CIS 275 Discrete Math I Summer 2019

Quiz 01

Name:

Time: Complete and submit to the instructor

Evaluation:

- As described in the syllabus, the Quiz is 20% of the overall grade.

<u>**Exercise1:**</u> select the correct answer.

❖ Let define the universe be U ={1, 2, 3 ,…, 10}, determine the element of the set $\overline{U}$

1. $\overline{U}$ = U – U ={1, 2, 3 ,…, 10}.
2. $\overline{U} = \emptyset$
3. $\overline{U}$ = {1, 10}

❖ Let A = {1, 4, 7, 10}, B = {1, 2, 3, 4, 5}, C = {2, 4, 6, 8} and U ={1, 2, 3 ,…, 10}, determine $\bar{B} \cap$ (C − A)

1. {6, 8}
2. {2, 6, 8}
3. {6, 7, 8, 9, 10}

❖ Let A ={1, 3, 6, 9, 10}, determine A ∪ $\emptyset$

1. $\emptyset$
2. $\{\emptyset, 1, 10\}$
3. A

❖ Let define A $=\{x \mid x^2 - 4x + 4 = 1\}$ and B = {1, 3, 5}, determine the correct answer

1. A is not a subset of B
2. $\Delta < 0$: There is not solution in the set A.
3. A is a subset of B

❖ Let consider the Cartesian product X × Y = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}.

Determine the domain X and the codomain Y.

1. X =_{1, 2} and Y = {a, b, c}
2. X =_{1} and Y = {a, c}
3. X =_{1, 2} and Y = {a, b}

❖ Let A ={0, 1, 1, 2} and B = {1, 2, 2}. Determine the correct answer

1. The sets A and B are equal.
2. The sets A and B have the same elements.
3. The sets A and B aren't equal.

❖ Let consider the set A = {1, 3} and B = {x | $3x^2$ + x – 2 =0}

1. A is a proper subset of B.
2. A is a subset of B.
3. A is neither a subset nor a proper subset of B.

❖ Let consider the set A = {1, 2, 4, 6, 8, 9, 11}. Determine the partition X of the set A.

1. X ={[11], [8, 9], [2, 6], [1]}
2. X ={[9, 11], [8], [2, 6], [4]}
3. X ={[8, 11], [9], [2, 4, 6], [1]}

❖ Let consider the set X = {1} and Y = {y| y^2 + y – 2 =0}. Determine the answer.

1. X is a subset of Y.
2. X is a proper subset of Y.
3. X and Y aren't equal.

- Let consider the set A = {1, 2, 7} and B = {0, 3, 8}. Determine the correct answer.
  1. B – A = {3, 8}
  2. A – B = {1, 7}
  3. B – A ≠ A – B

- Let A = {1, 5, 8, 9}, B = {1, 2, 3, 4, 5, 6}, C = {3, 5, 6, 8} and U = {1, 2, 3,…, 10}, determine B $\cap$ ($\overline{C \cup A}$)
  1. {2, 4}
  2. {2, 6, 7}
  3. {2, 7, 10}

- Let define A = $\{x|x^2 + x = 2\}$ and B ={2, 1}, determine the correct answer
  1. A is not equal to B
  2. $\Delta < 0$: There is not solution in the set A.
  3. A is equal to B

- The element in the set X × Y = {(1, a), (1, b), (1, c)}. Determine the domain X and the codomain Y.
  1. X = {1, 2} and Y = {a, b, c}
  2. X = {1} and Y = {a, c}
  3. X = {1} and Y = {a, b, c}

- Determine $\overline{\emptyset}$
  1. $\overline{U}$
  2. $\emptyset$
  3. U

- Let A ={1, 3, 6, 9, 10}, determine A $\cap$ $\emptyset$
  1. $\emptyset$
  2. $\{\emptyset, 1, 6, 10\}$
  3. A

<u>Exercise 2:</u> Let the universe be the set U ={1, 2, 3,…, 10}.

Let A = {1, 2, 3, 4, 7, 10}, B = {1, 2, 3, 4, 5, 6} and C = {2, 3, 6, 8}.

List the elements of each set

- $A \cap (B \cup C) - \bar{A}$
- $\bar{B} \cap (C - A) \cap (\overline{A \cap B})$
- $(A \cap B) - C \cup (B \cap U)$
- $\overline{A \cup B} \cup (C - A) - \bar{U}$
- $(A \cup B) - (C - B) \cap (A \cup \bar{B})$

Exercise 3: Using the induction, verify that each equation is true for every positive integer n,

$n \geq 1$.

a) $\mathbf{1 * 2 + 2 * 3 + 3 * 4 + \ldots + n\,(n + 1) = \frac{n\,(n+1)(n+2)}{3}}$

b) $1(1!) + 2(2!) + \ldots + n(n!) = (n+1)! - 1$

c) $\mathbf{1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2}$

<u>Exercise 4:</u> Using the induction, verify the inequality

$$2n + 1 \leq 2^n, \ n = 3, 4, \ldots.$$

Exercise 5: Use the induction to prove the statement

$6 * 7^n - 2 * 3^n$ is divisible by 4, for all $n \geq 1$

Formula

The Sets

Let U be a universal set and let A, B, and C be subsets of U. The following properties hold.

(a) Associative laws:

$$(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$$

(b) Commutative laws:

$$A \cup B = B \cup A, \quad A \cap B = B \cap A$$

(c) Distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(d) Identity laws:

$$A \cup \varnothing = A, \quad A \cap U = A$$

(e) Complement laws:

$$A \cup \overline{A} = U, \quad A \cap \overline{A} = \varnothing$$

(f) Idempotent laws:

$$A \cup A = A, \quad A \cap A = A$$

(g) Bound laws:

$$A \cup U = U, \quad A \cap \varnothing = \varnothing$$

(h) Absorption laws:

$$A \cup (A \cap B) = A, \quad A \cap (A \cup B) = A$$

(i) Involution law:

$$\overline{\overline{A}} = A$$

(j) 0/1 laws:

$$\overline{\varnothing} = U, \quad \overline{U} = \varnothing$$

(k) De Morgan's laws for sets:

$$\overline{(A \cup B)} = \overline{A} \cap \overline{B}, \quad \overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

Proof The proofs are left as exercises (Exercises 44–54, Section 2.1) to be done after more discussion of logic and proof techniques.

$$\boldsymbol{\overline{B} = U - B}$$

Quadratic equation

Theorem:

$$For\ ax^2 + bx + c = 0$$

$$Delta = \Delta = b^2 - 4ac$$

If $\sqrt{\Delta} \geq 0$ *: we have two solutions.*

If $\sqrt{\Delta} < 0$ *: we don't have any solution.*

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} \quad and \quad x_2 = \frac{-b+\sqrt{\Delta}}{2a}$$

$$ax^2 + bx + c = a\,(x - x_1)(x - x_2)$$

Mathematical induction

The Principle of Mathematical Induction consists of two steps:

- *Basic step :* Prove that S(1) is true.
- *Inductive step :* Assuming that S (n) is true for n $\geq$1, prove that S(n+1) is true

Then, S(n) is true for every positive integer n.