UNIVERSITY OF MICHIGAN-DEARBORN

# Chapter 3 : Algorithms

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# Recursive Algorithms

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A *recursive algorithm* is an algorithm that contains a recursive function. Recursive is a powerful, elegant and natural way to solve a large class of problems.

# Recursive Algorithms

A recurrence describes a sequence of numbers. Early terms are specified explicitly and later terms are expressed as a function of their predecessors. As a trivial example, this recurrence describes the sequence 1, 2, 3, etc.:

$$T_1 = 1$$

$$T_n = T_{n-1} + 1 \qquad \text{(for all } n \geq 2\text{)}$$

Here, the first term is defined to be 1 and each subsequent term is one more than its predecessor.

# Recursive Algorithms

- *Recurrences are one aspect of a broad theme in computer science:* reducing a big problem to progressively smaller problems until easy base cases are reached. This same idea underlies both *induction proofs and recursive algorithms.*
- *For example*, *one might describe the running time of a recursive algorithm with a recurrence and use induction to verify the solution.*

# Recursive Algorithms

- **Definition :** The *Fibonacci sequence* $\{ f_n \}$ is defined by the equations

$$f_0 = 0$$

$$f_1 = 1$$

$$f_2 = 1$$

$$f_n = f_{n-1} + f_{n-2} \qquad \text{for all } n \geq 3$$

The Fibonacci sequence begins

1, 1, 2, 3, 5, 8, 13, …

In mathematical terms, the sequence $F_n$ of Fibonacci numbers is defined by the recursive relation $\mathbf{f_n = f_{n-1} + f_{n-2}}$

By definition, the first two numbers in the Fibonacci sequence are either 1 and 1, or 0 and 1, depending on the chosen starting point of the sequence, and each subsequent number is the sum of the previous two numbers.

# Recursive Algorithms

**Exercise 1:** Use the mathematical induction to show that

$$\sum_{k=1}^{n} f_k = f_{n+2} - 1$$

for all n ≥ 1

# Exercises

**Exercise 2:** Use the mathematical induction to show that

$$f_n^2 = f_{n-1} f_{n+1} + (-1)^{n+1} \quad \text{for all } n \geq 2$$

# Exercises

**Exercise 3:** Use the mathematical induction to show that

$$\sum_{k=1}^{n} f_k^2 = f_n f_{n+1}$$

for all $n \geq 1$

# Exercises

**<u>Exercise 4:</u>** Let consider that

$$1 + 2 + \ \ldots + n = An^2 + Bn + C$$

For all n, and for some constant A, B and C.

1) Assuming that this is true, plug in $n = 1, 2, 3$ to obtain three equations in the three unknowns A, B and C.
2) Solve for A, B and C with the three equations obtained in the previous question.
3) Prove using the mathematical induction that the statement is true.