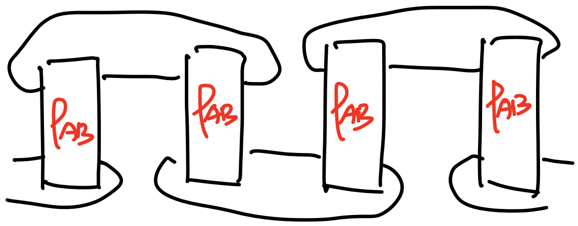
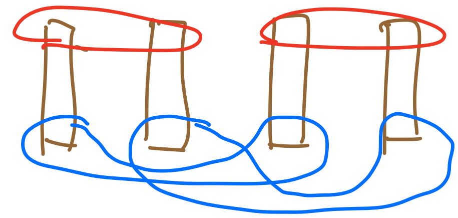
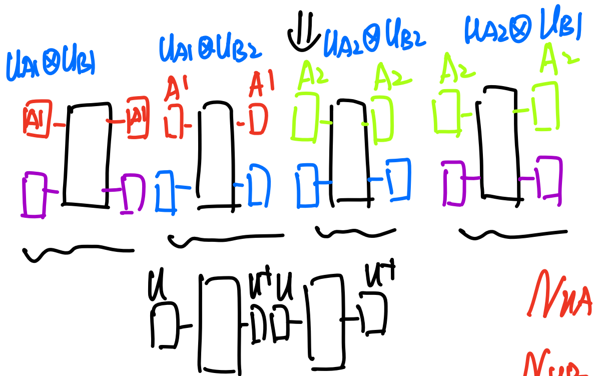
$U_A \otimes U_B$





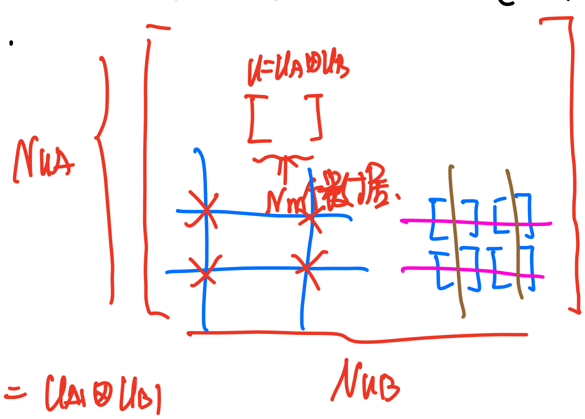


N_u 个 U. N_M 个 S.

N_{UA} 个 U_A

N_{UB} 个 U_B } $N_{UA} \times N_{UB}$ 个 U.

$(N_{UA}\times N_{UB})(N_{UA}-1)(N_{UB}-1)/2$



[$U_1 \rho U_1^\dagger$] $U_1 = U_{A1} \otimes U_{B1}$

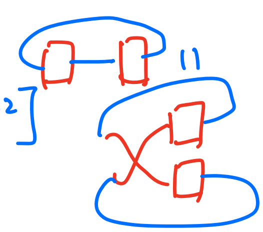
[$U_2 \rho U_2^\dagger$] $U_2 = U_{A1} \otimes U_{B2}$

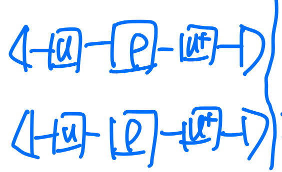
[$U_3 \rho U_3^\dagger$] $U_3 = U_{A2} \otimes U_{B2}$

[$U_4 \rho U_4^\dagger$] $U_3 = U_{A2} \otimes U_{B1}$

$\mathrm{Tr}\,\rho^2$

$= \mathrm{Tr}[\hat{S}\, \rho^{\otimes 2}]$





: $P(s|u)$

$$\frac{\sum_{ss'} X(s,s') P(s|u) P(s'|u)}{}\Big|_u = \mathrm{Tr}\,\rho^2$$

N_u 个 U, N_M 次:

s_1: $\delta(s,s_1)$

从 N_M 中挑两个结果 $\sum X(s_1,s_2) \binom{N_M}{2}^{-1}$

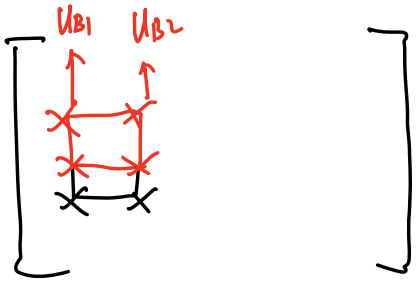
Min:

① Var ∝ qubit number (LS GS LR GR)

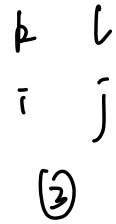
② Var ∝ Nu Var $\propto \frac{1}{Nu^2}$ (LR GR)

③ Var ∝ NM (LR, GR)

④ Var ∝ M (LS, LG)

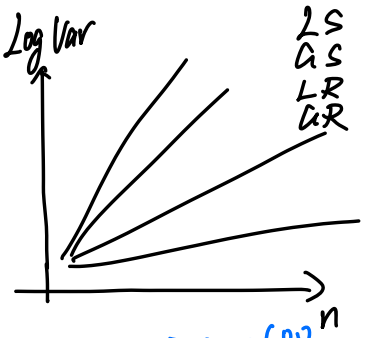


0719 debug:

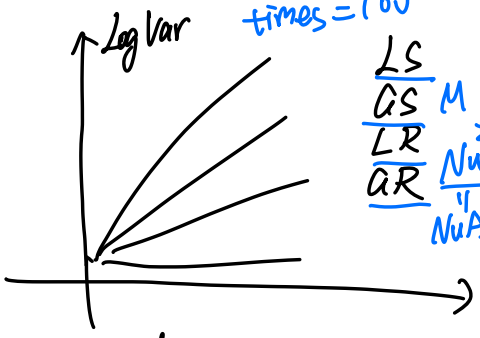


程序图：

$Tr\rho^2$ =



Mφ：

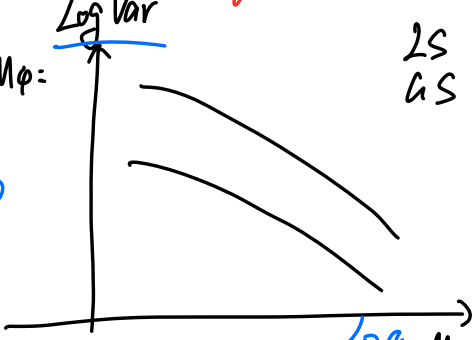


times = 100

LS
GS M = 90
LR
GR $Nu^2 Nm = 3^2 \times 10$

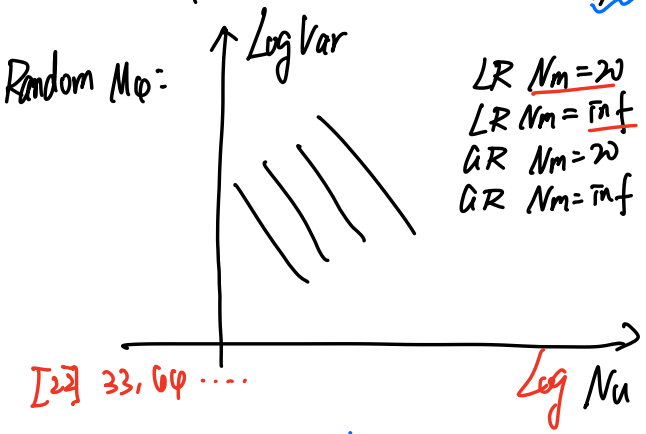
NuA = NuB

Shadow Mφ：



φ-qubit

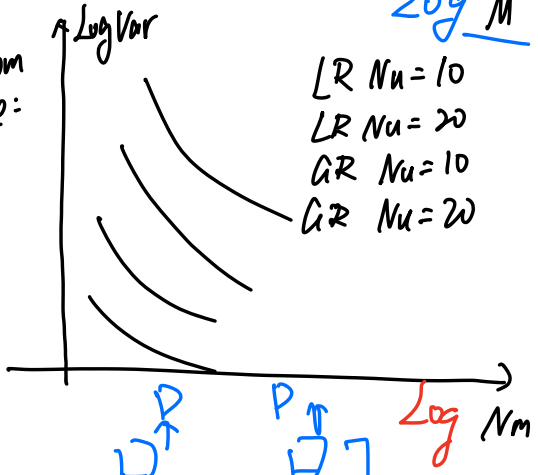
Random Mφ：



LR Nm = 20
LR Nm = Inf
GR Nm = 20
GR Nm = Inf

[22] 33, 6φ ····

Random Mφ：



LR Nu = 10
LR Nu = 20
GR Nu = 10
GR Nu = 20

$$Var = \frac{1}{Nu^2}\left[\square + \frac{\square}{Nm} + \frac{\square}{Nm^2} + \frac{\square}{Nm^3} + \frac{\square}{Nm^4}\right]$$

$$Var = \frac{\square}{M} + \frac{\square}{M^2} + \frac{\square}{M^3} + \frac{\square}{M^4}$$

D^2 D^4 D^6 D^8