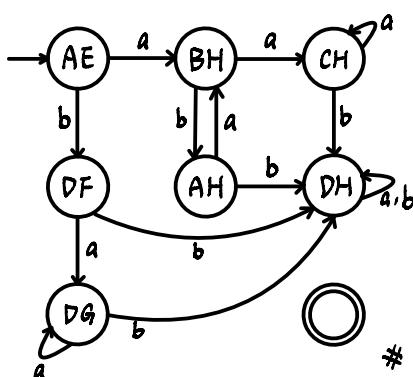
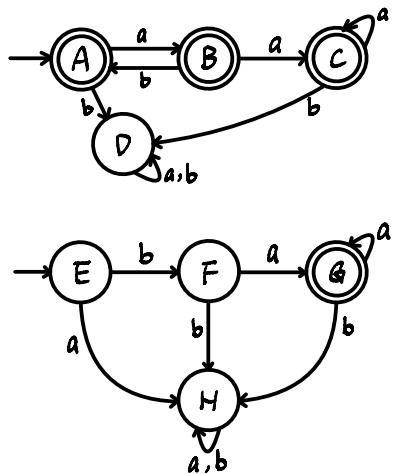


6. Use the construction in Theorem 4.1 to find nfa's that accept

(a) $L((ab)^* a^*) \cap L(baa^*) = \emptyset$

(b) $L(ab^* a^*) \cap L(a^* b^* a)$



6. Determine whether or not the following languages on $\Sigma = \{a\}$ are regular:

(a) $L = \{a^n : n \geq 2, n \text{ is a prime number}\}.$

(b) $L = \{a^n : n \text{ is not a prime number}\}.$

(c) $L = \{a^n : n = k^3 \text{ for some } k \geq 0\}.$

(d) $L = \{a^n : n = 2^k \text{ for some } k \geq 0\}.$

(e) $L = \{a^n : n \text{ is the product of two prime numbers}\}.$

(f) $L = \{a^n : n \text{ is either prime or the product of two or more prime numbers}\}.$

(g) $L^*, \text{ where } L \text{ is the language in part (a).}$

For given m , pick $w = a^{2^m} = xyz, |xyz| \leq m, 1 \leq k \leq m$, pick $i=2$, $w = a^{2^m+k}, \therefore k \leq m < 2^m$

$\therefore \not\exists k \text{ s.t. } 2^m+k = 2^n, n \text{ is an integer.}$

$\Rightarrow w \notin L \Rightarrow L \text{ is not regular.}$

9. Find context-free grammars for the following languages (with $n \geq 0, m \geq 0$).

(a) $L = \{a^n b^m : n \leq m+3\}.$

(b) $L = \{a^n b^m : n = m-1\}.$

(c) $L = \{a^n b^m : n \neq 2m\}.$

(d) $L = \{a^n b^m : 2n \leq m \leq 3n\}.$

(e) $L = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}.$

(f) $L = \{w \in \{a, b\}^* : n_a(v) \geq n_b(v), \text{ where } v \text{ is any prefix of } w\}.$

(g) $L = \{w \in \{a, b\}^* : n_a(w) = 2n_b(w)+1\}.$

(h) $L = \{w \in \{a, b\}^* : n_a(w) = n_b(w)+2\}.$

(b) $S \rightarrow Ab, A \rightarrow aAb \mid \lambda$

(c) $S \rightarrow aaSB \mid A \mid B, A \rightarrow aA \mid a$

$B \rightarrow CBb \mid \lambda, C \rightarrow a \mid \lambda$

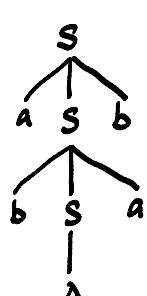
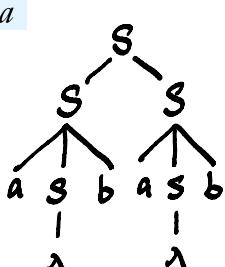
(d) $S \rightarrow aSAbb \mid \lambda, A \rightarrow b \mid \lambda$

19. Show that the grammar in Example 1.13 is ambiguous.

$S \rightarrow SS, S \rightarrow aSb,$

Input string: abab

$S \rightarrow \lambda, S \rightarrow bSa$



\Rightarrow parsing tree 不唯一 \Rightarrow ambiguous

21. Use the exhaustive search parsing method to parse the string $abbbbb$ with the grammar in Example 5.5. In general, how many rounds will be needed to parse any string w in this language?

$S \rightarrow aAB, \quad S \rightarrow aAB, A \rightarrow bBb, B \rightarrow A, A \rightarrow bBb, B \rightarrow A, A \rightarrow bBb, B \rightarrow A, B \rightarrow \lambda, B \rightarrow \lambda$

$A \rightarrow bBb, \quad B \rightarrow \lambda, B \rightarrow \lambda$

$B \rightarrow A | \lambda. \quad |w| + 1 \text{ rounds in general.}$

8. Eliminate all λ -productions from $S \rightarrow aSSS$,

$$S \rightarrow bb | \lambda.$$

$$S \rightarrow aSSS | ass | as | a | bb$$

9. Eliminate all λ -productions from $S \rightarrow AaB | aaB$,

$$\begin{aligned} A &\rightarrow \lambda, \\ B &\rightarrow bbA | \lambda. \end{aligned}$$

$$S \rightarrow a | aa | abb | aabb$$

26. Use the result of the preceding exercise to rewrite the grammar $A \rightarrow Aa | aBc | \lambda, B \rightarrow Bb | bc$ so that it no longer has productions of the form $A \rightarrow Ax$ or $B \rightarrow Bx$.

$$\begin{aligned} A &\rightarrow aBc | aBcA' | \lambda | A' \cdot A' \rightarrow aA' | a \\ B &\rightarrow bc | bcB' \cdot B' \rightarrow bB' | b \end{aligned}$$

2. Convert the grammar $S \rightarrow aSb | Sab | ab$ into Chomsky normal form.

$$S \rightarrow Av_1 | Sv_2 | AB, \quad v_1 \rightarrow SB, \quad v_2 \rightarrow AB, \quad A \rightarrow a, \quad B \rightarrow b$$

4. Transform the grammar with productions $S \rightarrow baAB$,

$$\begin{aligned} A &\rightarrow bAB | \lambda, \\ B &\rightarrow BAa | A | \lambda \end{aligned}$$

into Chomsky normal form.

$$\begin{array}{ll} S \rightarrow ba | baA | baB | baAB & S \rightarrow V_2V_1 | V_3A | V_3B | V_3V_4 \\ A \rightarrow b | bA | bB | bAB & V_1 \rightarrow a, \quad V_2 \rightarrow b, \quad V_3 \rightarrow V_2V_1, \quad V_4 \rightarrow AB \\ B \rightarrow a | Ba | Aa | BAA & A \rightarrow b | V_2A | V_2B | V_2V_4 \\ B \rightarrow b | bA | bB | bAB & B \rightarrow a | BV_1 | AV_1 | V_5V_1 | b | V_2A | V_2B | V_2V_4 \\ & V_5 \rightarrow BA \end{array}$$

4. Use the CYK method to determine if the string $w = aaabbbb$ is in the language generated by the grammar $S \xrightarrow{G} aSb \mid b$.

$$S \rightarrow A V_i \mid b, V_i \rightarrow SB, A \rightarrow a, B \rightarrow b$$

a a a b b b b

A A A B,S B,S B,S B,S

aa aa ab bb bb bb

V_i V_i V_i

aaa aab abb bbb bbb

S

aaab aabb abbb bbbb

V_i

aaabb qabbb abbbb

S

aaabbbb aabb

V_i

aaabbbb

S

↪ $w \in L(G)$