UNIVERSITY OF MICHIGAN-DEARBORN

CIS-275
Discrete Mathematics I

Outline

- Introduction
- Syllabus
- Chapter 1: Sets
- Homework

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Introduction

- Me
- You

Why Study Discrete Mathematics ?

- Discrete Mathematics solves problems that continuous mathematics such as Calculus cannot.
- Discrete Mathematics forms the basis for Computer Science.
- A Computer system consists of three major components: users, software, and hardware (tools, machinery, and other durable equipment)

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Syllabus

- **CIS-275, Discrete Mathematics I,**
  - Summer 2019
  - 10:00 am - 11:45 am and 2:00 pm – 3:45 pm, Monday and Wednesday,
  - HPEC1180
  - http://canvas.umd.umich.edu/

Instructor

- **Instructor: Professor Sana Neji**
  - Office Location: CIS-246
  - Phone Number: 313-583-6366
  - E-Mail: sananeji@umich.edu
  - Office Hours: Monday and Wednesday 11:45 A.M to 2:00 P.M

Topic of Interests

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- This course introduces students to various topics in discrete mathematics, such as :
  - set theory,
  - mathematical logic,
  - trees, and graph theory.
  - Applications to relational databases, modeling reactive systems and program verification are also discussed.

Prerequisite and References

- **Prerequisite:**
  - Successful completion of CIS 200 or MATH 115

- **References:**
  - Richard Johnsonbaugh, Discrete Mathematics, 7th edition, ISBN-10: 0131593188

Exams and Assessment

- **Homework (around 7 assignments): 20%**
- **Homework (around 3 assignments): 20%**
- **Mid-term Exam: 30% (2 exams)**
- **Final Exam: 30% (2 exams)**

- **Late submissions will not be accepted unless with a prior approval from the instructor.**

Grade Scale

A+ >= 95,

A >= 92,

A- >= 90,

B+ >= 85,

B >= 82,

B- >= 80,

C+ >= 75,

C >= 72,

C- >= 70,

D+ >= 65,

D > 62,

D>= 60,

E<=59.

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Tentative Schedule

Chapter 1.	Sets
Chapter 2.	Mathematical Induction
Chapter 3.	Functions
Chapter 4	Algorithms
Chapter 5	Introduction to Number Theory
Chapter 6	Counting Methods
Chapter 7	Recurrence Relations
Chapter 8	Graph Theory
Chapter 9	Trees
Chapter 10	Boolean Algebra

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Writing Center

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Writing Center

Writing Center consultants can help you:

- Understand assignment goals
- Formulate an approach to an assignment
- Develop a thesis
- Be aware of audience, purpose, genre, and context
- Articulate and organize ideas
- See the need for additional evidence
- Generate revision strategies
- Improve control of grammar and mechanics
- Identify and use appropriate resources
- Become more skilled in using APA, MLA, Chicago, and other documentation styles

What do I need to do before coming to the Writing Center?

- Clarify any questions you have about the assignment with your professor; if you are coming to work on a previously graded piece, get feedback from the prof before you start revising
- Read through your draft or graded paper, writing down your thoughts, questions, and ideas as you read
- Bring to your appointment: the assignment sheet, syllabus, class notes, relevant books, sources, and a draft (if you have one)

How do I make an appointment?

- Make an appointment online at: http://www.casl.umd.umich.edu/writ_center/
- Note: We try to accommodate drop-ins, but we prefer that students make appointments

Fall 2013 Hours and Locations:

3035 CASL (CB)
M-R 8:30 – 7:00
F 8:30 – 1:00

138 Fairlane Ctr. North (FCN)
M-R 2:00 – 8:00

Mardigian Library, 1st floor
(past the Library Research Center)
M-R 10:00 – 1:00

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- Questions?

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Chapter 1 : Sets

SETS

- The concept of set is basic to all of mathematics and mathematical applications.
- A set is a collection of objects (elements or members).
- ***Example:***

$$A = \{1, 2, 3, 4\}$$

In this set, we list all the elements or an infinite set.

- ***Example:***

$$B = \{x \mid x \text{ is a positive, even integer}\}$$

In this example, we list the property of the elements where *"B equals the set of all x such that x is a positive, even integer"*.

SETS

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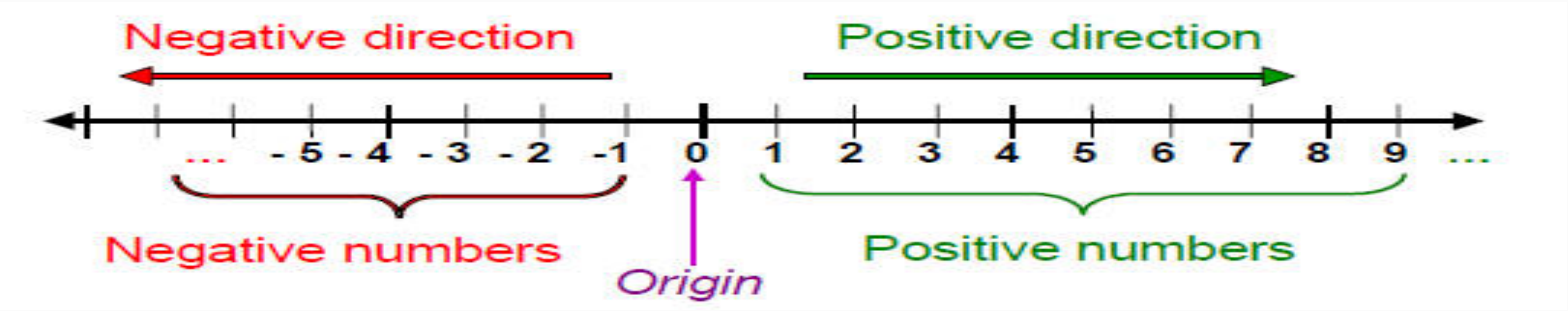
Some sets that occur frequently in mathematics generally, and in discrete mathematics such as:

Symbol	Set	Example of members
Z	Integers	-3, 0, 2, 154
Q	Rational numbers	-1/3, 0, 24/15
R	Real numbers	-3, -1.766, 0, 4/15, √2, 2.666, …, π

the real number line



SETS

If X is a finite set, we let

$$|X| = \text{the number of elements in } X.$$

We call $|X|$ the cardinality of X.

- **Example:**

We have $|A| = 4$, and the cardinality of A is 4.

The cardinality of the set $\{\mathbf{R}, \mathbf{Z}\}$ is 2 since it contains two elements, namely the two sets **R** and **Z**.

SETS

According to the real number line, we can determinate whether or not x belongs to X.

- If x is a member (or an element) of X, then we write $x \in X$.
- If x is not a member of X, we write $x \notin X$ (x does not belong to X).
- The set with no element is called the **empty** set and is denoted by ϕ
- The set X and Y are **equal**, we write X = Y.

Examples of Sets

- If A = {1, 3, 2} and B = {2, 2, 1,3}

  Then, A and B have the same elements.

  Therefore, A = B

- If A = { x | x^2 +x-6=0} and B = {2, -3}

  Then A = B

- Let A = {1, 3, 2} and B = {2, 4}

Then A ≠ B since there is at least one element in A that is not in B.

SETS

- Suppose X and Y are sets. If every element of X is an element of Y, we say X is a subset of Y and write

$$X \subset Y.$$

- **Example:** If C = {1, 3} and A = {1, 2, 3, 4}

Then, every element of C is an element of A. Therefore, C is a subset of A and we write $C \subset A$.

- **Example :** $X = \{ x \mid 3x^2+x-2=0\}$

If $x \in X$, then $3x^2+x-2=0$

Solving for x, we obtain x=-1 and x=2/3, we have

$x \in X$ but $x \notin Z$ (integers). Therefore, X is not a subset of Z

SETS

- Any set X is a subset of itself, since any element in X is in X. Also, The empty set is a subset of every set, and is denoted by ϕ .
- If X is a subset of Y and X does not equal Y. we say that X is a proper subset of Y and we write

$$X \subset Y$$

- **<u>Example:</u>** Let C = {1, 3} and A = {1, 2, 3, 4}

Then C is a proper subset of A since C is a subset of A but C does not equal to A. we write $C \subset A$

SETS

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- *Definition :* The set of all subsets (proper or not) of a set X, is called the power set of X and is denoted by *P*(X).

  |X|= n, then |*P*(X)| = 2^n.

- **Example**: Let A = {a, b, c}. Then the *P*(A) are

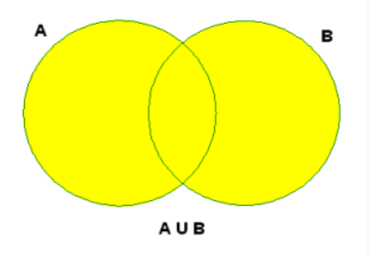
  P(A) = { ϕ , {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c} }

  All but {a, b, c} are proper subsets of A

  So, |A|=3, then |P(A)| = 2^3

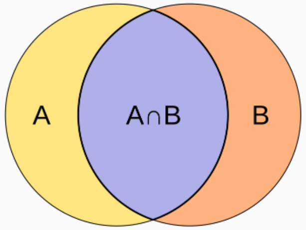
SETS

- If A and B are sets, there are various set operations involving A and B that can produce a new set. The set :



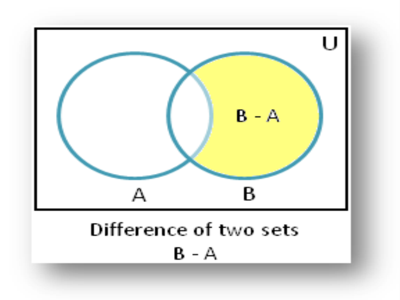
1. the union of A and B

$$\mathbf{A \cup B = \{ x \mid x \in A \text{ or } x \in B\}}$$



2. The intersection of A and B

$$\mathbf{A \cap B = \{ x \mid x \in A \text{ and } x \in B \}}$$



3. The difference (or relative complement) of A and B

$$\mathbf{A - B = \{ x \mid x \in A \text{ or } x \notin B\}}$$

- The sets X and Y are disjoint if A $\cap$ B = ϕ

SETS

- **Example:** If A = {1, 3, 5} and B = {4, 5, 6}, then

  $A \cup B = \{1, 3, 4, 5, 6\}$

  $A \cap B = \{5\}$

  $A - B = \{1, 3\}$

  $B - A = \{4, 6\}$

In general, $A - B \neq B - A$

SETS

- Let U be the universal set and X the subset of U, the set U – X is called the complement of X , denoted by $\overline{X}$
- **<u>Example:</u>** let A = {1, 3, 5}, a universal set is specified as U= {1, 2, 3, 4, 5}, then $\bar{A}$= {2, 4}.

If U= {1, 3, 5, 7, 9} , then $\bar{A}$= {7, 9}.

Theorem 1.21

Let U be a universal set and let A, B, and C be subsets of U. The following properties hold.

(a) Associative laws:

$$(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$$

(b) Commutative laws:

$$A \cup B = B \cup A, \quad A \cap B = B \cap A$$

(c) Distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(d) Identity laws:

$$A \cup \varnothing = A, \quad A \cap U = A$$

(e) Complement laws:

$$A \cup \overline{A} = U, \quad A \cap \overline{A} = \varnothing$$

(f) Idempotent laws:

$$A \cup A = A, \quad A \cap A = A$$

(g) Bound laws:

$$A \cup U = U, \quad A \cap \varnothing = \varnothing$$

(h) Absorption laws:

$$A \cup (A \cap B) = A, \quad A \cap (A \cup B) = A$$

(i) Involution law:

$$\overline{\overline{A}} = A$$

(j) 0/1 laws:

$$\overline{\varnothing} = U, \quad \overline{U} = \varnothing$$

(k) De Morgan's laws for sets:

$$\overline{(A \cup B)} = \overline{A} \cap \overline{B}, \quad \overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

Proof The proofs are left as exercises (Exercises 44–54, Section 2.1) to be done after more discussion of logic and proof techniques.

SETS

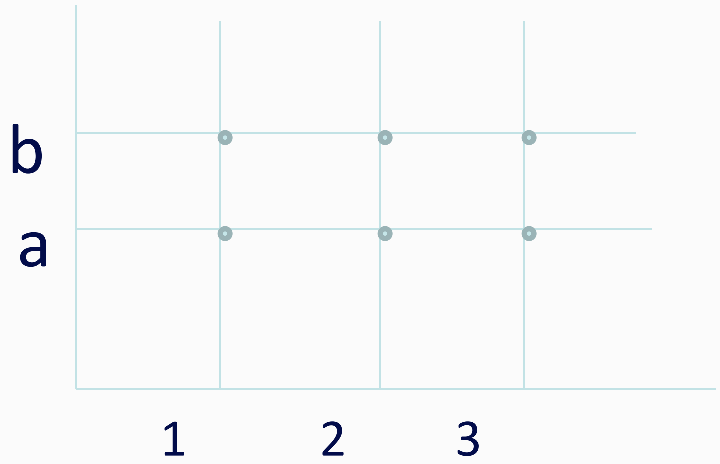
- *Definition :* A partition of a set X is every element in X belongs to exactly one member of S of sets.
- If X = {1, 2, 3, 4,5, 6, 7, 8}.

  S= {[1,4,5], [2,6],[3], [7,8])

S is a partition of X.

SETS

- If X and Y are sets and $X \times Y$ the set of all ordered pairs (x, y) where $x \in X$ and $y \in Y$. We call $X \times Y$ the Cartesian Product of X and Y.
- Let set X = {1, 2, 3} and Y = {a,b}. Then the Cartesian Product $X \times Y$ is defined as



$$X \times Y = \{ (1,a), (1,b),(2,a),(2,b),(3,a), (3,b) \}$$

- **$|X \times Y| = |X| \times |Y|$**
- **$X \times Y \neq Y \times X$**

Sets

- The Cartesian Product of set

$$\mathbf{R^3} = R \times R \times R,$$

where R = { [- ∞, + ∞]}

- ∞ + ∞

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Homework

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Exercise 1: Let the universe be the set U ={1, 2, 3,…, 10}.
Let A = {1, 4, 7, 10}, B = {1, 2, 3, 4, 5} and C = {2, 4, 6, 8}.
List the elements of each set

- **A U B**
- **B ∩ C**
- **A – B**
- **B – A**
- $\bar{A}$
- U **– C**
- $\bar{U}$
- **A U** $\oint$
- **B ∩** $\oint$
- **A U** U
- **B ∩** U
- **A ∩ (B U C)**
- $\overline{B} \cap (C - A)$
- **(A ∩ B) − C**
- $\overline{A \cap B}\ \cup C$
- **(A U B) – (C – B)**

Homework

Exercise 2: Determinate $A \subseteq B$

- $A = \{1, 2\}$ and $B = \{3, 2, 1\}$
- $A = \{1, 2\}$ and $B = \{x \mid x^3 - 6x^2 + 11x = 6\}$

Exercise 3: Show that A is not a subset of B.

- $A = \{1, 2, 3\}$ and $B = \{1, 2\}$
- $A = \{1, 2, 3\}$ and $B = \emptyset$

Exercise 4: Let X = {1, 2} and Y = {a, b, c}. List the element of each seat.

- **X × Y**
- **X × X**