Predefined Time Prescribed Performance Backstepping Control for Robotic Manipulators with Input Saturation

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摘要

本文研究了具有未知动力学、有界扰动以及执行器饱和条件下的机器人机械臂轨迹跟踪控制问题，提出了一种立于任何初始状态条件的预定义时间预设性能约束的自适应反步控制方法。针对传统预设性能控制中存在的初值奇异性和误差变换非光滑问题，针对每个通道独立设计了结合多项式性能函数与误差缩放函数的预定义时间误差变换结构，确保系统状态在预定义时间内严格且平滑地满足全局预设性能约束。为处理反步控制及执行器饱和输入引起的计算复杂度爆炸问题，设计了一种预定义时间饱和补偿器，以同时消除命令滤波与输入饱和的不利影响。此外，引入一阶滑模扰动观测器实时估计并补偿系统中未知的有界扰动，利用径向基函数神经网络对系统未建模动态进行辨识和补偿，从而提高了控制系统的鲁棒性和跟踪精度。同时，结合预定义时间理论和自适应动态障碍李雅普诺夫函数，严格证明了闭环系统在预定义时间内的全局稳定性和误差收敛性。数值和实验结果结果验证了所提出方法的有效性，与现有方法相比，本文方法在响应速度、稳态精度及鲁棒性能方面表现出明显优势。

Keywords: Prescribed performance control, predefined-time stability, adaptive dynamic barrier Lyapunov function, backstepping control, input saturation, robotic manipulators' trajectory tracking.

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0.1 System description

求V_1 的时间导数为

$$\dot{V}_1 = \sum_{i=1}^{n} \left[\frac{\rho_{1,i}^4 \, z_{1,i}}{\left(\rho_{1,i}^2 - z_{1,i}^2\right)^2} \dot{z}_{1,i} - \frac{\rho_{1,i} \, z_{1,i}^4}{\left(\rho_{1,i}^2 - z_{1,i}^2\right)^2} \dot{\rho}_{1,i} \right] \tag{1}$$

记 $P_{j,i} = \frac{\rho_{j,i}^4}{(\rho_{j,i}^2 - z_{j,i}^2)^2} > 0, Q_{j,i} = \frac{z_{j,i}^4}{(\rho_{j,i}^2 - z_{j,i}^2)^2} > 0, \Phi_{j,i} = \frac{z_{j,i}\left(\rho_{j,i}^2 - z_{j,i}^2\right)}{\rho_{j,i}^4} = \frac{z_{j,i}(\rho_{j,i}^2 - z_{j,i}^2)}{P_{j,i}}, j \in \{1, 2\}$, $\dot{z}_{1,i} = z_{2,i} - \dot{q}_{d,i} + \alpha_i^f + \zeta_i$, 将它们代入 BLF 导数后，并根据公式（），得到

$$\dot{V}_1 = \sum_{i=1}^{n} \Big[P_{1,i} z_{1,i} z_{2,i} + P_{1,i} z_{1,i} (\alpha_i + \zeta_i - \dot{q}_{d,i}) - P_{1,i} z_{1,i} \tilde{\alpha}_i - Q_{1,i} \rho_{1,i} \dot{\rho}_{1,i} \Big]$$

Let $V_{j,i}=\frac{1}{2}\frac{\rho_{j,i}^2 z_{j,i}^2}{\rho_{j,i}^2-z_{j,i}^2}$, $\Psi(V_{j,i})=\frac{\pi}{\eta T_p}\Big((V_{j,i})^{\,1-\eta/2}+n^{-\frac{\eta}{2}}(V_{j,i})^{\,1+\eta/2}\Big)$, 我们设计

$$\mathcal{K}_{1,i}(z_{j,i},\rho_{j,i})=\frac{\rho_{j,i}^2}{2}\frac{\Psi(V_{j,i})}{V_{j,i}}=\frac{\pi\,\rho_{j,i}^2}{2\eta T_p}\Big((V_{j,i})^{-\eta/2}+n^{-\frac{\eta}{2}}(V_{j,i})^{\eta/2}\Big).$$

把它代回，可设计虚拟控制律为：

$$\alpha_i=\dot q_{d,i}-\zeta_i+\frac{z_{1,i}^3}{\rho_{1,i}^3}\,\dot\rho_{1,i}-\mathcal{K}_{1,i}(z_{1,i},\rho_{1,i})\Phi_{1,i}-k_{1,i}\rho_{1,i}^2\Phi_{1,i}, \tag{2}$$

where $k_1=\mathrm{diag}\{k_{1,1},k_{1,i},\cdots,k_{1,n}\}>0$, $k_{1,i}>0$,

则一步 Lyapunov 导数化简为

$$\dot V_1\le\sum_{i=1}^{n}\Big[P_{1,i}z_{1,i}z_{2,i}-\Psi(V_{1,i})-P_{1,i}z_{1,i}\tilde\alpha_i-k_{1,i}\frac{\rho_{1,i}^2z_{1,i}^2}{\rho_{1,i}^2-z_{1,i}^2}\Big].$$

In the second step, a BLF is used as an energy function for the error dynamics. We choose:

$$V_2=\frac{1}{2}\sum_{i=1}^{n}\frac{\rho_{2,i}^2z_{2,i}^2}{\rho_{2,i}^2-z_{2,i}^2}. \tag{3}$$

Using $\dot z_{2,i}=\dot x_{2,i}-\dot\alpha_i^f-\dot\zeta_i$, we obtain the time derivative of V_2 is as

$$\begin{aligned}\dot V_2&=\sum_{i=1}^{n}\Big[\,P_{2,i}z_{2,i}\,(\dot x_{2,i}-\dot\alpha_i^f-\dot\zeta_i)-Q_{2,i}\rho_{2,i}\,\dot\rho_{2,i}\,\Big]\\&=\sum_{i=1}^{n}\Big[\,P_{2,i}z_{2,i}\,\Big(f_i(x)+h_i(x,t)+g_i(u_i-\Delta u_i)+d'(t)-\dot\alpha_i^f-\delta_i\;+\;\mu_2\,\zeta_i\;+\;g_i\,\Delta u_i-\tfrac{z_2^3}{\rho_{2,i}^3}\dot\rho_{2,i}\Big)\Big]\end{aligned} \tag{4}$$

Let $Q_2=\mathrm{col}\{Q_{2,i}\}$, $P_1=\mathrm{col}\{P_{1,i}\}$, $\Phi_2=\mathrm{col}\{\Phi_{2,i}\}$. The control input torque is designed as

$$\begin{aligned}u=&C(q,\dot q)x_2+G(q)\\&+M(q)\Big[\dot\alpha^f+\delta-\mu_2\,\zeta-\frac{P_1}{P_2}z_1+\tfrac{z_2^3}{\rho_{2,i}^3}\dot\rho_{2,i}-\mathcal{K}_2(z_2,\rho_{2,i})\Phi_2-\hat h(\chi)-\hat\omega-k_s\,\mathrm{sgn}(z_2)-k_2\rho_{2,i}^2\Phi_{2,i}\Big]\end{aligned} \tag{5}$$

where $k_2=\mathrm{diag}\{k_{2,1},k_{2,i},\cdots,k_{2,n}\}>0$, $k_{2,i}>0$, $k_s>0$.

Let $\chi=\{q,\dot q\}\in\mathbb{R}^m$ be the regressor and $\psi(\chi)\in\mathbb{R}^N$ the normalized basis. Approximate the structured uncertainty by $\hat h_i(\chi)=\hat\theta_i^\top\psi_i(\chi)$, with $\theta_i\in\mathbb{R}^N$ adapted online by

$$\dot{\hat\theta}_i=\varrho_i\,P_{2,i}z_{2,i}\psi(\chi)-\kappa_i\,\hat\theta_i,\qquad \varrho_i,\kappa_i>0. \tag{6}$$

将结构不确定估计误差和外扰的剩余项定义为 $\omega(x,t)=h(x)-\hat h(\chi)+M^{-1}(q)d(t)$。

为避免直接微分速度，采用一阶跟踪微分器作为观测器

$$\begin{cases}\dot\vartheta=-\omega_d\,\vartheta+\omega_d\,x_2,\\ \hat{\dot x}_2=\omega_d(x_2-\vartheta),\\ \dot{\hat\omega}=-\Lambda_o\,\hat\omega+\Lambda_o\Big(\hat{\dot x}_2-f(x)-M^{-1}(q)\,\mathrm{sat}(u)-\hat h(\chi)\Big),\end{cases} \tag{7}$$

where $\Lambda_o=\mathrm{diag}\{\lambda_{o,i}\}>0,\omega_d>0,\vartheta(0)=x_2(0),\hat\omega(0)=0$, 设观测误差为 $\tilde\omega=\omega-\hat\omega$. 其动力学为

$$\dot{\tilde\omega}=-\Lambda_o\,\tilde\omega\;+\;\Delta_\omega,\qquad \Delta_\omega:=\dot\omega-\Lambda_o\,\varepsilon_d,$$

where $\Lambda_o=\mathrm{diag}\{\lambda_{o,i}\}>0,w_d>0,\vartheta(0)=x_2(0)$,$\varepsilon_d:=\hat{\dot x}_2-\dot x_2$ 为跟踪微分器误差。

根据公式可得到

$$\dot V_2\le+\sum_{i=1}^{n}\Big(-k_{2,i}\frac{\rho_{2,i}^2z_{2,i}^2}{\rho_{2,i}^2-z_{2,i}^2}-\Psi(V_{2,i})-P_{2,i}z_{2,i}\tilde\omega_i\Big)-k_s\sum_{i=1}^{n}P_{2,i}\,|z_{2,i}| \tag{8}$$

$$
\begin{aligned}
\dot{V} \leq & \sum_{i=1}^{n}\left(-k_{1,i}\frac{\rho_{1,i}^2 z_{1,i}^2}{\rho_{1,i}^2 - z_{1,i}^2} - \Psi(V_{1,i}) - P_{1,i}z_{1,i}\tilde{\alpha}_i\right) + \sum_{i=1}^{n}\left(-k_{2,i}\frac{\rho_{2,i}^2 z_{2,i}^2}{\rho_{2,i}^2 - z_{2,i}^2} - \Psi(V_{2,i}) - P_{2,i}z_{2,i}\tilde{\omega}_i\right) \\
& - k_s\sum_{i=1}^{n} P_{2,i}\left|z_{2,i}\right| + \tilde{\alpha}^\top\dot{\tilde{\alpha}} + \tilde{\omega}^\top\dot{\tilde{\omega}} + \sum_{i=1}^{n}\tfrac{1}{\varrho_i}\tilde{\theta}_i^\top\dot{\tilde{\theta}}_i \\
\leq & \sum_{i=1}^{n}\left(-k_{1,i}\frac{\rho_{1,i}^2 z_{1,i}^2}{\rho_{1,i}^2 - z_{1,i}^2} - \Psi(V_{1,i})\right) + \sum_{i=1}^{n}\left(-k_{2,i}\frac{\rho_{2,i}^2 z_{2,i}^2}{\rho_{2,i}^2 - z_{2,i}^2} - \Psi(V_{2,i})\right) \\
& - \sum_{i=1}^{n} P_{2,i}z_{2,i}\tilde{\omega}_i + \tilde{\omega}^\top\dot{\tilde{\omega}} - k_s\sum_{i=1}^{n} P_{2,i}\left|z_{2,i}\right| + \tilde{\alpha}^\top\dot{\tilde{\alpha}} - \sum_{i=1}^{n} P_{1,i}z_{1,i}\tilde{\alpha}_i + \sum_{i=1}^{n}\tfrac{1}{\varrho_i}\tilde{\theta}_i^\top\dot{\tilde{\theta}}_i
\end{aligned} \tag{9}
$$

$$
\rho_{2,i}^2 - z_{2,i}^2 \leq \rho_{2,i}^2
$$

Let

$$
\mathcal{L} = -\sum_{i=1}^{n} P_{2,i}z_{2,i}\tilde{\omega}_i \; + \; \tilde{\omega}^\top\dot{\tilde{\omega}} \; - \; k_s\sum_{i=1}^{n} P_{2,i}|z_{2,i}|
$$

利用不等式 $ab \leq k\,a + \frac{b^2}{4k}$（$(\sqrt{k\,a} - \frac{b}{2\sqrt{k}})^2 \geq 0$）并取 $a = P_{2,i}|z_{2,i}|$，$b = |\tilde{\omega}_i|$，$k = k_s$，对每个通道有

$$
-P_{2,i}z_{2,i}\tilde{\omega}_i \;\leq\; P_{2,i}|z_{2,i}|\,|\tilde{\omega}_i| \;\leq\; k_s P_{2,i}|z_{2,i}| + \frac{1}{4k_s}\tilde{\omega}_i^2.
$$

于是

$$
-k_s\sum_{i=1}^{n} P_{2,i}|z_{2,i}| - \sum_{i=1}^{n} P_{2,i}z_{2,i}\tilde{\omega}_i \;\leq\; \frac{1}{4k_s}\,\|\tilde{\omega}\|^2
$$

由观测器误差$\dot{\tilde{\omega}} = -\Lambda_o\,\tilde{\omega} + \Delta_\omega, \Lambda_o = \mathrm{diag}\{\lambda_{o,i}\} > 0$, 并根据Young不等式得

$$
\tilde{\omega}^\top\dot{\tilde{\omega}} = -\tilde{\omega}^\top\Lambda_o\tilde{\omega} + \tilde{\omega}^\top\Delta_\omega \;\leq\; -\tilde{\omega}^\top\Lambda_o\tilde{\omega} + \frac{\varepsilon}{2}\tilde{\omega}^\top\Lambda_o\tilde{\omega} + \frac{1}{2\varepsilon}\Delta_\omega^\top\Lambda_o^{-1}\Delta_\omega \leq\; -\Big(1 - \tfrac{\varepsilon}{2}\Big)\tilde{\omega}^\top\Lambda_o\tilde{\omega} + \frac{1}{2\varepsilon}\left\|\Lambda_o^{-1/2}\Delta_\omega\right\|^2,
$$

把（A）与（B）相加：

$$
\mathcal{L} \leq -\Big(1 - \tfrac{\varepsilon}{2}\Big)\tilde{\omega}^\top\Lambda_o\tilde{\omega} + \frac{1}{4k_s}\|\tilde{\omega}\|^2 + \frac{1}{2\varepsilon}\left\|\Lambda_o^{-1/2}\Delta_\omega\right\|^2.
$$

利用 $\tilde{\omega}^\top\Lambda_o\tilde{\omega} \geq \lambda_{o,\min}\|\tilde{\omega}\|^2$，得到

$$
\mathcal{L} \leq\; -\Big(\big(1 - \tfrac{\varepsilon}{2}\big)\lambda_{o,\min} - \tfrac{1}{4k_s}\Big)\|\tilde{\omega}\|^2 + \frac{1}{2\varepsilon}\left\|\Lambda_o^{-1/2}\Delta_\omega\right\|^2.
$$

令 $\varepsilon = \frac{1}{2}$（简洁且常用），净负系数为 $\frac{3}{4}\lambda_{o,\min} - \frac{1}{4k_s}$。只要$\boxed{k_s \;>\; \dfrac{1}{3\,\lambda_{o,\min}}}$
$\frac{1}{2\varepsilon}\|\Lambda_o^{-1/2}\Delta_\omega\|^2$ 作为输入项（由 $\Delta_\omega = \dot{\omega} - \Lambda_o\varepsilon_d$ 决定）并入 $\sigma(\|r(t)\|)$。

Statements and Declarations

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Conflicts of Interest

The authors declare no conflict of interest.

Author contribution

Shuli Liu: Writing-Original Draft, Methodology, Software, Visualization, Data curation. Yi Liu:Resources, Investigation, Formal analysis, Conceptualization, Writing-Review & Editing. Jingang Liu: Project administration, Validation, Funding acquisition, Writing-Review & Editing. Yin Yang: Supervision, Funding acquisition, Writing-Review & Editing. All authors have reviewed and approved the final version of the manuscript.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.