Chapter 6: Counting Methods
Correction of Exercises

I. <u>Introduction:</u>

Exercise 1: Use the Multiplication Principle:

The menu for Kay's Quick lunch is shown in this table.

Appetizers	**2.15**
Nachos	**1.90**
Salad	
Main courses	**3.25**
Hamburger	**3.65**
Cheeseburger	**3.15**
Fish filet	
Beverages	**0.70**
Tea	**0.85**
Milk	**0.75**
Cola	**0.75**
Root Beer	

How many dinners at Kay's Quick Lunch consist of one appetizer and one beverage?

$$\mathbf{2 \times 4 = 6}$$

Exercise 2: Use the Multiplication Principle: A man has eight shirts, four pairs of pants, and five pairs of shoes. How many different outfits are possible?

$$\mathbf{8 \times 4 \times 5 = 160}$$

Exercise 3: **Use the Multiplication Principle:** Two dice are rolled, one blue and one red. How many outcomes are possible?

$$\mathbf{6^2 = 36}$$

<u>**Exercise 4: Use the addition Principle:**</u> Three departmental committees have 6, 12 and 9 members with no overlapping membership. In how many ways can these committees send one member to meet with the president?

$$\mathbf{6 + 12 + 9 = 27}$$

Exercise 5: Two dices are rolled, one blue and one red.

a) How many outcomes give the sum of 4?

There is 3 ways to get the sum of 4, which are (1, 3), (2, 2), (3, 1), where (b, r) means the blue die shows b and the red die shows r.

b) How many outcomes have the blue die showing 2?

There is 6 ways to have the blue die showing 2, which are (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), where (b, r) means the blue die shows b and the red die shows r.

c) How many outcomes have neither die showing 2?

Since each die can show any one of five values, by the Multiplication Principle there are $5 \times 5 = 25$ outcomes in which neither die shows 2.

Exercise 6: In group of 191 students, 10 are taking French, business, and music; 36 are taking French and Business; 20 are taking French and music; 18 are taking business and music; 65 are taking French; 76 are taking business; and 63 are taking music.

Use the inclusion-Exclusion Principle for three finite sets to determine how many students are not taking any of the three courses.

Let F be the set of students taking French, let B be the set of students taking Business, and let M be the set of students taking Music. We give that $|F \cap B \cap M| = 10$, $|F \cap B| = 36$, $|F \cap M| = 20$, $|B \cap M| = 18$, $|F| = 65$, $|B| = 76$, and $|M| = 63$.

$$|F \cup B \cup M| = |F| + |B| + |M| - |F \cap B| - |F \cap M| - |B \cap M| + |F \cap B \cap M|$$
$$= 65 + 76 + 63 - 36 - 20 - 18 + 10 = 140$$

Thus 140 students are taking French or business or music. Since there are 191 students, $191 - 140 = 51$ are not any of the three courses.

II. Permutations and Combinations:

Exercise 1: How many permutations are there of a. b, c, d?

$$4! = 24$$

Exercise 2: In how many ways can we select a chairperson, vice-chairperson, and recorder from a group of 11 persons?

$$P(11, 3) = 11 \times 10 \times 9$$

Exercise 3:

Determine how many strings can be formed by ordering the letters ABCDE :

a) Contains the substring ACE

$$3!$$

b) Contains either the substring AE or the substring EA or both.

4! Contain the substring AE and 4! Contain the substring EA; therefore, the total is

$$2 \times 4!$$

Exercise 4: In how many ways can five distinct Martians and eight distinct Jovians wait in line if no two Martians stand together?

Fix a seat for a Jovian. There are 7! arrangements for the remaining Jovians. For each of these arrangements, we can place the Martians in five of the eight in-between positions, which can be done in P(8, 5) ways. Thus there are 7! P(8, 5) such arrangements.

III. Discrete Probability Theory:

Exercise 1: A die is loaded so that the number 2 through 6 are equally likely to appear, but 1 is three times as any other number to appear. One die is rolled.

What is the probability of getting a 5?

the probability of getting a 5 = 1/8

Exercise 2: A dice that are loaded so that the number 2, 4 and 6 are equally likely to appear. 1, 3 and 5 are also equally likely to appear, but 1 is three times as likely as 2 is to appear.

a) One dice is rolled. What is the probability of not getting a 5? 1 – (1/4) = 3/4
b) Two dice is rolled. What is the probability of getting double?

$$3\ (1/12)^2 + 3\ (3/12)^2$$

Exercise 3: Suppose that a coin is flipped and a die is rolled. Let E1 denote the event "the coin shows a tail", let E2 denote the event "the die shows a 3", let E3 denote the event "the coin shows heads and the die shows an odd number".

a) List the element of the event E1 or E2.

(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6), (H, 3)

b) Are E1 and E3 mutually exclusive?

Yes, E1 and E3 are mutually exclusive.

Exercise 4: Six microprocessors are randomly selected from 100 microprocessors among which 10 are defective. Find the probability of obtaining no defective microprocessors.

C(90, 6) / C(100, 6)

IV. Binomial coefficients and Combinational Identities:

Exercise 1: Expand $(x+y)^4$ using the Binomial Theorem.

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Exercise 2: Prove $n(1+x)^{n-1} = n\sum_{k=0}^{n-1} C(n-1,k)x^k$

Set a = 1 and b = x and replace n by n-1 in the Binomial Theorem to obtain

$$n(1+x)^{n-1} = \sum_{k=0}^{n-1} C(n,k)kx^{k-1}$$

Now multiply by n to obtain

$$n(1+x)^{n-1} = n\sum_{k=0}^{n-1} C(n-1,k)x^k = n\sum_{k=1}^{n} C(n-1,k-1)x^{k-1}$$

$$= \sum_{k=1}^{n} \frac{n(n-1)!}{(n-k)!(k-1)!}x^{k-1} = \sum_{k=1}^{n} \frac{n!}{(n-k)!k!}kx^{k-1} = \sum_{k=1}^{n} C(n,k)kx^{k-1}$$