

4. Transform the grammar with productions $S \rightarrow baAB$,

$$\begin{aligned} A &\rightarrow bAB \mid \lambda, \\ B &\rightarrow BAa \mid A \mid \lambda \end{aligned}$$

into Chomsky normal form.

Remove λ :

$$\begin{aligned} S &\rightarrow ba \mid baA \mid baB \mid baAB \\ A &\rightarrow b \mid bA \mid bB \mid bAB \\ B &\rightarrow a \mid Ba \mid Aa \mid BAa \\ &\quad \mid b \mid bA \mid bB \mid bAB \end{aligned}$$

$$\Rightarrow \begin{aligned} S &\rightarrow V_b V_a \mid V_b A \mid V_b B \mid V_b V_i \\ V_i &\rightarrow V_b V_a \quad , \quad V_i \rightarrow AB \\ V_a &\rightarrow a \quad , \quad V_b \rightarrow b \end{aligned}$$

$$\begin{aligned} A &\rightarrow b \mid V_b A \mid V_b B \mid V_b V_i \\ B &\rightarrow a \mid BV_a \mid AV_a \mid V_b V_a \\ &\quad \mid b \mid V_b A \mid V_b B \mid V_b V_i \\ V_i &\rightarrow BA \end{aligned}$$

5. Convert the grammar $S \rightarrow AB \mid aB$,

$$\begin{aligned} A &\rightarrow abb \mid \lambda, \\ B &\rightarrow bbA \end{aligned}$$

into Chomsky normal form.

Remove λ :

$$\begin{aligned} S &\rightarrow B \mid AB \mid aB \\ A &\rightarrow abb \\ B &\rightarrow bb \mid bbA \end{aligned}$$

Remove unit production

$$\begin{aligned} S &\rightarrow bb \mid bbA \mid AB \mid aB \\ A &\rightarrow abb \\ B &\rightarrow bb \mid bbA \end{aligned}$$

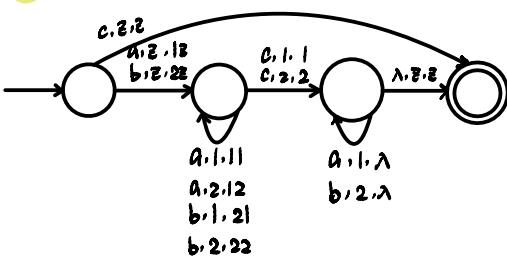
\Rightarrow

$$\begin{aligned} S &\rightarrow V_b V_b \mid V_b A \mid AB \mid V_a B \\ A &\rightarrow V_a V_b \\ B &\rightarrow V_b V_b \mid V_b A \\ V_a &\rightarrow a \quad , \quad V_b \rightarrow b \quad , \quad V_b \rightarrow V_b \rightarrow V_b V_b \end{aligned}$$

6. Construct npda's that accept the following languages on $\Sigma = \{a, b, c\}$:

(a) $L = \{a^n b^{3n} : n \geq 0\}$.

(b) $L = \{wcw^R : w \in \{a, b\}^*\}$.



6. Show that the language $L = \{a^{n^2} : n \geq 0\}$ is not context free.

Pick $w = a^{m^2} = uvkgz \in L$. Assume that L is context free.

$$|vxy| \leq m, |vy| \geq 1, \text{ let } v = a^{k_1}, y = a^{k_2}$$

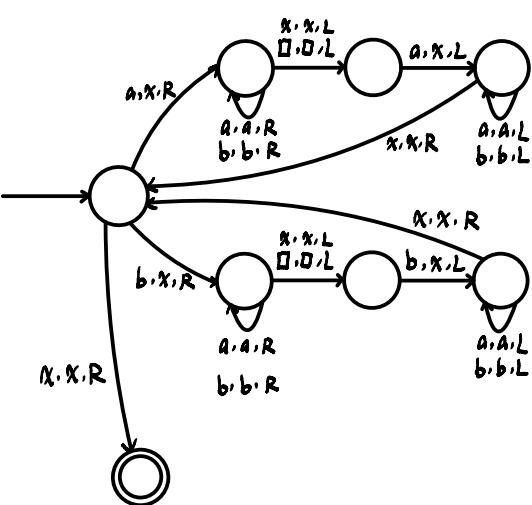
$$w = uv^k y^i z, \text{ pick } i=2, \text{ then } w = a^{m^2+k_1+k_2}$$

$$k_1 + k_2 \leq m \rightarrow m^2 + k_1 + k_2 \leq m^2 + m < m^2 + 2m + 1 = (m+1)^2$$

$$\rightarrow m^2 + k_1 + k_2 \text{ 不是完全平方數} \rightarrow w \notin L \rightarrow L \text{ is not context free.}$$

9. Design a Turing machine that accepts the language

$$L = \{ww : w \in \{a, b\}^+\}.$$



1. Give a formal definition of a two-tape Turing machine; then write programs that accept the languages below. Assume that $\Sigma = \{a, b, c\}$ and that the input is initially all on tape 1.

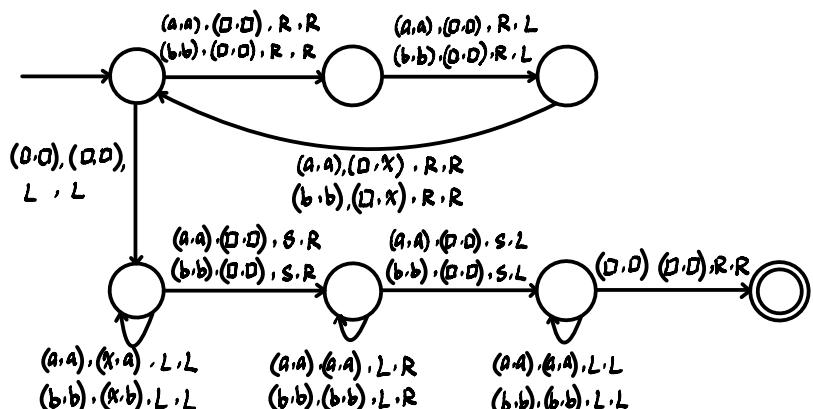
(a) $L = \{a^n b^n c^n : n \geq 1\}$.

(b) $L = \{a^n b^n c^m : m > n\}$.

(c) $L = \{ww : w \in \{a, b\}\}$.

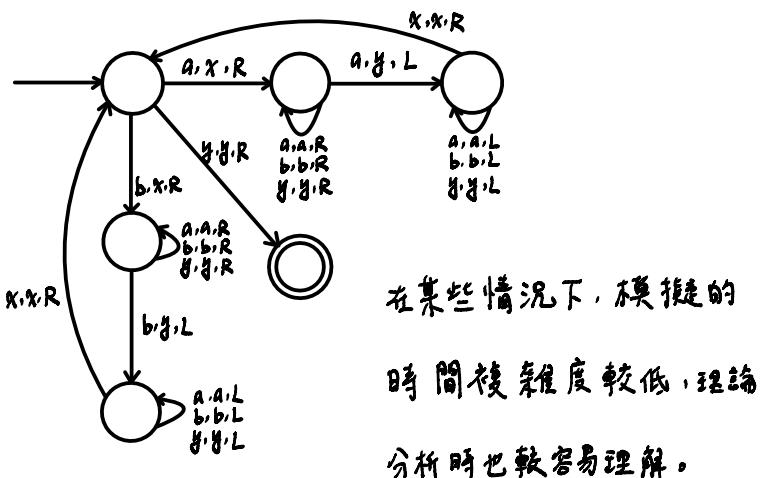
(d) $L = \{ww^R w : w \in \{a, b\}\}$.

$$S = Seq$$



2. Write programs for nondeterministic Turing machines that accept the languages below. In each case, explain if and how the nondeterminism simplifies the task.

(a) $L = \{ww : w \in \{a, b\}^+\}$.



在某些情況下，模擬的時間複雜度較低，理論分析時也較容易理解。