QNT 402 Quantum Information
Homework 7

1. [35] *A simple model test.* In this problem we'll generate some fake data and then try to fit it to determine if modeling it as linear or quadratic function is more reasonable.
   a.) [5] First generate some fake data using the Python code:
   ```
   slope = 1
   offset = 0.25
   time = np.linspace(0,10,10)
   data = np.random.normal(slope*time + offset,1)
   sigma = 1*np.random.normal(np.ones(len(data)),0.01)
   ```
   and plot it with the pyplot errorbar() function. This is obviously linear data but it's pretty noisy as you'll see.
   b.) [15] Now use curve_fit to fit a linear function ($bx + a$) and a quadratic function ($cx^2 + bx + a$) and add the plot of the best fits to the plot with the data on it.
   c.) [15] Now calculate the χ^2/ν of each of these and try to determine which is a more likely model.

2. [30] *Fisher Information.*
   a.) [15] In class we found that using the typical phase sensing protocol with N_q qubits in a GHZ state produced the state $|\psi\rangle = \sin\frac{N_q\phi}{2}|0\rangle + \cos\frac{N_q\phi}{2}|1\rangle$. We showed that this would allow sensing of the phase with precision: $\sigma_\phi = 1/\,(N_q t\sqrt{N})$, where t is the evolution time of the Ramsey sequence and N is the number of repetitions of the experiment. Calculate the Cramer-Ráo bound, $\sigma_\phi^{(CR)}$, for the phase sensitivity and compare it to σ_ϕ.
   b.) [15] Suppose we are trying to measure the oscillator phase ϕ using a coherent state $|\alpha\rangle$ by measuring the position x. Writing $\alpha = |\alpha|e^{\iota\phi}$, the wavefunction of a coherent state is
   $$\psi(x, |\alpha|, \phi, t) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}(x-\langle x(|\alpha|,\phi)\rangle)^2+\frac{i}{\hbar}\langle p(|\alpha|,\phi)\rangle x-\frac{i}{2}(\omega t-|\alpha|^2\sin(2\omega t-\phi))}$$
   where $\langle x(|\alpha|,\phi)\rangle$ and $\langle p(|\alpha|,\phi)\rangle$ are the expectation value of the position and moment of the coherent state and I've made their dependence on $|\alpha|$ and ϕ explicit. Calculate the Cramer-Ráo bound for the measurement of ϕ.

3. [50] *Alright, alright, alright, let's work with some quantum data.* In this problem we will generate data from a quantum sensing experiment. We know that for a Ramsey sequence (we worked it out last quarter) that for a qubit starting in $|0\rangle$ the probability of finding the qubit in state 1 is given as:
   $$P_1 = 1 - \frac{\Omega^2}{\Omega'^4}\left(\Omega' \cos\left(\frac{\delta t_w}{2}\right)\sin\left(\Omega' t_p\right) - 2\delta \sin\left(\frac{\delta t_w}{2}\right)\sin^2\left(\frac{\Omega' t_p}{2}\right)\right)^2$$
   This expression is for two $\pi/2$ pulses of length t_p separated by a free evolution time of t_w.

a.) [20] Write a Python function that generates realistic data for N repetitions of such a sequence. This function will output a value for P_1 that comes from a binomial distribution described by the true value of P_1 and the number of trials N. Use this function to make a plot of the "measured" values of P_1 versus t_w for the parameters $\Omega = 2\pi$ and $\delta = \frac{\pi}{4}$ for N = 1,10,100,1000.

b.) [20] Now simulate the signal for what is called "slope detection". Find the wait time for the first $P_1 = \frac{1}{2}$ point (i.e. the one near $t_w = 2$) and plot P_1 vs Φ for $-1 \leq \Phi \leq 1$. Assume $\gamma = 1$. Make the plots for N = 1,10,100,1000. Compare your result to that derived in Section IV.E.1 in of Reviews of Modern Physics, Volume 89, 035002 (2017)

c.) [10] Take the plot you made for N = 100 and add the expected signal plot to it as well as the expected signal plus σ_p and the expected signal minus σ_p. Is it what you expect?

4. [60] *One last dance with the QHO.* Here, we're going to examine the effect of squeezing on a QHO.

   a.) [10]Define a Python function that that takes time, t, as its input and returns the time evolution operator a time t for the Hamiltonian $H_o = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$. Use that function to calculate $|\alpha(t)\rangle = \hat{U}(t)|\alpha\rangle$ for various times during the oscillation period of the oscillator. Plot the expectation value of position and momentum in the (x,p) plane for $\alpha = 1$ – make sure to make your basis is big enough for this size coherent state. Choose $\omega = 2\pi$ and $m = 1$. Here, you'll likely find QuTip's displace() function handy.

   b.) [10] Calculate σ_x and σ_p at each of the times you chose in part (a) and plot σ_x, σ_p, and $\sigma_x\sigma_p$ versus time.

   c.) [15] QuTip has a function called squeeze() that implements the squeeze operator we discussed in class. Create a squeezed coherent state by first applying the squeeze operator (with squeeze parameter r = 1; fyi, QuTip uses z for the parameter r from class) and then applying the displacement operator used in part (a). Now, remake the plots of part (a) and part(b). What do you notice?

   d.) [20] The squeeze operator is a bit of shortcut, so let's make sure we know how to implement it on real hardware. Add a drive to QHO of the form $V = \beta_1\, x \sin(2\omega t) + \beta_2\, x^2 \sin(2\omega t)$ use e.g. sesolve to find the evolution. Choose $\beta_1 = \beta_2 = 1$. Make the same plots you made in part (a) and part (b). Is this squeezing?

   e.) [5] What are the two terms in the potential V? Try setting e.g. $\beta_1 = 0$ and describe what you see.