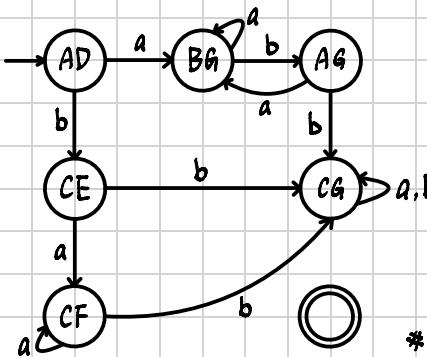
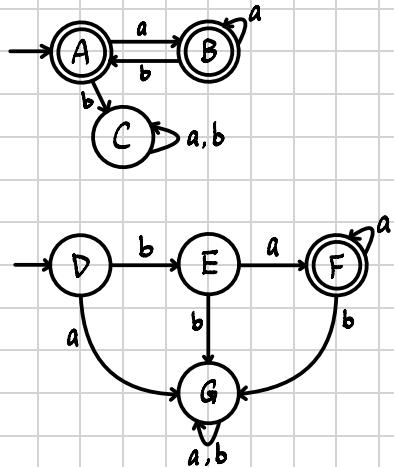


6. Use the construction in Theorem 4.1 to find nfa's that accept

$$(a) L((ab)^* a^*) \cap L(baa^*) = \emptyset$$

$$(b) L(ab^*a^*) \cap L(a^*b^*a)$$



6. Determine whether or not the following languages on $\Sigma = \{a\}$ are regular:

$$(a) L = \{a^n : n \geq 2, \text{ is a prime number}\}.$$

$$(b) L = \{a^n : n \text{ is not a prime number}\}.$$

$$(c) L = \{a^n : n = k^3 \text{ for some } k \geq 0\}.$$

$$(d) L = \{a^n : n = 2^k \text{ for some } k \geq 0\}.$$

$$(e) L = \{a^n : n \text{ is the product of two prime numbers}\}.$$

$$(f) L = \{a^n : n \text{ is either prime or the product of two or more prime numbers}\}.$$

$$(g) L^*, \text{ where } L \text{ is the language in part (a).}$$

9. Find context-free grammars for the following languages (with $n \geq 0, m \geq 0$).

$$(a) L = \{a^n b^m : n \leq m+3\}.$$

$$(b) L = \{a^n b^m : n = m-1\}.$$

$$(c) L = \{a^n b^m : n \neq 2m\}.$$

$$(d) L = \{a^n b^m : 2n \leq m \leq 3n\}.$$

$$(e) L = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}.$$

$$(f) L = \{w \in \{a, b\}^* : n_a(v) \geq n_b(v), \text{ where } v \text{ is any prefix of } w\}.$$

$$(g) L = \{w \in \{a, b\}^* : n_a(w) = 2n_b(w)+1\}.$$

$$(h) L = \{w \in \{a, b\}^* : n_a(w) = n_b(w)+2\}.$$

For given m , pick $n = a^{2^m} = xy^iz, |xy| \leq m, 1 \leq k \leq m$, pick $i=2, n = a^{2^m+k}, \therefore k \leq m < 2^m$

$\therefore \nexists k \text{ s.t. } 2^m+k = 2^n, n \text{ is an integer.}$

$\Rightarrow n \notin L \Rightarrow L \text{ is not regular.}$

$$(b) S \rightarrow Ab, A \rightarrow aAb \mid \lambda$$

$$(c) S \rightarrow AE \mid EB, E \rightarrow aaEb \mid \lambda$$

$$A \rightarrow aA \mid a, B \rightarrow bB \mid b$$

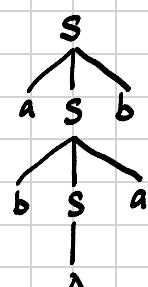
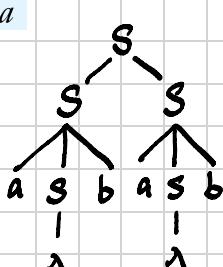
$$(d) S \rightarrow aSAbb \mid \lambda, A \rightarrow b \mid \lambda$$

19. Show that the grammar in Example 1.13 is ambiguous.

$$S \rightarrow SS, \quad S \rightarrow aSb,$$

Input string: abab

$$S \rightarrow \lambda, \quad S \rightarrow bSa$$



\Rightarrow parsing tree 不唯一 \Rightarrow ambiguous

21. Use the exhaustive search parsing method to parse the string $abbbbb$ with the grammar in Example 5.5. In general, how many rounds will be needed to parse any string w in this language?

$S \rightarrow aAB, \quad S \rightarrow aAB, \quad A \rightarrow bBb, \quad B \rightarrow A, \quad A \rightarrow bBb, \quad B \rightarrow A, \quad A \rightarrow bBb, \quad \cancel{B \rightarrow A}, \quad \cancel{B \rightarrow A}$

$A \rightarrow bBb, \quad \cancel{B \rightarrow \lambda}$

$B \rightarrow A|\lambda. \quad |w| + 1 \text{ rounds in general.} \quad \cancel{B \rightarrow \lambda} \quad B \rightarrow \lambda \quad B \rightarrow \lambda$

8. Eliminate all λ -productions from $S \rightarrow aSSS$,

$$S \rightarrow bb|\lambda.$$

$$S \rightarrow aSSS \mid ass \mid as \mid a \mid bb$$

9. Eliminate all λ -productions from $S \rightarrow AaB \mid aaB$,

$$A \rightarrow \lambda,$$

$$B \rightarrow bbA \mid \lambda.$$

$$S \rightarrow a \mid aa \mid abb \mid aabb$$

26. Use the result of the preceding exercise to rewrite the grammar $A \rightarrow Aa \mid aBc \mid \lambda, B \rightarrow Bb \mid bc$

so that it no longer has productions of the form $A \rightarrow Ax$ or $B \rightarrow Bx$.

$$A \rightarrow aBcA' \mid A', \quad A' \rightarrow aA' \mid \lambda. \quad B \rightarrow bcB' \mid B', \quad B' \rightarrow bB$$

2. Convert the grammar $S \rightarrow aSb \mid Sab \mid ab$ into Chomsky normal form.

$$S \rightarrow Av_1 \mid Sv_2 \mid AB, \quad v_1 \rightarrow SB, \quad v_2 \rightarrow AB, \quad A \rightarrow a, \quad B \rightarrow b$$

4. Transform the grammar with productions $S \rightarrow baAB$,

$$A \rightarrow bAB \mid \lambda,$$

$$B \rightarrow BAa \mid A \mid \lambda$$

into Chomsky normal form.

$$S \rightarrow ba \mid baA \mid baB \mid baAB$$

$$S \rightarrow V_2V_1 \mid V_3A \mid V_3B \mid V_3V_4$$

$$A \rightarrow b \mid bA \mid bB \mid bAB$$

$$V_1 \rightarrow a, \quad V_2 \rightarrow b, \quad V_3 \rightarrow V_2V_1, \quad V_4 \rightarrow AB$$

$$B \rightarrow a \mid Ba \mid Aa \mid BAA$$

$$A \rightarrow b \mid V_2A \mid V_2B \mid V_2V_4$$

$$B \rightarrow b \mid bA \mid bB \mid bAB$$

$$B \rightarrow a \mid BV_1 \mid AV_1 \mid V_4V_1 \mid b \mid V_2A \mid V_2B \mid V_2V_4$$

#

4. Use the CYK method to determine if the string $w = aaabbbb$ is in the language generated by the grammar G $S \rightarrow aSb \mid b$.

$$S \rightarrow AV_i \mid b, V_i \rightarrow SB, A \rightarrow a, B \rightarrow b$$

a a a b b b b

A A A B,S B,S B,S B,S

aa aa ab bb bb bb

V_i V_i V_i

aaa aab abb bbb bbb

S

aab aabb abbb bbbb

V_i

aaabb qabbb abbbb

S

aaa bbb aabbbb

V_i

aaabbbb

S

↪ $w \in L(G)$