(b) Find the relation between sigmoid function and hyperbolic function.

We will find the relationship between the sigmoid function, $\sigma(x)$, and the hyperbolic tangent function, $\tanh(x)$.

Define the functions: The sigmoid function is defined as:

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

The hyperbolic tangent function is defined as:

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Manipulate the $\tanh(x)$ function: Let's consider the function $\tanh\left(\frac{x}{2}\right)$:

$$\tanh\left(\frac{x}{2}\right) = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}}$$

To simplify this expression and relate it to $\sigma(x)$, we can multiply the numerator and the denominator by $e^{x/2}$:

$$\tanh\left(\frac{x}{2}\right) = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} \cdot \left(\frac{e^{x/2}}{e^{x/2}}\right)$$

$$\tanh\left(\frac{x}{2}\right) = \frac{(e^{x/2} \cdot e^{x/2}) - (e^{-x/2} \cdot e^{x/2})}{(e^{x/2} \cdot e^{x/2}) + (e^{-x/2} \cdot e^{x/2})} = \frac{e^x - 1}{e^x + 1}$$

Manipulate the $\sigma(x)$ function: Now, let's manipulate the sigmoid function. We can start by multiplying the numerator and denominator by e^x:

$$\sigma(x) = \frac{1}{1+e^{-x}} \cdot \left(\frac{e^x}{e^x}\right) = \frac{e^x}{e^x + e^0} = \frac{e^x}{e^x + 1}$$

Let's consider the expression $2\sigma(x) - 1$:

$$2\sigma(x) - 1 = 2\left(\frac{e^x}{e^x+1}\right) - 1$$

$$2\sigma(x) - 1 = \frac{2e^x}{e^x+1} - \frac{e^x+1}{e^x+1}$$

$$2\sigma(x) - 1 = \frac{2e^x - (e^x+1)}{e^x+1}$$

$$2\sigma(x) - 1 = \frac{2e^x - e^x - 1}{e^x+1}$$

$$2\sigma(x) - 1 = \frac{e^x - 1}{e^x+1}$$

State the relation: By comparing the results from step 2 and step 3, we can see that:

$$\frac{e^x-1}{e^x+1} = \tanh\left(\frac{x}{2}\right) \quad \text{and} \quad \frac{e^x-1}{e^x+1} = 2\sigma(x) - 1$$

Therefore, the relation between the sigmoid function and the hyperbolic tangent function is:

$$2\sigma(x) - 1 = \tanh\left(\frac{x}{2}\right)$$

This can also be rearranged to express the sigmoid function in terms of $\tanh$:

$$\sigma(x) = \frac{1 + \tanh\left(\frac{x}{2}\right)}{2}$$

This shows that the sigmoid function is essentially a linearly transformed version of the hyperbolic tangent function (scaled horizontally by $1/2$, scaled vertically by $1/2$, and shifted vertically by $1/2$).