第5章 相对论基础

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数学与应用数学(师)

5-1力学相对性原理(The relativity principle of mechanics)

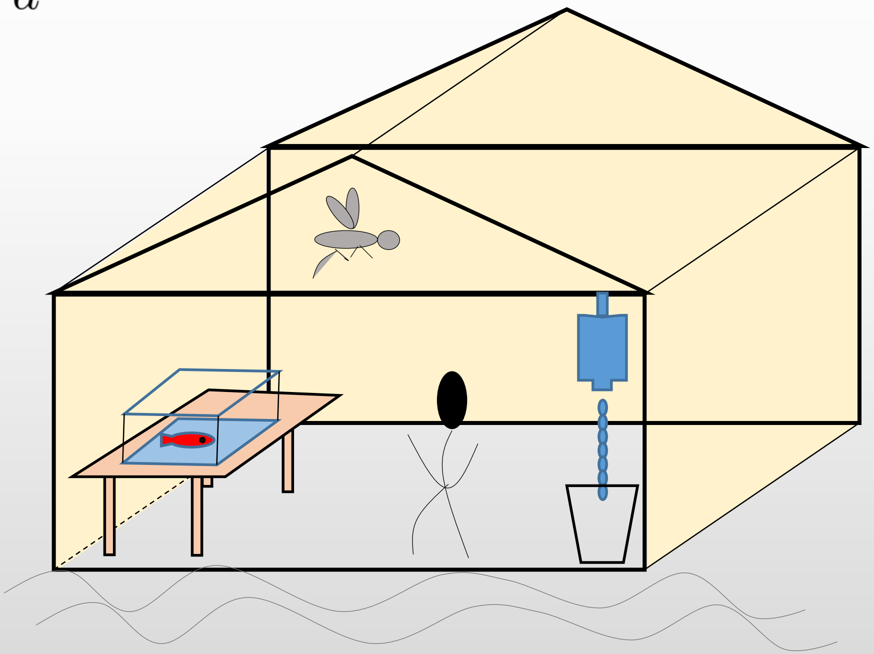
1. 力学相对性原理—伽利略相对性原理

力学定律在一切惯性系中都具有相同的数学表达形式.

$$\vec{a} = \vec{a}'$$

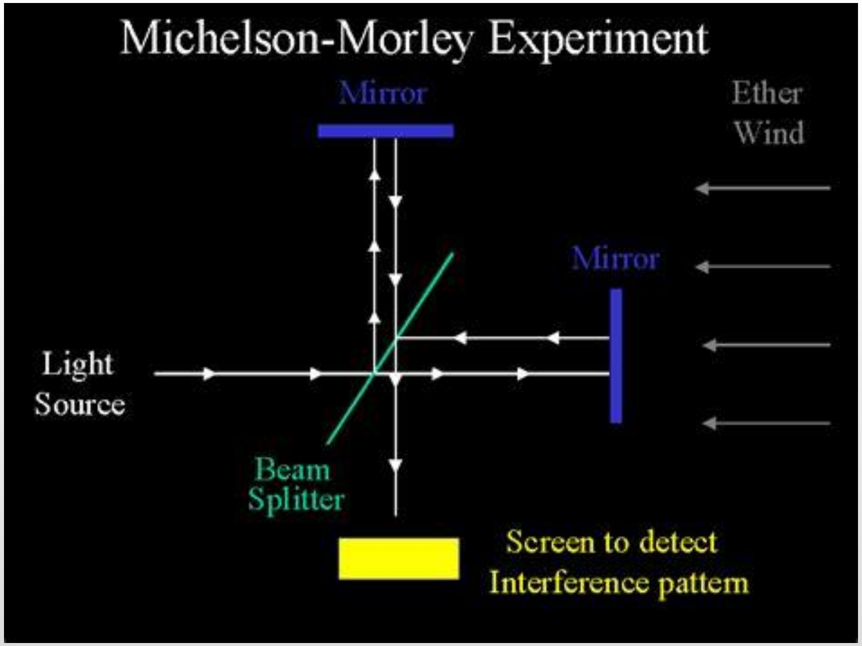
- 一切惯性系是等价的
- 无法通过力学实验判断惯性系是静止还是匀速直线运动
- 力和参考系无关

伽利略坐标变换的基础是绝对时空观: 时间和空间是独立的, 且不依赖于物质的运动.



5-2 狭义相对论(special theory of relativity)

1. 迈克尔逊-莫雷实验
   伪绝对参考系—以太(无质量,透明,不阻碍运动,非常刚性)
   在电磁学中, 即便是低速, 伽利略变换也不适用.

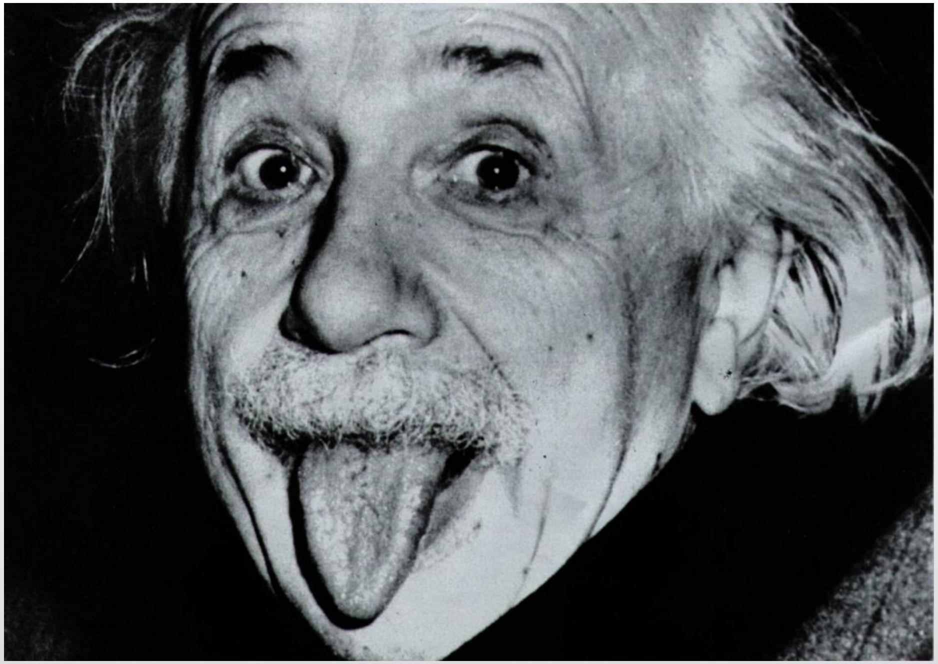


5-2 狭义相对论(special theory of relativity)

2. 狭义相对论的基本原理

(1) 相对性原理: 物理定律在一切惯性系中都有相同的数学表达形式

(2) 光速不变原理: 任何惯性系中, 光在真空中的速度恒为*c* (人为定义的常数)



在所有的学科你可以找到不止一个牛人的身影, 但是唯有相对论

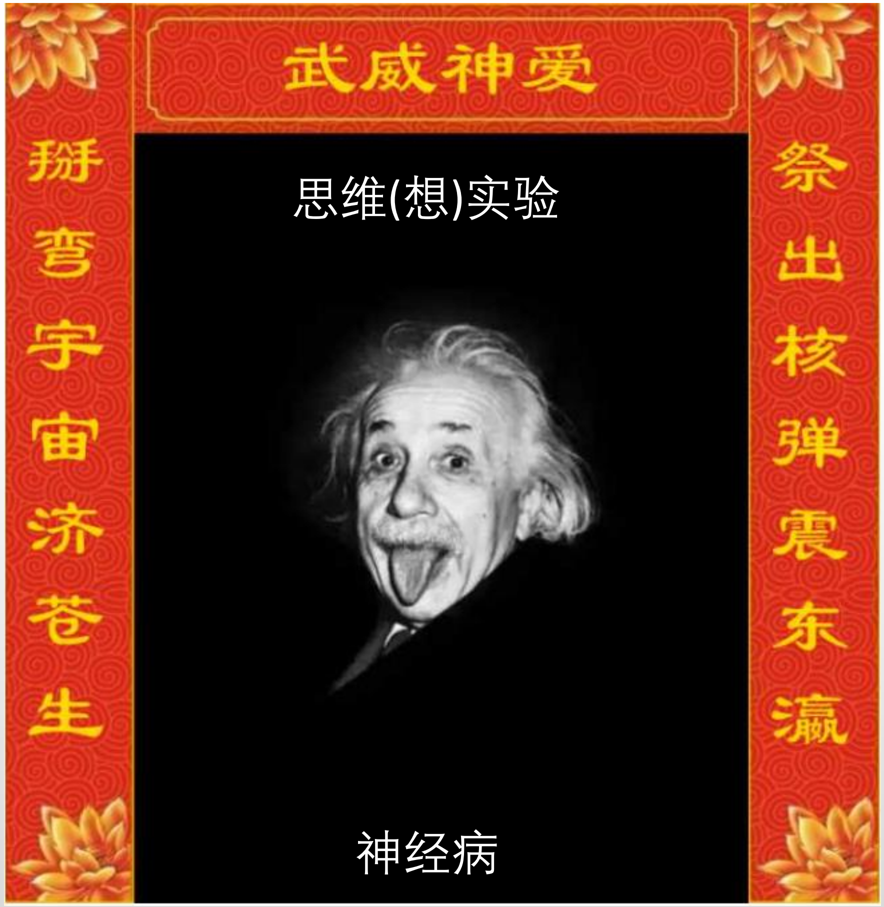
爱因斯坦的4篇划时代文章

Title (translated)	Area of focus	Received	Published	Significance
On a Heuristic Viewpoint Concerning the Production and Transformation of Light	Photoelectric effect	18 March	9 June	Resolved an unsolved puzzle by suggesting that energy is exchanged only in discrete amounts (quanta).[144] This idea was pivotal to the early development of quantum theory.[145]
On the Motion of Small Particles Suspended in a Stationary Liquid, as Required by the Molecular Kinetic Theory of Heat	Brownian motion	11 May	18 July	Explained empirical evidence for the atomic theory, supporting the application of statistical physics.
On the Electrodynamics of Moving Bodies	Special relativity	30 June	26 September	Reconciled Maxwell's equations for electricity and magnetism with the laws of mechanics by introducing major changes to mechanics close to the speed of light, resulting from analysis based on empirical evidence that the speed of light is independent of the motion of the observer.[146] Discredited the concept of a "luminiferous ether".[147]
Does the Inertia of a Body Depend Upon Its Energy Content?	Matter–energy equivalence	27 September	21 November	Equivalence of matter and energy, $E = mc^2$ (and by implication, the ability of gravity to "bend" light), the existence of "rest energy", and the basis of nuclear energy.

Only in **1905** 1905, 一个神奇的数字

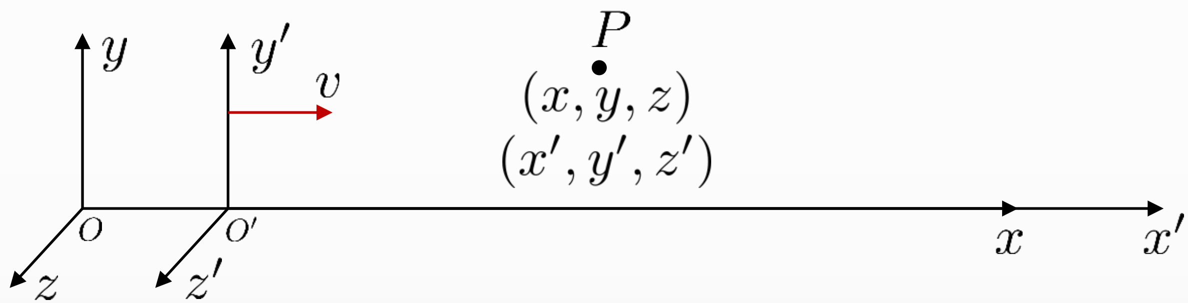
网传的爱神~~~

“我思考问题时，不是用语言进行思考，而是用活动跳跃的形象进行思考。而为了阐述这种思考，我还要费力将他们转换成语言……”



5-2 狭义相对论(special theory of relativity)

3. 洛伦兹坐标变换



$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad\qquad x = \frac{x' + vt}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad\qquad y = y$$

$$\beta = \frac{v}{c}$$

$$z' = z \qquad\qquad z = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\beta^2}} \qquad\qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\beta^2}}$$

5-2 狭义相对论(special theory of relativity)

2. 洛伦兹坐标变换—例题5-1

甲乙两人所乘飞行器沿x轴做相对运动.甲测得两个时间的时空坐标为
$x_1 = 6\times10^4$ m, $t_1 = 2\times10^{-4}$ s;
$x_2 = 12\times10^4$ m; $t_2 = 1\times10^{-4}$ s.
但是乙测得这两个事件同时发生于t'时刻. 问:

(1) 乙相对于甲的速度是多少?
(2) 乙测得两个事件的空间间隔是多少?

$$t_2' - t_1' = \frac{(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)}{\sqrt{1-\beta^2}}$$

$$x_2' - x_1' = \frac{(x_2 - x_1) - v(t_2 - t_1)}{\sqrt{1-\beta^2}}$$

$$v = -\frac{1}{2}c$$

$$x_2' - x_1' = 5.20\times10^4\ \mathrm{m}$$

5-3 相对论速度变换公式

1. 相对论速度变换公式推导

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\beta^2}}$$

$$u_x = \frac{dx}{dt} \qquad u_y = \frac{dy}{dt} \qquad u_z = \frac{dz}{dt}$$

$$u'_x = \frac{dx'}{dt'} \qquad u'_y = \frac{dy'}{dt'} \qquad u'_z = \frac{dz'}{dt'}$$

$$dx' = \frac{1}{\sqrt{1-\beta^2}}(dx - vdt) \qquad dy' = dy$$

$$dt' = \frac{1}{\sqrt{1-\beta^2}}\left(dt - \frac{v}{c^2}dx\right) \quad dz' = dz$$

$$u'_x = \frac{u_x - v}{1 - \frac{v}{c^2}u_x} \quad u'_y = \frac{u_y\sqrt{1-\beta^2}}{1 - \frac{v}{c^2}u_x} \quad u'_z = \frac{u_z\sqrt{1-\beta^2}}{1 - \frac{v}{c^2}u_x}$$

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x} \quad u_y = \frac{u'_y\sqrt{1-\beta^2}}{1 + \frac{v}{c^2}u'_x} \quad u_z = \frac{u'_z\sqrt{1-\beta^2}}{1 + \frac{v}{c^2}u'_x}$$

5-3 相对论速度变换公式

2. 相对论速度变换—<u>例题5-2</u>

$$u'_x = \frac{u_x - v}{1 - \frac{v}{c^2}u_x} \quad u'_y = \frac{u_y\sqrt{1-\beta^2}}{1 - \frac{v}{c^2}u_x} \quad u'_z = \frac{u_z\sqrt{1-\beta^2}}{1 - \frac{v}{c^2}u_x}$$

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x} \quad u_y = \frac{u'_y\sqrt{1-\beta^2}}{1 + \frac{v}{c^2}u'_x} \quad u_z = \frac{u'_z\sqrt{1-\beta^2}}{1 + \frac{v}{c^2}u'_x}$$

地面上测得两个飞船a, b分别以+0.9c和-0.9c的速度沿相反方向飞行.求**飞船a相对于飞船b的速度有多大**?(相当于在b上测a的速度)

b为K’系, 则v=-0.9c, u=0.9c, 代入速度变换可得:

$$u' = \frac{u - v}{1 - \frac{v}{c^2}u} = \frac{0.9c - (-0.9c)}{1 - \frac{-0.9c}{c^2}\cdot 0.9c} \simeq 0.994c$$

5-4 狭义相对论时空观

1. 同时的相对性

经典力学时空观: 同时和杆长度是绝对的.

$$t = t' \to \Delta t = \Delta t' \qquad x = x' + x_0 \to \Delta x = \Delta x'$$

狭义相对论时空观: 同时和杆长度是相对的.

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\beta^2}} \qquad x' = \frac{x - vt}{\sqrt{1-\beta^2}}$$

$$t_2' - t_1' = \frac{(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)}{\sqrt{1-\beta^2}} \qquad x_2' - x_1' = \frac{(x_2 - x_1) - v(t_2 - t_1)}{\sqrt{1-\beta^2}}$$

5-4 狭义相对论时空观

2. 尺缩钟缓一是相对运动的效应, 并非事物内部结构发生变化

运动的杆变短.

$$\ell = \ell_0 \sqrt{1-\beta^2}$$

运动的钟变慢.

$$\tau = \frac{\tau_0}{\sqrt{1-\beta^2}}$$

5-5 狭义相对论动力学基础

质量增大

$$m = \frac{m_0}{\sqrt{1-\beta^2}}$$

质能方程 (Mass–energy equivalence)

$$E = mc^2$$

$$E_k = mc^2 - m_0 c^2 = E - E_0$$

动量和能量的关系

$$vdp = dE_k$$

$$E^2 = c^2 p^2 + m_0^2 c^4$$

5-6 广义相对论

时空弯曲