

# CS 589 Fall 2021 Lecture 4

## Inverted Index

**Monday 6:30-9:00**  
**Babbio 122**

All zoom links in Canvas  
Most slides adapted from Stanford CS276




photo: <https://www.scubedstudios.com/information-retrieval/>

# Lecture 4: Information retrieval infrastructure

How does Google  
know cs 589 refers  
to a course?  
How does Google

cs 589 **stevens**

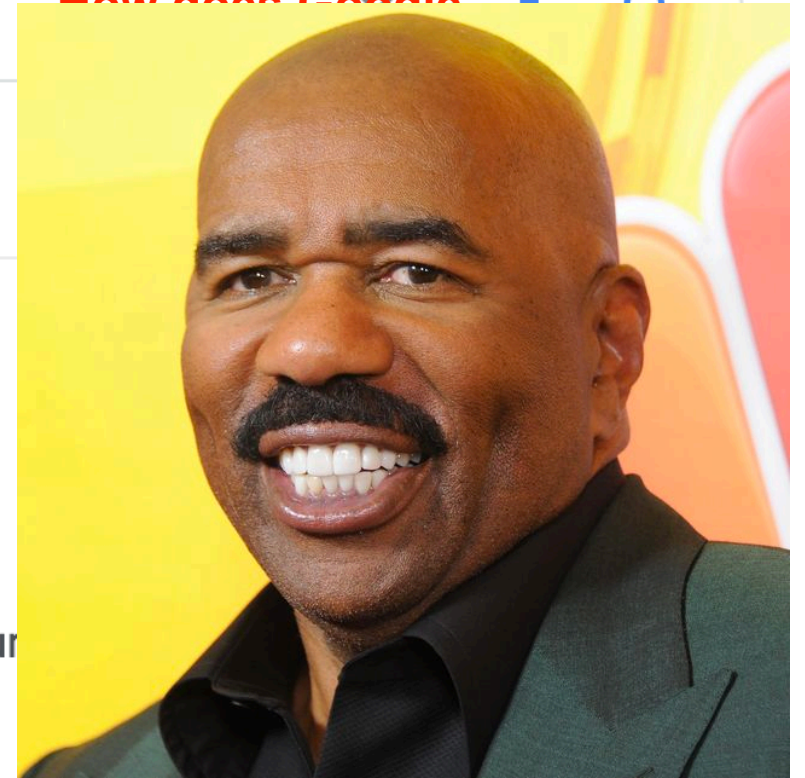
 All  Images  Maps  News  Videos  More

About 3,220,000 results (0.58 seconds)

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**cs589**

CS 589: Text Mining and Information Retrieval. Home | Canvas | Resources  
Stevens' guidelines on the coronavirus emergency (COVID-19), ...



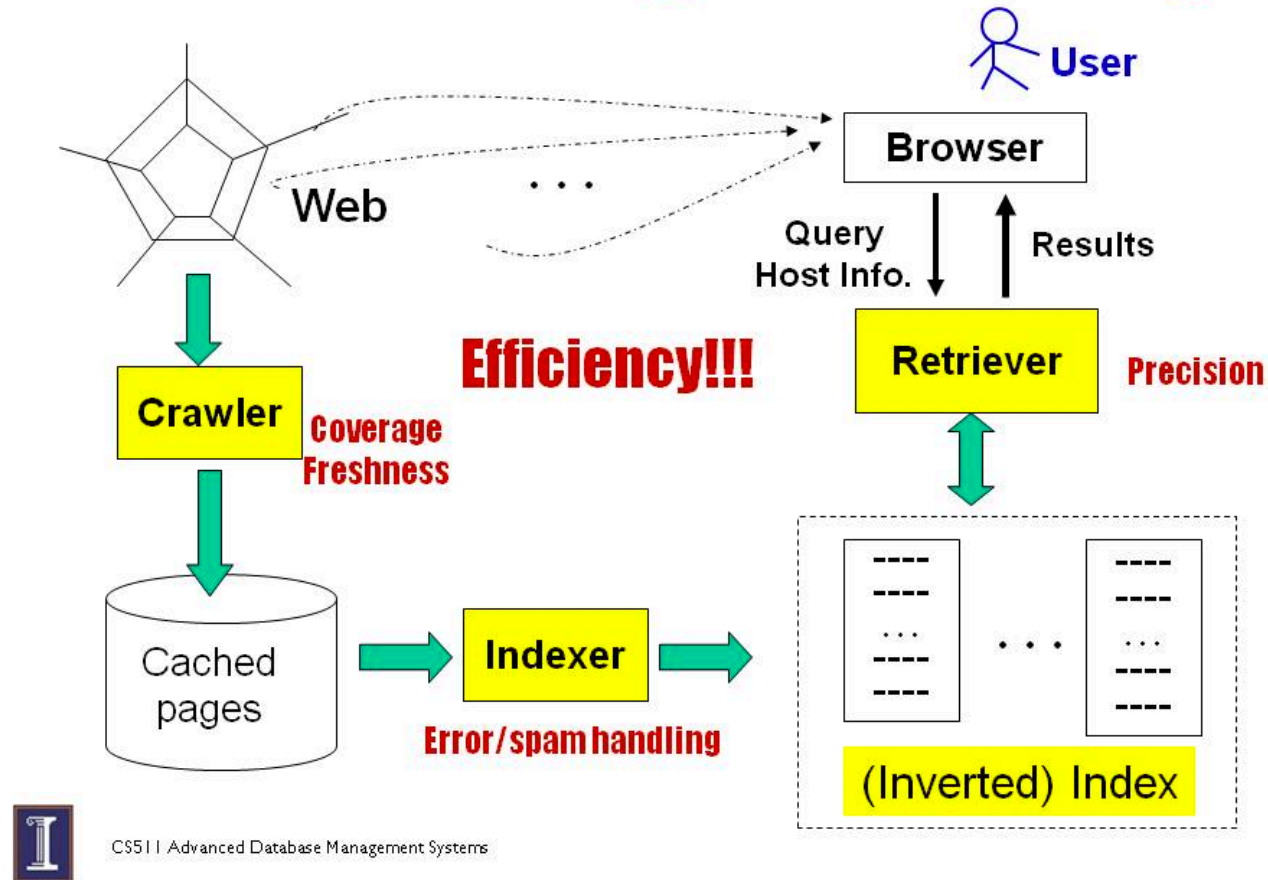
# Review of Lecture 1-3: Retrieval models

- Vector space model
  - Use the cosine score of TF-IDF to retrieve documents
- BM25:
  - Approximated 2-Poisson model derived from PRP, i.e., rank by  $p(rel = 1|q, d)$
  - Adding document length pivoting and IDF
- LM-based retrieval model
  - Rank by  $p(q|d)$  based on i.i.d. assumption for words and unigram LM

$$p(w_i|d) = \begin{cases} p_{seen}(w_i|d) & \text{if } w_i \text{ is seen in } d \\ \alpha_d p(w_i|C) & o.w. \end{cases}$$

# Lecture 4: Information retrieval infrastructure

## Basic Search Engine Technologies



7

# Pop quiz (IR Evaluation)

- Suppose a query has a total of 4 relevant documents in a collection with 100 documents. System A and System B have each retrieved 10 documents, and the relevance status of the two ranked lists of results is:

System A: [+,-,-,-,-,-,-,-,-]

System B: [+,-,+,-,-,-,-,-,-]

1. What is the MAP of System A and System B?
2. What is the NDCG@5 of System A?

# Lecture 4: Inverted index

- Key data structure underlying all modern IR systems
  - Systems run on a single machine
  - Massive systems for the biggest commercial search engines
- Exploiting the sparsity of the term-document matrix
- Inverted index can generally be applied to retrieval models
  - TF-IDF, BM25, LM-based retrieval model

## Lecture 4: Motivating example

- Which plays of Shakespeare contain the words *Brutus AND Caesar* but *NOT Calpurnia*?
- `grep -r ./* ".* brutus .* caesar.*"` then remove all documents containing *calpurnia*
  - Slow when the data size is large
  - Ranked retrieval

## Lecture 4: Motivating example

- Which plays of Shakespeare contain the words *Brutus AND Caesar* but *NOT Calpurnia*? *Brutus AND Caesar AND NOT Calpurnia*

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	1	0	1	1	1	1
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0



# Lecture 4: Motivating example

- Which plays of Shakespeare contain the words *Brutus AND Caesar* but *NOT Calpurnia*? *Brutus AND Caesar AND NOT Calpurnia*
  - 110100 AND
  - 110111 AND
  - 101111 =
  - 100100**

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

# Lecture 4: Motivating example

- Which plays of Shakespeare contain the words **Brutus AND Caesar** but **NOT Calpurnia**?

- Antony and Cleopatra, Act III, Scene ii

*Agrippa* [Aside to DOMITIUS ENOBARBUS]: Why, Enobarbus,  
When Antony found Julius **Caesar** dead,  
He cried almost to roaring; and he wept  
When at Philippi he found **Brutus** slain.

- Hamlet, Act III, Scene ii

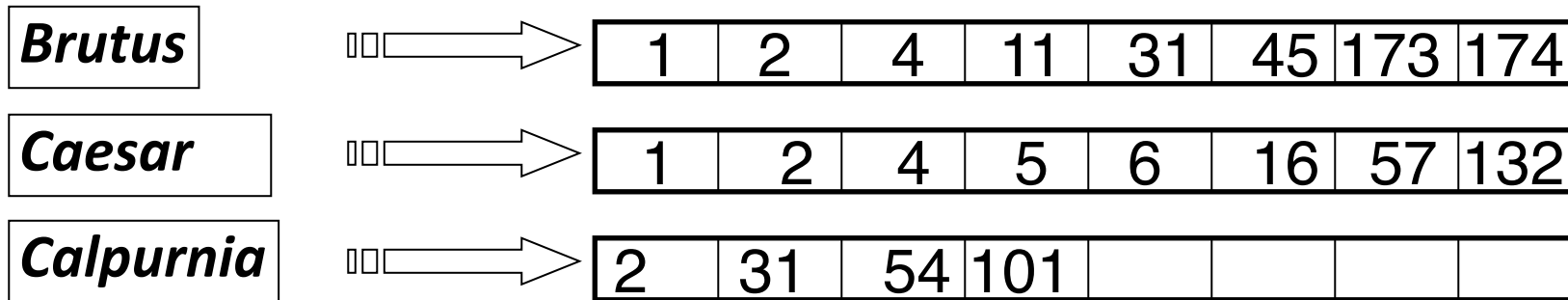
*Lord Polonius*: I did enact Julius **Caesar** I was killed i' the  
Capitol; **Brutus** killed me.

# What if we have a bigger collection?

- Consider  $N = 1$  million documents, each with about 1000 words
  - Avg 6 bytes/word including spaces/punctuation, 6GB
- Say there are  $M = 500K$  *distinct* terms among these.
- 500K x 1M matrix has half-a-trillion 0's and 1's
  - But it has no more than one billion 1's (**why?**)
- A better representation: only record the 1's

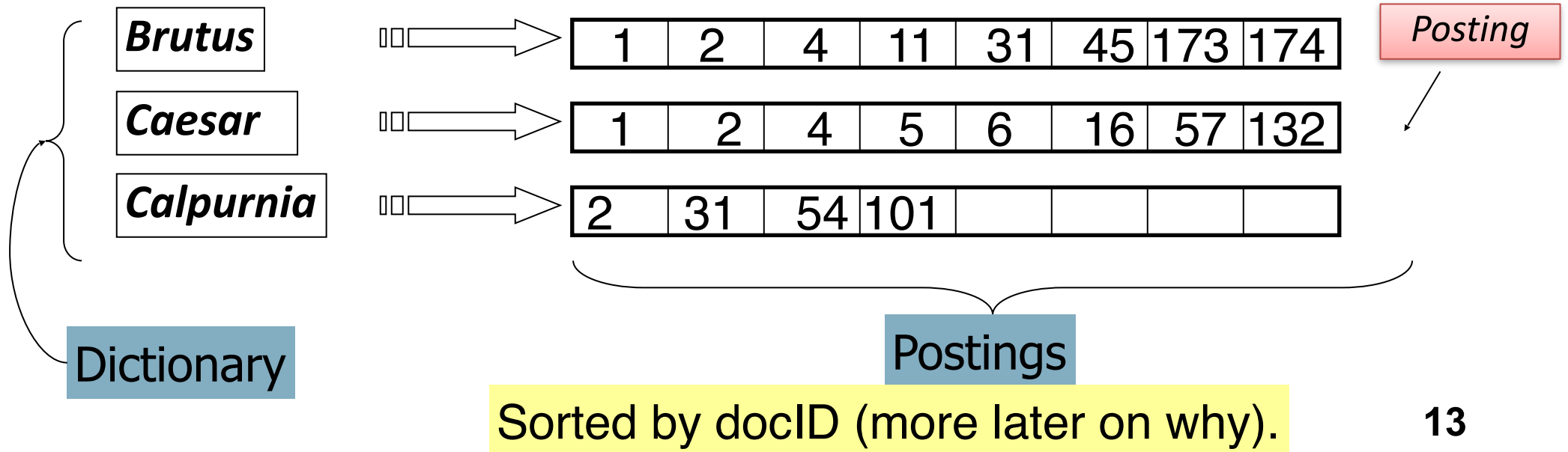
# Inverted index

- For each term  $t$ , we must store a list of all documents that contain  $t$ .
  - Identify each doc by a **docID**, a document serial number
- Can we use **fixed-size arrays** for this?

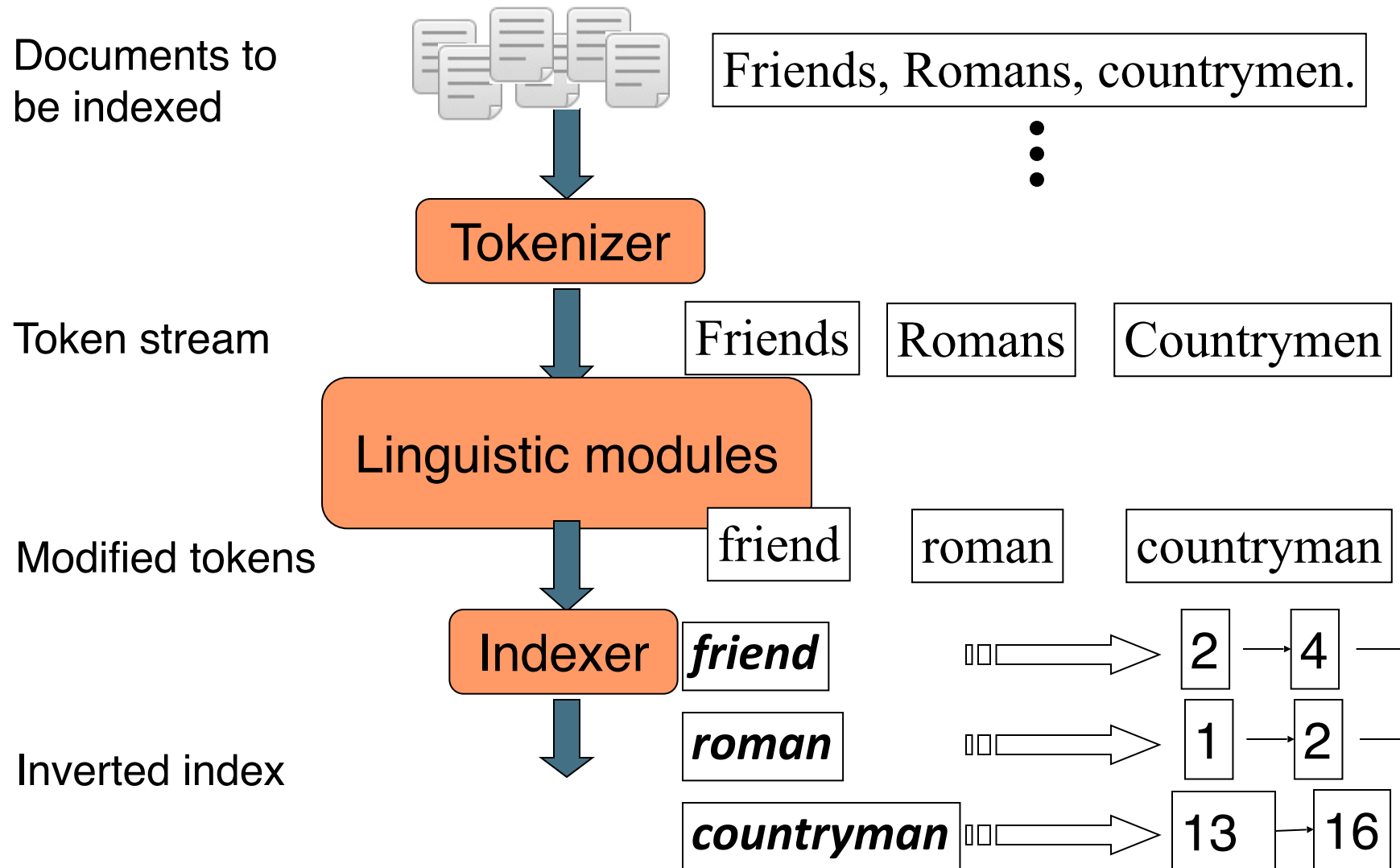


# Inverted index

- We need variable-size **postings lists**
  - On disk, a continuous run of postings is normal and best
  - In memory, can use linked lists or variable length arrays
    - Some tradeoffs in size/ease of insertion (**what's the insertion complexity?**)



# Building inverted index



# Text preprocessing for building inverted index

- Tokenization
  - Cut character sequence into word tokens
    - Deal with “***John’s***”, ***a state-of-the-art solution***
- Normalization
  - Map text and query term to same form
    - You want ***U.S.A.*** and ***USA*** to match
- Stemming
  - We may wish different forms of a root to match
    - ***authorize, authorization***
- Stop words
  - We may omit very common words (or not)
    - ***the, a, to, of***

# Indexer step: Tokenize sequence

- Sequence of (Modified token, Document ID) pairs.

Doc 1

I did enact Julius  
Caesar I was killed  
i' the Capitol;  
Brutus killed me.

Doc 2

So let it be with  
Caesar. The noble  
Brutus hath told you  
Caesar was ambitious



Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2



# Indexer step: Sorting index terms

- First sort by term then by docID

Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2



Term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2

# Indexer step: Dictionary and postings

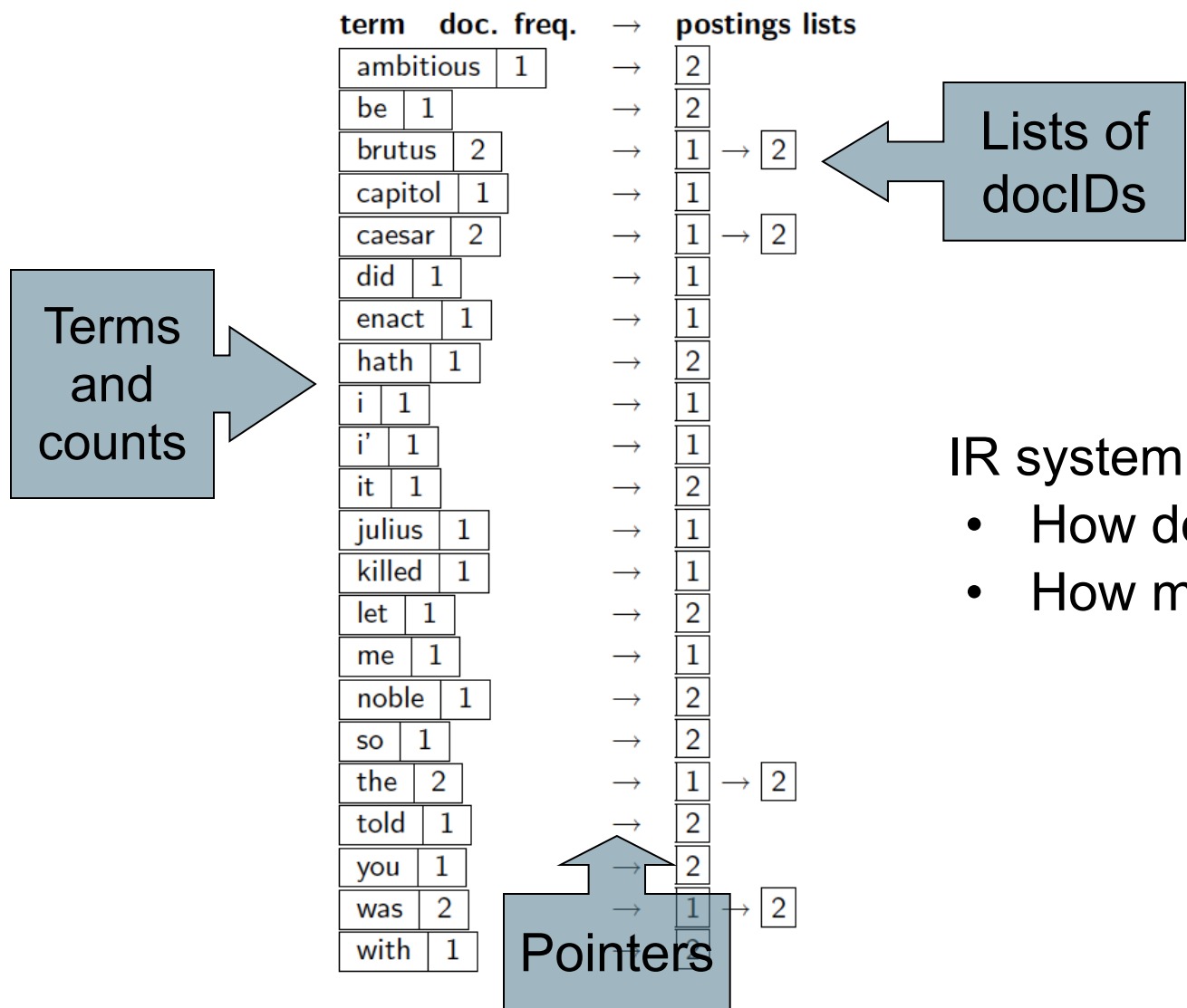
- Multiple term entries in a single document are merged
- Split into Dictionary and Postings
- Doc. frequency information is added.

Term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2



term	doc. freq.	→	postings lists
ambitious	1	→	2
be	1	→	2
brutus	2	→	1 → 2
capitol	1	→	1
caesar	2	→	1 → 2
did	1	→	1
enact	1	→	1
hath	1	→	2
i	1	→	1
i'	1	→	1
it	1	→	2
julius	1	→	1
killed	1	→	1
let	1	→	2
me	1	→	1
noble	1	→	2
so	1	→	2
the	2	→	1 → 2
told	1	→	2
you	1	→	2
was	2	→	1 → 2
with	1	→	2

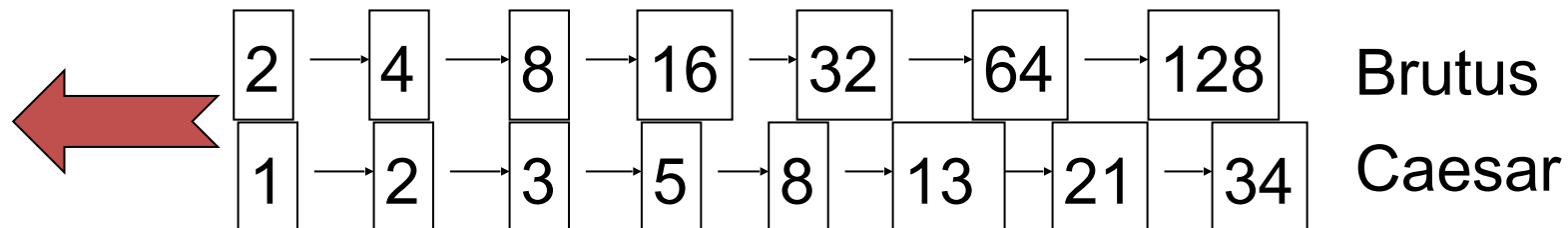
# Where do we pay in storage?



- IR system implementation
- How do we index efficiently?
  - How much storage do we need?

# Query processing for inverted index

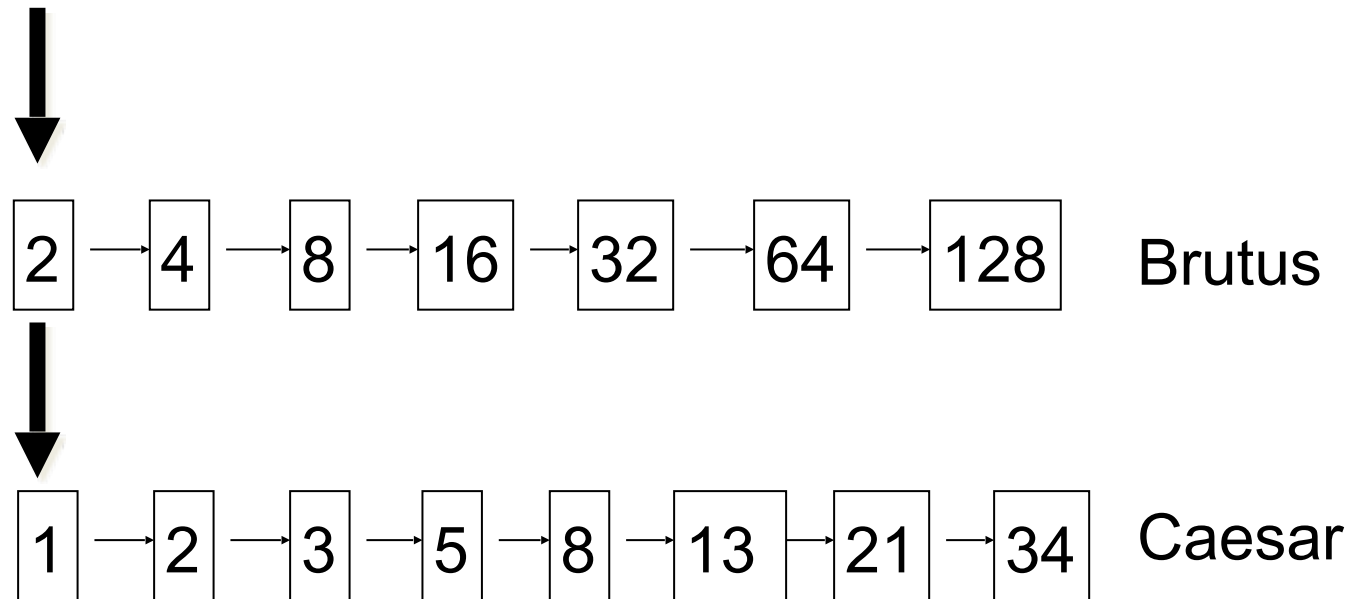
- Suppose we have constructed the inverted index, what query can we answer?
- Consider processing the query: **Brutus** AND **Caesar**
  - Locate **Brutus** in the Dictionary;
    - Retrieve its postings.
  - Locate **Caesar** in the Dictionary;
    - Retrieve its postings.
  - “Merge” the two postings (**intersect** the document sets):



# Merging two posting lists

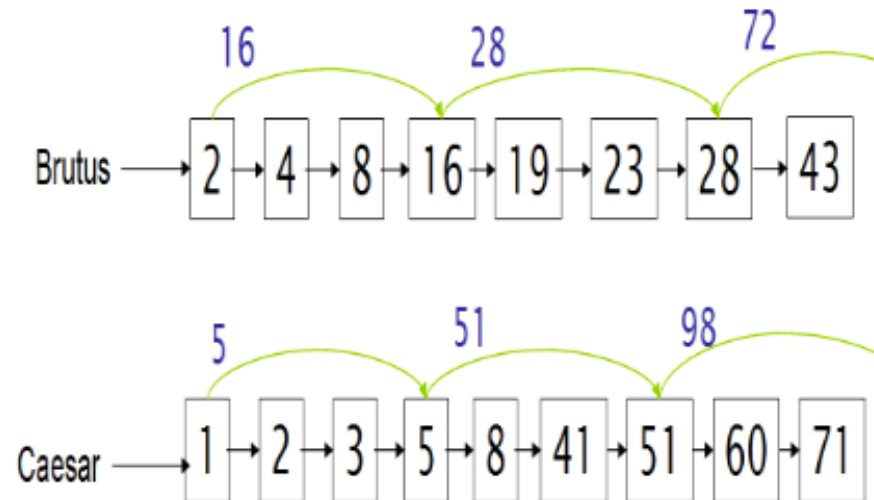
- Walk through the two postings simultaneously, in time linear in the total number of postings entries (using two pointers, without skipping  $O(x + y)$ )

```
INTERSECT( $p_1, p_2$ )  
1   $answer \leftarrow \langle \rangle$   
2  while  $p_1 \neq \text{NIL}$  and  $p_2 \neq \text{NIL}$   
3  do if  $docID(p_1) = docID(p_2)$   
4      then  $ADD(answer, docID(p_1))$   
5           $p_1 \leftarrow next(p_1)$   
6           $p_2 \leftarrow next(p_2)$   
7  else if  $docID(p_1) < docID(p_2)$   
8      then  $p_1 \leftarrow next(p_1)$   
9      else  $p_2 \leftarrow next(p_2)$   
10 return  $answer$ 
```



# Merging two posting lists: skipping lists

- Speeding up the merge by skipping every k pointers



# Inverted index for the Boolean Retrieval System

- The **Boolean retrieval model** is being able to ask a query that is a Boolean expression:
  - Boolean Queries are queries using *AND*, *OR* and *NOT* to join query terms
    - Views each document as a set of words
    - Is precise: document matches condition or not.
  - Perhaps the simplest model to build an IR system on
- Primary commercial retrieval tool for 3 decades.
- Many search systems you still use are Boolean:
  - Email, library catalog, macOS Spotlight

# Boolean queries: more general merges

- Exercise: Adapt the merge for the queries:  
*Brutus AND NOT Caesar*  
*Brutus OR NOT Caesar*
- Can we still run through the merge in time  $O(x+y)$ ? What can we achieve?

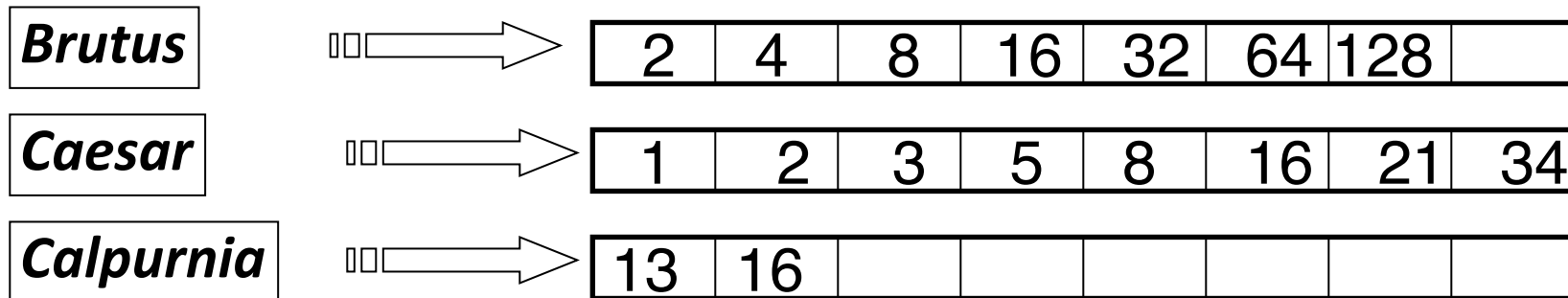


# What about arbitrary Boolean formula?

*(Brutus OR Caesar) AND NOT*

*(Antony OR Cleopatra)*

- Can we always merge in “linear” time?
- Consider a query that is an *AND* of  $n$  terms, what is the best way of processing?



Query: *Brutus AND Calpurnia AND Caesar*

# What about arbitrary Boolean formula?

- Process in order of increasing freq:
  - *start with smallest set, then keep cutting further.*

This is why we kept  
document freq. in dictionary

<b>Brutus</b>	→	2	4	8	16	32	64	128	
<b>Caesar</b>	→	1	2	3	5	8	16	21	34
<b>Calpurnia</b>	→	13	16						

Query: (**Calpurnia** AND **Brutus**) AND **Caesar**

# Query processing exercise

- Recommend a query processing order for

*(tangerine OR trees) AND  
(marmalade OR skies) AND  
(kaleidoscope OR eyes)*

Term	Freq
eyes	213312
kaleidoscope	87009
marmalade	107913
skies	271658
tangerine	46653
trees	316812

- Which terms should we process first?

# Phrase indexing

- We want to be able to answer queries such as “***Stevens Institute of Technology***” – as a phrase
- Sentence 1= “I went to ***Stevens Institute of Technology***”
- Sentence 2= “The ***Technology Institute*** that ***Steve*** Harvey went to. ”
- For this, it no longer suffices to store only
- *<term : docs>* entries

# A first attempt: bi-gram indexing

- Index every consecutive pair of terms in the text as a phrase
- For example the text “Friends, Romans, Countrymen” would generate the biwords
  - *friends romans*
  - *romans countrymen*
- Each of these biwords is now a dictionary term
- Two-word phrase query-processing is now immediate.

# Issues with bi-gram indexing

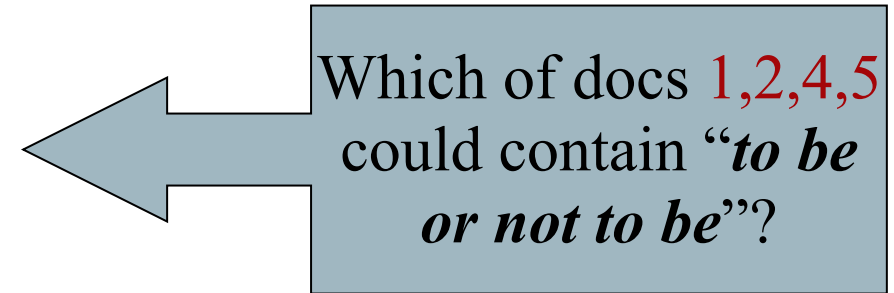
- False positives, as noted before
- Index blowup due to bigger dictionary
  - Infeasible for more than bigrams, big even for them
- Bigram indexes are not the standard solution (for all bigrams) but can be part of a compound strategy

## Solution 2: Positional indexing

- In the postings, store, for each ***term*** the position(s) in which tokens of it appear:
  - <***term***, number of docs containing ***term***;
  - *doc1*: position1, position2 ... ;
  - *doc2*: position1, position2 ... ;
  - etc.>

## Solution 2: Positional indexing

<*be*: 993427;  
*1*: 7, 18, 33, 72, 86, 231;  
*2*: 3, 149;  
*4*: 17, 191, 291, 430, 434;  
*5*: 363, 367, ...>



- For phrase queries, we use a merge algorithm recursively at the document level
- But we now need to deal with more than just equality



# Proximity search

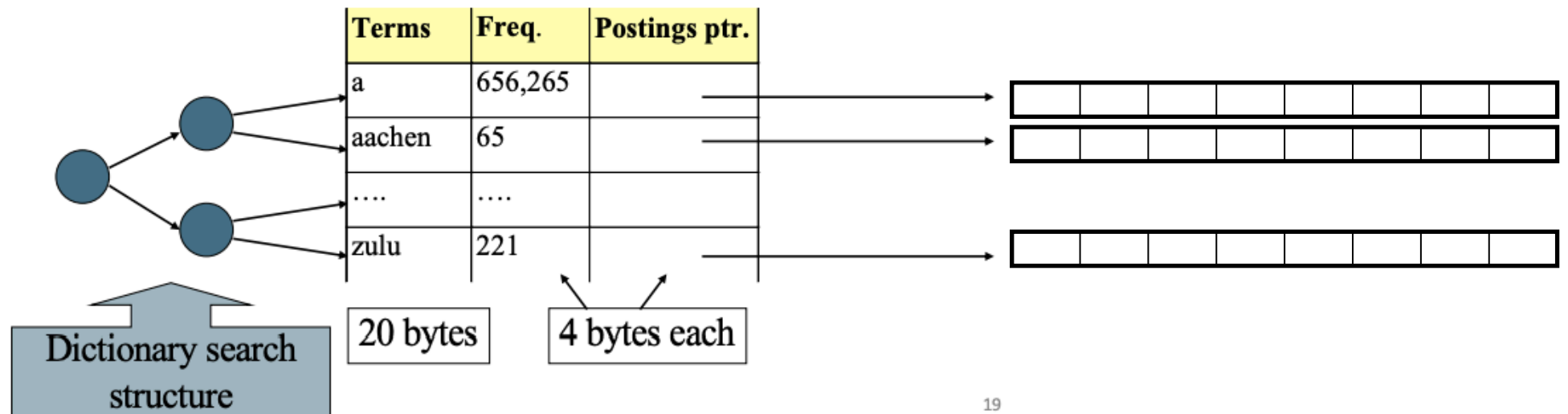
- Extract inverted index entries for each distinct term: ***to, be, or, not.***
- Merge their *doc:position* lists to enumerate all positions with “***to be or not to be***”.
  - ***to:***
    - 2:1,17,74,222,551; 4:8,16,190,429,433; 7:13,23,191; ...
  - ***be:***
    - 1:17,19; 4:17,191,291,430,434; 5:14,19,101; ...
- LIMIT! /3 to /3 be /2 or /2 not
  - Here, /*k* means “within *k* words of”.

# Positional indexing size

- A positional index expands postings storage *substantially*
  - Even though indices can be compressed
- Nevertheless, a positional index is now standardly used because of the power and usefulness of phrase and proximity queries ... whether used explicitly or implicitly in a ranking retrieval system
- A positional index is 2–4 as large as a non-positional index
- Positional index size 35–50% of volume of original text

# Index compression (for non-positional indexing)

- Compressing the posting pointer table
- Compressing the posting lists
- Speeding up the dictionary search with trie



19

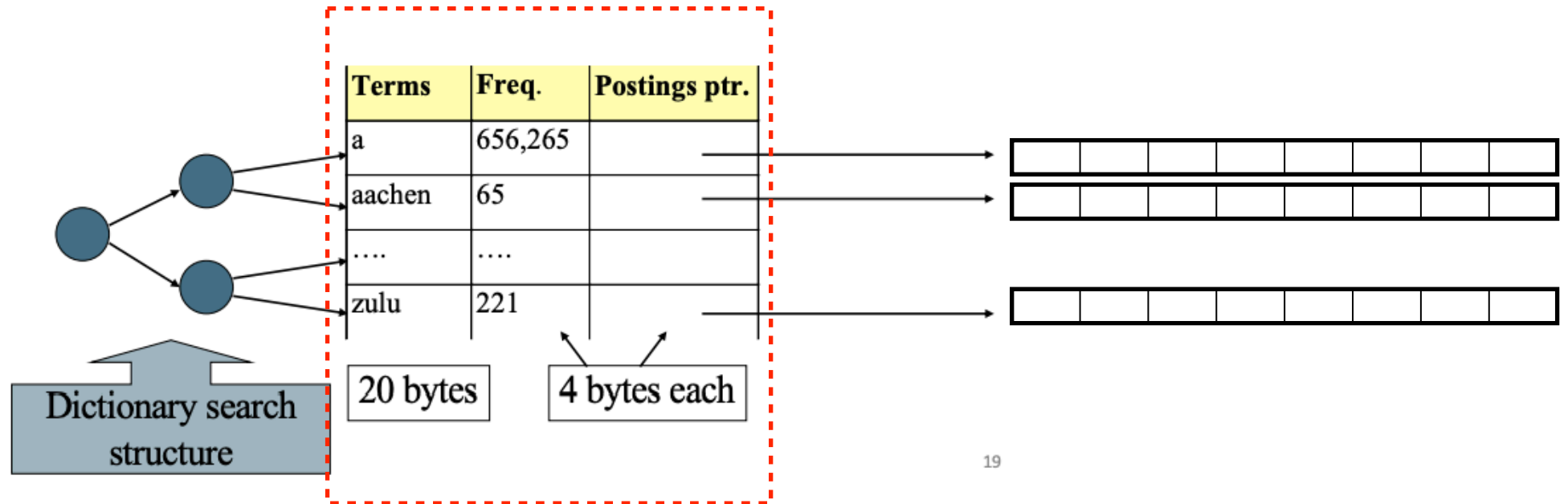
Speeding up the dictionary search

Compressing the posting ptr table

Compressing the posting list **35**

# Compressing the postings pointer table

- Most of the space in the table is wasted
  - Most words < 20 bytes
  - Table storage =  $28N$

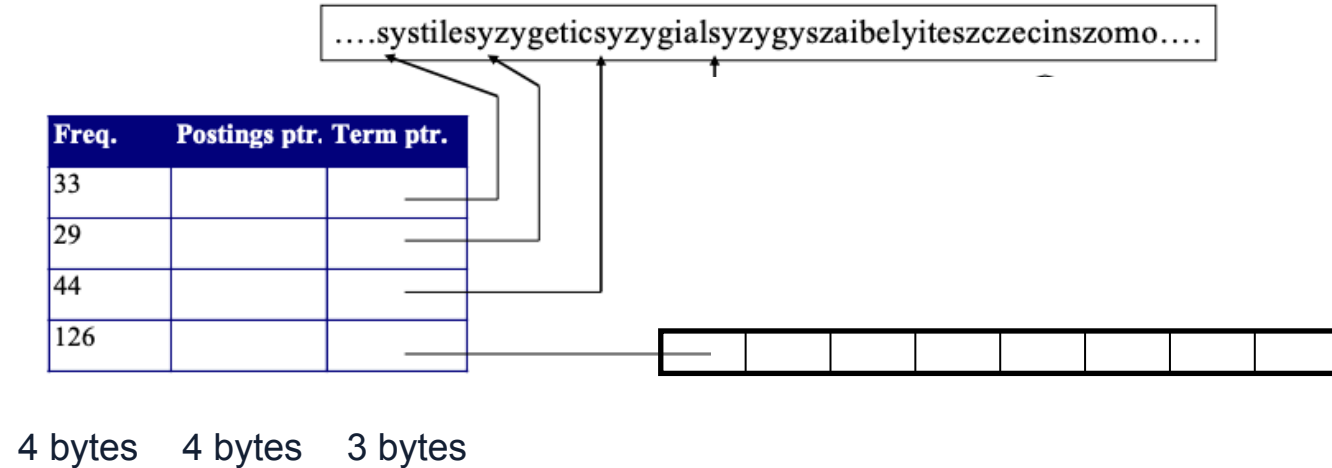


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Compressing the posting ptr table

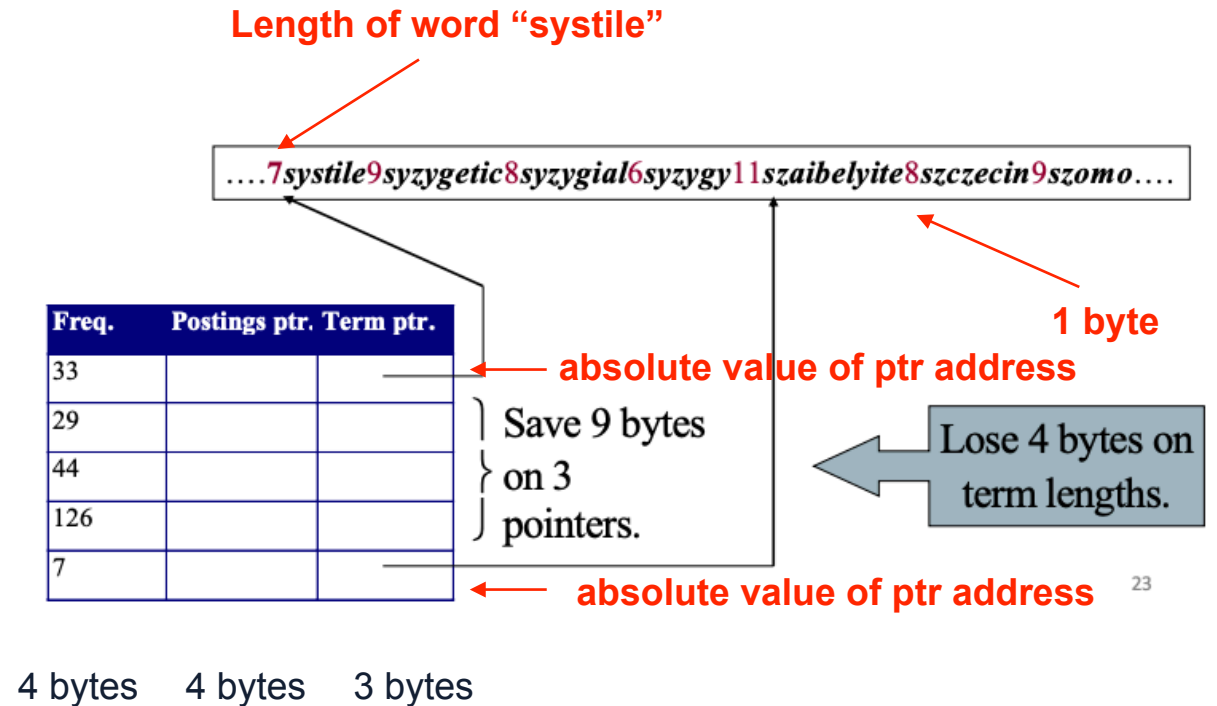
# Compressing the postings pointer table

- Concatenate the dictionary as one string
  - Table storage  $28N \rightarrow 11N$
- How to further improve the storage space?
  - Instead of storing absolute term pointers, store the gaps



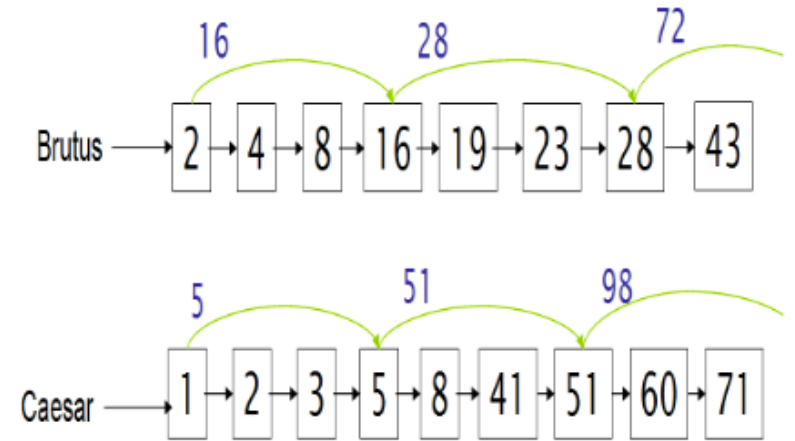
# Compressing the postings pointer table

- Save more space by skipping (k-1) pointers for every k pointers
  - Recover the skipped pointers by adding the **length of words**
- Table storage is further reduced to  $8N + 3N * (3+k/3*k)$ . When  $k=4$ , the storage required is  $9.75N < 11N$
- Trade-off between saving space (skipping more) vs. saving time (skipping less)



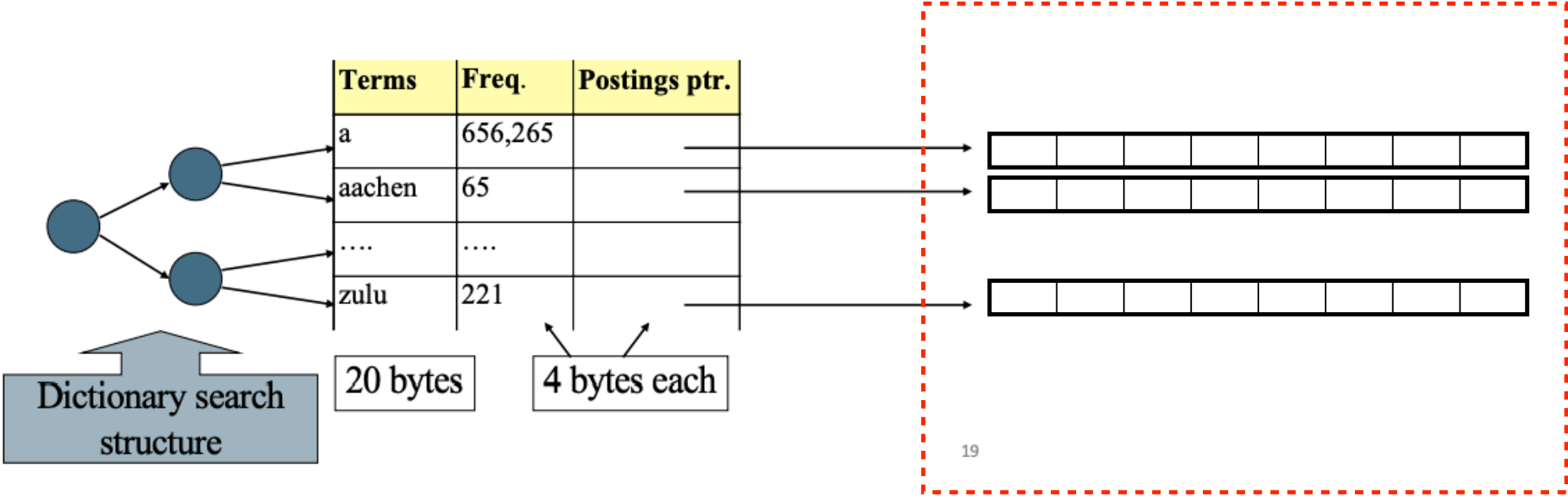
# Compressing the posting lists

- Observations of posting files
  - Instead of storing docID, store gaps
  - Brutus: 2,4,8,3,4,5,15
  - Binary seq: 10,100,1000,11,100,101,1111



- Prefix encoding
  - Binary encoding such that the sequence can be uniquely decoded (**why do we need this uniqueness?**)
  - e.g., Huffman encoding
  - Unary encoding: {2:110,4:11110, ...}
  - A uniquely decodable seq: 110111101111111101110...

# Compressing the posting lists



Compressing the posting list

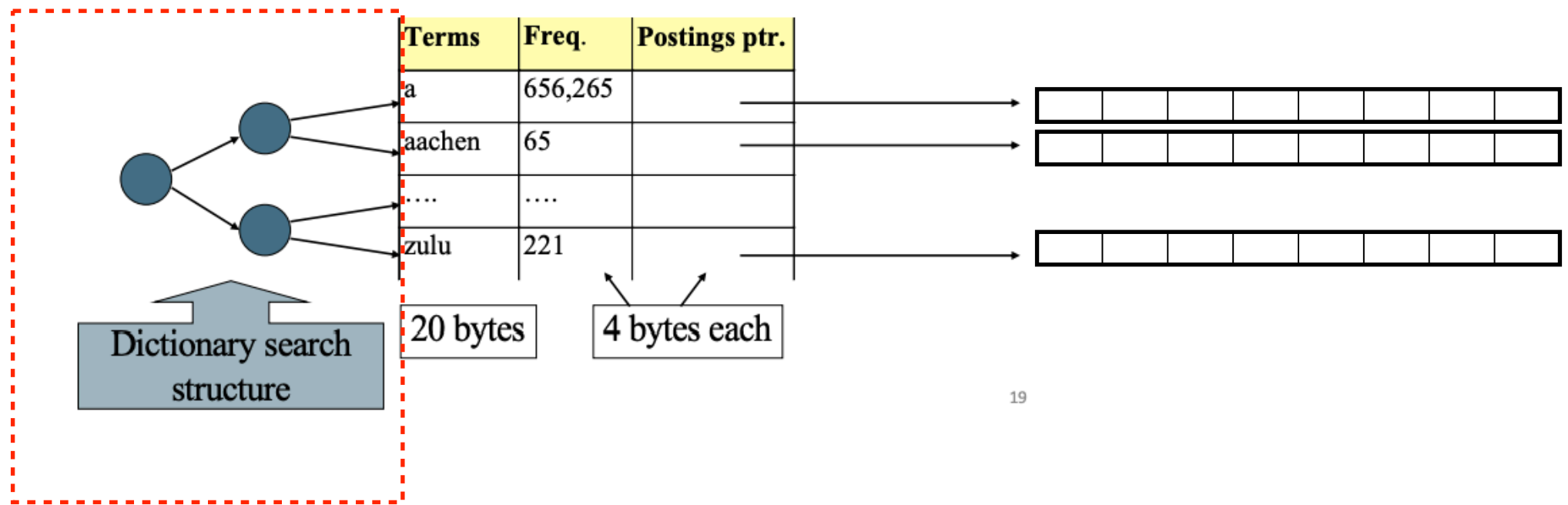


# Compressing the posting lists

number	length	offset	$\gamma$ -code
0			none
1	0		0
2	10	0	10,0
3	10	1	10,1
4	110	00	110,00
9	1110	001	1110,001
13	1110	101	1110,101
24	11110	1000	11110,1000
511	111111110	11111111	111111110,11111111
1025	1111111110	0000000001	1111111110,0000000001

- Unary encoding is too long!
- **Gamma code** of 13: 1110,101
  - Binary code for  $\{\text{length} - 1\}$  followed by 0: 1110
  - Offset (last  $\{\text{length} - 1\}$  bits of the binary value):  $13 = 1101 \rightarrow 101$
- What is the gamma code of 5?  $101 \rightarrow 110,01$
- We can prove gamma code is uniquely decodable!
- Gamma code compression rate: 11.7%

# Speeding up the dictionary search

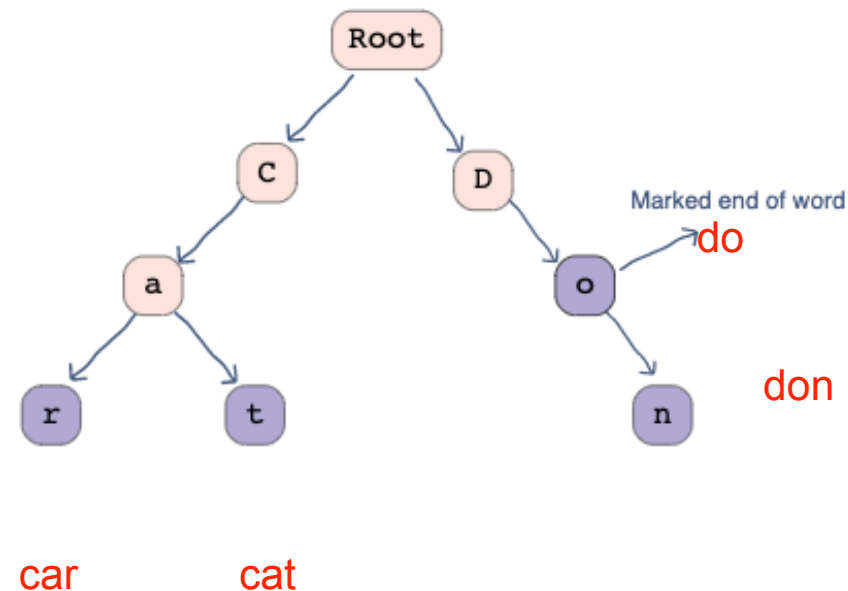
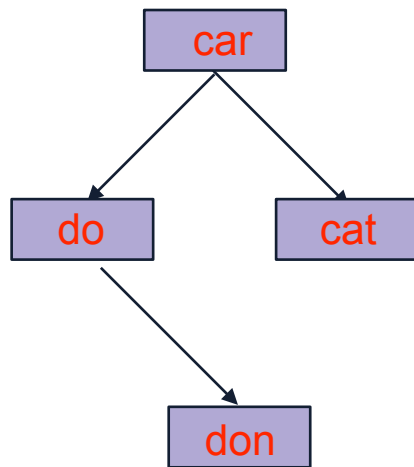


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Speeding up the dictionary search

# Speeding up the dictionary search with prefix tree

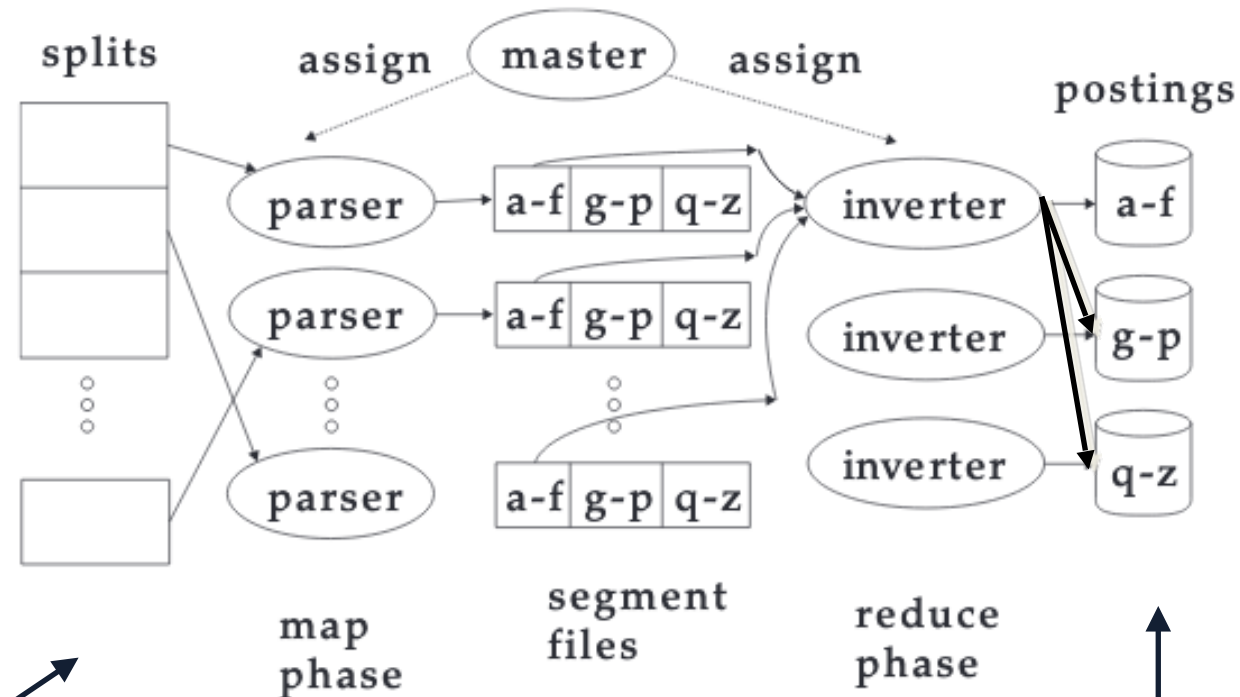
- Time complexity for searching/insertion:
  - BST:  $O(m * \log n)$ ,  $m$  is the maximum word length,  $n$  is the number of words in the vocabulary
  - Prefix-tree:  $O(m)$ ,  $m$  is the maximum word length



# Handling web scale indexing

- Web-scale indexing must use clusters of servers
  - Google had 1 million servers in 2011
- Fault tolerance of a massive data center
  - If a non-fault tolerance system has 1000 nodes, each has 99.9% uptime, then 63% of the time one or more servers is down
- Solution
  - Maintain a “master” server
  - Break indexing into parallel tasks
  - Assign each task to an idle machine

# Map-reduce



master assigns split  
to idle machine

parser emits  
(term,doc) pair

merge partitions  
in inverter

complete the  
index

# Examples of map-reduce

map:  $d_2$  : C died.  $d_1$  : C came, C c'ed.



$(\langle C, d_2 \rangle, \langle \text{died}, d_2 \rangle, \langle C, d_1 \rangle, \langle \text{came}, d_1 \rangle, \langle C, d_1 \rangle, \langle \text{c'ed}, d_1 \rangle)$

reduce:  $(\langle C, (d_2, d_1, d_1) \rangle, \langle \text{died}, (d_2) \rangle, \langle \text{came}, (d_1) \rangle, \langle \text{c'ed}, (d_1) \rangle)$



$(\langle C, (d_1:2, d_2:1) \rangle, \langle \text{died}, (d_2:1) \rangle, \langle \text{came}, (d_1:1) \rangle, \langle \text{c'ed}, (d_1:1) \rangle)$

# MapReduce: Industry practice

- Term partition vs. document partition
  - Term-partitioned: one machine handles a subrange of terms
  - Document-partitioned: one machine handles a subrange of documents
- Most industry search engine use document-partitioned index
  - Better load balancing (**why?**)

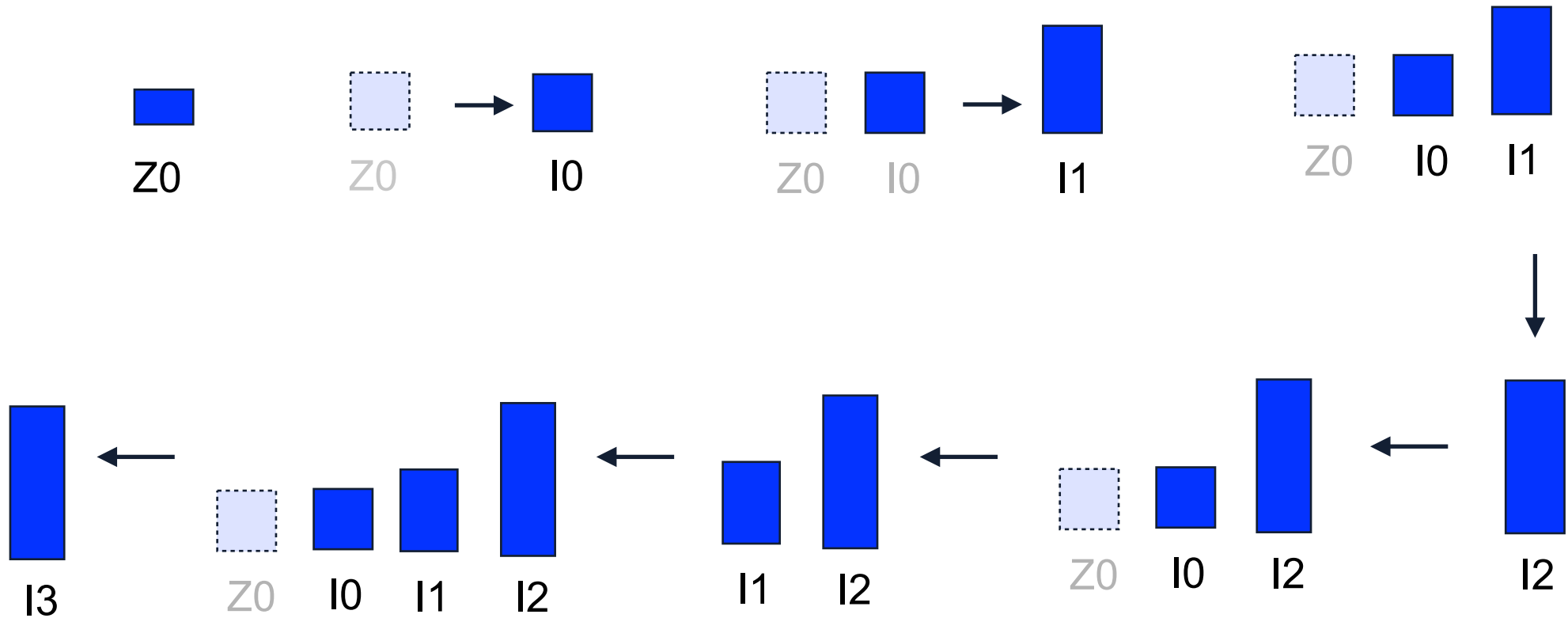
## MapReduce: Simplified Data Processing on Large Clusters

Jeffrey Dean and Sanjay Ghemawat

jeff@google.com, sanjay@google.com

*Google, Inc.*

# Logarithmic dynamic indexing
















# Real time search of Twitter

- Requires high real time search
  - Low latency, high throughput query evaluation
  - High ingestion rate and immediate data availability
  - Concurrent reads and writes of the index
- Solution: using segments
  - Each segment consists of  $2^{32}$  tweets (in memory)
  - New posts are appended to the posting lists
  - Only one segment can be written to at each time

# Search engine tools

- Apache Lucene
  - Free and open search engine library
  - First developed in 1999
- ElasticSearch
  - A search engine
  - based on Lucene

 **Elasticsearch vs Lucene**  
Which companies use these tools?

 Elasticsearch	 Uber	 9GAG
	 Asana	 Codecademy
<hr/>		
 Lucene	 Twitter	 Evernote
	 Slack	 Kifi
<hr/>		

# ElasticSearch

- Using a REST api

Dev Tools

Console

```
1 POST bibliography/novels/_bulk
2 {"create": {"_id": "1"}}
3 {"author": "Johann Wolfgang von Goethe", "title": "Die
4 Leiden des jungen Werther", "year": "1774"}
5 {"create": {"_id": "2"}}
6 {"author": "Umberto Eco", "title": "Il nome della rosa",
7 "year": "1980"}
8 {"create": {"_id": "3"}}
9 {"author": "Margaret Atwood", "title": "The Handmaid's Tale",
10 "year": "1985"}
```

Dev Tools

Console

```
1 GET /integrity/body/870595443049000/_termvectors
2 ?pretty=true
3 {
4   "fields": ["_all"]
5 }
```

# Homework 2: Using Elasticsearch to build a search engine

- Build an inverted index
- Evaluate three search algorithm's performance
  - TF-IDF
  - BM25
  - Dirichlet-LM

## Search Results

[Advanced Search Tips](#)[Ask Question](#)

Results for how to sort dictionary by value

[Search](#)

500 results

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3420  
votes

34  
answers

### Q: How do I sort a dictionary by value?

**how** can I **sort** based on the values? Note: I have read Stack Overflow question here **How** do I **sort** a list of dictionaries **by** a **value** of the **dictionary**? and probably could change my code **to** have a list of ... I have a **dictionary** of values read from two fields in a database: a string field and a numeric field. The string field is unique, so that is the key of the **dictionary**. I can **sort** on the keys, but ...

[python](#)[sorting](#)[dictionary](#)

asked Mar 5 '09 by [Gern Blanton](#)