The Global Implied Volatility Surface, Convexity, and Common Predictability of International Equity Premia*

Adlai Fisher† Terry Zhang‡

This draft: November 2023

Abstract

We construct a global implied volatility surface by combining information from the index options of twenty countries and regions. The convexity of the global surface positively predicts equity premia around the world, in- and out-of-sample, at horizons from one to twelve months. Semi-annually, R2 are 14.4% for S&P500 and 8.8% for twenty indexes on average, increasing to 20.8% and 11.4% out-of-sample. For U.S. forecasts, global convexity subsumes other option-based predictors, including global level and slope, U.S. convexity, VIX, SVIX, variance risk premium, and left-tail volatility. The predictability of global convexity comes from its left-tail contributions related to crash fears (left-tail volatility), and right-tail contributions related to speculative demand (short-sales and funding conditions). Our findings highlight the importance of global options markets for risk sharing and information aggregation.

Keywords: international return predictability, implied volatility surface, crash risk, speculation, funding conditions

*We thank the Natural Sciences and Engineering Research Council of Canada for financial support. We also thank Aurelio Vasquez (discussant) and participants at the 2023 CDI Conference on Derivatives for helpful comments.

†Sauder School of Business, University of British Columbia. Email: adlai.fisher@sauder.ubc.ca

‡Australian National University. Email: terry.zhang@anu.edu.au

1 Introduction

Option prices reflect state-price valuation of underlying risky streams (Cox and Ross, 1976),[1] implying that index options may naturally contain information about various contributors to the market risk premium. Empirical market-return predictors derived from index options include the variance-risk premium (Bollerslev, Tauchen, and Zhou, 2009; Carr and Wu, 2009), tail-risk premia (Bollerslev and Todorov, 2011; Bollerslev, Todorov, and Xu, 2015), and equity premium bounds (Martin, 2017; Chabi-Yo and Loudis, 2020). Other equity-return predictors such as short-interest (Rapach, Ringgenberg, and Zhou, 2016) relate most naturally to speculative demand, but from the logic of the law-of-one-price can also impact option prices.

International equity and option markets are further connected by global risks and cross-country risk sharing and information aggregation. Events such as the 2007-2008 financial crisis and Covid-19 pandemic have shown global events to be increasingly important. Integration of international equity markets has long been hypothesized and tested (Solnik, 1983; Harvey, 1991; Bekaert and Harvey, 1995). More recently, an emerging literature uses international option markets to better understand local and global equity risk premia (Bollerslev, Marrone, Xu, and Zhou, 2014; Andersen, Fusari, and Todorov, 2020).

We contribute to these efforts by combining information from the index options of twenty countries and regions to construct a single global implied volatility surface. Our procedure reveals a powerful and encompassing in- and out-of-sample equity-premium predictor, global-surface convexity. Implied volatilities of index options display two prominent empirical features.[2] The first, known as volatility smirk, captures that low-strike implied volatilities typically exceed high-strike implied volatilities. We measure smirk steepness with a slope factor. The second feature, known as volatility smile, captures that controlling for smirk, implied volatilities of options with low and high strikes typically exceed the implied volatilities of options with medium strikes. We measure the curvature of the volatility smile by

[1]See also Debreu (1959), Arrow (1964), Breeden and Litzenberger (1978), Ross (1978), and Ross (2015).

[2]See for example Bates (1991; 2000; 2022), Bakshi, Cao, and Chen (1997), Das and Sundaram (1999), Pan (2002), and Liu, Pan, and Wang (2005).

a convexity factor. In addition to slope and convexity, we also measure the level of the global volatility surface. The global-surface level, slope, and convexity effectively produce a low-dimensional representation of the global implied volatility surface.

Global-surface convexity is by far the most powerful option-based return predictor. It strongly forecasts equity premia around the world, in- and out-of-sample. When the global surface is more convex, it predicts higher market returns. In the US, global convexity predicts S&P 500 index returns one-month ahead with an R^2 of 3.7%, and six-months ahead with an R^2 of 14.4%, from 1996 to 2021. Predictability does not deteriorate out-of-sample, and in fact the one- and six-month ahead out-of-sample R^2 in the U.S. are larger, 4.1% and 20.8% respectively.[3] Internationally, global convexity significantly predicts the semi-annual return in 19 of 20 countries and regions, with average R^2 of 8.8%. Out-of-sample R^2 are all positive and average 11.4%. The predictability is also economically important. An investor using global convexity to time the market would have on average increased the Sharpe ratio by more than 60% from a buy-and-hold strategy across the twenty indexes in our sample.

Global convexity encompasses the predictability of other important option-based predictors and is highly robust to alternative specifications. For US returns, global convexity subsumes the predictive power of the global level and slope, the VIX index, SVIX, the variance risk premium, and left-tail volatility. We measure convexity using strikes of all maturities, but predictability changes little using only short (≤6 month) or long (¿6 month) maturities. Excluding or including the U.S. from the global surface has little impact on its predictive power. The measure is also robust to using calls-only or puts-only, in-the-money or out-of-the-money options, excluding highest and lowest strikes, and averaging across broad ranges of strikes to produce a robust measure of convexity. Global convexity captures important fundamental information from the option surface and is not sensitive to variations in measurement.

Why does global convexity predict market returns so strongly? We investigate the eco-

[3] We measure OOS R^2 according to Welch and Goyal (2008). It can be higher than in-sample R^2 because it compares MSE from our predictor to a historical-mean model. The higher OOS R^2 also reflects the fact that the predictability of our predictor becomes stronger in the later sample.

nomic source of return predictability. First, global convexity aggregates information from countries around the world. The global economy is increasingly interconnected, and shocks in one area can spread and affect other countries. A prominent example is the Covid-19 pandemic. During early 2020, global convexity responds more quickly to the spread of the virus than US convexity and leads the US by one month. This provides an important counter-example to the typical finding that the US leads the world informationally (e.g., Rapach, Strauss, and Zhou, 2013). In the case of the Covid-19 pandemic, the first signs of deteriorating fundamentals appeared in option markets outside the U.S.

Global convexity also effectively combines information from both the right and left tails of the risk-neutral distribution of returns. The left tail has been widely studied in prior literature (e.g., Andersen, Fusari, and Todorov, 2015; Bollerslev, Todorov, and Xu, 2015), and is appropriately associated with fears of negative jumps or market crashes. High prices in the left tail are commonly interpreted as demand for crash insurance through out-of-the-money puts and correspondingly large risk premia. The right-tail contribution to convexity, while smaller, is also economically important. Controlling for the left tail, high prices in the right tail positively predict returns, which has the natural interpretation of speculative demand for out-of-the-money calls. Consistent with this interpretation, the right-contribution to convexity is strongly negatively associated with short-interest and with funding conditions measured through the TED spread.

These contributions to global convexity from the left- and right-sides of the risk-neutral distribution have natural interpretations as the presumptive twin driving forces of financial markets – fear and greed. It is already well-known that low price in the left tail, or lack of fear, forecasts low future returns. New to the literature and controlling for the left tail, a low right-tail price signaling lack of speculative interest also forecasts low returns. Thus, fear and greed from the left and right tails of the risk-neutral density, while negatively correlated, are not opposites. The global convexity measure which combines both sources of information is required to optimize equity-premium predictability from the global option surface.

Our paper contributes to three strands of literature. The first is the literature on recovering the equity premium from option data. Various measures constructed based on index options have been proposed in this literature: for example, variance risk premium (Bollerslev, Tauchen, and Zhou, 2009; Carr and Wu, 2009), skew risk premium (Kozhan, Neuberger, and Schneider, 2013), left-tail volatility (Andersen, Fusari, and Todorov, 2015; Bollerslev, Todorov, and Xu, 2015), and equity premium bounds (Martin, 2017; Chabi-Yo and Loudis, 2020; Bakshi, Crosby, Gao, and Zhou, 2019; Jensen, Lando, and Pedersen, 2019; Liu, Lu, Xu, and Zhou, 2022; Back, Crotty, and Kazempour, 2022). This literature has shown that, both in theory and empirics, options data contain information about the short-to-medium-term equity premium. We contribute to this literature by discovering a new option-based predictor, namely, global convexity. This measure encompasses the return predictability of several previously documented indicators due to its strong ability to aggregate information, especially information from the international market and from the right-tail of risk neutral distribution. Convexity is also more symmetric and has smaller kurtosis than existing predictors, which contributes to its empirical success.

We contribute to the general debate on whether equity premium is predictable, especially out-of-sample. Since as early as Shiller (1981), a vast literature documents that the US equity premium is predictable, particularly at long horizons. Welch and Goyal (2008) cast doubt on whether the US equity premium is predictable out-of-sample (OOS). They show that the OOS R^2 of many predictors are negative. Campbell and Thompson (2008) show that imposing weak economic restrictions on predictors improves the OOS performance. We document a powerful short-term return predictor whose performance is robust out-of-the sample and in many countries around the world.

Lastly, we contribute to the study on the integration of the international financial market and international equity premium prediction. Henkel, Martin, and Nardari (2011) find that in the G7 countries, short-term return predictability exists only during economic contractions. Rapach, Strauss, and Zhou (2013) show that US stock return leads the stock return in other countries. Bollerslev, Marrone, Xu, and Zhou (2014) document that international variance risk premia predict stock returns in developed economies and Qiao, Xu, Zhang, and Zhou (2019) provide similar evidence for emerging markets. Miranda-Agrippino and Rey (2020) provide evidence on the co-movement in risky asset prices around the world, a phenomenon known as the Global Financial Cycle. Our measure, the global convexity significantly predicts changes in their Global Financial Cycle factor. This finding reinforces that the international options and equity market, or even other risky asset markets, are closely tied. A single measure from international options markets significantly predicts equity returns in 19 of 20 countries and regions, with similar coefficients, providing strong evidence of market integration and a common global risk premium.

2 Data

Our main data source for index options is OptionMetrics, which contains daily options data for major indices around the world. We select 20 indices from 20 different countries or regions where OptionMetrics has sufficient data coverage on corresponding index options. Table 1 lists the stock indexes in our sample and the availability of options data for each. The earliest options data in our sample is for the S&P 500 index, starting from January 4, 1996. Other index options are gradually included in the sample after 2002. The latest options data to be included is the OMXS 30 index option in Sweden, starting from May 14, 2007. Figure 1 plots the number of countries or regions with available index options in our sample from 1996 to 2021. Our 26-year sample period is long enough to span several episodes of market crisis around the world, such as the Asian debt crisis, the burst of the dot-com bubble, the subprime mortgage crisis, the European debt crisis, and the Covid-19 pandemic.

For each individual index, there are hundreds of options listed on option exchanges every day. Options differ based on their strike and maturity. The number of available strikes and maturities are symmetric for calls and puts. Different indexes have different number of available strikes and maturities. Table 1 shows the average number of options per day, including both calls and puts, for each index in our dataset, after we apply standard filters.[4] Indexes such as the S&P 500 or STOXX 50 have over one thousand different options outstanding on

[4]We drop options with non-positive implied volatility or with implied volatility above 200%. We drop options with fewer than 7 days to maturity or with longer than three years to maturity. We also drop options with non-positive bid or ask prices. If bid or ask prices are missing, we drop options with non-positive exchange settlement price.

a typical day, whereas indexes such as the BEL 20 or TAIEX have only around 200 options per day.

To facilitate comparison across markets, OptionMetrics provides an implied volatility surface on a standardized delta-maturity grid for all indexes. For each underlying index, the surface specifies the implied volatility of a hypothetical option with a particular delta and maturity. There is a separate surface for call options and for put options, although, if properly aligned, the two surfaces are close to each other due to put-call parity. To construct this surface, OptionMetrics applies a kernel function to compute the weighted average of all implied volatilities from options traded on each day. The kernel function puts a greater weight on options that are closer to a particular grid point. Intuitively, the implied volatility on each grid point is the interpolated implied volatility of options with deltas and maturities that are near the grid point. Appendix A provides details on the procedure to construct the standardized implied volatility surface. This standardized implied volatility surface makes it easy to analyze all markets without adjusting for the availability of strikes and maturities in each market. Our main empirical analysis is based on the standardized implied volatility surface provided by OptionMetrics. In the robustness section, we also construct our measures using the underlying individual options.

To conduct return predictability tests, we obtain monthly returns of the 20 selected indexes from FactSet. For 19 out of 20 indexes, we have return series from 1996 to 2022. For the MIB index in Italy, the return series starts from 1998. Table A1 provides summary statistics on the excess returns of each index. In our empirical tests, we convert all local-currency returns to US-dollar returns and subtract the US risk-free rate to compute excess returns.

2.1 Standardized implied volatility surface

This section discusses the characteristics of the implied volatility surface of the 20 indexes in our sample. Before we proceed, we take several steps to clean the data. First, we drop all grid points with a maturity of 10 days, because implied volatility data at this maturity are missing for most indexes. We also drop all grid points with a maturity of 730 days, because many indexes do not have options with a maturity longer than two years. The implied volatility at this maturity point is largely based on the extrapolated values, which could be biased. The standard set of maturities that we consider are 30, 60, 91, 122, 152, 182, 273, 365, and 547 days.

The OptionMetrics data for the Canadian S&P/TSX 60 index contain many missing values from Dec 27, 2019 to Mar 1, 2021. To fix this issue, we replace this part of the sample period for Canada with the implied volatility surface of MSCI Canada ETF, which does not have missing values. Also, available delta grid points for indexes in the Asian Pacific region (i.e., Australia, Japan, Taiwan, Hong Kong, and Korea) are slightly different from the US and European indexes. The available delta grid points for these Asian Pacific indexes are from 0.2 to 0.8 at 0.05 increment for call options and are from -0.2 to -0.8 at 0.05 increment for put options. The delta grid points for other indexes are from 0.1 to 0.9 at 0.05 increment for call options and are from -0.1 to -0.9 at 0.05 increment for put options. Following OptionMetrics' methodology, we extend the implied volatility surface of indexes in the Asian Pacific region to be the same as other indexes. In the robustness check, we only keep implied volatilities with a delta from 0.2 to 0.8 for calls (or -0.2 to -0.8 for puts) to construct our measures.

The next step we take is to change the labelling of delta grid points to align the call option surface with put option surface. Specifically, we multiply the delta of put options by -1. We multiply the delta of all call options by -1 and then add 1. After applying this one-to-one transformation, both call option surface and put option surface have the same set of delta grid points. Another convenience brought by this transformation is that the transformed deltas align with strike prices, e.g., lower values of the transformed delta corresponds to lower strike prices. Lastly, we winsorize all implied volatilities such that they have a minimum of 1% and a maximum of 200%.[5] Table A2 reports the summary statistics of the cleaned implied volatility surfaces. Most indexes have their average implied volatilities around 20%. The index with the highest average implied volatility, at 26.65%, is the MSCI

[5]Note that all implied volatilities are quoted on an annualized basis.

Emerging Market index. Implied volatilities have positive skewness and heavy tail, reflecting the fact that they tend to spike during a short period of time.

Table 2 shows unconditional global implied volatility surface averaged across all indexes and across both call and put options. To construct this surface, we first take the average implied volatility for a given grid point across all indexes on each day and then average across all sample days. We pool together both call option surfaces and put option surfaces, because their average values are similar to each other at a given grid point. Table 2 shows that the average implied volatilities monotonically decrease with the transformed delta. In other words, average implied volatilities monotonically decrease with the level of the strike price. This pattern is commonly known as the volatility smirk. On the other hand, variation in the average implied volatilities across maturities is small, displaying an unconditional flat term-structure. Table 3 shows the unconditional implied volatility surface of the S&P500 index. Its implied volatilities also decrease with the transformed delta (i.e., strike), but they display a slightly increasing term structure. As can be seen in both Table 2 and Table 3, the majority of variations in implied volatility on the surface is along the delta dimension, which is our primary research focus.[6]

2.2 Level and slope of the standardized implied volatility surface

The implied volatility surface is a high-dimensional object. To reduce its dimensionality, we construct three measures to capture the shape of the implied volatility surface in each time period, namely, the level, slope, and convexity of the surface. Alternative methods to reduce the dimensionality of the volatility surface include estimating a quadratic function (Dumas, Fleming, and Whaley, 1998) or using principal component analysis (Cont, Fonseca, and Durrleman, 2002). These methods are closely related to each other. We choose to use the level, slope, and convexity measures to summarize the surface because of their ease of interpretation. To measure the level of the standardized implied volatility surface of each index, we take the simple average of all implied volatilities on the surface, including both

[6] In the appendix, Table A3 shows that even conditionally maturity information explains little about the variation in the implied volatility.

calls and puts, on each day. To smooth out the daily variations, we aggregate the daily level measure into a monthly measure by taking the simple average of available trading days in each month. This gives us twenty monthly time series, one corresponding to each index. When constructing the monthly measure, we drop the last trading day of each month in each country to avoid any overlap in time between the construction of the index in the current month and the measurement of index return in the next month.[7] We further aggregate the information from twenty countries and regions into a single global level index by taking the simple average of all available country-level measures in each month. Before 2002, the only country in our sample with available options data is the US, so this global level index coincides with the US level index. After 2002, as we include additional countries in the sample, the global level index begins to diverge from the US level index.

Figure 2 plots both the global level and the US level index on a monthly frequency. We observe the two series co-move strongly with each other. The correlation between the two is above 95% in the entire sample. This is also true in the later sample period when we have 20 different countries or regions to construct the global level index. This strong co-movement indicates that the option markets across these countries are integrated and they experience similar shocks over time. Table 4 Panel A shows that the global level index has positive skewness and high kurtosis, which means that it tends to have large jumps during turbulent times, for example, during the global financial crisis and the Covid-19 pandemic period.

We also construct a slope index based on the standardized implied volatility surface. We measure the slope of the implied volatility surface with respect to the transformed deltas. We only focus on the slope on deltas because the variation of implied volatilities across maturities is relatively small. Specifically, on each day, we regress all the implied volatilities of a surface on the transformed deltas and estimate the coefficient on the delta.[8] This gives us a daily slope measure for each country. We then aggregate them into a monthly measure by taking

[7]Different index options are traded in different time zones. To avoid potential look-ahead bias for future month returns beginning in different time zones, we drop the last trading day of each month from the option surface calculations.

[8]We ignore the information on maturities or the type of the option (call or put) when estimating the slope. Controlling for maturity and the type of the option does not affect the estimated slope on deltas.

the simple average across available trading days, except the last trading day. Similar to the level index, we construct a global slope index by averaging all available slope measures from each country. Figure 3 plots the global slope and the US slope index. Across the entire sample period, the slope is negative, indicating that options with lower strike prices have higher implied volatilities, displaying a volatility smirk. The US slope index co-moves with the global slope index. The US slope index is more negative than the global slope index on average, which suggests that the volatility smirk is steeper in the US. The slope index has a strong negative correlation with the level index. As shown in Table 4 Panel B, the correlation between the global slope index and the global level index is 82%. This is because during periods of market crisis, all implied volatilities increase and the implied volatilities of options with low strike prices increase more than the average, causing the volatility surface to have a higher level and a more negative slope.

2.3 Measuring the convexity of the global surface

Our third measure is the convexity of implied volatility surface. A prominent feature of the implied-volatility curve is the presence of volatility smile in addition to volatility smirk. Volatility smile refers to the phenomenon that both low-strike and high-strike options have higher than average implied volatilities, making the implied-volatility curve a convex function. We propose a measure to capture the convexity of the standardized implied-volatility surface. Specifically, let $IV(\Delta, \tau)$ denote the function that represents the implied volatility surface, i.e. $IV(\Delta, \tau)$ is the implied volatility of options with a delta Δ and maturity τ. For any fixed maturity τ, we define the convexity of the implied volatility curve as

$$CV(\tau) = E\left[\frac{IV(\Delta_1, \tau) + IV(\Delta_2, \tau)}{2} - IV\left(\frac{\Delta_1 + \Delta_2}{2}, 2\right)\right] \tag{1}$$

In other words, we define the convexity of the implied volatility curve as the expected difference between the average implied volatility at two different delta points and the implied volatility at the point that equals to the average of the two previous deltas.

On the standardized implied volatility surface, we have 17 fixed delta grid points from

0.1 to 0.9 at 0.05 increments. A numerical approximation of the convexity of the implied volatility curve at any maturity t is

$$CV(\tau) = \frac{1}{64} \sum_{(i,j,k)} \frac{IV(\Delta_i, \tau) + IV(\Delta_j, \tau)}{2} - IV(\Delta_k, \tau) \tag{2}$$

where Δ_i, Δ_j, and Δ_k are three different delta points with $\Delta_i < \Delta_j$ and $\Delta_k = (\Delta_i + \Delta_j)/2$. There are 64 different sets of these delta-triples based on the availability of delta points. The above equation can be translated in the following equation:

$$\begin{aligned} CV = \frac{1}{64}[&4IV(0.1) + 2.5IV(0.15) + 2IV(0.2) + 0.5IV(0.25) - 1.5IV(0.35) \\ &- 2IV(0.4) - 3.5IV(0.45) - 4IV(0.5) - 3.5IV(0.55) - 2IV(0.6) \\ &- 1.5IV(0.65) + 0.5IV(0.75) + 2IV(0.8) + 2.5IV(0.85) + 4IV(0.9)] \end{aligned} \tag{3}$$

The above equation shows that the numerical approximation of our proposed convexity measures is the weighted average implied volatilities at different delta points, where the weights add up to 0. The weights are more positive at the tails and are more negative in the center.

We measure the convexity of the implied volatility curve at each maturity on the standardized surface. Table A4 in the appendix presents summary statistics of the convexity measures by maturity. Table A4 shows that the convexity of the volatility curve decreases with maturity. Short-term convexities are also more positively skewed and display fatter tail than the convexity at longer maturities. To construct the convexity index, we take the average convexity measure across all maturities on each day. This gives us a daily measure of convexity for each index. Similar to the slope and level indexes, we aggregate daily convexity measures into a monthly measure by taking the average value across all available days in a month, except the last trading day.

We construct the global convexity index by averaging monthly convexity measures from all available countries in our sample. Figure 4 shows the global convexity as well as the US convexity from 1996 to 2021.[9] Both the global and US convexity are positive throughout the sample period and co-move together. The correlation between the two is 82%, smaller than the correlation between the global and US level indexes. The figure also shows that the global convexity is less peaked than the global level index. In other words, the kurtosis of the global convexity is much smaller than the kurtosis of the global level index, which is shown in Table 4 Panel A.

Table 4 presents the summary statistics of the global level, slope, and convexity indexes and the US convexity index. In addition, we obtain data on other option-based return predictors, including the VIX index, the SVIX index proposed by Martin (2017), the left-tail volatility (LTV) index constructed by Bollerslev, Todorov, and Xu (2015), and the US variance risk premium (VRP) introduced by Bollerslev, Tauchen, and Zhou (2009). Details about these variables and their sources are in Appendix B. Notably, Table 4 Panel A shows that except the convexity indexes, other variables all display fat tail with kurtosis above 4 and strong degree of asymmetry with absolute skewness above 1. For comparison, the skewness and kurtosis of the S&P 500 monthly returns are -0.57 and 3.82, respectively. The closer match in skewness and kurtosis between the convexity index and the stock return contributes to the superior predictability of convexity indexes relative to other option-based predictors.

Table 4 Panel B shows the pairwise correlation among these variables. We observe significant correlations among some pairs of variables. For example, the global level index has a correlation of 93% with the VIX index and a correlation of 95% with the SVIX index. The global slope index has a correlation of -82% with the global level index, -80% with the VIX index, -85% with the SVIX index, and -72% with the left-tail volatility (LTV). The global convexity index has a correlation of -68% with the global slope index and a correlation of 68% with LTV. Variance risk premium (VRP) is least correlated with other variables.

[9] We notice a slight trend in the global convexity from 1996 to 2021. In the robustness checks, we use de-trended global convexity or annual change in global convexity to run predictive regressions and the predictability remains strong.

3 Equity premium predictability

This section documents and compares the return predictability of several option-based measures in the US and in the international setting.

3.1 US evidence

We test whether the option-based measures constructed in the previous section and from the existing literature predict the S&P 500 index returns during our sample from 1996 to 2021. Tables 5 and 6 regress the semi-annual cumulative excess return of the S&P 500 index on these variables. We use Newey-West standard errors with 6 lags of autocorrelation to calculate t-statistics. Table 5 shows that five of the eight option-based variables significantly predict the semi-annual S&P 500 returns from 1996 to 2021. The global convexity predicts the semi-annual S&P 500 returns with a t-statistic of 3.69 and an R^2 of 14.4%, making it the best univariate predictor in this group. The US convexity also significantly predicts the US return with a t-statistic of 2.35 and a 9.1% R^2, making it the second-best predictor in terms of R^2. The next best predictor is the left-tail volatility (LTV). It predicts the S&P 500 return with a t-statistic of 2.74 and an R2 of 7.2%. The global slope index and the SVIX index significantly predict the S&P 500 return with an R2 of 5.4% and 4.3%, respectively. The global level index, the VIX index, and the variance risk premium do not significantly predict the semi-annual return of S&P 500 during this sample period.

Table 6 runs multivariate predictive regressions with the global convexity as the first predictor and other additional predictors. The global convexity subsumes all predictive power of the other variables in multivariate regressions. The t-statistic on the global convexity remains high throughout all eight columns. The magnitude of the estimated coefficient on the global convexity is stable across all eight columns, centering around 0.3, which is similar to the estimated coefficient in the univariate regression. Comparing the R^2 in panel B with the univariate regression, the gain in R^2 from including other variables is small. Column 8 includes all eight variables in the predictive regression and the R^2 slightly improves from 14.4% in the univariate regression to 16.5%. This table shows the superior predictive power of the global convexity in predicting semi-annual S&P 500 returns. It subsumes the predictive power of several other option-based predictors and is not affected when controlling other predictors.

Tables 5 and 6 test the predictive relation using overlapping semi-annual returns. We verify the predictability of the global convexity using non-overlapping returns to avoid potential bias associated with overlapping returns (Stambaugh, 1999; Boudoukh, Israel, and Richardson, 2022). We run predictive regressions on monthly non-overlapping S&P 500 returns in Table 7. Each column regresses the return from month t+k, with k=1,2,...,12, on the global convexity measured in month t. Table 7 shows that the global convexity significantly predicts monthly stock returns up to month t+7. Both the R^2 and t-statistic are the strongest when predicting the return in the immediate month. Then, the predictability declines as the lag time increases. After month t+7, the coefficient on the global convexity becomes insignificant, although the signs of the coefficients remain mostly positive throughout. The magnitude of the coefficient on the global convexity diminishes gradually from column 1 to column 12, consistent with the idea that the information content of a predictor decays over time. This decay in coefficient also suggests that the market risk premium reverts back to its mean.

In the Appendix, Table A5 reports the predictability test of additional option-based variables, including the ones that capture the term-structure of implied volatility. None of these variables is significant in predicting 1 month or 6 month S&P 500 returns, and the predictability of global convexity is not affect by them. These tables present strong evidence on the ability of the global convexity in forecasting US equity premium. The next section evaluates the predictability of the global convexity in other markets around the world.

3.2 International evidence

This section evaluates the ability of the global convexity in predicting the return of other indexes in our sample. We present the results in Table 8. Table 8 regresses the semi-annual excess index return on the global convexity index. It shows that the global convexity significantly predicts 19 out of the 20 index returns in our sample. Table 8 Column 1 reproduces the predictive regression on the S&P 500 returns. The average coefficient across all 20 columns is 0.34, which is very close to that in the US. The average t-statistic of the coefficients is 2.63 and the average R^2 is 8.8%. The top 4 indexes with the highest R^2 are the S&P 500 in the US (14.4%), the Nikkei 225 in Japan (13.8%), the MSCI EAFE covering Europe, Australasia, and Middle East (13%), and the KOSPI 200 in Korea (12.8%). The bottom 4 indexes with the lowest R2 are the IBEX 35 in Spain (1.7%), the Hang Seng index in Hong Kong (4.6%), the DAX index in Germany (4.7%), and the BEL 20 index in Belgium (4.9%). Overall, this table shows that the global convexity index has a robust predictive power of market returns in many countries. One interpretation of this result is that the global convexity index reveals a significant amount of information on the global risk premium, which drives the market return around the world.

3.3 Alternative measures of the global convexity index

Having demonstrated the strong predictability of the baseline global convexity index, we change the construction of this index and report the performance of the index under alternative construction methods. This exercise sheds light on the source and the robustness of the index's predictive power.

We consider several variations in how we construct the global convexity by selecting only a subset of the data. Firstly, we only select call options or put options to measure the surface convexity. This exercise examines whether call or put options provide a stronger signal in predicting stock returns. We also separately examine the construction of the index using in-the-money options and out-of-the-money options. We investigate whether maturity plays a role in the predictability of the global convexity index. We use only part of the implied volatility surface with a maturity of less than 6 months or with a maturity of more than 6 months to construct the index. This test evaluates whether short-term options contribute more to the predictability than long-term options. Finally, we consider changing the set of countries in the construction of the index. In one specification, we only use the

S&P500 options. In another specification, we drop all S&P500 options after 2002 (when we have available option data from other indexes). This test examines whether the US or the collection of other countries contribute more to the predictability.

Table 9 reports the R^2 of predictive regressions using different versions of global convexity. The results of the baseline measure are reported in column 1. The average R^2 in all 20 predictive regressions in the baseline is 8.8%. Column 2 uses only call options to construct the index and reports a 4.6% average R^2. This shows that using only call options reduces the performance of the predictor. Column 3 uses only put options and produces a 10.2% average R^2. This is the best performance among all specifications in Table 9, which suggests that put options contain the most relevant information for predicting market returns. Column 4 uses only in-the-money options and column 5 uses only out-of-the-money options to measure global convexity. We define in-the-money or out-of-the-money options based on the transformed deltas. For call options, if the transformed delta is smaller than 0.5 (i.e., having low strike prices), we classify them as in-the-money; otherwise, out-of-the-money. We use the opposite definition to classify in-the-money or out-of-the-money put options. Columns 4 and 5 show that both in-the-money and out-of-the-money options contribute similarly to the predictability of the global convexity index with an R^2 of 7.8% and 7.9%, respectively. Columns 6 and 7 use short-term and long-term options only. The results are similar to the baseline. Column 9 uses only S&P 500 options. The average R^2 is 4.1%, the worst performance among all 9 specifications. This shows that focusing on the US options alone ignores a large amount of return relevant information. Column 9 excludes S&P 500 options after 2002. The performance of column 9 is very similar to the baseline performance. This shows that even without the US data, the global convexity index can still significantly predict equity returns around the world. The main benefit of having the US data is to extend our sample period from 2002 to 1996. Overall, this table shows that there is a significant amount of information that is relevant to market risk premium from index options outside of the US and from put options.

4 Dissecting return predictability of global convexity

Having documented the strong predictability of the global convexity index, we provide additional analysis on the economic source of its predictability. We first analyze the additional information content of the convexity index from non-US countries. We also decompose the global convexity into a convexity left and convexity right index to show that both tails of risk-neutral distribution reveal important information about the global risk premium.

4.1 International vs. US convexity

Table 9 shows that the convexity index measured from non-US countries contributes more than the US convexity to the predictability of equity premium. This is surprising given that the US has the biggest option market and the prior literature, e.g., Rapach, Strauss, and Zhou (2013), shows that the US stock return leads other countries. However, if traders that trade non-US index options incorporate information from around the world more efficiently or they are more sensitive to changes in the global risk premium, we could observe that the convexity measured from non-US countries reveals more information than the US convexity. One indication of this hypothesis, as shown in Figure 4, is that during the early spread of the coronavirus, the global convexity leads the US convexity by about 1 month. This is consistent with the spread of the pandemic. The severe effects of the virus first appeared in Asian countries, then in European countries, and later in the US.

To test the information content of the international convexity against the US convexity, we run lead-lag regressions in Table 10. Columns 1 to 3 show the autocorrelation coefficient of the US convexity, non-US convexity, and the global convexity index is around 0.9.[10] Columns 4 and 5 regress the US convexity on the lagged non-US and lagged global convexity in addition to the lagged US convexity. The estimated coefficients on the lagged non-US and global convexity are 0.17 and 0.15, respectively, and are both statistically significant at the 1 percent level. This indicates that the non-US convexity contains information about the next

[10] The non-US convexity is the average convexity of the remaining 19 countries and regions, which is available since 2002.

period US convexity, which suggests that the shocks on the risk premium around the world transmit to the US with some lag. Columns 6 and 7 regress the non-US and global convexity index on the lagged US convexity index, controlling their own lagged values. The coefficient on the US convexity is around 0 and insignificant, which indicates that shocks to the US risk premium is quickly reflected in the international market. This lead-lag relationship between the US and non-US convexity indexes explains why the global convexity contains much more information about the US risk premium than the US convexity.

4.2 Dissecting convexity measure into left tail and right tail

To further understand the source of return predictability of the global convexity index, we decompose it into two orthogonal components: the convexity left and the convexity right index. We take two steps to accomplish this. In the first step, we decompose the global convexity index into the contribution from the left-half and the right-half section of the implied volatility surface by splitting Equation (3) in the middle:

$$\begin{aligned} LH = \frac{1}{64}[&4IV(0.1) + 2.5IV(0.15) + 2IV(0.2) + 0.5IV(0.25) - 1.5IV(0.35) \\ &- 2IV(0.4) - 3.5IV(0.45) - 2IV(0.5)] \end{aligned} \tag{4}$$

$$\begin{aligned} RH = \frac{1}{64}[&-2IV(0.5) - 0.35IV(0.55) - 2IV(0.6) - 1.5IV(0.65) + 0.5IV(0.75) \\ &+ 2IV(0.8) + 2.5IV(0.85) + 4IV(0.9)] \end{aligned} \tag{5}$$

Hence, the convexity index is just the sum of contribution from the left-half and right-half of volatility surface. The left-half of the volatility surface consists of implied volatilities with a corresponding transformed delta below 0.5, while the right-half of the surface is based on the implied volatilities with a corresponding transformed delta above 0.5. In the next step, we orthogonalize the two components by regressing the right-half contribution on the left-half contribution and extracting the residual as the convexity right index:

$$RH_t = b_0 + b_1 LH_t + \epsilon_t \tag{6}$$

We define convexity right (CR) index as

$$CR_t = b_0 + \epsilon_t \tag{7}$$

and the convexity left (CL) index as

$$CL_t = LH_t(1 + b) \tag{8}$$

These two components are orthogonal to each other, and their sum equals to the global convexity index. The convexity left index is driven by the behavior of the left-tail of the risk-neutral distribution, while the convexity right index is driven by the behavior of the right-tail of neutral distribution that is independent from the left tail. Figure 5 plots the time series these two indexes. To be comparable with the prior literature, we also use the same methodology to construct convexity left and convexity right measures using US options only.

We test the return predictability of these variables. Table 11 reports the R^2 of using these variables to predict S&P 500 returns at different horizons. Column 1 of the table shows that US convexity left predicts monthly S&P 500 returns with an R^2 of 2.04%. The R^2 peaks at 9.87% when predicting index returns at the 7-month horizon. On the other hand, column 2 shows that US convexity right has little predictive power of S&P 500 returns. Its average R^2 is 0.2%. Turning to the global variables, global convexity left has similar predictability as US convexity left with an average R^2 of 7.35%. Interestingly, global convexity right also predicts US returns. At 1 month horizon, its R^2 is 1.29%. It peaks at 6.52% at the 7-month horizon. The average R^2 of global convexity right is 4.2%. The last column shows the R^2 of predicting US returns with our baseline global convexity measure. It achieves an average R^2 of 11.4%. The takeaway from this table is that both the US and global convexity left predict US returns similarly and their predictability explains roughly two thirds of the predictability of global convexity. Global convexity right also predicts US returns, which explains the remaining one third of our baseline result. In the appendix, Tables A6 and

A7 test the statistical significance of the return predictability of US left, global left and global right. These variables significantly predicts S&P 500 returns at the one or six-month horizons with or without controlling for other option-based predictors.

Table A8 reports the R^2 of predictive regressions using US and global versions of convexity left or convexity right as the predictor on international equity returns. It shows that US convexity left, global convexity left and global convexity right predict international returns, while US convexity right generally does not predict international returns. This table also shows that global convexity left have better predictability of international returns than US convexity left, even though they have roughly the same predictability in the US. Similar to the US setting, two thirds of our baseline predictability come from the left tail, and the remaining one third comes from the right tail. The average R^2 of using the global convexity left to predict international returns is 6.4% and the average R^2 of global convexity right is 3.2%. There are some cross-sectional variations in the strength of return predictability between the left and right measures. In some European countries, the convexity right index possesses a stronger return predictability than the left index.

4.3 Economic interpretation of convexity left and right index

What explains the variation in the convexity left and right? Table 12 regresses the left and right index on different state variables. Table 12 Panel A shows that the global slope index and the left-tail volatility (LTV) index explain the majority of variations in the convexity left index. The global slope index explains the convexity left index with an R^2 of 91.6%. The steeper the implied-volatility slope, the higher the convexity left index. The steepening of the implied-volatility slope is a sign that investors have strong demand for downside protection through out-of-the-money put options, which tends to happen when investors' fear towards market crash is elevated. The left-tail volatility index, another proxy of market fear, also explains the convexity left index with an R^2 above 50%. Therefore, we conclude that the convexity left captures investors' fear towards market crash and through this fear channel, it predicts the equity premium.

Table 12 Panel B shows that the global slope and the left-tail volatility index have little explanatory power of the convexity right index, but the TED spread and aggregate short-interest index in the US explain a sizable variation of the convexity right index. The TED spread measures the funding cost of financial intermediaries. Column 3 shows that the higher the TED spread, the lower the convexity right index, which predicts lower returns. One interpretation of this result is that when the funding condition of intermediaries tightens, they tend to sell risky assets, which generates downward pressure on equity returns. Column 4 shows that the aggregate short-interest index, introduced by Rapach, Ringgenberg, and Zhou (2016), is also negatively associated with the convexity right index. As shown by Rapach, Ringgenberg, and Zhou (2016), when the aggregate volume of short-interest increases, the market tends to decline in the future, suggesting that short-sellers have superior information about the market. Table 12 Panel B shows that the convexity right index reveals demand-relevant information about financial intermediaries and short sellers. It is through this informed demand channel that the convexity right index predicts stock returns.

To further distinguish the information content from the left and the right tail of risk neutral distribution, we regress contemporaneous US market returns on changes in the convexity left and right measures in Table 13. We also regress the future dividend growth rate on these measures in Table 13. Table 13 highlights a sharp distinction between the convexity left and right measures. Higher convexity left indicates elevated market fear, which is accompanied with large decline in the market return in column 1. This fear is rationally justified as higher convexity left predicts a decline in the US dividend growth rate in the next six months as shown in column 3. On the other hand, column 2 shows that an increase in the convexity right measure is associated with an increase in market returns. This is because the convexity right measure is negatively correlated with the funding cost and the aggregate short interests. Higher convexity right is good news to the market. This interpretation is also consistent with the fact that higher convexity right predicts higher dividend growth rate in the US as shown in column 4.

Overall, these tables show that both the left-tail and the right-tail of the implied volatility surface contain important information about equity premium. The left tail mainly contains information about the fear of market crash, while an increase in the right tail reveals good news about the market that is accompanied with lower funding cost and less short selling. The global convexity effectively combines the information from both the left tail and right tail, which is why it possesses superior return predictability.

5 Additional analysis

This section explores the information content of options with different maturities and the relationship between global convexity and Global Financial Cycle.

5.1 The information content of options at different maturities

As shown in the prior section, the US convexity left alone delivers very strong return predictability in the US and often generates higher R^2 than other option-based predictors proposed in the literature. Why does this variable perform better than others, when they all seem to capture the same economic force, i.e., fear of downside risk in the US? We hypothesize that one of its strength comes from combining information from options with different maturities. Other US option-based predictors extract information from options with a single specific maturity. For example, VIX index is constructed based on options with a 1-month maturity. Bollerslev, Todorov, and Xu (2015) constructs LTV index using ooptions with no more than 45 days until expiration. Martin (2017) creates SVIX indexes with different horizons, such as one month, two months, etc. For each version of SVIX index, he uses only options that are closest to that horizon. Even though short-maturity options are the most liquid (as shown by Golez (2014)), ex-ante it is not clear options with what maturity contain the most information about equity premium. The flexibility of our measure allows us to assess the information content of options with different maturities. Table 14 presents results on using options with different maturities to construct the US convexity left and its R^2 in predicting US returns with different horizons.

In Table 14, each row of the first column indicates which options we use to construct the US convexity left index. The next five columns use the specific US convexity left index to predict S&P 500 index return with one-, three-, six-, nine-, and twelve-month horizons. Three messages emerge from this table. First, the predictive power of the convexity left index has a inverse-U shaped relationship with the maturity of its underlying options. One-month maturity options produce an convexity left measure with the lowest R^2. Options with one and a half year maturity also produce low R^2. The convexity left measures constructed from options with five-, six-, or nine-month maturities have the highest R^2 in predicting US returns. This suggests that median term options contain most information about equity premium. The second message is that the maturity of options does not need to align with the horizon of returns to generate high R^2. In fact, medium-term options generate highest R^2 across all return horizons, while short-term and long-term options generate poor R^2 across all return horizons. The third message is that aggregating information across multiple maturities tend to improve R^2. These messages are also generally true when we examine international equity premium.

5.2 Common predictability and global financial cycle

Overall, we have documented that a single measure, global convexity, significantly predicts index returns around the world. This suggests that equity premia around the world co-move strongly and the global convexity is informative about this comovement. To formally test this hypothesis, we obtain data on the Global Financial Cycle factor, constructed by Miranda-Agrippino and Rey (2020), and test whether the global convexity predicts this factor. This factor is constructed to capture comovement in risky assets around the world. As expected, Table 15 shows that the global convexity significantly predicts changes in the Global Financial Cycle factor over 1 month, 2 months, ..., and up to 11 months. The highest R^2 is 11.1% in column 7, over 7 months horizon. The coefficient on the global convexity always increases, suggesting that the predictability does not revert. In the appendix, Table A9 shows that both the global convexity left and global convexity right can significantly predict changes in the Global Financial Cycle factor. The Global Financial Cycle factor is constructed based on a rich set of asset prices, including equities around the world, commodities, and corporate bonds. The fact that our measure predict changes in the Global Financial Cycle factor suggests that it can also predict the returns of many other assets that are not tested in this paper. Further investigating and understanding how global convexity relates to the global financial cycle both empirically and theoretically will be an exciting avenue of future research.

6 Out-of-sample analysis and robustness checks

This section reports the out-of-sample (OOS) analysis and additional robustness checks on the predictability of the global convexity index.

6.1 Out-of-sample analysis

We report the OOS R^2 of using the global convexity index to predict market return. Following Welch and Goyal (2008), we define OOS R^2 as

$$R^2_{OOS} = 1 - \frac{MSE_A}{MSE_N} \tag{9}$$

where MSE_A is the mean squared error from the global convexity predictive model and MSE_N is the mean squared error from the historical mean model. For each market index, we start with 10 years of in-sample data to train both the predictive model and the historical mean model. Then, we apply the predictive model and the historical mean model on a rolling basis to predict index return. MSE_A and MSE_N are computed during the out-of-sample period, i.e. after the initial 10 years. Table 16 shows the OOS R^2 of the global convexity index in predicting returns with different horizons in the twenty countries or regions in our sample. The OOS R^2 in predicting the 1-, 3-, 6-, 9-, and 12-month S&P 500 returns are 4.1%, 12.6%, 20.8%, 18.1%, and 17.7%, respectively. The average OOS R^2 in predicting the 1-, 3-, 6-, 9-, and 12-month in all countries are 2.2%, 6.2%, 11.4%, 8.3%, and 5.9%, respectively. The OOS R^2 peaks at the 6-month horizon, consistent with Table 7 that shows the predictability is significant up to month t+7.

Campbell and Thompson (2008) argue that an OOS R^2 of 1% in a predictive regression translates to an economically large gain for risk-averse investors. We compute the out-of-sample gain from the market timing strategy based on the global convexity in each country in Table A10. Table A10 column 1 shows the Sharpe ratio of a buy-and-hold strategy in each country during the out-of-sample testing period. The buy-and-hold Sharpe ratio in the US is 0.705 and the average Sharpe ratio across the twenty indexes is 0.384. Column 2 estimates an optimal market timing strategy assuming the equity weight is a linear function of the global convexity index x_t, i.e.

$$\omega = a + bx_t \tag{10}$$

Column 2 estimates parameters a and b using the entire testing data, which is an in-sample approach. Column 3 estimates the parameters a and b on a rolling basis, which is an out-of-sample approach. Both columns 2 and 3 show significant improvement in Sharpe ratio. The Sharpe ratio of market timing in the US is 1.085, if optimal a and b are known ex ante, or 1.011 if a and b are estimated on a rolling basis. The average Sharpe ratio across all countries almost doubles from 0.384 in column 1 to 0.743 in column 2 and 0.634 in column 3. This table indicates the convexity index creates significant utility gain to risk-averse investors when they use this variable to time the market.

6.2 Additional robustness checks

We run several robustness checks by using alternative specifications to measure the global convexity. The predictability of these alternative measures is qualitatively similar to our baseline result. In our first robustness check, we drop the part of the volatility surface associated with delta equal to 0.1, 0.15, 0.85, and 0.9. Effectively, we construct the global convexity index based on a reduced standardized implied volatility surface. Column 1 of Table 17 shows the R2 of using this alternative global convexity index to predict stock returns. The average R2 from all countries is 8.4%, similar to the baseline average of 8.8%.

We also consider only using index options from a smaller set of indexes to define the global convexity index. This is motivated by the fact that in some countries, the quality of index options data provided by OptionMetrics is poor. This could be due to illiquidity or limited availability. The indexes that we choose are from the US, Switzerland, Germany, Spain, France, the United Kingdom, Italy, Euro Stoxx 50, Australia, and Hong Kong. We choose this set of indexes because Gandhi, Gormsen, and Lazarus (2023) show that the index options from these countries and regions have better quality. Column 2 of Table 17 shows the predictive performance of this alternative global convexity index. The average performance improves relative to the baseline result.

We also consider a more robust way to measure the global convexity index. Specifically, we split the implied volatility surface into three sections based on the transformed delta: the left tail, the middle section, and the right tail. The cut-off point for the left tail is 0.2. We set the cut-off point for the right tail to be 0.8. Any grid point with a transformed delta between 0.25 and 0.75 is defined as the middle section of the surface. On each day, for each index, we compute the average implied volatility in the three sections. We take the average across all maturities. We measure the robust convexity as the average of the left and right tail volatility minus the implied volatility in the middle section. This robust convexity measure puts the same weight on implied volatilities in each section of the volatility surface. Column 3 shows the return predictability of this robust convexity measure. The average R^2 of this index in predicting stock returns is 8.8%, which is the same as our baseline result.

Moreover, we do not use the implied volatility surface data. Instead, we directly use option level implied volatilities to construct the convexity index. To do so, we first classify all options into five equal-length bins based on their transformed delta and take the average value of the implied volatilities in each bin. We then apply the same procedure from equation (1) to compute the volatility convexity. The predictive performance of the index based on individual option-level implied volatility is shown in column 4 of Table 17. The average R^2 is 7.1%, which is slightly lower than the baseline average. This demonstrates that initially standardizing the surface improves the empirical measure of the convexity index.

Our last set of robustness checks are to use detrended global convexity in column 5 and annual change in global convexity in column 6 as the return predictor. This is motivated by the observation that the global convexity increases over time during the sample period. We estimated the detrended global convexity by estimating a linear trend using the entire sample and use the residual as the return predictor. We also measure the change in global convexity over a 12-month period as the return predictor. As shown in the table, the average R2 of using these measures to predict market returns are greater than our baseline result.

7 Conclusion

We document that the convexity measured from the global implied volatility surface robustly predicts the stock market index return in the US and many other countries around the world. Our convexity index measures the degree of curvature of the implied-volatility curve. The convexity index is higher if the implied volatilities of options with both high and low strike prices are greater than the implied volatilities of options with medium strike prices. When this happens, the expected stock market return is higher. Empirically, the global convexity predicts the semi-annual S&P 500 returns with an in-sample and OOS R2 of 14.4% and 20.8%, respectively. The average R^2 of using this index to predict all 20 index returns in our sample is 8.8% in-sample and 11.4% out-of-sample. The global convexity subsumes the predictability of several existing option-based predictors, including the VIX index, SVIX index, variance risk premium, and left-tail volatility. Through various alternative specifications, we find the predictability of global convexity to be extremely robust.

The predictive power of the global convexity index comes from its ability to aggregate information from across the globe and combine information from both the left and right tail of the risk-neutral return distribution. Information contained in the left tail reveals investors' fear of market crash, while information contained in the right tail is associated with the funding cost of financial intermediaries and the amount of aggregate short interest, which reveal the speculative equity demand from financial intermediaries and short sellers.

Lastly, the fact that the global level, slope, and convexity index co-move strongly with their country-level counterparts indicates that the global options market are closely connected. It is plausible that the same group of marginal investors operate in all these markets. The fact that the information extracted from the global options market predicts stock returns around world also indicates that the marginal investors in the options market play a significant role in the pricing of the equity around the world. Understanding how information or preference is revealed and transmitted across different markets through these marginal investors is an interesting future research question.

References

Andersen, Torben G, Nicola Fusari, and Viktor Todorov, 2015, The risk premia embedded in index options, *Journal of Financial Economics* 117, 558–584.

——— , 2020, The pricing of tail risk and the equity premium: Evidence from international option markets, *Journal of Business & Economic Statistics* 38, 662–678.

Arrow, Kenneth J, 1964, The role of securities in the optimal allocation of risk-bearing, *The Review of Economic Studies* 31, 91–96.

Back, Kerry, Kevin Crotty, and Seyed Mohammad Kazempour, 2022, Validity, tightness, and forecasting power of risk premium bounds, *Journal of Financial Economics* 144, 732–760.

Bakshi, Gurdip, Charles Cao, and Zhiwu Chen, 1997, Empirical performance of alternative option pricing models, *The Journal of Finance* 52, 2003–2049.

Bakshi, Gurdip, John Crosby, Xiaohui Gao, and Wei Zhou, 2019, A new formula for the expected excess return of the market, *Fox School of Business Research Paper*.

Bates, David S, 1991, The crash of '87: was it expected? the evidence from options markets, *The Journal of Finance* 46, 1009–1044.

——— , 2000, Post-'87 crash fears in the s&p 500 futures option market, *Journal of Econometrics* 94, 181–238.

——— , 2022, Empirical option pricing models, *Annual Review of Financial Economics* 14, 369–389.

Bekaert, Geert, and Campbell R Harvey, 1995, Time-varying world market integration, *the Journal of Finance* 50, 403–444.

Bollerslev, Tim, James Marrone, Lai Xu, and Hao Zhou, 2014, Stock return predictability and variance risk premia: Statistical inference and international evidence, *Journal of Financial and Quantitative Analysis* 49, 633–661.

Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, Expected stock returns and variance risk premia, *The Review of Financial Studies* 22, 4463–4492.

Bollerslev, Tim, and Viktor Todorov, 2011, Tails, fears, and risk premia, *The Journal of Finance* 66, 2165–2211.

———, and Lai Xu, 2015, Tail risk premia and return predictability, *Journal of Financial Economics* 118, 113–134.

Boudoukh, Jacob, Ronen Israel, and Matthew Richardson, 2022, Biases in long-horizon predictive regressions, *Journal of Financial Economics* 145, 937–969.

Breeden, Douglas T, and Robert H Litzenberger, 1978, Prices of state-contingent claims implicit in option prices, *Journal of Business* pp. 621–651.

Campbell, John Y, and Samuel B Thompson, 2008, Predicting excess stock returns out of sample: Can anything beat the historical average?, *The Review of Financial Studies* 21, 1509–1531.

Carr, Peter, and Liuren Wu, 2009, Variance risk premiums, *The Review of Financial Studies* 22, 1311–1341.

Chabi-Yo, Fousseni, and Johnathan Loudis, 2020, The conditional expected market return, *Journal of Financial Economics* 137, 752–786.

Cont, Rama, Jose da Fonseca, and Valdo Durrleman, 2002, Stochastic models of implied volatility surfaces, *Economic Notes* 31, 361–377.

Cox, John C, and Stephen A Ross, 1976, The valuation of options for alternative stochastic processes, *Journal of Financial Economics* 3, 145–166.

Das, Sanjiv Ranjan, and Rangarajan K Sundaram, 1999, Of smiles and smirks: A term structure perspective, *Journal of Financial and Quantitative Analysis* 34, 211–239.

Debreu, Gerard, 1959, *Theory of value: An axiomatic analysis of economic equilibrium .* , vol. 17 (Yale University Press).

Dumas, Bernard, Jeff Fleming, and Robert E Whaley, 1998, Implied volatility functions: Empirical tests, *The Journal of Finance* 53, 2059–2106.

Gandhi, Mihir, Niels Joachim Gormsen, and Eben Lazarus, 2023, Does the market understand time variation in the equity premium?, *Working paper*.

Golez, Benjamin, 2014, Expected returns and dividend growth rates implied by derivative markets, *The Review of Financial Studies* 27, 790–822.

Harvey, Campbell R, 1991, The world price of covariance risk, *The Journal of Finance* 46, 111–157.

Henkel, Sam James, J Spencer Martin, and Federico Nardari, 2011, Time-varying short-horizon predictability, *Journal of Financial Economics* 99, 560–580.

Jensen, Christian Skov, David Lando, and Lasse Heje Pedersen, 2019, Generalized recovery, *Journal of Financial Economics* 133, 154–174.

Kozhan, Roman, Anthony Neuberger, and Paul Schneider, 2013, The skew risk premium in the equity index market, *The Review of Financial Studies* 26, 2174–2203.

Liu, Hong, Yueliang Jacques Lu, Weike Xu, and Guofu Zhou, 2022, Market risk premium expectation: Combining option theory with traditional predictors, *Available at SSRN*.

Liu, Jun, Jun Pan, and Tan Wang, 2005, An equilibrium model of rare-event premia and its implication for option smirks, *The Review of Financial Studies* 18, 131–164.

Martin, Ian, 2017, What is the expected return on the market?, *The Quarterly Journal of Economics* 132, 367–433.

Miranda-Agrippino, Silvia, and Hélene Rey, 2020, Us monetary policy and the global financial cycle, *The Review of Economic Studies* 87, 2754–2776.

Pan, Jun, 2002, The jump-risk premia implicit in options: Evidence from an integrated time-series study, *Journal of Financial Economics* 63, 3–50.

Qiao, Fang, Lai Xu, Xiaoyan Zhang, and Hao Zhou, 2019, Variance risk premiums in emerging markets, *Available at SSRN 3199522*.

Rapach, David E, Matthew C Ringgenberg, and Guofu Zhou, 2016, Short interest and aggregate stock returns, *Journal of Financial Economics* 121, 46–65.

Rapach, David E, Jack K Strauss, and Guofu Zhou, 2013, International stock return predictability: What is the role of the united states?, *The Journal of Finance* 68, 1633–1662.

Ross, Steve, 2015, The recovery theorem, *The Journal of Finance* 70, 615–648.

Ross, Stephen A, 1978, A simple approach to the valuation of risky streams, *Journal of Business* pp. 453–475.

Solnik, Bruno, 1983, International arbitrage pricing theory, *The Journal of Finance* 38, 449–457.

Stambaugh, Robert F, 1999, Predictive regressions, *Journal of Financial Economics* 54, 375–421.

Welch, Ivo, and Amit Goyal, 2008, A comprehensive look at the empirical performance of equity premium prediction, *The Review of Financial Studies* 21, 1455–1508.

Appendix A Construction of volatility surface

This section details how OptionMetrics constructs the standardized implied volatility surface. OptionMetrics first computes Black-Scholes implied volatility for options with available data. For European-style options, the Black-Scholes model:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

$$P = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left(\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{1}{2}\sigma^2\right)T\right)$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

C or P is the midpoint of the best closing bid price and best closing offer price for the call (put) option, S is the current underlying security price, K is the strike price, T is the time in years remaining to option maturity, r is the continuously compounded interest rate, q is the continuously compounded dividend yield, and σ is the implied volatility.

Then, OptionMetrics organizes the data by the log of days to maturity and by "call-equivalend delta" (i.e., delta for a call option, one plus delta for a put option). Then, at each grid point j on the volatility surface, the standardized implied volatility $\hat{\sigma}_j$ is calculated as a weighted sum of option implied volatilities:

$$\hat{\sigma}_j = \frac{\sum_i V_i \sigma_i \Phi(x_{ij}, y_{ij}, z_{ij})}{\sum_i V_i \Phi(x_{ij}, y_{ij}, z_{ij})}$$

where i is indexed over all available options on each day, V_i is the vega of the option, σ_i is the implied volatility, and $\Phi(\cdot)$ is the kernel function:

$$\Phi(x, y, z) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x^2}{2h_1} + \frac{y^2}{2h_2} + \frac{z^2}{2h_3}\right)}$$

The inputs to the kernel function measures the "distance" between an actual option i and the grid point j:

$$x_{ij} = \ln(T_i/T_j)$$

$$y_{ij} = \Delta_i - \Delta_j$$

$$z_{ij} = I_{\{CP_i = CP_j\}}$$

where T_i and T_j are measured in days; Δ_i and Δ_j are call-equivalent deltas; and z_{ij} is an indicator function, which equals one if both the option and the surface have the same call or put type. The kernel bandwidth parameters are set at $h_1 = 0.05$, $h_2 = 0.005$, and $h_3 = 0.001$. Options with fewer than 11 days to maturity are excluded from the sample.

Appendix B Variable definitions

Global level index: we take the average implied volatility of the standardized volatility surface in each country on each day. We drop the last trading day of each month to avoid overlapping with the next month. We then aggregate the daily average to a monthly average level index for each country. We average the level measures from all countries in our sample to obtain the global level index.

Global slope index: on each day, we regresses the implied volatilities of the standardized volatility surface on their transformed deltas and obtain the coefficient. We drop the last trading day of each month to avoid overlapping with the next month. We then aggregate the daily coefficient to a monthly average coefficient as the slope for each country. We average the slope measures from all countries in our sample to obtain the global slope index.

Global convexity index: we first measure the convexity of the implied volatility curve at any maturity τ as the following

$$CV(\tau) = \frac{1}{64} \sum_{\{i,j,k\}} \frac{\sigma(\Delta_i, \tau) + \sigma(\Delta_j, \tau)}{2} - \sigma(\Delta_k, \tau)$$

where Δ_i, Δ_j, and Δ_k are delta grid points on the standard implied volatility surface with $\Delta_i < \Delta_j$ and $\Delta_k = (\Delta_i + \Delta_j)/2$. There are 64 different sets of these delta triples based on the availability of grid points. To construct the convexity index, we then take the average convexity measure across all maturities on each day. This gives us a daily measure of convexity for each index. We aggregate daily convexity measures into a monthly measure by taking the average value across all available days in a month, except the last trading day. We construct the global convexity index by averaging monthly convexity index from all available countries in our sample.

VIX index: the implied-volatility index provided by the CBOE. We download the daily measure of the VIX index and aggregate it to a monthly measure by taking average.

CBOE SKEW index: Measures the average expected correlation between the top 50 stocks in the SPX index and is obtained from the CBOE website.

SVIX index: Martin (2017) introduces the SVIX index to measure the lower bound of US equity premium based on option prices. We download the six-month SVIX index from Ian Martin's website.[11] We extend the data to the end of 2021 based on the procedure in Martin (2017).

Left-tail volatility (LTV): LTV measures the return volatility generated by the left tail of the one-week risk-neutral return distribution introduced by Bollershlev, Todorov, and Xu (2015). We download the data from Viktor Todorov's website and extend it to the end of 2021.[12]

Variance risk premium (VRP): variance risk premium measures the difference between the squared VIX index and the realized variance of the market index. Bollershlev, Tauchen and Zhou (2009) document that VRP predicts US return from 1990 to 2007. We download the data on VRP from Hao Zhou's website, which updates the data to 2021.[13]

TED spread: measures the difference between the three-month LIBOR rate and the three-month yield on Treasury bills. We download daily TED spread from the Federal

[11] https://personal.lse.ac.uk/martiniw/
[12] https://tailindex.com/volatilityindex.html
[13] https://sites.google.com/site/haozhoupersonalhomepage/

Reserve database and aggregate it to a monthly measure by taking simple average.

Short-interest index: this is the detrended aggregate short-selling interest in the US. The measure is constructed by Rapach, Ringgenberg, and Zhou (2016). We download the data from Guofu Zhou's website.[14]

[14]http://apps.olin.wustl.edu/faculty/zhou/

Figure 1: Availability of index options

This figure plots the number of countries or regions in our sample for which we have available options data.

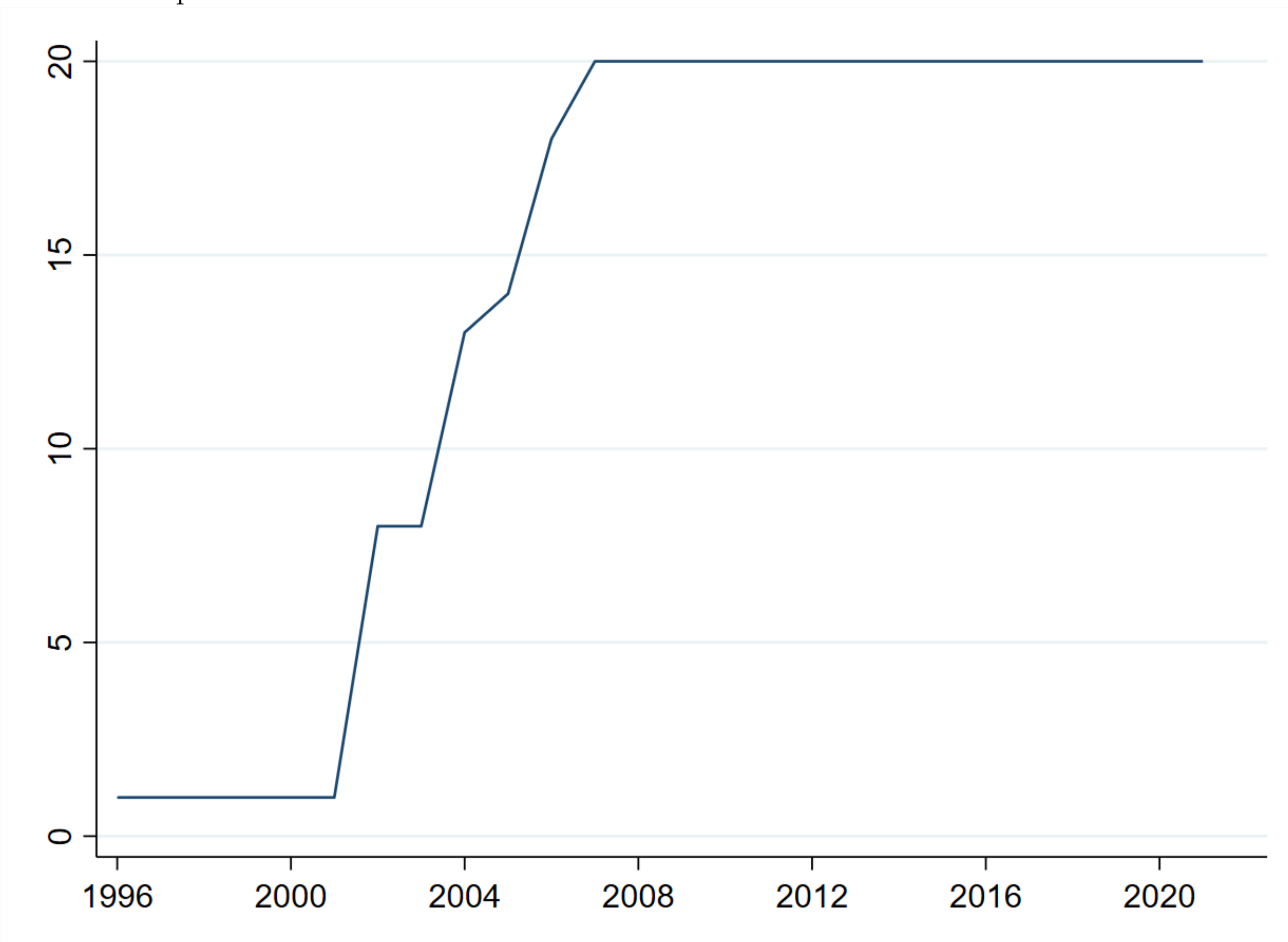


Figure 2: Level of the implied volatility surface

The figure plots the global and US average of the standardized volatility surface, i.e., the level index. The sample period is from 1996m1 to 2021m12.

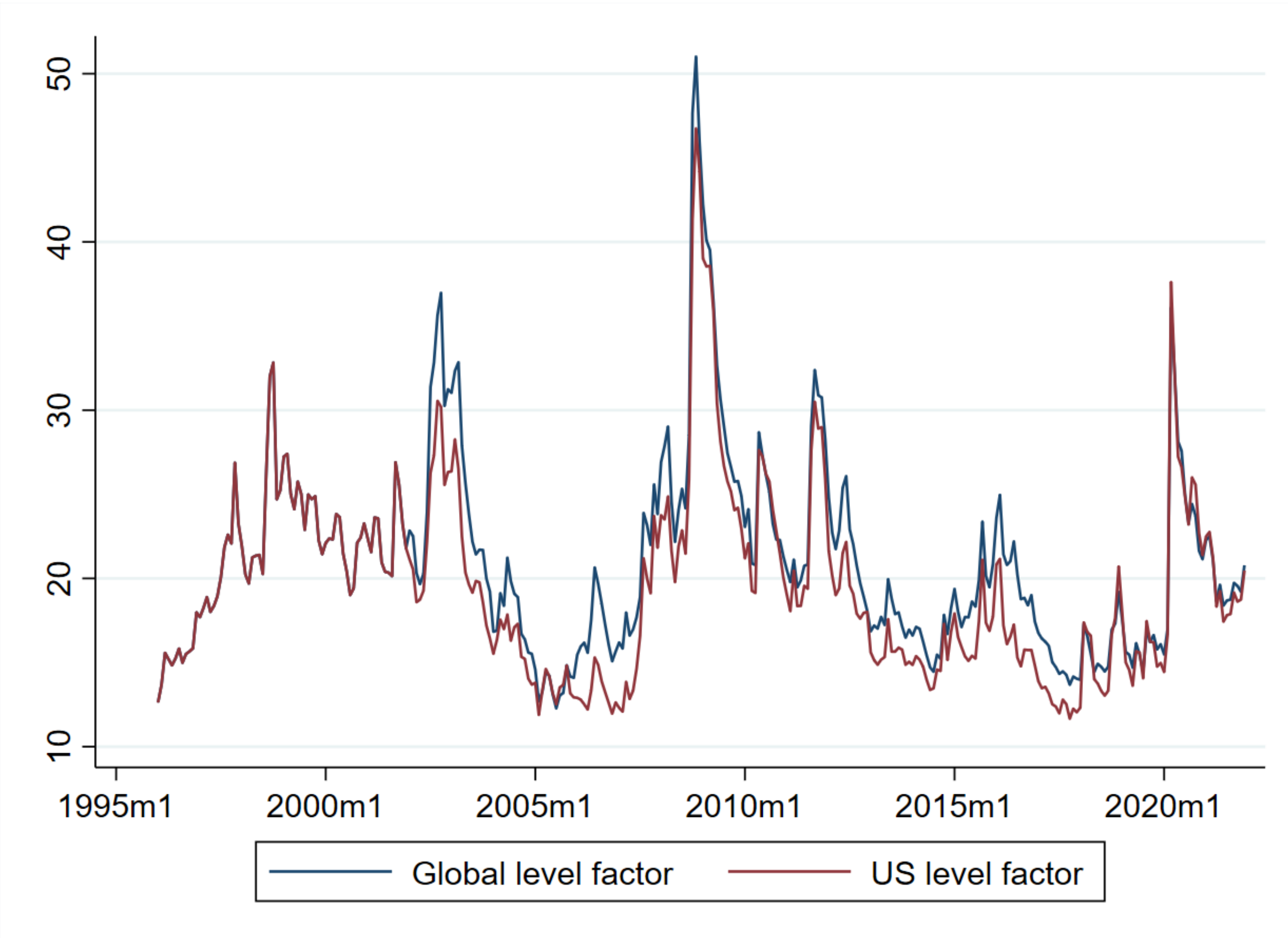


Figure 3: Slope of the implied volatility surface

The figure plots the slope of the global and US standardized volatility surface. The sample period is from 1996m1 to 2021m12.

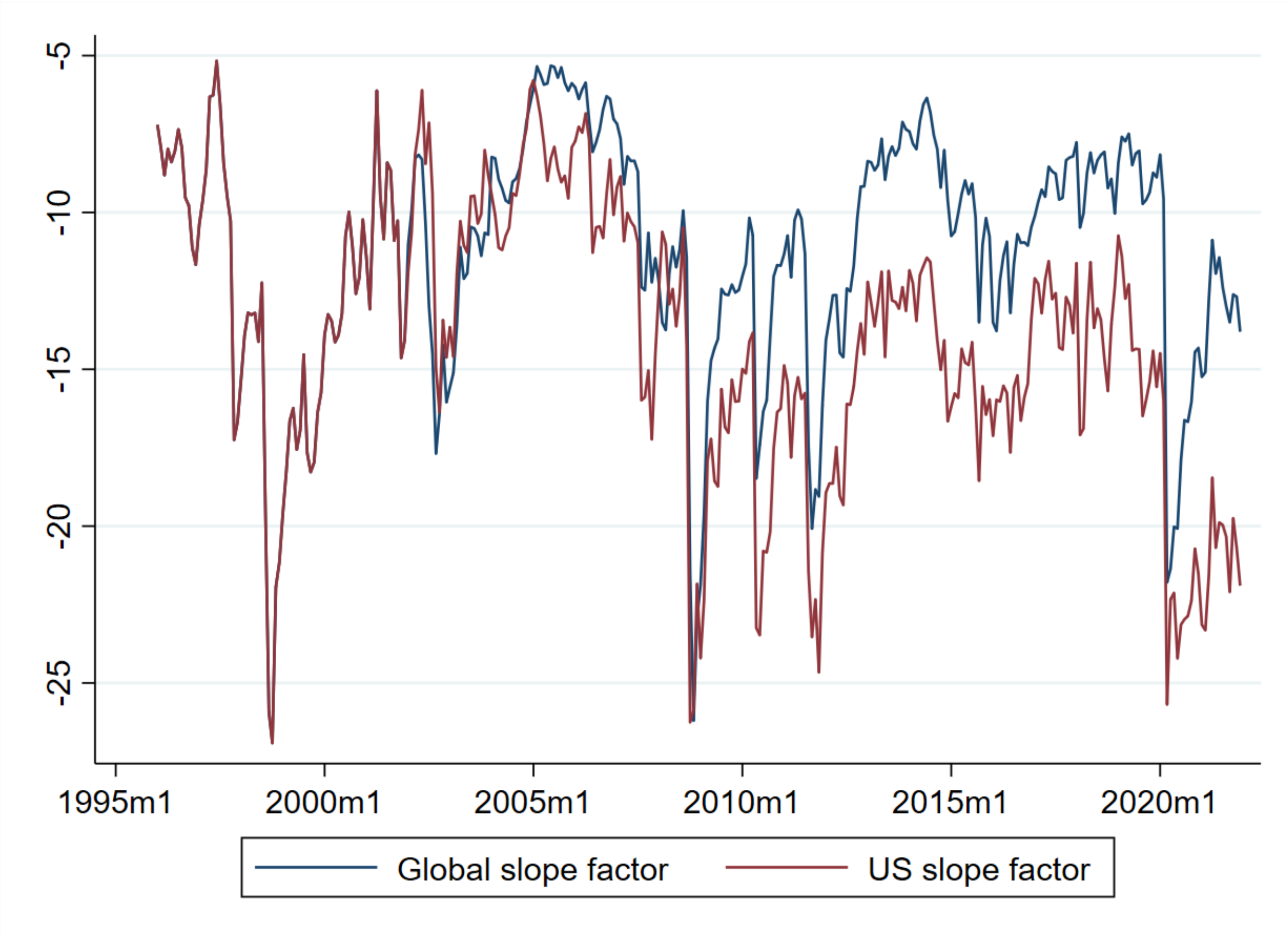


Figure 4: Convexity of the implied volatility surface

The figure plots the convexity of the global and US standardized volatility surface. The sample period is from 1996m1 to 2021m12.

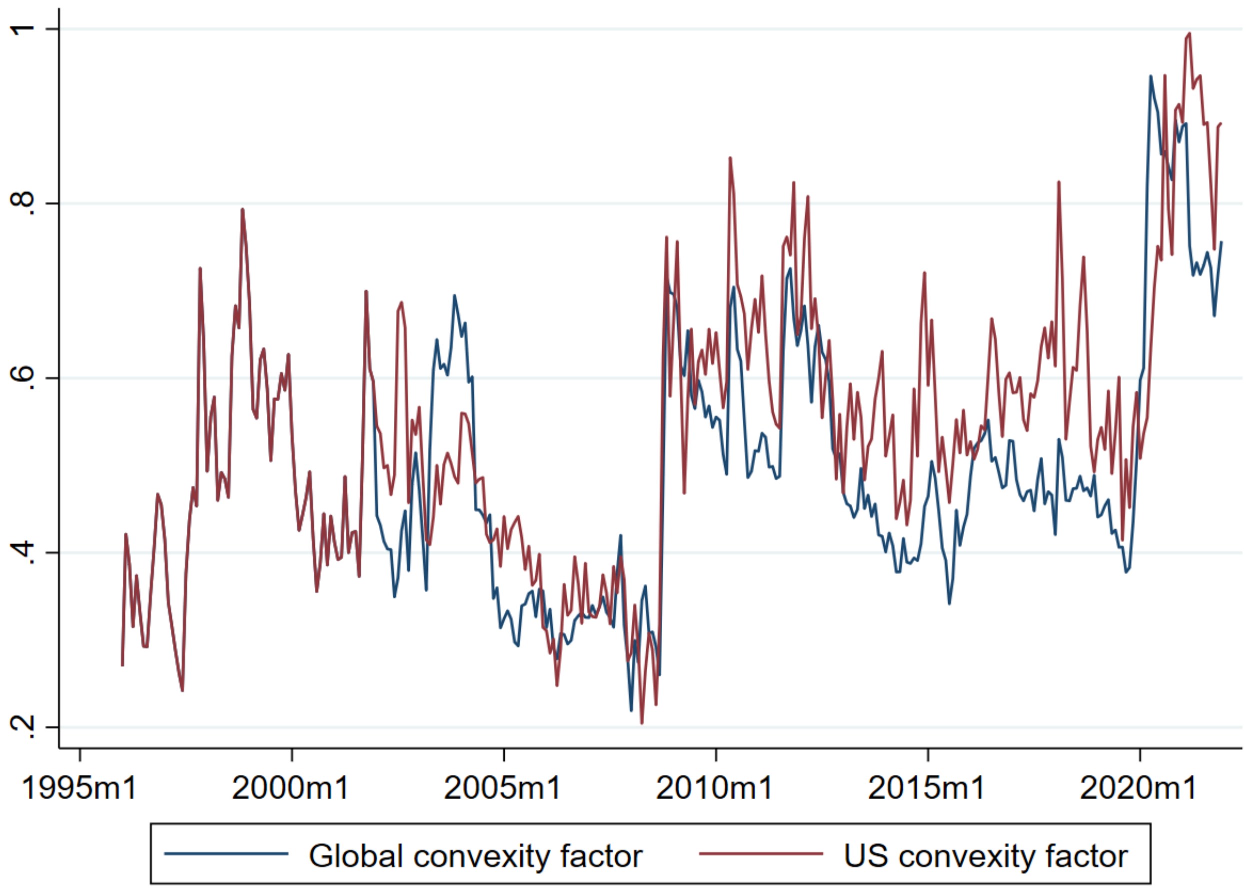
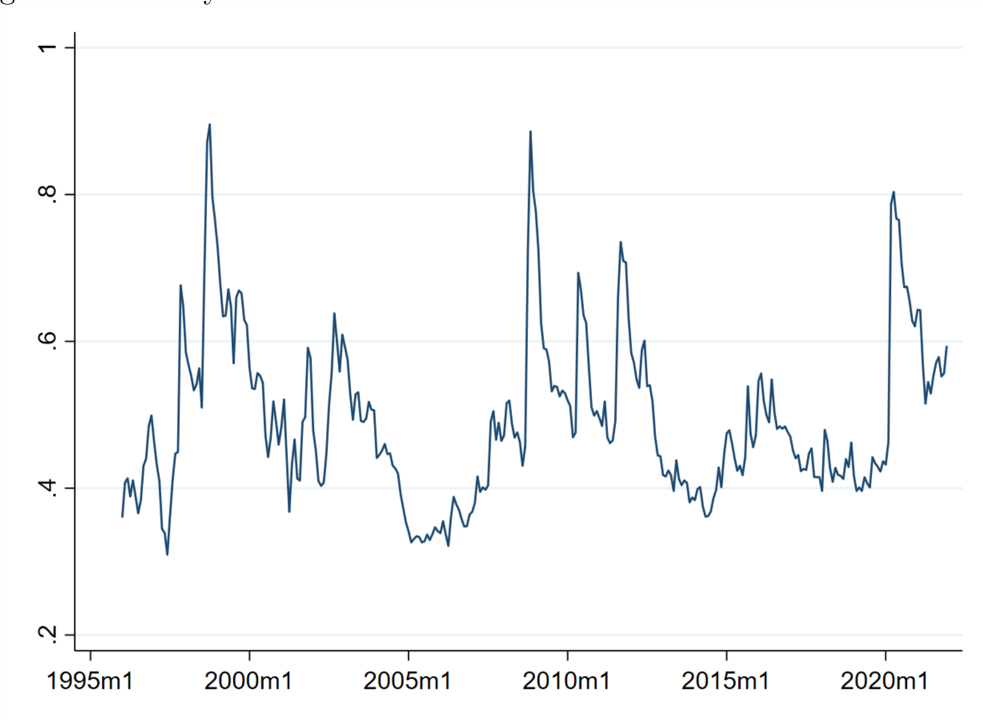


Figure 5: Global convexity left and global convexity right indexes

This figure plots global convexity left index in Panel A and global convexity right index in Panel B. Detailed definition of the measures are in the text and Appendix.

Panel A: global convexity left index



Panel B: global convexity right index

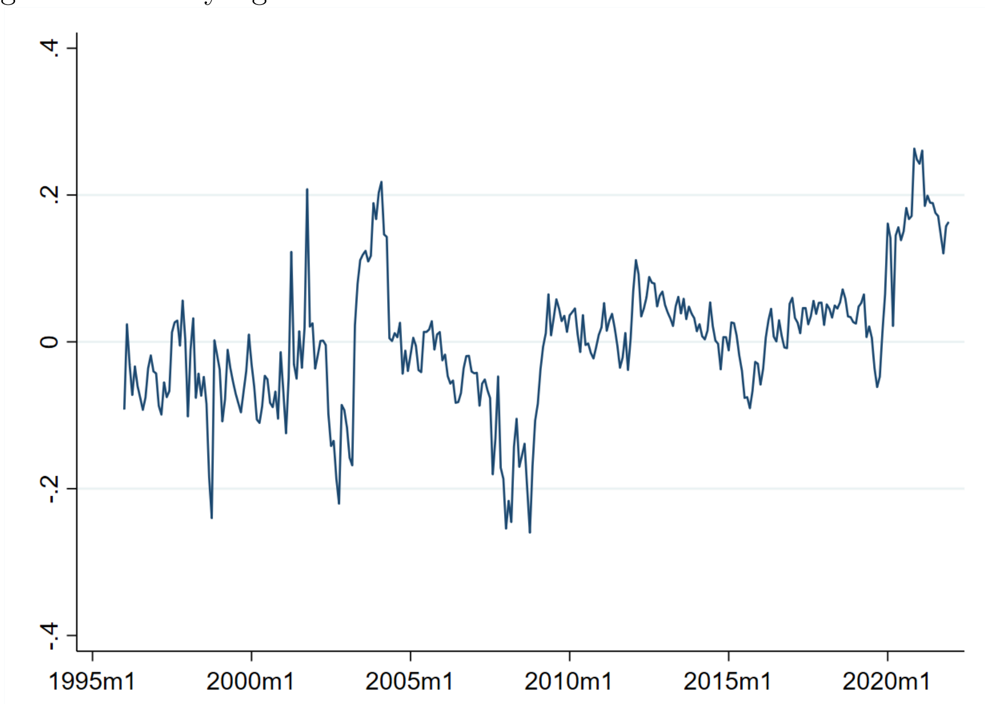


Table 1: list of index options in our sample

This table lists the availability of index options from each country or region in our sample. This table also lists the underlying market index in each region and the exchanges from which option quotes are obtained. The last column shows the average number of available options in each region per day.

Country / region	Short name	Market index	Exchange	Start	Finish	Num. obs./day
Australia	AUS	S&P/ASX 200	Australia Futures and Options	1/2/2004	12/31/2021	959
Belgium	BEL	BEL 20	Euronext Brussels	1/2/2002	12/31/2021	165
Canada	CAN	S&P/TSX 60	Montreal Exchange	3/26/2007	12/31/2021	356
Switzerland	CHE	SMI	EUREX, Frankfurt	1/2/2002	12/30/2021	927
Germany	DEU	DAX	EUREX, Frankfurt	1/2/2002	12/30/2021	1234
Europe, Australasia, and the Middle East	EAFE	MSCI EAFE	National Best BidOffer	9/25/2002	12/31/2021	493
Emerging market	EEM	MSCI EM	National Best BidOffer	3/9/2006	12/31/2021	669
Spain	ESP	IBEX 35	Mercado Espanol de Futuros	10/11/2006	12/30/2021	1212
Europe	EUR	STOXX 50	EUREX, Frankfurt	1/2/2002	12/30/2021	1339
Finland	FIN	HELSINKI 25	EUREX, Frankfurt	1/2/2002	12/30/2021	275
France	FRA	CAC 40	Euronext Monep	4/14/2003	12/31/2021	565
United Kingdom	GBR	FTSE 100	Euronext Liffe, London	1/2/2002	12/31/2021	1257
Hong Kong	HKG	HANG SENG	Hong Kong Futures Exchange	1/3/2006	12/31/2021	1070
Italy	ITA	MIB	Mercato dei Derivati, Milano	10/10/2006	12/30/2021	574
Japan	JPN	NIKKEI 225	Osaka Day Session	5/6/2004	12/30/2021	1016
Korea	KOR	KOSPI 200	Korea Futures Market	5/3/2004	12/30/2021	377
Netherlands	NLD	AEX	Euronext Amsterdam Options	7/1/2005	12/31/2021	338
Sweden	SWE	OMXS30	Stockholmborsen Options Market	5/14/2007	12/30/2021	448
Taiwan	TWN	TAIEX	Taiwan Futures Exchange	1/2/2004	12/30/2021	242
United States	USA	S&P 500 Index	National Best BidOffer	1/4/1996	12/31/2021	1378

Table 2: Average global implied volatility surface

This table reports the average global implied volatility at each delta-maturity grid point. Specifically, we first take the average implied volatility at each grid point across all indexes, including both call and put options, on each day and then average across all sample period.

	Maturity (days)									
delta	30	60	91	122	152	182	273	365	547	Average
0.10	28.11	27.61	27.65	27.69	27.56	27.43	27.19	27.09	26.82	27.46
0.15	26.15	26.01	26.10	26.19	26.14	26.07	25.92	25.88	25.75	26.02
0.20	24.52	24.62	24.75	24.86	24.86	24.83	24.76	24.75	24.71	24.74
0.25	23.31	23.49	23.64	23.76	23.78	23.78	23.75	23.76	23.77	23.67
0.30	22.40	22.60	22.73	22.84	22.88	22.90	22.89	22.91	22.95	22.79
0.35	21.68	21.86	21.98	22.07	22.11	22.13	22.15	22.17	22.23	22.04
0.40	21.07	21.22	21.32	21.40	21.44	21.46	21.48	21.51	21.58	21.39
0.45	20.55	20.67	20.75	20.81	20.84	20.86	20.88	20.92	20.99	20.81
0.50	20.08	20.17	20.23	20.27	20.30	20.31	20.34	20.38	20.46	20.28
0.55	19.67	19.72	19.77	19.79	19.80	19.82	19.86	19.90	19.98	19.81
0.60	19.29	19.31	19.34	19.35	19.36	19.37	19.41	19.46	19.57	19.38
0.65	18.96	18.94	18.95	18.95	18.95	18.96	19.01	19.06	19.19	19.00
0.70	18.67	18.60	18.59	18.58	18.57	18.58	18.64	18.70	18.86	18.64
0.75	18.45	18.31	18.28	18.25	18.24	18.25	18.31	18.39	18.55	18.34
0.80	18.33	18.09	18.03	17.98	17.97	17.97	18.04	18.13	18.30	18.09
0.85	18.35	17.98	17.88	17.81	17.78	17.78	17.85	17.93	18.10	17.94
0.90	18.53	17.98	17.84	17.74	17.69	17.67	17.72	17.81	17.95	17.88
Average	21.07	21.01	21.05	21.08	21.07	21.07	21.07	21.10	21.16	

Table 3: Average US implied volatility surface

This table reports the average US implied volatility at each delta-maturity grid point. Specifically, we first take the average implied volatility at each grid point across all indexes, including both call and put options, on each day and then average across all sample period.

	Maturity (days)									
delta	30	60	91	122	152	182	273	365	547	Average
0.10	26.76	27.00	27.27	27.43	27.49	27.53	27.55	27.51	27.37	27.32
0.15	24.54	25.07	25.40	25.62	25.75	25.85	25.99	26.03	26.01	25.58
0.20	22.75	23.42	23.77	24.03	24.21	24.34	24.56	24.64	24.71	24.05
0.25	21.43	22.11	22.47	22.73	22.93	23.07	23.32	23.43	23.55	22.78
0.30	20.46	21.08	21.42	21.67	21.86	22.01	22.27	22.38	22.53	21.74
0.35	19.67	20.22	20.53	20.77	20.95	21.09	21.35	21.47	21.64	20.85
0.40	19.00	19.48	19.77	19.98	20.15	20.28	20.52	20.64	20.83	20.07
0.45	18.41	18.83	19.09	19.28	19.43	19.55	19.77	19.89	20.09	19.37
0.50	17.88	18.25	18.47	18.64	18.78	18.89	19.09	19.20	19.40	18.74
0.55	17.39	17.71	17.91	18.06	18.18	18.28	18.46	18.57	18.78	18.15
0.60	16.94	17.21	17.39	17.51	17.63	17.71	17.88	17.98	18.20	17.61
0.65	16.52	16.74	16.89	17.00	17.10	17.18	17.33	17.44	17.67	17.10
0.70	16.14	16.30	16.43	16.52	16.60	16.67	16.82	16.93	17.17	16.62
0.75	15.81	15.90	16.00	16.07	16.14	16.21	16.35	16.45	16.71	16.18
0.80	15.59	15.57	15.64	15.69	15.75	15.80	15.93	16.02	16.29	15.81
0.85	15.58	15.38	15.40	15.42	15.46	15.49	15.58	15.67	15.92	15.54
0.90	15.82	15.37	15.32	15.31	15.31	15.31	15.33	15.40	15.63	15.42
Average	18.86	19.16	19.36	19.51	19.63	19.72	19.89	19.98	20.15	

Table 4: Summary statistics of option based predictors

This table shows the summary statistics of various option-based measures in Panel A and their pairwise correlation in Panel B. The sample period is from 1996m1 to 2021m12. Appendix and the main text contains detailed description of each variable.

Panel A: summary statistics

	mean	sd	min	p5	p25	p50	p75	p95	max	skewness	kurtosis
VIX index	20.30	8.02	10.13	11.53	14.47	19.00	23.84	35.03	62.67	1.95	9.06
Variance risk premium	14.82	32.48	-403.40	-3.36	6.71	13.11	24.02	49.10	115.85	-7.75	98.15
Left-tail volatility	7.92	3.16	2.39	4.62	5.87	7.07	8.89	13.91	25.65	1.81	7.88
SVIX index	4.29	2.66	1.50	1.75	2.46	3.64	5.22	8.50	20.63	2.46	12.15
Global level	21.08	5.95	12.28	14.30	16.78	20.04	23.67	32.34	51.15	1.61	7.08
Global slope	-11.31	3.99	-27.07	-19.18	-13.39	-10.57	-8.34	-6.09	-5.24	-1.12	4.42
Global convexity	0.49	0.14	0.22	0.30	0.39	0.47	0.59	0.75	0.95	0.76	3.33
US convexity	0.54	0.15	0.20	0.29	0.43	0.53	0.62	0.83	0.99	0.45	3.20

Panel B: correlation matrix

	VIX	VRP	LTV	SVIX	Global level	Global slope	Global convexity
VIX index	100%	-13%	73%	96%	93%	-80%	43%
Variance risk premium	-13%	100%	-13%	-4%	-3%	-4%	10%
Left-tail volatility	73%	-13%	100%	75%	65%	-72%	68%
SVIX index	96%	-4%	75%	100%	95%	-85%	51%
Global level	93%	-3%	65%	95%	100%	-82%	43%
Global slope	-80%	-4%	-72%	-85%	-82%	100%	-68%
Global convexity	43%	10%	68%	51%	43%	-68%	100%
US convexity	26%	11%	67%	34%	27%	-51%	82%

Table 5: Predicting semi-annual US equity premium (univariate regressions)

We regress the 6-month excess return of S&P500 index on different option-based predictors. The sample period of predictors is from 1996m1 to 2021m12. Standard errors are Newy-West standard errors with 6 lags. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES	Semi-annual S&P 500 excess return							
Global convexity	0.3002***							
	(3.69)							
VIX		0.0023						
		(1.61)						
VRP			0.0003					
			(0.59)					
LTV				0.0096***				
				(2.74)				
SVIX					0.0088**			
					(2.12)			
Global level						0.0027		
						(1.34)		
Global slope							-0.0066***	
							(-3.09)	
US convexity								0.2232**
								(2.35)
Constant	-0.1051**	-0.0048	0.0381**	-0.0333	0.0051	-0.0146	-0.0316	-0.0774
	(-2.25)	(-0.19)	(2.36)	(-1.04)	(0.30)	(-0.39)	(-1.23)	(-1.35)
Observations	312	312	312	312	312	312	312	312
R^2	0.144	0.028	0.008	0.072	0.043	0.021	0.054	0.091

Table 6: Predicting semi-annual US equity premium (multivariate regressions)

We regress the 6-month excess return of S&P500 index on different option-based predictors. The sample period of predictors is from 1996m1 to 2021m12. Standard errors are Newy-West standard errors with 6 lags. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES	Semi-annual S&P 500 excess return							
Global convexity	0.2997***	0.2959***	0.2908***	0.2924***	0.3078***	0.3273**	0.3212***	0.3346***
	(2.88)	(3.60)	(2.91)	(2.66)	(2.95)	(2.53)	(3.92)	(3.46)
VIX	0.0000							-0.0006
	(0.01)							(-0.10)
VRP		0.0002						0.0002
		(0.53)						(0.64)
LTV			0.0006					0.0010
			(0.15)					(0.15)
SVIX				0.0008				0.0182
				(0.15)				(1.30)
Global level					-0.0004			-0.0055
					(-0.18)			(-0.79)
Global slope						0.0014		0.0042
						(0.38)		(0.71)
US convexity							-0.0239	-0.0428
							(-0.20)	(-0.33)
Constant	-0.1053**	-0.1059**	-0.1055**	-0.1049**	-0.0998**	-0.1023**	-0.1026*	-0.0153
	(-2.51)	(-2.27)	(-2.25)	(-2.20)	(-2.30)	(-2.36)	(-1.82)	(-0.20)
Observations	312	312	312	312	312	312	312	312
R^2	0.144	0.147	0.145	0.145	0.145	0.146	0.145	0.165

Table 7: Predicting monthly US equity premium (non-overlapping periods)

We regress the 1-month excess return of S&P500 index in month t+1 up to t+12 on the global convexity index in month t. The sample period of the global convexity index is from 1996m1 to 2021m12. Standard errors are heteroscedasticity robust standard errors. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	1 month S&P 500 excess return											
VARIABLES	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10	t+11	t+12
Global convexity	0.0587***	0.0523***	0.0530***	0.0530***	0.0425**	0.0370**	0.0335*	0.0124	-0.0037	0.0247	0.0235	0.0081
	(3.24)	(3.02)	(3.05)	(3.11)	(2.47)	(2.11)	(1.85)	(0.68)	(-0.20)	(1.38)	(1.32)	(0.46)
Constant	-0.0217**	-0.0187**	-0.0190**	-0.0193**	-0.0142	-0.0117	-0.0095	0.0007	0.0081	-0.0057	-0.0051	0.0023
	(-2.37)	(-2.12)	(-2.10)	(-2.17)	(-1.57)	(-1.27)	(-1.00)	(0.07)	(0.84)	(-0.60)	(-0.56)	(0.25)
Obs.	312	312	312	312	312	312	312	312	312	312	312	312
R^2	0.037	0.029	0.030	0.029	0.019	0.014	0.011	0.002	0.000	0.006	0.006	0.001

Table 8: Predicting equity premium around the world

We regress the semi-annual excess return of the leading market index from 20 different countries and regions on the global convexity index. The sample period of the global convexity index is from 1996m1 to 2021m12. Standard errors are Newy-West standard errors with 6 lags. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
VARIABLES	USA	AUS	BEL	CAN	CHE	DEU	EAFE	EEM	ESP	EU
Global convexity	0.3002***	0.3773***	0.2535*	0.3304***	0.2050**	0.2594**	0.2892***	0.3109**	0.1674	0.2946**
	(3.69)	(2.83)	(1.88)	(2.87)	(2.28)	(2.01)	(3.52)	(2.33)	(1.18)	(2.48)
Constant	-0.1051**	-0.1412**	-0.0886	-0.1160*	-0.0621	-0.0904	-0.1183**	-0.1096	-0.0443	-0.1128*
	(-2.25)	(-2.03)	(-1.23)	(-1.79)	(-1.25)	(-1.27)	(-2.44)	(-1.61)	(-0.59)	(-1.73)
Obs.	312	312	312	312	312	312	312	312	312	312
R^2	0.144	0.116	0.049	0.095	0.058	0.047	0.130	0.083	0.017	0.071

	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
VARIABLES	FIN	FRA	GBR	HKG	ITA	JPN	KOR	NLD	SWE	TWN
Global convexity	0.3321**	0.3328***	0.2835***	0.2548*	0.3627***	0.3869***	0.6824***	0.2870**	0.4439***	0.4584***
	(2.29)	(2.99)	(2.74)	(1.75)	(2.73)	(3.42)	(3.06)	(2.19)	(3.02)	(3.34)
Constant	-0.1034	-0.1251**	-0.1133*	-0.0910	-0.1648**	-0.1814***	-0.2934***	-0.1025	-0.1716**	-0.1825**
	(-1.30)	(-2.02)	(-1.94)	(-1.22)	(-2.22)	(-3.10)	(-2.72)	(-1.41)	(-2.24)	(-2.56)
Obs.	312	312	312	312	288	312	312	312	312	312
R^2	0.059	0.094	0.100	0.046	0.086	0.138	0.128	0.065	0.116	0.116

Table 9: Alternative specifications to measure convexity and predictive R^2

This table reports the in-sample R^2 from predicting semi-annual index returns using convexity measures constructed based on different specifications. Column 1 uses our baseline global convexity index. Columns 2 and 3 uses call and put implied volatilities to measure convexity. Columns 4 and 5 use in-the-money (ITM) and out-of-the-money (OTM) implied volatilities to measure convexity. Columns 6 and 7 use implied volatilities with maturity no greater than or greater than 6 month to measure convexity. Column 8 uses only US implied volatilities and Column 9 excludes US implied volatilities after 2002.

Country	(1) Baseline	(2) Call	(3) Put	(4) ITM	(5) OTM	(6) <=6month	(7) >6month	(8) USA only	(9) Exclude USA
USA	14.4%	8.9%	14.3%	10.8%	15.7%	12.6%	14.6%	9.1%	14.6%
AUS	11.6%	3.3%	17.9%	12.2%	8.3%	12.4%	8.6%	5.0%	11.8%
BEL	4.9%	3.5%	4.4%	5.9%	2.8%	4.6%	4.5%	1.1%	5.3%
CAN	9.5%	3.8%	12.4%	6.8%	10.7%	8.3%	9.6%	5.0%	9.6%
CHE	5.8%	4.0%	5.2%	5.9%	4.3%	4.5%	6.8%	2.6%	6.0%
DEU	4.7%	2.7%	4.9%	4.2%	4.2%	3.3%	6.1%	0.7%	5.1%
EAFE	13.0%	8.7%	12.1%	10.2%	13.4%	10.6%	14.4%	8.0%	13.2%
EEM	8.3%	1.9%	14.0%	7.8%	6.9%	9.2%	5.8%	3.0%	8.6%
ESP	1.7%	0.9%	1.9%	1.8%	1.3%	1.4%	1.9%	0.1%	1.9%
EU	7.1%	4.8%	6.5%	5.9%	6.9%	5.8%	7.8%	2.1%	7.4%
FIN	5.9%	3.1%	6.6%	4.4%	6.5%	4.6%	7.0%	2.0%	6.1%
FRA	9.4%	7.3%	7.7%	7.5%	9.5%	7.7%	10.3%	3.7%	9.7%
GBR	10.0%	6.1%	10.1%	8.7%	9.2%	8.6%	10.5%	4.9%	10.3%
HKG	4.6%	1.0%	8.0%	5.0%	3.1%	3.9%	4.8%	1.3%	4.8%
ITA	8.6%	5.9%	7.9%	8.2%	7.0%	8.1%	7.8%	4.9%	8.7%
JPN	13.8%	8.2%	14.0%	14.2%	10.0%	12.2%	13.6%	8.5%	14.0%
KOR	12.8%	4.8%	17.4%	10.8%	12.1%	16.5%	6.6%	6.3%	12.9%
NLD	6.5%	4.4%	6.1%	5.2%	6.6%	5.5%	7.0%	2.3%	6.9%
SWE	11.6%	4.8%	14.9%	10.8%	9.7%	9.5%	12.7%	4.4%	11.9%
TWN	11.6%	3.4%	17.6%	10.6%	9.9%	11.6%	9.6%	6.2%	11.8%
Average	8.8%	4.6%	10.2%	7.8%	7.9%	8.1%	8.5%	4.1%	9.0%

Table 10: Auto-correlation and lead-lag relationship in convexity index

This table reports the autocorrelation and lead-lag relationship of the USA convexity index, Non-USA convexity index, and the global convexity index. Non-USA convexity index uses non-USA options and starts from 2002m1. The sample period of USA and global convexity indexes are from 1996m1 to 2021m12. Standard errors are heteroscedasticity robust standard errors. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

VARIABLES	(1) USA	(2) Non-USA	(3) Global	(4) USA	(5) USA	(6) Non-USA	(7) Global
USA convexity$_{t-1}$	0.8793***			0.7654***	0.7612***	0.0119	-0.0111
	(31.64)			(18.03)	(16.90)	(0.33)	(-0.31)
Non-USA convexity$_{t-1}$		0.9471***		0.1732***		0.9377***	
		(40.45)		(3.58)		(23.70)	
Global convexity$_{t-1}$			0.9172***		0.1535***		0.9269***
			(40.14)		(2.97)		(22.67)
Constant	0.0669***	0.0274**	0.0423***	0.0464***	0.0548***	0.0254**	0.0435***
	(4.51)	(2.34)	(3.68)	(2.62)	(3.47)	(2.02)	(3.61)
Obs.	311	239	311	239	311	239	311
R^2	0.768	0.887	0.839	0.794	0.774	0.887	0.839

Table 11: R^2 from predicting US returns with convexity left and right

We regress S&P 500 returns at different horizons on the US and global convexity left and right indexes and report the R^2 in this table. The last column uses the baseline global convexity as the predictor. Detailed definitions of these variables are in the text and appendix. The sample period is from 1996m1 to 2021m12.

Horizon (months)	(1) US Convexity left	(2) US Convexity right	(3) Global Convexity left	(4) Global Convexity right	(5) Baseline
1	2.04%	0.01%	2.40%	1.29%	3.66%
2	4.49%	0.01%	4.84%	1.48%	6.31%
3	5.95%	0.13%	6.43%	2.95%	9.35%
4	7.53%	0.20%	7.99%	4.12%	12.02%
5	9.04%	0.28%	9.00%	4.66%	13.56%
6	9.62%	0.37%	9.22%	5.42%	14.45%
7	9.87%	0.49%	9.30%	6.52%	15.43%
8	8.99%	0.29%	8.59%	6.05%	14.29%
9	7.83%	0.15%	7.62%	4.57%	12.02%
10	7.79%	0.15%	7.72%	4.55%	12.11%
11	7.43%	0.18%	7.68%	4.68%	12.18%
12	6.85%	0.13%	7.45%	4.12%	11.46%
Average	7.29%	0.20%	7.35%	4.20%	11.40%

Table 12: Explaining convexity left and right

This table regresses global convexity left and global convexity right index on global slope, SVIX, TED spread, and aggregate short-interest. Detailed definitions of these variables are in the text and appendix. The sample period is from 1996m1 to 2021m12. Standard errors are Newy-West standard errors with 6 lags. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

Panel A: Global convexity left

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Global convexity left				
Global slope	-0.0295***				-0.0295***
	(-26.44)				(-29.94)
Left-tail volatility		0.0289***			0.0019**
		(8.23)			(2.06)
TED spread			0.0592		-0.0426***
			(1.54)		(-4.91)
Short-interest				-0.0285**	-0.0119***
				(-2.40)	(-4.17)
Constant	0.0735***	0.1785***	0.3804***	0.4110***	0.0797***
	(6.72)	(6.62)	(17.86)	(26.13)	(13.40)
Obs.	312	312	312	312	312
R^2	0.916	0.550	0.032	0.079	0.961

Panel B: Global convexity right

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Global convexity right				
Global slope	0.0050**				0.0085***
	(2.32)				(4.04)
Left-tail volatility		0.0020			0.0098***
		(0.56)			(4.96)
TED spread			-0.1121***		-0.0761***
			(-5.99)		(-4.47)
Short-interest				-0.0285***	-0.0159***
				(-4.23)	(-2.73)
Constant	0.1426***	0.0696***	0.1363***	0.0897***	0.1413***
	(6.75)	(2.62)	(10.75)	(11.24)	(11.26)
Obs.	312	312	312	312	312
R^2	0.074	0.008	0.322	0.218	0.501

Table 13: Cash flow news and convexity left and right

This table regresses current month S&P 500 return on changes in global convexity left and right in columns 1 to 3. This table also predicts log US dividend growth over 6-month horizon with convexity left and right in columns 4 to 6. The sample period is from 1996m1 to 2021m12. Standard errors are Newy-West standard errors with 6 lags. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	S&P 500 current month return			Dividend growth$_{t\to t+6}$		
ΔGlobal convexity left	-0.2850***		-0.2451***			
	(-4.04)		(-3.65)			
ΔGlobal convexity right		0.2813***	0.2101**			
		(2.95)	(2.22)			
Global convexity left				-0.1189***		-0.1189***
				(-2.95)		(-2.82)
Global convexity right					0.1188*	0.1188**
					(1.86)	(2.19)
Constant	0.0093***	0.0089***	0.0092***	0.0770***	0.0184**	0.0668***
	(3.96)	(3.48)	(3.91)	(5.67)	(2.45)	(4.61)
Obs.	311	311	311	312	312	312
R^2	0.114	0.073	0.152	0.130	0.047	0.176

Table 14: Predictive R^2 of US convexity left with different maturities

We regress S&P 500 index returns at different horizons on the US convexity left measure constructed from options with different maturities and report the R^2 in this table. Detailed definitions of these variables are in the text and appendix. The sample period is from 1996m1 to 2021m12.

	Predicting S&P 500 returns				
Option maturities (days)	1 month	3 months	6 months	9 months	12 months
30	1.17%	3.37%	5.71%	4.10%	3.81%
60	1.56%	4.64%	8.16%	6.56%	5.61%
91	2.02%	5.89%	9.60%	7.87%	6.76%
122	2.24%	6.46%	10.69%	8.64%	7.75%
152	2.22%	6.49%	10.58%	8.47%	7.80%
182	2.24%	6.48%	10.19%	8.21%	7.45%
273	2.20%	6.61%	9.66%	8.32%	6.94%
365	1.78%	5.54%	8.51%	7.37%	6.10%
547	1.45%	3.80%	6.29%	5.32%	4.49%
30, 60, and 91	1.63%	4.76%	8.04%	6.30%	5.53%
122, 152, and 182	2.26%	6.54%	10.59%	8.53%	7.74%
273 365 and 547	1.82%	5.34%	8.22%	7.05%	5.89%
30 to 152	1.92%	5.60%	9.35%	7.41%	6.60%
152 to 547	2.03%	5.91%	9.26%	7.72%	6.70%
all	2.04%	5.95%	9.62%	7.83%	6.85%

Table 15: Predicting Global Financial Cycle factor

This table regresses changes in the Global Financial Cycle factor from month t to $t+k$, where $k = 1, 2, ..., 12$ on the global convexity in month t. The sample period is from 1996m1 to 2019m4. Standard errors are Newy-West standard errors with k lags. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Change in Global Financial Cycle factor over											
VARIABLES	1 month	2 months	3 months	4 months	5 months	6 months	7 months	8 months	9 months	10 months	11 months	12 months
Global convexity	0.3334*	0.6730**	0.9946**	1.4299**	1.8101**	2.1158**	2.3439**	2.4422*	2.4588*	2.6423*	2.7723*	2.8224
	(1.96)	(2.05)	(2.17)	(2.34)	(2.25)	(2.16)	(2.07)	(1.90)	(1.70)	(1.69)	(1.67)	(1.64)
Constant	-0.1609*	-0.3254*	-0.4816**	-0.6920**	-0.8779**	-1.0276**	-1.1399*	-1.1900*	-1.2016	-1.2917	-1.3577	-1.3860
	(-1.90)	(-1.95)	(-2.02)	(-2.14)	(-2.08)	(-2.00)	(-1.93)	(-1.77)	(-1.59)	(-1.57)	(-1.55)	(-1.51)
Obs.	279	278	277	276	275	274	273	272	271	270	269	268
R^2	0.025	0.042	0.056	0.080	0.096	0.106	0.111	0.108	0.099	0.105	0.108	0.105

Table 16: Out-of-sample predictive R^2

This table reports the OOS R2 of predicting stock returns at different horizons. The predictor is the global convexity index. The OOS R2 is computed as

$$R^2_{OOS} = 1 - \frac{MSE_A}{MSE_N}$$

where MSE_A is the mean squared error of the predictive model based on the global convexity index and MSE_N is the mean squared error of the historical mean model. To compute these mean squared errors, we use first ten years of observations to train both models and then update them on a rolling-basis.

Country	(1) 1 month	(2) 3 month	(3) 6 month	(4) 9 month	(5) 12 month
USA	4.1%	12.6%	20.8%	18.1%	17.7%
AUS	3.0%	7.6%	13.1%	11.6%	10.8%
BEL	0.4%	2.9%	7.0%	3.4%	-1.0%
CAN	2.4%	7.6%	14.4%	13.0%	14.0%
CHE	1.0%	4.3%	8.8%	-0.1%	-2.6%
DEU	1.8%	4.4%	7.4%	1.2%	-2.9%
EAFE	3.1%	8.9%	18.3%	18.3%	18.7%
EEM	3.1%	6.9%	9.7%	6.6%	3.2%
ESP	0.4%	1.0%	1.6%	-4.4%	-8.4%
EU	1.9%	4.8%	9.3%	6.1%	4.3%
FIN	1.4%	4.1%	8.6%	6.0%	4.6%
FRA	1.9%	5.7%	12.2%	10.6%	9.8%
GBR	2.1%	6.8%	13.2%	10.3%	10.6%
HKG	0.2%	0.7%	1.5%	-0.7%	-4.1%
ITA	1.8%	5.1%	10.2%	11.1%	12.0%
JPN	3.2%	6.7%	15.5%	14.2%	9.6%
KOR	1.8%	6.1%	11.3%	7.0%	-0.1%
NLD	2.0%	5.4%	9.5%	3.8%	-0.7%
SWE	3.8%	9.9%	15.8%	14.0%	12.2%
TWN	4.1%	12.6%	20.0%	16.3%	11.5%
Average	2.2%	6.2%	11.4%	8.3%	5.9%

Table 17: Robustness checks

This table reports the R^2 of predicting semi-annual excess returns with the global convexity index as the predictor. We use different ways to construct the global convexity index. Columns 1 drops the two outmost delta points to measure the global convexity. Column 2 uses only options from AUS, CHE, DEU, ESP, EU, FRA, GBR, HKG, ITA, and USA. Column 3 estimates the convexity index in a robust fashion, which equals the average difference between the implied volatility in the tail region and the implied volatility in the middle region of the volatility curve. Column 4 uses option-level implied volatility instead of the standardized implied volatility surface to construct the global convexity index. Column 5 uses detrended convexity index. Column 6 uses annual change in global convexity as the predictor. Details of the construction method are in the text.

Country	(1) Drop extreme tail	(2) Small set of countries	(3) Robust convexity	(4) Option level	(5) Detrended	(6) Δconvexity
USA	12.5%	11.0%	14.3%	13.2%	13.0%	11.5%
AUS	11.1%	17.7%	11.7%	11.9%	12.6%	13.9%
BEL	3.3%	7.7%	4.9%	6.9%	6.1%	9.6%
CAN	11.1%	14.5%	9.7%	5.9%	10.9%	6.3%
CHE	3.9%	6.5%	5.7%	6.1%	5.6%	6.5%
DEU	4.8%	5.6%	4.7%	3.0%	5.9%	8.4%
EAFE	11.6%	14.1%	12.8%	10.6%	12.4%	6.3%
EEM	9.3%	12.7%	8.4%	6.9%	9.2%	10.4%
ESP	1.6%	3.1%	1.8%	1.4%	3.3%	7.1%
EU	6.9%	7.8%	7.1%	4.3%	8.7%	10.6%
FIN	6.5%	7.7%	6.0%	2.6%	7.5%	10.0%
FRA	9.0%	10.5%	9.4%	5.6%	11.2%	11.3%
GBR	8.7%	11.9%	10.0%	8.1%	11.8%	11.6%
HKG	5.3%	8.0%	4.7%	2.8%	5.7%	9.8%
ITA	7.2%	11.1%	8.6%	7.4%	7.9%	7.3%
JPN	12.6%	16.9%	13.5%	12.9%	10.1%	12.1%
KOR	13.6%	15.6%	12.9%	8.4%	13.9%	13.4%
NLD	5.8%	6.3%	6.5%	5.9%	7.2%	11.8%
SWE	12.4%	14.7%	11.6%	7.6%	13.3%	13.3%
TWN	11.6%	12.2%	11.6%	10.0%	9.8%	15.9%
Average	8.4%	10.8%	8.8%	7.1%	9.3%	10.4%

Table A1: Summary statistics of index returns

This table lists the data coverage and summary statistics of monthly index returns in our sample. Returns are reported in percentage points.

Country/region	Market index	start	finish	mean	sd	skewness	kurtosis
AUS	S&P/ASX 200	1996m1	2022m12	0.71	6.12	-0.64	5.12
BEL	BEL 20	1996m1	2022m12	0.54	5.96	-0.56	5.58
CAN	S&P/TSX 60	1996m1	2022m12	0.70	5.80	-0.70	5.58
CHE	SMI	1996m1	2022m12	0.60	4.72	-0.40	3.76
DEU	DAX	1996m1	2022m12	0.60	6.77	-0.36	4.21
EAFE	MSCI EAFE	1996m1	2022m12	0.37	4.14	-0.75	4.30
EEM	MSCI EM	1996m1	2022m12	0.64	5.27	-0.84	5.92
ESP	IBEX 35	1996m1	2022m12	0.61	6.94	-0.13	4.50
EUR	STOXX 50	1996m1	2022m12	0.52	6.26	-0.31	3.79
FIN	HELSINKI 25	1996m1	2022m12	0.91	6.95	0.06	4.90
FRA	CAC 40	1996m1	2022m12	0.63	6.15	-0.29	3.85
GBR	FTSE 100	1996m1	2022m12	0.38	4.72	-0.36	4.40
HKG	HANG SENG	1996m1	2022m12	0.56	6.94	0.09	5.48
ITA	MIB	1998m1	2022m12	0.35	7.21	-0.10	3.94
JPN	NIKKEI 225	1996m1	2022m12	0.10	5.42	-0.20	3.39
KOR	KOSPI 200	1996m1	2022m12	0.54	10.18	0.97	9.77
NLD	AEX	1996m1	2022m12	0.61	6.16	-0.63	4.92
SWE	OMXS30	1996m1	2022m12	0.69	6.73	-0.18	4.20
TWN	TAIEX	1996m1	2022m12	0.60	7.34	0.10	4.01
USA	S&P 500 Index	1996m1	2022m12	0.66	4.50	-0.57	3.82

Table A2: Summary statistics of surface implied volatilities

This table reports the summary statistics of implied volatilities from standardized implied volatility surfaces for each index.

Country / region	mean	p50	sd	min	max	skewness	kurtosis
AUS	17.67	15.82	7.19	2.87	198.19	2.06	12.46
BEL	19.97	17.98	8.38	3.17	200.00	2.40	19.47
CAN	19.84	17.53	10.33	1.45	200.00	4.65	57.66
CHE	17.71	16.03	6.67	3.39	103.62	2.04	10.00
DEU	21.74	19.95	7.91	3.78	131.34	1.70	7.77
EAFE	19.64	17.67	8.21	3.12	135.16	1.76	8.35
EEM	26.65	23.97	10.63	8.52	180.29	2.34	12.45
ESP	23.31	21.91	7.62	3.25	103.55	1.29	6.12
EUR	22.03	20.28	8.18	5.20	115.25	1.55	7.07
FIN	22.14	20.06	8.62	1.08	97.20	1.22	5.44
FRA	20.91	19.40	7.38	3.04	117.92	1.64	8.00
GBR	18.57	16.81	7.39	5.05	108.23	1.72	7.84
HKG	22.87	19.95	9.17	7.99	148.64	2.51	12.16
ITA	24.25	22.81	7.59	5.09	110.83	1.42	6.91
JPN	22.51	20.69	7.99	1.11	200.00	2.71	16.49
KOR	19.82	17.87	8.45	4.74	137.58	2.38	12.62
NLD	20.81	18.71	9.48	1.02	200.00	1.50	7.69
SWE	21.04	19.20	7.54	4.57	177.11	1.76	8.78
TWN	19.95	17.70	8.49	1.20	108.98	1.34	5.49
USA	19.58	18.45	7.23	3.70	99.34	1.43	7.20

Table A3: Statistical models to represent implied volatility surface

This table reports average R^2 of different statistical models that can be used to explain variations of implied volatility on the implied volatility surface. Specifically, on each day, we regress the implied volatility of each delta and maturity grid point on any combination of delta δ, log of maturity (in days) $ln(\tau)$, their squared terms δ^2 and $\ln(\tau)^2$, and their interaction term $\delta \times \ln(\tau)$. We report the average R^2 of daily regressions for each model. Column 1 pools together the call and put implied volatility surfaces. Column 2 adds call/put fixed effects.

Model	(1) Pool together calls and puts	(2) With Call/Put fixed effects
δ	77%	80%
δ δ^2	82%	86%
$\ln(\tau)$	10%	13%
$\ln(\tau)$ $\ln(\tau)^2$	10%	13%
δ $\ln(\tau)$	87%	90%
δ $\ln(\tau)$ $\delta \times \ln(\tau)$	88%	91%
δ δ^2 $\ln(\tau)$ $\ln(\tau)^2$	92%	96%
δ δ^2 $\ln(\tau)$ $\ln(\tau)^2$ $\delta \times \ln(\tau)$	93%	97%

Table A4: Convexity of implied volatility by maturity

This table shows the summary statistics of the convexity of implied volatility curve for each maturity in Panel A and their pairwise correlation in Panel B. The sample period is from 1996m1 to 2021m12.

Panel A: convexity by maturity

Maturity (days)	mean	sd	min	p5	p25	p50	p75	p95	max	skewness	kurtosis
30	0.65	0.20	0.20	0.40	0.52	0.62	0.74	1.05	1.36	1.10	4.52
60	0.54	0.17	0.15	0.31	0.42	0.51	0.60	0.87	1.23	1.14	4.72
91	0.51	0.16	0.21	0.30	0.40	0.49	0.60	0.86	1.07	0.82	3.56
122	0.50	0.16	0.21	0.31	0.39	0.47	0.61	0.84	0.98	0.79	3.19
152	0.48	0.15	0.20	0.28	0.37	0.45	0.59	0.78	0.95	0.77	3.08
182	0.46	0.15	0.19	0.26	0.35	0.43	0.56	0.75	0.92	0.70	2.96
273	0.44	0.14	0.18	0.24	0.33	0.41	0.55	0.69	0.81	0.49	2.52
365	0.43	0.14	0.16	0.23	0.33	0.42	0.54	0.67	0.77	0.27	2.16
547	0.41	0.14	0.10	0.20	0.32	0.39	0.50	0.65	0.79	0.27	2.54

Panel B: correlation matrix

Maturity (days)	30	60	91	122	152	182	273	365	547
30	100%	82%	78%	77%	75%	73%	66%	59%	51%
60	82%	100%	93%	84%	84%	83%	76%	68%	60%
91	78%	93%	100%	94%	90%	89%	84%	76%	68%
122	77%	84%	94%	100%	97%	93%	88%	83%	75%
152	75%	84%	90%	97%	100%	98%	91%	86%	79%
182	73%	83%	89%	93%	98%	100%	94%	88%	81%
273	66%	76%	84%	88%	91%	94%	100%	96%	86%
365	59%	68%	76%	83%	86%	88%	96%	100%	93%
547	51%	60%	68%	75%	79%	81%	86%	93%	100%

Table A5: Testing the return predictability of other option-based variables

This table reports the results of using various option-based variables to predict S&P 500 returns. The sample period is from 1996m1 to 2021m12. Standard errors are heteroscedasticity robust standard errors when predicting 1-month returns or Newy-West standard errors with 6 lags when predicting semi-annual returns. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

	Panel A: Predicting 1-month S&P 500 return							
	Univariate: predictor only			Multivariate: predictor + global convexity				
Predictor	Coef. of predictor	t-stat of predictor	R^2	Coef. of predictor	t-stat of predictor	Coef. of global convexity	t-stat of global convexity	R^2
US term structure slope	-0.0015	(-0.58)	0.003	-0.0009	(-0.38)	0.0577***	(3.20)	0.038
Global term structure slope	-0.0019	(-0.65)	0.003	-0.0011	(-0.37)	0.0574***	(3.15)	0.038
US IVOL: 1 year-minus-1 month	-0.0005	(-0.52)	0.002	-0.0004	(-0.43)	0.0581***	(3.23)	0.038
Global IVOL: 1 year-minus-1 month	-0.0007	(-0.56)	0.002	-0.0004	(-0.36)	0.0577***	(3.18)	0.038
US convexity: 1 year-minus-1 month	0.0156	(1.04)	0.004	0.0183	(1.27)	0.0601***	(3.28)	0.043
Global convexity: 1 year-minus-1 month	-0.0095	(-0.55)	0.001	0.0025	(0.15)	0.0593***	(3.27)	0.037
CBOE SKEW index	0.0002	(0.85)	0.002	-0.0001	(-0.52)	0.0623***	(3.33)	0.037

	Panel B: Predicting semi-annual S&P 500 return							
	Univariate: predictor only			Multivariate: predictor + global convexity				
Predictor	Coef. of predictor	t-stat of predictor	R^2	Coef. of predictor	t-stat of predictor	Coef. of global convexity	t-stat of global convexity	R^2
US term structure slope	-0.0055	(-0.71)	0.005	-0.0025	(-0.37)	0.2974***	(3.52)	0.146
Global term structure slope	-0.0072	(-0.85)	0.007	-0.0027	(-0.36)	0.2968***	(3.46)	0.145
US IVOL: 1 year-minus-1 month	-0.0016	(-0.52)	0.003	-0.0010	(-0.37)	0.2988***	(3.62)	0.146
Global IVOL: 1 year-minus-1 month	-0.0021	(-0.63)	0.004	-0.0008	(-0.28)	0.2983***	(3.53)	0.145
US convexity: 1 year-minus-1 month	0.0527	(0.76)	0.008	0.0663	(1.08)	0.3054***	(3.66)	0.157
Global convexity: 1 year-minus-1 month	-0.0261	(-0.34)	0.001	0.0364	(0.60)	0.3094***	(3.64)	0.147
CBOE SKEW index	0.0014	(0.95)	0.013	-0.0005	(-0.37)	0.3124***	(4.37)	0.146

Table A6: Monthly S&P 500 return predictability of convexity left and right

This table reports the results of using various option-based variables to predict monthly S&P 500 returns. The predictive model is US convexity left or US convexity left combined with another predictor in Panel A, global convexity left or global convexity left combined with another predictor in Panel B, and global convexity right or global convexity right combined with another predictor in Panel C. The sample period is from 1996m1 to 2021m12. Standard errors are heteroscedasticity robust standard errors. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

Panel A: Predicting with US convexity left + predictor					
Predictor	Coef. of predictor	t-stat of predictor	Coef. of US convexity left	t-stat of US convexity left	R^2
US convexity left			0.0468**	(2.35)	0.020
VIX	0.0001	(0.22)	0.0440**	(2.04)	0.021
SVIX	0.0001	(0.03)	0.0463*	(1.96)	0.020
LTV	-0.0010	(-0.57)	0.0649**	(1.99)	0.023
VRP	0.0000	(0.14)	0.0466**	(2.32)	0.021
Panel B: Predicting with global convexity left + predictor					
Predictor	Coef. of predictor	t-stat of predictor	Coef. of global convexity left	t-stat of global convexity left	R^2
Global convexity left			0.0554**	(2.53)	0.024
VIX	-0.0003	(-0.50)	0.0700**	(2.21)	0.026
SVIX	-0.0015	(-0.71)	0.0808**	(2.24)	0.028
LTV	-0.0012	(-0.76)	0.0786**	(2.58)	0.027
VRP	0.0000	(0.08)	0.0551**	(2.38)	0.024
Panel C: Predicting with global convexity right + predictor					
Predictor	Coef. of predictor	t-stat of predictor	Coef. of global convexity right	t-stat of global convexity right	R^2
Global convexity right			0.0677**	(2.01)	0.013
VIX	0.0007**	(2.14)	0.0917***	(2.60)	0.026
SVIX	0.0020**	(2.03)	0.0876**	(2.51)	0.028
LTV	0.0009	(1.17)	0.0643*	(1.91)	0.027
VRP	0.0000	(0.30)	0.0672**	(1.99)	0.024

Table A7: Semi-annual S&P 500 return predictability of convexity left and right

This table reports the results of using various option-based variables to predict semi-annual S&P 500 returns. The predictive model is US convexity left or US convexity left combined with another predictor in Panel A, global convexity left or global convexity left combined with another predictor in Panel B, and global convexity right or global convexity right combined with another predictor in Panel C. The sample period is from 1996m1 to 2021m12. Standard errors are Newy-West standard errors with 6 lags. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

Panel A: Predicting with US convexity left + predictor					
Predictor	Coef. of predictor	t-stat of predictor	Coef. of US convexity left	t-stat of US convexity left	R^2
US convexity left			0.2614***	(2.98)	0.096
VIX	0.028	(0.32)	0.2469**	(2.22)	0.097
SVIX	0.043	(0.45)	0.2349**	(1.98)	0.099
LTV	0.072	(0.55)	0.2105	(1.64)	0.099
VRP	0.008	(0.61)	0.2595***	(2.92)	0.103
Panel B: Predicting with global convexity left + predictor					
Predictor	Coef. of predictor	t-stat of predictor	Coef. of global convexity left	t-stat of global convexity left	R^2
Global convexity left			0.2797***	(3.77)	0.092
VIX	-0.0012	(-0.53)	0.3366**	(2.37)	0.096
SVIX	-0.0023	(-0.28)	0.3182*	(1.95)	0.093
LTV	0.0034	(0.71)	0.2144**	(2.12)	0.096
VRP	0.0002	(0.58)	0.2746***	(3.53)	0.097
Panel C: Predicting with global convexity right + predictor					
Predictor	Coef. of predictor	t-stat of predictor	Coef. of global convexity right	t-stat of global convexity right	R^2
Global convexity right			0.3569*	(1.89)	0.054
VIX	0.0038***	(2.97)	0.4867**	(2.52)	0.118
SVIX	0.0126***	(3.52)	0.4838**	(2.57)	0.134
LTV	0.0090***	(2.69)	0.3234*	(1.76)	0.116
VRP	0.0003	(0.53)	0.3504*	(1.88)	0.060

Table A8: R^2 from predicting international returns with convexity left and right

We regress semi-annual index returns on the US and global convexity left and right indexes and report the R^2 in this table. Detailed definitions of these variables are in the text and appendix. The sample period is from 1996m1 to 2021m12.

Country	(1) US convexity left	(2) US convexity right	(3) Global convexity left	(4) Global convexity right
USA	9.6%	0.4%	9.2%	5.4%
AUS	4.9%	0.4%	9.5%	2.2%
BEL	0.3%	1.5%	1.1%	6.7%
CAN	4.5%	0.6%	8.5%	1.3%
CHE	1.3%	1.6%	1.2%	7.9%
DEU	1.2%	0.1%	3.0%	1.8%
EAFE	7.3%	0.9%	7.3%	6.3%
EEM	3.8%	0.0%	8.1%	0.7%
ESP	0.0%	0.6%	0.8%	1.1%
EU	2.0%	0.2%	4.8%	2.3%
FIN	1.9%	0.1%	4.5%	1.5%
FRA	3.0%	0.8%	6.3%	3.2%
GBR	3.0%	2.0%	4.8%	6.3%
HKG	2.9%	0.5%	6.7%	0.0%
ITA	3.0%	1.9%	4.6%	4.1%
JPN	11.0%	0.0%	10.3%	3.4%
KOR	9.2%	0.1%	16.1%	0.1%
NLD	1.9%	0.3%	3.0%	4.4%
SWE	5.8%	0.0%	10.8%	1.3%
TWN	7.2%	0.1%	8.2%	3.3%
Average	4.2%	0.6%	6.4%	3.2%

Table A9: Predicting Global Financial Cycle with convexity left and right

This table regresses changes in the Global Financial Cycle factor from month t to $t+k$, where $k = 1, 2, ..., 12$ on the global convexity left (Panel A) and convexity right (Panel B) in month t. The sample period is from 1996m1 to 2019m4. Standard errors are Newy-West standard errors with k lags. The t-statistics are reported in parentheses. Superscripts ***, **, * correspond to statistical significance at the 1, 5, and 10 percent levels, respectively.

Panel A: Predicting GFC with global convexity left

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Change in Global Financial Cycle factor over											
VARIABLES	1 month	2 months	3 months	4 months	5 months	6 months	7 months	8 months	9 months	10 months	11 months	12 months
Global convexity left	0.1287	0.3779	0.5844*	0.8818**	1.2148**	1.4664**	1.5596*	1.5449	1.5634	1.6857	1.7591	1.7999
	(0.95)	(1.43)	(1.69)	(2.13)	(2.19)	(2.04)	(1.84)	(1.60)	(1.45)	(1.45)	(1.45)	(1.42)
Constant	-0.0552	-0.1588	-0.2457*	-0.3694**	-0.5088**	-0.6148*	-0.6570*	-0.6555	-0.6673	-0.7201	-0.7545	-0.7760
	(-1.03)	(-1.46)	(-1.68)	(-2.01)	(-2.02)	(-1.90)	(-1.74)	(-1.53)	(-1.39)	(-1.38)	(-1.37)	(-1.33)
Obs.	279	278	277	276	275	274	273	272	271	270	269	268
R^2	0.004	0.013	0.018	0.029	0.041	0.048	0.047	0.041	0.038	0.041	0.041	0.041

Panel B: Predicting GFC with global convexity right

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Change in Global Financial Cycle factor over											
VARIABLES	1 month	2 months	3 months	4 months	5 months	6 months	7 months	8 months	9 months	10 months	11 months	12 months
Global convexity right	0.6602**	0.9825*	1.3822	1.8669	2.0686	2.2846	2.7347	3.1098	3.1266	3.3605	3.5682	3.6239
	(2.09)	(1.69)	(1.58)	(1.60)	(1.51)	(1.46)	(1.46)	(1.41)	(1.23)	(1.18)	(1.15)	(1.12)
Constant	-0.0536	-0.0824	-0.1169	-0.1583	-0.1804	-0.2020	-0.2398	-0.2705	-0.2749	-0.2950	-0.3143	-0.3222
	(-1.55)	(-1.27)	(-1.18)	(-1.19)	(-1.13)	(-1.08)	(-1.09)	(-1.06)	(-0.95)	(-0.92)	(-0.91)	(-0.89)
Obs.	279	278	277	276	275	274	273	272	271	270	269	268
R^2	0.032	0.028	0.034	0.043	0.040	0.039	0.047	0.055	0.050	0.053	0.055	0.054

Table A10: Economic gain of predicting the market return

This table reports the Sharpe ratio of three different investment strategies in each country from 2006m1 to 2021m12. The first strategy is to buy and hold each country's market index. The second strategy is to invest in the market index with a weight equal to

$$\omega_t = a + bx_t$$

where x_t is the average global convexity over the past six months. The coefficients a and b are optimized based on the in-sample data from 2006m1 to 2021m12. The third strategy is similar to the second except that the coefficients a and b in each month are estimated based on the data from 1996m1 until the month of trading on a rolling basis.

Country	(1) Buy-and-hold Sharpe ratio	(2) In-sample optimal Sharpe ratio	(3) OOS optimal Sharpe ratio
USA	0.705	1.085	1.011
AUS	0.378	0.823	0.712
BEL	0.271	0.614	0.432
CAN	0.392	0.780	0.671
CHE	0.571	0.860	0.695
DEU	0.358	0.621	0.498
EAFE	0.373	0.853	0.778
EEM	0.495	0.861	0.793
ESP	0.203	0.381	0.203
EU	0.276	0.600	0.477
FIN	0.492	0.727	0.647
FRA	0.329	0.644	0.544
GBR	0.256	0.739	0.585
HKG	0.355	0.515	0.396
ITA	0.160	0.580	0.446
JPN	0.369	0.759	0.682
KOR	0.268	0.783	0.742
NLD	0.387	0.800	0.671
SWE	0.443	0.855	0.795
TWN	0.591	0.973	0.901
Average	0.384	0.743	0.634