CIS 275 Discrete Math I Summer 2019

Assignment 01

Evaluation:

- As described in the syllabus, the assignments are 30% of the overall grade.

Submission:

- Submit your document on Canvas on Monday 05/20/19 by 10:00AM. No late submission will be accepted.

Exercise 1: Find the element of each set, draw a graph and determine if the function is one-to-one, onto or both. If it is one-to-one and onto, give the description of the inverse function as a set of ordered pairs, draw a graph and identify the element of each set.

- K = [(1,c), (2,d), (3,a), (4,b)]
- V = [(1,d), (2,d), (4,a)]

Exercise 2: Let f be the function from X = {0, 1, 2, 3, 4} to X defined by f(x) = 4x mod 5

1. Determine f as a set of ordered pairs.
2. Draw the arrow diagram of f.
3. Determine if f is one-to-one or onto.

Exercise 3: Consider the sequence Y and Z defined by

$$\mathbf{Yn = 2^n - 1} \qquad \mathbf{Zn = n(n-1)}$$

1. Find

$$\left(\sum_{i=1}^{3} Y_i\right)\left(\sum_{i=1}^{3} Z_i\right)$$

2. Find

$$\left(\sum_{i=1}^{5} Y_i\right)\left(\sum_{i=1}^{4} Z_i\right)$$

3. Find

$$\sum_{i=1}^{4} Y_i Z_{ii}$$

4. Find

$$\left(\sum_{i=3}^{4} Y_i\right)\left(\prod_{i=2}^{4} Z_i\right)$$

Exercise 4: Consider the relation R on the set {1, 2, 3, 4, 5} defined by the rule (x, y) ∈ R if x + y ≤ 6

1. List the element of R
2. List the element of R^{-1}
3. Is the element of R is reflexive, symmetric, antisymmetric, transitive and/or partial order?

Exercise 5: Determine whether the relation is an equivalence relation on x, y {1, 2, 3, 4, 5}. If the relation is an equivalence relation, list the equivalence classes.

1. [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 3), (3, 1)]
2. [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 5), (5, 1), (3, 5), (5, 3), (1, 3), (3, 1)]
3. {(x, y)| 3 divides by x +y }

Exercise 6: Let the relations R1 = {(1, x), (1, y), (2, x), (3, x)}; R2 = {(x, b), (y, b), (y, a), (y, c)} ordering of X ={1, 2, 3} , Y = {x, y} and Z = {a, b, c}.

1. Find the matrix A1 of the relation R1
2. Find the matrix A2 of the relation R2
3. Find the matrix product A1 A2
4. Find the relation R2 o R1
5. Find the matrix of the relation R2 o R1

Exercice 7:

Write an algorithm that outputs the smallest and largest values in the sequence

s1, …, sn

Exercice 8:

Write an algorithm that reverses the sequence s1, …, sn.
Example: If the sequence is

Amy Bruno Elie,

The reverse sequence

Elie Bruno Amy.

Exercise 9: Trace the algorithm of the Insertion Sort for the input

34 20 144 55

Algorithm: the insertion sort

```
Input: s, n
Output: s (sorted)
insertion_sort(s, n) {
   for i = 2 to n {
     val = si // save si so it can be inserted into the correct place
     j = i - 1
     // if val < sj, move sj right to make room for si
     while (j ≥ 1 ∧ val < sj) {
     sj+1 = sj
    j = j – 1
     }
     sj+1 = val // insert val
   }
}
```

Exercise 10: Trace the algorithm of the Shuffle for the input

34 57 72 101 135

Assume that the values of *rand* are

$$rand(1, 5) = 2, \qquad rand(2, 5) = 5$$

$$rand(3, 5) = 3, \qquad rand(4, 5) = 4$$

```
Input : A, n
Output : A (shuffled)
shuffle(A n) {
    for i = 1 to n - 1
        swap(Ai, Arand(i, n)
}
```

Exercise 11:

Trace the algorithm of the computing n Factorial for n = 3.

Algorithm: the computing n Factorial

This recursive algorithm computes n!

```
Input : n, an integer greater than or equal to 0
Output : n!
1. factorial(n) {
2.     if (n ==0)
3.         return 1
4.     return n * factorial(n – 1)
5. }
```

Exercise 12: Let consider that

$$\mathbf{1 + 2 + \dots + n = An^2 + Bn + C}$$ For all n and for some constant A, B and C.

① Assuming that this is true, plug in n = 1, 2, 3 to obtain three equations in the three unknowns A, B and C.

② Solve for A, B and C with the three equations obtained in the previous question.

③ Prove using the mathematical induction that the statement is true.