Data Structures

Stacks

CS284

Structure of this week's classes

Stacks

Applications

Implementing a Stack

Stack

- A stack is one of the most commonly used data structures in computer science
- A stack can be compared to a Pez dispenser
  - Only the top item can be accessed
  - You can extract only one item at a time
- The top element in the stack is the one added to the stack most recently
- The stack's storage policy is Last-In, First-Out, or LIFO

Operations on Stacks

- Only the top element of a stack is visible; therefore the number of operations performed by a stack are few
- We need the ability to
  - test for an empty stack (`boolean empty()`)
  - inspect the top element (`E peek()`)
  - retrieve the top element (`E pop()`)
  - put a new element on the stack (`E push(E obj)`)

`StackInt<E>` interface

```java
public interface StackInt<E> {

    E push(E obj);

    E peek();

    E pop();

    boolean empty();
}
```

An example: a stack of strings`java.util.List` interface

(a) (b) (c)

- "Rich" is oldest element on stack; "Jonathan" is youngest (Figure a)
- `String last = names.peek();`
  stores a reference to "Jonathan" in last
- `String temp = names.pop();`
  removes "Jonathan" and stores a reference to it in temp (Figure b)
- `names.push("Philip");`
  pushes "Philip" onto the stack (Figure c)

Stacks

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Implementing a Stack

Detecting Palindromes

- Palindrome: a string that reads identically in either direction, letter by letter
  - "kayak"
  - "I saw I was I"
  - "Able was I ere I saw Elba"
  - "Level, madam, level"
- We ignore case and whitespace
- Problem: Write a program that reads a string and determines whether it is a palindrome

Detecting Palindrome

- How should we tackle this problem?
- The solution depends on the data structure of that represents the palindrome!

If the Palindrome is a String/Array

- We can fetch the character based on the index;
- Use the two pointers approach:
  - If the char at the left pointer != that at the right pointer, return false
  - Move the left/right pointer;
- e.g., kayak

What if No Index?

- For example, detecting palindrome in a single linked list

$$k \rightarrow a \rightarrow y \rightarrow a \rightarrow k$$

- In a single-linked list, can only traverse in one direction
- Stack can help "reverse" a linked list:

```
public boolean isPalindrome(){
    Node node = this.head;
    Stack s = new Stack();
    while(node!= null){
        s.push(node.data);
        node = node.next;
    }
    node = this.head;
    while(node!= null){
        char this_char = s.pop();
        if(node.data != this_char)
            return false;
        node = node.next;
    }
    return true;
}
```

Balanced Parenthesis

- When analyzing arithmetic expressions, it is important to determine whether an expression is balanced with respect to parentheses

$$(a + b * (c/(d - e))) + (d/e)$$

- The problem is further complicated if braces or brackets are used in conjunction with parentheses
- The solution is to use stacks!

Code

```
public boolean isBalanced(String expression) {
   Stack<Integer> s = new Stack<Integer>();
   int index = 0;
   while (index < expression.length()) {
         char nextCh = expression.charAt(index);
         if (nextCh == '(') {
               s.push(1);
         }
         else if (nextCh == ')'){
               try
               {
                     s.pop();
               }
               catch(Exception e) {
                     return false;
               }
         }
         index++;
   }
   return s.empty();
}
```

Why is `isBalanced` Correct?

Context-free grammar for balanced-parenthesis:

- E$\to$ E E;
- E$\to$ (E);
- E$\to \oslash$;

Assertion 1. If the parenthesis is balanced, `isBalanced` must return true:

- `isBalanced` returns false only if it pops more times than the pushed times;
- We can prove using the CFG that $\#$ pushes is $\geq \#$ pops;

Why is `isBalanced` Correct?

Context-free grammar for balanced-parenthesis:

- E$\rightarrow$ E E;
- E$\rightarrow$ (E);
- E$\rightarrow$ $\oslash$;

Assertion 2. If `isBalanced` returns true, the parenthesis must be balanced;

A At each index i, count((, i) $\geq$ count( ) , i);

B At the last index, count((, $n-1$) $=$ count( ) , $n-1$);

- We call a sequence that satisfy both A and B a mountain sequence;

Why is `isBalanced` Correct?

Context-free grammar for balanced-parenthesis:

- E$\rightarrow$ E E;
- E$\rightarrow$ (E);
- E$\rightarrow \oslash$;

Theorem 2. Any mountain sequence must be a balanced sequence of parenthesis (i.e., they can be generated using the CFG grammar above).

Proof. If a sequence is a mountain sequence of length $n \geq 2$, the first and last parenthesis must be `(` and `)`.

- If the first parenthesis is `)`, it violates A;
- If the last parenthesis is `(`, it violates B (why?);

Why is `isBalanced` Correct?

Context-free grammar for balanced-parenthesis:

- E$\rightarrow$ E E; (rule 1)
- E$\rightarrow$ (E); (rule 2)
- E$\rightarrow \oslash$;

Theorem 2. Any mountain sequence must be a balanced sequence of parenthesis (i.e., they can be generated using the CFG grammar above).

Proof (cont.). By math induction, suppose the theorem is true for all sequences of length $n-1$ or shorter.

- For any sequence s of length n, consider the subsequence $s_{1:n-1}$ from the 1st to the $n-2$-th char;
- $s_{1:n-1}$ is either a mountain sequence or not a mountain sequence.
  - If it is a mountain sequence, we can generate s using rule 2;

Why is `isBalanced` Correct?

Context-free grammar for balanced-parenthesis:

- E$\rightarrow$ E E; (rule 1)
- E$\rightarrow$ (E); (rule 2)

Theorem 2. Any mountain sequence must be a balanced sequence of parenthesis (i.e., they can be generated using the CFG grammar above).

Proof (cont.). If $s_{1:n-1}$ is not a mountain sequence, consider count(`(`, j) and count(`)` , j) at the 2nd position of parenthesis;

- At the j-th position of $s_{1:n-1}$, $count(\texttt{(}, j) < count(\texttt{)}, j)$;
- At the j-th position of s, $count(\texttt{(}, j) + 1 \geq count(\texttt{)}, j)$;

$\Rightarrow count(\texttt{(}, j) = count(\texttt{)}, j) - 1 \Rightarrow count(\texttt{(}, j) = 0, count(\texttt{)}, j) = 1 \Rightarrow$ the second parenthesis is a `)`, therefore, the first two parentheses are `()`, by math induction, s can be generated using rule 1. $\square$

Stacks

Applications

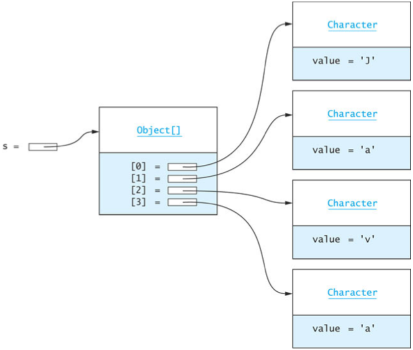
Implementing a Stack

A Stack as an Extension of Vector

- Java API includes Stack class as part of package `java.util`:

```
public class Stack<E> extends Vector<E>
```

- Elements of a Vector can be accessed using an integer index and the size can grow or shrink as needed to accommodate the insertion and removal of elements



Implementing a Stack as an Extension of Vector (cont.)

We can use Vector's add method to implement push:

```
public E push(obj E) {
  add(obj);
  return obj;
}
```

`pop` can be coded as

```
public E pop throws EmptyStackException {
  try {
    return remove(size() - 1);
  } catch (ArrayIndexOutOfBoundsException ex) {
    throw new EmptyStackException();
  }
}
```

Disadvantage of Stack as Subclass of Vector

- Because a Stack is a Vector, all of Vector operations can be applied to a Stack (such as searches and access by index)
- But, since only the top element of a stack should be accessible, this violates the principle of information hiding

```
Stack<Integer> s = new Stack<Integer>();
s.push(3);
s.push(4);
System.out.println(s.elementAt(0)); // prints 3
```

Implementing a Stack with a List Component

- Alternative: write a class, `ListStack`, that has a `List` component (in the example below, `theData`)

```
public class ListStack<E> implements StackInt<E>{
    private List<E> theData;
    public ListStack() {
        theData = new ArrayList<E>();
    }
...
}
```

Implementing a Stack with a List Component

- We can use `ArrayList`, `Vector`, or the `LinkedList` classes, as all implement the List interface.
- The `push` method, for example, can be coded as

```
public E push(E obj) {
  theData.add(obj);
  return obj;
}
```

- A class which adapts methods of another class by giving different names to essentially the same methods (push instead of add) is called an adapter class
- Writing methods in this way is called method delegation

Implementing a Stack Using an Array

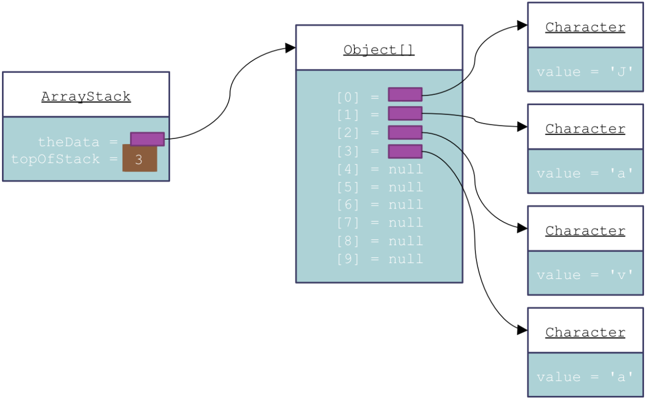
If we implement a stack as an array, we would need...

```
public class ArrayStack<E> implements StackInt<E> {
  private E[] theData;
  int topOfStack = -1;
  private static final int INITIAL_CAPACITY = 10;

  @SuppressWarnings("unchecked")
  public ArrayStack() {
    theData = (E[])new Object[INITIAL_CAPACITY];
  }
```

- Note: The `SuppressWarnings` annotation is just to avoid the compiler from warning: “Type safety: Unchecked cast from `Object[]` to `E[]`”

Implementing a Stack Using an Array

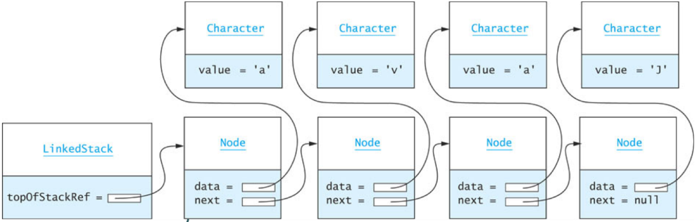



```java
public E push(E obj) {
  if (topOfStack == theData.length-1) { reallocate(); }
  topOfStack++;
  theData[topOfStack] = obj;
  return obj;
}
```

Implementing a Stack Using an Array

```java
public E pop() {
  if (empty()) {
    throw new EmptyStackException();
  }
  return theData[topOfStack--];
}
```

Implementing a Stack as a Linked List



when the list is empty, pop returns null

Implementing a Stack as a Linked List

```
import java.util.EmptyStackException;

public class LinkedStack<E> implements StackInt<E> {

  private static class Node<E> {
    private E data;
    private Node next;
```

Implementing a Stack as a Linked List

```
// Constructors
  private Node(E dataItem) {
    data = dataItem;
    next = null;
  }

  private Node(E dataItem, Node<E> nodeRef) {
    data = dataItem;
    next = nodeRef;
  }
} //end class Node
```

Implementing a Stack as a Linked List

```java
// Data Fields
  /** The reference to the first stack node. */
  private Node<E> topOfStackRef = null;

  /** Insert a new item on top of the stack.
      post: The new item is the top item on the stack.
            All other items are one position lower.
      @param obj The item to be inserted
      @return The item that was inserted
   */
  public E push(E obj) {
    topOfStackRef = new Node<E>(obj, topOfStackRef);
    return obj;
  }
```

Implementing a Stack as a Linked List

```java
/** Remove and return the top item on the stack.
      pre: The stack is not empty.
      post: The top item on the stack has been
        removed and the stack is one item smaller.
      @return The top item on the stack
      @throws EmptyStackException if stack is empty
   */
  public E pop() {
    if (empty()) {
      throw new EmptyStackException();
    }
    else {
      E result = topOfStackRef.data;
      topOfStackRef = topOfStackRef.next;
      return result;
    }
  }
```

Implementing a Stack as a Linked List

```java
/** Return the top item on the stack.
     pre: The stack is not empty.
     post: The stack remains unchanged.
     @return The top item on the stack
     @throws EmptyStackException if stack is empty
  */
 public E peek() {
   if (empty()) {
     throw new EmptyStackException();
   }
   else {
     return topOfStackRef.data;
   }
 }
```

Implementing a Stack as a Linked List

```java
/** See whether the stack is empty.
     @return true if the stack is empty
  */
 public boolean empty() {
   return topOfStackRef == null;
 }
}
```

Comparison of Implementations

- Extending a `Vector` (as is done by Java) is a poor choice for stack implementation, since all `Vector` methods are accessible
- The easiest implementation uses a `List` component (`ArrayList` is the simplest) for storing data
  - An underlying array requires reallocation of space when the array becomes full, and
  - An underlying linked data structure requires allocating storage for links
  - As all insertions and deletions occur at one end, they are constant time, $\mathcal{O}(1)$, regardless of the type of implementation used

More Applications

- Evaluating postfix expressions
- Converting postfix to infix expressions

Homework 3

- Given a sequence of balanced parentheses, print the trace of CFG generation rules it has used, e.g., $s_1 = ((()()))$, the output should be $s_1 \rightarrow (s_2), s_2 \rightarrow (s_3), s_3 \rightarrow s_4 s_5, s_4 \rightarrow (s_6), s_5 \rightarrow (s_7), s_6 \rightarrow \oslash, s_7 \rightarrow \oslash$

Stack Applications