CIS 275 Discrete Math I Summer 2019

Quiz 05

Name:

Time: Complete and submit to the instructor

Evaluation:

- As described in the syllabus, the Quiz is 20% of the overall grade.

Exercise 1:

A five persons committee composed of John, Ben, Alice, Connie, Francisco is to select a chairperson, vice-chairperson, secretary, and treasurer.

1) *How many ways can this be done?*

2) *How many ways can this be done if Alice or Ben must be the chair?*

3) *In how many ways can this be done if John must hold one of the offices?*

4) *In how many ways can this be done if both Connie and Francisco must hold office?*

Exercise 2:

A six persons committee composed of John, Ben, Emily, Alice, Connie, Francisco is to select a chairperson and secretary.

How many selections are there in which either Alice or Emily or both are officers?

Exercise 3:

In how many ways can we select a committee of three from a group of 10 distinct persons?

Exercise 4:

Two fair dice are rolled. What is the probability that the sum of the numbers on the dice is 8?

Exercise 5:

A six-person committee composed of Alice, Ben, Connie, Dolph, Egbert and Francisco is to select chairperson, secretary and Treasurer.

1) How many selections exclude Connie.

2) How many selections are there in which neither Ben nor Francisco is officer?

3) How many selections are there in which Dolph is an officer and Francisco in not an officer?

4) How many selections are there in which either Dolph is chairperson or he is not an officer?

5) How many selections are there in which Ben is either chairperson or treasurer?

Exercise 6:

A six-person committee composed of Alice, Ben, Connie, Dolph, Egbert and Francisco is to select a chairperson, secretary and treasure.

1) How many selections are there in which either Ben is chairperson or Alice is secretary or both?

2) How many selections are there in which either Connie is chairperson or Alice is an officer or both?

Exercise 7: In group of 191 students, 10 are taking French, business, and music; 36 are taking French and Business; 20 are taking French and music; 18 are taking business and music; 65 are taking French; 76 are taking business; and 63 are taking music.

Use the inclusion-Exclusion Principle for three finite sets to determine how many students are not taking any of the three courses.

Exercise 8: A Bag containing 20 balls: six red, six green and eight purple.

In how many ways can we select five balls if the balls are considered distinct?

Exercise 8: Suppose that a professional wrestler is selected at random among 90 wrestlers, where 35 are over 350 pounds, 20 are bad guys, and 15 are over 350 pounds, 20 are bad guys, and 15 are over 350 pounds and bad guys.
What is the probability that the wrestler selected in over 350 pounds or a bad guy?

<u>Exercise 9:</u> A company buys computers from three vendors and tracks the number of defective machines. The following table shows the results:

	Vendor		
	Acme	DotCom	Nuclear
Percent purchased	55	10	35
Percent defective	10	30	30

Let A denote the event "the computer was purchased from Acme", let D denote the event "the computer was purchased from Dotcom" and let N denote the event "the computer was purchased from Nuclear", and let B the event "the computer was defective".

1) Find P(A), P(D), and P(N).

2) Find P(B|A), P(B|D), and P(B|N).

3) Find P(A|B), P(D|B), and P(N|B).

4) Find P(B).

<u>Exercise 10:</u> A Dice are loaded so that the numbers 2, 4 and 6 are equally likely to appear, 1, 3, and 5 are also equally likely to appear, but 1 is three times as likely as 2 is to appear.

1) One die is rolled. Assign probabilities to the outcomes that accurately model the likelihood of the various numbers to appear.

2) One die is rolled. What is the probability of not getting a 5?

3) One die is rolled. What is the probability of getting an even number?

FORMULA

- **The Multiplication Principle**

$$n1 \times n2 \times \ldots. \times nt.$$

- **<u>Definition of the Addition Principle:</u>** Suppose that X1, …, Xt are sets and that the ith set Xi has ni elements. If {X1, …, Xt} is a pairwise disjoint family (if $i \neq j$, Xi Π Xj = ϕ), , the number of possible elements that can be selected from X1 or X2 or … or Xt is

$$n1 + n2 + \ldots + nt$$

(Equivalently, the union X1 U X2 U … U Xt contains n1 + n2 + … + nt elements)

- **The *Inclusion-Exclusion Principle***

$$|X \cup Y| = |X| + |Y| - |X \cap Y|$$

- **A *permutation***

$n (n - 1) (n - 2) \ldots 2 \times 1 = n!$ Permutations of n elements.

- **A *r-permutation***

$P(n, r) = n (n - 1) (n - 2) .. (n - r + 1) = n!/(n-r)!$, where r <= n.

$$P(n, n) = n!$$

- **A combination**

$C(n, r) = P(n, r)/r! = n!/r!(n - r)!$, where r <= n.

- **The probability P(E) of an event E from the finite sample space S is**

$$P(E) = |E| / |S|$$

- **<u>Definition:</u> A *probability function P* assigns to each outcome x in a sample space S a number P(x) so that**

  $$0 \leq P(x) \leq 1,$$

  And

  $$\sum_{x \notin S} P(x) = 1$$

- **<u>Theorem:</u>** Let E be an event. The probability of E, the complement of E, satisfies

  P(E) + P(E) = 1

- **<u>Theorem:</u>** <u>*The probability of the union of two events.*</u>

Let E1 and E2 be events, then

P(E1 U E2) = P(E1) + P(E2) – P(E1 ∩ E2)

- **<u>Definition:</u>** Bayes' Theorem is useful in computing the probability of a class given a set of features. **<u>Bayes' Theorem:</u>** Suppose that the possible classes are C1, …, Cn. Suppose further that each pair of classes is mutually exclusive and each item to be classified belongs to one of the classes. For a feature set F, we have

$$P(C_j \mid F) = \frac{P(F \mid C_j)P(C_j)}{\sum_{i=1}^{n} P(F \mid C_i)P(C_i)}$$