

Math 6a - Problem Set 6

1. Show that any isomorphism f from the group \mathbb{Q} of rational numbers under addition to itself must be of the form $f = cx$ for some c . Does this also hold for isomorphisms of the group $\{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ under addition to itself?
2. Consider a prime p and let q be a prime divisor of $2^p - 1$. Consider the group $\{1, 2, \dots, q - 1\}$ under multiplication mod q and show that 2 has order at most p in this group. Why does this result imply that there are infinitely many primes?
3. The sequence a_n is defined by $a_0 = 0$, $a_1 = 1$ and $a_n = 5a_{n-1} - 6a_{n-2} + 5^n$. By using generating functions, find a non-recursive formula for a_n .
4. Let b_n denote the number of sequences of ones and zeros that are of length n and have the ones only occurring in groups of three or more. Find a recurrence relation for b_n such that the number of elements in this relation does not depend on n .
5. Suppose that we write 2020 numbers on the blackboard. Show that we can select some subset of these numbers so that their sum is a multiple of 2020.