

---

Pre-lecture exercises will not be collected for credit. However, you will get more out of each lecture if you do them, and they will be referenced during lecture. We recommend **writing out** your answers to pre-lecture exercises before class. Pre-lecture exercises usually should not take you more than 30 minutes.

---

In this pre-lecture exercise, we will remember a little bit of probability!

1. Let  $X$  be a random variable which is 1 with probability  $1/100$  and 0 with probability  $99/100$ .
  - (a) What is the expected value  $\mathbb{E}[X]$ ?
  - (b) Suppose you draw  $n$  independent random variables,  $X_1, X_2, \dots, X_n$ , distributed like  $X$ . What is the expected value  $\mathbb{E}[\sum_{i=1}^n X_i]$ ?
  - (c) Suppose I draw independent random variables  $X_1, X_2, \dots$  and I stop when I see the first “1”. For example, if I draw

$$X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 1$$

then I would stop at  $X_4$ . Let  $N$  be the last index that we draw. (So in the previous example,  $N = 4$ ). How big do you expect  $N$  to be?

[**Note:** actually figuring out  $\mathbb{E}[N]$  from scratch is a bit tricky, although you may have seen it in CS109. But even if you don’t do it rigorously, intuitively how big do you expect  $N$  to be?]

2. Consider the following pseudocode, which is an in-place sorting algorithm for an array  $A$ .

```
def bogosort(A):  
    while A is not sorted:  
        A.shuffle() # this randomly permutes A  
    return A
```

- (a) Let  $X_i$  be a random variable which is 1 if `A.shuffle()` is sorted after the  $i$ ’th call, and 0 otherwise.
- (b) What is  $\mathbb{E}[X_i]$ ?
- (c) What is the *expected* number of times that `bogosort` executes the `while` loop?