Correction of Exercises
CHAPTER 1: SETS

Exercise 1: Let the universe be the set U ={1, 2, 3, … , 10}.
Let A = {1, 4, 7, 10}, B = {1, 2, 3, 4, 5} and C = {2, 4, 6, 8}.
List the elements of each set

- ❖ A U B = {1, 2, 3, 4, 5, 7, 10}

- ❖ B ∩ C = {2, 4}

- ❖ A – B = {7, 10}

- ❖ B – A = {2, 3, 5}

- ❖ Ā = U – A = {2, 3, 5, 6, 8, 9}

- ❖ U – C = {1, 3, 5, 7, 9, 10}

- ❖ $\bar{U}$ = $\emptyset$

- ❖ A U $\emptyset$ = A = {1, 4, 7, 10}

- ❖ B ∩ $\emptyset$ = $\emptyset$

- ❖ A U U = *{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}* = U

- ❖ B ∩ U = *{1, 2, 3, 4, 5}* = B

- ❖ A ∩ (B U C) = {1, 4, 7, 10} ∩ {1, 2, 3, 4, 5, 6, 8}= {1, 4}

- ❖ $\bar{B}$ ∩ (C – A) = (U – B) ∩ (C – A)

  = {6, 7, 8, 9, 10} ∩ {2, 6, 8} ={6, 8}

- ❖ (A ∩ B) – C = {1, 4} - {2, 4, 6, 8} = {1}

- ❖ $\overline{A \cap B}$ U C = ($\bar{A}$ U $\bar{B}$) U C = ({2, 3, 5, 6, 8, 9} U {6, 7, 8, 9, 10}) U C

$$= \{2, 3, 5, 6, 7, 8, 9, 10\} \cup \{2, 4, 6, 8\}$$
$$= \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

❖ $(A \cup B) - (C - B) = \{1, 2, 3, 4, 5, 7, 10\} - \{6, 8\}$
$$= \{1, 2, 3, 4, 5, 7, 10\} = (A \cup B)$$

Exercise 2: Determinate $A \subseteq B$

❖ $A = \{1, 2\}$ and $B = \{3, 2, 1\}$

Every element of A is an element of B.

Therefore, $A = \{1, 2\}$ is a subset of $B = \{3, 2, 1\}$ and we write $A \subseteq B$

❖ $A = \{1, 2\}$ and $B = \{x \mid x^3 - 6x^2 + 11x = 6\}$

$x^3 - 6x^2 + 11x = 6$

$x^3 - 6x^2 + 11x - 6 = 0$

$\color{red}{(x - 1)(x^2 + m\,x + 6) = 0}$

$(x - 1)(x^2 + m\,x + 6) = 0$

$x^3 + mx^2 + 6x - x^2 - m\,x - 6 = 0$

$x^3 + (m - 1)x^2 + x(6 - m) - 6 = 0$

As $x^3 - 6x^2 + 11x - 6 = 0$

We will define:

$$m - 1 = -6$$
$$6 - m = 11$$
$$\mathbf{m = -5}$$

$$For\ ax^2 + bx + c = 0$$

$$Delta = \Delta = b^2 - 4ac$$

If $\sqrt{\Delta} > 0$: we have two solutions.
If $\sqrt{\Delta} < 0$: we don't have any solution.

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} \quad and \quad x_2 = \frac{-b+\sqrt{\Delta}}{2a}$$

$$ax^2 + bx + c = a\ (x - x_1)(x - x_2)$$

We have a quadratic equation: $x^2 - 5x + 6 = 0$

$\Delta = b^2 - 4ac = \Delta = (-5)^2 - 4 \times 1 \times 6 = 25 - 24 = 1$

$\sqrt{\Delta} = \sqrt{1} = 1$

$x_1 = \frac{-(-5)-\sqrt{1}}{2\times 1} = \frac{4}{2} = 2$

$x_2 = \frac{-(-5)+\sqrt{1}}{2\times 1} = \frac{6}{2} = 3$

$x^3 - 6x^2 + 11x - 6 = (x-1)(x^2 - 5x + 6) = (x-1)(x-2)(x-3)$

So, **B = {1, 2, 3}**

Every element of A= {1, 2} is an element of B = {1, 2, 3}

Therefore, A is a subset of B and we write A ⊆ B

Exercise 3: Show that A is not a subset of B.

❖ A = {1, 2, 3} and B = {1, 2}

Only one element (the number 3) of A is not an element of B.

Since 3 ∈ A but 3 ∉ B

Therefore, A is not a subset of B and we write A ⊄ B.

❖ A = {1, 2, 3} and B = ∅

No element of A is an element of B, the empty set.

Therefore, A is not a subset of B and we write A ⊄ B.

Exercise 4: Let X = {1, 2} and Y = {a, b, c}. List the element of each seat.

❖ X × Y = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}

❖ X × X = {(1, 1), (1, 2), (2, 2), (2, 1)}