Quiz 04

Name:

Time: Complete and submit to the instructor

Evaluation:

- As described in the syllabus, the Quiz is 20% of the overall grade.

Exercise 1: Let the universe be the set U ={1, 2, 3, … , 10}.

Let A = {1, 4, 7, 9}, B = {1, 2, 3, 4, 5,6} and C = {2, 4, 7, 6, 8}.

List the elements of each set

- $A \cap (B \cup C)$
- $\bar{B} \cap (C - A)$
- $(A \cap B) - C$
- $\overline{A \cap B} \cup C$
- $(A \cup B) - (C - B)$

Exercise 2: Use the Mathematical induction to prove that the statement is verified.

1) Use the geometric sum to prove that

$$r^0 + r^1 + \ldots + r^n = \frac{r^{n+1}-1}{r-1}$$

For all $n \geq 0$ and $0 < r < 1$

2) $1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n+1)(2n+1)/6 \qquad n \geq 1$

Exercise 3:

1) Define a sequence S as

$$\mathbf{T_n = 2^n + 4 \times 3^n} \qquad n \geq 0$$

a) Find T_0

b) Find T_1

c) Find a formula of T_i

d) Find a formula for T_{n-1}

e) Find a formula for T_{n-2}

f) Prove that {Sn} satisfies: $T_n = 5\ T_{n-1} - 6\ T_{n-2}$

2) Consider the sequence A defined by $An = n^2 - 3n + 3$

1. Find

$$\sum_{i=1}^{4} A_i$$

2. Find

$$\sum_{j=3}^{5} A_j$$

3. Find

$$\prod_{i=1}^{2} A_i$$

4. Find

$$\prod_{x=3}^{4} A_x$$

5. Is A increasing?

6. Is A decreasing?

7. Is A nonincreasing?

8. Is A nondecreasing?

Exercise 4:

a) Use the mathematical induction to show that

$$\sum_{k=1}^{n} f_k^2 = f_n \, f_{n+1} \qquad \mathbf{n} \geq \mathbf{1}$$

b) Let consider the formula

$$s_1 = 2, \qquad s_n = s_{n-1} + 2n \qquad \text{for all } n \geq 2$$

Write the recursive algorithm that computes: $s_n = 2 + 4 + 6 + \ldots + 2n$.

Exercise 5:

1. Express each binary (base 2) number in decimal (base 10).

    a) 1001

    b) 100000

2. Express each decimal (base 10) number in binary (base 2).

    a) 43

    b) 400

3. Express each hexadecimal (base 16) number in decimal (base 10)

    a) 3A

    b) A03

4. Add the binary numbers with base 2.

   a) 1001 + 1110

5. Add the Hexadecimal numbers with base 16.

   $F0BA_{16}$ and $B8AD_{16}$

6. Use the Euclidean algorithm to find the greatest common divisor of each pair of integers.

   a) 60, 90

   b) 30, 105

Formula

The Sets

Let U be a universal set and let A, B, and C be subsets of U. The following properties hold.

(a) Associative laws:

$$(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$$

(b) Commutative laws:

$$A \cup B = B \cup A, \quad A \cap B = B \cap A$$

(c) Distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(d) Identity laws:

$$A \cup \varnothing = A, \quad A \cap U = A$$

(e) Complement laws:

$$A \cup \overline{A} = U, \quad A \cap \overline{A} = \varnothing$$

(f) Idempotent laws:

$$A \cup A = A, \quad A \cap A = A$$

(g) Bound laws:

$$A \cup U = U, \quad A \cap \varnothing = \varnothing$$

(h) Absorption laws:

$$A \cup (A \cap B) = A, \quad A \cap (A \cup B) = A$$

(i) Involution law:

$$\overline{\overline{A}} = A$$

(j) 0/1 laws:

$$\overline{\varnothing} = U, \quad \overline{U} = \varnothing$$

(k) De Morgan's laws for sets:

$$\overline{(A \cup B)} = \overline{A} \cap \overline{B}, \quad \overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

Proof The proofs are left as exercises (Exercises 44–54, Section 2.1) to be done after more discussion of logic and proof techniques.

Mathematical induction

The Principle of Mathematical Induction consists of two steps:

- *Basic step :* Prove that S(1) is true.
- *Inductive step :* Assuming that S (n) is true for n ≥1, prove that S(n+1) is true

Then, S(n) is true for every positive integer n.

The Sequences

A *sequence* is a special type of function in which the domain consists of a set of consecutive integers.

Let **Sn** denoted the entire sequence:

S1, S2, S3, S4, S5, …

We use the notation Sn to denote the single element of the sequence S at *index* n.

- A sequence S is **increasing** if Sn < Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **decreasing** if Sn > Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **nondecreasing** if Sn ≤ Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **nonincreasing** if Sn ≥ Sn+1 for all n for which n and n+1 are in the domain of the sequence.

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \ldots + a_n$$

$$\prod_{i=m}^{n} a_i = a_m \times a_{m+1} \times \ldots. \times a_n$$

The recursive algorithms :

The *Fibonacci sequence* { f_n } is defined by the equations

$$f_0 = 0$$

$$f_1 = 1$$

$$f_2 = 2$$

$$\mathbf{f_n = f_{n-1} + f_{n-2}} \qquad \text{for all } n \geq 3$$

The Fibonacci sequence begins

1, 1, 2, 3, 5, 8, 13, …

In mathematical terms, the sequence F_n of Fibonacci numbers is defined by the recursive relation $f_n = f_{n-1} + f_{n-2}$

Introduction to number theory :

The ***Euclidean algorithm*** is an odd, famous, and efficient algorithm for finding the greatest common divisor of two integers,

The Euclidean algorithm is based on the fact that if $r = a \bmod b$, then

$$\mathbf{gcd(a, b) = gcd(b, r)}$$

Hexadecimal to Decimal Conversion Chart

Hexadecimal	Decimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
A	10
B	11
C	12
D	13
E	14
F	15