Trajectory Planning for Connected and Automated Vehicles: Cruising and Platooning in Mixed Traffic

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Abstract

Autonomy and connectivity are considered to be among the most promising technologies to improve safety, mobility, and fuel consumption in transportation systems. Some of the fuel efficiency benefits of connected and automated vehicles (CAVs) can be realized through platooning. A platoon is a virtual train of CAVs who travel together following the platoon head, with small gaps between them. In this paper, we devise an optimal control-based trajectory model that can provide fuel-efficient trajectories for the subject vehicle and can incorporate platooning. We embed this trajectory planning model in a simulation framework to quantify its fuel efficiency benefits in a dynamic traffic stream. Furthermore, we perform extensive numerical experiments to investigate whether the vehicles upstream of the subject vehicle may also experience second-hand fuel efficiency benefits.

Keywords: Optimal control, Connected vehicles, Autonomous vehicles, Mixed traffic, Connected and automated vehicle, Platooning, Fuel efficiency

INTRODUCTION

It is envisioned that in the near future, transportation systems would be composed of vehicles with varying levels of connectivity and autonomy. Connected vehicle (CV) technology facilitates communication among vehicles, the infrastructure, and other road users (*1*), allowing vehicles to see beyond the drivers' line of sight, and the transportation infrastructure to be proactive in responding to stochastic changes in demand.

Autonomous vehicle technology enables automation of vehicles at different levels, where level 0 automation indicates no automation, and automation levels 1 and 2 refer to a single and multiple driving assistant systems being present in the vehicle, respectively. Level 3 automation allows transfer of control authority between the human driver and the autonomous entity when the automation fails. Level 4 autonomy allows for the vehicle to control all functionalities within specified regions. Finally, in level 5 autonomy, vehicles can travel anywhere without any intervention from human drivers (*2*).

Although each of the connected and automated vehicle technologies can be deployed separately in a vehicle, when combined they can provide a synergistic effect that goes beyond the sum of their individual benefits. It is expected that upon deployment, the connected and automated vehicle (CAV) technology could significantly improve mobility, enhance traffic flow stability, reduce congestion, and improve fuel economy, among other benefits. The degree to which such benefits can be realized in real world conditions depends on a wide array of factors, among which trajectory planning of CAVs plays a major role (*3*). The main purpose of trajectory planning is to provide a vehicle with a collision-free path, considering the vehicle dynamics, the surrounding traffic environment, and traffic rules (*4*). More advanced trajectory planning techniques could incorporate secondary objectives such as achieving fuel economy (*5, 6, 7, 8*).

Traditionally, trajectory planning has been mainly based on vehicle dynamics constraints, such as acceleration range, steering performance, etc. More advanced driving assistance systems (ADAS), e.g., adaptive cruise control (ACC), enhance trajectory planning through utilizing data collected by on-board sensors. CV technology provides an opportunity to incorporate more diverse types of data (e.g., weather condition) from a wider spatial range (e.g., from vehicles beyond the line of sight of the vehicle's on-board sensors). However, there is a need to develop algorithmic tools that can incorporate this information into trajectory planning. Several attempts have been made in the literature to incorporate vehicle-to-vehicle (V2V) communications into trajectory planning, such as Cooperative Adaptive Cruise Control (CACC) (*9, 10, 11*) and Connected Cruise Control (CCC) (*12, 13*). CACC is one of the most promising technologies that allows CVs to share information through V2V communications. Advanced communication protocols, such as Dedicated Short Range Communications (DSRC), LTE, and 5G are proposed and developed to improve the communication bandwidth of V2V communications (*14, 15, 16*).

TABLE 1 summarizes the recent studies in the literature that have focused on trajectory planning of CAVs, with different levels of automation. This table points out multiple attributes of these studies, including the degree to which they are reactive to the dynamics of the traffic stream, their capability to model vehicles with various levels of autonomy and connectivity, whether they are capable of platoon formation, and their cost functions. The rest of this section elaborates on the specifics of these attributes.

The ultimate goal of trajectory planning is to enable vehicles to travel safely and efficiently in real traffic conditions. Therefore, different trajectory planning algorithms are developed for implementation in different contexts, to capture different abstractions of real-world conditions, e.g.,

TABLE 1 : Overview of the trajectory planning literature

Study	Environmental dynamics	Platoon	Mixed traffic	Cost			
				tracking	fuel	time	comfort /safety
(17)	pedestrian/static object	✗	pedestrian/static object	✓	✗	✗	✓
(5)	stop signs/ traffic lights	✗	✗	✗	✓	✗	✗
(18)	multiple dynamic obstacles (3 surrounding vehicles)	✗	✗	✗	✓	✓	✓
(19)	fixed velocity of surrounding surrounding vehicles	✓	connected/non-connected	✓	✗	✗	✗
(20)	curvy road segment with one obstacle	✗	✗	✗	✗	✗	✓
(21)	fixed velocity of the leading vehicle	✓	all automated vehicles	✗	✓	✓	✓
(22)	✗	✗	✗	✓	✗	✗	✗
This paper	**all vehicles**	✓	**human driver/CAV**	✗	✓	✗	✓

obstacles, curved roads, signal lights, and mixed traffic components, etc. In (*17*), Gu et al. focus on the subject vehicle's movement around a single static obstacle, and its distance-keeping and overtaking of a single leading vehicle. (*5*) propose a dynamic programming (DP) algorithm for speed planning in a transportation network with stop signs and traffic lights. (*20*) presents a method that exploits the complete permissible road width in curvy road segments to increase driving comfort and safety through minimized steering actuation. (*18*) develops a hybrid intelligent optimization algorithm based on ordinal optimization for autonomous vehicles traveling in environments with multiple moving obstacles. (*19*) and (*21*) consider the impact of surrounding vehicles with fixed velocity on trajectory planning of the subject vehicle. In general, the degree to which different models are set to imitate real traffic conditions depends on research priorities. The closer the environment is to real-world conditions, the higher the accuracy and reliability of trajectory planning, but the more computational complexity and the worse the real-time performance. As such, literature is in general very limited in capturing the dynamics of the driving environment.

The capability to incorporate platooning is another factor that differentiates existing trajectory planning methods. Platooning is a specific application of CV technology that can introduce a wide range of vehicle- and system-level benefits. A platoon is a single-file line (i.e., a virtual train) of vehicles that, owing to constant communication, are able to travel with small gaps between them. Platoon formation can introduce many benefits including (i) energy efficiency through reducing the aerodynamic drag force on platoon members (*23, 24*), as well as reducing emissions (*25*); (ii) increasing road capacity through reducing the headways between vehicles; (iii) reducing stochasticity in the traffic stream by having platoon members follow the lead of the platoon head, thereby reducing the likelihood of highway traffic breakdown, improving travel times, and increasing travel time reliability (*26, 27*); and (iv) facilitating real-time management of traffic and improving mobility by aggregating the unit of traffic from an individual vehicle to a cluster of vehicles. TABLE 1 specifies the studies in the literature that incorporate platooning. Note that a check mark for the field 'platoon' in this table indicates the capability of platoon formation, rather than platoon control strategies (28, 29) or intra-platoon communication (30, 31).

The ability to capture the heterogeneity in the level of connectivity and autonomy of vehicles is another factor that differentiates different trajectory planning methods, as described in TABLE 1. Finally, trajectory planning methods are different in terms of their objective function. In general, the goal is to find the least-cost trajectory, where the cost function could include any combination of the following factors: time-cost of the trip (i.e., trip length), fuel consumption, comfort and safety of on-board passengers, and precision in tracking (i.e., the degree to which the vehicle deviates from a pre-specified ideal trajectory).

To the best of the our knowledge, there is not yet a study that comprehensively discusses trajectory planning for a connected and automated vehicle in a fully dynamic environment with mixed traffic, considering platoon formation for both the subject vehicle and its surrounding vehicles. The goal of this paper is to determine a platoon-joining policy that can reduce the fuel cost over a prediction horizon for the subject vehicle, which is considered to be a connected and automated vehicle, in a dynamic environment with mixed traffic. Here a ‘dynamic’ environment indicates that the surrounding vehicles can enter or exit from a two-lane highway through on- and off-ramps, merge into or split from a platoon, change lane, and adjust their velocities. The joint acceleration and platoon-joining decisions are made by the subject vehicle in such a dynamic traffic environment so as to minimize fuel cost. We run simulations to assess whether optimal control of a single vehicle can affect fuel consumption of its surrounding vehicles. Furthermore, we investigate the fuel efficiency benefits of platooning for both platoon members and other surrounding vehicles. These experiments allow us to quantify whether, and the extent to which, having a few CAVs in the traffic stream could have fuel efficiency benefits that go beyond the CAVs themselves.

The rest of the paper is organized as follows: First, We formulate an optimal control model for planning the trajectory of a CAV. Next, we present a simulation environment that consists of a two-lane highway with multiple on- and off-ramps and a dynamic traffic stream. In particular, we describe how vehicles with various levels of autonomy interact with each other in the simulation environment. Finally, in the NUMERICAL EXPERIMENTS section, we conduct a series of analyses under various traffic conditions to quantify the fuel-efficiency benefits of our approach for the subject vehicle as well as those of its surrounding vehicles within platoons and as free agents. We end the paper with summarizing the takeaways.

METHODS

The goal of this study is to design an optimal control-based trajectory planning model that can be utilized by an automated (level 2 or higher autonomy) vehicle, hereafter referred to as the “subject vehicle”. The optimal control model will be designed to incorporate microscopic traffic information from the traffic stream in the local neighbourhood of the subject vehicle, with the goal of devising fuel efficient trajectories that may include merging into a platoon. We start this section by describing the optimal control model. We then describe a simulation environment that we will use to quantify the fuel-efficiency implications of platooning for the subject vehicle as well as its surrounding traffic.

Optimal Control Model

In this section, we devise an optimal control model to determine the trajectory of the subject vehicle in real-time. The proposed optimal control model is provably safe, and is designed to account for fuel efficiency as well as comfort of on-board passengers.

The control parameters of the optimal control model include acceleration/deceleration and platoon membership. While adjusting acceleration can be considered as a single action that can be almost instantaneously carried out, a change in platoon membership is a lengthier process and may require multiple sub-actions, as described in TABLE 2. As demonstrated in this table, at each point in time the subject vehicle can be in one of the following three states: (i) 'free-agent', indicating that the vehicle is not part of any platoon, (ii) 'In platoon (active)', indicating that the subject vehicle is the platoon lead, and the scheduled platoon splitting position has not yet reached, and (iii) 'In platoon (passive)', indicating that the platoon splitting position has reached, and the platoon the subject vehicle was formerly leading is in the process of dissolving. TABLE 2 shows that at each point in time, regardless of its current state the subject vehicle can take an action to be a free agent or to be in a platoon . Depending on its current state and the selected action, the subject vehicle may need to complete a sequence of sub-actions, including 'wait', 'merge', and 'split'. The 'wait' sub-action indicates that the vehicle needs to maintain its state after completing its previous sub-action, and the sub-actions 'merge' and 'split' indicate merging into a platoon and splitting from a platoon, respectively. For example, if the action 'free agent' is the selected action under the current state 'In platoon (active)', then the subject vehicle needs to complete a sequence of sub-actions that start with 'split' and end with 'wait'.

Following the study in (*32*), we use a quintic function, based on time, as our trajectory function for each sub-action. The quintic function is selected because it guarantees a smooth overall trajectory, even with multiple different sub-actions. Eq. (1) shows the trajectory function,

$$x(t) = \sum_{i=1}^{N_{\text{act}}} (a_5^i t^5 + a_4^i t^4 + a_3^i t^3 + a_2^i t^2 + a_1^i t + a_0^i) f_i(t) \tag{1}$$

where N_{act} is the number of sub-actions the subject vehicle needs to complete. Function $f_i(t)$ may be formulated as

$$f_i(t) = \begin{cases} 1 & t_{i-1} \le t < t_i \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $[t_{i-1}\ ,\ t_i]$ is the time window for completing the i'th sub-action, and $t_{N_{\text{act}}}$ is the prediction horizon.

Boundary conditions

For every sub-action, the following boundary conditions must be satisfied,

$$x(t_{i-1}) = x_{t_{i-1}},\ \dot{x}(t_{i-1}) = v_{x,t_{i-1}},\ \ddot{x}(t_{i-1}) = a_{x,t_{i-1}}, \tag{3}$$

$$x(t_i) = x_{t_i},\ \dot{x}(t_i) = v_{x,t_i},\ \ddot{x}(t_i) = a_{x,t_i} \tag{4}$$

where t_{i-1} and t_i are the starting and ending time for the ith sub-action, respectively, and $x_{t_{i-1}}$, $v_{x,t_{i-1}}$ and $a_{x,t_{i-1}}$ are the longitudinal coordinate, velocity and acceleration for the starting point of the sub-action, respectively. These values are accordant with the ending point for the last sub-action. For each sub-action, the longitudinal coordinate, velocity and acceleration at the ending point of the sub-action as well as the duration of the sub-action are all free variables and are optimized.

TABLE 2 : Sub-action sequences for each state-action tuple

State	Action	Sub-action sequence
Free agent	free agent	wait
	in platoon	merge$\longrightarrow$wait
In platoon (active)	free agent	split$\longrightarrow$wait
	in platoon	wait
In platoon (passive)	free agent	split$\longrightarrow$wait
	in platoon	split$\longrightarrow$wait$\longrightarrow$merge$\longrightarrow$wait

Constraint sets

There are a number of constraints on the position, speed, acceleration, and jerk of the subject vehicle, elaborated in the following:

1. Speed limitation: The longitudinal speed of the subject vehicle should be no more than the maximum speed in its lane, and should always be positive, as in Eq. (5).

$$0 < v_x(t) < v^l_{x,\max} \tag{5}$$

where $v_x(t)$ denotes the longitudinal speed of the subject vehicle, l indicates the lane in which the vehicle is traveling, and $v^l_{x,\max}$ denotes the maximum vehicle speed in lane l.

2. Collision avoidance: The subject vehicle should maintain a minimum time gap (denoted by t_p) from its immediate downstream vehicle during all sub-actions, as in Eq. (6).

$$\min\big(x_L(t) - x_\text{sub}(t)\big) > t_p\, v_\text{sub}(t) \tag{6}$$

where $x_L(t)$ is the position of the immediate downstream vehicle (i.e., the leader), and $x_\text{sub}(t)$ and $v_\text{sub}(t)$ are the position and velocity of the subject vehicle, respectively.

3. Acceleration bound: During all sub-actions, the acceleration of the subject vehicle cannot exceed a maximum acceleration due to the mechanical performance limitations and safety considerations. This constraint is enforced in Eq. (7),

$$|a_x| < a_\text{max} \tag{7}$$

where a_max is the maximum acceleration.

4. Jerk bound: Since the subject vehicle's jerk directly influences the comfort level as well as the safety of on-board passengers, we bound the jerk by a maximum value as stated in Eq. (8),

$$|j_x| < j_\text{max} \tag{8}$$

where j_max is the maximum jerk.

Objective function

We consider fuel cost as the objective function in our optimization problem, as stated in Eq. (9) below,

$$C_{\text{overall}} = \eta_f \, \Sigma_{i=1}^{N_{\text{act}}} \beta(i) \int_{t_{i-1}}^{t_i} \left(\gamma_{\text{AR}} v^2(t) + \gamma_{\text{RR}} + \gamma_{\text{IR}}(a(t))_+ \right) v(t) \, dt \tag{9}$$

The three terms $\gamma_{\text{AR}} v^2(t)$, γ_{RR}, and $\gamma_{\text{IR}}(a(t))_+$ are the air resistance force, rolling resistance force, and inertia resistance force, respectively. For computation details of these forces, refer to (*33*). The parameter η_f is the fuel cost for a unit energy consumed by the vehicle, and is measured in dollars. The parameter $\beta(i)$ indicates the fuel saving coefficient for sub-action i. We set $\beta(i) = 1$ for a free agent, and $\beta(i) = 0.9$ for a platoon member. Furthermore, we set $\beta(i) = 0.95$ for split and merge sub-actions. The reason for assuming a β value less than one for these sub-actions will be explained in the next section.

Simulation Environment

In this study we consider a mixed traffic stream with various levels of autonomy. Specifically, we model both vehicles who are human-driven and not platoon-enabled, and platoon-enabled vehicles. A platoon-enabled vehicle is a vehicle that has level 2 or higher autonomy (and is equipped with adaptive cruise control) according to the Society of Automotive Engineer's (SAE's) classification. Furthermore, in this study we assume all vehicles are connected; that is, all vehicles can communicate with each other and with road side units (RSUs) using dedicated short range communication (DSRC) devices, with a reliable communication range of 300 meters. FIGURE 1 demonstrates the communication and control framework of our work.

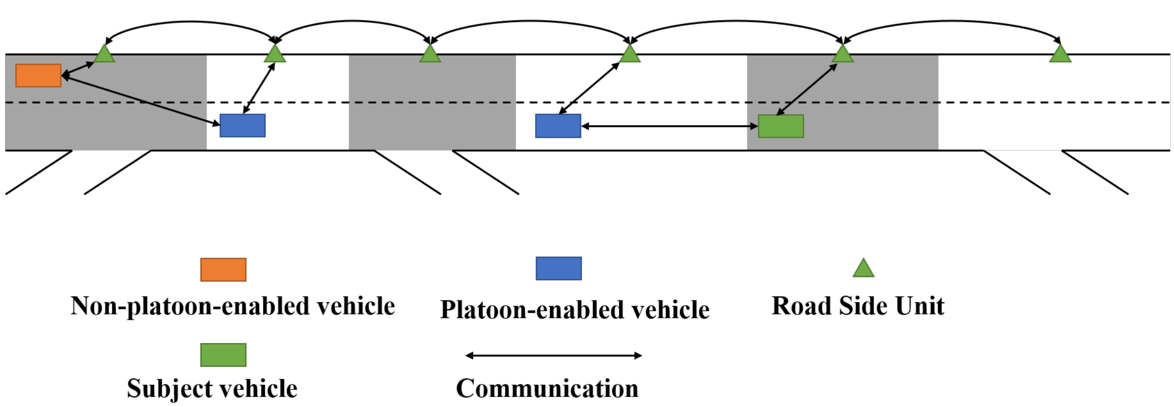

FIGURE 1 : The communication and control framework

To develop a simulation environment for the system, we divide the transportation network into a number of "road pieces". A road piece is a section of a road that satisfies the following two conditions: (i) the macroscopic traffic conditions, to which we refer as "traffic states", are likely to be homogeneous within a road piece. For example, in a highway section the traffic conditions around on- and off-ramps are typically different, indicating that on- and off-ramps require dedicated road pieces; and (ii) vehicles within a road piece are able to communicate, either directly or through RSUs. This requirement ensures that in case of DSRC-enabled communication, the length of a road piece cannot exceed 600 m, for all vehicles to become connected using a single

RSU located in the middle of the road piece. Limiting the length of a road piece ensures that, with strategic positioning of RSUs, all connected vehicles can receive microscopic traffic information of their neighbours (i.e., trajectories including longitude and latitude, velocity, acceleration, braking, steering angle, etc.), and use this information to plan more informed and efficient trajectories.

In our modeling of a traffic stream characterized with full connectivity and a heterogeneous level of autonomy, we account for the delay between the occurrence of a stimulus and the execution of an action in response to it. In case of a human being the driving authority, this delay is referred to as the perception-reaction time (*34*), and accounts for the perception delay (either by the driver or from the part of the vehicle sensors), the decision-making delay, and the persecution delay. In case of the autonomous entity being in charge, this delay can be attributed to sensory delay, delay in the communication network, computational time, and actuation delay.

Surrounding vehicles

Surrounding vehicle trajectories will be simulated based on a microscopic car-following model so as to reflect a realistic and dynamic traffic environment. The surrounding traffic information will get updated every $\tau_s = 0.4$ seconds. At each updating step, four functions will be executed by surrounding vehicles in the following sequence: join/exit from the highway, merge into/split from a platoon, change lane, and adjust velocity based on the car-following model. These functions are elaborated in the following:

1. Join/exit from the highway: We assume that the probability that a vehicle enters the highway from an on-ramp at each updating epoch is p_{on}. The vehicle is assumed to be able to join the highway if it can maintain a minimum time gap of length t_p from the vehicles both upstream and downstream of the ramp entry point in the right lane of the highway. We set the speed of this entering vehicle similar to the speed of its downstream vehicle. Furthermore, we set the probability of the vehicle not being a platoon-enabled vehicle as p_{npe}.

We assume at each update step each vehicle may take an off-ramp with the probability p_{off}. A vehicle who is marked to leave the highway based on this probability may do so if it is traveling on the right lane of the highway, on the upstream of an off-ramp point, and the time gap between the vehicle and the off-ramp point is smaller than the update step, τ_s. This exiting vehicle and its profile is directly taken off the current iteration.

2. Merge into/split from a platoon: It is assumed that a vehicle could hold only a single platoon membership status (either a member or not a member) throughout a road piece, i.e., the merging or splitting process can only commence in the transition point between two road pieces. (Note that that is the reason for assuming a β value of 0.95 for the merge and split sub-actions in Eq. (9).) A vehicle can merge into a platoon when it is already a platoon leader (resulting in merging of two platoons), or a platoon-enabled free agent. Among all vehicles who qualify to merge into a platoon, the probability that a vehicle intends to merge is assumed to be p_{merge}. There are two cases regarding the profile of the vehicle in the immediate downstream of the merging vehicle. If it is a platoon member, then the new merging vehicle will have the same scheduled splitting position as other vehicles in the platoon. If it is a single vehicle, the scheduled splitting position P_{sch} will be decided at this time using a normal distribution. For more details, see the section *Platoon Membership*.

Every time when a platoon passes the transition point of two road pieces, the scheduled splitting position will decrease by 1 unit until this value reaches -1, at which point the platoon would split into free agents.

3. Lane change: (*35*) provide a comprehensive review of prior work on lane changing models. For simplicity, in this paper we adopt the random lane changing (RLC) model, in which vehicles may change lane once a minimum gap criterion is satisfied. We assume that in every update step at most a single vehicle can change lane. Furthermore, for safety considerations, we require a minimum time (no less than $t_{\mathbf{lc}} = 5$ seconds) between two successive lane changes by two successive vehicles (immediate follower/leader) traveling in the same lane. We allow only free agents, and not platoons, to change lane. The gap between the lane changing vehicle and surrounding vehicles (the leading vehicle in the same lane, and the leading and following vehicles in the target lane) should be large enough (larger than $d_{\mathbf{cg}}$) to ensure safety. Finally, the following vehicle in the target lane cannot be a follower in a platoon, indicating that the lane changing process cannot insert vehicles into a platoon.

Not all vehicles that satisfy the conditions above intend to change lane. Among all qualified vehicles, the probability that a vehicle intends to change lane is p_{change}. The lane changing process is assumed to be completed within the period τ_s, after which the lateral position of the lane changing vehicle would not change, and its longitudinal speed has to reach the speed of the leading vehicle in the target lane.

4. Adjusting velocity using a car-following model: Each vehicle needs to continuously adjust its velocity to maintain a large enough safety gap from its leading vehicle. Adjustment of velocity depends on the platoon membership status of the vehicle. We use the Intelligent Driver Model (IDM), proposed in (*36*), for adjusting velocity. The parameters used to calibrate the IDM are summarized in the TABLE 3.

Subject vehicle

The subject vehicle updates its motion plan every $t_{\text{upd}} = 0.4$ seconds. It is assumed that surrounding vehicles' motion information is available to the subject vehicle in real time. Due to the long computational time of trajectory planning and control in a dynamic driving environment, it is problematic for the subject vehicle to obtain the latest traffic information and then plan its own trajectory for the immediate next period; that is, after the trajectory planning process is completed, the planed trajectory is already outdated. Thus, we put forward a revised computational process in this paper, as shown in the FIGURE 2. During this process, the subject vehicle perceives the environment, estimates other vehicles' motions for the next $2t_{\text{upd}}$ period, and makes its own trajectory plan for the second following period, i.e., $[t + t_{\text{upd}}\ ,\ t + 2t_{\text{upd}}]$, where t is the current time. This results in a trajectory that can still be effectively followed during this window.

As discussed in (*32*), the subject vehicle may get involved in a collision due to the surrounding vehicles' sudden speed fluctuations during the lane changing process. More generally, the subject vehicle may not be able to take any action without violating the constraints of the optimal control model for the following reasons: (i) the sudden speed change of the surrounding vehicles; (ii) the comfort-related maximum acceleration and jerk constraints in the optimal control model; and (iii) the conservative constraints regarding the safely time gap between the subject vehicle and any surrounding vehicles. In case there is no feasible solution for the optimal control model, the Intelligent Driver car-following Model is utilized to provide a longitudinal motion reference for the subject vehicle.

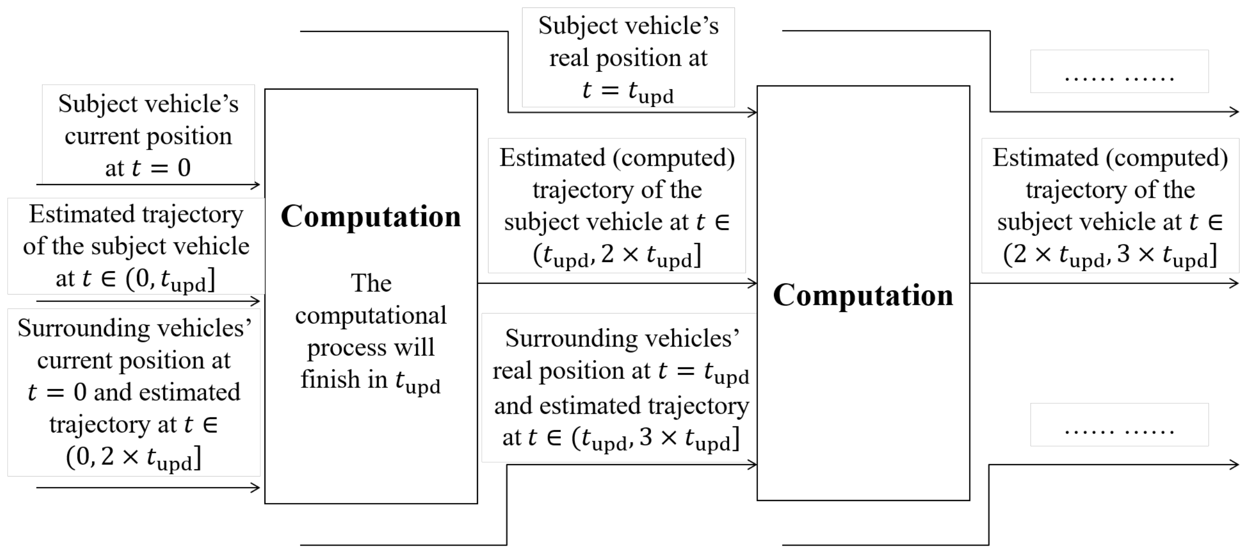

FIGURE 2 : The computational process inputs and outputs: Each computational process for the optimal trajectory of the subject vehicle is finished within the time t_{upd}. The output of the last computational process serves as an input to the next computational process. Once a computational process is completed, the last trajectory of the subject vehicle is updated based on the new trajectory.

Platoon membership

This section elaborates on platoon formations. When merging, we assume a free agent or a platoon can merge with its immediate downstream free agent or platoon. That is, merging can occur between two free agents, two platoons, or a free agent and a platoon. We assume a finite number of possible scheduled splitting positions, ℓ^1_{sch}, ℓ^2_{sch}, $\cdots$, ℓ^n_{sch}, in an ascending order of time. Given the mean μ_{sch} and the standard deviation σ_{sch}, we draw a random number p_{sch} from the normal distribution $\mathcal{N}(\mu_{\text{sch}}, \sigma_{\text{sch}})$ to schedule a splitting time, where $\ell^{i-1}_{\text{sch}} < p_{\text{sch}} \leq \ell^i_{\text{sch}}$ indicates selecting the scheduling time $P_{\text{sch}} = \ell^{i-1}_{\text{sch}}$. We set $P_{\text{sch}} = \ell^1_{\text{sch}}$ for $i = 0$. At the scheduled splitting position, platoon members will detach one by one, starting from the platoon tail, by increasing their gap with their immediate downstream vehicles.

NUMERICAL EXPERIMENTS

In this section we conduct experiments in the simulation framework laid out in the previous section, where the trajectory of the subject vehicle is controlled by the proposed optimal control model. The simulation framework consists of a two-lane highway, where the subject vehicle is assumed to travel on the right lane. The traveled path is composed of 20 road pieces, with two on-ramps in the first and eighteenth road pieces, and three off-ramps on the fourth, twelve, and the destination of the trip. The travel path is 10.8 km in length, where the first, fourth, twelfth and eighteenth road pieces are 400, 300, 200 and 300 meters in length, respectively, and the rest of the road pieces are 600 meters in length. Recall that we consider a road piece to be homogeneous in macroscopic traffic conditions.

We quantify the implications of the optimal control model with and without platooning under different traffic environments. Specifically, we consider three traffic states, namely, free-flow

TABLE 3 : Summary of parameters

Parameter	Value	Definition
t_{upd}	0.4 secs	updating period of trajectory of the subject vehicle
p_{on}	0.6	the possibility of that a vehicle is interested in joining the freeway from on-ramp
p_{off}	0.6	the possibility of that a vehicle is interested in taking at off-ramp
p_{npe}	0.5	the possibility of that the vehicle is a non-platoon-enabled vehicle
p_{merge}	0.6	the possibility of that a vehicle intends to merge
p_{change}	0.1	the probability of that the vehicle intends to change lane
t_p	3.5 secs	time gap between two successive vehicles that are not in a platoon
t_g	0.55 secs	time gap between two successive vehicles in a platoon
t_{lc}	5 secs	the minimum time interval between two successive lane changes by two successive vehicles in the same lane
τ_s	0.4 secs	reaction time delay in the car-following model
$t_{N_{\text{act}}}$	10 secs	prediction horizon in optimal control model
v_{m}^{le}	20 m/s	the velocity in left lane when it reaches the maximum flow
v_{m}^{ri}	14 m/s	the velocity in right lane when it reaches the maximum flow
$v_{\text{max}}^{\text{le}}$	30 m/s	maximum velocity in left lane
v_{max}^{ri}	20 m/s	maximum velocity in right lane
a_{max}	m/s^2	maximum acceleration for subject vehicle
j_{max}	m/s^3	maximum jerk for subject vehicle
d_{cg}	50 m	critical gap decide whether it is feasible to change lane
l_{car}	5 m	length of a vehicle
h_{st}	5 m	vehicle would stop at headway of this value
a	$2\ \text{m}/\text{s}^2$	the maximum desired acceleration
b	$3\ \text{m}/\text{s}^2$	the comfortable deceleration
γ_{AR}	0.3987	coefficient for air resistance force
γ_{RR}	281.547	coefficient for rolling resistance force
γ_{IR}	1750	coefficient for inertia resistance force
η_f	5.98×10^{-8} dollars/Joule	fuel cost for a unit energy consumed by the vehicle
P_{sch}	{2,10,50}	the scheduled splitting position can be 2, 10 or 50 road pieces later
$\mathcal{N}(\mu_{\text{sch}}, \sigma_{\text{sch}})$	$\mathcal{N}(2,5)$, left, $\mathcal{N}(-1,5)$, right	the norm distribution of the scheduled splitting position in two lanes, respectively

traffic, onset-of-congestion traffic, and congested traffic. In order to provide a realistic simulation environment under each traffic state, we setup a warming-up process, during which we use a fundamental diagram of traffic flow to create simulation instances under each traffic state. According to (*37*), many different models have been proposed to capture the relationship among the three fundamental parameters of traffic flow—traffic flow, speed, and traffic density. Here, we adopt the Greenshied's model, which presents one of the earliest and most well-known speed-density models (*38, 39*). Let v_m and k_m be the corresponding velocity and density when the flow reaches its maximum value, which is $\frac{1}{t_\mathrm{p}}$. We set $k_1 = 0.3\, k_\mathrm{m}$, $k_2 = 0.8\, k_\mathrm{m}$, $k_3 = 2\, k_\mathrm{m}$ as the maximum density under the free-flow, onset-of-congestion, and congested traffic states, respectively. We then use the Greenshield's speed-density relationship in Eq. (10) to compute the corresponding velocity of each of the three density cut-off points,

$$v = v_\mathrm{m} \ln(\frac{k_\mathrm{j}}{k}) \tag{10}$$

where v denotes the space-mean-speed, k denotes the traffic density, v_m indicates the velocity when the flow reaches its maximum value, and k_j indicates the jam density. Value of k_j is determined by parameters in the IDM model,

$$k_\mathrm{j} = \frac{1}{l_\mathrm{car} + h_\mathrm{st}} \tag{11}$$

where l_car is the average vehicle length, and h_st is the minimum headway at which point vehicles are at complete stop. After generating vehicle trajectories using the IDM model, we perturb these trajectories using Gaussian noise to incorporate random deviations from an idealized model. During the warm-up process all surrounding vehicles run for 2 minutes following the IDM model.

For each traffic state, we run four simulation scenarios, each scenario using a different controller for the subject vehicle, as follows: (1) the subject vehicle travels according to the intelligent driver (*36*) car-following model and cannot join a platoon (CF), (2) the subject vehicle travels according to the optimal control model and cannot join a platoon (OC), (3) the subject vehicle travels according to the optimal control model and can join a platoon; however, the platoon can dissolve at any point in time after formation (OC_merge_0km), and (4) the subject vehicle travels according to the optimal control model and can join a platoon; however, a platoon has to travel for at least 6 km before it can dissolve (OC_merge_6km). For each traffic state, we run 25 random instances of each simulation scenario and report the fuel costs.

Fuel Efficiency Results for the Subject Vehicle

In this section we report the fuel consumption of the subject vehicle under the four introduced controllers and the three traffic states. Figure 3 displays the fuel cost of all four simulation scenarios under all three traffic states. To facilitate the comparison of scenarios, the results of the first scenario (i.e., CF) is repeated next to the three other scenarios. The values of fuel consumption under all scenario pairs are compared using two-tailed Student's t-tests at 5% significance level to identify fuel savings that are statically significant.

The top plot in FIGURE 3 presents the results for the free-flow traffic state. Results suggest that the optimal control model (both with an without the ability to form a platoon) can result in statistically significant reductions in fuel cost (at the 5% significance level), compared to the car-following model. If the subject vehicle is enabled to be a platoon member and forced to keep its platoon membership for at least 6 km (i.e., the OC_merge_6km scenario), the fuel savings are

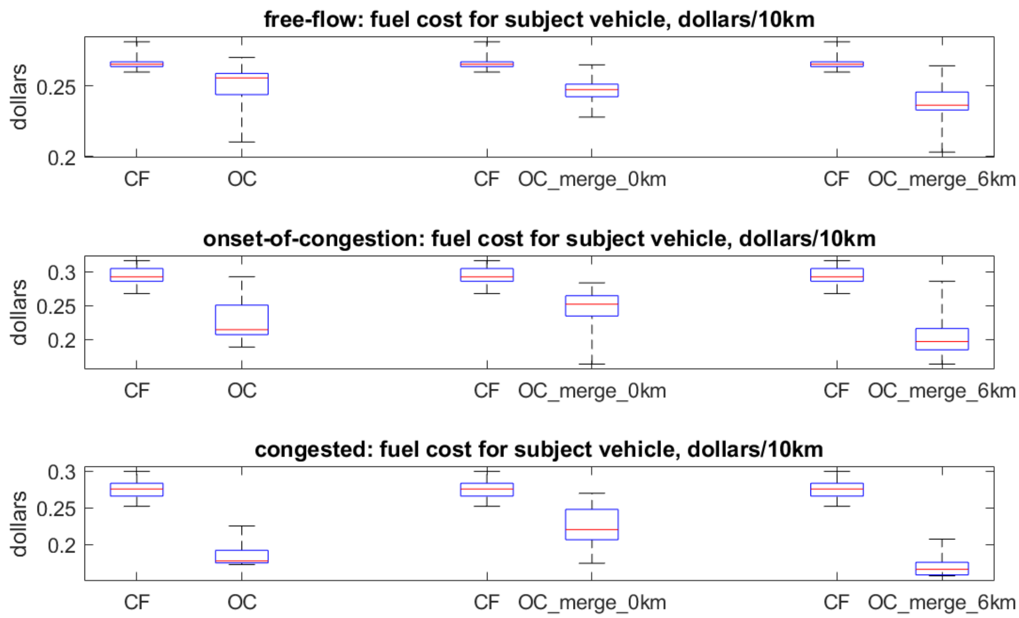

FIGURE 3 : The top, middle, and bottom figures represent the free-flow, onset-of-congestion, and congested traffic states, respectively. The vertical axes in these figures show the fuel cost in dollars for 10 km long trips. Along the horizontal axes, fuel cost of the subject vehicle under different controllers are compared. Here 'CF' and 'OC' denote the car-following and the optimal control models, respectively, where platooning is disabled. 'OC_merge_xkm' indicates that the subject vehicle is enabled to merge into a platoon using the optimal control model, and that it needs to keep staying in the platoon for at least x km after the platoon is formed.

more significant compared to OC alone. However, scenario OC_merge_0km, where the platoon can dissolve at any point in time after its formation, does not produce statistically significant fuel savings compared to OC.

The middle plot in FIGURE 3 demonstrates the results for the onset-of-congestion traffic state. Results indicate that similar to the free-flow case, optimal control offers statistically significant fuel savings compared to car-following for all control-based scenarios (with and without platooning). However, comparison of OC, OC_merge_0km, and OC_merge_6km scenarios in the onset-of-congestion traffic state shows that OC_merge_0km results in the least fuel saving, OC holds the second place, while OC_merge_6km earns the most energy saving. These results are not surprising since the frequent splitting of the subject vehicle from platoons in the onset-of-congestion state leads to higher energy consumption in the OC_merge_0km scenario, and the energy savings from a short-lived platoon cannot make up for this loss.

Finally, the bottom figure in Figure 3 displays the results for the congested traffic state. Results indicate that similar to the two previous traffic states, optimal control offers lower fuel cost compared to car-following. The OC_merge_0km scenario does not offer statistically significant improvements over the OC scenario for the same reason stated above; however, OC_merge_6km can still offer statistically significant fuel savings over both OC and OC_merge_0km controllers.

In summary, results indicate that the optimal control model consistently produces more fuel-efficient trajectories for the subject vehicle compared to the IDM, under all traffic states. In general, the scenario OC_merge_0km cannot produce statistically significant savings compared to OC. This can be attributed to the 'shortsightedness' nature of the optimal control model, which only includes local information and optimizes over a prediction horizon when making platoon-merging and keeping decisions. Lastly, OC_merge_6km can result in lower fuel cost than OC_merge_0km in all traffic states.

Fuel Efficiency Results for the Surrounding Vehicles

It this section, we analyze the simulation results to investigate whether the different controllers used by the subject vehicle have a significant impact on the fuel consumption of its upstream traffic. The fuel costs of $N_{\text{sur}} = 15$ upstream vehicles in the same lane as the subject vehicle are summed and averaged to reflect the fuel cost of surrounding vehicles.

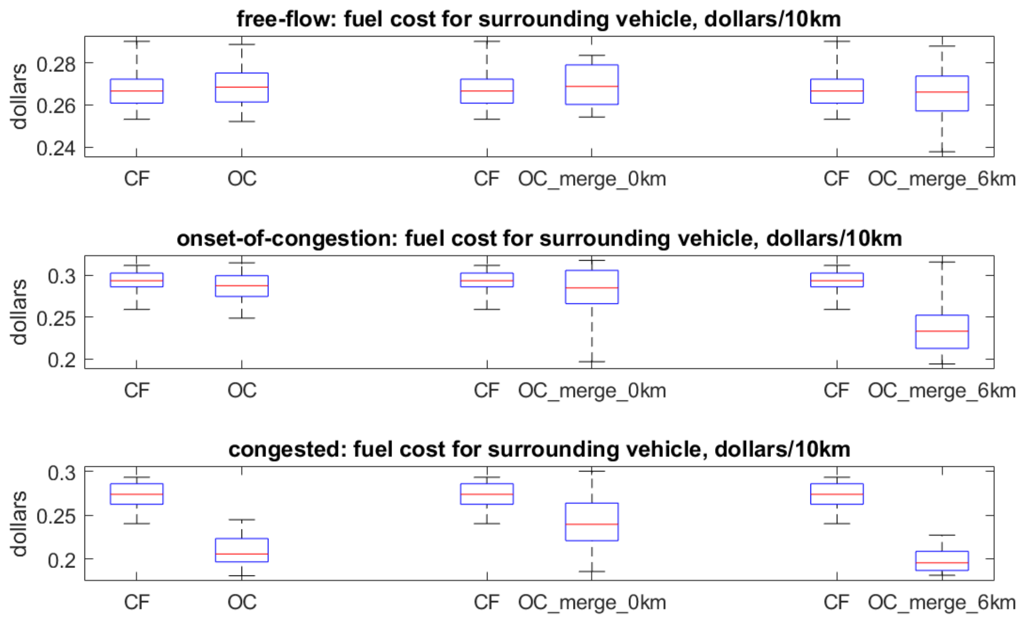

FIGURE 4 : The vertical axis shows the fuel cost per distance, the unit is dollars per 10 kilometers. Along the horizon axis, fuel costs of the subject vehicle under different controllers are compared. Here 'CF', 'OC' and 'OC_merge_xkm' have the same meaning as in FIGURE 3. 15 vehicles in the upstream of the subject vehicle are considered respectively, their fuel costs are summed and averaged. We also considered three traffic situations for surrounding vehicles' fuel costs.

FIGURE 4 displays fuel consumption of the upstream vehicles to the subject vehicle under the three traffic states and the four controllers. This figure suggests that changing the subject vehicle controller from the car-following model to the optimal control model does not lead to any significant changes in fuel consumption of the upstream vehicles in the free-flow and onset-of-congestion traffic states, but results in significant fuel savings in the congested state. Following a similar trend with subject vehicle, OC_merge_0km cannot produce statistically significant savings

TABLE 4 : fuel cost for subject vehicle and immediate upstream vehicle in an onset-of-congestion scenarios

fuel cost,	first 500 seconds		the whole trip	
dollars per 10 km	subject vehicle	following vehicle	subject vehicle	following vehicle
CF	0.2537	0.2494	0.2659	0.2606
OC	0.2356	0.2343	0.2099	0.2094
OC_merge_0km	0.2440	0.3008	0.2601	0.2826
OC_merge_6km	0.2036	0.2196	0.1745	0.1810

compared to OC, and could even result in higher costs in congested cases. Finally, this figure demonstrates that platooning maintained for a minimum of 6 km can lead to a statistically significant reduction in fuel consumption of upstream vehicles in onset-of-congestion and congested traffic states, compared to OC. Generally speaking, the subject vehicle provides the most fuel-efficiency benefits for its surrounding vehicles when it is forced to keep its platoon membership status for at least 6 km.

Finally, FIGURE 5 allows us to pinpoint the source of fuel efficiency induced by the OC model. This figure shows an example of the velocity curves of the subject vehicle and its immediate upstream vehicle in the onset-of-congestion traffic state. The points at the bottom of the plots in this figure mark the platoon membership status of the subject vehicle under the OC_merge_6km and OC_merge_0km controllers at each time epoch. In FIGURE 5, only the first 500 seconds of the trip are presented, and the fuel costs for this 500-second section of the trip as well as the entire trip are computed and shown in TABLE 4. This figure shows that, compared to CF, OC provides smoother velocity trajectories, thereby resulting in fuel savings for both the subject vehicle and its immediate upstream vehicle. This figure also demonstrates that the OC_merge_6km controller provides the smoothest trajectories, and therefore can provide the highest fuel-saving benefits.

CONCLUSION

In this paper we proposed an optimal control model for trajectory planning of a CAV in a mixed traffic environment. The optimal controller was developed to plan the trajectory of the subject vehicle and make platoon formation decisions, explicitly accounting for computation delay. We developed a simulation framework to quantify the effectiveness of the optimal control model in providing first-hand energy savings for the subject vehicle as well as second-hand energy savings for the vehicles traveling upstream of the subject vehicle. Our experiments suggest that, generally speaking, the optimal controller performs better than the IDM car-following model. Results suggest that making platooning decisions based on local information does not necessary lead to fuel savings compared to the same controller being implemented while restricting the subject vehicle to always remain a free agent; however, if a minimum platoon-keeping distance is imposed by the model, platooning can offer significant fuel-efficiency benefits. Our experiment results also indicate that, under the controller with minimum platoon-keeping distance requirement, the vehicles upstream of the subject vehicle, who are human-driven, will experience second-hand fuel savings that are statistically significant at the 5% significance level.

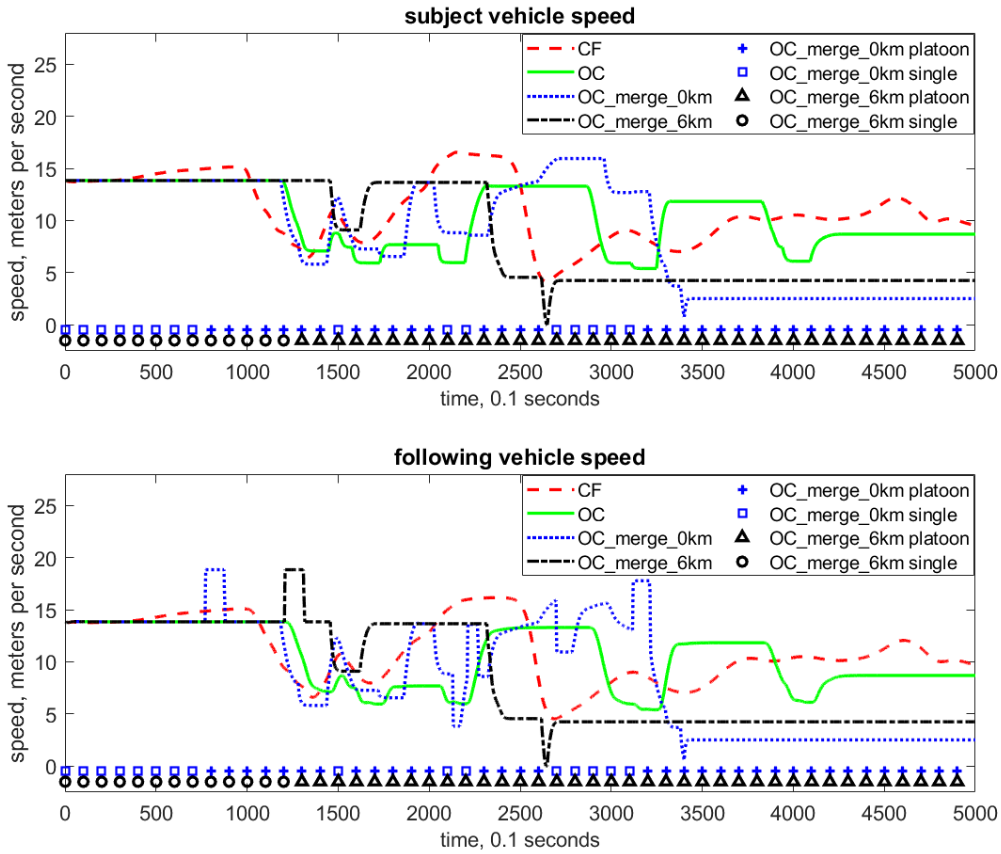

FIGURE 5 : The vertical axis shows velocity, with unit of meters per second. The horizontal axis is time, with unit of 0.1 seconds. The top plot compares the speed curves of the subject vehicle under different controllers, and the bottom plot shows the corresponding speed curves of the upstream vehicle to the subject vehicle. Here 'CF', 'OC' and 'OC_merge_xkm' have the same meaning as in FIGURE 3. The platoon membership status of the subject vehicle is presented along the horizontal axis for the OC_merge_0km and OC_merge_6km controllers.

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AUTHOR CONTRIBUTION STATEMENT

The authors confirm contribution to the paper as follows: study conception and design: X. Liu, G. Zhao, N. Masoud. Analysis and interpretation of results: X. Liu, G. Zhao, N. Masoud. Draft manuscript preparation: X. Liu, G. Zhao, N. Masoud, Q. Zhu. All authors reviewed the results and approved the final version of the manuscript.

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