CIS 275 Discrete Math I Summer 2019

Quiz 02

Name:

Time: Complete and submit to the instructor

Evaluation:

- As described in the syllabus, the Quiz is 20% of the overall grade.

<u>Exercise 1:</u> select the correct answer.

- ❖ Let consider a function

$$\mathbf{f(u) = 70 + 15 \lceil u - 1 \rceil} \qquad 0 \le u \le 13$$

If u = 2.5. Find the value of f(u).

1. f(u) = 100
2. f(u) = 85
3. f(u) = 70

- ❖ Let consider the sequence

$$\mathbf{A_i = {}^{1}/_{i}} \qquad i \ge 1$$

Is A_i decreasing or increasing or nonincreasing ?

1. A_i is decreasing ($S_n > S_{n+1}$)
2. A_i is nonincreasing ($S_n \ge S_{n+1}$)
3. A_i is increasing ($S_n < S_{n+1}$)

❖ Consider the sequence A defined by

$$\mathbf{An = n^2 - 3n + 3}$$

Find the product $\prod_{i=1}^{2} A_i$:

1. $\prod_{i=1}^{2} A_i = 1$
2. $\prod_{i=1}^{2} A_i = 2$
3. $\prod_{i=1}^{2} A_i = 0$

❖ Let consider a function

$$\mathbf{f = \{(1,c),(2,a),(3,b)\}}$$

We define the domain X = {1, 2, 3} and the codomain Y = {a, b, c}.

Is the function f one-to-one, onto or a bijection?

1. This function is not one-to-one
2. This function is not onto
3. This function is called a bijection.

❖ Let consider the sequence

b , c

This sequence is a subsequence of the sequence T_n. We define $1 \leq n \leq 5$.

Find the element of the sequence T_n :

1. $T_n = \{ T_1 = a , T_2 = a , T_3 = b , T_4 = c , T_5 = d \}$
2. $T_n = \{ T_1 = b , T_2 = b , T_3 = c , T_4 = a , T_5 = d \}$
3. $T_n = \{ T_1 = c , T_2 = b , T_3 = a , T_4 = b , T_5 = d \}$

- Let consider the function g and f

$$\mathbf{g = \{(1, a), (2, a), (3, c)\}}$$

$$\mathbf{f = \{(a, y), (b, x), (c, z)\}}$$

We define the function f from X = {1, 2, 3} to Y = {a, b, c}, and the function g from Y = {a, b, c} to Z = {x, y, z}.

Find the composition function from f to g.

1. f o g = {(1, y), (2, y), (3, z)}
2. f o g = {(1, y), (2, y), (2, x)}
3. f o g = {(1, y), (1, z), (2, z)}

- Consider the sequence T defined by

$$\mathbf{T_n = 2n - 1}$$

Find the sum $\sum_{i=1}^{3} T_i$.

1. $\sum_{i=1}^{3} T_i = 8$
2. $\sum_{i=1}^{3} T_i = 10$
3. $\sum_{i=1}^{3} T_i = 9$

- Let consider the function

$$\mathbf{f = \{(1, a), (2, c), (3, b)\}}$$

We define the domain X = {1, 2, 3} and the range Y = {a, b, c}.

Find the inverse of the function f.

1. $f^{-1} = \{(a, 1),(c, 2), (3, b)\}$
2. $f^{-1} = \{(a, 1), (c, 2), (b, 3)\}$
3. $f^{-1} = \{(a, 1), (2, c), (3, b)\}$

<u>Exercise 2:</u> Consider the matrix

$$
\begin{array}{c|cccc}
 & w & x & y & z \\
\hline
a & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\
b & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
c & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\
d & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1}
\end{array}
$$

1. Write the relation R, given by the matrix, as a set of ordered pairs. Determine the domain and the range of the relation R.

2. Find the matrix of the product R^2.

3. Write the inverse of the relation R, given by the matrix, as a set of ordered pairs. Determine the domain and the range of the inverse of the relation R.

4. Find the matrix of the inverse of the relation R.

Exercise 3:

Let the relations
R1 = $\{(x, y) | x \text{ divides } y\}$, R1 is from X to Y. R2 = $\{(y, z) | y > z\}$, R2 is from Y to Z, ordering of X and Y: 2, 3, 4, 5; ordering of Z: 1, 2, 3, 4

1. Find the matrix A1 of the relation R1

2. Find the matrix A2 of the relation R2

3. Find the matrix product A1 A2

4. Find the relation R2 o R1

5. Find the matrix of the relation R2 o R1

<u>Exercise 4:</u> Let each function is one-to-one on the specified domain X. If Y = range of f, we obtain a bijection from X to Y. Find each inverse function

- $f(x) = 4x + 2$ x = set of real numbers
- $f(x) = 3^x$ x = set of real numbers
- $f(x) = 3 + 1/x$ x = set of nonzero real numbers

<u>Exercise 5:</u> Consider the relation R on the set {1, 2, 3, 4, 5} defined by the rule $(x, y) \in R$ if 3 divides $x - y$

1. List the element of R
2. List the element of R^{-1}
3. Is the element of R is reflexive, symmetric, antisymmetric, transitive and/or partial order?

<u>Exercise 6:</u> Consider the sequence A defined by $A_n = n^2 - 3n + 3$

1. Find

$$\sum_{i=1}^{4} A_i$$

2. Find

$$\prod_{i=1}^{2} A_i$$

3. Is A increasing?
4. Is A decreasing
5. Is A nonincreasing?
6. Is A nondecreasing?

Formula

The Sequences

A *sequence* is a special type of function in which the domain consists of a set of consecutive integers.

Let **Sn** denoted the entire sequence:

S1, S2, S3, S4, S5, …

We use the notation Sn to denote the single element of the sequence S at *index* n.

- A sequence S is **increasing** if Sn < Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **decreasing** if Sn > Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **nondecreasing** if Sn ≤ Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **nonincreasing** if Sn ≥ Sn+1 for all n for which n and n+1 are in the domain of the sequence.

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \ldots + a_n$$

$$\prod_{i=m}^{n} a_i = a_m \times a_{m+1} \times \ldots\ldots \times a_n$$