CIS 275 Discrete Math I Summer 2019

**Correction of Quiz 02**

**Name:**

**Time:** Complete and submit to the instructor

**Evaluation:**

- As described in the syllabus, the Quiz is 20% of the overall grade.

**Exercise 1:** select the correct answer.

- Let consider a function

$$\mathbf{f(u) = 70 + 15 \lceil u - 1 \rceil} \qquad 0 \leq u \leq 13$$

If u = 2.5. Find the value of f(u).

1. f(u) = 100
2. f(u) = 85
3. f(u) = 70

- Let consider the sequence

$$\mathbf{A_i = {}^{1}/_{i}} \qquad i \geq 1$$

Is $A_i$ decreasing or increasing or nonincreasing ?

1. $A_i$ is decreasing ($S_n > S_{n+1}$)
2. $A_i$ is nonincreasing ($S_n \geq S_{n+1}$)
3. $A_i$ is increasing ($S_n < S_{n+1}$)

- Consider the sequence A defined by

**$A_n = n^2 - 3n + 3$**

Find the product $\prod_{i=1}^{2} A_i$ :

1. $\prod_{i=1}^{2} A_i = 1$
2. $\prod_{i=1}^{2} A_i = 2$
3. $\prod_{i=1}^{2} A_i = 0$

- Let consider a function

**f = {(1,c),(2,a),(3,b)}**

We define the domain X = {1, 2, 3} and the codomain Y = {a, b, c}.

Is the function f one-to-one, onto or a bijection?

1. This function is not one-to-one
2. This function is not onto
3. This function is called a bijection.

- Let consider the sequence

**b , c**

This sequence is a subsequence of the sequence $T_n$. We define $1 \leq n \leq 5$.

Find the element of the sequence $T_n$ :

1. $T_n = \{ T_1 = a , T_2 = a , T_3 = b , T_4 = c , T_5 = d \}$
2. $T_n = \{ T_1 = b , T_2 = b , T_3 = c , T_4 = a , T_5 = d \}$
3. $T_n = \{ T_1 = c , T_2 = b , T_3 = a , T_4 = b , T_5 = d \}$

- Let consider the function g and f

$$\mathbf{g = \{(1, a), (2, a), (3, c)\}}$$

$$\mathbf{f = \{(a, y), (b, x), (c, z)\}}$$

We define the function f from X = {1, 2, 3} to Y = {a, b, c}, and the function g from Y = {a, b, c} to Z = {x, y, z}.

Find the composition function from f to g.

1. f o g = {(1, y), (2, y), (3, z)}
2. f o g = {(1, y), (2, y), (2, x)}
3. f o g = {(1, y), (1, z), (2, z)}

- Consider the sequence T defined by

$$\mathbf{T_n = 2n - 1}$$

Find the sum $\sum_{i=1}^{3} T_i$.

1. $\sum_{i=1}^{3} T_i = 8$
2. $\sum_{i=1}^{3} T_i = 10$
3. $\sum_{i=1}^{3} T_i = 9$

- Let consider the function

$$\mathbf{f = \{(1, a), (2, c), (3, b)\}}$$

We define the domain X = {1, 2, 3} and the range Y = {a, b, c}.

Find the inverse of the function f.

1. $f^{-1}$ = {(a, 1),(c, 2), (3, b)}
2. $f^{-1} = \{(a, 1), (c, 2), (b, 3)\}$
3. $f^{-1}$ = {(a, 1), (2, c), (3, b)}

**Exercise 2:** Consider the matrix

$$\begin{array}{c|cccc} & w & x & y & z \\ a & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ b & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ c & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ d & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{array}$$

1. Write the relation R, given by the matrix, as a set of ordered pairs. Determine the domain and the range of the inverse of the relation R.

$$R = \{(a, w), (a, y), (c, y), (d, w), (d, x), (d, y), (d, z)\}.$$

The domain = (a, b, c, d) and the codomain = (w, x, y, z)

2. Find the matrix of the product $R^2$.

$$\begin{array}{c|cccc} & w & x & y & z \\ a & \mathbf{1} & \mathbf{0} & \mathbf{2} & \mathbf{0} \\ b & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ c & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ d & \mathbf{2} & \mathbf{1} & \mathbf{3} & \mathbf{1} \end{array}$$

3. Write the inverse of the relation R, given by the matrix, as a set of ordered pairs. Determine the domain and the range of the inverse of the relation R.

$$R^{-1} = \{(w, a), (y, a), (y, c), (w, d), (x, d), (y, d), (z, d)\}.$$

The domain = (w, x, y, z) and the codomain = (a, b, c, d)

4. Find the matrix of the inverse of the relation R.

$$\begin{array}{c|cccc} & a & b & c & d \\ w & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ x & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ y & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ z & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{array}$$

**Exercise 3:**

Let the relations
R1 = $\{(x, y) | x \text{ divides } y\}$, R1 is from X to Y. R2 = $\{(y, z) | y > z\}$, R2 is from Y to Z, ordering of X and Y: 2, 3, 4, 5; ordering of Z: 1, 2, 3, 4

1. Find the matrix A1 of the relation R1

R1 = {(2, 2), (2, 4), (3, 3), (4, 2), (4, 4), (5, 5)}

| | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 2 | **1** | **0** | **1** | **0** |
| 3 | **0** | **1** | **0** | **0** |
| 4 | **1** | **0** | **1** | **0** |
| 5 | **0** | **0** | **0** | **1** |

2. Find the matrix A2 of the relation R2

R2 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)}

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 2 | **1** | **0** | **0** | **0** |
| 3 | **1** | **1** | **0** | **0** |
| 4 | **1** | **1** | **1** | **0** |
| 5 | **1** | **1** | **1** | **1** |

3. Find the matrix product A1 A2

4. Find the relation R2 o R1

R2 o R1 = {(2, 1), (2, 2), (3, 1), (5, 1), (4, 1), (5, 2), (5, 3), (5, 4), (3, 2), (4, 2), 4, 3)}

5. Find the matrix of the relation R2 o R1

**Exercise 4:** Let each function is one-to-one on the specified domain X. If Y = range of f, we obtain a bijection from X to Y. Find each inverse function

- $f(x) = 4x + 2$ x = set of real numbers

*Let* $y = 4x + 2$ *we find* $x = (x-2)/4$ $f^{-1} = (y-2)/4$

- $f(x) = 3^x$ x = set of real numbers

*Let* $y = 3x$ *we find* $x = \log y/\log 3$ $f^{-1} = \log y/\log 3$

- $f(x) = 3 + 1/x$ x = set of nonzero real numbers

*Let* $y = 3 + 1/x$ *we find* $x = 1/(y-3)$ $f^{-1} = 1/(y-3)$

**Exercise 5:** Consider the relation R on the set {1, 2, 3, 4, 5} defined by the rule $(x, y) \in R$ if 3 divides $x - y$

1. List the element of R

R = {(1, 1), (1, 4), (2, 2), (2, 5), (3, 3), (4, 1), (4, 4), (5, 2), (5, 5)}

2. List the element of $R^{-1}$

$R^{-1}$ = {(1, 1), (4, 1), (2, 2), (5, 2), (3, 3), (1, 4), (4, 4), (2, 5), (5, 5)}

3. Is the element of R is reflexive, symmetric, antisymmetric, transitive and/or partial order?

The relation **R from X to Y is a partial order** because R is reflexive, symmetric, antisymmetric and transitive.

- *Reflexive:* (1, 1), (2, 2), (3, 3), (4, 4) and (5, 5) are each in R.
- symmetric: (1, 4), (4, 1), (2, 5) and (5, 2) are each in R.
- Antisymmetric: (1, 1), (2, 2), (3, 3), (4, 4) and (5, 5) are each in R.
- Transitive: (1, 1), (2, 2), (3, 3), (4, 4) and (5, 5) are each in R.

**<u>Exercise 6:</u>** Consider the sequence A defined by An = $n^2$ - 3n +3

1. Find

$\sum_{i=1}^{4} A_i = A_1 + A_2 + A_3 + A_4 = 1 + 1 + 3 + 7 = 12$

2. Find

$\sum_{j=3}^{5} A_j = A_3 + A_4 + A_5 = 3 + 7 + 13 = 23$

3. Find

$\prod_{i=1}^{2} A_i = A_1 \times A_2 = 1 \times 1 = 1$

4. Find

$\prod_{x=3}^{4} A_x = A_3 \times A_4 = 3 \times 7 = 21$

5. Is A increasing?

A is not increasing because A1 < A2 but A2 = A1.

6. Is A decreasing?

A is not decreasing because A1 $\ngtr$ A2 and A3 $\ngtr$ A4

7. Is A nonincreasing?

A isn't nonincreasing because A2 $\ngeq$ A3 and A4 $\ngeq$ A5

8. Is A nondecreasing?

A is nondecreasing because A2 $\leq$ A3 and A2 = A3

# Formula

## The Sequences

A *sequence* is a special type of function in which the domain consists of a set of consecutive integers.

Let **Sn** denoted the entire sequence:

S1, S2, S3, S4, S5, …

We use the notation Sn to denote the single element of the sequence S at *index* n.

- A sequence S is **increasing** if Sn < Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **decreasing** if Sn > Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **nondecreasing** if Sn ≤ Sn+1 for all n for which n and n+1 are in the domain of the sequence.
- A sequence S is **nonincreasing** if Sn ≥ Sn+1 for all n for which n and n+1 are in the domain of the sequence.

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \ldots + a_n$$

$$\prod_{i=m}^{n} a_i = a_m \times a_{m+1} \times \ldots\ldots \times a_n$$