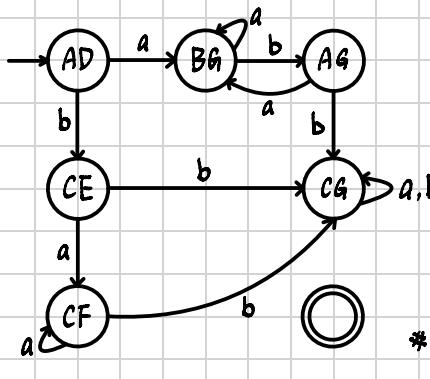
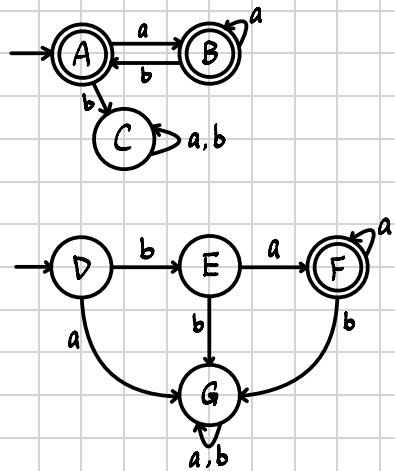


6. Use the construction in Theorem 4.1 to find nfa's that accept

$$(a) L((ab)^* a^*) \cap L(baa^*) = \emptyset$$

$$(b) L(ab^*a^*) \cap L(a^*b^*a)$$



6. Determine whether or not the following languages on  $\Sigma = \{a\}$  are regular:

$$(a) L = \{a^n : n \geq 2, \text{ is a prime number}\}.$$

$$(b) L = \{a^n : n \text{ is not a prime number}\}.$$

$$(c) L = \{a^n : n = k^3 \text{ for some } k \geq 0\}.$$

$$(d) L = \{a^n : n = 2^k \text{ for some } k \geq 0\}.$$

$$(e) L = \{a^n : n \text{ is the product of two prime numbers}\}.$$

$$(f) L = \{a^n : n \text{ is either prime or the product of two or more prime numbers}\}.$$

$$(g) L^*, \text{ where } L \text{ is the language in part (a).}$$

9. Find context-free grammars for the following languages (with  $n \geq 0, m \geq 0$ ).

$$(a) L = \{a^n b^m : n \leq m + 3\}.$$

$$(b) L = \{a^n b^m : n = m - 1\}.$$

$$(c) L = \{a^n b^m : n \neq 2m\}.$$

$$(d) L = \{a^n b^m : 2n \leq m \leq 3n\}.$$

$$(e) L = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}.$$

$$(f) L = \{w \in \{a, b\}^* : n_a(v) \geq n_b(v), \text{ where } v \text{ is any prefix of } w\}.$$

$$(g) L = \{w \in \{a, b\}^* : n_a(w) = 2n_b(w) + 1\}.$$

$$(h) L = \{w \in \{a, b\}^* : n_a(w) = n_b(w) + 2\}.$$

For given  $m$ , pick  $w = a^{2^m} = xy^iz, |xy| \leq m, 1 \leq k \leq m$ , pick  $i=2, w = a^{2^m+k}, \therefore k \leq m < 2^m$

$\therefore \nexists k \text{ s.t. } 2^m+k = 2^n, n \text{ is an integer.}$

$\Rightarrow w \notin L \Rightarrow L \text{ is not regular.}$

$$(b) S \rightarrow Ab, A \rightarrow aAb \mid \lambda$$

$$(c) S \rightarrow aaSB \mid A \mid B, A \rightarrow aA \mid a$$

$$B \rightarrow CBb \mid \lambda, C \rightarrow a \mid \lambda$$

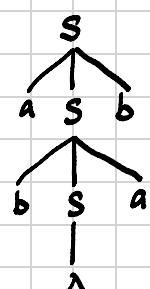
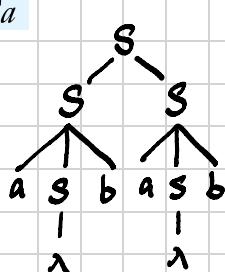
$$(d) S \rightarrow aSAbb \mid \lambda, A \rightarrow b \mid \lambda$$

19. Show that the grammar in Example 1.13 is ambiguous.

$$S \rightarrow SS, \quad S \rightarrow aSb,$$

Input string: abab

$$S \rightarrow \lambda, \quad S \rightarrow bSa$$



$\Rightarrow$  parsing tree 不唯一  $\Rightarrow$  ambiguous

21. Use the exhaustive search parsing method to parse the string  $abbbbb$  with the grammar in Example 5.5. In general, how many rounds will be needed to parse any string  $w$  in this language?

$S \rightarrow aAB, \quad S \rightarrow aAB, \quad A \rightarrow bBb, \quad B \rightarrow A, \quad A \rightarrow bBb, \quad B \rightarrow A, \quad A \rightarrow bBb, \quad \cancel{B \rightarrow A}, \quad \cancel{B \rightarrow A}$

$A \rightarrow bBb, \quad \cancel{B \rightarrow \lambda}$

$B \rightarrow A | \lambda. \quad |w| + 1 \text{ rounds in general.} \quad \cancel{B \rightarrow \lambda} \quad B \rightarrow \lambda \quad B \rightarrow \lambda$

8. Eliminate all  $\lambda$ -productions from  $S \rightarrow aSSS$ ,

$$S \rightarrow bb | \lambda.$$

$$S \rightarrow aSSS | ass | as | a | bb$$

9. Eliminate all  $\lambda$ -productions from  $S \rightarrow AaB | aaB$ ,

$$A \rightarrow \lambda,$$

$$B \rightarrow bbA | \lambda.$$

$$S \rightarrow a | aa | abb | aabb$$

26. Use the result of the preceding exercise to rewrite the grammar  $A$

$$\rightarrow Aa | aBc | \lambda, \quad B \rightarrow Bb | bc$$

so that it no longer has productions of the form  $A \rightarrow Ax$  or  $B \rightarrow Bx$ .

$$A \rightarrow aBc | aBcA' | \lambda | A' \cdot A' \rightarrow aA' | a$$

$$B \rightarrow bc | bcB' \cdot B' \rightarrow bB' | b$$

2. Convert the grammar  $S \rightarrow aSb | Sab | ab$  into Chomsky normal form.

$$S \rightarrow Av_1 | Sv_2 | AB, \quad v_1 \rightarrow SB, \quad v_2 \rightarrow AB, \quad A \rightarrow a, \quad B \rightarrow b$$

4. Transform the grammar with productions  $S \rightarrow baAB$ ,

$$A \rightarrow bAB | \lambda,$$

$$B \rightarrow BAa | A | \lambda$$

into Chomsky normal form.

$$S \rightarrow ba | baA | baB | baAB$$

$$A \rightarrow b | bA | bB | bAB$$

$$B \rightarrow a | Ba | Aa | BAA$$

$$B \rightarrow b | bA | bB | bAB$$

$$S \rightarrow V_2V_1 | V_3A | V_3B | V_3V_4$$

$$V_1 \rightarrow a, \quad V_2 \rightarrow b, \quad V_3 \rightarrow V_3V_1, \quad V_4 \rightarrow AB$$

$$A \rightarrow b | V_2A | V_2B | V_2V_4$$

$$B \rightarrow a | BV_1 | AV_1 | V_4V_1 | b | V_2A | V_2B | V_2V_4$$

\*

$$S \rightarrow bA B \mid bAAB$$
$$A \rightarrow bB \mid bAB$$
$$B \rightarrow Ba \mid BAA \mid A \mid \lambda$$

) remove  $A \rightarrow \lambda$

$$S \rightarrow ba \mid baB \mid bAA \mid bAAB$$
$$A \rightarrow b \mid bB \mid bA \mid bAB$$
$$B \rightarrow a \mid Ba \mid Aa \mid BAA \mid A$$

) remove  $B \rightarrow \lambda$

$$S \rightarrow ba \mid bAB \mid bAA \mid baAB$$

remove  $B \rightarrow A$

$$A \rightarrow b \mid bB \mid bA \mid bAB$$
$$B \rightarrow a \mid Ba \mid Aa \mid BAA$$
$$+ B \rightarrow b \mid bB \mid bA \mid bAB$$

4. Use the CYK method to determine if the string  $w = aaabbbb$  is in the language generated by the grammar  $G$   $S \rightarrow aSb \mid b$ .

$$S \rightarrow AV_i \mid b, V_i \rightarrow SB, A \rightarrow a, B \rightarrow b$$

a a a b b b b

A A A B,S B,S B,S B,S

aa aa ab bb bb bb

V<sub>i</sub> V<sub>i</sub> V<sub>i</sub>

aaa aab abb bbb bbb

S

aab aabb abbb bbbb

V<sub>i</sub>

aaabb qabbb abbbb

S

aaa bbb aabbbb

V<sub>i</sub>

aaabbbb

S

↪  $w \in L(G)$