CIS 275 Discrete Math I Winter 2019

Correction-Quiz 01

Name:

Time: Complete and submit to the instructor

Evaluation:

- As described in the syllabus, the Quiz is 20% of the overall grade.

<u>**Exercise1:** </u>select the correct answer.

❖ Let define the universe be U ={1, 2, 3 ,…, 10}, determine the element of the set $\overline{U}$

1. $\overline{U}$ = U – U ={1, 2, 3 ,…, 10}.
2. $\overline{U} = \emptyset$
3. $\overline{U}$ = {1, 10}

❖ Let A = {1, 4, 7, 10}, B = {1, 2, 3, 4, 5}, C = {2, 4, 6, 8} and U ={1, 2, 3 ,…, 10}, determine $\overline{B} \cap (C - A)$

1. {6, 8}
2. {2, 6, 8}
3. {6, 7, 8, 9, 10}

❖ Let A ={1, 3, 6, 9, 10}, determine A ∪ $\emptyset$

1. $\emptyset$
2. $\{\emptyset, 1, 10\}$
3. A

❖ Let define A ={$x \mid x^2 - 4x + 4 = 1$} and B = {1, 3, 5}, determine the correct answer

1. A is not a subset of B
2. $\Delta < 0$: There is not solution in the set A.
3. A is a subset of B

❖ Let consider the Cartesian product X × Y = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}.

Determine the domain X and the codomain Y.

1. X =_{1, 2} and Y = {a, b, c}
2. X =_{1} and Y = {a, c}
3. X =_{1, 2} and Y = {a, b}

❖ Let A ={0, 1, 1, 2} and B = {1, 2, 2}. Determine the correct answer

1. The sets A and B are equal.
2. The sets A and B have the same elements.
3. The sets A and B aren't equal.

❖ Let consider the set A = {1, 3} and B = {x | $3x^2$ + x – 2 =0}

1. A is a proper subset of B.
2. A is a subset of B.
3. A is neither a subset nor a proper subset of B.

❖ Let consider the set A = {1, 2, 4, 6, 8, 9, 11}. Determine the partition X of the set A.

1. X ={[11], [8, 9], [2, 6], [1]}
2. X ={[9, 11], [8], [2, 6], [4]}
3. X ={[8, 11], [9], [2, 4, 6], [1]}

❖ Let consider the set X = {1} and Y = {y| y^2 + y – 2 =0}. Determine the answer.

1. X is a subset of Y.
2. X is a proper subset of Y.
3. X and Y aren't equal.

❖ Let consider the set A = {1, 2, 7} and B = {0, 3, 8}. Determine the correct answer.

1. B – A = {3, 8}
2. A – B = {1, 7}
3. B – A ≠ A – B

❖ Let A = {1, 5, 8, 9}, B = {1, 2, 3, 4, 5, 6}, C = {3, 5, 6, 8} and U = {1, 2, 3,…, 10}, determine B ∩ ($\overline{C \cup A}$)

1. {2, 4}
2. {2, 6, 7}
3. {2, 7, 10}

❖ Let define A = $\{x|x^2 + x = 2\}$ and B ={2, 1}, determine the correct answer

1. A is not equal to B
2. $\Delta < 0$: There is not solution in the set A.
3. A is equal to B

❖ The element in the set X × Y = {(1, a), (1, b), (1, c)}. Determine the domain X and the codomain Y.

1. X = {1, 2} and Y = {a, b, c}
2. X = {1} and Y = {a, c}
3. X = {1} and Y = {a, b, c}

❖ Determine $\overline{\emptyset}$

1. $\overline{U}$
2. $\emptyset$
3. U

❖ Let A ={1, 3, 6, 9, 10}, determine A ∩ ∅

1. $\emptyset$
2. $\{\emptyset, 1, 6, 10\}$
3. A

Exercise 2: Let the universe be the set U ={1, 2, 3,…, 10}.

Let A = {1, 2, 3, 4, 7, 10}, B = {1, 2, 3, 4, 5, 6} and C = {2, 3, 6, 8}.

List the elements of each set

- $A \cap (B \cup C) - \bar{A}$

{1, 2, 3, 4}

- $\bar{B} \cap (C - A) \cap (\overline{A \cap B})$

{8}

- $(A \cap B) - C \cup (B \cap U)$

{∅}

- $\overline{A \cup B} \cup (C - A) - \bar{U}$

{6, 8, 9}

- $(A \cup B) - (C - B) \cap (A \cup \bar{B})$

{1, 2, 3, 4, 7, 10}

Exercise 3: Using the induction, verify that each equation is true for every positive integer n, n $\geq$ 1.

a) $1 * 2 + 2 * 3 + 3 * 4 + \ldots + n\,(n + 1) = \frac{n\,(n+1)(n+2)}{3}$

1. Basic step: If n = 1, we have $\frac{n\,(n+1)(n+2)}{3} = \frac{1\,(1+1)(1+2)}{3} = \mathbf{2}$

For n = 1, the statement is true.

2. Inductive step:

Let assume that the statement is true for n = 1. We must prove then show that the statement is true for n+1, n $\geq$ 1.

$$1 * 2 + 2 * 3 + 3 * 4 + \ldots + n\,(n + 1) + (n+1)\,(n+2) = \frac{n\,(n+1)(n+2)}{3} + (n+1)\,(n+2)$$

$$= \frac{n\,(n+1)(n+2)+3\,(n+1)\,(n+2)}{3} = \frac{(n+1)(n+2)+(n+3)}{3}$$

- ***Since the Basic step and the Inductive step have been verified, the Principle of Mathematical Induction tells us that the geometric sum is true for n ≥ 1.***

b) 1(1!) + 2(2!) + … + n(n!) = (n+1)! – 1

1. Basic step : The statement is true if n = 1.

This is easily accomplished, since (1+1)! - 1= 1

For n = 1, the statement is true.

2. Inductive step : If the statement is true. We must then prove that the inequality is true for n+1:

1(1!) + 2(2!) + … + n(n!) + (n+1) ((n+1)!) = (n+1)! – 1 + (n+1) ((n+1)!)

= (n+1)!(1 + (n+1)) – 1 = (n+1)!(n+2) – 1 = (n+2) * (n+1) * n! – 1 = **(n+2)! – 1**

So, the statement is true for all n $\geq$ 1.

c) $\mathbf{1^3 + 2^3 + 3^3 + \ldots + n^3} = \left(\frac{\boldsymbol{n(n+1)}}{\mathbf{2}}\right)^{\mathbf{2}}$

1. Basic step: If n = 1, we have $\left(\frac{\boldsymbol{n(n+1)}}{\mathbf{2}}\right)^{\mathbf{2}} = \left(\frac{\mathbf{1(1+1)}}{\mathbf{2}}\right)^{\mathbf{2}} = \mathbf{1}$

For n = 1, the statement is true.

2. Inductive step:

Let assume that the statement is true for n = 1. We must prove then show that the statement is true for n+1, n ≥ 1.

a) $\mathbf{1^3 + 2^3 + 3^3 + \ldots + n^3 + (n+1)^3} = \left(\frac{\boldsymbol{n(n+1)}}{\mathbf{2}}\right)^{\mathbf{2}} + \mathbf{(n+1)^3} = \frac{\boldsymbol{(n^2 * (n+1)^2) + 4\,(n+1)^3}}{\mathbf{4}}$

$= \frac{\boldsymbol{(n+1)^2(n^2+4\,(n+1)^1)}}{\mathbf{4}} = \frac{\boldsymbol{(n+1)^2(n^2+4\,n+4)}}{\mathbf{4}} = \frac{\boldsymbol{(n+1)^2(n+2)^2}}{\mathbf{4}} = \left(\frac{\boldsymbol{(n+1)(n+2)}}{\mathbf{2}}\right)^{\mathbf{2}}$

- ***Since the Basic step and the Inductive step have been verified, the Principle of Mathematical Induction tells us that the geometric sum is true for n ≥ 1.***

Exercise 4: Using the induction, verify the inequality

$$2n + 1 \leq 2^n,\ n = 3, 4, \ldots.$$

1. Basic step : The statement is true if n = 3.

This is easily accomplished, since $2 * 3 + 1 = 5 < 2^3 = 8$

2. Inductive step : If $2n + 1 \leq 2^n$ is true. We must then prove that the inequality is true for n+1: $2(n+1) + 1 \leq 2^{n+1}$

$2^{n+1} = 2^n * 2 = (2n + 1) * 2 = 4n + 2$

since n = 3, 4, ….

$4*3 + 2 = 14 > 2*(3+1) + 1 = 9$

We can conclude that $2^{n+1} = 2^n * 2 = (2n + 1) * 2 = 4n + 2 > 2(n+1) + 1$

So, the statement is true for all n = 3, 4, ….

<u>Exercise 5:</u> Use the induction to prove the statement

$6 * 7^n - 2 * 3^n$ is divisible by 4, for all n ≥ 1

1. *<u>Basic Step:</u>*

If n = 1, we have $6 * 7^n - 2 * 3^n = 6 * 7^1 - 2 * 3^1 = 36$, which is divisible by 4.

2. *<u>Inductive step:</u>*

We assume that $6 * 7^n - 2 * 3^n$is divisible by 4. We must then show that $6 * 7^{n+1} - 2 * 3^{n+1}$ is divisible by 4.

$$6 * 7^{n+1} - 2 * 3^{n+1} = 6 * 7^n * 7^1 - 2 * 3^n * 3^1 = 6 * 7^n * (3 + 4) - (2 + 4) * 3^n$$

If n = 2, $6 * 7^{2+1} - 2 * 3^{2+1} = 2058 - 54 = 2004$ $\Rightarrow$ 2004/2 = 1002

By the inductive assumption, $6 * 7^n - 2 * 3^n$is divisible by 4, the sum $6 * 7^n * (3 + 4) - (2 + 4) * 3^n = 6 * 7^{n+1} - 2 * 3^{n+1}$ is divisible by 4.

- ***Since the Basic step and the Inductive step have been verified, the Principle of Mathematical Induction tells us that*** $6 * 7^n - 2 * 3^n$***is divisible by 4 for all n ≥ 1.***

Formula

<u>The Sets</u>

Let U be a universal set and let A, B, and C be subsets of U. The following properties hold.

(a) Associative laws:

$$(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$$

(b) Commutative laws:

$$A \cup B = B \cup A, \quad A \cap B = B \cap A$$

(c) Distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(d) Identity laws:

$$A \cup \varnothing = A, \quad A \cap U = A$$

(e) Complement laws:

$$A \cup \overline{A} = U, \quad A \cap \overline{A} = \varnothing$$

(f) Idempotent laws:

$$A \cup A = A, \quad A \cap A = A$$

(g) Bound laws:

$$A \cup U = U, \quad A \cap \varnothing = \varnothing$$

(h) Absorption laws:

$$A \cup (A \cap B) = A, \quad A \cap (A \cup B) = A$$

(i) Involution law:

$$\overline{\overline{A}} = A$$

(j) 0/1 laws:

$$\overline{\varnothing} = U, \quad \overline{U} = \varnothing$$

(k) De Morgan's laws for sets:

$$\overline{(A \cup B)} = \overline{A} \cap \overline{B}, \quad \overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

Proof The proofs are left as exercises (Exercises 44–54, Section 2.1) to be done after more discussion of logic and proof techniques.

$$\overline{B} = U - B$$

Quadratic equation

Theorem:

For $ax^2 + bx + c = 0$

$Delta = \Delta = b^2 - 4ac$

If $\sqrt{\Delta} \geq 0$ *: we have two solutions.*

If $\sqrt{\Delta} < 0$ *: we don't have any solution.*

$x_1 = \frac{-b-\sqrt{\Delta}}{2a}$ *and* $x_2 = \frac{-b+\sqrt{\Delta}}{2a}$

$ax^2 + bx + c = a\,(x - x_1)(x - x_2)$