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## Introduction

Oceanic flow at scales greater than few km is largely two-dimensional (2D). In such a system, kinetic energy (KE) is transferred upscale, which is seen in altimetry data and ocean models [3,6]. On the ocean surface, the inverse cascade of energy can be explained with both quasi-geostrophic (QG) dynamics [1], i.e., conservation of potential vorticity (PV), and surface-quasi-geostrophic (SQG) dynamics [2], i.e., conservation of buoyancy. The mechanisms predict different scaling laws for KE spectrum and evidence for both have been found in the observations and model analyses [3-6].

In this work, we compute spectra and fluxes of KE, enstrophy and buoyancy variance using high-resolution model data in different geographical locations. The idea is to compare the relative strengths of the fluxes of enstrophy and buoyancy variance and, if possible, find out which of the mechanisms dominate at the ocean surface.

## Geographical Locations

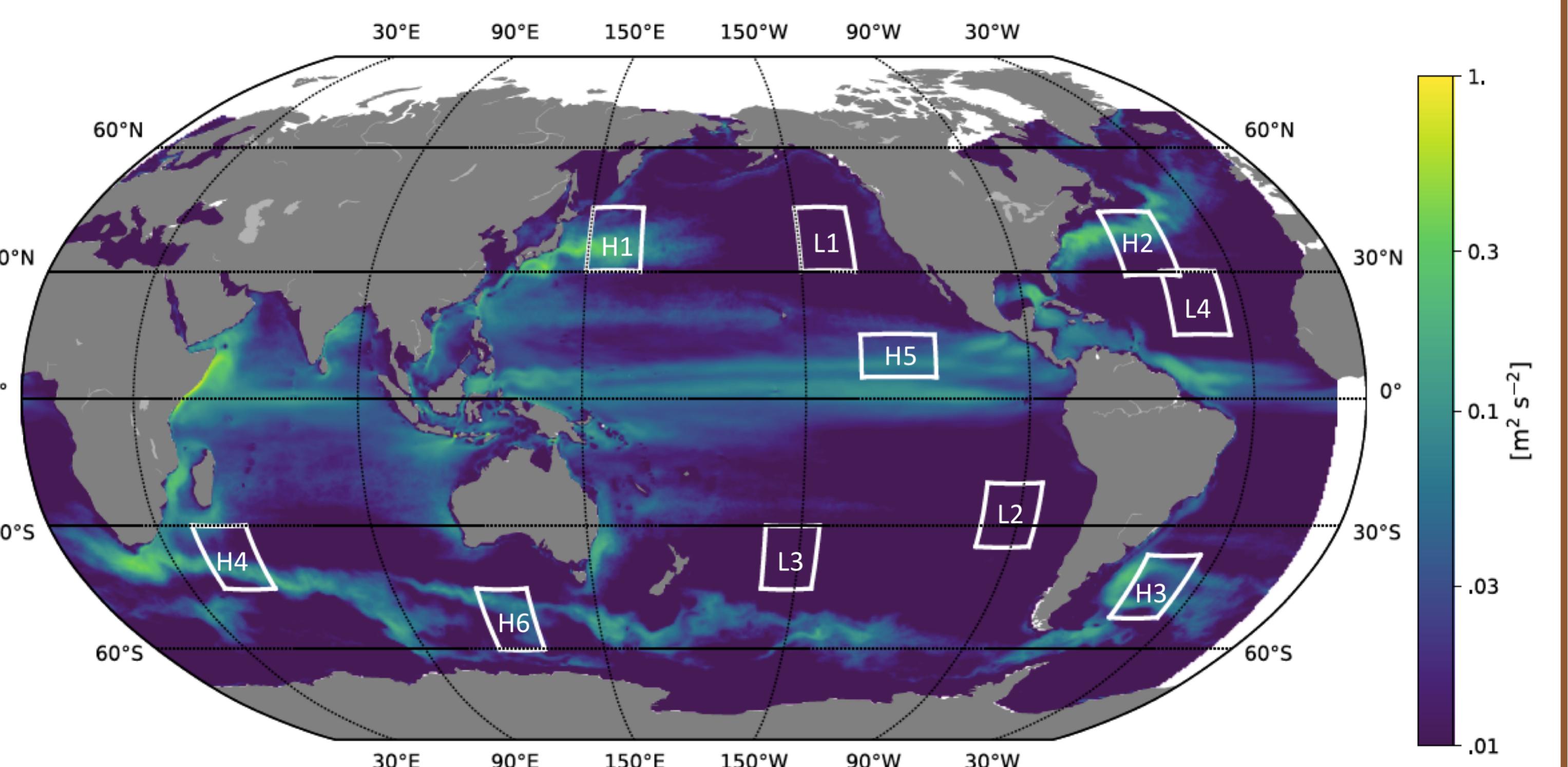


Figure 1: Domains chosen for the analysis. H1-H6 and L1-L4 represent the high and low eddy activity regions. The time-mean eddy kinetic energy is shown in color.

Ten different domains (Fig. 1) are chosen in different parts of the oceans for the analysis. All domains are roughly  $10^\circ \times 10^\circ$  in area. H1-H6 are high eddy activity regions, in which strong currents are present, e.g., the Gulf Stream and Kurishio current. Rest of the domains (L1-L4) are low eddy activity regions, in which the dynamics are largely isotropic. We expect that our analysis in the two kinds of regions would be helpful in understanding the dynamics in almost all parts of the global ocean.

## Conclusions and Future Work

- In POP data, KE spectra follow a scaling  $\sim k^{-3}$  and  $\sim k^{-2}$  in the low and high eddy activity regions, respectively, whereas buoyancy variance spectra are close to  $k^2$ . KE flux is upscale and strongest in 500-100 km scales. On the other hand, enstrophy and buoyancy variance are transferred to scales smaller than about 200 km and 1000 km, respectively.
- Dynamics in low eddy activity regions may have a significant contribution from buoyancy conservation. A quantitative measure is required to assess the significance of QG vs SQG dynamics in different regions. Work is in progress in that direction.
- In the channel model data, KE (enstrophy and buoyancy variance) flux is upscale (downscale) at all depths. KE spectra steepen ( $\sim k^{-2}$  to  $\sim k^{-3}$ ) with depth. Also, the flux magnitude decreases with depth.

## Model Description

We use 30 years of data from a coupled Earth system climate model which was run under present-day greenhouse gas conditions [7]. The eddy-permitting ocean component of the model (POP) is a primitive equation model that produces daily data at a spatial resolution of approximately  $0.1^\circ \times 0.1^\circ$ . POP model resolves scales up to 50 km fairly well and the viscous dissipation effects are negligible in scales larger than 80 km [8].

We use the geostrophic component of the ocean surface currents from POP model to compute spectra and spectral fluxes of KE, enstrophy and buoyancy variance. If we consider the evolution a field  $\phi$  in a flow field  $\mathbf{u}$ , then a conservation equation for  $\phi^2$  (let's call it  $\phi$ -variance) in the spectral domain at any wavenumber  $k$  can be represented as

$$\frac{\partial \mathcal{A}(k)}{\partial t} = \mathcal{T}(k) + \mathcal{F}(k) - \mathcal{D}(k)$$

where  $\mathcal{A}(k) = \frac{1}{2}|\phi(k)|^2$  and  $\mathcal{T}(k) = \mathcal{R}[\phi^*(k)\mathbf{u}\cdot\nabla\phi]$  are the spectrum and nonlinear transfer term (real part), respectively. Here, the hat symbol represents the Fourier transform. The rest of the terms represent the forcing and dissipation at wavenumber  $k$ . Spectral flux, which measures the  $\phi$ -variance passing through wavenumber  $k$ , is defined as

$$\text{Flux} = - \int_0^k \mathcal{T}(k') dk'$$

Here, we compute spectrum and flux for three different quantities, i.e., KE ( $\mathbf{u}^2/2$ ), enstrophy ( $(\nabla \times \mathbf{u})^2/2$ ) and buoyancy variance ( $(-\mathbf{g}\rho'/\rho_0)^2/2$ ). Note that linear trends in velocity and buoyancy fields were removed before the computations. More details on the dataset and computations can be found in [8,9]. Additionally, we use geostrophic currents from a channel model configured to simulate the Southern Ocean, which was forced with monthly temperature relaxation and wind stress. The model has a horizontal grid spacing of 10 km and 40 layers in the vertical. We compute spectra and fluxes at different depths.

## Results

- Both KE and buoyancy variance spectra from POP data seem to follow a power-law (Fig. 2) in the range roughly 200-50 km. KE spectra follow close to  $k^{-3}$  scaling in high eddy activity regions (H1-H6 except H5, which is very close to the equator) whereas the spectra are relatively shallow, quite close to  $k^{-2}$ , in low eddy activity regions (L1-L4). Buoyancy variance spectra don't show any significant differences between the high and low eddy activity regions, and follow  $\sim k^{-2}$  scaling in all regions.
- Analyzing a spectrum in terms of a power-law is only useful if there exists an inertial range. Hence, we also computed spectral fluxes (Fig. 3). In all regions, KE flux is upscale and strongest in 500-100 km scales, whereas enstrophy is transferred to scales smaller than  $\sim 200$  km. With the observed  $k^{-3}$  like scaling in KE spectrum in high eddy activity regions, the fluxes are in accord with QG turbulence.
- On the other hand, the flux of buoyancy variance is downscale at all wavenumbers, in accord with SQG turbulence. However, buoyancy variance flux shows a lot of variability and the mean flux picture is not as robust as KE and enstrophy fluxes.
- Just with a visual inspection, buoyancy seems to have a significant contribution in L1-L4 as the scaling in KE spectra is close to  $k^{-2}$ . Another way is to look at the ratio of domain averaged buoyancy variance to enstrophy in each region. The ratio is generally larger in L1-L4 than in H1-H6 (Fig. 4). Work is in progress to derive a quantitative measure for comparison.

### Spectra and Fluxes in the Channel Model

- Spectra and fluxes from the channel model are in agreement with the results from POP model.
- The fluxes are strongest at the near surface and the flux magnitude decreases with depth. Scaling in KE spectrum is close to  $k^{-2}$  near the surface and is roughly  $k^{-3}$  in the interior. Buoyancy variance spectrum also shows similar steepening.
- Note that here we define buoyancy as the vertical derivative of potential pressure. Thus, flux buoyancy variance comes out to be much cleaner in comparison to POP results.

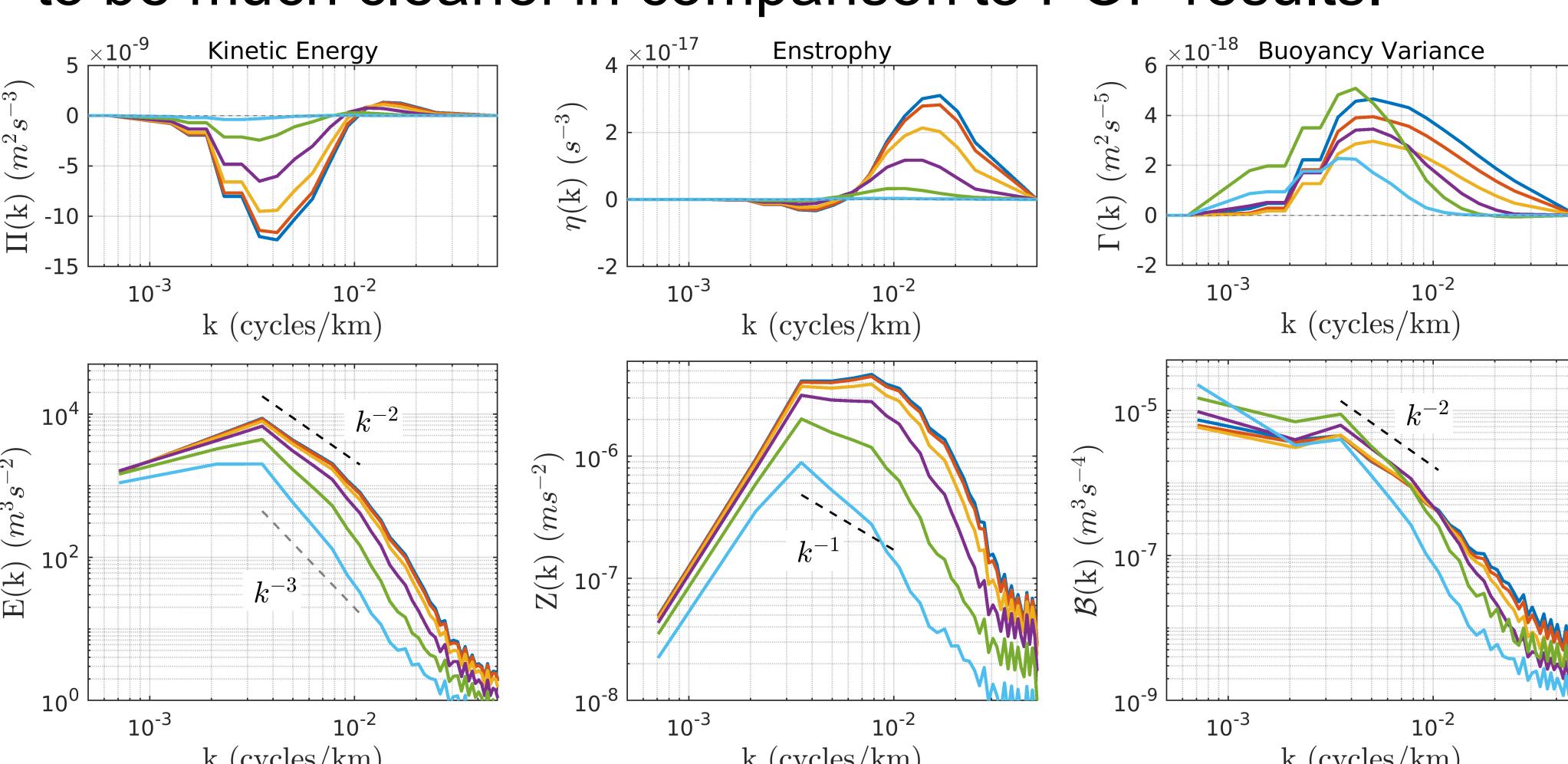


Figure 2: Spectra (average over 30 years) of KE (blue,  $m^2 s^{-2}$ ) and buoyancy variance (red,  $m^3 s^{-4}$ ) in all regions. Power-laws of  $k^{-2}$  and  $k^{-3}$  are shown for reference.

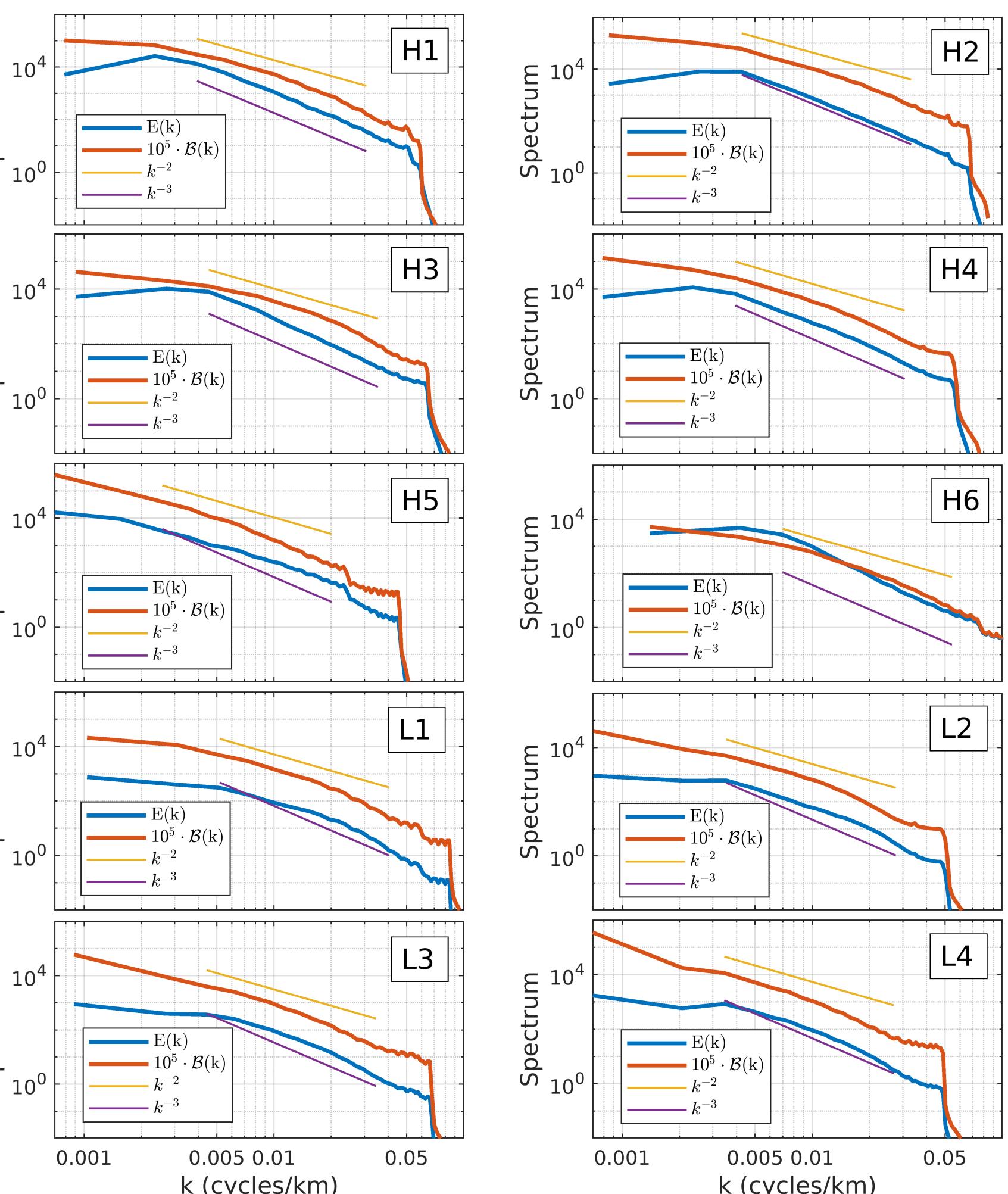


Figure 3: Spectral fluxes (averaged over 30 years) of KE (blue,  $m^2 s^{-3}$ ), enstrophy (red,  $s^{-3}$ ) and buoyancy variance (yellow,  $m^2 s^{-5}$ ) in all regions.

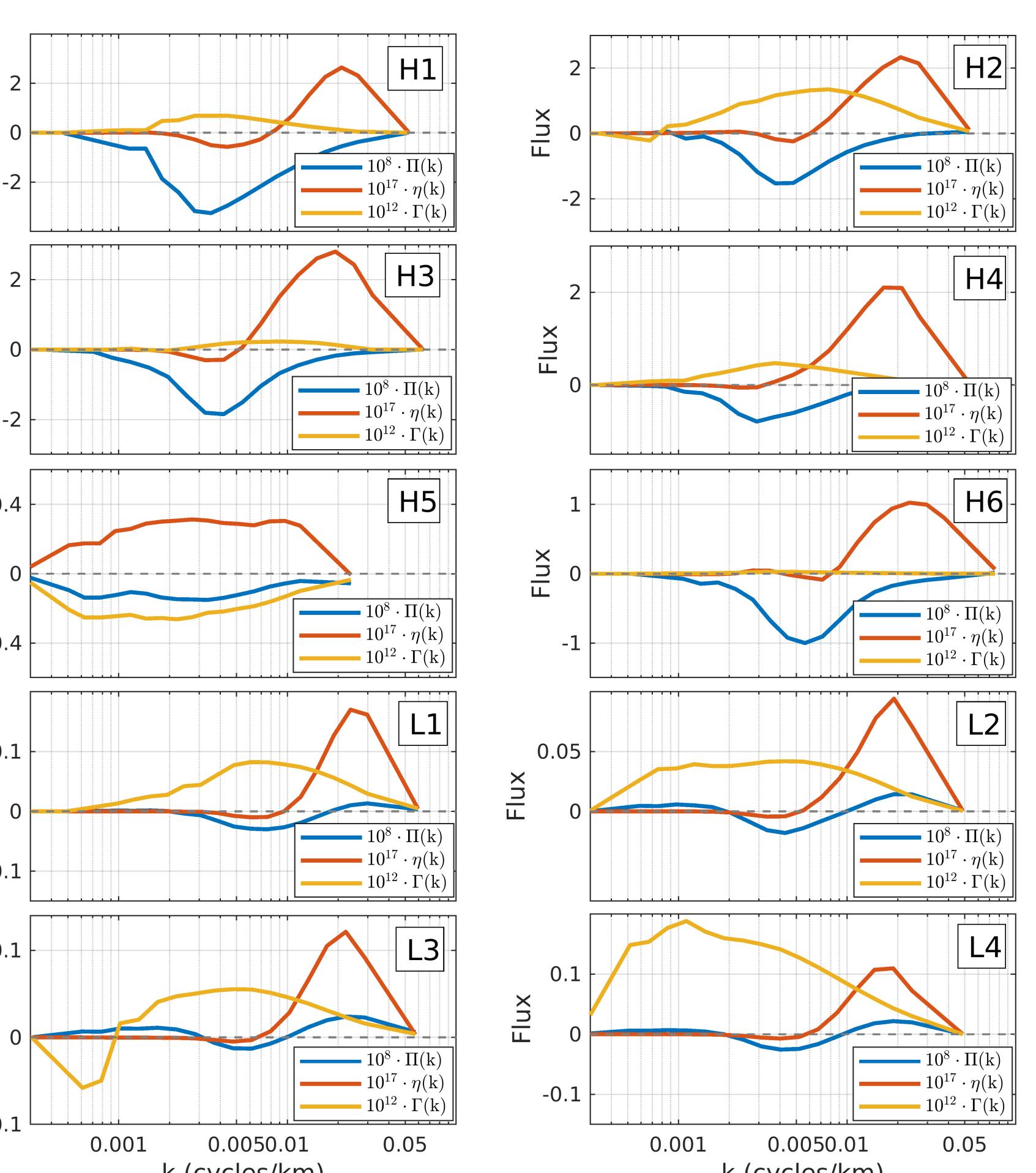


Figure 4: Ratio of domain averaged buoyancy variance ( $(-\mathbf{g}\rho'/\rho_0)^2/2$ ) and enstrophy ( $(\nabla \times \mathbf{u})^2/2$ ) computed using the detrended POP data in each region.

## References

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