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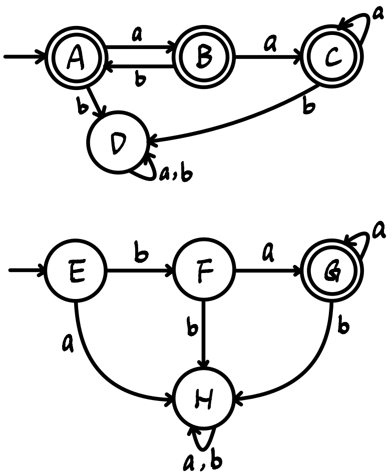
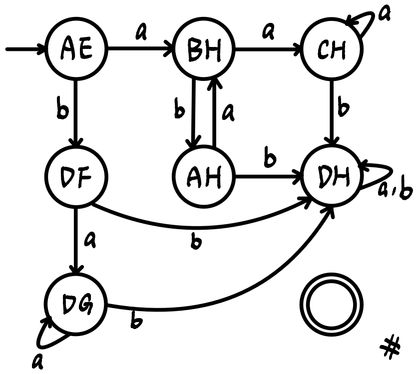
6. Use the construction in Theorem 4.1 to find nfa's that accept

(a) $L((ab)^* a^*) \cap L(baa^*)$ $= \varnothing$

(b) $L(ab^*a^*) \cap L(a^*b^*a)$





6. Determine whether or not the following languages on $\Sigma = \{a\}$ are regular:

(a) $L = \{a^n : n \geq 2$, is a prime number$\}$.

(b) $L = \{a^n : n$ is not a prime number$\}$.

(c) $L = \{a^n : n = k^3$ for some $k \geq 0\}$.

(d) $L = \{a^n : n = 2^k$ for some $k \geq 0\}$.

(e) $L = \{a^n : n$ is the product of two prime numbers$\}$.

(f) $L = \{a^n : n$ is either prime or the product of two or more prime numbers$\}$.

(g) L^*, where L is the language in part (a).

For given m, pick $w = a^{2^m} = xy^iz$, $|xy| \leq m$,

$1 \leq k \leq m$, pick $i=2$, $w = a^{2^m+k}$, $\because k \leq m < 2^m$

$\therefore \nexists k$ s.t. $2^m + k = 2^n$, n is an integer.

$\Rightarrow w \notin L \Rightarrow L$ is not regular.

9. Find context-free grammars for the following languages (with $n \geq 0$, $m \geq 0$).

(a) $L = \{a^nb^m : n \leq m+3\}$.

(b) $L = \{a^nb^m : n = m-1\}$.

(c) $L = \{a^nb^m : n \neq 2m\}$.

(d) $L = \{a^nb^m : 2n \leq m \leq 3n\}$.

(e) $L = \{w \in \{a,b\}^* : n_a(w) \neq n_b(w)\}$.

(f) $L = \{w \in \{a,b\}^* : n_a(v) \geq n_b(v)$, where v is any prefix of $w\}$.

(g) $L = \{w \in \{a,b\}^* : n_a(w) = 2n_b(w)+1\}$.

(h) $L = \{w \in \{a,b\}^* : n_a(w) = n_b(w)+2\}$.

(b) $S \to Ab$, $A \to aAb \mid \lambda$

(c) $S \to aaSB \mid A \mid B$, $A \to aA \mid a$

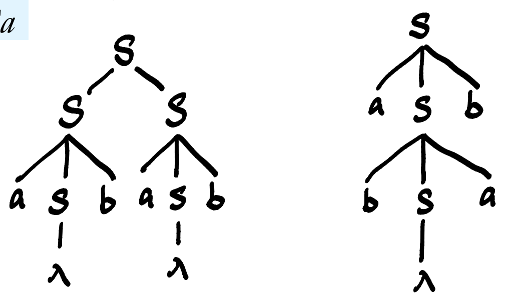
$B \to CBb \mid \lambda$, $C \to a \mid \lambda$

(d) $S \to aSAbb \mid \lambda$, $A \to b \mid \lambda$

19. Show that the grammar in Example 1.13 is ambiguous.

$S \to SS$,	$S \to aSb$,
$S \to \lambda$,	$S \to bSa$

Input string : abab



$\Rightarrow$ parsing tree 不唯一 $\Rightarrow$ ambiguous

21. Use the exhaustive search parsing method to parse the string *abbbbbb* with the grammar in Example 5.5. In general, how many rounds will be needed to parse any string w in this language?

$$S \to aAB,$$
$$A \to bBb,$$
$$B \to A|\lambda.$$

$S \to aAB$, $A \to bBb$, $B \to A$, $A \to bBb$, $B \to A$, $A \to bBb$, $\cancel{B \to A}$, $\cancel{B \to A}$

$\cancel{B \to \lambda}$ $\cancel{B \to \lambda}$ $B \to \lambda$ $B \to \lambda$

$|w| + 1$ rounds in general.

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8. Eliminate all λ-productions from $S \to aSSS$,

$$S \to bb \mid \lambda.$$

$S \to aSSS \mid aSS \mid aS \mid a \mid bb$

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9. Eliminate all λ-productions from $S \to AaB \mid aaB$,

$$A \to \lambda,$$

$$B \to bbA \mid \lambda.$$

$S \to a \mid aa \mid abb \mid aabb$

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26. Use the result of the preceding exercise to rewrite the grammar $A \to \underset{x}{Aa} \mid \underset{y}{aBc} \mid \underset{y}{\lambda}$, $B \to \underset{x}{Bb} \mid \underset{y}{bc}$
so that it no longer has productions of the form $A \to Ax$ or $B \to Bx$.

$A \to aBc \mid aBcA' \mid \lambda \mid A'$, $A' \to aA' \mid a$

$B \to bc \mid bcB'$, $B' \to bB' \mid b$

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2. Convert the grammar $S \to aSb \mid Sab \mid ab$ into Chomsky normal form.

$S \to AV_1 \mid SV_2 \mid AB$, $V_1 \to SB$, $V_2 \to AB$, $A \to a$, $B \to b$

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4. Transform the grammar with productions $S \to baAB$,

$$A \to bAB \mid \lambda,$$

$$B \to BAa \mid A \mid \lambda$$

into Chomsky normal form.

$S \to ba \mid baA \mid baB \mid baAB$

$A \to b \mid bA \mid bB \mid bAB$

$B \to a \mid Ba \mid Aa \mid BAa$

$B \to b \mid bA \mid bB \mid bAB$

$S \to V_2V_1 \mid V_3A \mid V_3B \mid V_3V_4$

$V_1 \to a$, $V_2 \to b$, $V_3 \to V_2V_1$, $V_4 \to AB$

$A \to b \mid V_2A \mid V_2B \mid V_2V_4$

$B \to a \mid BV_1 \mid AV_1 \mid V_5V_1 \mid b \mid V_2A \mid V_2B \mid V_2V_4$ #

$V_5 \to AB$

4. Use the CYK method to determine if the string $w = aaabbbb$ is in the language generated by the grammar G $S \to aSb \mid b$.

$S \to AV_1 \mid b$, $V_1 \to SB$, $A \to a$, $B \to b$

a	a	a	b	b	b	b
A	A	A	B,S	B,S	B,S	B,S
aa	aa	ab	bb	bb	bb	
			V_1	V_1	V_1	
aaa	aab	abb	bbb	bbb		
		S				
aaab	aabb	abbb	bbbb			
		V_1				
aaabb	aabbb	abbbb				
	S					
aaabbb	aabbbb					
	V_1					
aaabbbb						
S						

$\hookrightarrow w \in L(G)$