.gov/divisions/marketreg/rule611faq.pdf

To examine how retail and institutional order flows evolved over time, we divide the dataset into distinct periods as follows: 1) the pre-surge period, denoted as before January 1, 2021, is captured by the constant α; 2) I_t^S is a dummy that is one during the surge period, which spans from January 1, 2021 to March 31, 2021, and zero otherwise; 3) I_t^{PS} is a dummy that is one after the surge period, which extends from April 1, 2021 to December 31, 2021, and zero otherwise. We estimate the following regression model:

$$\text{Order Flow}_t = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + Controls_{t-1} + \varepsilon_{i,t}, \tag{1}$$

where the order flows measures come from either retail traders or institutions. To control market trading activity and market performance, we include the daily trading volume of SPY, the daily return of the SPX index, and the VIX index. All control variables are lagged for one trading day.

[Insert Table 2 here.]

Panel A of Table 2 reveals a remarkable and statistically significant increase in all four types of order flows: after-hours institutional, intraday institutional, after-hours retail, and intraday retail. In particular, daily intraday institutional and retail trading volumes have grown significantly, with an increase of 9.96 million and 8.39 million shares during the surge period, respectively. Compared to the total intraday trading volume before the surge period, which stood at only 5.98 million shares (as shown in Table 1), the surge in intraday institutional and retail trading volume is thus quite substantial. A similar trend is also observed in after-hours trading sessions, where increases in trading activity are also statistically significant, although of a smaller magnitude. Specifically, the daily after-hours institutional order flow increases by 1.68 million shares, while the daily after-hours retail order flow increases by 0.05 million shares after the market closes.

The coefficients for the post-surge period's trading volumes of all four order types suggest that the activity has returned to pre-surge levels. Of particular interest is the divergence in behavior between institutional and retail order flows. Institutional order flows have insignificant but positive post-surge coefficients, indicating that institutional traders are still engaging in trading activities in post-surge period. On the other hand, retail order flows have negative post-surge coefficients, suggesting that retail traders have chosen to divest from GME stocks after the surge period. This is further supported by the post-surge coefficients results in Panel B.

Furthermore, we discover a dramatic shift in trading volume composition. Panel B in Table 2 reveals that during intraday trading, both institutional and retail trades increase in their market shares by 4% and 5%, respectively. However, the after-hours session reveals a stark reversal. Institutional trade market share surge by 14% (from 22% to 36%), while retail trade market share plummets by 13% (from 22% to 9%).

The results of Panel B demonstrate that institutional traders have taken a substantial amount of the retail traders' market share in after-hours sessions during and after the surge, resulting in a lasting change in the makeup of after-hours trading volume.[22] Before the surge, both institutional and retail trading volumes made up around 22% of the total trading volume. Following the short squeeze, institutional trade volume became dominant, accounting for 36% of total after-hours activity. In contrast, retail trader contribution dropped sharply to a mere 9%.

Because most significant GameStop price movements occurred during after-hours session, this shift in volume composition emphasizes the impact of institutional trading on the short squeeze. With institutions dominating after-hours trading, the overall influence of retail traders on price action was greatly diminished.

4.4 Which Trades Move the Price?

Although increased institutional trading and significant stock price surges occur concurrently in after-hours sessions, it remains unclear how these price movements are primarily driven by retail

[22] We observe a significant increase in institutional trading behavior during the surge period only for GameStop. Institutional trading for the matched firm, BestBuy, remain the same during this time. The non-squeezed meme stocks experience little changes in intraday institutional trading and a rise in after-hours institutional trading, however the size is only one-third of that of GameStop. The results for matched firm and meme stocks can be found in Appendix A1.

or institutional trades. In this session, we present in three aspects to demonstrate that institutional trades are the main driving force behind the upward movement of the stock price. This direct evidence confirms the presence of a short squeeze in GameStop.

4.4.1 Cumulative Signed Volume

We begin our investigation using the Lee and Ready (1991) algorithm to differentiate between buyer-initiated and seller-initiated institutional trades. For retail trades, we adopt Boehmer et al. (2021)'s approach to label them as buy or sell. We then use all trade data, which includes intraday and after-hours trading sessions, to calculate the daily signed share volume, the difference between buy and sell volume, for both institutional and retail trades.[23] The cumulative signed volumes for institutional investors and retail traders are shown in Figure 4.

[Insert Figure 4 here.]

Examining the left side of Figure 4, a pattern is evident: institutional investors began to increase their ownership of GameStop shares from late August 2020. This is in line with the large purchase by Ryan Cohen, the former CEO of Chewy.[24] Following this, retail traders also began to buy more of the stock in early October, which coincides with GameStop's announcement of a multiyear strategic partnership with Microsoft.[25] This news caused a remarkable one-day price jump, taking the stock from \$9.54 to \$13.49 on October 8, 2020. At the same time, GameStop's short interest exceeded 100% of its total float.[26]

When we take a closer look at the surge period (as seen in the right panel of Figure 4), an interesting pattern emerges: retail traders significantly decreased their ownership just before the

[23] By employing Barber et al. (2023) method to identify the direction of retail trades, our findings become more significant. The results are presented in Appendix A2.

[24] Ryan Cohen submitted a Form 13-D disclosing ownership of 5.8 million GameStop shares on August 28. Subsequent amendments to the form were made on August 31, September 21 and December 17. His September 21 (December 17) filing indicated ownership of 6.5 (9.0) million shares, reflecting a 9.98% (12.9%) stake.

[25] Refer to https://news.microsoft.com/2020/10/08/gamestop-announces-multiyear-strategic-partnership-with

[26] Refer to https://seekingalpha.com/news/3620910-gamestop-jumps-21-percent-after-microsoft-partnership-a

dramatic increase in price. From January 13 to January 22, 2021, they sold off the stock, while its price rose from \$19.95 (closing price on January 12) to \$65.01 (closing price on January 22). On the other hand, institutional investors kept up their considerable net buying of GameStop shares until January 26, 2021, when the price skyrocketed from January 26's close price of \$147.98 to January 27's open price of \$354.83.

Interestingly, after the short squeeze was triggered during after-hours session on January 26, the total amount of shares bought and sold by both institutional investors and retail traders remained steady from January 27 to February 24. This implies that there was no significant daily order discrepancy between institutional trades and retail trades during this period. This result may initially appear to be in opposition to the expected large buying from short sellers during the short squeeze, as demonstrated in Mitts et al. (2022), wherein the number of shares returned by short sellers experienced a substantial increase on January 27 and stayed high until February 1.

Our model, building upon the work of Brunnermeier and Pedersen (2005) (presented in Section 7), sheds light on this phenomenon. Before the short squeeze, both short sellers and institutional speculators are net buyers. Additionally, options market makers exhibit a similar buying pattern due to the positive gamma exposure of options investors (explained in Section 7). However, when the short squeeze triggers and the price spikes, this serves as a signal for institutional speculators to exit their positions. They then sell their holdings to short sellers, explaining the flat cumulative order imbalance in the institutional order flow observed in Figure 4 after January 27th.

In summary, institutional net buying emerges as the likely dominant force behind the surge period's large price movement, according to an analysis of institutional and retail daily signed volume. Retail trading appears to have played a lesser role.

4.4.2 Weighted Price Contribution

In this section, we aim to identify the types of trades that are responsible for the price fluctuations observed during intraday and after-hours trading sessions. To do this, we calculate the weighted price contribution (WPC) to measure the extent to which a particular type of trades affects the total price change of a stock, as previously done in studies such as Barclay and Warner (1993), Barclay and Hendershott (2003), and O'Hara et al. (2014). Specifically, the WPC is defined as

$$WPC_t^i = \frac{\sum_{n=1}^{N} \delta_{n,i} r_{n,t}}{\sum_{n=1}^{N} r_{n,t}}, \tag{2}$$

where suppose N trades are executed on day t, $\delta_{n,i}$ is an indicator variable that takes the value of one if the nth trade is type i trade, and zero otherwise. $r_{n,t}$ is the difference between the price of trades n and $n-1$. The weighted price contribution for particular types of trades is defined as the sample mean of the daily WPC_t^i.

[Insert Table 3 here.]

The positive WPCs for institutional trades in Table 3 reveal that institutional investors are the main force behind the price movements of GameStop's stock, both during intraday and after-hours trading, in both surge and non-surge periods. On the other hand, retail investors have a restraining effect on the price, as indicated by the negative WPC values for their trades. The WPC of 44.96 for institutional trades during after-hours sessions in the surge period is much higher than the WPC of -1.13 for retail trades during the same period, which demonstrates the greater informativeness of institutional trades. Even during normal times, after-hours institutional trades have a significant impact on the stock price, with a WPC of 3.37, while retail trades only account for 0.33 of the WPC. These results are in line with the findings of Chakravarty et al. (2012), which suggest that informed institutions predominantly use ISO trades.

Retail investors are more active in their trading during regular trading hours, but their trades usually go against the current price trend, leading to a negative WPC for intraday trades. This is especially true during surge periods, where the WPC for retail trades is much lower (-155.65) than during normal times (-5.84). On the other hand, institutional trades tend to follow the intraday price movement, resulting in positive WPC values of 3.64 during normal times and 8.90 during surge periods.

These findings not only validate the observations made through cumulative signed volume analysis, but also provide novel insights into the distinct roles played by institutional and retail investors in influencing GameStop's price movements during various trading sessions.

4.4.3 Intraday High-Frequency Regression

In this section, we try to answer whether institutional and/or retail trading contribute to the increases in high-frequency prices. We first construct the scaled 1-minute order imbalance measure for different types of trades, which is defined as the ratio of 1-minute signed volume to 1-minute total volume. Next, we estimate the following model:

$$\begin{aligned} R_t =& \alpha + \beta_1 Retail_t + \beta_2 Inst_t + \beta_3 Retail_t \times I_t^S + \beta_4 Inst_t \times I_t^S \\ &+ \beta_5 Retail_t \times I_t^{PS} + \beta_6 Inst_t \times I_t^{PS} + \varepsilon_t, \end{aligned} \tag{3}$$

where R_t are 1-minute midpoint quote returns measured by basis points, $Retail_t$ and $Inst_t$ are 1-minute order imbalance ratios for retail trades and institutional trades, I_t^S and I_t^{PS}, as before, are dummy variables for surge period and post-surge period. We restrict our analysis to intraday data exclusively because the after-hours trading session exhibits low liquidity, even during the surge period. Using the scaled order imbalance measure in such illiquid conditions could yield misleading results.

[Insert Table 4 here.]

Table 4 reveals a clear pattern. During normal times, a 1% rise in institutional order imbalance leads to a 0.1555 bps return on GameStop, while the same increase in retail order imbalance yields a smaller 0.0396 bps return. The surge period amplifies this effect: a 1% rise in institutional order imbalance generates an additional 0.9073 bps return, compared to 0.1474 bps for retail order imbalance. This demonstrates that both retail and institutional net trading directions align with corresponding price movements, but institutional trades exert a stronger influence on GameStop's price.

This influence is further emphasized by the consistently larger coefficients for institutional trades. Regressions focusing solely on retail or institutional variables underscore this point: the high R^2 value (0.111) in column 3 is primarily driven by institutional variables (0.108 in column 2). While retail order imbalance measures correlate significantly with contemporaneous returns, their explanatory power remains limited.

To summarize, our analysis encompasses results from various time frame levels, including daily, different intraday trading sessions, and intraday 1-minute intervals, and consistently shows that institutional trading had a positive effect on the price of GameStop, particularly during the surge period. This is in contrast to the comparatively limited influence of retail trading. This evidence supports the idea that a short squeeze occurred in GameStop in early 2021, and institutional trading played an important role in the price increase.

5 The Role of the Options Market

Table 1 illustrates the pronounced increase in options trading volume during the surge period. In this section, we show that as implied volatility escalates within surge period, options trading exerts a notable influence on the stock market. This impact encompasses overnight return predictability and subsequent market quality. Importantly, we provide evidence that option market makers mitigate their gamma exposure before the surge period with retail option trades. In contrast, sophisticated option traders maintain elevated gamma levels throughout surge periods, exposing market makers to substantial overnight gamma risk. The model presented in Section 7 elucidates how positive gamma exposure benefits institutional speculators by not only enhancing their short squeeze profits but also by aligning the trading behavior of option market makers with their own. Consequently, despite the absence of an intraday gamma squeeze, our analysis reveals a potent after-hours gamma squeeze on the evening of January 26th. Our analysis centers exclusively on regular option trades, excluding trades that violate put-call parity.[27]

5.1 How Does the Implied Volatility Change During Surge Period?

Option prices can provide us with an insight into what the market expects the volatility of the underlying return to be. By examining a cross section of option prices for different strikes, we can estimate the expected volatility of an asset return under the risk-neutral probability measure without specific modeling assumptions. This model-free implied volatility (MFIV) is a sign of the economic uncertainty anticipated by the market and can be beneficial for volatility trading, hedging, and portfolio management. Following the method of Bakshi et al. (2003), we estimate the intraday 7-day annualized MFIV using trade data for all intraday non-overlapped 30-minute time intervals.

[Insert Figure 5 here.]

Figure 5 shows the daily MFIV at the end of the trading day. Since April 2020, the MFIV has remained mostly below 1.50. However, on January 25, 2021, it first surpassed 1.50, and continued to climb steadily, peaking at 3.31 on January 28, 2021. We will explore these four days in more depth in Section 6. Moreover, MFIV remains elevated (mean 1.44) throughout the surge period, gradually declining to 0.50 after March 2021.

As proposed in Easley et al. (1998), an asymmetric information model suggests that informed traders can strategically trade in the options market first. Because the options market closes before the stock market, professional traders might continue trading in the after-hours stock market. This pattern could explain the initial surge in MFIV observed within the options market. Typically, rising MFIV signals upcoming price volatility without indicating directionality. An

[27] Appendix A3 shows a significant increase in put-call parity violation trade execution during the surge period. However, these trades still represent a small proportion of overall options trading activity (0.72% before the surge, 1.05% during).

example is the increase in options implied volatility before earnings announcements, reflecting fundamental uncertainty (as studied in Gao et al. (2018)). However, if options-informed traders possess directional knowledge and participate in both options and stock markets, the MFIV may incorporate directional cues. This could explain its ability to significantly predict overnight returns during the surge period. To test this hypothesis, we run the following regression:

$$R_{t+1} = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + \beta_3 MFIV_t \times I_t^S + \beta_4 MFIV_t \times_t^{PS} + Controls_t + \varepsilon_t, \tag{4}$$

where R_{t+1} can be overnight, intraday and close-to-close return, $MFIV_t$ is the end of day model-free implied volatility on day t. I_t^S and I_t^{PS} are dummy variables for surge and post-surge period, respectively. To control market trading activity and market performance, we include the daily trading volume of SPY, the daily return of the SPX index and the VIX index.

[Insert Table 5 here.]

Table 5 shows that there is a significant overnight and intraday return predictability during the surge period. We observe significant coefficients of β_1 and β_3 for the prediction of overnight and intraday returns but insignificant coefficients of β_2 and β_4 for all cases. This finding confirms that this predictability is short-lived and disappears in the post-surge period. This fleeting predictability can be due to two possible sources: option informed traders trading their directional information in the stock market directly after the option market closes, and/or option market makers transmit the directional information via delta hedging in the after-hours session. In the unlisted results, we included the net open interest delta in the regression, the coefficients of $MFIV_t$ change little. Therefore, we propose that informed option traders are more likely to trade in both the options market and during after-hours stock sessions.

5.2 Do Put Options Become More Popular During Surge Period?

Table 1 highlights a dramatic shift in option trading preferences. During the pre-surge period, call options were favored, with an average daily volume of 28.16 thousand compared to 16.81 thousand for puts. This pattern reversed in the surge period, as put option volume surged to 237.06 thousand, outpacing calls at 172.27 thousand.

In our analysis, we not only identify the direction of option trades, but also use the method proposed by Bryzgalova et al. (2022) to label the options trade with the "SLAN" trade flag, which indicates the use of a single-leg price improvement mechanism, as retail option trades. Furthermore, Moussawi et al. (2024) reveals that options traders who place multi-leg orders display contrasting trading patterns compared to those who use single-leg orders in the SPX options market, particularly when S&P 500-based ETFs experience liquidity shocks. Since multi-leg trades are usually submitted by experienced options traders, we use them as a proxy to comprehend the trading behavior of this particular group.

As we observe a substantial increase on trading volume across different types of option trades during surge period in Table 1,[28] to determine the significance of this change, we estimate the following regression model:

$$OV_t = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + Controls_{t-1} + \varepsilon_{i,t}, \tag{5}$$

where OV_t is the specific type of options volume that is of particular interest to us, I_t^S and I_t^{PS} are dummy variables for surge and post-surge period, respectively. To control market trading activity and market performance, we include the daily trading volume of SPY, the daily return of the SPX index, and the VIX index. All control variables are lagged for one trading day.

[Insert Table 6 here.]

[28] We report the time series of various types of option trades in Appendix A4.

Table 6 reveals a dramatic surge in options trading across various trade types (calls/puts, buys/sells) and investor groups (retail/sophisticated) during and after the surge period. While activity levels moderate post-surge, trading volumes remain significantly higher than pre-surge levels (though with a smaller β_2 than β_1). This pattern contrasts with the stock market (Table 1), where volumes returned to pre-surge levels.

Digging into Table 6, the surge period have particularly high options volume (as indicated by β_1). Put option trades dominate across all investor groups, with retail (sophisticated) traders adding 37.36 (72.40) thousand contracts daily, over half of which (20.71 and 37.36 thousand, respectively) were puts. Interestingly, this pattern shifts in the post-surge period, with call options becoming more popular. Retail (sophisticated) traders add 8.92 (13.96) thousand contracts daily, with the majority (5.68 and 9.08 thousand) being calls. This change in strategy aligns with the SEC report's findings on retail investors' tendency toward puts.

Table 6 also shows increased volume across all options trade types (buying/selling calls/puts) during and after the surge. Curiously, both retail and sophisticated traders (Panels B and C) demonstrate a preference for writing calls and puts during both the surge and post-surge periods. This contrasts with the typical preference for buying options pre-surge. Importantly, with no significant difference in at-the-money implied volatility between the pre and post-surge periods (t-stat=0.20), rational option pricing alone cannot explain this change. This suggests other factors, like investor sentiment and speculation, may be influencing the shift in options trading behavior.

5.3 How Does Options Trading Affect the Stock Market?

Option market makers link the option market and stock market tightly through their delta-hedging activities. To investigate the influence of options trading on the stock market, we must first measure the open interest and options risk exposure, as measured by the options greeks, for the option market makers. In agreement with prior research (Garleanu et al. (2008), Christoffersen et al. (2018)), we assume that option market makers take on the net demand from option end users.

Consequently, we propose a method to calculate the daily net open interest and daily net option greeks exposure for the option end users based on tick-by-tick options trades and daily market close information, as outlined follow:

Step 1: We classify option trades as buyer-initiated or seller-initiated, following the methodology established in Savickas and Wilson (2003). Afterward, we calculate the daily net trading volume, the difference between the buy and sell volume, for each option contract.

Step 2: We sum the cumulative net open interest from the previous trading day with the net trading volume obtained from Step 1 to determine the cumulative net open interest for each option contract on the current day. We get the overall net open interest by summing the net open interest across all unexpired option contracts.

Step 3: We multiply the cumulative net open interest obtained from Step 2 by the corresponding greeks (such as delta) from OptionMetrics. The resulting calculation yields the net greeks exposure for each option contract.

Step 4: We conclude by adding together the net greeks values from Step 3 for all unexpired option contracts. This summation yields the net open interest greeks (such as delta) for the day.

In this section, we analyze net open interest and net open interest delta to highlight the distinct behaviors of sophisticated and retail investors within the options market. Additionally, we demonstrate how option market makers' delta hedging amplifies underlying volatility, significantly impacting after-hours price movements and the quality of the next day's market.

5.3.1 Do sophisticated and retail investors act differently in option trading?

Section 5.2 revealed a surge in put and call trading activity (both buying and selling) during the surge period. Interestingly, traders of all experience levels preferred selling options. This raises the question: Do they hold net long or net short option inventory? As Ni et al. (2021) explain, option market makers' net buying stabilizes prices, while net selling increases volatility. By analyzing the net open interest of option end users, we aim to determine whether market makers' hedging ultimately amplifies or dampens underlying volatility.

[Insert Figure 6 here.]

Figure 6 reveals contrasting trading behaviors between sophisticated and retail option traders. Notably, retail traders tend to buy calls prior to the surge, while sophisticated traders favor buying puts. This aligns with trends observed during the short squeeze, where Reddit users shared their long call positions, and institutional short-sellers revealed their long put positions on 13-F forms. During the surge, both groups decrease their net open interest across all option types. However, this net open interest remains positive, suggesting option market makers hold a net selling position. This, in turn, likely contributes to an increased underlying volatility through their delta hedging trades. After the surge, sophisticated and retail traders diverge in their outlook. Sophisticated traders lean towards purchasing puts, anticipating a decline, while retail traders tend to buy calls, expecting further price increases.

In conclusion, sophisticated and retail traders exhibit distinct preferences before and after price surges. During the surge, high volatility (as seen in Figure 5) prompts all traders to sell options. Although options market makers significantly reduce their net holdings, they still maintain a net selling position. This implies that their delta hedging activities continue to amplify the underlying volatility.

5.3.2 Does Net Open Interest Delta Predict After-Hours Return?

Options market makers actively trade the underlying stock to manage their delta exposure, a tactic particularly important when they hold large delta positions that could influence market liquidity and stock prices. As the counterparty to options traders, market makers possess a net open interest delta that is opposite in direction. To maintain a delta-neutral position, they must hold an amount of the underlying asset equivalent to the net open interest delta; this makes the net open interest delta a crucial metric reflecting the hedging needs of options market makers.

[Insert Figure 7 here.]

The graph in Figure 7 indicates that the net open interest delta has been significantly greater than the cumulative signed volume of retail and institutional traders (as presented in Figure 4) since late 2020. This suggests that delta hedging by option market makers could have a substantial effect on the stock price.

Our analysis reveals that a significant portion of GameStop's price volatility occurs during after-hours trading. However, the unavailability of options trading in these sessions raises the question of how net open interest delta influences after-hours returns. Options market makers typically maintain delta-neutral positions at market close. This suggests that with low gamma and/or minimal after-hours price fluctuations, they may not actively rebalance their holdings in the underlying asset. Nonetheless, substantial after-hours movements leading to significant delta exposure would necessitate trading the underlying stock to mitigate delta risk, even with the increased costs associated with after-hours transactions.

As we observe that the net open interest delta presents different patterns during the pre-surge, surge, and post-surge period, we introduce three dummy variables to explore the relationship between the daily net open interest delta and the subsequent after-hours return via the following model:

$$R_{t+1}^{AH} = \alpha + \beta_1 \Delta_t^i I_t^{BS} + \beta_2 \Delta_t^i I_t^{S} + \beta_3 \Delta_t^i I_t^{PS} + Controls_t + \varepsilon_t, \tag{6}$$

where Δ_t^i is the net open interest delta based on type i trades at the daily market close (4 PM EST), R_{t+1}^{AH} is the after-hours return from 4 PM to next day's market open at 9:30 AM. I^{BS}, I^{S} and I^{PS} are dummy variables representing the before-surge, surge and post-surge period, respectively. To control market trading activity and market performance, we include the daily trading volume of SPY, the daily return of the SPX index, and the VIX index.

[Insert Table 7 here.]

Table 7 reports the regression results of Equation (6). The significant and positive coefficients β_2 indicate that the net open interest delta during surge period can reliably predict after-hours returns. Moreover, This observation holds consistently across various investor groups and different types of options. However, the insignificant coefficients β_1 and β_3 reveal that such predictability is only present during the surge period. Such short-lived predictability may be mainly driven by the trading behavior of institutional speculators who trade stock and options at the same time.

Motivated by the absence of options trading and the hedging requirements of options market makers during after-hours sessions, institutional speculators with long delta options positions or knowledge of this dynamic may strategically trigger price surges. This strategy exploits the reduced liquidity of the after-hours market, where a limited number of trades can generate significant price movements. Furthermore, speculators leverage the delta-neutral positioning of market makers by initiating after-hours trades that create substantial delta imbalances. To maintain delta neutrality, market makers are compelled to hedge these imbalances by buying or selling the underlying stock, further amplifying the price surge. This creates a feedback loop where increased hedging activity by market makers drives prices even higher. Consequently, we may observe a temporary but statistically significant association between the net open interest delta and after-hours returns during periods of price surges.

5.3.3 Does Net Open Interest Delta Affect the Next Day's Market Quality?

The magnitude of the net open interest delta is significant. Therefore, the substantial underlying held by option market makers could decrease the supply in the stock market, potentially leading to a deterioration in market quality in the following days. To test this hypothesis, we obtain five daily market quality measures from the Intraday Indicators database by WRDS and estimate the following model:

$$L_{t+1} = \alpha + \beta_1 \Delta_t I_t^{BS} + \beta_2 \Delta_t I_t^{S} + \beta_3 \Delta_t I_t^{PS} + Controls_t + \varepsilon_t, \tag{7}$$

where Δ_t is the net open interest delta based on all options trades at the daily market close (4 PM EST), L_{t+1} is one of the daily market quality measures, including dollar quoted spread, percentage quoted spread, dollar price impact, percent price impact, and intraday quote-based volatility. I^{BS}, I_S and I^{PS} are dummy variables representing the before-surge, surge and post-surge period, respectively. To control market trading activity and market performance, we include the daily trading volume of SPY, the daily return of the SPX index, and the VIX index.

[Insert Table 8 here.]

Table 8 presents the regression results of Equation (7). The results indicate significant and positive coefficients β_2, which support our hypothesis that a substantial net open interest delta during surge periods worsens the market quality the following day. This is evidenced by observed higher quoted spreads, larger price impacts, and increased intraday volatility.

Furthermore, significant coefficients β_3 associated with dollar-type market quality measures (dollar quoted spread and dollar price impact) can be attributed to the high stock prices during the post-surge period. Larger price stocks tend to exhibit wider dollar quoted spreads, and a similar pattern is observed in the case of dollar price impact. The other insignificant coefficients β_1 and β_3 suggest that during normal times, the net open interest delta does not appear to have a significant impact on the market quality the following day.

Our findings align with prior research by Allen et al. (2023) and the October 2021 SEC report, which both confirm a deterioration in market quality during surge period. By identifying the large net open interest delta as one of the potential reasons for this deterioration, our study contributes additional evidence to the existing body of literature.

5.4 Is There a Gamma Squeeze?

A gamma squeeze occurs when options market makers, seeking to hedge positive delta positions, purchase the underlying stock. This increased demand can propel the stock price upward, potentially explaining the surge observed in GameStop's case. While the October 2021 SEC report did not find conclusive evidence of a gamma squeeze, researchers such as Muravyev (2016) suggest that GameStop may have been vulnerable to the gamma squeeze.

In this section, we first utilize the net open interest gamma measure to evaluate the likelihood of an intraday gamma squeeze on GameStop. Our findings indicate a low probability of such an event during regular trading hours. However, we subsequently present empirical evidence suggesting the potential for gamma squeeze activity during after-hours trading sessions.

5.4.1 Net Open Interest Gamma

Gamma, an option metric, measures how rapidly the option's delta changes in response to fluctuations in the underlying asset's price. From an investor's perspective, a significant positive gamma is indicative of a potential gamma squeeze. In this scenario, the positive gamma compels options market makers to increase (decrease) their holdings of the underlying asset as its price rises (falls) to maintain delta neutrality. Consequently, this hedging activity by market makers can reduce market liquidity and exacerbate upward price movements during an already bullish trend.

[Insert Figure 8 here.]

To identify gamma squeezes, we can monitor the cumulative net gamma and the underlying liquidity. Following a similar approach that calculates the net open interest delta, we calculate the daily net open interest gamma. Figure 8 displays the time series plots of the net open interest gamma at the close of the market. Remarkably, the net open interest gamma consistently maintains a positive value throughout the entire sample period. This implies that delta hedging trades from options market makers may increase the underlying volatility because they buy(sell) the underlying when the price increases(decreases).

While the net open interest gamma remained stable in late 2020, a dramatic shift occurred thereafter. It sharply declined from 16.03 million shares on January 4, to 0.16 million shares at market close on January 26. The net gamma further contracted to 2,270 shares on January 27 and 840 shares on January 28, remaining low for the remaining sampled period. This pattern strongly suggests that options market makers proactively reduced their gamma exposure before the short squeeze event. One tactic to achieve this reduction is a slight increase in implied volatility, attracting more option sellers. Increased option writing observed during and after the surge period (detailed in Table 6) supports this interpretation. This aligns with the findings of Lakonishok et al. (2007), who empirically demonstrate that while investors both buy and sell more options under high volatility, the effect is more pronounced for option writing.

While large daily net gamma values exceeding 16 million shares were observed in late 2020, it is crucial to note that a substantial net gamma alone does not guarantee a gamma squeeze. Options market makers often possess the capacity to manage delta hedging activities without substantially disrupting underlying asset liquidity. To further investigate this dynamic, we directly analyzed GameStop's intraday liquidity for indications of a potential shortage. The maximum daily percentage quoted spread from January 4 to January 22 (marking the final instance where net gamma exceeded the 1 million threshold prior to the short squeeze) reaches only 0.25%. This value aligns closely with the average daily quoted spread observed during the first half of 2020, suggesting a consistently healthy market environment.

Our findings indicate both the presence of a substantial net gamma and a well-functioning intraday stock market environment leading up to January 22, 2021, followed by a subsequent decrease in net gamma. This evidence suggests that options market makers effectively managed their gamma exposure throughout the sample period. Consequently, our analysis provides limited support for the occurrence of a intraday gamma squeeze, aligning with the conclusions of the October 2020 SEC report.

5.4.2 Who Reduce the Net Gamma During Surge Period?

Having observed the significant decrease in net open interest gamma during and after the surge period, we seriously test the significance of this change and investigate what types of options trade underlie the decrease in gamma. To answer these questions, we estimate the following model:

$$\Gamma_t^i = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + Controls_{t-1} + \varepsilon_{i,t}, \tag{8}$$

where Γ_t^i is the net gamma calculated based on the specific type i of options. To control market trading activity and market performance, we include the daily trading volume of SPY, the daily return of the SPX index, and the VIX index. All control variables are lagged for one trading day.

[Insert Table 9 here.]

Table 9 reports the estimation results for Equation (8). In particular, significant coefficients β_1 and β_2 are evident across various types of options trade. This robustly confirms that the consistent reduction in gamma across different options trade both during surge and post-surge periods.

We observe a significant decrease in average net gamma for all options trades during the surge period, with a reduction of 4.44 million shares compared to the pre-surge average. Notably, call options account for the majority of this decline, contributing 68% (3.01/4.44) to the net gamma decrease. This shift is likely attributable to two key factors. First, as the underlying price surges, long ATM (at-the-money) and OTM (out-of-the-money) call options with positive gamma become deep in-the-money (ITM) positions with close-to-zero gamma. Second, increased call writing activity by traders further contributes to net gamma reduction. Examining the post-surge period, this pattern intensifies. The average net gamma registers a further reduction of 5.01 million shares, predominantly driven by call option trades, mirroring the trend observed during the surge period.

Analysis of trades executed by sophisticated traders reveals a divergent pattern. For this group, the gamma reduction is primarily driven by put option trading (0.61/1.01=60%). Notably, the post-surge period witnesses an even more significant gamma reduction (1.65 million shares), representing a 63% increase (1.65/1.01-1) compared to the surge period. However, for all options trades combined, the post-surge reduction in gamma is just 13% higher (5.01/4.44-1). This observation suggests that sophisticated traders maintained a consistently positive gamma throughout the sample, perhaps anticipating the potential for the options they hold to contribute to a gamma squeeze scenario. Our model in Section 7 confirms that holding positive gamma is the optimal choice for institutional speculators.

Analyzing retail trader behavior (seventh to ninth columns) reveals a pattern similar to the overall scenario. In particular, call options drive 72% (0.49/0.68) of their gamma reduction. Additionally, the surge and post-surge gamma reductions are comparable in size, differing by only 13% (0.77/0.67-1=13%). While the Bryzgalova et al. (2022) method captures only a subset of retail trades, the striking similarity between this subset and the aggregate data suggests that retail traders play a substantial role in driving the overall gamma reduction.

In sum, the early months of 2021 saw options market makers keenly addressing gamma risk, successfully controlling net gamma exposure ahead of the short squeeze. Certain sophisticated traders chose to strategically maintain elevated gamma levels during surge periods, possibly with the intent of stimulating a gamma squeeze scenario. Ultimately, options market makers skillfully managed their gamma exposure through involvement with retail trades.

5.4.3 Overnight Gamma Risk

Option market makers appear to significantly reduce their gamma exposure prior to a short squeeze. However, the overnight price movement during the surge period is still considerable. Does the gamma exposure reduction allow them to avoid overnight delta trading despite the surge? We can answer this question by assuming market makers only engage in discrete delta hedging at two times: 4 PM EST (market close) and 9:30 AM EST (market open). This type of hedging leads to a hedging loss for the market makers since they are holding a short gamma position.[29] Following the methodology of Boyle and Emanuel (1980), who demonstrate the gamma risk inherent in discrete delta hedging, we term this loss "overnight gamma risk" and approximate its measure as:

$$Risk_t = \frac{1}{2}|\Gamma_t \sigma_t^2 (S_{t+1,open}^2 - S_{t,close}^2)|dt, \tag{9}$$

where Γ_t is the net open interest gamma on day t, σ_t is the implied volatility selected by option market makers for delta hedging calculation. In this case, we select the 7-day MFIV on day t. $S_{t,close}$ and $S_{t+1,open}$ are the closing price of the stock on day t and the open price on day $t+1$. dt is the time interval between two discrete hedgings, and we make it equal to $1/365$.

[Insert Figure 9 here.]

Figure 9 reveals a significant and unexpected surge in overnight gamma risk on January 26, 2021, reaching approximately $145 million (calculated using Equation (9)). This surge implies that option market makers would incur this loss if they had solely rebalanced their delta positions at the January 27 opening price of $354.83. This scenario assumes that they neglected to perform more precise delta hedging throughout the overnight period while the price surged from its closing value of $147.98 on January 26th.

The evidence strongly suggests that option market makers are compelled to perform delta rebalancing on the night of January 26, despite their attempts to effectively manage their gamma exposure. In other words, a gamma squeeze occurs on the night of January 26.

6 What make matters worse?

We present evidence to demonstrate the presence of a short squeeze and a gamma squeeze on GameStop. In this section, we draw attention to two facts suggesting that the extraordinary surge

[29] Figure 8 present that the option end users hold net long gamma positions, therefore, option market makers hold short gamma position.

during January 26 to 28 may be linked to something that had been overlooked. First, institutional investors keep a close eye on the social media platform, allowing them to respond much faster than retail traders. Second, the components of options market maker holdings has a major impact on their overnight delta hedging, leading to a special type of gamma squeeze in the GameStop case, which we term an "after-hours gamma squeeze".

[Insert Figure 10 here.]

Figure 10 depicts the intraday price movement and trading activities on three big move days (January 26, 27 and 28). We observe that the stock price becomes more volatile during the overnight session, which is contrary to the typical volatility seasonality.[30] In contrast to the active intraday session, the overnight session is relatively quiet measured by trading activities. As we have demonstrated previously, the overnight institutional trading drives the overnight price surge.

6.1 Who Follows Influencers More Closely?

Previous research has suggested that amateur investors who post on Reddit and Twitter have the upper hand over professional traders who short sell, resulting in a significant loss for the latter in the GameStop case, thus creating a David-versus-Goliath story. However, institutional speculators, who may not be active on Reddit and Twitter, can still keep track of these social media platforms in real time. This allows them to engage in front-running, which involves buying from the market before retail investors and then selling to retail investors who read those influencers' post later.

[Insert Figure 11 here.]

At 4:08:02 PM EST on January 26, Elon Musk tweeted "Gamestonk!!" with a Reddit link. Figure 11 displays the second-by-second level trading activities and the price chart from 4:07:00

[30] On average, stock intraday volatility is much higher than overnight. See Oldfield and Rogalski (1980) and Stoll and Whaley (1990).

to 4:09:00. As demonstrated in the figure, three ISO orders were executed three seconds after the tweet was sent. Additionally, for the following 10 seconds (4:08:06 to 4:08:15), institutions traded a total of 89.5 thousand shares, while retail traders were inactive during this period.[31] This example indicates that institutions are able to keep track of the renowned social media platform in real time, and can act on the opinion of influencers within seconds by trading in the market.

[Insert Figure 12 here.]

Keith Gill, a famous Reddit influencers and individual retail trader, attended a US House hearings on GameStop and posted his GameStop holdings daily during the period of significant price movement.[32] Figure 12 reveals distinct trading patterns on days with large price movements. Institutional traders are active in the morning and after-hours sessions, while retail traders dominate the regular trading session. While some brokers allow overnight orders, these are processed at 8 AM, explaining the retail spike then. Interestingly, the cumulative signed volume figure shows jumps after 3 PM, coinciding with Keith Gill's posts. Additionally, institutional investors tend to sell upon market open, while retail traders buy. In the afternoon, this flips - institutions buy as retail traders close their longs. This pattern only emerges on big move days. Combined with the overnight price surge information, it suggests that institutions may be buying after Gill's posts and then selling as retail traders pile in at market open. This reveals a novel "pump and dump" dynamic driven by cooperation between institutional speculators and influencers, aligning with the social network effect model proposed by Pedersen (2022).

In essence, these observations suggest a potential trend: institutional traders might be more responsive to influencer activity than retail traders.

[31] During pre-surge period, the daily overnight trading volume is 260 thousand shares reported in Table 1.

[32] At 15:05:36 on January 25, Keith Gill shared his GameStop stake. The following day, at 15:08:03 on January 26, he posted it again. On January 27, he posted it a third time at 15:15:35, and on January 28, he posted it for the fourth time at 15:06:23.

6.2 After-Hours Gamma Squeeze

While our analysis demonstrates effective gamma management by option market makers during regular hours, the overnight gamma squeeze of January 26 highlights a critical point. When faced with high overnight volatility, the composition of option market makers' holdings, a factor often overlooked, becomes a key factor for managing risk.

Long option positions exhibit positive gamma that changes with the underlying price, peaking near at-the-money (ATM) with moderate implied volatility. Notably, when investors hold large amounts of deep out-of-the-money (OTM) calls, an overnight price surge can unexpectedly transform them into ATM calls. This shift creates significant gamma exposure for option market makers even if their gamma exposure is low at the closing of the option market. To further complicate matters, the overnight closure of the options market means that they can only manage their delta exposure by trading the underlying asset in the illiquid after-hours session. This confluence of factors sets the stage for what we term an "after-hours gamma squeeze," exemplified by the events of January 26th and 27th, 2021.

[Insert Figure 13 here.]

Figure 13 illustrates the relationship between signed volume for high strike price options and GameStop's overnight price fluctuations on two key dates. On January 26, 2021, GameStop closed at $147.98 (intraday high: $150). The upper left chart tracks signed volume across option contracts at different strike prices (excluding $150 contracts, all others are newly issued on that day). The upper right graph shows the overnight midpoint quote price at 1-minute intervals.

Elon Musk's "Gamestonk!!" tweet at 4:08 PM triggered a rapid price surge from $144.55 to a peak of $247.445 at 5:04 PM, followed by stabilization between $200 and $250 until 8 PM. This sustained activity suggests a potential gamma squeeze.

Options theory dictates that the gamma of long OTM call options increases with price, peaks when options become ATM, then decreases as calls go ITM. While total net gamma (sum of individual trades) also fluctuates, its behavior depends on the composition of holdings. For instance, similar positive signed volumes for $175 and $190 strike prices could lead to a relatively stable positive net gamma between $170 and $190, as options move in and out of the money. This demand for hedging between $170 and $190 could drive market makers to continue to buy the underlying. However, above $230, total gamma might decline as all calls become ITM, potentially easing the sustained buying pressure. Therefore, the price kept around $230.

Contrasting this, January 27's overnight session shows substantial positive signed volumes for newly-issued calls with $400 and $430 strike prices, and negative volume for $500 calls. The stock price surged through $400, peaked at $503, and stabilized around $430 by 7 AM. The presence of $500 call options might have exerted downward pressure on total gamma, making it harder to break the $500 barrier.

In summary, changes in total gamma as the underlying price fluctuates overnight have significant consequences. Market makers are limited to managing delta risk in an illiquid stock market. Crucially, the composition of holdings shapes total net gamma exposure and can even trigger an ”after-hours gamma squeeze.” This phenomenon warrants further theoretical study and the attention of regulatory bodies.

7 A Model on Short-Selling Predators

The empirical evidence presented in the preceding sections confirms three key points. First, institutional order flows move the stock price. Second, certain sophisticated options traders maintain a positive gamma during surge period. Third, the occurrence of an after-hours gamma squeeze was evident on the night of January 26. In this section, we further develop the model proposed by Brunnermeier and Pedersen (2005) to elucidate the behavior of short selling predators. Our model is consistent with the empirical evidence we have observed.

7.1 Equilibrium Price

Following Brunnermeier and Pedersen (2005) setup, we consider a continuous-time economy with two assets, a riskless bond with zero risk-free rate and a risky asset. The risky asset has an aggregate supply of $S > 0$ and a final random payoff v with $E(v) = \mu$ at time T. The price of the risky asset at time t is denoted by $p(t)$. Diverging from Brunnermeier and Pedersen (2005)[33], we extend their model by accommodates three types of agents: large strategic traders encompassing both short sellers and predators, long-term investors, and delta-hedging traders.

Strategic traders, $i \in 1, 2, \cdots, I$, are risk neutral and seek to maximize their expected profit. Each strategic trader is sizable, and thus, their trading has an effect on the equilibrium price. Therefore, they act strategically and consider their price impact when trading. Each strategic trader i has a given initial holding, $x^i(0)$, of the risky asset and can continuously trade with his trading itensity, $a^i(t)$. Hence, his holding at time t, $x^i(t)$, of the risk asset is

$$x^i(t) = x^i(0) + \int_0^t a^i(\tau) d\tau, \tag{10}$$

and the aggregate holding for all strategic traders is $X(t) = \sum_i x^i(t)$.

In addition to strategic traders, the market is populated by long-term investors such as pension funds and hedged funds. The long-term investors are price-takers and have an aggregate demand

$$Y(p) = \frac{1}{\lambda}(\mu - p), \tag{11}$$

where λ is the price impact which is measured the liquidity of the risky asset. The demand (11) depends only on the current price p, that is, long-term investors do not attempt to profit from price fluctuations. Finally, the delta-hedging traders have an aggregate hedging demand

$$D(t) = \int_0^t d\Delta(\tau) = \int_0^t \Gamma(\tau) dp(\tau) \approx \Gamma(p(t) - p_0). \tag{12}$$

[33] Their model includes large strategic traders and long-term investors.

We assume that the cumulative net open interest gamma is stable in $t \in [0,T]$.

Therefore, the total demand is $Y(p(t))+X(t)+D(t)$, and the total supply is $S-\Delta_0$, where Δ_0 is the net open interest delta at $t=0$. The market clearing price $p(t)$ solves $Y(p(t))+X(t)+D(t) = S-\Delta_0$, and so the price is

$$
\begin{aligned}
p(t) =& \frac{1}{1-\Gamma\lambda}\mu - \frac{\lambda}{1-\Gamma\lambda}[S-\Delta_0+\Gamma p_0 - X(t)] \\
=& \frac{1}{1-\Gamma\lambda}\mu - \frac{\lambda}{1-\Gamma\lambda}(S-X(t)) + \frac{\lambda}{1-\Gamma\lambda}(\Delta_0-\Gamma p_0).
\end{aligned}
\tag{13}
$$

At the equilibrium price, $\Gamma\lambda < 1$. λ is Kyle (1985) price impact, defined as $\lambda = \frac{|dP|}{dVolume_t} > 0$. On the other hand, $\Gamma = \frac{d\Delta_t}{dP}$. Therefore, if Γ is positive, $\Gamma\lambda = \frac{d\Delta_t}{dVolume_t}$ must be less than 1 in equilibrium, since the additional hedging demand $d\Delta_t$ should be less than the additional liquidity supply $dVolume_t$. Note that in practice, if the hedging demand is temporarily higher than the liquidity supply, the price will increase, then the positive Γ will decrease with the price, thus the price will reach a new equilibrium where $\Gamma\lambda < 1$. If $\Gamma = 0$ and $\Delta_0 = 0$, the equilibrium price (13) is in agreement with the equilibrium established by Brunnermeier and Pedersen (2005).

7.2 Optimal Strategy for Predators

We categorize strategic traders into two groups: short sellers and predators. The number of short sellers is $I^S > 0$, and the number of predators is $I^P > 1$, thus $I^S + I^P = I$. At t_0, short sellers are exposed to financial distress and begin to buy to cover. Predators can easily know the moment t_0 by monitoring the order flow in the stock market. We are interested in the optimal strategy for predators for the time period $[t_0, T]$.

Unlike Brunnermeier and Pedersen (2005),[34] we expect the holdings are different for these two

[34] They assume all strategic traders have the same holdings at t_0.

groups

$$x^i(t_0)\begin{cases} <0 & \text{if trader } i \text{ is a short seller.} \\ \geq 0 & \text{if trader } i \text{ is a predator.} \end{cases} \tag{14}$$

Predators are short-term investors who expect to profit from short sellers. In contrast to short-sellers, predators exhibit either a lack of opinion or a positive perspective on the underlying. This also provides a straightforward method to distinguish between predators and short sellers by analyzing their initial holdings before the occurrence of a short squeeze. Moreover, we assume that both short-sellers and predators hold their opinions during $[t_0, T]$, that is,

$$\text{For all } t \in [t_0, T] \quad \begin{cases} x^i(t) \leq 0 & \text{if trader } i \text{ is a short seller.} \\ x^i(t) \geq 0 & \text{if trader } i \text{ is a predator.} \end{cases} \tag{15}$$

As mentioned earlier, the price impact should be considered due to the size of these strategic traders. We assume that the liquidity capacity for all strategy traders is a given A and strategy traders suffer temporary impact costs at time t if

$$|\sum_i a^i(t)| > A. \tag{16}$$

Moreover, the temporary price impact cost for trader i is $G^i(t,\gamma)$, where γ is a significant large temporary price impact. In other words, the trader i has no temporary price impact if $a^i \in [\underline{a}(i), \bar{a}(i)]$.

We assume that at time T, predators have the position limit, $\bar{x}$, where $\bar{x} \geq x^i(t_0)$, and short sellers have zero position. Furthermore, we also assume that T is large enough to allow short sellers to cover their short positions. Brunnermeier and Pedersen (2005) have position limit throughout the period, but we do not set a position limit in $[t_0, T)$. First, margin trading is popular and the capital of strategic traders is large. Second, the liquidity capacity is limited, and strategic traders would suffer the significant temporary price impact cost if they place large market orders.

The strategic trader i is to maximize his expected wealth subject to the constraints described above, and a strategic trader's objective function is

$$\max_{a^i} E\left(x^i(T)p(T) - \int_0^T [a^i(t)p(t) + G^i(t,\gamma)]dt\right). \tag{17}$$

Following Brunnermeier and Pedersen (2005), we define an equilibrium set as a set of processes $(a^1, \cdots, a^I)$ s.t, for each i, a^i solves Equation (17), taking $a^{-i} = (a^1, \cdots, a^{i-1}, a^{i+1}, \cdots, a^I)$ as given.

Two considerations can help us simplify the objective function. First, it is not optimal to incur the temporary impact cost. Therefore, the optimal trader intensity follows $a^i \in [\underline{a}(i), \bar{a}(i)]$. Second, because $X(t_0) < 0 < X(T)$, from equation (13), the price follows $p(t_0) < p(T)$, therefore, any optimal trading strategy satisfies $x^i(T) = \bar{x}$ for $i \in \mathbf{I^P}$. Based on these two considerations and put equation (13) into equation (17), we rewrite our objective function as

$$\begin{aligned}
&\text{For large } T \text{ and each } i \\
&\operatorname*{Min}_{a^i} E\left(\int_0^T [a^i(t)p(t)]dt\right) \Rightarrow \operatorname*{Min}_{a^i} E\left(\int_0^T [a^i(t)X^{-i}(t)]dt\right), \\
\text{s.t. } & a^i(t) \in [\underline{a}, \bar{a}]
\end{aligned} \tag{18}$$

where $X^{-i}(t) = \sum_{j \neq i} x^j(t)$. This implies that strategic traders (both short sellers and predators) try to minimize their own trading cost, but not taking into account their price impact. These traders, particularly predators, generate profits by capitalizing on the influence of other traders on prices. This fundamental distinction serves as a means to distinguish predatory trading from price manipulation, where a trader's own transactions directly impact prices. Furthermore, the rewritten objective function in (18) indicates that once short sellers decide to close their short positions, which is the time t_0 in the model, optimal strategies for strategic traders, including short sellers and predators, are not related to the underlying price, but only related to the trading intensity.

Until now, we have the same objective function as Brunnermeier and Pedersen (2005) but with slightly different constraints. Following their method, we get the unique equilibrium for our case. To simplify, at t_0, all short sellers' positions are $x^S(t_0) < 0$, and all predators' positions are $x^P(t_0)$, satisfying $\bar{x} \geq x^P(t_0) \geq 0$. The optimal strategy for short sellers is to repurchase at constant speed A/I until time $t^{end} = t_0 - x^S(t_0)/(A/I)$. The optimal strategy for predators is to buy at constant speed A/I during $[t_0, t_0 + \tau)$, then sell their holdings to short sellers during $[t_0 + \tau, t_0 - x^S(t_0)/(A/I)]$.

The optimal trading intensity for predators at different period is

$$a^{i*}(t) = \begin{cases} \frac{A}{I} & \text{for } t \in [t_0, t_0 + \tau); \\ -\frac{(A/I)I^S}{I^P - 1} & \text{for } t \in [t_0 + \tau, t_0 - \frac{x^S(t_0)}{A/I}); \\ 0 & \text{for } t \geq t_0 - \frac{x^S(t_0)}{A/I}, \end{cases} \tag{19}$$
$$\text{where} \quad \tau = \frac{(I^P - 1)(\bar{x} - x^P(t_0)) - x^S(t_0)I^S}{I - 1} / \frac{A}{I}.$$

To ensure $t_0 + \tau \leq t_0 - x^S(t_0)/(A/I)$, $\bar{x} \leq x^P(t_0) - x^S(t_0)$ must hold. This condition implies that the holding change for predators, $\bar{x} - x^P(t_0)$, must be smaller than the demand of short sellers, $-x^S(t_0)$, which should be held in practice. Under this condition, predators have the opportunity to sell their holdings to short sellers before short sellers finish covering their short positions.

With the optimal trading intensity, the price dynamics are

$$p^*(t) = \begin{cases} p(t_0) + \frac{\lambda}{1 - \Gamma\lambda} A[t - t_0] & \text{for } t \in [t_0, t_0 + \tau); \\ p(t_0) + \frac{\lambda}{1 - \Gamma\lambda} \left(A\tau - \frac{(A/I)I^S}{I^P - 1}[t - (t_0 + \tau)] \right) & \text{for } t \in [t_0 + \tau, t_0 - \frac{x^S(t_0)}{A/I}); \\ p(t_0) + \frac{\lambda}{1 - \Gamma\lambda} [I^P(\bar{x} - x^P(t_0)) - x^S(t_0)I^S] & \text{for } t = t^{end} = t_0 - \frac{x^S(t_0)}{A/I}. \end{cases} \tag{20}$$

Equation (20) demonstrates that the price reaches its highest point at $t = \tau$. This indicates the point at which predators complete their acquisitions and transition to selling shares to short-sellers. Furthermore, a comparison of the price function's slope during the predator buying and selling periods reveals a sharp price increase during predator buying, followed by a more gradual decline during the selling phase. Finally, the negativity of $x^S(t_0)$ and the condition $\bar{x} \geq x^P(t_0)$ ensure that the second term of the after-squeeze price, $p(t^{end})$, is positive. This aligns with the observed phenomenon of a relatively high price following a short squeeze. As the social network model proposed by Pedersen (2022) suggests, this elevated price may be attributed to investor overoptimism fueled by social network effects.

Proposition 1 (Predators Move the Price)**.** *The price movement is driven by predators' trading because* $\frac{\partial p^*(t)}{\partial t} = \frac{\lambda I}{1-\Gamma\lambda} a^*(t)$ *for* $t \in [t_0, t_0+\tau)$ *and* $\frac{\partial p^*(t)}{\partial t} = \frac{\lambda}{1-\Gamma\lambda} a^*(t)$ *for* $t \in [t_0+\tau, t^{end})$*, where* $a^*(t)$ *is optimal trading intensity for predators. Moreover, the predators holding,* $x^P(t)$*, over time is*

$$x^P(t) = x^P(t_0) + \int_{t_0}^{t} a^*(t)dt = \begin{cases} x^P(t_0) + \frac{A}{I}[t-t_0] & \text{for } t \in [t_0, t_0+\tau); \\ x^P(t_0) + \frac{A}{I}\tau - \frac{(A/I)I^S}{I^P-1}[t-(t_0+\tau)] & \text{for } t \in [t_0+\tau, t_0 - \frac{x^S(t_0)}{A/I}); \\ \bar{x} & \text{for } t = t^{end} = t_0 - \frac{x^S(t_0)}{A/I}. \end{cases} \tag{21}$$

Proposition 1 suggests that predator trades generate a positive permanent price impact directly proportional to $\frac{\lambda}{1-\Gamma\lambda}$. Importantly, this impact intensifies with increasing λ, making after-hours sessions (where λ is high) attractive targets for predators seeking to significantly influence prices.

7.3 When to Initiate the Short Squeeze?

We first calculate potential predator profits and identify the optimal timing for a short squeeze that would maximize those profits.

Proposition 2 (Predators Profit)**.** *Based on the trading intensity in Equation* (19) *and the price* *dynamics in Equation (20), the predator's profit during $[t_0, t^{end}]$ is*

$$
\begin{aligned}
W^P &= p^*(t^{end})\bar{x} - \int_{t_0}^{t^{end}} p^*(t) a^{i*}(t) dt - p(t_0) x^P(t_0) \\
&= \frac{\lambda}{2(1-\Gamma\lambda)} g(\bar{x}, x^P(t_0), x^S(t_0), I^S, I^P, A),
\end{aligned}
\tag{22}
$$

where $g(\cdot)$ function is always positive.

This demonstrates that predators can always profit from a successfully triggered short squeeze. Moreover, Equation (22) reveals that this profit is independent of the trigger price, $p(t_0)$. Since the $g(\cdot)$ function depends on predetermined market factors $(x^P(t_0), x^S(t_0), I^S, I^P, A)$ and the predators' final holding $\bar{x}$, predators can strategically maximize their profit by influencing Γ and λ.

Proposition 3 (Short Squeeze Timing)**.** *To maximize profit, predators should trigger the short squeeze when λ and Γ are large, because*

$$
\begin{aligned}
\frac{\partial W^P}{\partial \lambda} &= \frac{1}{2(1-\Gamma\lambda)^2} g(\cdot), \\
\frac{\partial W^P}{\partial \Gamma} &= \frac{\lambda^2}{2(1-\Gamma\lambda)^2} g(\cdot),
\end{aligned}
\tag{23}
$$

where $g(\cdot)$ function is defined in Equation (22) and is always positive.

The positive $\frac{\partial W^P}{\partial \lambda}$ means that predators benefit by initiating a short squeeze when λ is higher (indicating more illiquid). This aligns with our observation of large price movements during less liquid after-hours sessions. Several large overnight price movements before January 26 suggest that predators may make multiple attempts to trigger the short squeeze. Similarly, a positive $\frac{\partial W^P}{\partial \Gamma}$ indicates that predators holding options with positive gamma can also increase profits. Our empirical results confirm this, highlighting that sophisticated option traders maintain positive gamma positions during the surge period.

Proposition 4 (Demand for Delta-hedging traders)**.** *Based on delta-hedging traders' aggregate demand function in Equation (12) and the price dynamics in Equation (20), the aggregate demand* *function $D(t)$ is*

$$D(t)=\begin{cases}\frac{\Gamma\lambda}{1-\Gamma\lambda}A[t-t_0] & \textit{for } t\in[t_0,t_0+\tau);\\ \frac{\Gamma\lambda}{1-\Gamma\lambda}\left(A\tau-\frac{(A/I)I^S}{I^P-1}[t-(t_0+\tau)]\right) & \textit{for } t\in[t_0+\tau,t_0-\frac{x^S(t_0)}{A/I});\\ \frac{\Gamma\lambda}{1-\Gamma\lambda}[I^P(\bar{x}-x^P(t_0))-x^S(t_0)I^S] & \textit{for } t=t^{end}=t_0-\frac{x^S(t_0)}{A/I}.\end{cases} \tag{24}$$

Comparing the demand for delta-hedging traders in Equation (24) with predators holding described in Equation (21) reveals a complex relationship. Their trading intensities may be highly correlated, meaning delta-hedging traders can either amplify or counter the effects of predator trading.[35] When $\Gamma > 0$, delta-hedging traders worsen the short squeeze by effectively becoming teammates with predators. In contrast, when $\Gamma < 0$, they can partially mitigate predator influence. Consequently, predators are strongly incentivized to maintain a positive gamma position in the options market, manipulating options market makers into unintentionally supporting their aims. To counteract this, options market makers should carefully manage their Γ exposure, aiming to keep it low before and during the squeeze, as illustrated in Figure 8.

In summary, predators can strategically amplify a short squeeze by targeting periods of market illiquidity and exploiting positive gamma positions within the options market. The after-hours session often presents this ideal scenario, especially when price movements push significant amount of out-of-the-money options into in-the-money positions. This shift serves as a key signal for a potential after-hours gamma squeeze.

8 Conclusion

The GameStop short squeeze stands out as arguably the most dramatic event in the modern history of finance. In January 2021, a staggering 140 percent of the float shares had been sold short,

[35] Option market makers may maintain acceptable delta exposure, eliminating the need to immediately satisfy aggregate demand at each time t. Therefore, perfect alignment of trading intensities occurs only when option market makers actively engage in continuous delta-hedging rebalancing.

leading to a frenzied rush of buying that propelled the price far beyond all expectations. This surge resulted in substantial losses for short sellers and even led to the bankruptcy of certain hedge funds that had taken positions against the bubble.

In our study, we reveal several crucial aspects of GameStop price dynamics that highlights its remarkable dependency on overnight trading activities. Our empirical results show that the prices during the surge period are mainly driven by institutional trades during both the day and after-hour sessions. Moreover, the after-hours volatility prominently hinges on institutional order flows rather than on retail order flows. With an in-depth examination, we find that the prices charge ahead sharply in the illiquid after-hours market, when it is beneficial for institutional speculators to strategically orchestrate a short squeeze in a situation in which short sellers have little possibility of covering their short positions at night.

Furthermore, our analysis identifies the participation of sophisticated option traders who have consistently maintained long gamma positions throughout the surge period. In particular, our evidence suggests that the remarkable increase during the after-hours session on January 26, 2021, is attributable to an after-hours gamma squeeze. The squeeze is caused by sudden price jumps during after-hour sessions and a large positive net open interest in OTM call options, leading to amplified gamma exposure and suboptimal hedging by option market makers who, among the short seller, are forced to buy shares at rising prices.

Although our available data do not permit a definitive identification of which institutional speculators are responsible, we extend the model of Brunnermeier and Pedersen (2005) to explain their potential collective behaviors. Our model predicts that these institutional speculators would optimize their gains by orchestrating the short squeeze within an environment of limited liquidity while simultaneously maintaining a positive gamma position within the options market. Interestingly, these model predictions are well supported by our empirical evidence.

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Figure 1. Time Series of Close Price and Volume

The figures depict the daily close price for GameStop, S&P 500 index and volume for GameStop during full sample period. The sample period covers January 1, 2020, to December 31, 2021.

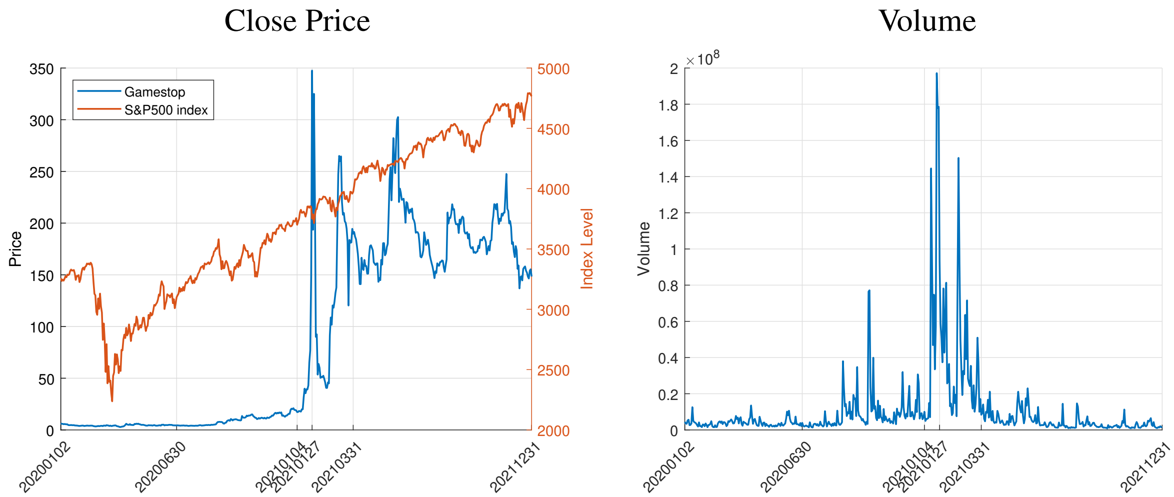
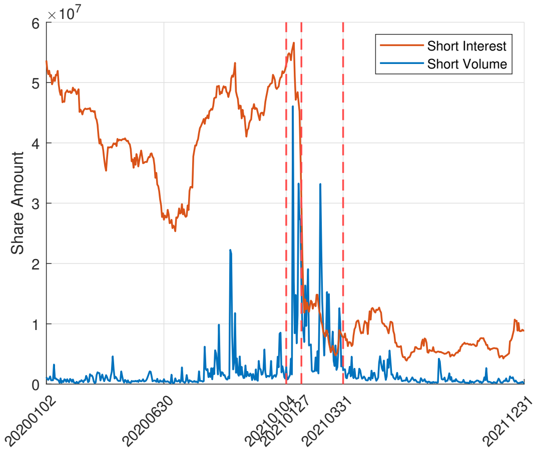


Figure 2. Daily Short Interest and Short Volume

The figures depict the daily short interest and short volume for GameStop (GME) during the full sample period and the surge period, respectively. Short interest is the 'quantity on loan' reported in IHS Markit data. The short volume is obtained from FINRA. The sample period is from January 1, 2020, to December 31, 2021.

Full Sample Period



Surge Period

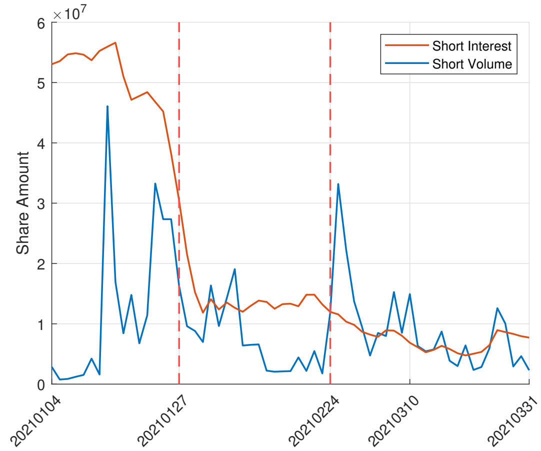


Figure 3. Cumulative Overnight and Intraday Returns

The figures depict the cumulative overnight return and intraday return for GameStop (GME) and BestBuy (BBY) during the full sample period and the surge period, respectively. The overnight return is defined as the return between the closing time of the last trading day at 4:00 PM and the opening time of the current trading day at 9:30 AM. The intraday return is defined as the return from market open to close. The sample period is from January 1, 2020, to December 31, 2021.

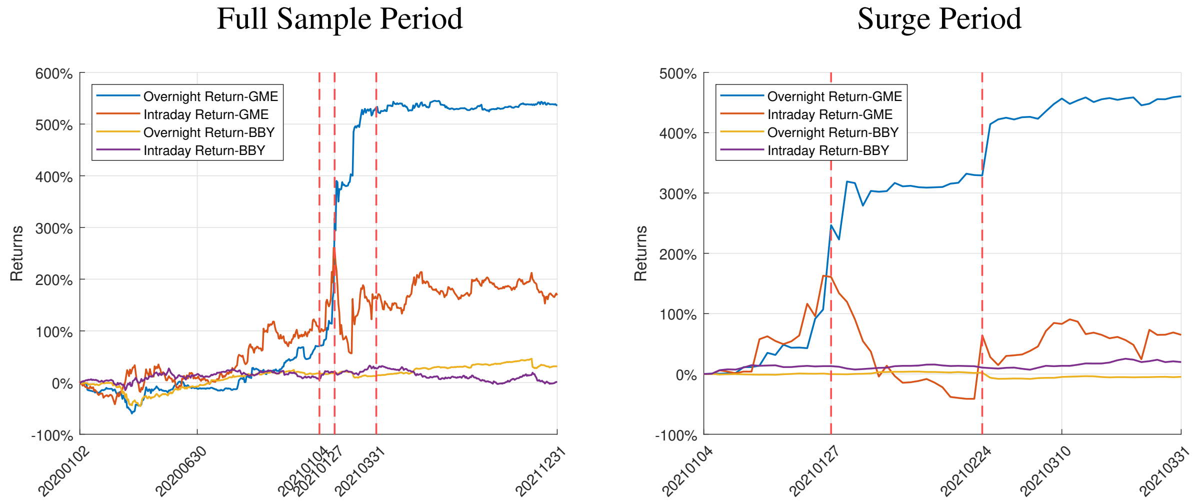


Figure 4. Cumulative Signed Volume for Different Types of Trades

The figures show the cumulative signed share volume for institution investors and retail traders during the full sample period and the surge period, respectively.

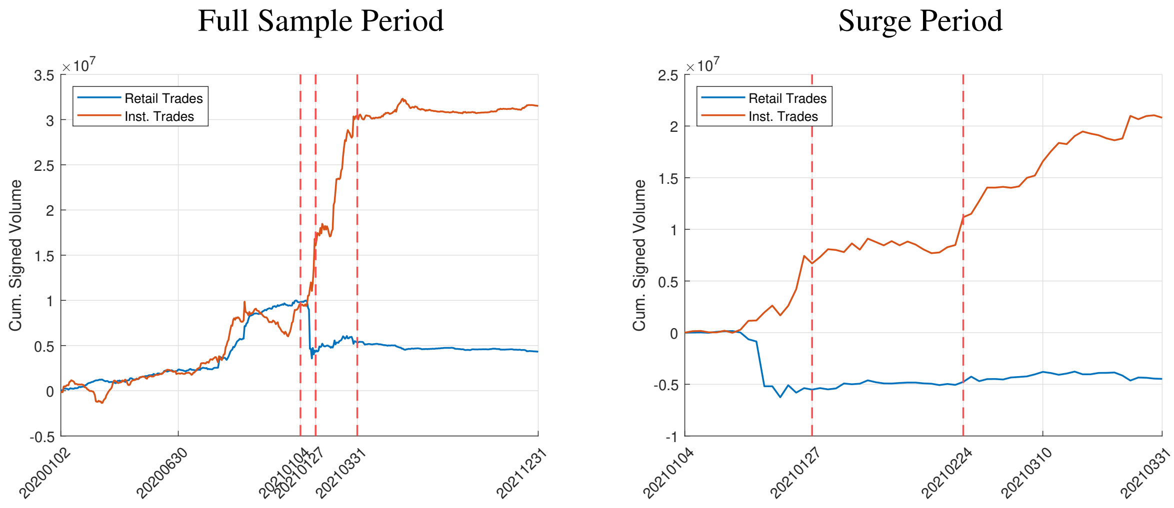


Figure 5. 7-Day Annualized MFIV Over Time

The figures depict the 7-day annualized MFIV during the full sample period and the surge period, respectively. The sample period covers from January 1, 2020, to December 31, 2021.

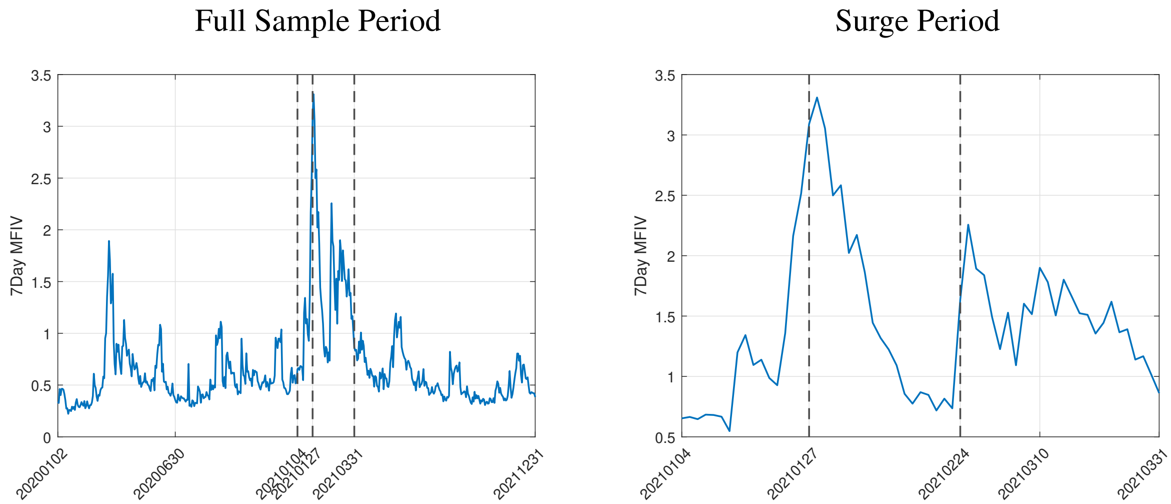


Figure 6. Net Open Interest for Different Group of Traders

The figures plot daily calls and puts net open interest from retail trades and multi-leg trades during the full sample period and the surge period, respectively. The sample period covers from January 1, 2020, to December 31, 2021.

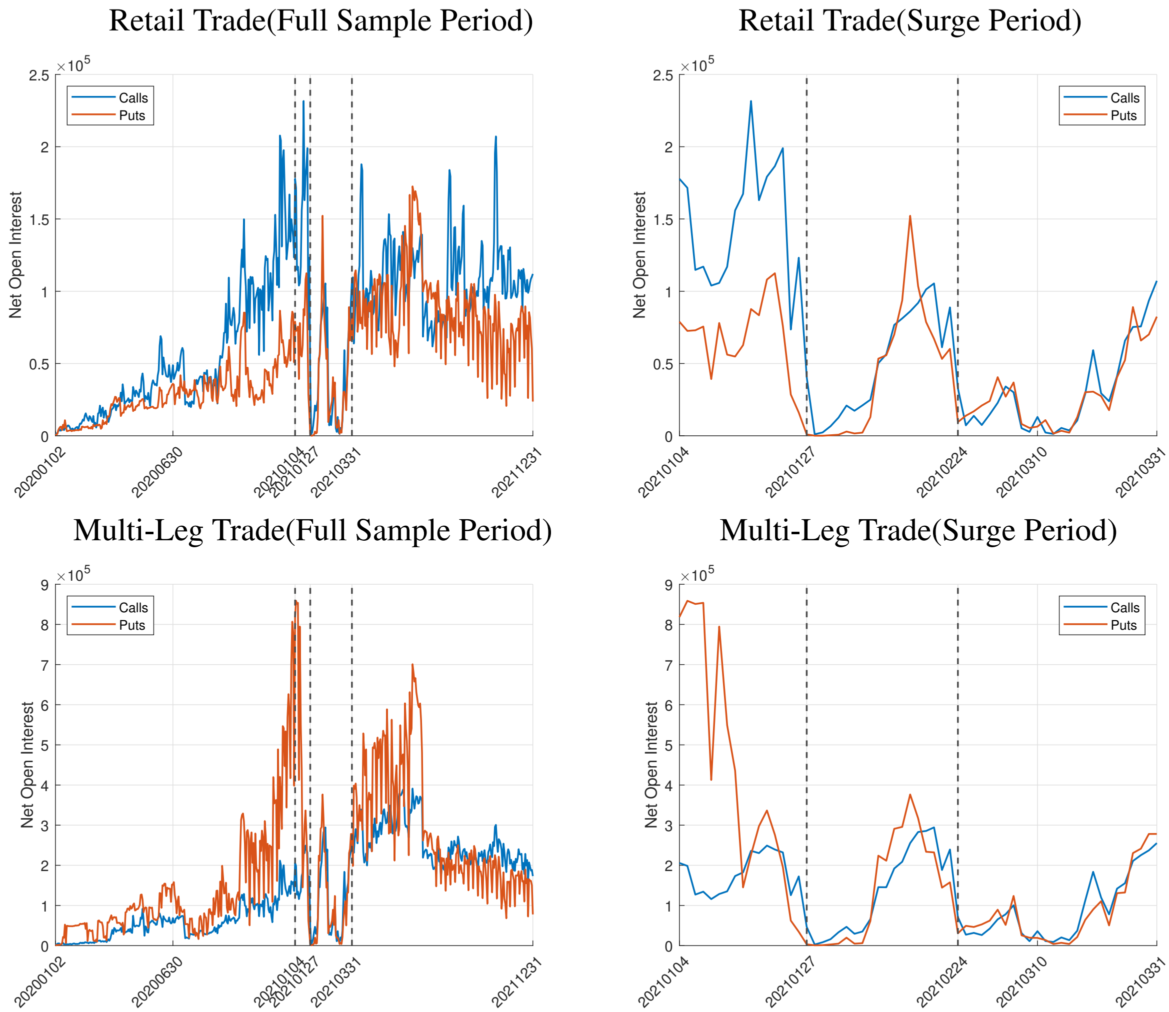
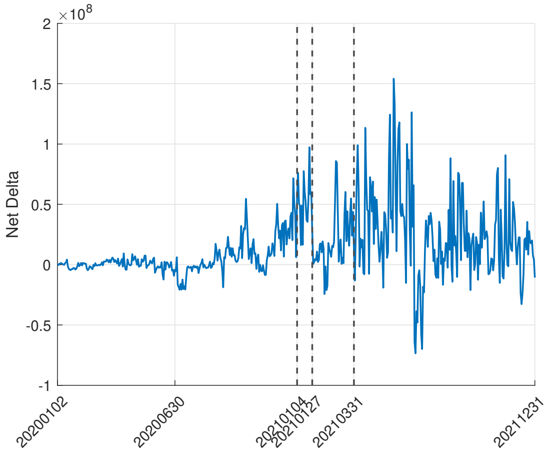


Figure 7. Net Open Interest Delta Over Time

The figures depict the net open interest delta during the full sample period and the surge period, respectively. The sample period covers from January 1, 2020, to December 31, 2021.

Full Sample Period



Surge Period

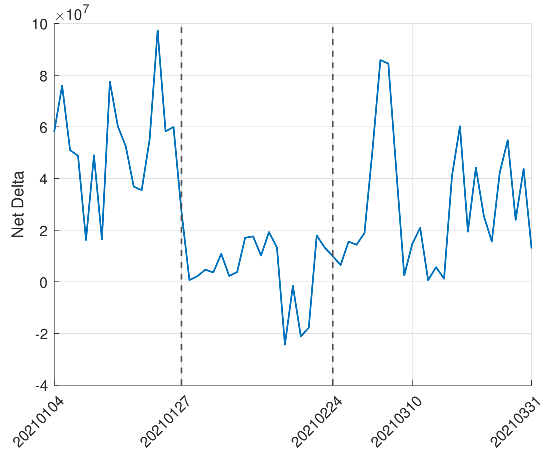


Figure 8. Net Open Interest Gamma Over Time

The figures depict the net open interest gamma during the full sample period and the surge period, respectively. The sample period covers from January 1, 2020, to December 31, 2021.

Full Sample Period | Surge Period

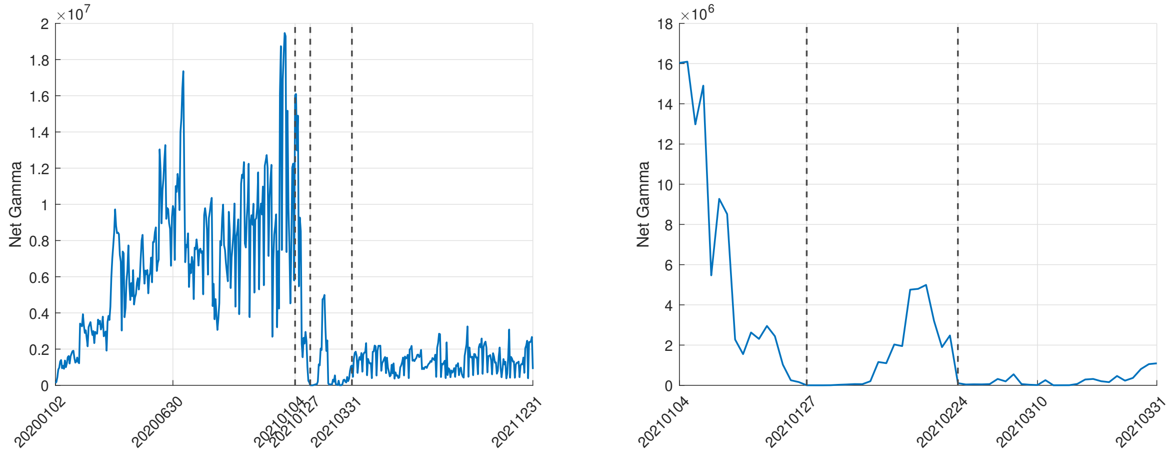


Figure 9. Overnight Gamma Risk Over Time

The figures depict the overnight gamma risk during the full sample period and the surge period, respectively. The sample period covers from January 1, 2020, to December 31, 2021.

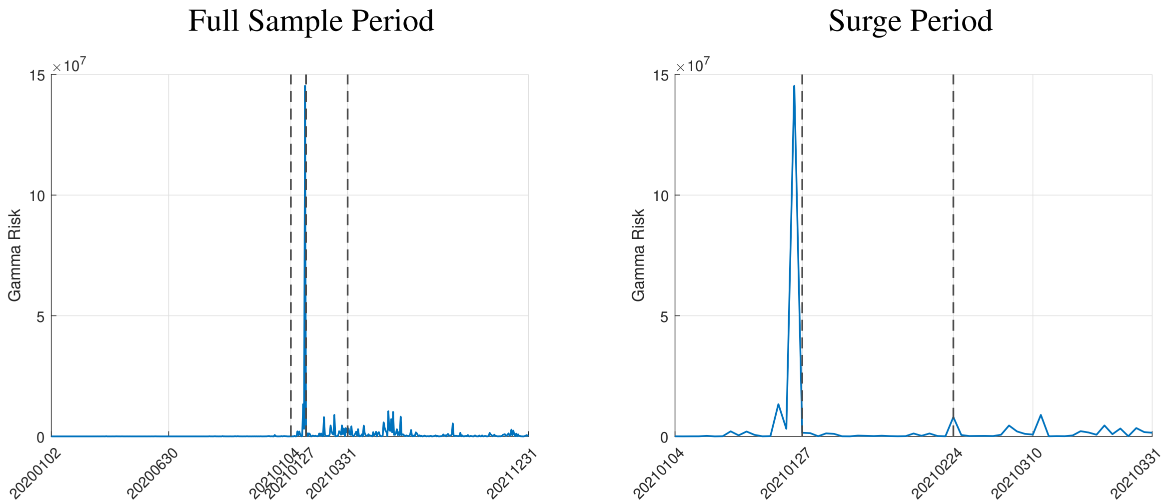


Figure 10. Intraday Price and Trading Activities on Three Big Move Days

The figures present the intraday price movement and trading activities on January 26, 27 and 28, 2021.

Price Volume

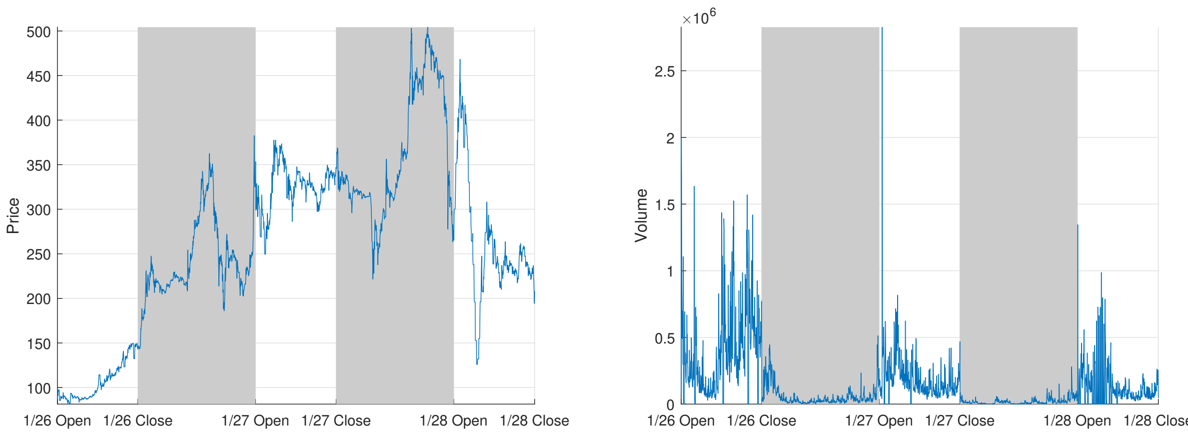
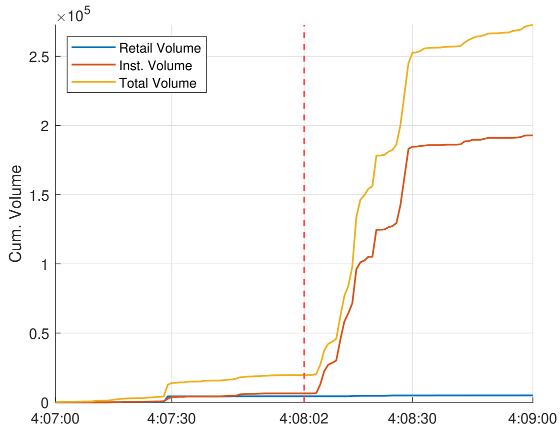


Figure 11. Reaction on Elon Musk's Tweet

The figures present the cumulative trading volume for various order types and price chart one minute before and after Elon Musk tweeted on January 26, 2021.

Volume



Price Chart

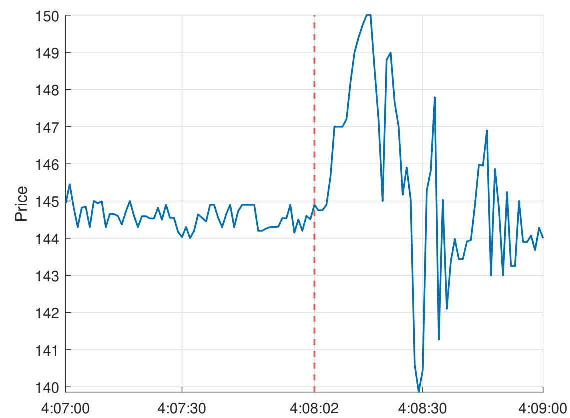


Figure 12. Intraday Trading Pattern on Big Move Days

The figures present the intraday trading density and cumulative signed trading volume for institutional trader and retail traders. Sample covers from January 25, 2021 to January 28, 2021.

Trading Density | Cum. Signed Volume

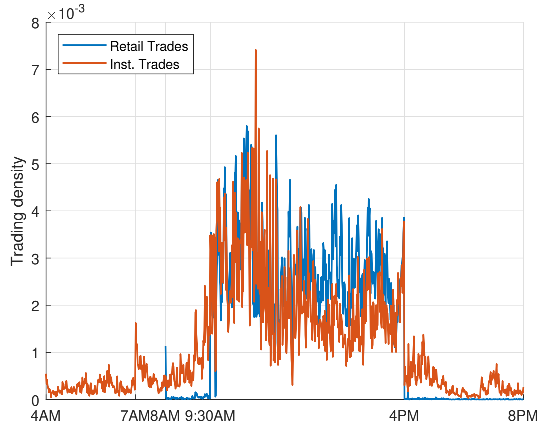


Figure 13. Options Estimated Net Option Interest and Overnight Price Chart on Jan 26 and 27

The figures show the estimated net option interest of high strike price options at market close and corresponding overnight price chart on January 26 and 27.

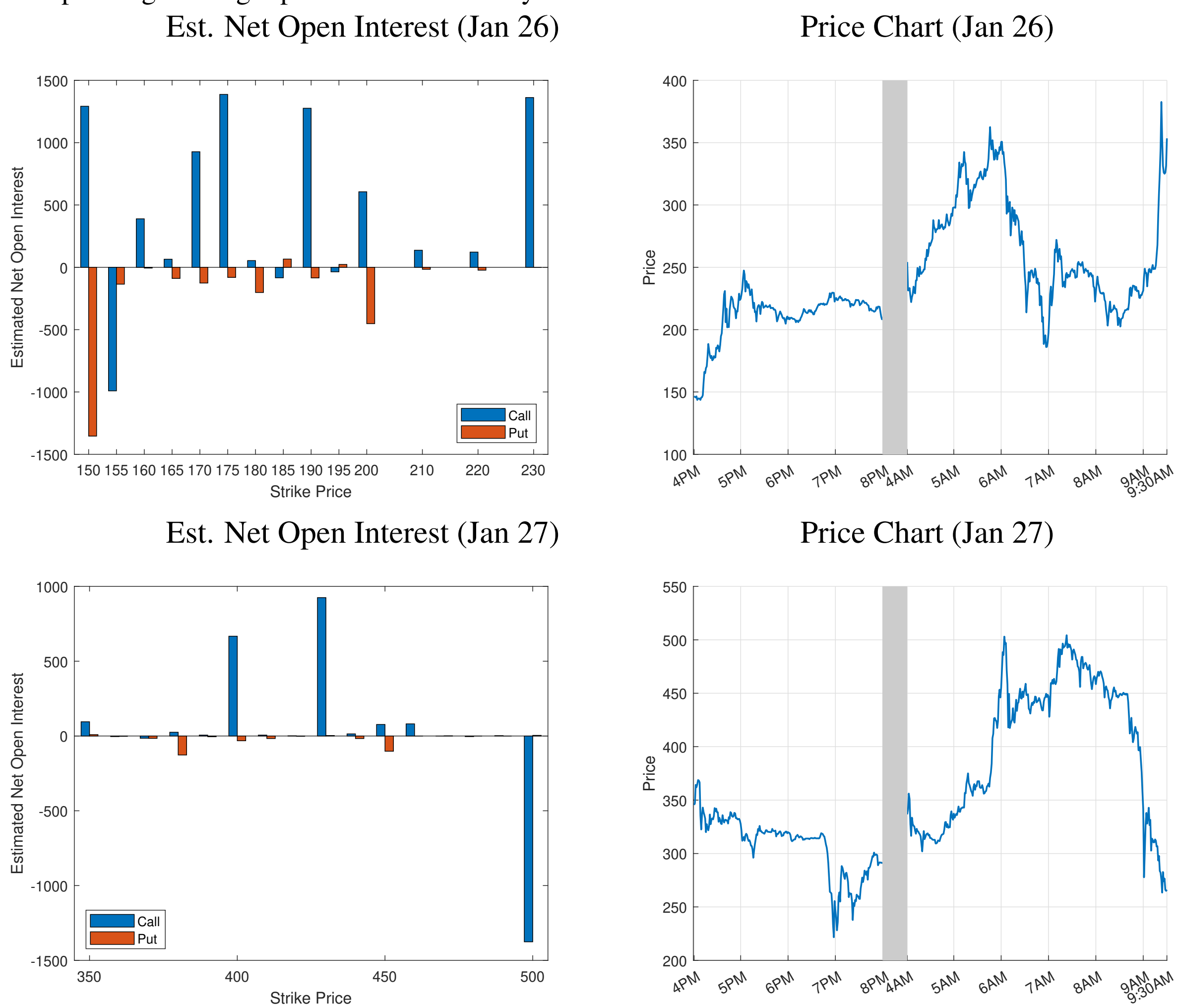


Table 1 Descriptive Statistics

The table reports the summary statistics of return and volume for both the stock and options markets. The pre-surge period is from January 1, 2020, to December 31, 2020, the surge period is from January 1, 2021 to March 31, 2021, and the post-surge period is from April 1, 2021 to December 31, 2021. The sample period covers from January 1, 2020, to December 31, 2021. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	Pre-Surge(1)			Surge Period(2)			Post-Surge(3)			Diff	
	mean	median	std	mean	median	std	mean	median	std	(2)-(1)	(3)-(2)
Panel A: Stock market											
Return(Unit:percentage)											
All Day	0.71	-0.23	7.45	8.02	0.17	32.36	0.04	-0.22	5.87	7.32***	-7.98***
Overnight	0.28	0.00	3.68	7.56	1.01	25.83	0.02	-0.02	2.52	7.28***	-7.54***
Intraday	0.42	-0.22	6.29	0.91	-1.64	23.98	0.03	-0.57	5.55	0.49	-0.88
Volume(Unit:million shares)											
All Day	6.24	3.74	8.44	45.26	30.61	45.16	4.32	2.73	4.14	39.01***	-40.94***
Overnight	0.26	0.06	1.04	4.00	1.62	8.36	0.22	0.08	0.46	3.74***	-3.78***
Intraday	5.98	3.65	7.81	41.27	29.76	41.55	4.10	2.67	3.90	35.29***	-37.17***
Signed volume(Unit:thousand shares)											
All Day	101.98	-6.92	587.36	45.73	-3.76	2072.58	21.17	-7.05	271.11	-56.25	-24.56
Overnight	-0.67	-0.26	148.23	100.53	27.16	412.56	6.30	-0.35	117.50	101.20***	-94.23***
Intraday	100.72	-0.66	552.28	-38.86	-6.38	2003.89	14.79	-3.75	252.27	-139.59	53.65
Panel B: Options market											
Realized volatility	0.74	0.67	0.33	3.69	3.85	1.67	0.86	0.78	0.64	2.95***	-2.83***
ATM implied volatility	1.20	1.11	0.45	2.01	1.96	0.51	1.15	1.00	0.47	0.81***	-0.86***
Volume (Unit:thousand contracts)											
Total	44.96	22.01	68.26	409.33	377.16	294.74	103.01	76.35	74.34	364.37***	-306.32***
Call	28.16	12.34	40.50	172.27	135.25	127.34	65.58	49.20	48.47	144.11***	-106.69***
Put	16.81	7.84	29.88	237.06	185.82	199.74	37.43	25.48	32.32	220.26***	-199.64***
Buy Call	13.13	6.33	19.34	83.72	64.38	65.02	31.28	23.15	24.08	70.59***	-52.44***
Sell Call	13.66	5.89	19.60	83.46	67.94	59.17	31.95	23.97	23.25	69.80***	-51.50***
Buy Put	8.00	3.29	15.37	114.82	85.93	100.79	17.16	11.56	15.58	106.82***	-97.66***
Sell Put	7.92	3.84	13.33	114.80	94.42	95.51	18.61	13.05	16.00	106.87***	-96.19***

Table 2 Retail and Institutional Order Flow During Surge Period

Panel A of this table reports the results from the order flow regression

$$\text{Order Flow}_t = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + Controls_{t-1} + \varepsilon_{i,t},$$

where dependent variables are the volumes associated with various order flow: after-hours and intraday institutional order flow, after-hours and intraday retail order flows. Panel B reports the same results except now the dependent variables in Panel B are the proportion of total volumes for each of the four types of order flow. The sample period covers from January 1, 2020, to December 31, 2021. Controls include the SPY daily trading volume, SPX index daily return and daily VIX index, all control variables are lagged for one trading day. The t-statistics are reported in parentheses. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

Panel A: Volume (unit: million shares)

	Institutions		Retail Traders	
	After-hours (1)	Intraday (2)	After-hours (3)	Intraday (4)
Surge(β_1)	1.68***	9.96***	0.05***	8.39***
	(13.19)	(16.44)	(13.64)	(15.30)
Post-Surge(β_2)	0.10	0.06	-0.006**	-0.30
	(1.02)	(0.13)	(-2.42)	(-0.72)
Obs	505	505	505	505
Controls	Yes	Yes	Yes	Yes
Adj R^2	0.271	0.379	0.337	0.358

Panel B: Proportional volume

	Institutions		Retail Traders	
	After-hours (5)	Intraday (6)	After-hours (7)	Intraday (8)
Surge(β_1)	0.14***	0.04***	-0.13***	0.05***
	(7.70)	(6.99)	(-8.80)	(9.34)
Post-Surge(β_2)	0.17***	0.05***	-0.14***	-0.02***
	(12.06)	(10.95)	(-12.30)	(-5.17)
Obs	505	505	505	505
Controls	Yes	Yes	Yes	Yes
Adj R^2	0.332	0.306	0.267	0.371

Table 3 Weighted Price Contribution

This table reports the weighted price contribution of both the retail and institutional trades, during the surge and non-surge period, intraday and after hours sessions.

	After-hours		Intraday	
	Surge period	Non-surge period	Surge period	Non-surge period
Retail trades	-1.13	0.33	-155.65	-5.84
Institutional trades	44.96	3.37	8.90	3.64

Table 4 Intraday Returns and Order Imbalance Ratio Regression

This table reports the results from the return and order imbalance ratio regression

$$\begin{aligned} R_t = &\alpha + \beta_1 Retail_t + \beta_2 Inst_t + \beta_3 Retail_t \times I_t^S + \beta_4 Inst_t \times I_t^S \\ &+ \beta_5 Retail_t \times I_t^{PS} + \beta_6 Inst_t \times I_t^{PS} + \varepsilon_t. \end{aligned}$$

The sample period covers from January 1, 2020, to December 31, 2021. The t-statistics are reported in parentheses. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)
Constant	0.07	0.24*	0.13
	(0.47)	(1.77)	(0.97)
$Retail_t$	4.72***		3.96***
	(16.38)		(14.96)
$Retail_t \times I_t^S$	38.19***		14.74***
	(29.93)		(13.15)
$Retail_t \times I_t^{PS}$	-0.10		-0.96**
	(-0.19)		(-2.00)
$Inst_t$		15.20***	15.55***
		(55.53)	(53.71)
$Inst_t \times I_t^S$		93.42***	90.73***
		(103.49)	(97.29)
$Inst_t \times I_t^{PS}$		4.22***	3.67***
		(9.45)	(7.98)
Obs	186,416	182,908	177,815
Adj R^2	0.0083	0.108	0.111

Table 5 Forecasting Return Using MFIV

This table reports the results from the return prediction regression

$$R_{t+1} = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + \beta_3 MFIV_t \times I_t^S + \beta_4 MFIV_t \times I_t^{PS} + Controls_t + \varepsilon_t.$$

The dependent variables are overnight returns, intraday returns and close-to-close returns reported in (1), (2) and (3) columns. The sample period covers from January 1, 2020, to December 31, 2021. Controls include the SPY daily trading volume, SPX index daily return and daily VIX index. The t-statistics are reported in parentheses. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	Overnight (1)	Intraday (2)	Close-to-Close (3)
I_t^S	-0.08***	0.18***	0.10**
	(-2.76)	(5.81)	(2.38)
I_t^{PS}	0.002	0.003	0.005
	(0.08)	(0.14)	(0.16)
$MFIV_t \times I_t^S$	0.11***	-0.12***	-0.02
	(5.89)	(-6.38)	(-0.70)
$MFIV_t \times I_t^{PS}$	-0.006	-0.02	-0.02
	(-0.17)	(-0.49)	(-0.51)
Obs	505	505	505
Controls	Yes	Yes	Yes
Adj R^2	0.115	0.072	0.027

Table 6 Options Trading Volume During Surge Period

This table reports the results from the order flow regression

$$OV_t = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + Controls_t + \varepsilon_{i,t}.$$

The dependent variables in Panel A are the options volumes for various trade types, including all trades, call options trades, put options trades, buy call options trades, sell call options trades, buy put options trades and sell put options trades. The dependent variables in Panel B are options volumes from retail traders, while Panel C examines options volumes from sophisticated traders. The sample period covers from January 1, 2020, to December 31, 2021. Controls include the SPY daily trading volume, SPX index daily return and daily VIX index. The t-statistics are reported in parentheses. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

Panel A: Total Volume (unit: thousand contracts)

	All	Call Only	Put Only	Buy Call	Sell Call	Buy Put	Sell Put
Surge(β_1)	367.50***	143.99***	223.52***	70.58***	69.70***	108.59***	108.34***
	(20.81)	(16.42)	(20.50)	(16.07)	(16.84)	(19.77)	(20.83)
Post-Surge(β_2)	64.31***	37.50***	26.82***	18.27***	18.26***	12.50***	13.47***
	(4.71)	(5.54)	(3.18)	(5.38)	(5.71)	(2.95)	(3.35)
Obs	505	505	505	505	505	505	505
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj R^2	0.476	0.369	0.469	0.357	0.382	0.451	0.476

Panel B: Volume from retail traders (unit: thousand contracts)

	All	Call Only	Put Only	Buy Call	Sell Call	Buy Put	Sell Put
Surge(β_1)	37.36***	16.65***	20.71***	7.87***	8.41***	9.30***	10.30***
	(15.92)	(10.83)	(19.15)	(10.57)	(11.46)	(18.48)	(19.30)
Post-Surge(β_2)	8.92***	5.68***	3.24***	2.79***	2.97***	1.40***	1.80***
	(4.92)	(4.78)	(3.87)	(4.85)	(5.24)	(3.61)	(4.36)
Obs	505	505	505	505	505	505	505
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj R^2	0.357	0.227	0.434	0.217	0.247	0.417	0.436

Panel C: Volume from sophisticated traders (unit: thousand contracts)

	All	Call Only	Put Only	Buy Call	Sell Call	Buy Put	Sell Put
Surge(β_1)	72.40***	35.04***	37.36***	16.41***	16.69***	15.57***	19.43***
	(17.54)	(19.42)	(14.26)	(18.48)	(19.71)	(11.93)	(15.66)
Post-Surge(β_2)	13.96***	9.08***	4.88**	3.97***	4.37***	1.62***	2.74***
	(4.38)	(6.51)	(2.41)	(5.79)	(6.68)	(1.61)	(2.86)
Obs	505	505	505	505	505	505	505
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj R^2	0.476	0.369	0.469	0.357	0.382	0.451	0.476

Table 7 Forecasting After-Hours Return Using Net Open Interest Delta

This table reports the results from the after-hours return prediction regression

$$R^{AH}_{t+1} = \alpha + \beta_1 \Delta^i_t I^{BS}_t + \beta_2 \Delta^i_t I^S_t + \beta_3 \Delta^i_t I^{PS}_t + Controls_t + \varepsilon_t.$$

We run the regression using various net open interest deltas (Δ^i_t) derived from different types of options trades. The dependent variable in each regression is the after-hours returns, which remains consistent across all the different regressions. The net open interest delta is measured in million shares, while the after-hours returns are quantified in basis points (bps). The sample period covers from January 1, 2020, to December 31, 2021. Controls include the SPY daily trading volume, SPX index daily return and daily VIX index. The t-statistics are reported in parentheses. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	All traders			Sophisticated traders			Retail traders		
	All	Call Only	Put Only	All	Call Only	Put Only	All	Call Only	Put Only
Pre-Surge(β_1)	-0.45	0.06	-2.04	-3.91	-1.01	-5.46	6.66	2.02	27.08
	(-0.10)	(0.01)	(-0.22)	(-0.30)	(-0.03)	(-0.36)	(0.25)	(0.06)	(0.36)
Surge(β_2)	18.76***	49.67***	15.45***	51.63***	269.31***	31.05**	216.08***	303.15***	412.78***
	(5.78)	(7.93)	(2.94)	(4.69)	(8.95)	(2.18)	(7.58)	(7.94)	(4.81)
Post-Surge(β_3)	0.23	0.87	-0.56	1.68	3.89	0.51	7.41	6.87	8.64
	(0.13)	(0.19)	(-0.27)	(0.19)	(0.18)	(0.04)	(0.31)	(0.22)	(0.18)
Obs	505	505	505	505	505	505	505	505	505
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj R^2	0.054	0.103	0.007	0.033	0.130	-0.001	0.094	0.103	0.035

Table 8 The Impact of Net Open Interest Delta on Market Quality

This table reports the results from the market quality regression

$$L_{t+1} = \alpha + \beta_1 \Delta_t I_t^{BS} + \beta_2 \Delta_t I_t^{S} + \beta_3 \Delta_t I_t^{PS} + Controls_t + \varepsilon_t.$$

The dependent variable is one of five market quality measures, including dollar quoted spread, percentage quoted spread, dollar price impact, percentage price impact, and intraday quote-based volatility. The net open interest delta is measured in million shares. The sample period covers from January 1, 2020, to December 31, 2021. Controls include the SPY daily trading volume, SPX index daily return and daily VIX index. The t-statistics are reported in parentheses. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	Quoted Spread		Price Impact		Intraday Volatility
	Dollar(10^{-3})	Percentage(10^{-6})	Dollar(10^{-3})	Percentage(10^{-6})	(10^{-9})
Pre-Surge(β_1)	-2.52***	-23.47***	-1.50	-12.11	-1.21
	(-2.79)	(-6.55)	(-1.09)	(-1.50)	(-1.60)
Surge(β_2)	3.17***	9.25***	4.16***	13.88**	2.16***
	(4.93)	(3.63)	(4.26)	(2.41)	(4.01)
Post-Surge(β_3)	2.24***	-1.83	0.87*	-4.35	-0.003
	(6.62)	(-1.37)	(1.69)	(-1.44)	(-0.01)
Obs	504	504	504	504	504
Controls	Yes	Yes	Yes	Yes	Yes
Adj R^2	0.232	0.258	0.042	0.054	0.067

Table 9 Net Open Interest Gamma During Surge Period

This table reports the results from the net open interest gamma regression

$$\Gamma_t^i = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + Controls_{t-1} + \varepsilon_{i,t}.$$

The dependent variables are various net open interest gamma (Γ_t^i) derived from different types of options trades. The net open interest gamma is measured in million shares. The sample period covers from January 1, 2020, to December 31, 2021. Controls include the SPY daily trading volume, SPX index daily return and daily VIX index, all control variables are lagged for one trading day. The t-statistics are reported in parentheses. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	All traders			Sophisticated traders			Retail traders		
	All	Call Only	Put Only	All	Call Only	Put Only	All	Call Only	Put Only
Surge(β_1)	-4.44***	-3.01***	-1.42***	-1.01***	-0.40***	-0.61***	-0.68***	-0.49***	-0.19***
	(-10.39)	(-11.56)	(-6.23)	(-5.55)	(-9.96)	(-3.91)	(-12.67)	(-11.25)	(-9.58)
Post-Surge(β_2)	-5.01***	-3.28***	-1.73***	-1.65***	-0.44***	-1.21***	-0.77***	-0.55***	-0.22***
	(-15.19)	(-16.27)	(-9.83)	(-11.72)	(-14.10)	(-10.02)	(-18.51)	(-16.35)	(-14.20)
Obs	505	505	505	505	505	505	505	505	505
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj R^2	0.491	0.507	0.317	0.326	0.470	0.246	0.569	0.487	0.490

Appendix

A1 Retail and Institutional Order Flow in Other Firms

Table 2 reveals that the proportion of institutional order flow is significantly higher during surge periods. This result is consistent across both regular trading sessions and after-hours sessions. In this section, we conduct a similar analysis on other companies to determine whether the rise in institutional trading activity is present in other non-squeezed stocks.

Table A1 presents the results for two groups of stocks. Panel A displays the findings for the matched firm, BestBuy. The results of the analysis suggest that institutional and retail trading do not experience significant pattern changes during the GameStop surge period, as indicated by insignificant coefficients β_1. Panel B shows the results for six non-squeezed meme stocks which are examined in Allen et al. (2023). These six non-squeezed meme stocks are Black Berry, Castor, Koss, Nokia, Sundial Growers Inc. and Trivago NV. We observe that the coefficients β_1 are significantly higher in the after-hours session for institutional trading, similar to the case of GameStop. However, compared to the coefficient in Table 2 (0.14 for institutional order flow in the after-hours session), the six non-squeezed meme stocks have much lower coefficients (0.06). Furthermore, the intraday patterns of the six meme stocks during the GameStop surge period are similar to those observed before the surge. This is in stark contrast to the GameStop case, where we observe a significant increase in both intraday institutional and retail trading during surge period. This difference in behavior between GameStop and the six meme stocks may explain why none of the meme stocks experiences a short squeeze during the GameStop surge.

Table A1 Retail and Institutional Order Flow During Surge Period

This table reports the results from the order flow regression

$$\text{Order Flow}_t = \alpha + \beta_1 I_t^S + \beta_2 I_t^{PS} + Controls_{t-1} + \varepsilon_{i,t},$$

where dependent variables are the volumes proportion associated with various order flow: after-hours and intraday institutional order flow, after-hours and intraday retail order flows. Panel A reports the results for matched firm, BestBuy. Panel B reports the same results for six non-squeezed meme stocks discussed in Allen et al. (2023). The sample period covers from January 1, 2020, to December 31, 2021. Controls include the SPY daily trading volume, SPX index daily return and daily VIX index, all control variables are lagged for one trading day. The t-statistics are reported in parentheses. The symbols, ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

Panel A: BestBuy

	Institutions		Retail Traders	
	After-hours (1)	Intraday (2)	After-hours (3)	Intraday (4)
Surge(β_1)	0.01	0.01**	0.004	-0.01*
	(0.45)	(2.16)	(0.41)	(-1.96)
Post-Surge(β_2)	0.02	0.02***	-0.008	-0.004
	(1.29)	(5.89)	(-1.23)	(-1.53)
Obs	505	505	505	505
Controls	Yes	Yes	Yes	Yes
Adj R^2	0.051	0.138	0.022	0.049

Panel B: Six non-squeezed meme stocks

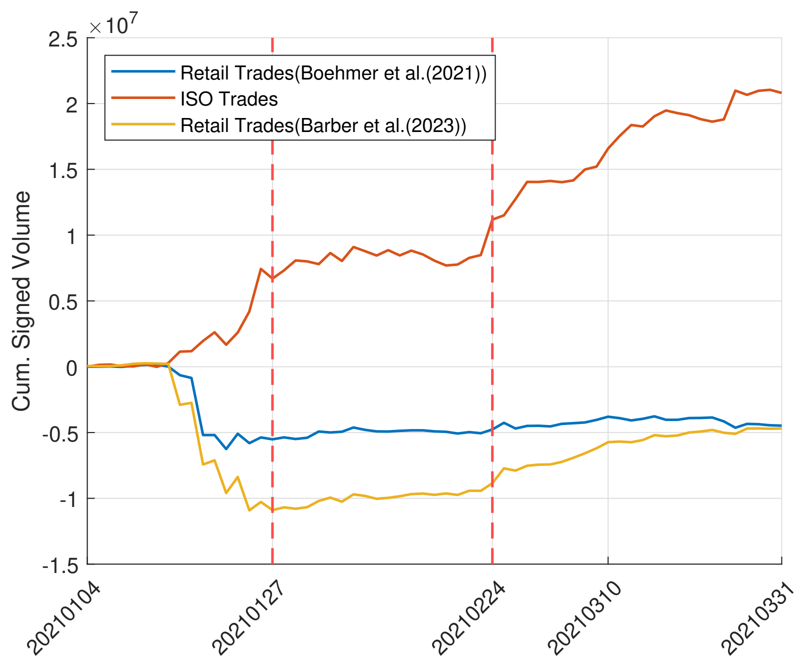
	Institutions		Retail Traders	
	After-hours (5)	Intraday (6)	After-hours (7)	Intraday (8)
Surge(β_1)	0.06***	0.0009	-0.05***	-0.007
	(4.76)	(0.22)	(-6.33)	(-1.46)
Post-Surge(β_2)	0.09***	0.009***	-0.06***	-0.02***
	(8.46)	(3.03)	(-4.6)	(-6.39)
Obs	505	505	505	505
Controls	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes
Adj R^2	0.059	0.306	0.180	0.557

A2 Retail Trades Identification

Barber et al. (2023) suggest a new quote midpoint method, which eliminates off-exchange trades that are between 40 and 60% of the NBBO to identify retil trades and the direction of retail trades. This section follows their method to distinguish between buy and sell retail trades and displays the signed volume of the retail traders using different methods in Figure A1. The results of either Boehmer et al. (2021) or Barber et al. (2023) demonstrate that retail traders begin to offload their investments prior to a short-squeeze.

In Barber et al. (2023), the positive retail order-imbalance ratio for GameStop during early 2021 is reported in Internet Appendix (Figure IA.1). Their ratio is determined based on the number of trade measures, which is calculated as the difference between the number of buy and sell trades divided by the total number of buy and sell trades. When using the share volume-based measure, we can get the negative retail order-imbalance ratio during early 2021.

Figure A1. Cumulative Signed Volume for Different Types of Trades
The figures show the cumulative signed share volume for institution investors and retail traders during the surge period.



A3 Are There More Put-Call Parity Violations?

Allen et al. (2023) use daily market-close options data to construct the synthetic stock price based on the put-call parity. They observe a considerable rise in the gap between the synthetic stock price and the stock close price, which serves as a proxy for the put-call parity violation, during the squeeze period. Cremers and Weinbaum (2010) suggest that such put-call parity violations are caused by a non-zero implied volatility spread between calls and puts, and that the implied volatility spread can significantly forecast future stock returns. Furthermore, Muravyev et al. (2022) develop a formula that links this implied volatility spread to the borrow fee. Therefore, the proxy used in Allen et al. (2023) may capture the information of the borrow fee, thus an increase of this proxy is observed during the squeeze period.

In this section, we can use tick-by-tick options trade data to detect trades that violate the put-call parity, which could potentially generate a risk-free arbitrage gain before taking into account transaction costs. These arbitrage trades are usually submitted by inexperienced option traders, thus giving us some understanding of the trading habits of novice option traders during the GME event.

The GME options are of the American style, and, under mild no-arbitrage assumptions, it is not optimal to exercise an American call before the expiration date. However, it may be advantageous to exercise an American put before the expiration date. This results in the put-call parity between an American call and put being expressed as an inequality

$$Call + PV(K) \leq S + Put \leq Call + K,$$

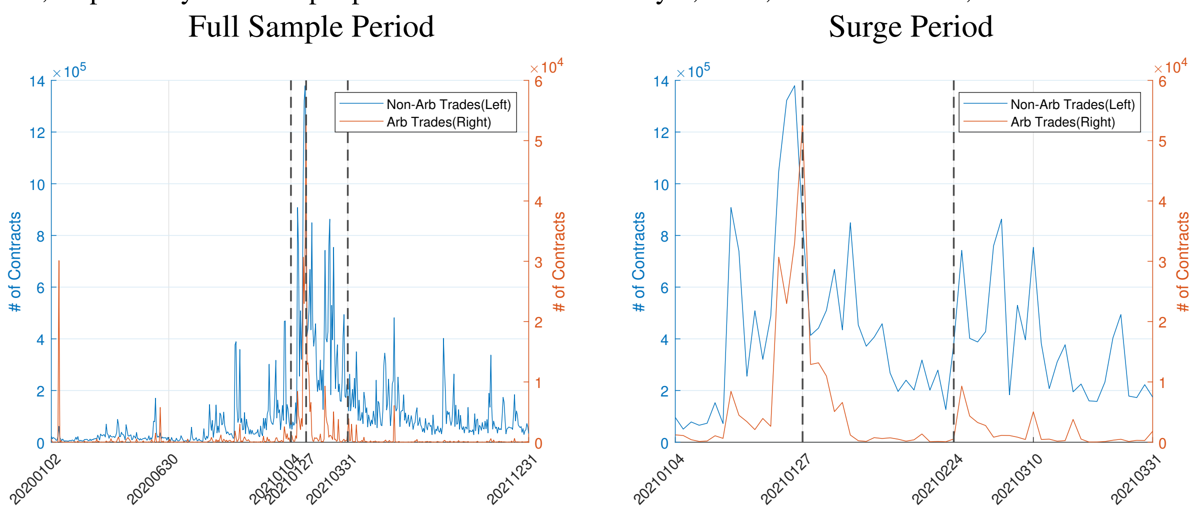
where S is the stock price, K and $PV(K)$ are the strike price and its present value. *Call* and *Put* are option transaction prices.

Our options trade data only include the contemporary underlying stock quote and do not encompass contemporary quotes for other options. Given that options' prices are nonnegative, we define the options risk-free arbitrage trades which are the put-call parity violations as follows[36]

$$S_{Ask} + Put < K \qquad \text{or} \qquad S_{Bid} > Call + K.$$

Figure A2. Option Arbitrage Trades Over Time

The figures depict the option arbitrage trades and regular trades during the full sample period and the surge period, respectively. The sample period covers from January 1, 2020, to December 31, 2021.



Before the surge, the daily average of arbitrage trades was 324 contracts, with the majority (86.14%) being call options. However, when the surge began, these numbers increased dramatically, reaching an average of 4,325 contracts per day, with call options making up 96.62% of all trades. As seen in Figure A2, the peak of arbitrage trades occurred on January 27, 2021, following the short squeeze.

It is evident that those who purchase options can make risk-free profits by engaging in arbitrage trades, as option sellers sell in-the-money (ITM) options at a lower price by mistake. However, naked writing ITM options is not a popular practice due to the high margin requirements. We believe that option sellers in these arbitrage trades may be closing their positions inadvertently,

[36] If $S_{Ask} + Put < K$, then the buyer of the put option can buy the stock at ask price and early exercise the option to sell the stock at price K, then the total risk-free payoff is $K - (S_{Ask} + Put) > 0$. If $S_{Bid} > Call + K$, then the buyer of the call option can early exercise the option to buy the stock at price K, and sell the stock at bid price, then the total risk-free payoff is $S_{bid} - (Call + K) > 0$.

when they could potentially make more money by exercising their options early, although early exercise might not always be the optimal choice. It appears that these risk-free arbitrage opportunities are usually taken advantage of by less experienced options traders. The prevalence of call options in these trades suggests that novice option traders are more likely to take long positions on call options, which could lead to market makers buying more underlying stock.

Figure A2 shows a dramatic rise in the number of regular option trades, from an average of 44,965 contracts per day before the surge to 409,330 contracts per day during the surge. On January 22 to 26, the daily option volume even surpassed 1 million contracts before the short squeeze occurred. On January 27, the volume dropped to 849,609 contracts.

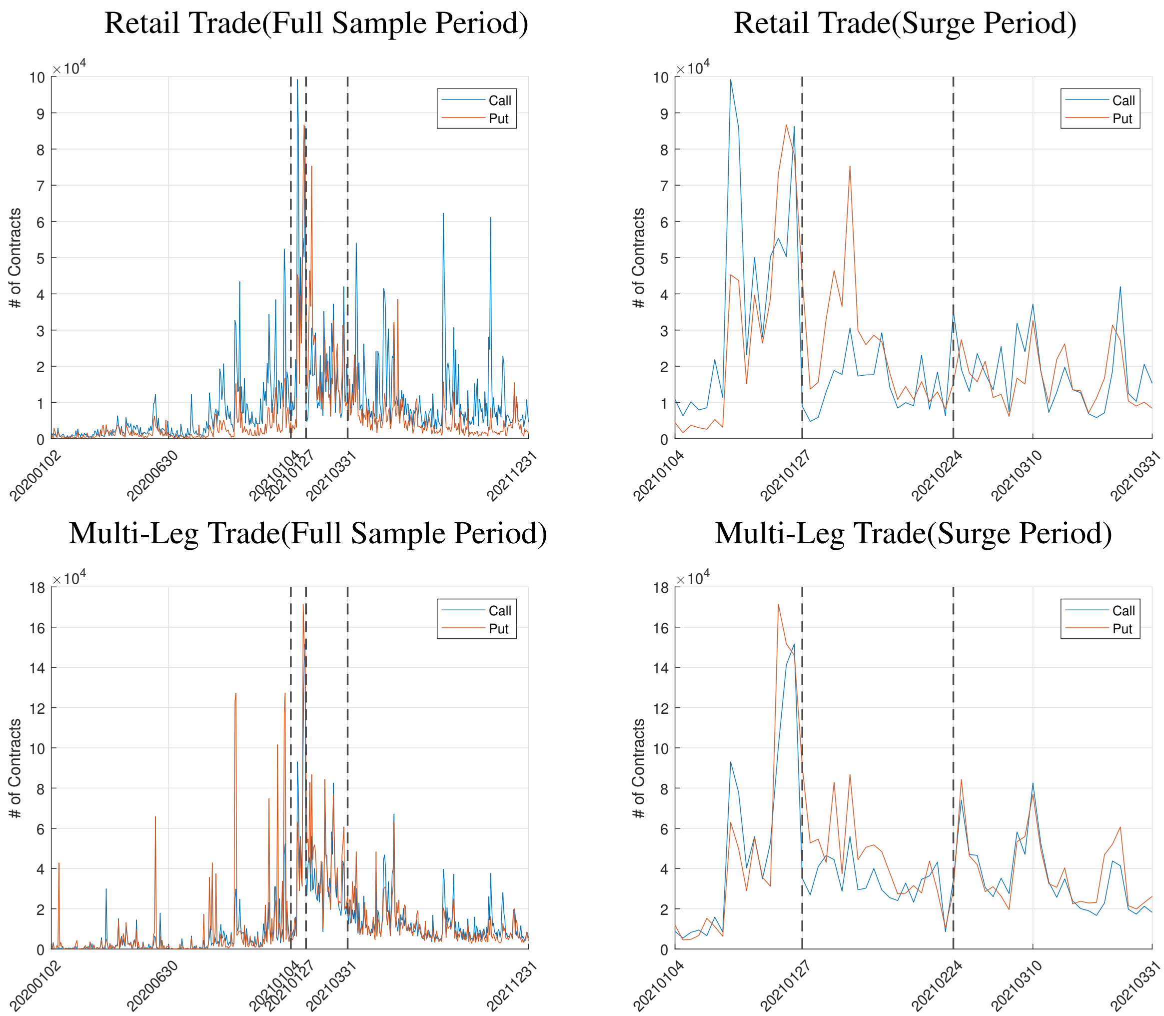
A4 Daily Option Volume for Various Trade Types

In this section, we provide the time series of option trading volume for various trade types. In Figure A3, we can observe that the trading volume of both retail and multi-leg trades experienced a significant increase during the surge period. Both groups of traders showed similar trading volumes for both call and put options during this period. However, prior to the surge, retail traders had a preference for call options, while sophisticated traders tended to favor put options. This is in line with the observation in Figure 4 that retail stock traders continued to net buy on GameStop since late 2020, as retail option traders also showed a preference for buying call options during the same period. The average buy-to-sell call ratio increased from 88.73% before October 8, which marks the announcement of the partnership with Microsoft, to 94.26% in late 2020.

Sophisticated traders may have a preference for put options during the pre-surge period for two possible reasons. First, short sellers may move to the options market due to the high cost of short selling in the stock market, and use put options to create a short position on the stock. Second, institutional speculators may write put options as a way to increase their profits before a potential short squeeze is triggered. This strategy also implies a positive outlook on future prices, which is in line with the expectations of these speculators.

Figure A3. Different Types of Option Trades

The figures plot daily calls and puts volume from retail trades and multi-leg trades during the full sample period and the surge period, respectively. The sample period covers from January 1, 2020, to December 31, 2021.



A5 Proofs

Proof of Proposition 1:

Following the definition of $x^P(t) = x^P(t_0) + \int_{t_0}^{t} a^*(t)dt$, we can get the results for $t \in [t_0, t_0 - \frac{x^S(t_0)}{A/I})$ directly. When $t^{end} = t_0 - \frac{x^S(t_0)}{A/I}$, we apply the relation $I = I^S + I^P$, and have

$$
\begin{aligned}
x^P(t^{end}) =& x^P(t_0) + \int_{t_0}^{t^{end}} a^*(t)dt \\
=& x^P(t_0) + \frac{A}{I}\tau + \frac{(A/I)I^S}{I^P-1}\left(\frac{x^S(t_0)}{A/I} + \tau\right) \\
=& x^P(t_0) + \frac{(I^P-1)(\bar{x}-x^P(t_0)) - x^S(t_0)I^S}{I-1} + \frac{I^S}{I^P-1}\left(x^S(t_0) + \frac{(I^P-1)(\bar{x}-x^P(t_0)) - x^S(t_0)I^S}{I-1}\right) \\
=& x^P(t_0) + \frac{(I^P-1)(\bar{x}-x^P(t_0)) - x^S(t_0)I^S}{I-1} + \frac{I^S}{I^P-1}\frac{(I^P-1)(\bar{x}-x^P(t_0)+x^S(t_0))}{I-1} \\
=& x^P(t_0) + \bar{x} - x^P(t_0) = \bar{x}
\end{aligned}
$$

This is consistent with our definition of $x^P(t^{end}) \equiv \bar{x}$. Therefore, we get

$$
\int_{t_0}^{t^{end}} a^*(t)dt = \frac{A}{I}\tau + \frac{(A/I)I^S}{I^P-1}\left(\frac{x^S(t_0)}{A/I} + \tau\right) = \bar{x} - x^P(t_0). \tag{A1}
$$

Proof of Proposition 2:

Inserting Equation (20) and Equation (19) into Equation (22), we get

$$
\begin{aligned}
W^P =& p^*(t^{end})\bar{x} - \int_{t_0}^{t^{end}} p^*(t)a^{i*}(t)dt - p(t_0)x^P(t_0) \\
=& p(t_0)\bar{x} - \int_{t_0}^{t^{end}} p(t_0)a^{i*}(t)dt - p(t_0)x^P(t_0) + \frac{\lambda}{1-\Gamma\lambda}[I^P(\bar{x}-x^P(t_0)) - x^S(t_0)I^S]\bar{x} \\
& - \frac{\lambda}{1-\Gamma\lambda}\int_{t_0}^{t_0+\tau}\frac{A^2}{I}(t-t_0)dt - \frac{\lambda}{1-\Gamma\lambda}\int_{t_0+\tau}^{t^{end}}\frac{(A/I)I^S}{I^P-1}\left(\frac{(A/I)I^S}{I^P-1}[t-(t_0+\tau)] - A\tau\right)dt
\end{aligned}
\tag{A2}
$$

Using Equation (A1), the first three terms equal 0, thus Equation (A2) becomes

$$\begin{aligned}
W^P =& \frac{\lambda}{1-\Gamma\lambda}[I^P(\bar{x}-x^P(t_0))-x^S(t_0)I^S]\bar{x} \\
&-\frac{\lambda}{1-\Gamma\lambda}\left[\int_{t_0}^{t_0+\tau}\frac{A^2}{I}(t-t_0)dt+\int_{t_0+\tau}^{t^{end}}\frac{(A/I)I^S}{I^P-1}\left(\frac{(A/I)I^S}{I^P-1}[t-(t_0+\tau)]-A\tau\right)dt\right] \\
=& \frac{\lambda}{1-\Gamma\lambda}[I^P(\bar{x}-x^P(t_0))-x^S(t_0)I^S]\bar{x} \\
&-\frac{\lambda}{1-\Gamma\lambda}\left[\frac{(A\tau)^2}{2I}+\left(\frac{(A/I)I^S}{I^P-1}\right)^2\frac{(\frac{x^S(t_0)}{A/I}+\tau)^2}{2}+(A\tau)\frac{(A/I)I^S}{I^P-1}\left(\frac{x^S(t_0)}{A/I}+\tau\right)\right]
\end{aligned} \tag{A3}$$

We define $m \equiv \frac{(A/I)I^S}{I^P-1}\left(\frac{x^S(t_0)}{A/I}+\tau\right)$, and because $t_0-\frac{x^S(t_0)}{A/I} \geq t_0+\tau$, $\frac{x^S(t_0)}{A/I}+\tau \leq 0$, that is, $m \leq 0$.

From Equation (20), we have $I^P(\bar{x}-x^P(t_0))-x^S(t_0)I^S = A\tau+m$ and from Equation (A1), we have $\bar{x}-x^P(t_0)=\frac{A}{I}\tau+m$. Therefore,

$$\begin{aligned}
[I^P(\bar{x}-x^P(t_0))-x^S(t_0)I^S]\bar{x} =& \frac{1}{2}[I^P(\bar{x}-x^P(t_0))-x^S(t_0)I^S][(\bar{x}-x^p(t_0))+(\bar{x}+x^p(t_0))] \\
=& \frac{1}{2}(A\tau+m)(\frac{A}{I}\tau+m)+\frac{1}{2}(A\tau+m)(\bar{x}+x^p(t_0))
\end{aligned} \tag{A4}$$

On the other hand,

$$\begin{aligned}
&\frac{(A\tau)^2}{2I}+\left(\frac{(A/I)I^S}{I^P-1}\right)^2\frac{(\frac{x^S(t_0)}{A/I}+\tau)^2}{2}+(A\tau)\frac{(A/I)I^S}{I^P-1}\left(\frac{x^S(t_0)}{A/I}+\tau\right) \\
=&\frac{(A\tau)^2}{2I}+\frac{m^2}{2}+(A\tau)m=\frac{1}{2}(A\tau+m)(\frac{A}{I}\tau+m)+\frac{1}{2}(A\tau)m(1-\frac{1}{I})
\end{aligned} \tag{A5}$$

Putting Equations (A4) and (A5) in Equation (A5), we get

$$W^P=\frac{\lambda}{2(1-\Gamma\lambda)}\left[(A\tau+m)(\bar{x}+x^p(t_0))-(A\tau)m(1-\frac{1}{I})\right] \tag{A6}$$

Note that $A\tau+m=I^P(\bar{x}-x^P(t_0))-x^S(t_0)I^S>0$, $\bar{x}\geq x^P(t_0)\geq 0$, the first term in Equation (A6) is positive. Moreover, $\tau>0$, $m\leq 0$ and $I>1$, so $(A\tau)m(1-\frac{1}{I})\leq 0$. To sum up, $g(\cdot)\equiv (A\tau+m)(\bar{x}+x^p(t_0))-(A\tau)m(1-\frac{1}{I})$ is always positive.