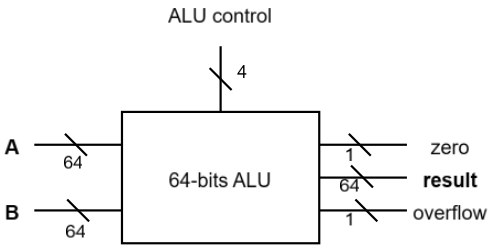
CS4100 Computer Architecture

Spring 2024, Homework 3

Due: 23:59, 4/21/2024

1. (18 points) Please modify the ALU design introduced in the class to satisfy the following requirements:

Ainvert	Bnegate	Operation	Function
0	1	01	AND
0	1	10	OR
0	1	00	add
0	0	00	sub
1	0	01	NOR
			add-ext
			sub-ext



Here, "add-ext" and "sub-ext" refer to 32-bit addition and subtraction with sign-extension to 64 bits. You are required to draw the circuit diagrams for each 1-bit ALU and the 64-bit ALU. For each 1-bit ALU, use only one full adder to perform an addition or subtraction operation, similar to the method demonstrated in class. Additionally, show the ALU control signals for "add-ext" and "sub-ext" in your design.

2. (14 points) Consider two unsigned binary numbers: $M = 1110$ and $N = 1001$.
   (a) (7 points) Write down each step of $M \times N$ according to version 1 of the multiply algorithm.
   (b) (7 points) Write down each step of $M \times N$ according to version 2 of the multiply algorithm.
3. (14 points) Consider two unsigned binary numbers: $M = 0111$ and $N = 0101$.
   (a) (7 points) Write down each step of $M \div N$ according to version 1 of the division algorithm.
   (b) (7 points) Write down each step of $M \div N$ according to version 2 of the division algorithm.
4. (12 points) Answer the following questions in detail. You will receive 0 point if you only write down the answers.
   (a) (4 points) What decimal number does the bit pattern $05948DEC_{16}$ represent if it's a two's complement integer? If it's an unsigned number, is the result the same as the two's complement? If they are different, why?
   (b) (4 points) Answer problem (a) with a different bit pattern $FA6B7214_{16}$.
   (c) (4 points) What decimal numbers do $05948DEC_{16}$ and $FA6B7214_{16}$ represent if they are IEEE 754 floating point numbers.

5. (10 points) Consider two decimal numbers: X = 88.4375 and Y = −7.3125.
   (a) (6 points) Write down X and Y in the IEEE 754 single precision format. You must detail how you get your answer, or you will receive 0 point.
   (b) (4 points) Assuming X and Y are given in the IEEE 754 single precision format. Show all the steps to perform X × Y and write the solution in the IEEE 754 single precision format.
6. (20 points) Consider a new floating-point number representation that is only 16 bits wide. The leftmost bit is still the sign bit, the exponent is 9 bits wide and has a bias of 255, and the fraction is 6 bits long. A hidden 1 to the left of the binary point is assumed. In this representation, any 16-bit binary pattern having 000000000 in the exponent field and a non-zero fraction indicates a denormalized number: $(-1)^S \times (0 + \text{Fraction}) \times 2^{-254}$. Write the answers of (a), (b) and (c) in scientific notation, e.g., 1.0101×2^2.
   (a) (3 points) What is the smallest positive "normalized" number, denoted as a0?
   (b) (6 points) What is the largest positive "denormalized" number, denoted as a1? What is the second largest positive "denormalized" number, denoted as a2?
   (c) (4 points) Find the differences between a0 and a1, and between a1 and a2.
   (d) (3 points) What binary number does the binary pattern 1011110110100111 represent?
   (e) (4 points) Let U be the nearest representation of the decimal number 1.31; that is, U has the smallest approximation error. What is U? What is the actual decimal number represented by U?
7. (12 points) **X** is a 32-bit signed integer variable, **&** is the bitwise-AND operator, and ">>" is the sra (shift right arithmetic) operator. For the following options, determine whether they provide the correct result for (**X** / 4) and explain the reasons.
   (a) (**X** + 3) >> 2
   (b) ((**X** >= 0) ? **X** >> 2 : (**X** + 3) >> 2)
   (c) **X** >> 2
   (d) (**X** + ((**X** >> 31) **&** 3)) >> 2