Quiz 06

Name:

Time: Complete and submit to the instructor

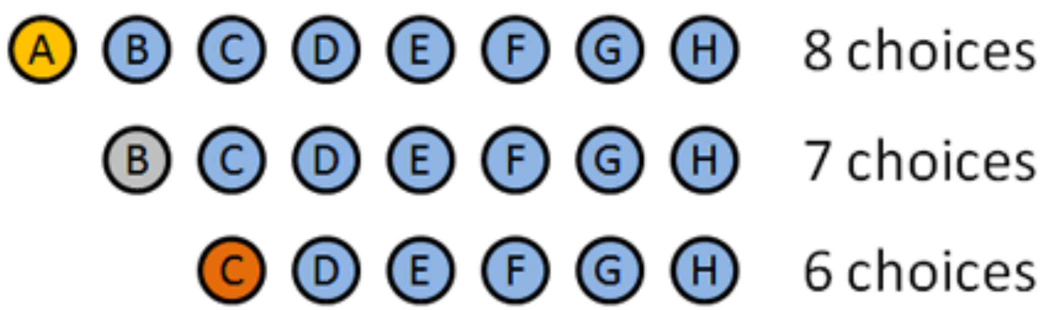
Evaluation:

- This Quiz will be considered as an extra credit of 10 points in the midterm exam.

Chapter 6: Counting Methods

Exercise 1:

How many ways can we award a 1st, 2nd and 3rd place prize among eight contestants? (Gold / Silver / Bronze)



Exercise 2: In how many ways can we select a committee of two women and three men from a group of five distinct women and six distinct men?

Chapter 7: Recurrence relations

Exercise 1: Solve the recurrence relation

1) $F_n = 10F_{n-1} - 25F_{n-2}$ where $F_0 = 3$ and $F_1 = 17$

2) $F_n = 5F_{n-1} - 6F_{n-2}$ where $F_0 = 1$ and $F_1 = 4$

<u>Exercise 2:</u> Solve the recurrence relation for *the Fibonacci sequence:*

The Fibonacci sequence is defined by the linear homogeneous, second-order recurrence relation $\mathbf{f_n - f_{n-1} - f_{n-2} = 0}$ for $n \geq 3$

And the initial conditions $f_0 = 0$, $f_1 = 1$, $f_2 = 1$

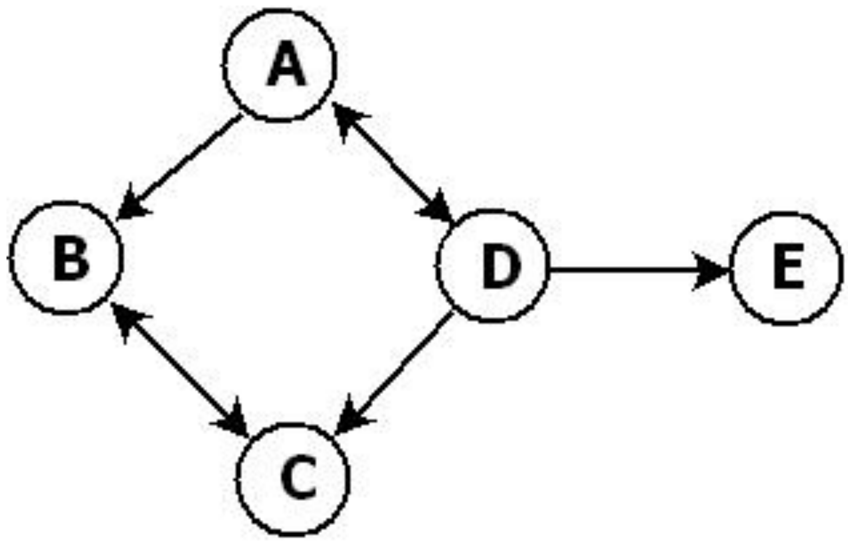
Exercise 3: Professor Larry proposes the following version of binary search:

```
binary_search3(s, i, j, key){
  while ( i ≤ j) {
    k = |_(i+j)/2_|
    if (key==Sk)
       return k
      if (key < Sk)
          j = k
   else
       i = k
    }
    return 0
  }
```

Is professor's version correct (does it find key if it is present and return 0 if it is not present)? If the professor's version is correct, what is the worst-case time? Justify your answer.

Chapter 8: Graph theory

Exercise 1: Consider the graph G:

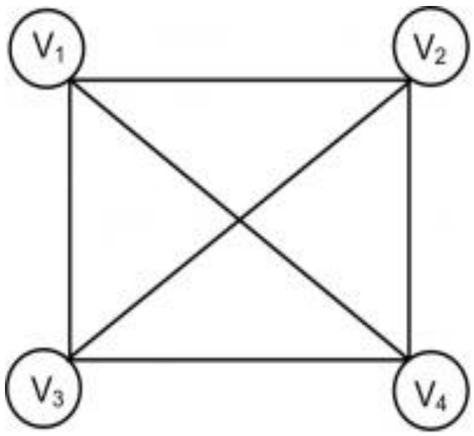


1) Find the adjacency Matrix of the graph G

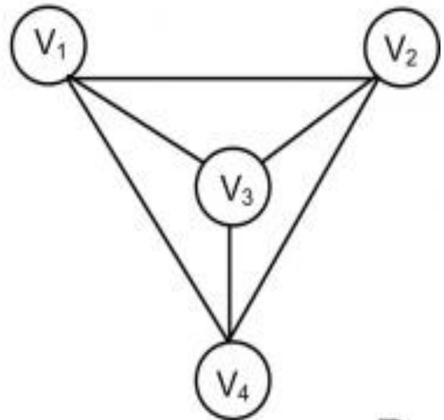
2) Find the square of the Matrix A

Exercise 2:

I. Let consider the Graphs A and B:



A

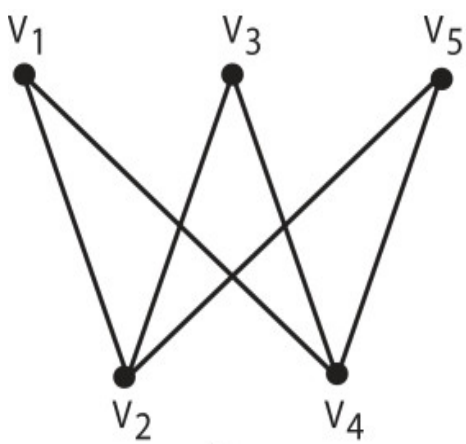


B

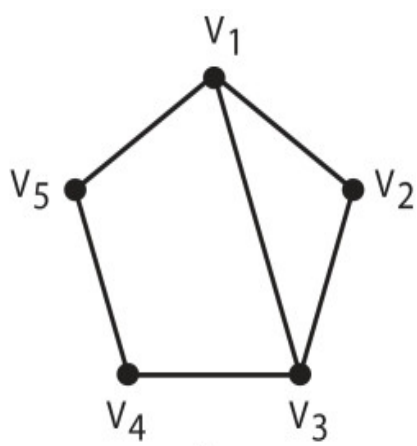
1) Can we define A and B as Isomorphic graphs. Why?

2) Find the Adjacency Matrix of the graphs A and B.

II. Let consider the Graphs G1 and G2:



G1



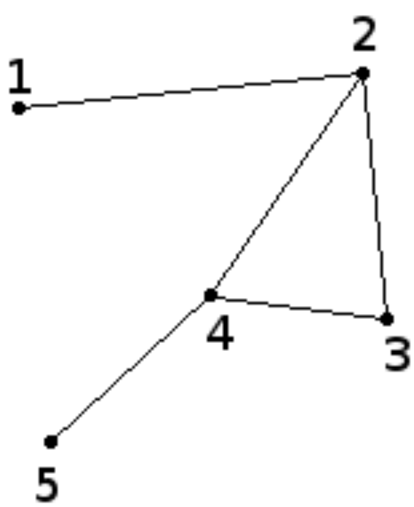
G2

1) Can we define G1 and G2 as Isomorphic graph. Why?

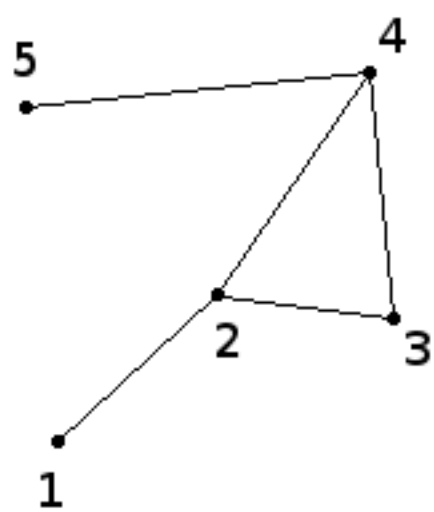
2) Find the Adjacency Matrix of the graphs G1 and G2.

III. Let consider the Graphs G1 and G2

G1



G2

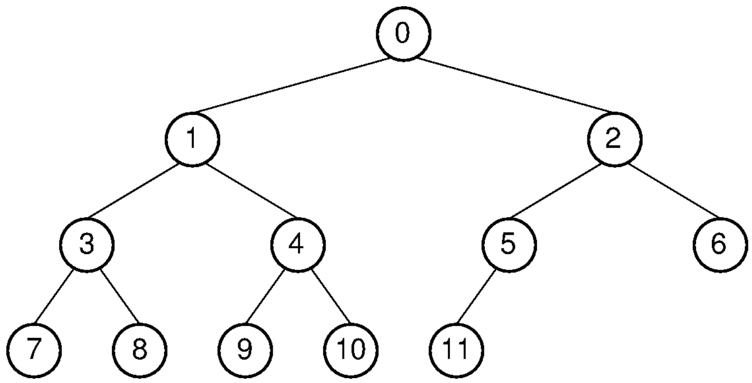


1) Can we define G1 and G2 as Isomorphic graph. Why?

2) Find the Adjacency Matrix of the graphs G1 and G2.

Chapter 9: Trees.

Exercise 1: Let consider this graph:



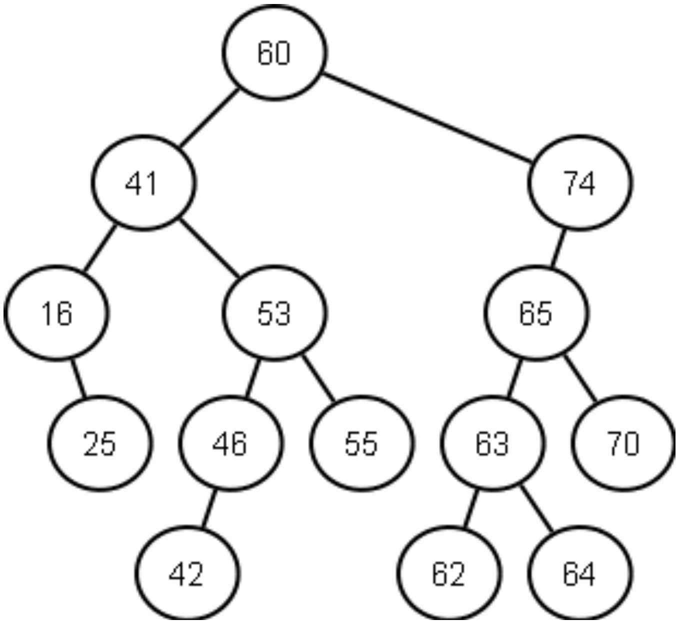
(a)

1) Is this graph a full tree or a complete tree? Explain.

2) Determine the root node, the internal node and the terminal node

Exercise 2:

Let consider this tree:



1. Is this tree a binary search tree? Why?

2. Define the internal node, the terminal node, the height and the root node.

Exercise 3:

Make the construction of this Huffman Code:

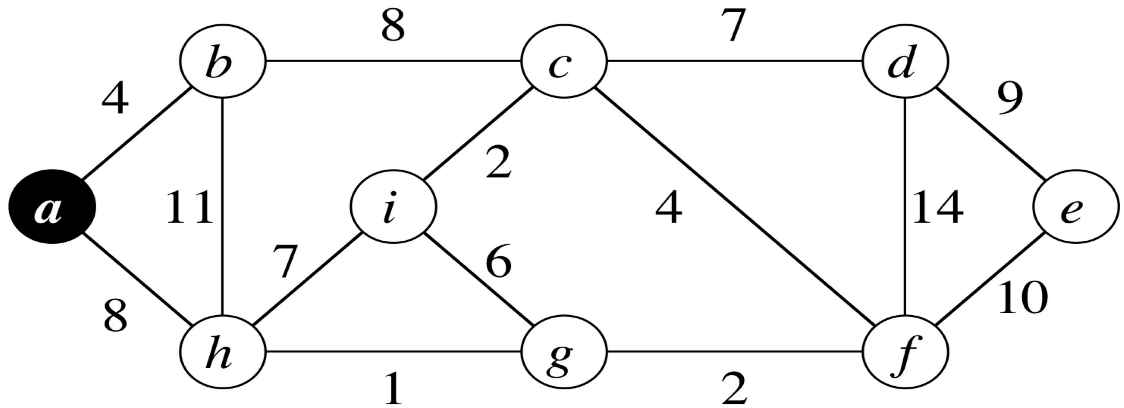
f:5 e:9 c:12 b:13 d:16 a:45

Exercise 4: Construct an optimal Huffman Code.

2	3	5	7	11	13	17	19	23	29	31	37	41

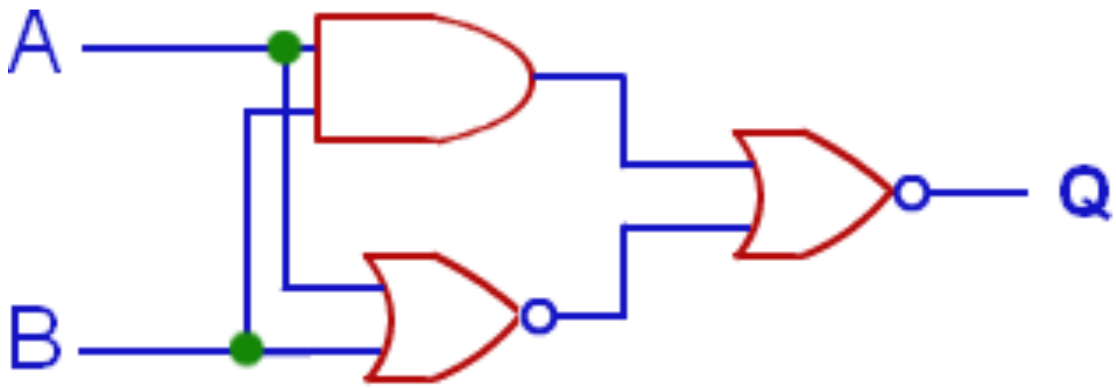
Exercise 5:

Find the Minimum Spanning Tree using Kruskal's algorithm.



Chapter 10: Boolean algebra

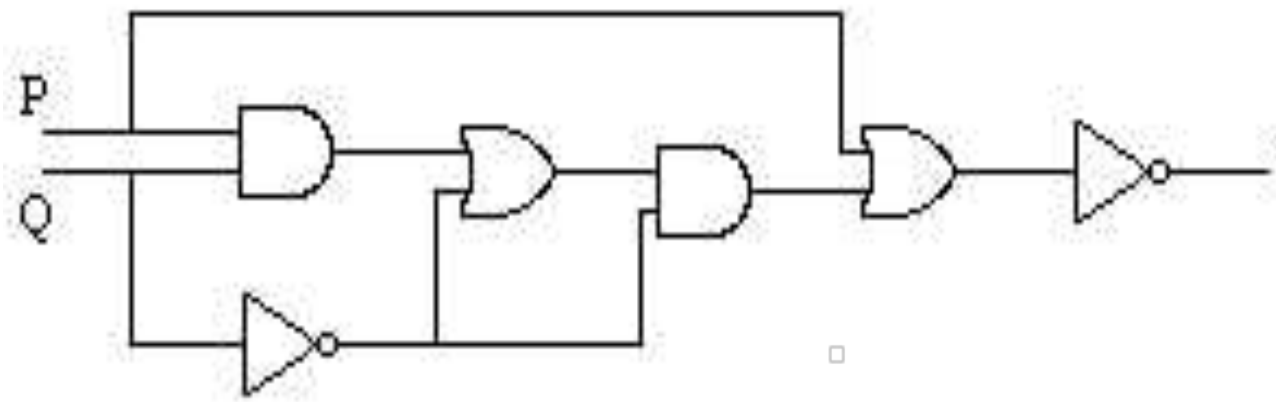
Exercise 1: Let consider the following system.



1) Determine the final expression of the Boolean algebra.

2) Determine a table to represent the Boolean algebra.

Exercise 2: Let consider the following system .



1) Determine the final expression of the Boolean algebra Q.

2) Determine a table to represent the Boolean algebra Q.

Formula

Counting Methods

- Theorem : There are n! permutations of n elements. By the Multiplication Principle, there are $n(n-1)(n-2)\ldots 2 \times 1 = n!$ Permutations of n elements.

- Theorem r-permutation: $P(n, r) = n(n-1)(n-2) .. (n-r+1) = n!/(n-r)!$, where $r \le n$. $P(n, n) = n!$

- Theorem Combination: $C(n, r) = P(n, r)/r! = \dfrac{n!}{r!(n-r)!}$, where $r \le n$.

- The *Inclusion-Exclusion Principle*

$$|X \cup Y| = |X| + |Y| - |X \cap Y|$$

- The ***Fibonacci sequence*** $\{ f_n \}$ is defined by the equations

$$f_0 = 0$$
$$f_1 = 1$$
$$f_2 = 1$$
$$\mathbf{f_n = f_{n-1} + f_{n-2}} \quad \text{for all } n \ge 3$$

The Fibonacci sequence begins 0, 1, 1, 2, 3, 5, 8, 13, …
In mathematical terms, the sequence F_n of Fibonacci numbers is defined by the recursive relation $\mathbf{f_n = f_{n-1} + f_{n-2}}$

Recurrence Relations

Theorem:

- Let $\mathbf{a_n = c_1 a_{n-1} + c_2 a_{n-2}}$

be a second order linear homogeneous recurrence relation with constant coefficients.

- If S and T are solution a_n , then $U = bS + dT$ is also a solution a_n .
- If r_1 and r_2 are solutions of $t^2 + c_1 t + c_2 = 0$, then the sequence r^n, n = 0, 1, … is a solution of a_n.
- If a is a sequence defined by a_n: $a_0 = C_0$ and $a_1 = C_1$

- ❖ And r_1 and r_2 are roots of $t^2 + c_1 t + c_2 = 0$ with $r_1 \neq r_2$, then there exist constant b and d such that $\mathbf{a_n = br_1^n + dr_2^n}$ $\quad n = 0, 1, \ldots$

- Let $\mathbf{a_n = c_1\, a_{n-1} + c_2\, a_{n-2}}$.

Be a *second-order linear homogeneous recurrence relation* with constant coefficients.
Let a be the sequence satisfying a_n and $a_0 = c_0$ $\quad a_1 = c_1$
If both roots of $\mathbf{t^2 - c_1 t - c_2 = 0}$
Are equal to r, then there exist constants b and d such that

$$\mathbf{a_n = br^n + dnr^n}, \qquad n=0, 1, 2, ,,,,, v$$

Trees

Minimum Spanning Tree (MST)

Below are the steps for finding MST using Kruskal's algorithm

a. Sort all the edges in non-decreasing order of their weight.
b. Pick the smallest edge. Check if it forms a cycle with the spanning tree formed so far. If cycle is not formed, include this edge. Else, discard it.
c. Repeat step#2 until there are (V-1) edges in the spanning tree.

Trees

The first step of *Huffman encoding* is building the Huffman tree. Given a set of characters and their associated frequencies, we can build an optimal Huffman tree as follows:

- Construct leaf Huffman trees for each character/frequency pair
- Repeatedly choose two minimum-frequency Huffman trees and join them together into a new Huffman tree whose frequency is the sum of their frequencies.
- When only one Huffman tree remains, it represents an optimal encoding.

Boolean Algebra