assignment2

April 21, 2023

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import minimize
from scipy.optimize import newton
```

[ ]:
```
student_number = 15655200
np.random.seed(student_number)
g0 = np.random.uniform(1,2)
A0 = np.random.rand()
s = np.random.uniform(0.5, 1.5)
print(g0, A0, s)
```

```
1.1143123187976574 0.96523303908l4144 1.1292321723475864
```

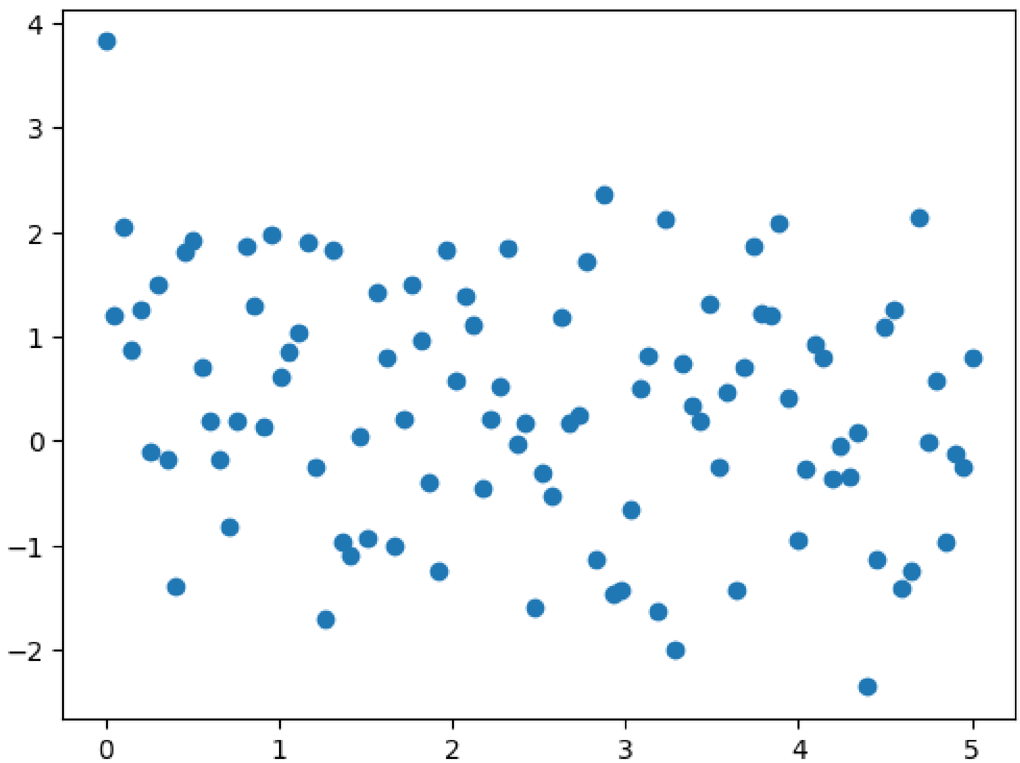
1 2A

[ ]:
```
def line(A, g, t):
    return A*np.exp(-g*t)

def data(start, end, N, A = A0, g = g0, sigma = s):
    time = np.linspace(start,end, N)
    data = line(A, g, time) + sigma*np.random.randn(N)
    return time, data


t0, tend, N = 0, 5, 100
t1, d1 = data(t0, tend, 100)

plt.scatter(t1, d1)
plt.show()
```

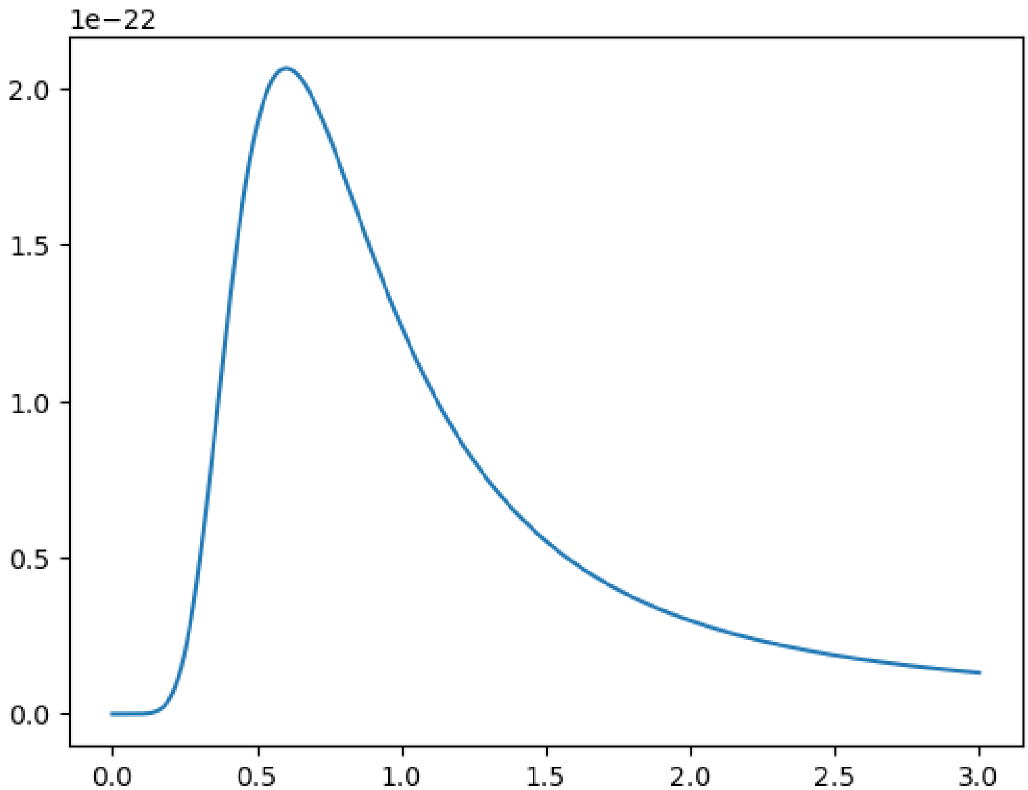


2 2B

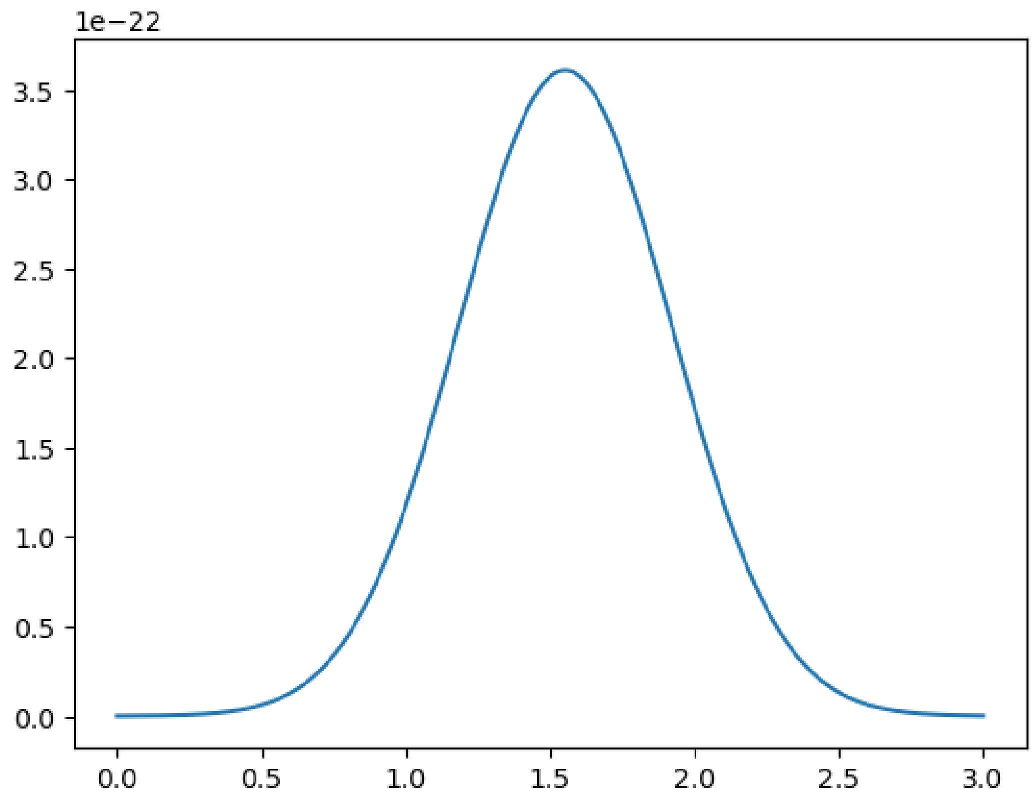
[ ]:
```
# plotting the distributon function
def Pos_line(data, A, g, t, sigma = s):
    noise2 = ((data - A*np.exp(-g*t))**2).sum() # summing noise^2
    return np.exp(-noise2/(2*sigma**2)) # normalize

pnt = 2000

# plotting for g
g_r = np.linspace(0, 3, pnt)
Pos_r = np.zeros(pnt)
for i in range(pnt):
    Pos_r[i] = Pos_line(d1, A0, g_r[i], t1)
plt.plot(g_r, Pos_r)
plt.show()
```



[ ]:
```
# plotting for A
A_r = np.linspace(0, 3, pnt)
Pos_r = np.zeros(pnt)
for i in range(pnt):
    Pos_r[i] = Pos_line(d1, A_r[i], g0, t1)
plt.plot(A_r, Pos_r)
plt.show()
```



3 2C

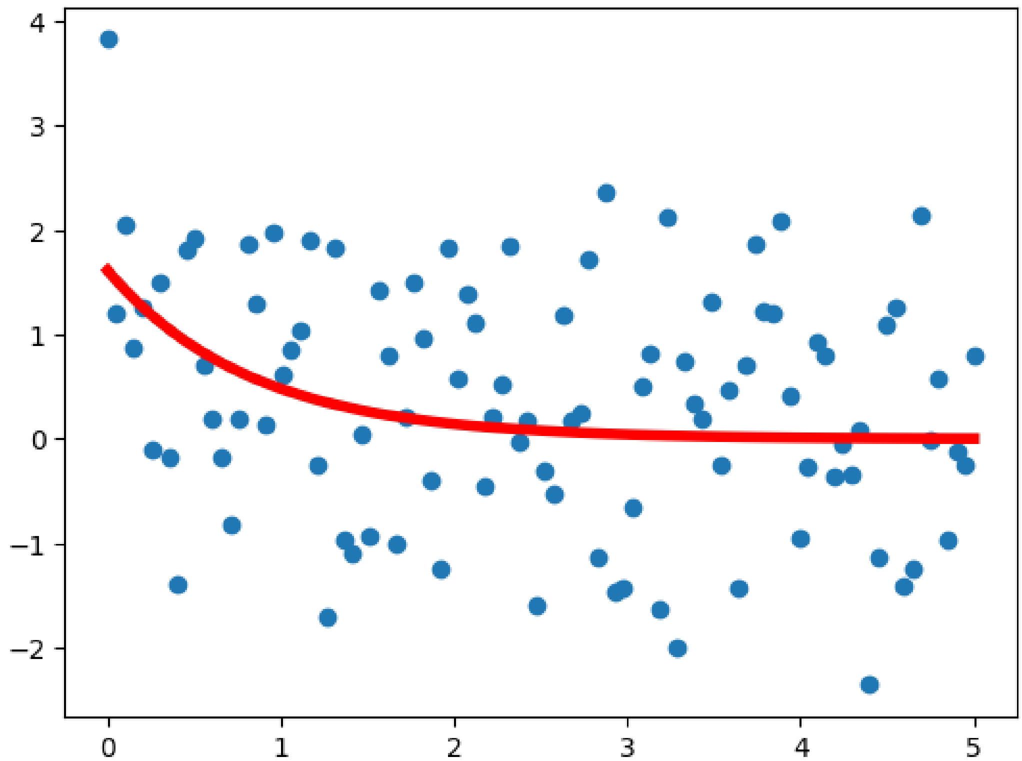
[ ]:
```python
# calculate the average chi^2 min
def chi_line(data, A, g, t, sigma = s):
    noise2 = ((data - A*np.exp(-g*t))**2).sum()
    return noise2/(sigma**2)

print(chi_line(d1, A0, g0, t1))
```

101.28028953613614

[ ]:
```python
# using scipy.minize to get the best fit line
line_sol = minimize(lambda X:chi_line(d1, X[0], X[1], t1), [A0, 0],␣
 ↪method="Nelder-Mead", tol=1e-6)['x']
plt.scatter(t1, d1)
print(line_sol[0], line_sol[1])
plt.plot(t1, line_sol[0]*np.exp(-line_sol[1]*t1), lw=4, color = "red")
plt.show()
```

1.613742239789833 1.2101493326852069



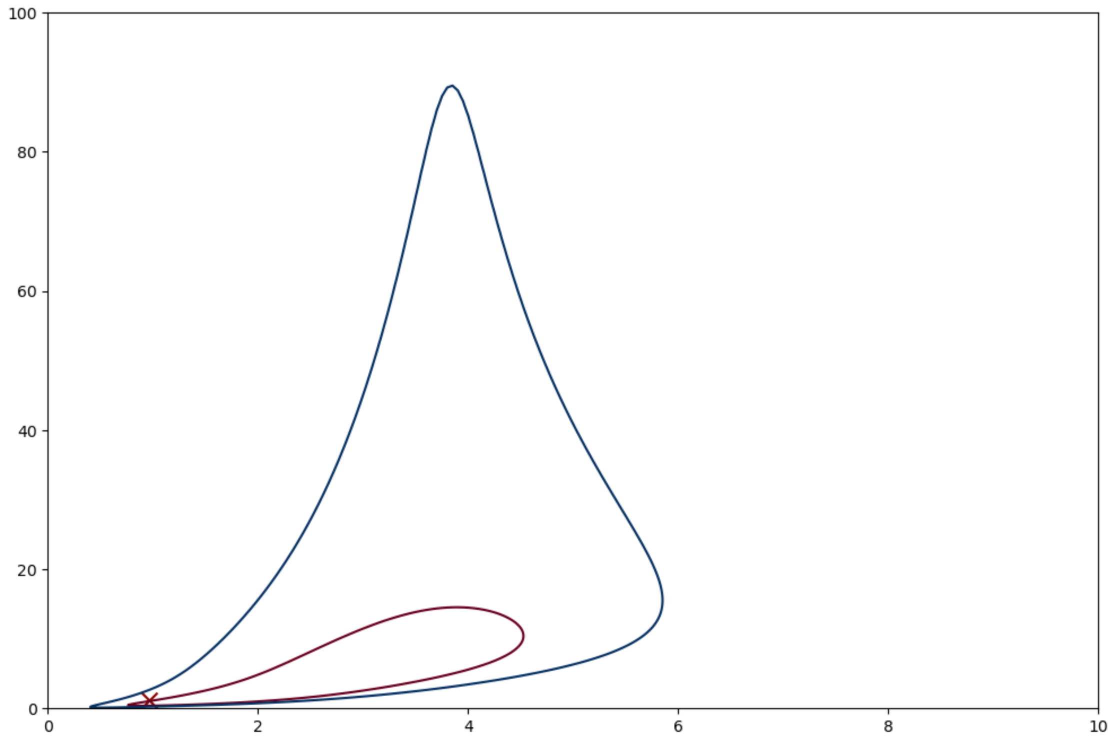
[ ]:
```
chi_min_line=chi_line(d1,line_sol[0],line_sol[1],t1)
print(chi_min_line)

A_r = np.linspace(0, 100 , pnt)
g_r = np.linspace(0, 100 , pnt)

# creating meshgrid for contour
A, B = np.meshgrid(A_r, g_r)
logP=np.zeros(A.shape)
for i in range(pnt):
    for j in range(pnt):
        logP[i,j]=chi_line(d1,A[i,j],B[i,j],t1)

# plotting contour
fig,ax=plt.subplots(figsize=(12,8))
ax.contour(A,B,logP,cmap='RdBu',levels=[chi_min_line+2.30,chi_min_line+6.18])
ax.set_xlim(0,10)
ax.set_ylim(0,100)
ax.scatter(A0,g0,marker='x',s=100,color='darkred')
plt.show()
```

```
98.73721571883131
```



4 2D

From part 1, we see that the best fit value for A and g should be A__0 and g__0. However, looking at the simulation, the best fit value for the data for the two parameters does not exactly match with A__0 and g__0, being 1.613742239789833 and 1.2101493326852069.

5 3a

$$\langle \chi^2_{min} \rangle = \sum_{i=0}^{N-1} \frac{1}{\sigma^2} \int_{-\infty}^{\infty} x^2 * norm(\mu = 0, \sigma = \sigma) dx$$

Since the mean of the normal is zero, the integral can be simplifed as followed:

$$\int_{-\infty}^{\infty} x^2 * norm(\mu = 0, \sigma = \sigma) dx = \langle x^2 \rangle - \langle x \rangle^2 = \sigma^2$$

plugging this back in, we get

$$\sum_{i=0}^{N-1} 1 = N$$

6 3b

The property of the chi-square distribution dictates that the mean is its degree of freedom. In this problem, the degree of freedom is

$$m + 1 - 1 = m$$

We can thus calculate

$$\sigma^2_{\chi^2} = \langle (\chi^2)^2 - 2\chi^2 * \langle \chi^2_{min} \rangle + \langle \chi^2_{min} \rangle^2 \rangle$$

which is

$$\langle (\chi^2)^2 \rangle - 2\langle \chi^2 \rangle \langle \chi^2_{min} \rangle + \langle \chi^2_{min} \rangle^2 = m^2 - 2mN + N^2$$

This quantity gives us information about if we repeat the same generation with the same noise source many times, the variation between samples will be govern by the above quantity.

7 3c

[ ]:
```
t0, tend, N = 0, 5, 100
t2, d2 = data(t0, tend, 100, sigma = 0.5)
t3, d3 = data(t0, tend, 100, sigma = 1.5)
t4, d4 = data(t0, tend, 100, sigma = 2)

print(chi_line(d2, A0, g0, t2, sigma = 0.5))
print(chi_line(d3, A0, g0, t3, sigma = 1.5))
print(chi_line(d4, A0, g0, t4, sigma = 2))
```

```
105.70889260177715
96.8341475175958
101.75025438819391
```

[ ]:
```

```