

discretization of 3D fokker planck

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The constraints

$$-\frac{\partial m}{\partial t} + \mathcal{L}^* m - \mu(t, a, x) = 0 \quad (1)$$

$$-\frac{\partial \tilde{m}}{\partial t} + \mathcal{L}^{0*} \tilde{m} + m(t, \bar{a}, x) - \tilde{\mu}(t, a, x) = 0 \quad (2)$$

$$-\frac{\partial \hat{m}}{\partial t} + \mathcal{L}^{0*} \hat{m} - \hat{\mu}(t, a, x) = 0 \quad (3)$$

$$t \in [0, \bar{t}], a \in [0, \bar{a}], X \in [0, \bar{x}]$$

$$\mathcal{L}^{0*} m = -\frac{\partial}{\partial x} (mk(\theta - x)) + \frac{\delta^2}{2} \frac{\partial^2}{\partial x^2} (\tilde{m}x) \quad (4)$$

$$\mathcal{L}^* m = -\frac{\partial}{\partial x} (mk(\theta - x)) + \frac{\delta^2}{2} \frac{\partial^2}{\partial x^2} (\tilde{m}x) - \frac{\partial m}{\partial a} \quad (5)$$

$$i = 0 \dots N_t - 2, t = (i + 1)\Delta t$$

$$z = 0 \dots N_a - 1$$

$$j = 0 \dots N_X - 1$$

$$\begin{aligned} & \frac{m[i][z][j] - m[i-1][z][j]}{\Delta t} \\ & + \frac{m[i][z][j+1]k(\theta - x[j+1]) - m[i][z][j-1]k(\theta - x[j-1])}{2\Delta x} \\ & - \frac{\delta^2}{2} \frac{m[i][z][j+1]x[j+1] - 2m[i][z][j]x[j] + m[i][z][j-1]x[j-1]}{\Delta x^2} \end{aligned} \quad (6)$$

$$+ \frac{m[i][z][j] - m[i][z-1][j]}{\Delta a} + \mu = 0 \quad (7)$$

$$\begin{aligned} & \frac{\hat{m}[i][j] - \hat{m}[i-1][j]}{\Delta t} \\ & + \frac{\hat{m}[i][j+1]k(\theta - x[j+1]) - \hat{m}[i][j-1]k(\theta - x[j-1])}{2\Delta x} \\ & - \frac{\delta^2}{2} \frac{\hat{m}[i][j+1]x[j+1] - 2\hat{m}[i][j]x[j] + \hat{m}[i][j-1]x[j-1]}{\Delta x^2} + \hat{\mu} = 0 \end{aligned} \quad (8)$$

$$\begin{aligned}
& \frac{\tilde{m}[i][j] - \tilde{m}[i-1][j]}{\Delta t} \\
& + \frac{\tilde{m}[i][j+1]k(\theta - x[j+1]) - \tilde{m}[i][j-1]k(\theta - x[j-1])}{2\Delta x} \\
& - \frac{\delta^2}{2} \frac{\tilde{m}[i][j+1]x[j+1] - 2\tilde{m}[i][j]x[j] + \tilde{m}[i][j-1]x[j-1]}{\Delta x^2} + \tilde{\mu} - m[i][Na-1][j] = 0
\end{aligned} \tag{9}$$

$$V[j] = 1 + \frac{\Delta t \delta^2 x[j]}{\Delta x} + \frac{\Delta t}{\Delta a} \tag{10}$$

$$V_0[j] = 1 + \frac{\Delta t \delta^2 x[j]}{\Delta x} \tag{11}$$

$$V_1[j] = -\frac{\Delta t \delta^2 x[j]}{2\Delta x^2} + \frac{\Delta t k(\theta - x[j])}{2\Delta x} \tag{12}$$

$$V_2[j] = -\frac{\Delta t \delta^2 x[j]}{2\Delta x^2} - \frac{\Delta t k(\theta - x[j])}{2\Delta x} \tag{13}$$

$$V[j]m[i][z][j] + V_1[j+1]m[i][z][j+1] + V_2[j-1]m[i][z][j-1] - m[i-1][z][j] - \frac{\Delta t}{\Delta a}m[i][z-1][j] + \Delta t\mu = 0 \tag{14}$$

$$V_0[j]\hat{m}[i][j] + V_1[j+1]\hat{m}[i][j+1] + V_2[j-1]\hat{m}[i][j-1] - \hat{m}[i-1][j] + \Delta t\hat{\mu} = 0 \tag{15}$$

$$V_0[j]\tilde{m}[i][j] + V_1[j+1]\tilde{m}[i][j+1] + V_2[j-1]\tilde{m}[i][j-1] - \tilde{m}[i-1][j] + \Delta t\tilde{\mu} - \Delta tm[i][Na-1][j] = 0 \tag{16}$$

When $i = 0$, $t = \Delta t$

$$V[j]m[0][z][j] + V_1[j+1]m[0][z][j+1] + V_2[j-1]m[0][z][j-1] - \frac{\Delta t}{\Delta a}m[0][z-1][j] + \Delta t\mu = m_0[j] \tag{17}$$

$$V_0[j]\hat{m}[0][j] + V_1[j+1]\hat{m}[0][j+1] + V_2[j-1]\hat{m}[0][j-1] + \Delta t\hat{\mu} = \hat{m}_0[j] \tag{18}$$

$$V_0[j]\tilde{m}[0][j] + V_1[j+1]\tilde{m}[0][j+1] + V_2[j-1]\tilde{m}[0][j-1] + \Delta t\tilde{\mu} - \Delta tm[0][Na-1][j] = 0 \tag{19}$$

capacity:

$$c(t) = \int_{\mathcal{O}} \int_{\mathcal{A}} m \, dadx \tag{20}$$

conventional offer:

$$\int_{\mathcal{O}} \int_{\mathcal{A}} Fm \, dadx + \int_{\mathcal{O}} \int_{\mathcal{A}} F\tilde{m} \, dx \tag{21}$$

for given i (t)

$$\sum_{j=0}^{N_X-1} F[j] \left(\left(\sum_{z=0}^{N_a-1} m[i][z][j] \Delta a \Delta x \right) + \tilde{m}[i][j] \Delta x \right) \tag{22}$$

Renewable offer:

$$\int_{\mathcal{O}} \int_{\mathcal{A}} Xm \, dadx + \int_{\mathcal{O}} \int_{\mathcal{A}} X\tilde{m} \, dx \tag{23}$$

for given $i(t)$

$$\sum_{j=0}^{N_X-1} X[j] \left(\left(\sum_{z=0}^{N_a-1} m[i][z][j] \Delta a \Delta x \right) + \tilde{m}[i][j] \Delta x \right) \quad (24)$$

objective function

$$\int_t \int_{\mathcal{O}} \int_{\mathcal{A}} e^{-\rho t} G m dt dx da + \int_t \int_{\mathcal{O}} \int_{\mathcal{A}} e^{-(\rho+\gamma)t} f \mu dt dx da - \int_{\mathcal{O}} \int_{\mathcal{A}} e^{-(\rho+\gamma)t} s \hat{\mu} dx da \quad (25)$$

$$\sum_{i=0}^{N_t-2} \sum_{j=0}^{N_a-1} \sum_{z=0}^{N_X-1} e^{-\rho t} G m[i][z][j] \Delta t \Delta x \Delta a + \sum_{i=0}^{N_t-2} \sum_{j=0}^{N_a-1} \sum_{z=0}^{N_X-1} e^{-(\rho+\gamma)t} f \mu[i][z][j] \Delta t \Delta x \Delta a - \sum_{i=0}^{N_t-2} \sum_{j=0}^{N_a-1} e^{-(\rho+\gamma)t} s \hat{\mu}[i][j] \Delta t \Delta x \quad (26)$$

When $z = 0$

$$m[i][0][j] = \hat{\mu}[i][j]$$

When $z = 1$

$$V[j]m[i][1][j] + V_1[j+1]m[i][1][j+1] + V_2[j-1]m[i][1][j-1] - m[i-1][1][j] + \Delta t \mu[i][j] = \frac{\Delta t}{\Delta a} \hat{\mu}[i][j] \quad (27)$$

When $z = 2, \dots, N_a - 1$

$$V[j]m[i][z][j] + V_1[j+1]m[i][z][j+1] + V_2[j-1]m[i][z][j-1] - m[i-1][z][j] - \frac{\Delta t}{\Delta a} m[i][z-1][j] + \Delta t \mu[i][j] = 0 \quad (28)$$

with

$$V_0[j]\tilde{m}[i][j] + V_1[j+1]\tilde{m}[i][j+1] + V_2[j-1]\tilde{m}[i][j-1] + \Delta t \tilde{\mu} - \Delta t m[i][N_a-1][j] = 0 \quad (29)$$

When $z \neq 0, i = 0, j \neq 0, j \neq N_X - 1$

$$V[j]m[0][z][j] + V_1[j+1]m[0][z][j+1] + V_2[j-1]m[0][z][j-1] - \frac{\Delta t}{\Delta a} m[0][z-1][j] + \Delta t \mu[0][j] = m_0[i][j] \quad (30)$$

$$V_0[j]\tilde{m}[0][j] + V_1[j+1]\tilde{m}[0][j+1] + V_2[j-1]\tilde{m}[0][j-1] + \Delta t \tilde{\mu} - \Delta t m[0][N_a-1][j] = 0 \quad (31)$$

When $z \neq 0, j = 0, i \neq 0$

$$V[0]m[i][z][0] + V_1[1]m[i][z][1] - m[i-1][z][0] - \frac{\Delta t}{\Delta a} m[i][z-1][0] + \Delta t \mu[i][z][0] = 0 \quad (32)$$

$$V_0[0]\hat{m}[i][0] + V_1[1]\hat{m}[i][1] - \hat{m}[i-1][0] + \Delta t \hat{\mu}[i][0] = 0 \quad (33)$$

$$V_0[0]\tilde{m}[i][0] + V_1[1]\tilde{m}[i][1] - \tilde{m}[i-1][0] + \Delta t \tilde{\mu}[i][0] - \Delta t m[i][N_a-1][0] = 0 \quad (34)$$

When $z \neq 0, j = N_X - 1, i \neq 0$

$$V[N_X-1]m[i][z][N_X-1] + V_2[N_X-2]m[i][z][N_X-2] - m[i-1][z][N_X-1] - \frac{\Delta t}{\Delta a} m[i][z-1][N_X-1] + \Delta t \mu[i][z][N_X-1] = 0 \quad (35)$$

$$V_0[N_X-1]\hat{m}[i][N_X-1] + V_2[N_X-2]\hat{m}[i][N_X-2] - \hat{m}[i-1][N_X-1] + \Delta t \hat{\mu}[i][N_X-1] = 0 \quad (36)$$

$$V_0[N_X - 1]\tilde{m}[i][N_X - 1] + V_2[N_X - 2]\tilde{m}[i][N_X - 2] - \tilde{m}[i - 1][N_X - 1] + \Delta t \tilde{\mu}[i][N_X - 1] - \Delta t m[i][N_a - 1][N_X - 1] = 0 \quad (37)$$

When $z \neq 0, i = 0, j = 0$

$$V[0]m[0][z][0] + V_1[1]m[0][z][1] - \frac{\Delta t}{\Delta a}m[0][z - 1][0] + \Delta t \mu[0][z][0] = 0 \quad (38)$$

$$V_0[0]\hat{m}[i][0] + V_1[1]\hat{m}[0][1] + \Delta t \hat{\mu}[0][0] = 0 \quad (39)$$

$$V_0[0]\tilde{m}[0][0] + V_1[1]\tilde{m}[0][1] + \Delta t \tilde{\mu}[0][0] - \Delta t m[0][N_a - 1][0] = 0 \quad (40)$$

When $z \neq 0, i = 0, j = N_X - 1$

$$V[N_X - 1]m[0][z][N_X - 1] + V_2[N_X - 2]m[0][z][N_X - 2] - \frac{\Delta t}{\Delta a}m[0][z - 1][N_X - 1] + \Delta t \mu[0][z][N_X - 1] = 0 \quad (41)$$

$$V_0[N_X - 1]\hat{m}[0][N_X - 1] + V_2[N_X - 2]\hat{m}[0][N_X - 2] + \Delta t \hat{\mu}[0][N_X - 1] = 0 \quad (42)$$

$$V_0[N_X - 1]\tilde{m}[0][N_X - 1] + V_2[N_X - 2]\tilde{m}[0][N_X - 2] + \Delta t \tilde{\mu}[0][N_X - 1] - \Delta t m[0][N_a - 1][N_X - 1] = 0 \quad (43)$$

Summing up with z from 1 to $N_a - 1$. Take

$$\begin{aligned} \bar{m}[i][j] &= \left(\sum_{z=1}^{N_a-1} m[i][j]\Delta a \right) + \tilde{m}[i][j] \\ \bar{\mu}[i][j] &= \left(\sum_{z=1}^{N_a-1} \mu[i][j]\Delta a \right) + \tilde{\mu}[i][j] \end{aligned}$$

$$V_0[j]\bar{m}[i][j] + V_1[j + 1]\bar{m}[i][j + 1] + V_2[j - 1]\bar{m}[i][j - 1] - \bar{m}[i - 1][j] + \Delta t \bar{m}u[i][j] - \Delta t \hat{\mu}[i][j] = 0 \quad (44)$$