

# COMP4418

## Knowledge Representation and Reasoning

### Assignment 3

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1.

a)

Uncovered set:

$\{a, b, c\}$

Top Cycle:

$\{a, b, c, d, e, f\}$

Copeland Winner:

$\{c\}$

Banks winners:

$\{b, c\}$

Condorcet winners:

$\{\}$

There's no Condorcet winner because no alternative dominates all others pair-wisely.

b)

Pure Nash equilibrium:

	D	E
A		8,5
B	6,6	

Mixed Nash equilibrium:

Player1 has chance  $x$  to take action A, then  $1-x$  chance to take action B:

$$2x + 6(1-x) = 8x + 4(1-x)$$

$$\text{So } x = 1/4$$

Player2 has chance  $y$  to take action D then  $1-y$  chance to take action E:

$$4y + 5(1-y) = 6y + 4(1-y)$$

$$\text{So } y = 1/3$$

The mixed Nash equilibrium is Player1:(1/4,3/4) Player2:(1/3,2/3).

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2.

a)

### 1. Blackjack POMDP

The player does not know the coming card and the bank can not take action, so it is pomdp.

### 2. Candy Crush MDP

The player can observe the full board and need to take actions, so it is mdp.

### 3. Chess

None/Other

There are two players involved, so it is not Markov process.

### 4. Minesweeper

POMDP

Player need to take action and the board is not fully observable.

### 5. Snakes and Ladders

Markov process

Player can not take actions. All actions are purely from the dice with known probability.

### vi. Texas Hold'em Poker

None/Other

There are multiple players involved, so it is not Markov process.

b)

If discount factor is very high, The optimal policy for  $s_3$  is either Leave or Stay because the reward will remain the same for both action. But for  $s_2$  and  $s_3$  the optimal policy is to choose Stay because if they leave, they will have a chance to get to  $s_3$  which the reward in that state is negative.

If discount factor is very low, The optimal policy for  $s_3$  is the same because either Leave or Stay have the same reward. The optimal policy for  $s_1$  will be Stay since it brings reward 1 which is larger than Leave, for  $s_2$  will be Leave since the reward is 5 which is larger than Stay.

d)

For each step we have:

Initialization:

$$V(s_{i+1}) = \max\{u(S) + \delta \sum P(s, S, s') V_i(s')\} \quad V_0(s_1) = 0; V_0(s_2) = 0; V_0(s_3) = 0$$

In first step:

$$V_1(s_1) = \max\{1; 0\} = 1 \quad V_1(s_2) = \max\{0; 5\} = 5 \quad V_1(s_3) = \max\{-2; -2\} = -2$$

In second step:

$$V_2(s_1) = \max\{1 + 0.6 \cdot 1; 0 + 0.6 \cdot 5\} = 3 \quad V_2(s_2) = \max\{0 + 0.6 \cdot 5; 5 + 0.6 \cdot (0.5 \cdot 1 + 0.5 \cdot (-2))\} = 4.7$$

$$V_2(s_3) = \max\{-2 + 0.6 \cdot (-2); -2 + 0.6 \cdot (-2)\} = -3.2$$

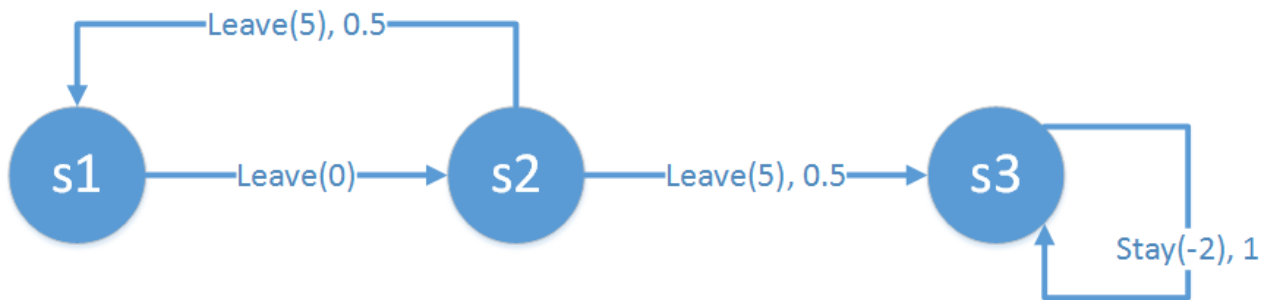
In third step:

$$V_3(s_1) = \max\{1 + 0.6 \cdot 3; 0 + 0.6 \cdot 4.7\} = 2.82 \quad V_3(s_2) = \max\{0 + 0.6 \cdot 4.7; 5 + 0.6 \cdot (0.5 \cdot 3 + 0.5 \cdot (-3.2))\} = 4.94$$

$$V_3(s_3) = \max\{-2 + 0.6 \cdot (-3.2); -2 + 0.6 \cdot (-3.2)\} = -3.92$$

	$V_0(s)$	$V_0(s, S)$	$V_0(s, L)$	$V_1(s)$	$V_1(s, S)$	$V_1(s, L)$	$V_2(s)$	$V_2(s, S)$	$V_2(s, L)$	$V_3(s)$
s1	0	1	0	1	1.6	3	3	2.8	2.82	2.82
s2	0	0	5	5	3	4.7	4.7	2.82	4.94	4.94
s3	0	-2	-2	-2	-3.2	-3.2	-3.2	-3.92	-3.92	-3.92

e)



According to the process above, we have the following formulas.

$$V(s_{i+1}) = u(S) + \delta \sum P(s, S, s') V_i(s')$$

	$V_0(s)$	$V_1(s)$	$V_2(s)$	$V_3(s)$
s1	0	1	3	2.4
s2	0	5	4	4.9
s3	0	-2	-3.2	-3.72

f)

$$V_{i+1}(s_1) = \delta V_i(s_2)$$

$$V_{i+1}(s_2) = 5 + \delta(0.5 \cdot V_i(s_1) + 0.5 \cdot V_i(s_3)) \quad V_{i+1}(s_3) = -2 + \delta V_i(s_3)$$

From the equation above, we can solve s3 first:

$$V_{n \rightarrow \infty}(s_3) = 1 / 1 - \delta$$

Then the others:

$$V_{n \rightarrow \infty}(s_2) = 5 - 6\delta / (1 - \delta)(1 - 0.5\delta^2)$$

$$V_{n \rightarrow \infty}(s_1) = \delta(5 - 6\delta) / (1 - \delta)(1 - 0.5\delta^2)$$

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Now we can tell from the equation above that if  $\delta$  is very high, both  $V_{n \rightarrow \infty}(s_2)$  and  $V_{n \rightarrow \infty}(s_1)$  will become negative infinity. So we can not choose Leave for  $s_1$  and  $s_2$ . Meanwhile, if  $\delta$  is very low,  $V_{n \rightarrow \infty}(s_2)$  will become 5 and  $V_{n \rightarrow \infty}(s_1)$  will become 0, so for  $s_2$  we should choose Leave and for  $s_1$  we should choose Stay. The conclusion is the same with the answer in the previous questions.