CS4100 Computer Architecture

Spring 2024, Homework 3

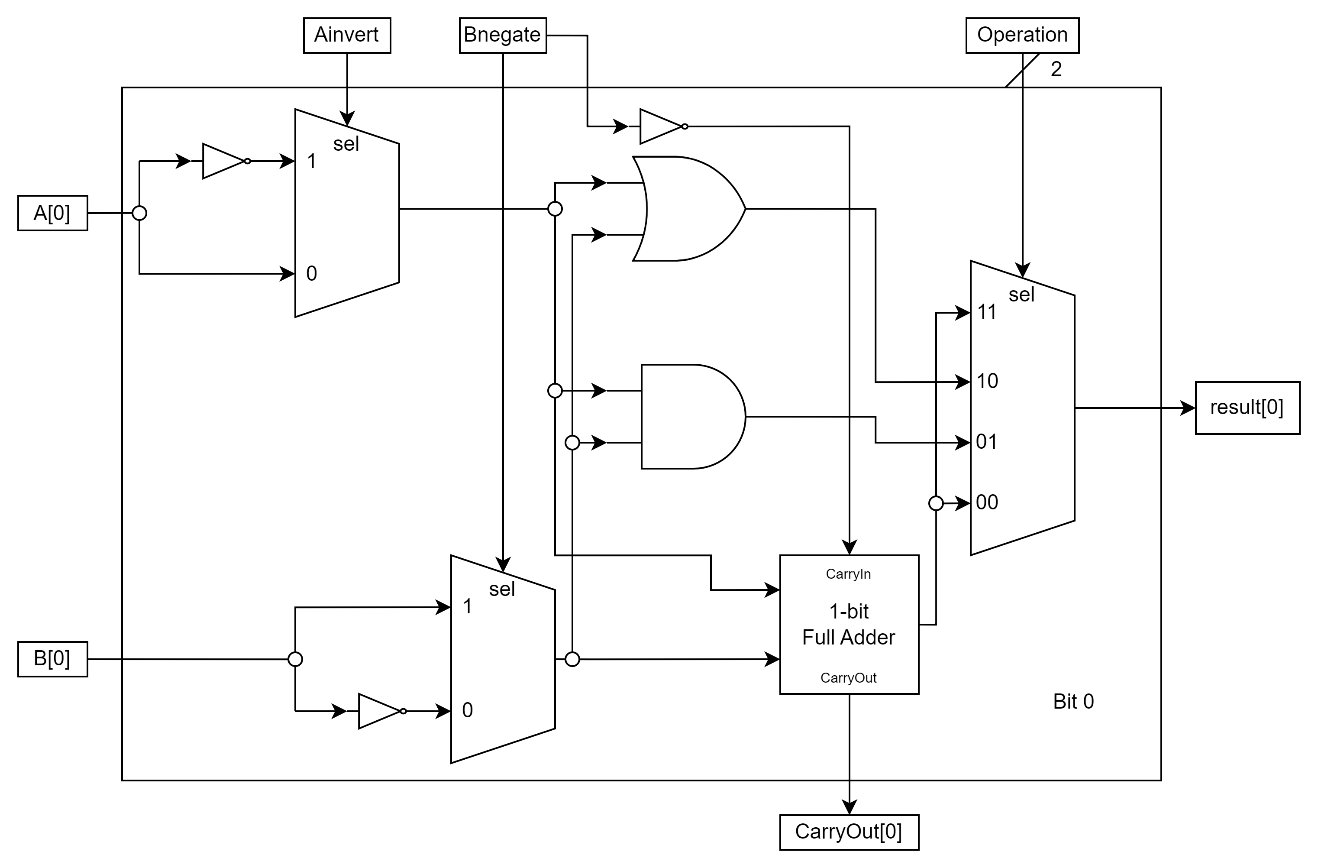
Due: 23:59, 4/21/2024

1. (18 points) Please modify the ALU design introduced in the class to satisfy the following requirements:

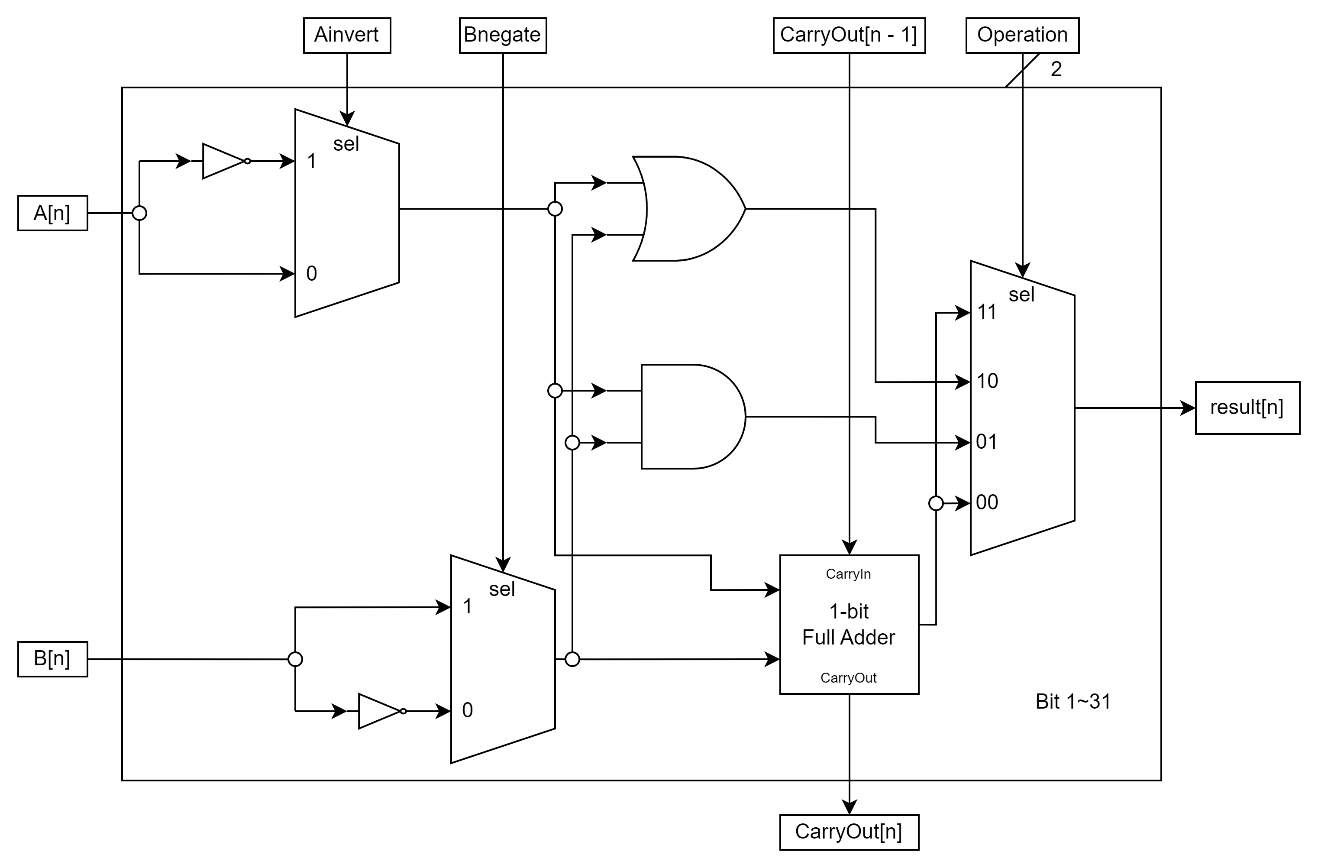
|  |  |  |  |
| --- | --- | --- | --- |
| Ainvert | Bnegate | Operation | Function |
| 0 | 1 | 01 | AND |
| 0 | 1 | 10 | OR |
| 0 | 1 | 00 | add |
| 0 | 0 | 00 | sub |
| 1 | 0 | 01 | NOR |
| 0 | 1 | 11 | add-ext |
| 0 | 0 | 11 | sub-ext |

Here, "add-ext" and "sub-ext" refer to 32-bit addition and subtraction with sign-extension to 64 bits. You are required to draw the circuit diagrams for each 1-bit ALU and the 64-bit ALU. For each 1-bit ALU, use only one full adder to perform an addition or subtraction operation, similar to the method demonstrated in class. Additionally, show the ALU control signals for "add-ext" and "sub-ext" in your design.

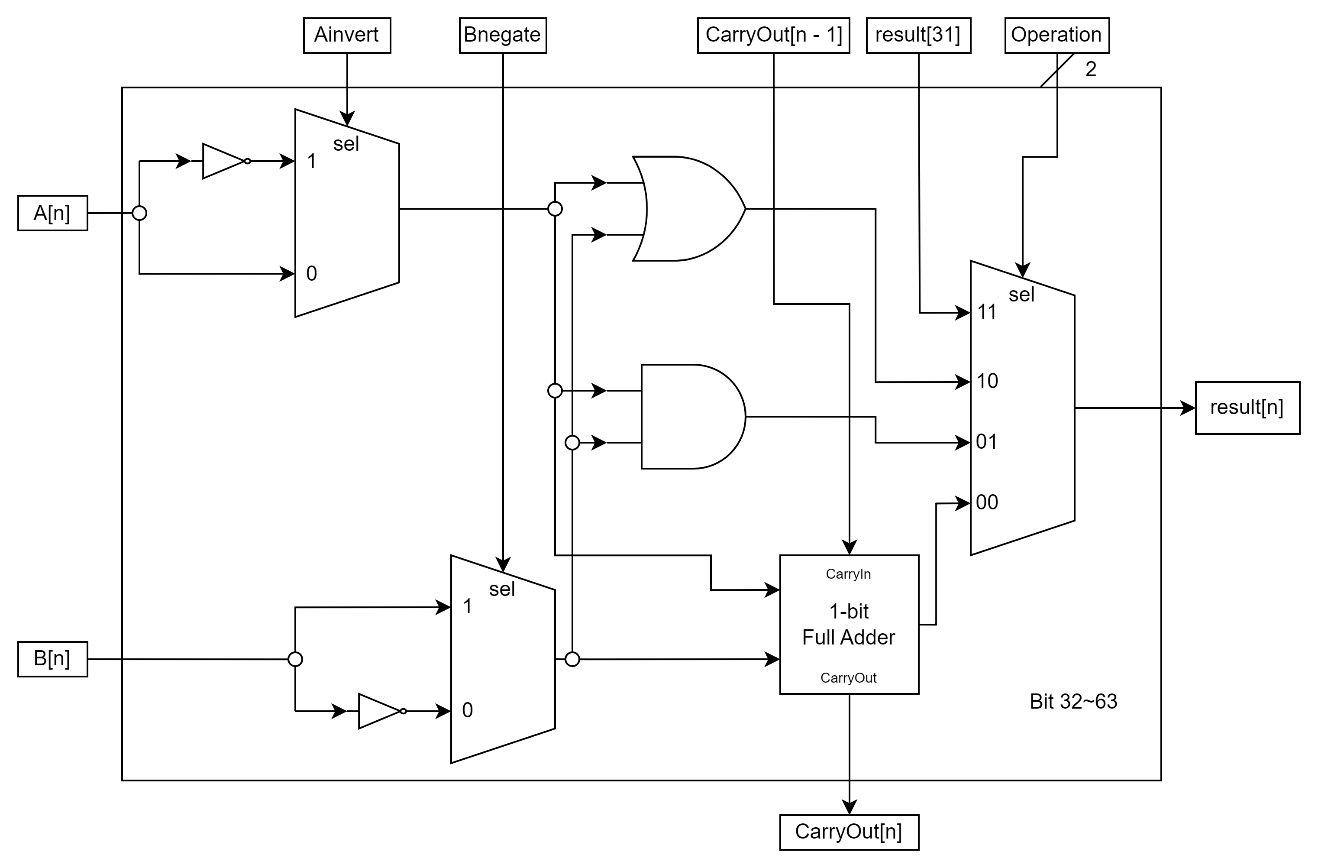




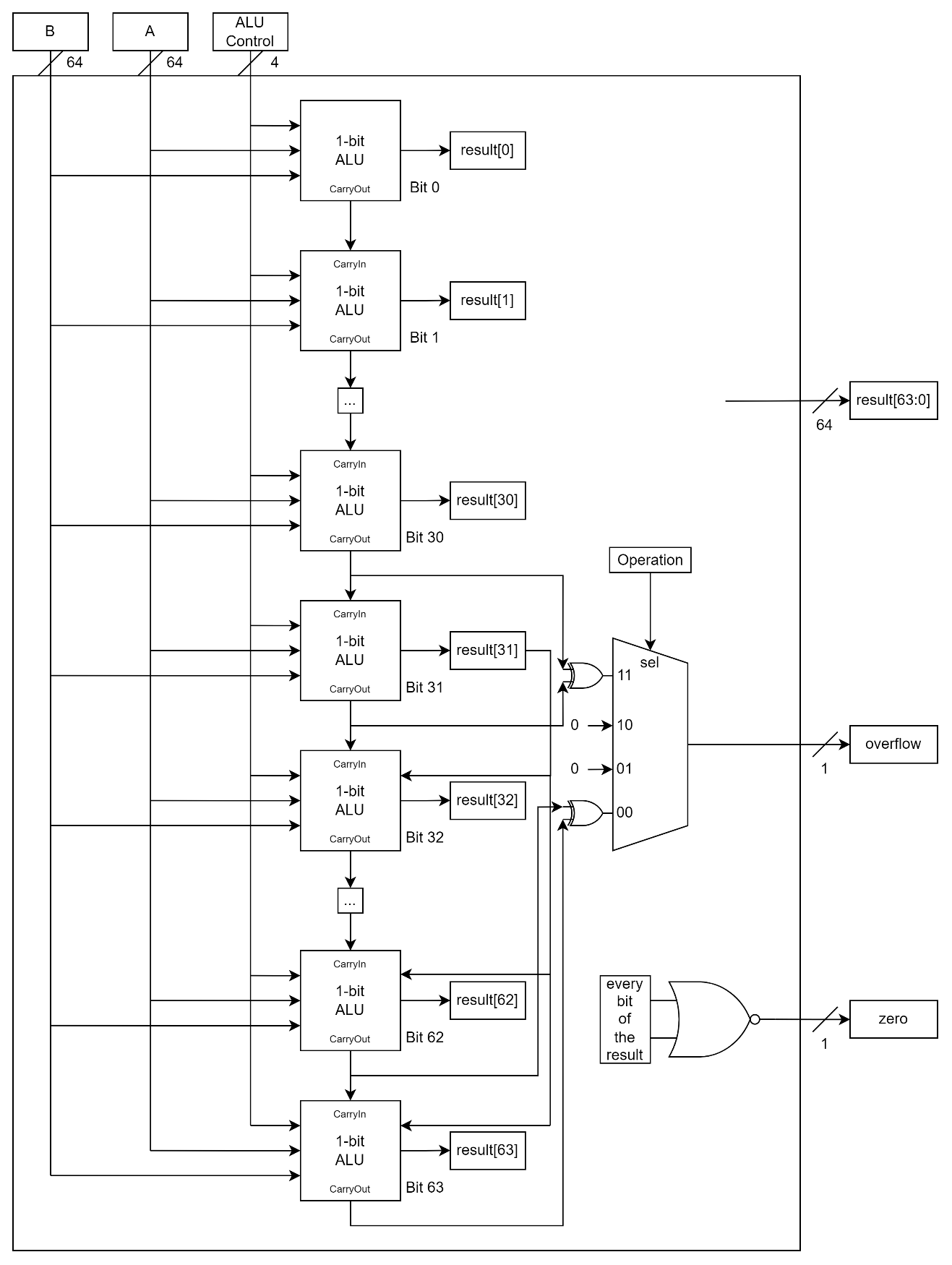
1-bit ALU



1-bit ALU



1-bit ALU



64-bit ALU

1. (14 points) Consider two unsigned binary numbers: M = 1110 and N = 1001.
   1. (7 points) Write down each step of M × N according to version 1 of the multiply algorithm.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Iteration** | **Step** | **Multiplier** | **Multiplicand** | **Product** |
| **0** | Initial values | 1001 | 0000 1110 | 0000 0000 |
| **1** | Prod = Prod + Mcand | 1001 | 0000 1110 | 0000 1110 |
| Shift left multiplicand | 1001 | 0001 1100 | 0000 1110 |
| Shift right multiplier | 0100 | 0001 1100 | 0000 1110 |
| **2** | Prod = Prod + 0 | 0100 | 0001 1100 | 0000 1110 |
| Shift left multiplicand | 0100 | 0011 1000 | 0000 1110 |
| Shift right multiplier | 0010 | 0011 1000 | 0000 1110 |
| **3** | Prod = Prod + 0 | 0010 | 0011 1000 | 0000 1110 |
| Shift left multiplicand | 0010 | 0111 0000 | 0000 1110 |
| Shift right multiplier | 0001 | 0111 0000 | 0000 1110 |
| **4** | Prod = Prod + Mcand | 0001 | 0111 0000 | 0111 1110 |
| Shift left multiplicand | 0001 | 1110 0000 | 0111 1110 |
| Shift right multiplier | 0000 | 1110 0000 | 0111 1110 |

M \* N = 0111 1110

* 1. (7 points) Write down each step of M × N according to version 2 of the multiply

algorithm.

|  |  |  |  |
| --- | --- | --- | --- |
| **Iteration** | **Step** | **Multiplicand** | **Product** |
| **0** | Initial values | 1110 | 0000 1001 |
| **1** | LeftProd += Mcand | 1110 | 1110 1001 |
| Shift right multiplier | 1110 | 0111 0100 |
| **2** | LeftProd += 0 | 1110 | 0111 0100 |
| Shift right multiplier | 1110 | 0011 1010 |
| **3** | LeftProd += 0 | 1110 | 0011 1010 |
| Shift right multiplier | 1110 | 0001 1101 |
| **4** | LeftProd += Mcand | 1110 | 1111 1101 |
| Shift right multiplier | 1110 | 0111 1110 |

M \* N = 0111 1110

1. (14 points) Consider two unsigned binary numbers: M = 0111 and N = 0101.
   1. (7 points) Write down each step of M ÷ N according to version 1 of the division algorithm.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Iteration** | **Step** | **Quotient** | **Divisor** | **Remainder** |
| **0** | Initial values | 0000 | 0101 0000 | 0000 0111 |
| **1** | Rem = Rem - Div | 0000 | 0101 0000 | 1011 0111 |
| Rem < 0   * Rem += Div * Shift left quotient | 0000 | 0101 0000 | 0000 0111 |
| Shift right divisor | 0000 | 0010 1000 | 0000 0111 |
| **2** | Rem = Rem - Div | 0000 | 0010 1000 | 1101 1111 |
| Rem < 0   * Rem += Div * Shift left quotient | 0000 | 0010 1000 | 0000 0111 |
| Shift right divisor | 0000 | 0001 0100 | 0000 0111 |
| **3** | Rem = Rem - Div | 0000 | 0001 0100 | 1111 0011 |
| Rem < 0   * Rem += Div * Shift left quotient | 0000 | 0001 0100 | 0000 0111 |
| Shift right divisor | 0000 | 0000 1010 | 0000 0111 |
| **4** | Rem = Rem - Div | 0000 | 0000 1010 | 1111 1101 |
| Rem < 0   * Rem += Div * Shift left quotient | 0000 | 0000 1010 | 0000 0111 |
| Shift right divisor | 0000 | 0000 0101 | 0000 0111 |
| **5** | Rem = Rem - Div | 0000 | 0000 0101 | 0000 0010 |
| Rem >= 0   * Shift left quotient * quotient[0] = 1 | 0001 | 0000 0101 | 0000 0010 |
| Shift right divisor | 0001 | 0000 0010 | 0000 0111 |

M / N = 0001…0010

* 1. (7 points) Write down each step of M ÷ N according to version 2 of the division algorithm.

|  |  |  |  |
| --- | --- | --- | --- |
| **Iteration** | **Step** | **Divisor** | **Remainder / Quotient** |
| **0** | Initial values | 0101 | 0000 0111 |
| **1** | Shift left remainder | 0101 | 0000 1110 |
| LeftRem -= Dsivisor | 0101 | 1011 1110 |
| Rem < 0   * LeftRem += Divisor * Shift left Rem | 0101 | 0001 1100 |
| **2** | LeftRem -= Dsivisor | 0101 | 1100 1100 |
| Rem < 0   * LeftRem += Divisor * Shift left Rem | 0101 | 0011 1000 |
| **3** | LeftRem -= Dsivisor | 0101 | 1110 1000 |
| Rem < 0   * LeftRem += Divisor * Shift left Rem | 0101 | 0111 0000 |
| **4** | LeftRem -= Dsivisor | 0101 | 0010 0000 |
| Rem >= 0   * Shift left Rem * Rem[0] = 1 | 0101 | 0100 0001 |
| **5** | Shift right leftRem | 0101 | 0010 0001 |

M / N = 0001…0010

1. (12 points) Answer the following questions in detail. You will receive 0 point if you only write down the answers.
   1. (4 points) What decimal number does the bit pattern 05948DEC16 represent if it’s a two’s complement integer? If it’s an unsigned number, is the result the same as the two’s complement? If they are different, why?

05948DEC16 = 12 + 14\*16 + 13\*162 + 8\*163 + 4\*164 + 9\*165 + 5\*166

= 9362174010

因為在 signed 的情況下 sign bit 是 0，因此 signed 與 unsigned 的結果會是一樣的。

* 1. (4 points) Answer problem (a) with a different bit pattern FA6B721416.

signed number:

FA6B721416 = 1111\_1010\_0110\_1011\_0111\_0010\_0001\_01002

= - (0000\_0101\_1001\_0100\_1000\_1101\_1110\_10112 + 1)

= - (0000\_0101\_1001\_0100\_1000\_1101\_1110\_11002)

= - 9362174010

unsigned number:

FA6B721416 = 1111\_1010\_0110\_1011\_0111\_0010\_0001\_01002

= 420134555610

以 signed 的方式來看，sign bit 為 1 代表是負數，因此結果會與 unsigned 的結果不一樣。

* 1. (4 points) What decimal numbers do 05948DEC16 and FA6B721416 represent if they are IEEE 754 floating point numbers.

05948DEC16 = 0000\_0101\_1001\_0100\_1000\_1101\_1110\_11002

* 0\_00001011\_00101001000110111101100
* sign bit: 0, exponent: 1110 – 12710 (bias) = -11610,

fraction: 001010010001101111011002

* 1 + 2-3 + 2-5 + 2-8 + 2-12 + 2-13 + 2-15 + 2-16 + 2-17 + 2-18 + 2-20 + 2-21

= 1 + 0.160581 = 1.160581

* Decimal number: 1.160581 \* 2-116

FA6B721416 = 1111\_1010\_0110\_1011\_0111\_0010\_0001\_01002

* 1\_11110100\_110101101110010000101002
* sign bit: 1, exponent: 24410 – 12710 = 11710

fraction: 110101101110010000101002

* 1 + 2-1 + 2-2 + 2-4 + 2-6 + 2-7 + 2-9 + 2-10 + 2-11 + 2-14 + 2-19 + 2-21

= 1 + 0.839419 = 1.839419

* Decimal number: -1.839419 \* 2117

1. (10 points) Consider two decimal numbers: X = 88.4375 and Y = −7.3125.
   1. (6 points) Write down X and Y in the IEEE 754 single precision format. You must detail how you get your answer, or you will receive 0 point.

88.437510 = 8810 + 0.2510 + 0.12510 + 0.062510

= 01011000.01112 = 1.01100001112 \* 2106

X is positive => sign bit = 0

exponent = 610 + 12710 = 13310 = 100001012

fraction = 011000011100000000000002

X = 01000010101100001110000000000000

7.312510 = 710 + 0.2510 + 0.062510

= 000001112 + 0.01012 = 00000111.01012

= 1.1101012 \* 2102

Y is negative => sign bit = 1

exponent = 210 + 12710 = 100000012

fraction = 110101000000000000000002

Y = 11000000111010100000000000000000

* 1. (4 points) Assuming X and Y are given in the IEEE 754 single precision format. Show all the steps to perform X × Y and write the solution in the IEEE 754 single precision format.

Add exponent: 2 + 6 = 8，加上 bias 為 135

1.011000011100000000000002 \* 1.110101000000000000000002

= 10.10000110101100110000000000000000000000000000002

* 10.100001101011001100000002 \* 28 = 1.010000110101100110000002 \* 29

sign bit: 1, exponent: 9 + 127 = 13610 = 100010002,

fraction: 01000011010110011000000

X \* Y = 11000100001000011010110011000000

1. (20 points) Consider a new floating-point number representation that is only 16 bits wide.

The leftmost bit is still the sign bit, the exponent is 9 bits wide and has a bias of 255, and the fraction is 6 bits long. A hidden 1 to the left of the binary point is assumed. In this representation, any 16-bit binary pattern having 000000000 in the exponent field and a non- zero fraction indicates a denormalized number: (−1)𝑆 × (0 + Fraction) × 2−254 . Write the answers of (a), (b) and (c) in scientific notation, e.g., 1.0101 × 22.

* 1. (3 points) What is the smallest positive “normalized” number, denoted as a0?

sign bit: 0, exponent: 000000001, fraction: 000000

exponent – bias = 1 – 255 = 254 => a0 = 1.0000002 \* 2-254.

* 1. (6 points) What is the largest positive “denormalized” number, denoted as a1? What is the second largest positive “denormalized” number, denoted as a2?

sign bit: 0, exponent: 000000000, fraction: 111111

0.1111112 \* 26 = 1111112 = 6310

a1 = 63 \* 2-260 = 1111112 \* 2-260 = 1.1111102 \* 2-255

sign bit: 0, exponent: 000000000, fraction: 111110

0.1111102 \* 26 = 1111102 = 6210

a2 = 62 \* 2-260 = 1111102 \* 2-260 = 1.1111002 \* 2-255

* 1. (4 points) Find the differences between a0 and a1, and between a1 and a2.

a0 – a1 = 1.0000002 \* 2-254 – 1.1111102 \* 2-255 = 0.0000012 \* 2-254 = 1.0000002 \* 2-260

a1 – a2 = 1.1111102 \* 2-255 – 1.1111002 \* 2-255 = 0.0000102 \* 2-255 = 1.0000002 \* 2-260

* 1. (3 points) What binary number does the binary pattern 1011110110100111 represent?

sign bit: 1, exponent: 011110110, fraction: 100111

exponent - bias = 0111101102 – 25510= 24610 – 25510 = -910

1.1001112 = 11001112 \* 210-6 = 10310 \* 210-6

This binary number is -11001112 \* 210-15.

* 1. (4 points) Let U be the nearest representation of the decimal number 1.31; that is, U has the smallest approximation error. What is U? What is the actual decimal number represented by U?

0.31 \* 2 => (0).62 \* 2 => (1).24 \* 2 => (0).48 \* 2 => (0).96 \* 2 => (1).92 \* 2 => (1).84 \* 2 => (1).68 (進位)。 => 1.3110 = 1.0101002。255 + 1 = 25610

=> sign bit: 0, exponent: 100000001 (25610), fraction: 010100

U is 0000000001010100. Its actual decimal number is 1.3125

1. (12 points) **X** is a 32-bit signed integer variable, **&** is the bitwise-AND operator, and "**>>**" is the sra (shift right arithmetic) operator. For the following options, determine whether they provide the correct result for (**X** / 4) and explain the reasons.

(a) (**X** + 3) **>>** 2

(b) ((**X** >= 0) ? **X >>** 2 : (**X** + 3) **>>** 2)

(c) **X >>** 2

(d) (**X** + ((**X >>** 31) **&** 3 )) **>>** 2

Let A be a nonnegative 32-bit signed integer. Obviously, A >> 2 = A / 4.

(-A) / 4 = - (A / 4) = - (A >> 2) = ~ (A >> 2) + 1 = ((~A) >> 2) + 1 = (~A + 4) >> 2

= (~A + 1 + 3) >> 2 = (-A + 3) >> 2

Therefore, X / 4 = X >> 2 if X >= 0. Otherwise, X / 4 = (X + 3) >> 2.

Counter case:

* Use X / 4 = (X + 3) >> 2: For X = 1 >= 0, (X + 3) >> 2 = 1 ≠ X / 4 = 0
* Use X / 4 = X >> 2: For X = -1 < 0, X >> 2 = -1 ≠ X / 4 = 0

1. 由上述可知，若 X >= 0 則 (X + 3) >> 2 不一定等於 X / 4
2. 由上述可知，((X >= 0) ? X >> 2 : (X + 3) >> 2) = X / 4
3. 由上述可知，若 X < 0 則 X >> 2 不一定等於 X / 4
4. 若 X < 0 則 (X + ((X >> 31) & 3)) >> 2 = (X + 3) >> 2 = X / 4

若 X >= 0 則 (X + ((X >> 31) & 3)) >> 2 = (X + 0) >> 2 = X / 4

(X + ((X >> 31) & 3 )) >> 2 = X / 4