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Spatial accessibility to amenities, natural areas and urban green spaces: using a multiscale, multifractal simulation model for managing urban sprawl

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Abstract. We are confronted with rising energy consumption inter alia due to increasing worldwide mobility contributing to the greenhouse effect and global warming. Thus, one of the challenges in planning is to manage urban sprawl by introducing an efficient distribution of agglomerations and an optimal urban pattern incorporating economic, ecological, and social expectations of sustainable regional and urban development. In order to tackle these challenges we have taken a specific interest in the benefits of using a multifractal logic combined with measures of accessibility to urban and rural amenities including temporal settings for planning. Herein, we propose a multiscale, multifractal simulation model named Fractalopolis for simulating and evaluating scenarios consistently from a regional to a neighbourhood scale. Access to shops, services, and facilities can be improved by altering the location, whereas access to natural areas and urban green spaces can be improved by suggesting different areas for urbanization. The urbanization strategy will impact on the future regional layout and urban form. The computer application supports GIS data for incorporating the simulation system into planning support systems to support planning processes and assist with choice processes.

Keywords: regions, cities, multifractal simulation, scenarios, accessibility, amenities, landscape

1 Introduction

The phenomenon of urban sprawl arises due to a combination of land prices, commuting, and the social demand for homeownership in periurban or quiet residential areas close to green areas (Gault and Bedeau, 2007; Guo and Bhat, 2002; Schwanen et al, 2004) resulting in damage to nature and the generation of increasing traffic volume and trip lengths. The challenge in urban planning is to promote consistently efficient and easy access across scales (pedestrian-friendly environments, well-balanced public and private transport modes, efficient road networks; access for all to jobs, retail, services, healthcare, culture, and leisure) (Bolitzer and Netusil, 2000; Bonaiuto et al, 2003; De Clerq et al, 2006; Feldman, 1990). Schwanen et al (2004) showed that households usually optimize their residential choice with respect to accessibility to various types of amenities. Academic research has proved the influence of land-use patterns on travel behaviour (Geurs and van Wee, 2006).

In this paper we provide new insights into how we can find a solution for managing and avoiding urban sprawl while at the same time interlinking built-up and green area/space consistently across scales using a multiscale, multifractal approach—taking into account spatial accessibility to various amenities including temporal settings. The multiscale approach arises from the recognition that cities are complex systems (Batty and Longley, 1986; 1994);

therefore every inconsistency at a smaller scale influences the bigger scale and vice versa (see also multilevel interactions; Johnson, 2007; Pumain, 2006; Salingaros, 2005). Thus, problems such as urban sprawl must be addressed at both a regional and a local scale in order to make a metropolitan system work.

In town planning and urban design authors such as Hillman (1996), who favour the compact city concept, interpret the idea in the context of its social effects. The discussion around transport efficiency was given impetus by the work of Newman and Kenworthy (1989). The overused phrase 'access for all' encapsulates the benefit of a compact city concept as it allows easy access to urban amenities. Conversely, a strict compact city policy gives limited access to natural areas and will encourage suburbanization of households to a lower density in the long term (urban sprawl) due to higher travel times resulting in traffic congestion (Breheny, 1997; Schwanen et al, 2004). A decentralized concentration (Gordon and Richardson, 1997; Jenks et al, 1996; Levinson and Kumar, 1994)—which fractal structures can offer—provides high efficiency for transport and reduces the number of car journeys.

1.1 Research content

This paper introduces the planning simulation model *Fractalopolis* (developed as a computer application) that allows multifractal urban forms consistent across scales to be generated on the basis of existing patterns using an iterative logic, while at the same time ensuring good accessibility to various amenities and facilities as well as green areas at different temporal scales. At a regional scale the fractal logic allows the implementation of power-law logic to generate a hierarchical city/town/village system, while at a local scale (multi)fractal shapes allow identification of leisure areas for neighbourhoods. Thus, the model is operational for urbanization across scales in order to manage urban sprawl and structure agglomerations and the urban fabric.

2 Theoretical background and methodology

The spatial organization of urban patterns is rather consistent with fractal ordering principles [eg, Batty and Longley (1986; 1994), Batty and Xie (1999), Frankhauser (1994; 2008), Salingaros (2003), Shen (2002), Tannier and Pumain (2005), Thomas et al (2008; 2012), or see Batty (2013) for a synthesis]. To some extent it is reminiscent of planning strategies such as the Copenhagen Finger Plan (1947) and the Eberstadt–Möhring–Petersen Plan for Berlin (1910). Urbanization is based on the complex interaction of economic, social, and individual preferences and, at least partly, on a process of self-organization. This gives rise to processes that contribute to urban sprawl. Thus, fractal geometry offers features which can be related to spatial efficiency and optimization criteria that meet environmental, social, and economic expectations of sustainable development. Fractals have nonuniform distribution of mass (potential urbanization areas) and are neither dense nor diluted [decentralized centralization logic (see Calthorpe and Fulton, 2001)]. They have a strong hierarchical order [optimization principle and efficiency (see Pumain, 2006)] and are based on a scaling law [nonlinear logic across scales (see De Roo, 2012)]. Using fractal geometry for urban planning assumes implicitly that fractality corresponds to underlying optimization criteria that natural structures (eg, clouds, leaves, or the human vascular system) have been shown to have. Indeed, fractal surfaces seem to be optimal for spatial systems requiring a high articulation between subsystems.

In addition, a well-known property of urban systems is the emergence of a central place hierarchy known as rank-size distribution, which corresponds to a fractal hierarchy. The concept presented herein for the planning model refers to such a hierarchical organization of metropolitan areas. The hierarchical structure of an agglomeration, developed on the basis of social and economic interaction and interdependency between the locations (eg, villages), has been investigated in urban geography for a long time. These observations served Christaller (1933) as the foundation for his central place theory, which is based on a reflection about the catchment areas of different levels of services depending on how often the services are used. That is why the services for everyday life (eg, supermarkets) are close to housing, whereas weekly or monthly services require bigger catchment areas. Christaller`s theory is constrained to a functional hierarchy, and does not reflect the spatial structure (topography). This explains why, in Christaller's theory, locations are distributed evenly across the spatial surface plane. The accessibility of such a distribution is disadvantageous for several reasons. On the one hand, it demands a pseudo-homogeneous traffic infrastructure; on the other hand, all the remaining free spaces are approximately the same size. In this context, Frankhauser (2008; 2012) proposed some initial reflections which modify Christaller's hexagonal central place system by concentrating development areas around nodes of a hierarchically structured public transport network reminiscent of the logic of Calthorpe's transit-oriented development (TOD) (Calthorpe, 1993). The simulation model Fractalopolis was inspired by this theoretical reasoning. It allows scenarios to be developed for different planning strategies, such as growth, urban infill, and shrinkage, which all contribute to managing and avoiding urban sprawl. For Fractalopolis' basic spatial model, fractal measures become norms for planning. The decentralized centralization logic (see fractal features) can be best implemented using a multifractal approach, where centres and subcentres of different sizes are introduced. The advantage of using a multifractal is that it allows the generation of a figure that consists of elements of different sizes within an iteration step, whereas a unifractal is composed of elements of the same size (figure 1). Hence, we can generate a more differentiated spatial hierarchy with a multifractal. Thus, for the basic spatial model (Czerkauer-Yamu and Frankhauser, 2011; 2013) a multifractal approach with two different reduction factors (herein r_1, r_2) is introduced in order to generate centres and subcentres of different sizes reminiscent of a central place hierarchy (Sierpinksi carpet and Fournier dust, figure 1). This allows a variety of densely and less densely populated zones to be favoured. Further, the fractal logic allows the generation of an interwoven system of built-up areas and green areas.

In addition to the basic spatial model we introduce five main rule sets (table 1) which define and constrain accessibility to amenities and facilities as well as to natural areas and urban green spaces, including constraints on the existing (or newly introduced) road network, morphological standards for a multiscale intensity of occupation and 'landscape view' (lacunarity[1] rule), development standards, and preservation and density standards. Access to amenities and facilities as well as to natural areas and urban green spaces, is considered for daily, weekly, monthly, and occasional potential frequentation (levels 4, 3, 2, and 1, respectively, with analogous distance and diversity rules). Thus, access is considered in spatial terms with an inherent link to temporal terms. However, we have to be aware that frequentation can only be approached in a normative way as it depends on individual behaviour. The computer application Fractalopolis (in its current version 0.6.1; programmed by Gilles Vuidel) applies to metropolitan areas in a multiscale manner with no restriction on inhabitants. Access is considered equal for all inhabitants of the urbanized cells.

[1] lacuna = a gap or blank space in something or missing part.

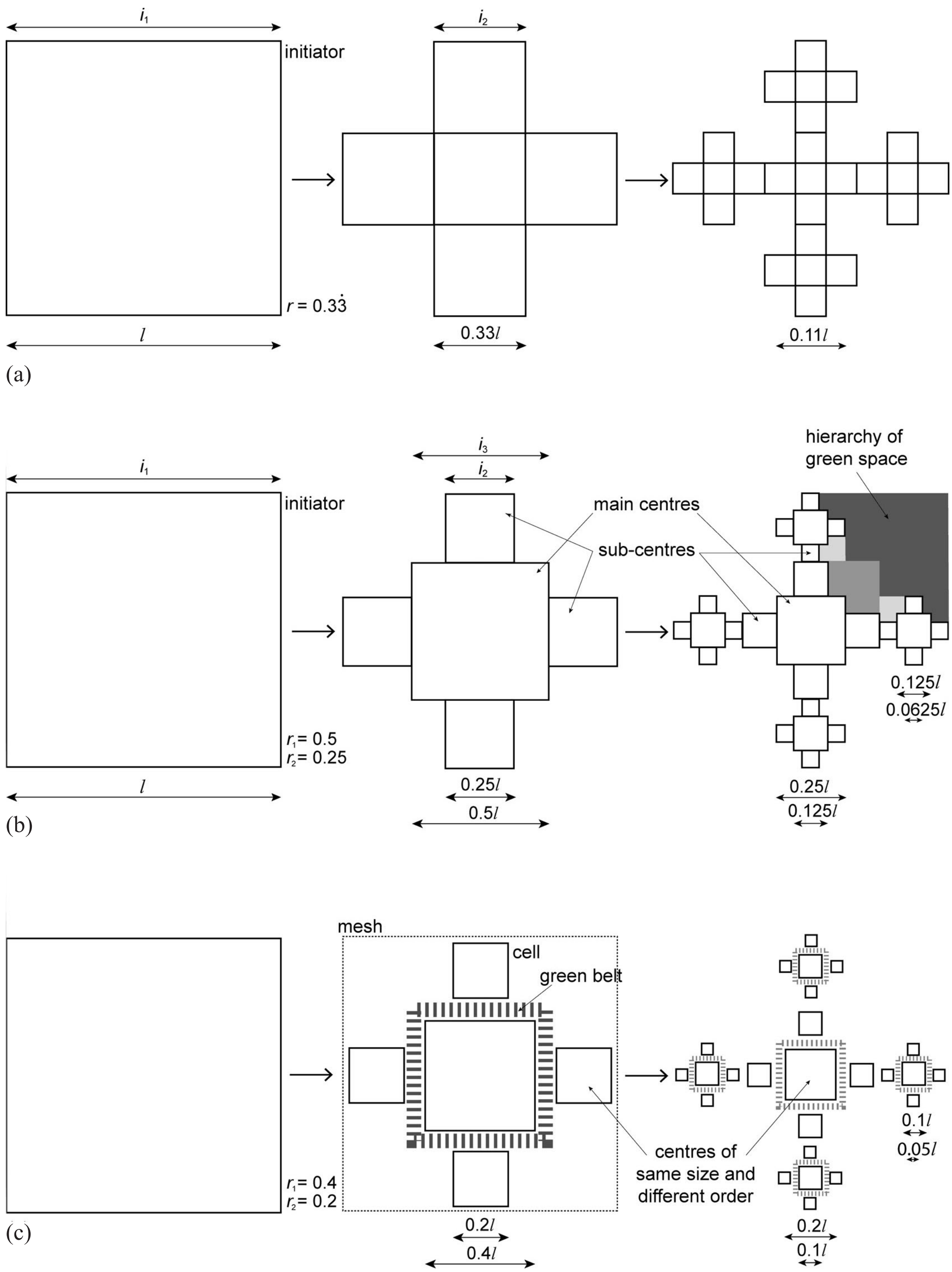

Figure 1. Generating a unifractal and two different multifractals: (a) unifractal Sierpinksi carpet where $r = 0.33$ and no different-sized centres can be distinguished; (b) a multifractal Sierpinksi carpet where $r_1 = 0.5$ and $r_2 = 0.25$, and (c) a multifractal Fournier dust where $r_1 = 0.4$ and $r_2 = 0.2$, all with the same number of elements $N_{urb} = 5$ for iteration steps 0–2 and different green-space strategies. Both (b) and (c) represent growth along axes. The chosen reduction factors are constant for the iterative mapping procedure.

Table 1. Standards and rule set for Fractalopolis.

Challenge	Rule
Morphological standards	
Multiscale intensity of occupation	Fractal iteration rule
No fragmentation of development	Neighbourhood rule
Environmental quality	Lacunarity rule including landscape view
Accessibility standards	
Access to shopping, services, and facilities at different levels	Distance rule (street network); diversity rule, cluster rule
Access to leisure and sports facilities at different levels	Distance rule (street network); diversity rule
Access to natural areas and urban green spaces at different levels	Distance rule (street network); size rule
Development standards	
Central place hierarchy	Distance rule (street network); diversity rule
Preservation standards (natural environment)	
Avoid fragmentation of green areas; protected zones	Neighbourhood rules
Respect hierarchy of green areas	(Multi)Fractal logic
Density standards	
Moderate density according to intensity of occupation	Ponderation rule (population)

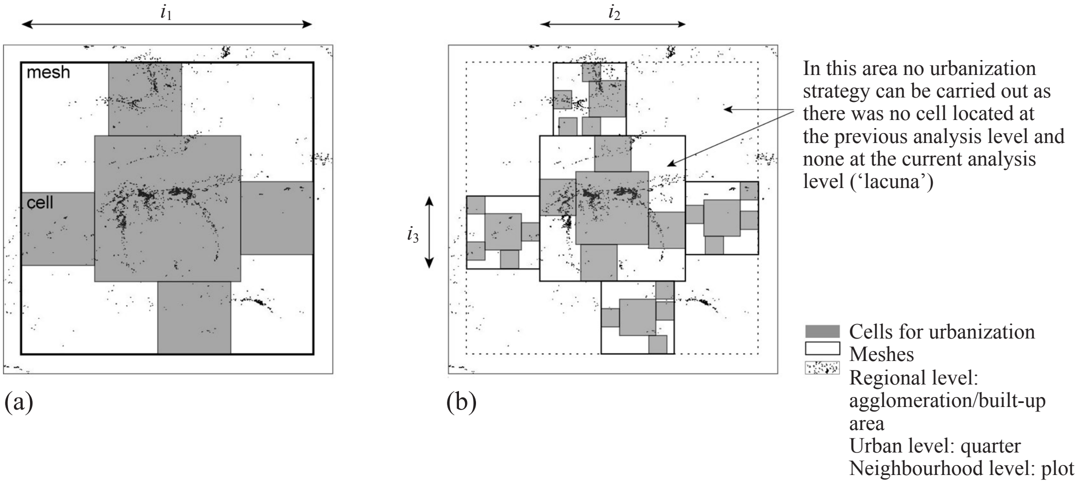

Figure 2. Sample application for the fractal urbanization rule with a Sierpinski carpet $N_{\mathrm{urb}} = 5$ (number of potential cells for urbanization) based on multiscale spatial modelling. (a) First step of iteration: one mesh where $i_1 = l$. In this mesh $N_{\mathrm{urb}} = 5$ (= $N_{\max}$) are available for urbanization. (b) Second step of iteration: one mesh where $i_2 = 0.5\ i_1$ and four meshes where $i_3 = 0.25\ i_1$. In five meshes $N_{\mathrm{urb}} = 25$ (= $N_{\max}$) are available for urbanization.

2.1 Morphological approach: application of the multifractal modelling and rules

For developing scenarios we use a cell-based approach referring to the devleopment of zones. The position of all generated cells can be chosen freely depending on the real-world situation and the planning strategy implemented. As we will see later in the application example (Vienna–Bratislava metropolitan area, figure 7 in section 3) the number of centres can also vary according to diverse morphological settings for simulation scenarios. For each iteration step the cells can be placed individually within the mesh (figure 2). The only restriction is that the cells and meshes are not allowed to intersect and must lie within the a priori defined and positioned initiator defining the size of the area under scrutiny [figure 2(b)]. At a regional scale the generated cells are most likely positioned upon existing urban structures in order to identify potential development areas, whereas from an urban scale to a neighbourhood scale potential urbanization zones are identified (eg, accessibility and morphological evaluations of non-built-up zones for urban growth scenarios; evaluation of existing fabrics for infill development scenarios). To manage urban sprawl across scales the spatial model only allows urbanization strategies for areas that have been covered across scales by cells (iteration procedure; figure 2). In general, we prefer areas for development which are served by public transport networks (this information is given by a separate GIS layer) and thus are in line with the concept of TOD.

For the study area, using an iterated function system (IFS) allows definition of a freely chosen setting for the multifractal iteration and its reduction factors for the morphological model. With the IFS the fractal rule for urbanization is defined (self-similarity dimension $D_s = D_0$; see fractal rule). This can be done separately for 'macrolevel' (iteration: regional scale to urban scale) and 'microlevel' (iteration: urban scale to neighbourhood scale). In the next step, the study area is covered by the initial square. This initiator can be varied in size to define the area under scrutiny. The number of meshes and number of cells are determined by the reduction factors $r_1, r_2, \ldots, r_n$. Iteration reduces the size of cells and meshes, which corresponds to a multiscale modelling approach (centres of different orders are generated at each iteration step; figure 1). The sizes of cells can be calculated for each level according to the reduction factors for the initial mesh i_1 and the cells for main centres i_2 and subcentres i_3 (figure 2):

$$i_2 = \frac{1}{r_1} i_1 \, , \tag{1}$$

$$i_3 = \frac{1}{r_2} i_1 \, . \tag{2}$$

By the iteration process for the case of a multifractal Sierpinski carpet, as shown here [figure 1(b)], the reduction factors r_1 and r_2 are combined in all possible ways. For the second iteration step (figures 1 and 2), for example, we have:

$$r_1 . r_1 = r_1 . r_2 \, . \tag{3}$$

Because of the commutability property we have:

$$r_1 . r_2 = r_2 . r_1 \, . \tag{4}$$

This is why areas assigned to second-order centres are the same as those for third-order centres, as described above, which differs from the Christallerian hierarchy (figure 1). Further, multifractals embrace a peculiarity which lies in the possible combination of reduction factors, explaining why catchment areas (in our case the cell's surface for each iteration step) of second-order centres and third-order centres belong to the highest-ranked centre. Another important feature of multifractals is the different sizes of cells of the same order. For a multifractal figure we find small elements (cells) of base length $r_2 . r_2$ and large ones with base length $r_2 . r_1$. Ongoing iteration adds further hierarchical levels, where we again discover that the cells' sizes issuing from different iteration steps, and thus corresponding to different hierarchical levels, are the same, while cells of the same order are different in size (figure 1). Through the iterative mapping procedure we generate an increasingly differentiated hierarchical morphospatial system, representing a centre hierarchy in the sense of Christaller and consistent with the logic of fractal structures. Using this hierarchical centre logic we introduce four amenity or facility levels, assigned to a temporal setting represented by a potential purchase rate; levels 1–4 being occasional, monthly, weekly, and daily potential frequency of use, respectively.

To implement this idea in Fractalopolis we use a binary coding system (0;1) to allow the different centres (in our case the cells) to be distinguished according to their amenity level. We are aware that by introducing codes we have given up the previously discussed commutability. For example, codes 101, 110, or 011 are not equivalent (figure 3), even though the cells are the same size. Thus, our code introduces a noncommutative operation and the system displays certain properties corresponding to unifractals. However, the code informs us directly about facility levels. This hierarchical logic forms the basis for the accessibility rules described later.

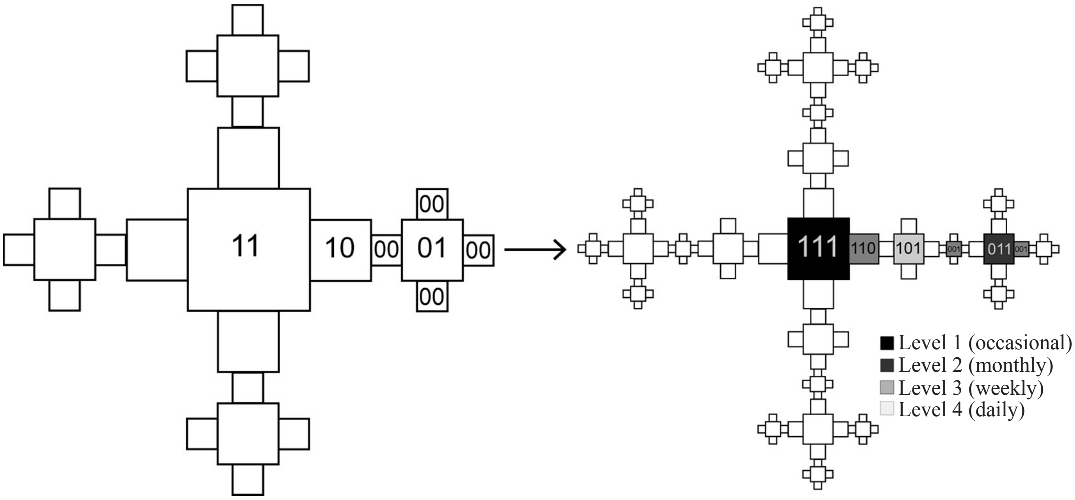

Figure 3. For a better understanding we illustrate the coding system by means of a theoretical multifractal (iteration steps 2 and 3); illustration of a centre hierarchy by potential frequencies of purchase (Frankhauser, 2012).

On the basis of the iterative mapping logic and the decrease in the size of cells, we find for each iteration step at a certain level a relationship between decreasing potential of urbanization areas and area coverage (figure 2).

2.2 Fractal rule for urbanization and lacunarity rule (additional morphological rule)

For the simulation model the self-similarity dimension D can be used as a spatial measure for predefining more or less 'compact' scenarios (within the IFS as described above). The IFS allows us to work separately with different predefined multifractals at the regional and urban scale.

In order to use the self-similarity dimension D as a fractal rule, we have to recall that multifractals are neither universally nor statistically self-similar but possess an uneven distribution of complexity for a certain object or set (Smith et al, 1996). More precisely, the geometrical structure consists of subsets where each subset has its own scaling behaviour. These subsets are spatially mixed in size according to their fractal logic. Thus, these sets can only be characterized by a series of fractal dimensions that vary from point to point, similar to static moments. For our example of the multifractal Sierpinski carpet the most elementary dimension D_0 can be calculated by means of a formula which corresponds to that used for defining the usual fractal dimension for unifractals:

$$\sum_i N_i r_i^{\,D} = 1 \ . \tag{5}$$

As a general rule, it can only be solved numerically (Mandelbrot, 1982). For our Sierpinksi carpet the result can be obtained analytically; the dimension amounts to $D_0 = 1.36$. In addition to the fractal morphological logic of the simulation model, we introduce a lacunarity rule ('gap rule') useful at the urban and neighbourhood scale. On the one hand this rule pertains to the homogeneity ('no buildings on defined green space') and interconnectivity of vacant green areas (lacunae) and on the other to the perception of natural scenery ('landscape view'—no building shall block the beautiful view) (figure 4). This in turn influences economic realities such as land prices in neighbouring potential urbanization cells. Further, the rule ensures easy access to vacant space. The theoretical configuration of the multifractal Sierpinksi carpet in figure 1(b) illustrates the biggest possible vacant space in the context of well-connected spaces consistent across scales.

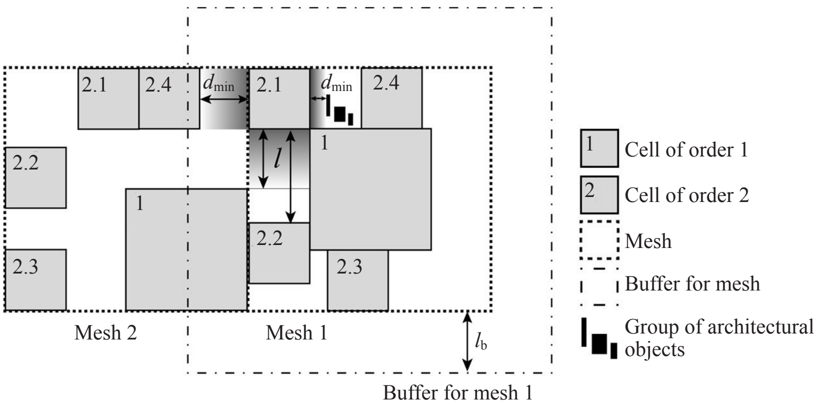

Figure 4. Lacunarity rule at an urban level: evaluation of two equal-sized meshes of $d_{\min}$ (2.1; 2.2), $d_{\min}$ (2.1; 2.4) = $d_{\min}$ (2.1; 2.4) to ensure green spaces remain interconnected and prevent the view being blocked by buildings ('maintaining landscape').

For a generalized approach we take the mean value of the minimum distance $d_{\min}$ of all present distances to buildings from a potential cell for urbanization to its neighbouring cells and the best evaluation (ie, biggest possible vacant space; $l = 1$) (figure 4). This takes into account buildings blocking the view of the landscape. By applying the rule at different decomposition steps for different levels of analysis we take into account green spaces of decreasing size (regional to neighbourhood scale). Note that the size of open and green spaces is related directly to their potential frequencies of use. The evaluation $d_{\min}$ corresponds to a linear declining function:

$$\mu(d_{\min}) = \frac{d_{\min}}{l}, \qquad \text{for } d_{\min} \leqslant 1\,, \tag{6}$$

$$\mu(d_{\min}) = 1, \qquad \text{for } d_{\min} \quad 1\,. \tag{7}$$

In the event that neighbouring cells (belonging to different meshes; eg, meshes 1 and 2; figure 4) are of the same order but different sizes (see multifractal features), the smallest cell size corresponds to l. Distances to cells of the same order are measured (regardless of their size). The minimum distance is always taken in all directions from the assessed cell according to the logic of a Manhattan metric. An introduced buffer for the mesh corresponds to l. The buffer l_b is potentially different for each mesh assessed. As well as the basic morphological multifractal rule and the additional lacunarity rule, accessibility rules are used to identify potential areas and zones for urbanization.

2.3 Accessibility rules and combination of criteria

The accessibility rules not only introduce constraints regarding access to shops, facilities, natural areas, urban green spaces, and proximity to the road network, but also a combination of criteria for a 'microlevel' (urban to neighbourhood scale) and 'macrolevel' (regional to urban scale) for each defined centre according to the centre hierarchy (figure 3) and potential levels of purchase or frequentation for amenities or facilities and green space. The accessibility rules are described by fuzzy logic and for the urban level (for daily and weekly use) introduce a combination of variables involving aggregation operators derived from fuzzy set theory (Tannier et al, 2012; Zimmermann and Zysno, 1983) to compensate for and manipulate imprecise knowledge. Depending on the iteration level of analysis (regional scale versus neighbourhood scale; figure 7) each cell is evaluated between 0 and 1 according to either eight rules ('macrolevel') or ten rules ('microlevel'). For an overall evaluation all criteria (see subsection 2.3.4) are combined according to centrality levels in a Christallerian logic. This combination can be adapted for any spatial system.

2.3.1 *Rules 1–4 (potential occasional and monthly frequentation rate of amenities)*

At a regional (in the following defined as macrolevel M) and urban (microlevel m) level the same types of shopping, services, and leisure amenities are distinguished according to their occasional and monthly potential frequentation rates. Shopping and services include universities and colleges, central public administrations, cultural centres such as opera house, theatre, museum; specialized shops, shopping malls, hospitals and health centres, DIY and garden centres, and casinos. Leisure amenities include skiing and watersports (windsurfing, kitesurfing, sailing) facilities. Green areas include golf courses as well as moors and heathlands, forests, mountains, alluvial forests, and UNESCO World Heritage sites. Transport mode is public transport (rail stations).

2.3.1.1 *Regional level (M)*. The potential of shopping, services and leisure amenities are identified by the presence, number, and diversity of services and facilities as well as the size of an area. For services including shopping S_{1M} (occasional) and S_{2M} (monthly) distances are not taken into account (identification of potential). The logic follows a quality evaluation and checks whether or not services are present. For diversity δ of services no preference is assigned to specific types of services (all types have the same weight). The presence of several services within a cell is taken into account by means of a linear increase. Green and leisure amenities L_{1M} (occasional) and L_{2M} (monthly) follow the same logic as services.

$$\mu(S_{1M}, S_{2M}) = \frac{1}{4}\delta\,, \tag{8}$$

$$\mu(S_{1M}, S_{2M}) = 1, \qquad \text{for } \delta \quad 3\,, \tag{9}$$

$$\mu(L_{1M}, L_{2M}) = \frac{1}{4}\delta\,, \tag{10}$$

$$\mu(L_{1M}, L_{2M}) = 1, \qquad \text{for } \delta \quad 3\,. \tag{11}$$

2.3.1.2 *Urban level (m)*. Access from a cell i is defined by distance d_i from the cell and the size of an area (figure 5). Distance for shopping and services S_{1m} (occasional) and S_{2m} (monthly) ranges from 0 km–20 km (evaluation value = 1) and 20 km–40 km (evaluation value = 1–0) and for leisure amenities L_{1m} (occasional) and L_{2m} (monthly) including areas >150 ha from 0 km–60 km (evaluation value = 1) and 60 km–100 km (evaluation value = 1–0). The logic for S_{1m}, L_{1m}, and L_{2m} are the same (see example):

$$\mu_i(S_{1m}) = 1, \qquad \text{for } d_i \leqslant 20 \text{ km}\,, \tag{12}$$

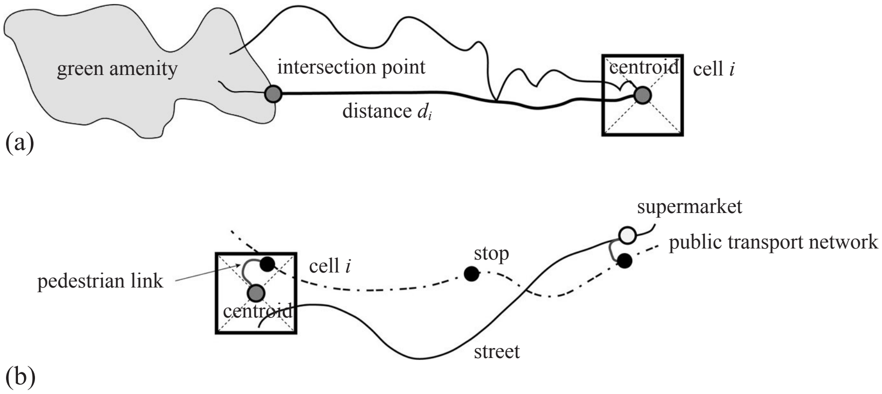

Figure 5. Processing distance.

and

$$\mu_i(S_{1\mathrm{m}}) = 2 - \frac{1}{20} d_i, \qquad \text{for 20 km} \quad d_i \leqslant 40 \text{ km} , \tag{13}$$

$$\mu_i(S_{1\mathrm{m}}) = 0, \qquad \text{for } d_i \leqslant 40 \text{ km} . \tag{14}$$

In addition, for a monthly frequentation for shopping and services $S_{2\mathrm{m}}$ diversity δ comes into play. Diversity is of importance for distinguishing the attractiveness of different central places of the same level. This formalization has been chosen since diversity is more important than distance.

$$\mu_i(S_{2\mathrm{m}}) = \mu(d_i)^{\mu(\delta)} . \tag{15}$$

2.3.2 *Rules 5 and 6 (potential weekly frequentation rate of amenities)*
Following the same classification logic as for rules 1–4, shopping, services, facilities, and leisure amenities potentially visited by people weekly include post offices, secondary schools, banks, hairdressers, florists, cafés, restaurants, bars, pharmacies, car repair workshops, bicycle shops, dentists, cinemas, household shops, drugstores, places of worship, libraries, DIY and garden stores, farmers' markets, clothes shops, beauty salons, spa centres (as competitive leisure service including beauty salon), sports centres (competitive leisure service), supermarkets, local cultural centres, and local public administration (including social facilities).

Leisure amenities are defined as small recreation areas and sports areas for activities such as tennis, soccer, and basketball and public swimming pools, with an area ranging from 2 ha–150 ha. The reason for this wide area range is the possibility of a combination of sports grounds with recreation areas for the evaluation. Transport mode is public transport (rail, bus) and cycling.

2.3.2.1 *Regional level (M).* Shopping, services, and facilities (S_3) used weekly: diversity δ and presence of several services n are taken into account. Both criteria are presumed to be 'equivalent'. The mathematical product corresponds to a rather 'pessimistic' evaluation; this seems realistic since people tend to be interested in both criteria for potential weekly use (How many different services can I use?). The number of services n ranges from 0 services (evaluation value = 0) to 4 services (evaluation value = 1) and diversity δ from 0 services (evaluation value = 0) to 5 services (evaluation value = 1).

$$\mu(S_{3\mathrm{M}}) = \mu(n)\mu(\delta) . \tag{16}$$

The rules for leisure amenities $L_{3\mathrm{M}}$ that are important in a regional context follow the same logic as in rules 1–4. For urban and neighbourhood scales only the introduced distance range differs: 0 km–2 km (evaluation value = 1), 2 km–15 km (evaluation value = 1–0), and > 15 km (evaluation value = 0).

$$\mu(L_{3\mathrm{M}}) = \frac{1}{4}\delta \,, \tag{17}$$

$$\mu(L_{3\mathrm{M}}) = 1, \qquad \text{for } \delta \quad 3 \,. \tag{18}$$

2.3.2.2 *Urban level (m).* For the urban scale we introduce the logic of clusters. The formalization follows that proposed by Tannier et al (2012) for the MUP City simulation model. According to Tannier et al, businesses (here we take businesses, services, and leisure facilities into account) are assumed to form clusters for weekly frequentation. A cluster is composed of either one service or facility or at least two that are less than 800 m apart. The accessibility evaluation of a cell is defined by the attractiveness of a cluster and its distance from the cell. A cluster's attractiveness is constrained by the number of services and their diversity. On the basis of the accessibility evaluation multiple clusters can be present. For the evaluation one highly accessible cluster scores more highly than two moderately accessible ones, two equally accessible clusters are deemed better than just one, and a single very highly accessible cluster is more beneficial than a very accessible cluster plus one that is not very accessible. The distance ranges are 0 km–3 km (evaluation value = 1), 3 km–10 km (evaluation value = 1–0), and > 10 km (evaluation value = 0). Formally, the rule is defined as follows:

- a set of cells: $i = (1, 2, f, k)$;
- a set of clusters of shops and services: $j = (1, 2, f, l)$;
- the number of shops and services in cluster j is n_j;
- the diversity of the businesses (number of different types of businesses) in cluster j is δ_j;
- the distance (measured over the network) between cell i and cluster j is d_{ij};
- accessibility from cell i to cluster j is Y_{ij}.

$$Y_{ij} = [\mu(n_j)^{\mu(\delta_j)}\mu(d_{ij})]^{1-\mu(d_{ij})}\{1 - [1 - \mu(n_j)^{\mu(\delta_j)}][1 - \mu(d_{ij})]\}^{\mu(d_{ij})} \,. \tag{19}$$

The operator $\mu(S_{3\mathrm{m}})$ evaluates the accessibility of the cell i to the set of service clusters visited weekly.

$$\mu(S_{3\mathrm{m}}) = 1 - \prod(1 - Y_{ij}) \,. \tag{20}$$

Calculating Y_{ij} and $\mu(S_{3\mathrm{m}})$ involves applying Zimmermann's aggregation operator (Zimmermann and Zysno, 1983). For the definition of functions $\mu(n_j)$, $\mu(d_j)$, and $\mu(\mathrm{d}_{ij})$ see Tannier et al (2012, page 819).

For green and leisure amenities $L_{3\mathrm{m}}$ the presence of several amenities is taken into account without evaluation of numbers of the same category. We use the same logic as for $L_{1\mathrm{m}}$ and $L_{2\mathrm{m}}$ (rules 1–4). Distance ranges from 0 km to 2 km (evaluation value = 1) and 2 km to 15 km (evaluation value = 1–0) and areas 2 ha to 150 ha.

$$\mu_i(L_{3\mathrm{m}}) = 1, \qquad \text{for } d_i \leqslant 2 \text{ km} \,, \tag{21}$$

$$\mu_i(L_{3\mathrm{m}}) = \frac{15}{13} - \frac{1}{13}d_i, \qquad \text{for } 2 \text{ km} \quad d_i \leqslant 15 \text{ km} \,, \tag{22}$$

$$\mu_i(L_{3\mathrm{m}}) = 0, \qquad \text{for } d_i \quad 15 \text{ km} \,. \tag{23}$$

2.3.3 *Rules 7 and 8 (potential daily frequentation rate of amenities)*
Rules 7 and 8 apply the same logic across iteration levels as described in rules 5–6, taking into account the following amenities: for daily use, corner shops and minimarkets, organic stores, primary schools, kindergartens and crèches, newsagents and tobacconists, bakeries, butchers, general medical practitioners, cash machines; for leisure, playgrounds, dog exercise areas; small parks with an area from 0 ha to 2 ha.

At a regional level $\mu(S_{4\mathrm{M}})$ for daily use is the same as $\mu(S_{3\mathrm{M}})$ for weekly use. $\mu(L_{4\mathrm{M}})$ follows the same logic as $\mu(L_{3\mathrm{M}})$ for daily frequentation with the only difference being that $\delta = 0.33$.

At an urban level the cluster logic is used with different distances, ranging from 0 m–600 m (evaluation value = 1), 600 m–1200 m (evaluation value = 1–0), and > 1200 m (evaluation value = 0); cluster size 200 m for potential daily use of shopping and services $S_{4\mathrm{m}}$. For leisure amenities for daily use $L_{4\mathrm{m}}$ follows the same logic as $L_{3\mathrm{m}}$; distance ranges from 0 m to 400 m (evaluation value = 1) and 400 m–800 m (evaluation value = 1–0).

Transport mode is public transport (bus), cycling and walking.

2.3.4 *Combination of criteria*
Finally, all criteria are combined according to centrality levels. This combination can be adapted for any spatial system.

We use following symbols:
Amenities (*services and facilities*):
F_4: daily frequentation—services S_4 and leisure amenities L_4, morphological rule M_4;
F_3: weekly frequentation—services S_3 and leisure amenities L_3, morphological rule M_3;
F_2: monthly frequentation—services S_2 and leisure amenities L_2;
F_1: occasional frequentation—services S_1 and leisure amenities L_1.
Centrality levels:
P_1: important central place—Oberzentrum;
P_2: intermediate central place—Mittelzentrum;
P_3: small central place—Unterzentrum;
P_4: very small central place—Kleinzentrum.
For the overall accessibility A at each level (important central place P_1, intermediate central place P_2, small central place P_3, very small central place P_4) three or two types of criteria (services, and leisure and morphology) are weighted and an arithmetic mean is computed,[(2)] for example, for the Vienna–Bratislava metropolitan region for P_3:

$$A(P_3)[F_4] = \mu(S_4) \cap \mu(L_4) \cap \mu(M_4) = 0.33\mu(S_4) \cap 0.33\mu(L_4) \cap 0.33\mu(M) \,, \tag{24}$$

$$A(P_3)[F_3] = \mu(S_3) \cap \mu(L_3) \cap \mu(M_3) = 0.4\mu(S_3) \cap 0.4\mu(L_3) \cap 0.2\mu(M) \,, \tag{25}$$

$$A(P_3)[F_2] = \mu(S_2) \cap \mu(L_2) = 0.45\mu(S_2) \cap 0.55\mu(L_2) \,, \tag{26}$$

$$A(P_3)[F_1] = \mu(S_1) \cap \mu(L_1) = 0.45\mu(S_1) \cap 0.55\mu(L_1) \,. \tag{27}$$

The accessibility A is hierarchically structured. From a functional point of view the explicit hierarchical approach allows a relational link to be made between potential frequentation of different amenities (F_1, F_2, F_3, F_4) and the corresponding distances: for example, from P_1 to P_4 for the Vienna–Bratislava metropolitan region:

$$A(P_1) = 0.25A(P_1)[F_4] + 0.25A(P_1)[F_3] + 0.25A(P_1)[F_2] + 0.25A(P_1)[F_1] \,, \tag{28}$$

(2) The combination of criteria was first formulated by Tannier (ThéMA, CNRS, France) and later adapted, tested, and verified by Yamu and Frankhauser (2013).

$$A(P_2) = 0.25A(P_2)[F_4] + 0.25A(P_2)[F_3] + 0.25A(P_2)[F_2] + 0.25A(P_2)[F_1]\,, \tag{29}$$

and

$$A(P_3) = 0.3A(P_3)[F_4] + 0.3A(P_3)[F_3] + 0.20A(P_3)[F_2] + 0.20A(P_3)[F_1], \tag{30}$$

$$A(P_4) = 0.5A(P_4)[F_4] + 0.25A(P_4)[F_3] + 0.1875A(P_4)[F_2] + 0.0625A(P_4)[F_1]\,. \tag{31}$$

The following rules define the variables and types of services, facilities, and leisure amenities according to their potential frequentation based on their purpose. These rules will feed into the overall evaluation as criteria.

2.3.5 *Processing distances*

For the accessibility evaluation the shortest metric distances on the network are chosen. Three main categories can be evaluated: access via the road network (car, bicycle, pedestrian); public transport network (PTN) access or a combination of, for example, car and PTN access, allowing the best evaluation to be taken for the evaluation function (usual behaviour). See figure 5.

2.4 Nondevelopable zones

The multiscale logic incorporates nondevelopable areas. Zones which were not chosen to be part of an urbanization strategy cannot be developed in the subsequent decomposition steps (iterative logic; figure 2). In addition, Fractalopolis allows cells to be readjusted to a higher level (eg, metropolitan scale) when it is realized at a lower scale (eg, neighbourhood scale) that a certain area needs to be urbanized. The advantage of this strategy is that the user can develop 'best fit' scenarios consistent across scales (bottom-up and top-down logic).

2.5 Restricted zones

There is allowance for defining a priori restricted zones which are not part of an urbanization strategy: for example, alluvial forests, World Heritage areas, and alpine areas.

3 Study area

For the study area we chose the Vienna–Bratislava metropolitan region (figure 6) which has a current population of 4.5 million people. The rationale for choosing this cross-border study area (Austria, Czech Republic, Slovak Republic) was due to its historical context and the former 'Iron Curtain' separating Western and Eastern Europe. This region has suffered until now—especially in the areas close to the former Iron Curtain—from an irregular and fragmented morphology, lack of development, and low access to a number of amenities. We were seeking to find out if a spatial simulation model such as Fractalopolis can support the development of scenarios enabling the two regions (Vienna region and Bratislava region) to be interwoven in accordance with a sustainable planning approach. We created four different scenarios (two at a regional scale and two at a neighbourhood scale) using two different reference models with combinations (IFS): that is, a Sierpinski carpet and a Fournier dust.

We argue that a multifractal Sierpinski carpet supports a more compact scenario compared with a Fournier dust (figure 1). At a regional scale, the multifractal Sierpinksi carpet supports an axes-based development incorporating the logic of different sized centres and subcentres, which are important for the Vienna–Bratislava metropolitan region (eg, enhancing urbanization close to existing rail routes and historic routes alongside the Danube River), whereas a Fournier dust is closer to the emergence of agglomerations. At an urban scale, a Fournier dust creates, for example, more potential green spaces between districts compared with a Sierpinksi carpet. This geometric logic is fuzzier at a neighbourhood scale (where plot sizes are predominant). We have to be aware that more dense and diluted scenarios are strongly influenced by N_{urb}, as the latter predefines the potential development areas (figure 7). We are aware that a series of tests needs to be carried out for various metropolitan areas as well as using various fractal geometries for the underlying spatial and morphological logic.

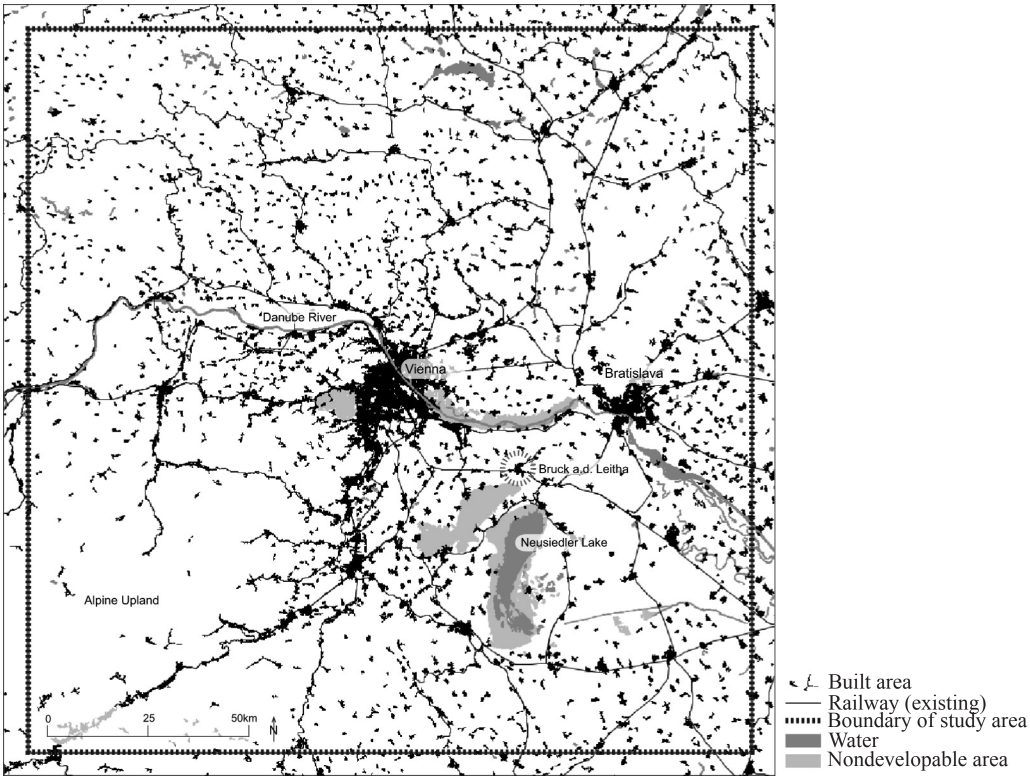

Figure 6. The study area: Vienna–Bratislava metropolitan area.

For our case study all accessibility rules and the morphological rules were activated each time. The base length of the initiator (iteration step 0) was always 175 km in order to cover the study including a buffer (figure 6); the final smallest cell size at iteration step 8 was 27.35 m (approximately 750 m² for a plot of building land). Shops, businesses, and services were equally distributed according to their potential monthly, weekly, and daily frequentation rates. For assembling the database the area of the cross-border region between the cities and towns of Brno (Czech Republic, north), Graz (Austria, south); St. Pölten (Austria, west), and Trnava (Slovakia, east) was chosen.

For all scenarios the position of the initial square was the same—Vienna's historic core. This choice was based on preanalyses of the region using the rank-size rule (Auerbach 1913; Zipf, 1949) and space syntax. The number of cells worth urbanizing was identified with the aid of Fractalopolis software for each scenario.

3.1 Scenarios and ex post assessment

For each scenario, the accessibility of shops, services, and leisure amenities including green areas/spaces for daily, weekly, monthly, and occasional use was evaluated. For the Sierpinski carpet scenario the IFS was defined with the maximum number of cells within a mesh set at $N_{urb} = 5$; 1 main cell with a reduction factor $r_1 = 0.5$ and 4 subcells where $r_2 = 0.25$, and a preponderation (ranks) for the main cell (main centre) of 1 and the subcells (subcentres) of 0; main and subcells are topologically contiguous [figures 7(a), 7(c)]. For the Fournier dust scenario IFS was defined with $N_{urb} = 9$ [regional scale: 1 main cell; 8 subcells; figure 7(b)], $N_{urb} = 6$ [urban scale: 1 main cell, 5 subcells; figure 7(d)] where $r_1 = 0.4$ and $r_2 = 0.25$; preponderation was the same as for the Sierpinski carpet scenario. Main cells and subcells are not topologically contiguous, hence predefined open (green) spaces are introduced between main cells and subcells (figure 1).

At a regional scale, the model is used in a normative way as the global evaluation for each criterion (table 1 and rule sets) mainly ranges between 0.75–1.00. All scenarios take the regional and local topography and nondevelopable zones into consideration.

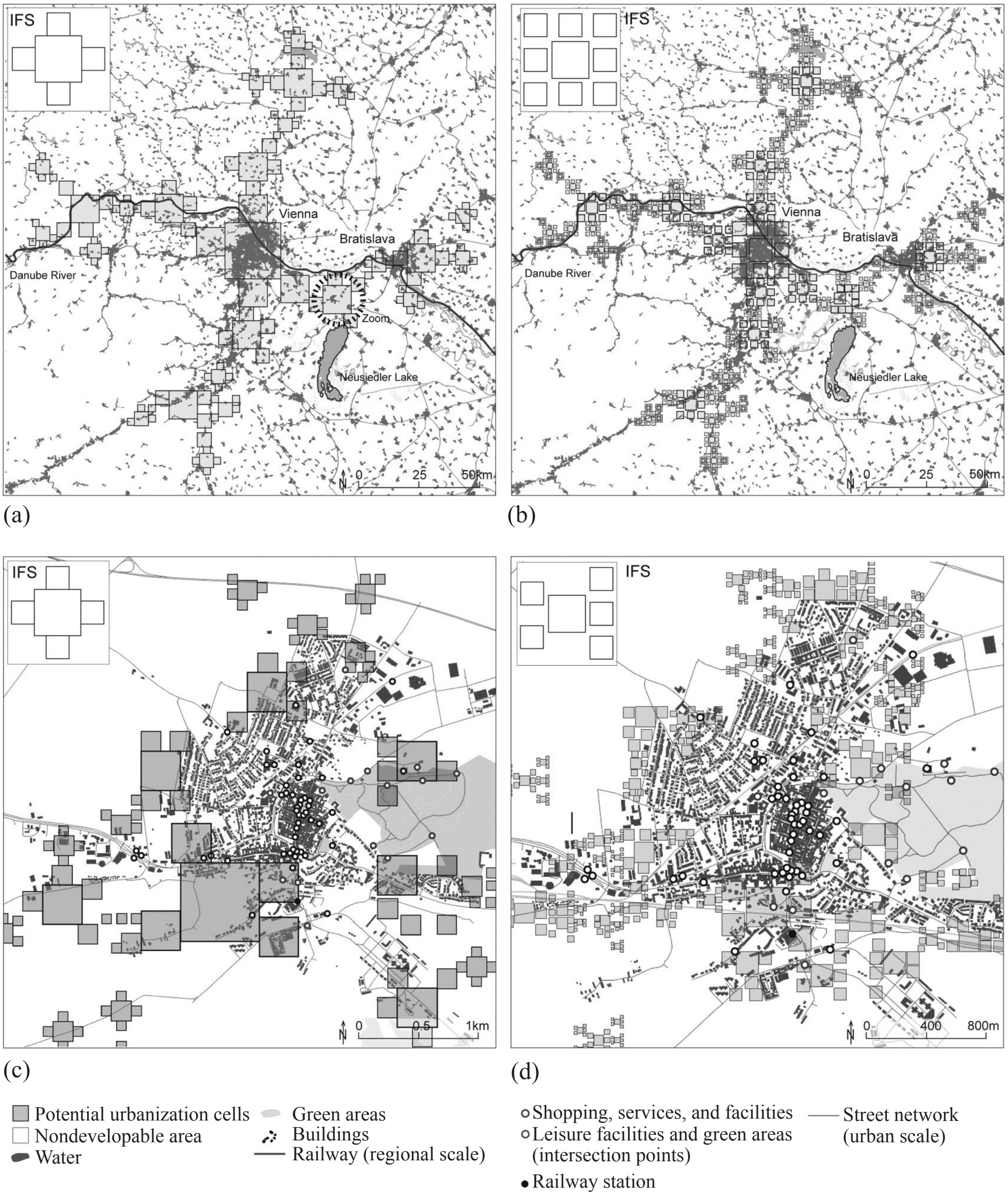

Figure 7. Interactive developed scenarios for Vienna–Bratislava metropolitan region with a zoom for Bruck Leitha, Austria. (a) Regional scenario at iteration step 3 using a Sierpinski carpet with $N_{\mathrm{urb}} = 5$, $r_1 = 0.5$, $r_2 = 0.25$, cell sizes vary from 21.86 km–2.73 km, 125 cells. (b) Implementing at iteration step 3 using a Fournier dust with $N_{\mathrm{urb}} = 9$ to adjust better to the existing situation, $r_1 = 0.4$, $r_2 = 0.25$, cell sizes vary from 8.74 km–0.68 km, 1125 cells. (c) Urban scenario (zoom) at iteration step 7 using Sierpinski carpet with $N_{\mathrm{urb}} = 5$, $r_1 = 0.5$, $r_2 = 0.25$, cell sizes vary from 847.6 m–54.7 m, 125 cells. (d) Urban scenario (zoom) at iteration step 8 using a Fournier dust with $N_{\mathrm{urb}} = 6$, $r_1 = 0.4$, $r_2 = 0.25$, cell sizes vary from 21.35 m–276 m, 513 cells.

Table 2. Sierpinski carpet scenario ($N_{urb} = 5$) for iteration step 3 (regional scale); accessibility evaluation for 23 sample cells, no morphological rule, see figure 8(a).

Cell code (iteration step 3)	Cell base length (m)	Presence and diversity				Population (P)
		monthly facilities [a] ($S_1 = S_2$)	weekly facilities (S_3)	monthly leisure amenities [a] ($L_1 = L_2$)	weekly leisure amenities (L_3)	
111	21 859	1.00	1.00	1.00	1.00	1 440 254
110	10 929	1.00	1.00	1.00	1.00	62 485
101	10 929	1.00	1.00	1.00	1.00	5 700
011	10 929	0.75	1.00	1.00	1.00	20 305
100	5 465	1.00	1.00	1.00	1.00	3 807
011	10 929	1.00	1.00	0.75	1.00	6 821
101	10 929	1.00	1.00	1.00	0.75	7 798
000	2 733	1.00	0.40	0.75	0.25	318
010	5 465	1.00	1.00	0.75	1.00	2 254
000	2 732	1.00	1.00	0.75	0.75	462
000	2 732	0.75	0.80	1.00	0.75	7
101	10 929	1.00	1.00	1.00	1.00	13 349
010	5 465	1.00	1.00	1.00	0.75	89 006
011	10 929	0.75	1.00	0.75	1.00	11 573
001	5 465	0.75	0.75	1.00	0.75	1 584
001	5 465	1.00	0.80	1.00	1.00	3 080
000	2 732	0.75	0.60	0.75	0.50	892
000	2 733	1.00	1.00	1.00	1.00	189
001	5 465	1.00	0.80	1.00	0.25	5 779
011	10 929	0.75	1.00	1.00	1.00	20 305
000	5 465	0.75	0.80	0.75	1.00	10 507

Cell code	Cell (m)	($S_1 = S_2$)	(S_3)	(S_4)[b]	($L_1 = L_2$)	(L_3)	(L_4)[b]	(P)	Overall evaluation
For a small central place ('Unterzentrum' P_3)—Bruck an der Leitha and surrounding area									
101	10 929	1.00	1.00	1.00	1.00	1.00	0.50	13 349	0.73
100	5 465	1.00	1.00	1.00	1.00	1.00	0.25	4 925	0.75
For an important central place ('Oberzentrum' P_1)—Vienna and surrounding area (south)									
111	21 859	1.00	1.00	1.00	1.00	1.00	0.25	1 440 254	0.66
110	10 929	1.00	1.00	1.00	1.00	1.00	0.25	62 485	0.75

[a] In terms of calculation the monthly level equals the occasional level (see rules in text).
[b] S_4 = daily shopping and service facilities; L_4 = daily sport facilities, leisure amenities—on 'macrolevel'.

At iteration step 8 the size of the smallest cell approximates the average European plot size of 500 m² for single houses. Thus, the distance to a given set of services and amenities indicates the distance to these from an individual resident's point of view (location choice) and from residential neighbourhoods and quarters. For the evaluation a sample set of cells was chosen for scenario A for different iteration steps (see figures 8 and 9 and tables 2–4).

All scenarios were calculated using the road network for the accessibility evaluation. Fractalopolis offers an 'overall' evaluation combining all criteria as described in subsection 2.3.4. In addition, for each cell the individual accessibility according to temporal settings (see rules above) including the morphological rule (lacunarity rule) can be evaluated.

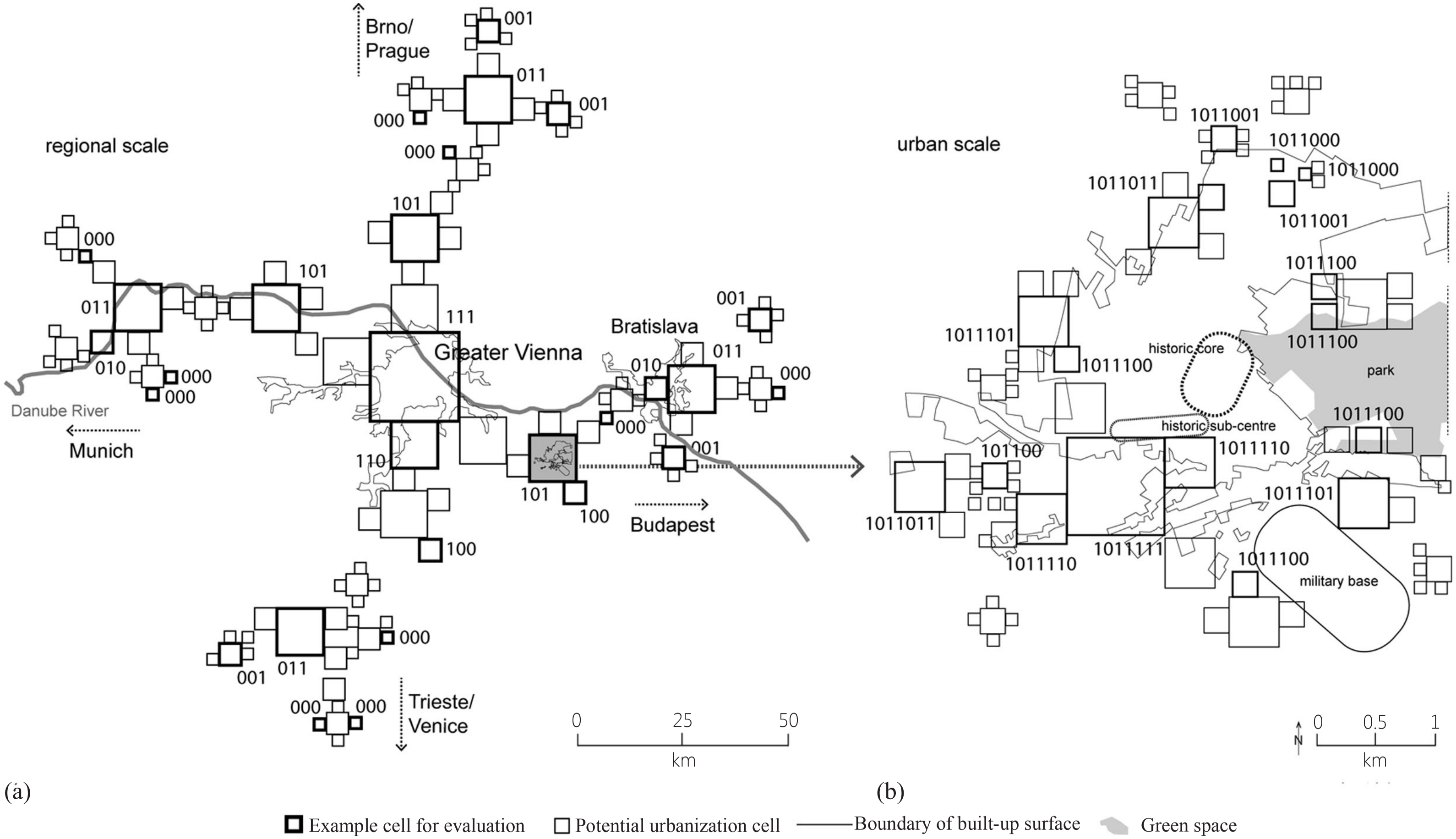

Figure 8. (a) Regional level: Sierpinski carpet scenario ($N_{\text{urb}} = 5$) for iteration step 3; accessibility evaluation for 23 sample cells, no morphological rule (see table 2). (b) Urban level: Bruck an der Leitha, Lower Austria scenario A Sierpinski carpet ($N_{\text{urb}} = 5$) for iteration step 7; accessibility evaluation for 18 cells including morphological rule (see table 3).

Table 3. Sierpinski carpet scenario ($N_{urb} = 5$) for iteration step 7 (urban scale); accessibility evaluation for 18 sample cells, including morphological rule, see figure 8(b).

Cell code (iteration step 7)	Cell base length (m)	Presence and diversity						Morphological rule	Population	Overall evaluation
		monthly facilities[a] ($S_1 = S_2$)	weekly facilities (S_3)	daily facilities[b] (S_4)	monthly leisure amenities[a] ($L_1 = L_2$)	weekly leisure amenities (L_3)	daily leisure amenities[c] (L_4)			
For small central place (Unterzentrum P_3)—Bruck an der Leitha (urban scale m)										
1011111	683	0	1	0.00	0.67	0.41	0.00	1.00	296	0.51
1011110	342	0	1	0.00	0.67	0.40	0.00	0.00	28	0.22
1011110	342	0	1	1.00	0.67	0.42	0.20	0.00	214	0.43
1011100	171	0	1	1.00	0.67	0.41	0.14	0.15	0	0.46
1011101	342	0	1	0.48	0.67	0.41	0.00	1.00	3	0.56
1011100	171	0	1	0.10	0.66	0.41	0.00	0.00	16	0.24
1011100	171	0	1	0.26	0.66	0.42	0.42	0.33	0	0.36
1011100	171	0	1	1.00	0.66	0.42	0.00	0.40	0	0.40
1011001	171	0	1	1.10	0.66	0.41	0.00	1.00	7	0.50
1011000	85	0	1	0.21	0.66	0.41	0.00	0.00	4	0.26
1011000	85	0	1	0.00	0.66	0.41	0.00	0.00	2	0.22
1011001	171	0	1	0.00	0.66	0.41	0.41	1.00	0	0.48
1011010	171	0	1	0.50	0.66	0.41	0.00	0.00	66	0.31
1011011	342	0	1	0.73	0.67	0.41	0.00	1.00	65	0.61
1011101	342	0	1	0.00	0.67	0.41	0.00	1.00	15	0.48
1011100	171	0	1	0.13	0.67	0.41	0.00	0.00	39	0.24
1011001	171	0	1	0.00	0.68	0.40	0.00	1.00	0	0.48
1011011	342	0	1	0.00	0.68	0.39	0.00	1.00	18	0.48

[a] In terms of calculation the monthly level equals the occasional level (see rules in text).
[b] Daily shopping and service facilities at "microlevel".
[c] Daily sports facilities and leisure amenities at "microlevel".

Table 4. Bruck an der Leitha, Lower Austria. Scenario A Sierpinski carpet ($N_{urb} = 5$) for iteration step 8 (neighbourhood and plot scale); accessibility evaluation for 35 sample cells, including morphological rule, see figure 9.

Cell code (iteration step 8)	Cell base length (m)	Presence and diversity						Morphological rule	Population
		monthly facilities [a] ($S_1 = S_2$)	weekly facilities (S_3)	daily facilities [b] (S_4)	monthly leisure amenities [a] ($L_1 = L_2$)	weekly leisure amenities (L_3)	daily leisure amenities [c] (L_4)		
For small central place (Unterzentrum P_3)—Bruck an der Leitha (urban scale m)									
10111111	342	0	1	0.000	0.67	0.41	0.000	1.000	2.3
10111110	171	0	1	0.000	0.67	0.41	0.000	0.000	0.0
10110001	43	0	1	0.000	0.68	0.40	0.000	1.000	0.0
10110011	86	0	1	0.000	0.68	0.40	0.000	1.000	0.0
10110010	43	0	1	0.000	0.68	0.40	0.000	2.700	0.0
10111110	14	0	1	1.000	0.67	0.41	0.090	0.000	14.0
10111101	171	0	1	1.000	0.67	0.42	0.170	1.000	45.0
10111100	85	0	1	1.000	0.67	0.41	0.120	0.210	0.0
10111101	171	0	1	1.000	0.67	0.41	0.063	1.000	0.0
10111010	85	0	1	0.000	0.66	0.39	0.000	0.002	0.0
10111010	85	0	1	1.000	0.67	0.41	0.000	4.610	0.0
10111010	85	0	1	0.430	0.67	0.40	0.000	4.614	0.0
10111001	85	0	1	0.010	0.66	0.41	0.000	1.000	0.0
10111000	43	0	1	1.000	0.67	0.41	0.100	0.100	0.0
10111001	43	0	1	1.000	0.67	0.42	0.100	0.100	0.0
10111000	43	0	1	0.850	0.66	0.41	0.120	0.000	0.0
10111001	85	0	1	0.300	0.66	0.41	0.060	0.000	0.0
10110011	85	0	1	0.100	0.66	0.41	0.000	1.000	0.0
10110010	43	0	1	0.080	0.66	0.41	0.000	0.030	0.0
10110000	21	0	1	0.150	0.66	0.41	0.000	0.001	0.0
10110011	85	0	1	0.000	0.66	0.41	0.000	1.000	0.0
10110001	43	0	1	0.000	0.66	0.41	0.000	1.000	0.0

Table 4 (continued).

Cell code (iteration step 8)	Cell base length (m)	Presence and diversity						Morphological rule	Population
		monthly facilities [a] ($S_1 = S_2$)	weekly facilities (S_3)	daily facilities [b] (S_4)	monthly leisure amenities [a] ($L_1 = L_2$)	weekly leisure amenities (L_3)	daily leisure amenities [c] (L_4)		
10110100	43	0	1	0.490	0.66	0.41	0.000	0.000	2.0
10110101	85	0	1	1.000	0.66	0.41	0.000	1.000	1.0
10110111	171	0	1	0.370	0.66	0.41	0.000	1.000	0.0
10110101	85	0	1	1.000	0.66	0.41	0.000	1.000	2.0
10111001	85	0	1	0.000	0.69	0.41	0.000	1.000	0.0
10111101	171	0	1	0.000	0.67	0.41	0.000	1.000	1.0
10111001	85	0	1	0.131	0.67	0.41	0.000	1.000	5.0
10111101	171	0	1	0.000	0.67	0.41	0.000	1.000	18.0
10111100	85	0	1	0.000	0.67	0.41	0.000	0.000	6.0
10111100	85	0	1	0.250	0.67	0.41	0.000	0.000	0.5
10110111	171	0	1	0.000	0.68	0.38	0.000	1.000	0.0
10110110	85	0	1	0.000	0.68	0.39	0.000	0.000	0.0
10110101	85	0	1	0.000	0.68	0.39	0.000	1.000	0.0

[a] In terms of calculation the monthly level equals the occasional level (see rules in text).
[b] Daily shopping and service facilities at "microlevel".
[c] Daily sports facilities and leisure amenities at "microlevel".

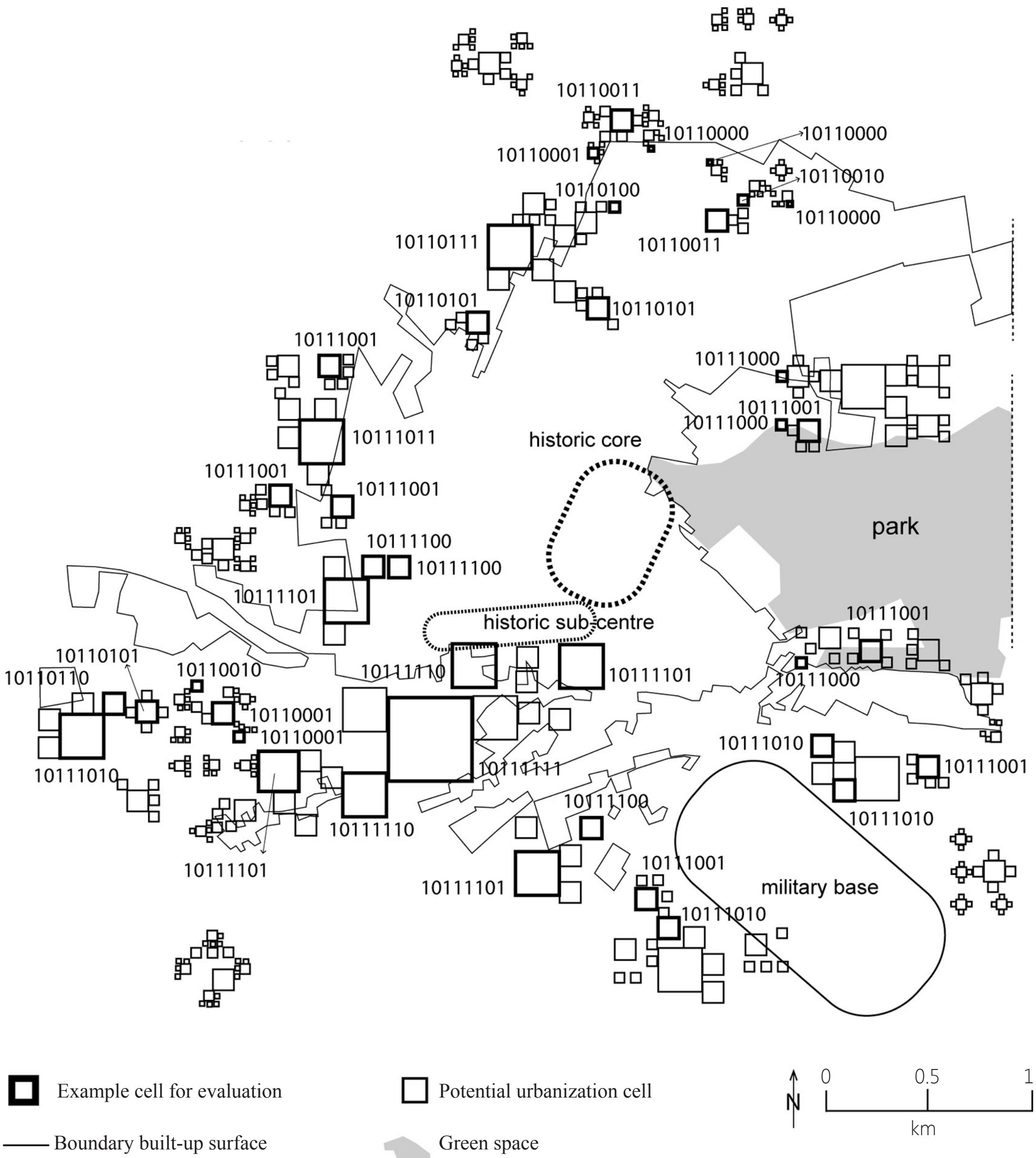

Figure 9. Neighbourhood level: Bruck an der Leitha, Lower Austria scenario A Sierpinski carpet (N_{urb} = 5) for iteration step 8; accessibility evaluation for 35 sample cells including morphological rule (see table 4).

At a regional scale the 'overall' evaluation is interesting as it is functionally linked to the central place logic, whereas at an urban to a neighbourhood scale the individual accessibility to daily, weekly, monthly, and occasionally visited amenities and morphological evaluation for each cell is of interest. For our case study we show the individual evaluations as well as the overall evaluation for each scenario outlined here. The evaluation is normalized from 0–1. This supports the comparison between different scenarios and applications.

The evaluation shows that Fractalopolis works at a regional scale as a normative model supporting the identification of constraints on and opportunities for potential developments. At an urban and neighbourhood scale the model works more as an interactive decision support simulation in real time for identifying in greater detail potential cells for urbanization.

However, the morphological rule with its lacunarity rule is a strong rule dominating the overall evaluation (see also generic process flow diagram in the appendix).

4 Conclusion

We set out to answer the main question: how can we find a solution to managing urban sprawl across scales while at the same time linking built-up and green areas in a nonhomogeneous manner?

In order to answer this question we have developed the simulation model Fractalopolis based on a multiscale approach incorporating a multifractal principle. The multiscale logic reflects a stringent hierarchy of urban systems (cities, towns, villages, hamlets) reminiscent of Christaller by introducing an uneven distribution of settlements. At an urban scale it distinguishes between different-sized neighbourhoods and at a neighbourhood scale between different-sized building plots.

Briefly, Fractalopolis combines a multifractal morphological logic with accessibility rules (plus a population model which was not discussed here) in order to develop scenarios aimed at a highly efficient distribution of agglomerations within a region, while at an urban scale it enables the development of scenarios that take into account environmental and social compatibility including some social realities (eg, residential choice criteria, such as individual housing and landscape quality). Further, the simulation model enables the development of scenarios taking into account a sustainable planning approach (moderate density, urban microclimate, city of short distances, and interweaving of built-up and green space) as well as the idea of resilience for urban and suburban journeys (train instead of car; bicycle instead of car; walking), supplying people with the closest possibility for shopping, services, sports, and leisure activities according to their potential frequentation rate (eg, convenience and organic store for daily use near home; hypermarket for weekly shopping; small parks for daily walks; skiing for monthly leisure events).

In summary, the interactive tool Fractalopolis performs consistently across scales (from regions to neighbourhoods). Thus, the development of specific methods, such as aggregation and the flexibility (sensitivity) of parameters, was important in order to allow the user to adapt to any regional or urban system at all scales in order to validate (or invalidate) various regional and urban development strategies.

We have shown that a multifractal model can limit urban sprawl while at the same time avoiding the nonefficiency of a compact city model (Schwanen et al, 2004). Moreover, the tool allows the assessment of accessibility to urban green spaces and natural areas. The additional rule for the 'landscape view' (feeding into the multifractal morphological rule) strengthens the idea of an environmental quality assessment. At a local iteration level it takes into account the idea of different housing types, while the incorporation of restricted zones allows the preservation of non-built-up areas.

The application of Fractalopolis to more study areas would be useful for confirming alterability, variability, and stability with regard to different topological, morphological, and topographic conditions. Other particularly interesting applications would be to explore the integration of economic realities such as ticket prices and land prices and also time constraints and activity chains, as well as interactive 3D models with different levels of details (eg, for virtual-reality environments) in the context of real-time decision making for laypersons, experts, and the public in the framework of participative and collaborative planning.

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Appendix

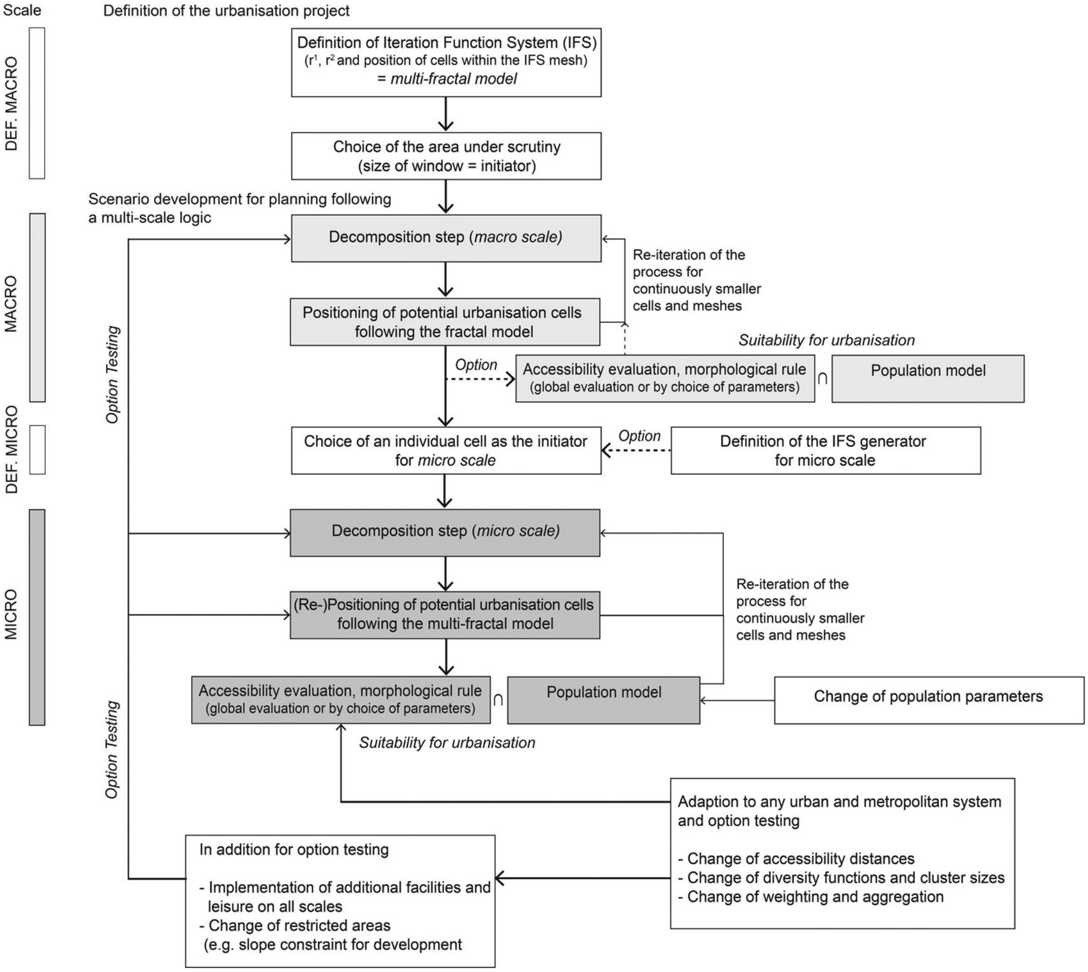

Figure A1. Generic flow chart for Fractalopolis.