14. Find grammars for $\Sigma = \{a, b\}$ that generate the sets of

(a) all strings with exactly two a's.

(b) all strings with at least two a's. $= \{(b)\}$

(c) all strings with no more than three a's. $= \{(c)\}$

(d) all strings with at least three a's. $= \{(d)\}$

(e) all strings that start with a and end with b. $= \{(e)\}$

(b) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow S'aS'aS'$, $S' \rightarrow aS' \mid bS' \mid \lambda$

(c) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow S'aS'aS'aS' \mid S'aS'aS' \mid S'aS' \mid S'$, $S' \rightarrow bS' \mid \lambda$

(d) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow S'aS'aS'aS'$, $S' \rightarrow aS' \mid bS' \mid \lambda$

(e) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow aS'b$, $S' \rightarrow aS' \mid bS' \mid \lambda$

Argument (d)

$\rightarrow L(G) \in \{(d)\}$

If $n_a(str) < 3$, then str must not be accept by $S \rightarrow S'aS'aS'aS'$

Let $W_{ijkl} = A_i a B_j a C_k a D_l$, $G' = \{\{S'\}, \{a, b\}, S', P'\}$, $P' = S' \rightarrow aS' \mid bS' \mid \lambda$

Base case: $A_0 = \lambda \in L(G')$, $A_1 = A_0 a \mid aA_0 \mid A_0 b \mid bA_0 \in L(G')$. Assume that $n \geq 0$,

$A_n \in L(G')$, $A_{n+1} = aA_n \mid A_n a \mid bA_n \mid A_n b \in L(G')$, which means that $\forall n \geq 0$, $A_n \in L(G')$.

Similarly, $B_j, C_k, D_l \in L(G')$ $\forall j \geq 0, k \geq 0, l \geq 0$, which implies that $W_{ijkl} \in L(G)$

$\forall\ i, j, k, l \geq 0 \Rightarrow \{(d)\} \subseteq L(G) \Longrightarrow L(G) = \{(d)\}$

Argument (b)

$\rightarrow L(G) \subseteq \{(b)\}$

If $n_a(str) < 2$, then str must not be accept by $S \rightarrow S'aS'aS'$

$A_i, B_j, C_k \in \{a, b\}^*$

$W_{ijk} = A_i a B_j a C_k$, i, j, k represent the length of A, B, C respectively. base case: aa is equivalent to $W_{000} = A_0 a B_0 a C_0$. $G' = \{\{S'\}, \{a, b\}, S', P'\}$, $P' = S' \rightarrow aS' \mid bS' \mid \lambda$

$A_0 \in L(G')$, $A_1 = aA_0 \mid A_0 a \mid bA_0 \mid A_0 b \in L(G')$. Assume that $A_n \in L(G')$ for $n \geq 0$,

$A_{n+1} = aA_n \mid A_n a \mid bA_n \mid A_n b \in L(G') \Rightarrow \forall n \geq 0$ $A_n \in L(G')$. Similarly, B and $C \in L(G')$.

which implies that $\{(b)\} \subseteq L(G') \Longrightarrow L(G) = \{(b)\}$

Argument (c)

$\rightarrow L(G) \subseteq \{(c)\}$

If $n_a(str) > 3$, str can't be generated by G since at least one S' contains a by pigeon-hole principle.

There are only 4 cases:

Case 1: (no $a \Rightarrow$ string $= A_l$)

A_l is a string. l is the length. $A_0 = \lambda$

$L(G') = \{\{S'\}, \{b\}, S', P'\}$, $P' = S' \rightarrow bS' \mid \lambda$

Base case: $A_0 \in G'$, $A_1 = bA_0 \mid A_0 b \in L(G')$

Assume that for $n \geq 0$, $A_n \in L(G')$

$A_{n+1} = bA_n \mid A_n b \in L(G')$, which means that $\forall n \geq 0$, $A_n \in L(G') \Longrightarrow \{A_l : \forall l \geq 0\} \subseteq L(G)$

Case 2: (one $a \Rightarrow A_i a B_j$)

We can prove that $A_i, B_j \in L(G')$ $\forall i, j \geq 0$ similarly from case 1, which means that $\{A_i a B_j : \forall i, j \geq 0\} \subseteq L(G)$

Case 3: (two a's $\Rightarrow A_i a B_j a C_k$)

Case 4: (three a's $\Rightarrow A_i a B_j a C_k a D_l$)

These two cases can also be proved similarly from case 1 and case 2.

$\Rightarrow \{A_i a B_j a C_k : \forall i, j, k \geq 0\} \subseteq L(G)$

$\{A_i a B_j a C_k a D_l : \forall i, j, k, l \geq 0\} \subseteq L(G)$

$\rightarrow \{(c)\} \subseteq L(G) \Rightarrow L(G) = \{(c)\}$

Argument (f)

$\rightarrow L(G) \subseteq \{(e)\}$

If str is not begin by a or end by b, it isn't accepted by $S \rightarrow aS'b$. Let W_l be a string, l is the length, $W_0 = \lambda$

base case: $ab = aW_0 b$, $G' = \{\{S'\}, \{a, b\}, S', P'\}$, $P' = S' \rightarrow aS' \mid bS' \mid \lambda$

$W_0 \in L(G')$, $W_1 = aW_0 \mid W_0 a \mid bW_0 \mid W_0 b \in L(G')$. Assume that for $n \geq 0$, $W_n \in L(G')$, $W_{n+1} = aW_n \mid W_n a \mid bW_n \mid W_n b \in L(G')$

which implies that $\forall n \geq 0$ $aW_n b \in L(G) \equiv \{(e)\} \subseteq L(G)$

$\Rightarrow L(G) = \{(e)\}$

17. Let $\Sigma = \{a, b\}$. For each of the following languages, find a grammar that generates it.

(a) $L_1 = \{a^n b^m : n \geq 0, m < n\}$.

(b) $L_2 = \{a^{3n} b^{2n} : n \geq 2\}$.

(c) $L_3 = \{a^{n+3} b^n : n \geq 2\}$.

(d) $L_4 = \{a^n b^{n-2} : n \geq 3\}$.

(e) $L_1 L_2$.

(f) $L_1 \cup L_2$.

(g) L_1^3.

(h) L_1^*.

(i) $L_1 - L_4$.

(a) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow aS'$, $S' \rightarrow aS'b \mid aS' \mid \lambda$

(b) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow aaaaaaS'bbbb$, $S' \rightarrow aaaS'bb \mid \lambda$

(c) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow aaaaaS'bb$, $S' \rightarrow aS'b \mid \lambda$

(d) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow aaaS'b$, $S' \rightarrow aS'b \mid \lambda$

(e) $G = \{\{S, A, B\}, \{a, b\}, S, P\}$, $P = S \rightarrow aA\,aaaaaaBbbbb$, $A \rightarrow aAb \mid aA \mid \lambda$, $B \rightarrow aaaBbb \mid \lambda$

(f) $G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P = S \rightarrow aS'$, $S' \rightarrow aS'b \mid aS' \mid \lambda$

(g) $G = \{\{S, S', S''\}, \{a, b\}, S, P\}$, $P = S \rightarrow S'S'S'$, $S' \rightarrow aS''$, $S'' \rightarrow aS''b \mid aS'' \mid \lambda$

(h) $G = \{\{S, A, B\}, \{a, b\}, S, P\}$, $P = S \rightarrow AS \mid \lambda$, $A \rightarrow aB$, $B \rightarrow aBb \mid aB \mid \lambda$

(i) $G = \{\{S, A, B, C, D\}, \{a, b\}, S, P\}$, $P = S \rightarrow aaaAb \mid aab \mid aC$, $A \rightarrow aBb \mid B$, $B \rightarrow aC \mid bD$, $C \rightarrow aC \mid \lambda$, $D \rightarrow bD \mid \lambda$

19. Find a grammar that generates the language

$$L\left\{ww^R : w \in \{a,b\}^+\right\}.$$

Give a complete justification for your answer.

(i) S_l : l 表示長度, $S_2 = aa$ or bb $\in L\{ww^R : w \in \{a,b\}^+\}$,

$S_4 = aS_2a$ or $bS_2b \in L\{ww^R : w \in \{a,b\}^+\}$, Let $S_k \in L\{ww^R : w \in \{a,b\}^+\}$

$S_{k+2} = aS_ka$ or $bS_kb \in L\{ww^R : w \in \{a,b\}^+\} \Rightarrow G \subseteq L\{ww^R : w \in \{a,b\}^+\}$ #

(ii) W_l : l 表示長度, $W_1 = a$ or $b \rightarrow W_1W_1^R = aa$ or $bb \in G$,

$W_2W_2^R = aW_1W_1^Ra$ or $bW_1W_1^Rb \in G$, Let $W_kW_k^R \in G$

$W_{k+1}W_{k+1}^R = aW_kW_k^Ra$ or $bW_kW_k^Rb \in G \Rightarrow L\{ww^R : w \in \{a,b\}^+\} \in G$ #

$G = \{\{S, S'\}, \{a, b\}, S, P\}$, $P =$ $S \rightarrow aS'a \mid bS'b$, $S' \rightarrow aS'a \mid bS'b \mid \lambda$

4. For $\Sigma = \{a, b\}$, construct dfa's that accept the sets consisting of

(a) all strings with exactly one a.

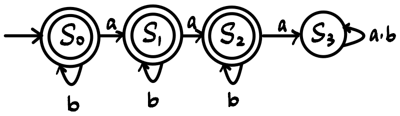
(b) all strings with at least two a's.

(c) all strings with no more than two a's.

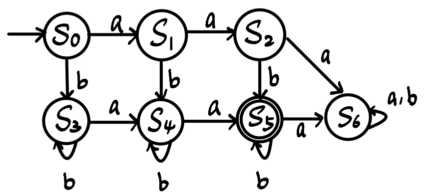
(d) all strings with at least one b and exactly two a's.

(e) all the strings with exactly two a's and more than three b's.

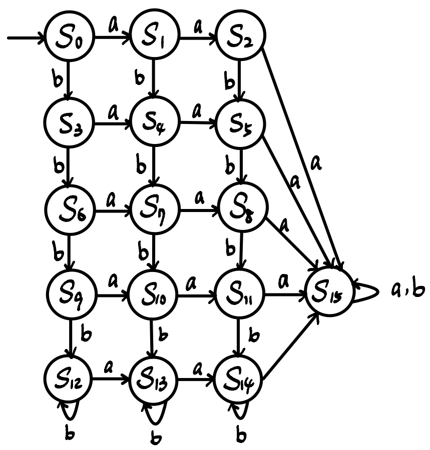
(c)



(d)



(e)



11. Consider the set of strings on {0, 1} defined by the requirements below. For each, construct an accepting dfa.

(a) Every 00 is followed immediately by a 1. For example, the strings 101, 0010, 0010011001 are in the language, but 0001 and 00100 are not.

(b) All strings that contain the substring 000, but not 0000.

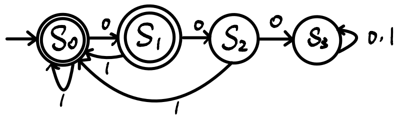
(c) The leftmost symbol differs from the rightmost one.

(d) Every substring of four symbols has, at most, two 0's. For example, 001110 and 011001 are in the language, but 10010 is not because one of its substrings, 0010, contains three zeros.

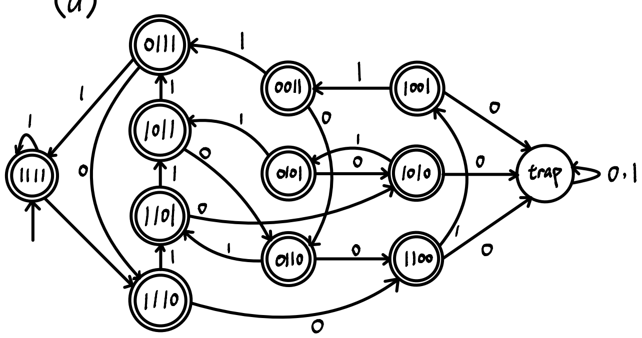
(e) All strings of length five or more in which the third symbol from the right end is different from the leftmost symbol.

(f) All strings in which the leftmost two symbols and the rightmost two symbols are identical.

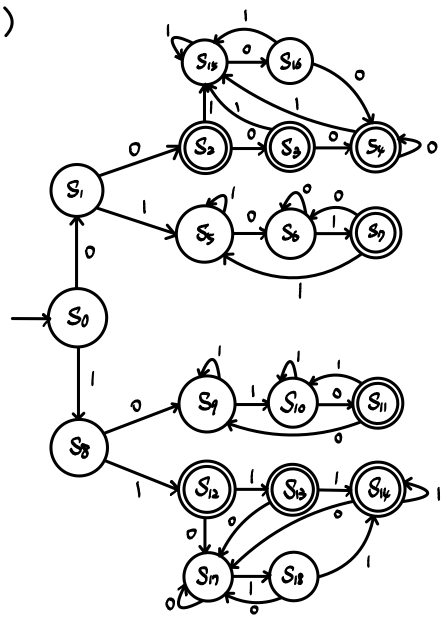
(a)



(d)

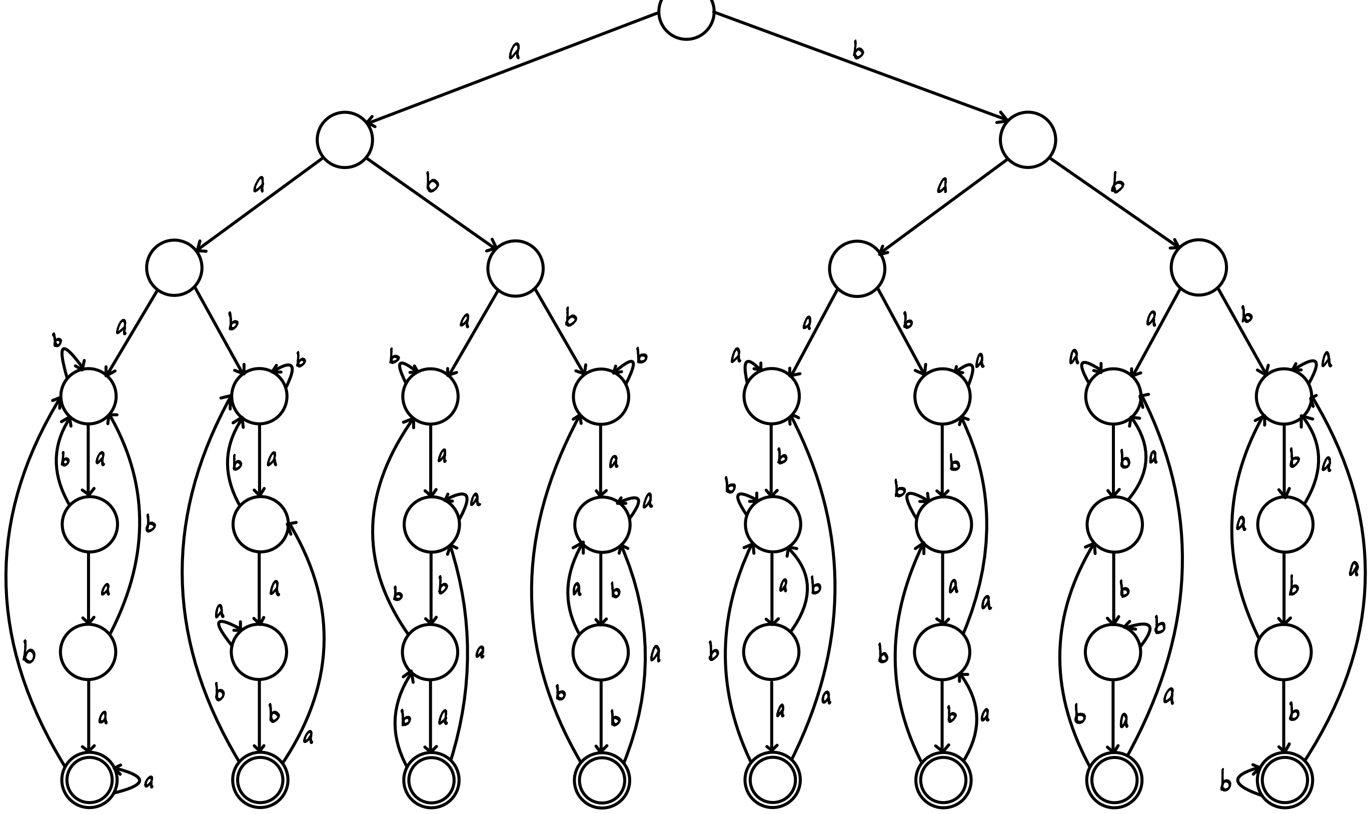


(f)

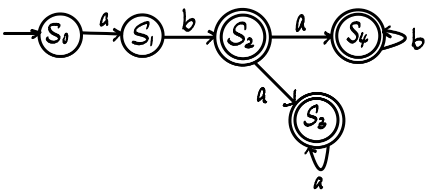


13. Show that the language $L = \{vwv : v, w \in \{a, b\}^*, |v| = 3\}$ is regular.

L can be represented by DFA. Therefore, L is regular.



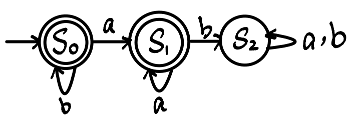
8. Design an nfa with no more than five states for the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$.

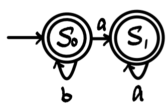


11. (a) Find an nfa with three states that accepts the language

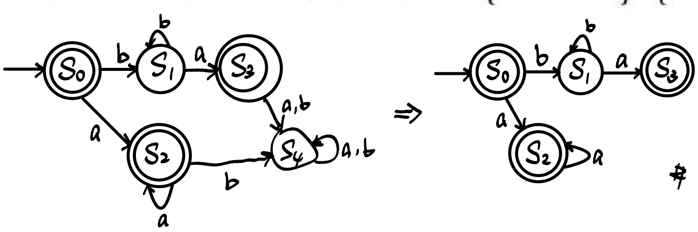
$$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}.$$

(b) Do you think the language in part (a) can be accepted by an nfa with fewer than three states?

(a) 

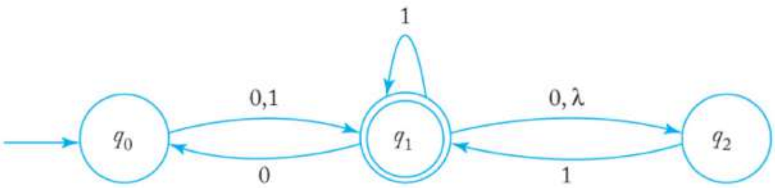
(b) 

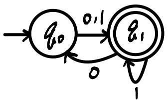
12. Find an nfa with four states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$.



2. Convert the nfa in Exercise 13, Section 2.2, into an equivalent dfa.

13. Which of the strings 00, <u>01001</u>, 10010, <u>000</u>, 0000 are accepted by the following nfa?

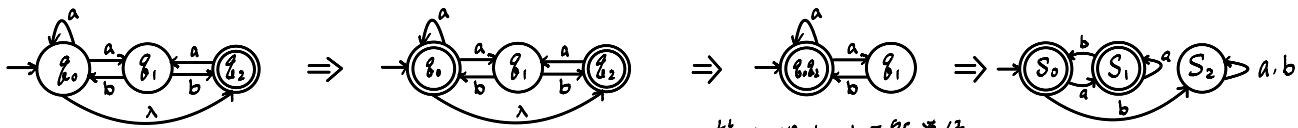




4. Convert the nfa defined by

$$
\begin{aligned}
\delta(q_0, a) &= \{q_0, q_1\} \\
\delta(q_1, b) &= \{q_0, q_2\} \\
\delta(q_2, a) &= \{q_1\} \\
\delta(q_0, \lambda) &= \{q_2\}
\end{aligned}
$$

with initial state q_0 and final state q_2 into an equivalent dfa.



第一個不是 b，b不能連續

#