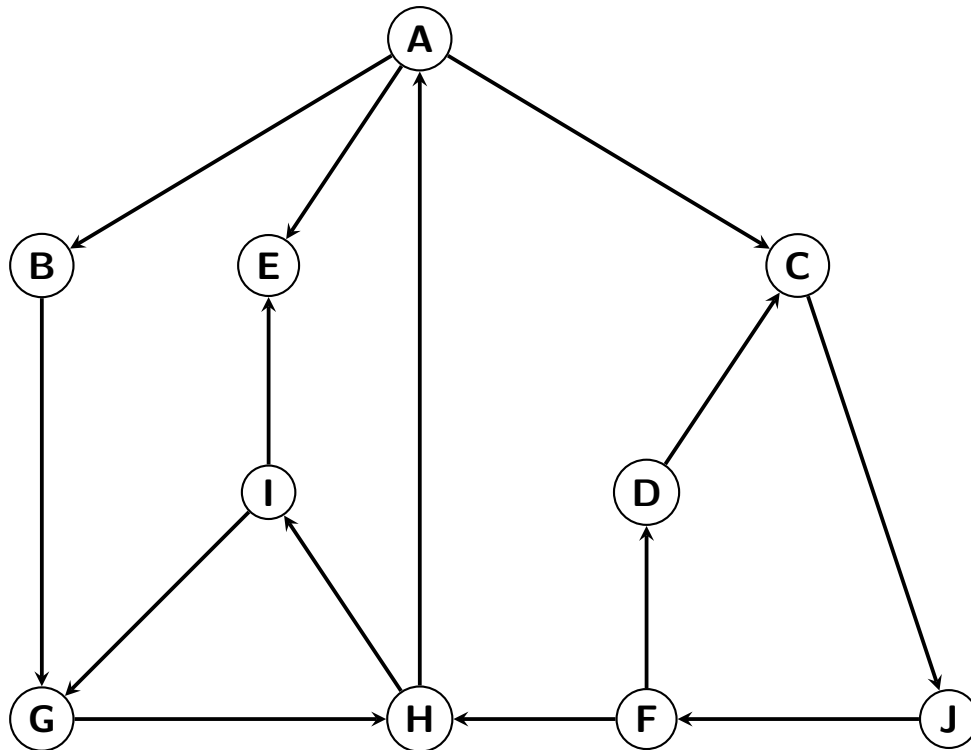


CS 161 Winter 2020 Section 5

February 13-14, 2020

1 Strongly Connected Components

Ten “friends” decided to make a Valentine’s Day graph in which each person is a node, and an edge points from v_i to v_j if Person j has a crush on Person i . Below is their graph:



What are all the strongly connected components of this graph? (i.e., groups of vertices such that there exists a path between any two vertices in the group)

2 Russian Boxes

Through hard work and a small stroke of good luck you have glowed up over the past year and are suddenly inundated with admirers for Valentine’s Day, who have each sent you a box of chocolates (if you are allergic to chocolates, you may assume they sent you strawberries or something else nice that comes in a box). After you finish consuming the spoils of your attractiveness, you are left with n empty rectangular boxes (you may assume n VERY large), and you decide to nest some of them within each other for easy storage. The i -th

box has dimensions $w_i \times h_i$. Box i can fit inside box j if and only if $w_i < w_j$ and $h_i < h_j$. A sequence of boxes b_1, b_2, \dots, b_k form a chain if box b_i fits inside box b_{i+1} for each $1 \leq i < k$. Design an algorithm which takes as input a list of dimensions $w_i \times h_i$ and returns the length of the longest possible chain of boxes. You must construct a directed graph as part of your solution.

BONUS: Having found the length of the longest possible chain, how can you use your directed graph to return the chain itself? (If there is more than one longest chain, you may return any of them)

3 Bipartite Graphs

A Bipartite Graph is a graph whose vertices can be divided into two independent sets, U and V such that every edge (u, v) either connects a vertex from U to V or a vertex from V to U . A bipartite graph is possible if the graph coloring is possible using two colors such that vertices in a set are colored with the same color.

You are attending a talk on Valentine's Day in hopes of making an intellectual connection with someone. The speaker is presenting empirical research on romantic relationships in classroom settings, with a case study of students who attend CS 161 section. They are presenting their findings using a graph of student relationships, but you are somewhat skeptical that they may have not observed everything. Your first step to testing your hypothesis is to check if the graph they present is bipartite.

1. Design an algorithm using DFS to determine whether or not a graph is bipartite.
2. Design an algorithm using BFS to determine whether or not a graph is bipartite.