MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1. (a) Government issue bonds to raise fund from investors for financing day to day projects.
   (b) If the price sensitivity to interest rate is similar for long-term bonds, those bonds tends to also have similar yield which cause the yield curve flatten at the long-term part.
   (c) Quantitative easing is a monetary policy which increase money supply through central bank buying long term securities, and it (QE) has been announced by U.S. Fed twice during March 2020 which had negative but statistical significant single-day impact on U.S. 10-year Treasury yield.[2]

ISNB	Coupon %	Issue date	Maturity	ISNB	Coupon %	Issue date	Maturity
k296	1.5	5-6-2019	8-1-2021	J546	2.25	10-5-2018	3-1-2024
k601	1.5	11-4-2019	2-1-2022	J967	1.50	4-5-2019	9-1-2024
L286	0.25	5-4-2020	8-1-2022	k528	1.25	10-11-2019	3-1-2025
L773	0.25	10-26-2020	2-1-2023	k940	0.50	4-3-2020	9-1-2025
A610	1.50	7-20-2012	6-1-2023	L518	0.25	10-9-2020	3-1-2026

2. Here are the 10 bonds selected for constructing yield curve. Notice that the maturity dates of these bonds are close to ten maturity date $d + i$ where d are the ten weekdays from Jan 18 to Jan 29 and $i = 1/2, 1, \ldots, 5$.

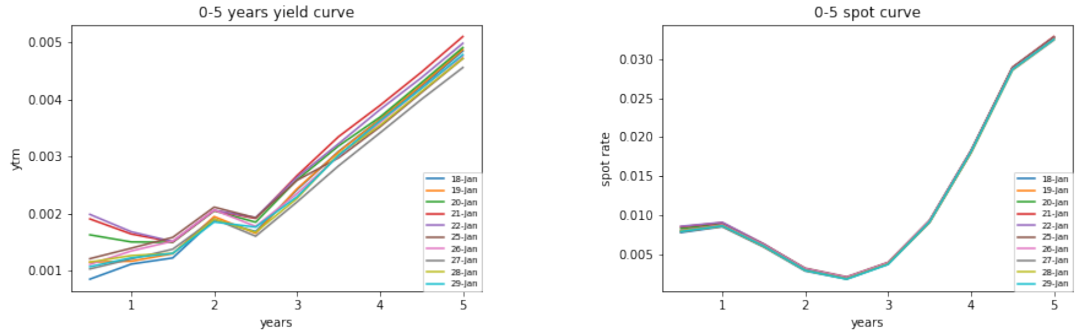
3. For a covariance matrix Σ, we have $\Sigma\lambda = v\lambda$ for every corresponding eigenvector v and eigenvalue λ. We want to fine a linear combination of some eigenvectors that has a larger covariance. After ranking eigenvalue in descending order, we select the top few one's corresponding eigenvectors to form a projection matrix. After projecting, we will preserve our outcomes which has higher covariances and eliminate the data which carries least amount of information.

Empirical Questions - 75 points

4.

   (a) Figure 1(a) shows the 5 year ytm of ten dates.[1] Before calculating ytm, we first calculate the dirty price for every bond by finding out it's last payment from it's maturity date. We use formula $P_{\text{dirty}} = (n/365)\text{coupon} + P_{\text{clean}}$. For example, the bond `CA13508K296` which mature at 8-1-2021, so for each date, we have the time from the last payment n equals to 6 month minus the date left in January. After calculating dirty price, we assume today is 2-1-2021, but we still count the payment at 2-1-2021 with discount rate 0. By using discrete semi annual compounding

[1] All the code for this assignment is uploaded to my GitHub under repository `math-fin`. The repository will remain private before 2pm, Feb-8th. The link is listed below under References.



(a) yield curve (b) spot curve

Figure 1: 5-year yield (ytm) curve and spot curve

formula $P_n = \sum_i p_i(1 + r/n)^{-t_i n}$ where $n = 2$, we got all yield for 10 bonds in 10 different day price and then interpolate them into the correct date and plot them [1].

(b) The spot curve is shown in Figure 1(b). Here is the explanation for the calculation. The main formula we used is discrete bootstrapping.

$$P = \sum_{i=1}^{n-1} \frac{p_i}{(1 + r_{t_i}/2)^{t_i}} + \frac{p_n}{(1 + r_{t_n}/2)^{t_n}}, \quad r_{t_n} \text{ unknown.}$$

If the mature date equals to our target date, we use above formula simply. If the mature date not equals to our target date and not excess 6 months from last bonds maturity, (for example: bonds `CA13508A610`), we use linear interpolation to find spot rate with corresponding time to each coupon payments and then use our formula. If the mature date not equals to our target date and excess 6 months from the last bonds maturity (for example: bond `CA13508J546`), we need to use linear interpolation to express the missing rate as a function of our about-to-solve rate. And then use the formula. Among our bonds, the first four and the last four fall into the first case. The fifth bond fall into the second case and the sixth bond fall into the third case.

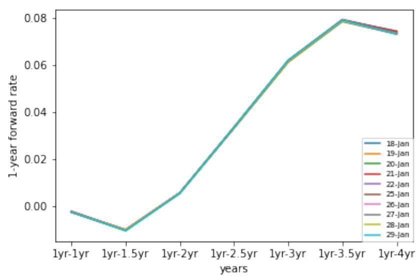


Figure 2: 1-year forward rate for year 2-5

(c) Figure 2 shows the 1-year forward curve from year 2 to 5. Since in this case, our spot rate is semi annual compounded, we should also semi-annual compound our forward rate as well. Let's say for each date (weekday between 18-Jan and 29-Jan), we have spot rate $s_{t_1}, s_{t_2}, \cdot, s_{t_{10}}$ from (b). Therefore, we have formula as follow.

$$(1 + s_{t_{2(i+1)}})^{2(i+1)} = (1 + f_{1,i})^2(1 + s_{t_{2i}})^{2i}, \; i = 1, 2, 3, 4.$$

Since we are calculating 1 year forward rate for year 2 to year 5, we only need spot rate at time year 1 to year 5 which are $s_{t_2}, s_{t_4}, \ldots, s_{t_{10}}$. Using this, we calculate our 4 rates for each ten dates and interpolate them.

5. Here are the two covariance matrix. For the following, we have $i = 1, \ldots, 5$, $j = 1, \ldots, 9$ and $m = 2, \ldots, 5$.

$$\mathrm{Cov}(\log(r_{i,j+1}/r_{i,j})) = \begin{bmatrix} 0.01562699 & 0.00244882 & 0.0042514 & 0.00355386, & 0.00262593 \\ 0.00244882 & 0.00142582 & 0.00119534 & 0.00015939 & 0.00022766 \\ 0.0042514 & 0.00119534 & 0.00236184 & 0.00083723 & 0.00073678 \\ 0.00355386 & 0.00015939 & 0.00083723 & 0.00188512 & 0.0013994 \\ 0.00262593 & 0.00022766 & 0.00073678 & 0.0013994 & 0.00107074 \end{bmatrix}$$

$$\mathrm{Cov}(\log(f_{(1,m),j+1}/f_{(1,m),j}) =$$
$$\begin{bmatrix} 5.56481951e-03 & -1.80668308e-05 & 3.20291525e-05 & 1.23684323e-04 \\ -1.80668308e-05 & 3.35390571e-04 & -5.46000809e-07 & 1.49522866e-05 \\ 3.20291525e-05 & -5.46000809e-07 & 2.62233332e-05 & 1.47034481e-06 \\ 1.23684323e-04 & 1.49522866e-05 & 1.47034481e-06 & 1.05320705e-05 \end{bmatrix}$$

6. Figure 3 shows the results of the calculation of the eigenvalues and the eigenvectors of the two covariance matrix from Question 5. From the results of both, we observe the largest eigenvalue being the third one for both covariance matrix, which indicates that most of our data points in our two time series move in the direction of the third eigenvector of their covariance matrix respectively.

```
w1, v1 = la.eig(lr_cov)
w2, v2 = la.eig(fr_cov)
print(w1) # eigenvalues of log-return(yield)
print(v1) # eigenvectors of .....
print(w2) # eigenvalues of log-return(forward)
print(v2) # eigenvectors of .....
```

```
[1.86364834e-02 1.15476443e-03 8.61212668e-06 5.21277030e-04
 2.04936240e-03]
[[ 0.90993442  0.41139799  0.03476347 -0.03891644 -0.00693195]
 [ 0.15233595 -0.26575041 -0.07454578  0.78178605 -0.5379814 ]
 [ 0.26785753 -0.64934782 -0.04874403 -0.56946938 -0.42417967]
 [ 0.22182605 -0.41155301 -0.61906662  0.17503387  0.60624829]
 [ 0.16690911 -0.41121763  0.77949662  0.17989958  0.40381008]]
[5.56781788e-03 3.36048834e-04 7.02960331e-06 2.60691620e-05]
[[ 9.99730123e-01 -2.35304617e-03 -2.21223241e-02  6.68928039e-03]
 [-3.38896915e-03 -9.98901990e-01 -4.67223041e-02  5.95892041e-04]
 [ 5.78444762e-03  1.29513087e-03 -4.08519289e-02 -9.99147628e-01]
 [ 2.22426522e-02 -4.67717832e-02  9.97827014e-01 -4.07297892e-02]]
```

Figure 3: eigenvalues and eigenvectors of the two covariance matrices

References

[1] URL: https://github.com/liuton23.

[2] Jonathan S Hartley and Alessandro Rebucci. *An Event Study of COVID-19 Central Bank Quantitative Easing in Advanced and Emerging Economies.* Working Paper 27339. National Bureau of Economic Research, June 2020. DOI: 10.3386/w27339. URL: http://www.nber.org/papers/w27339.