

How to Write Scientific Papers

Nick Higham
Department of Mathematics
The University of Manchester

<https://nhigham.com/>

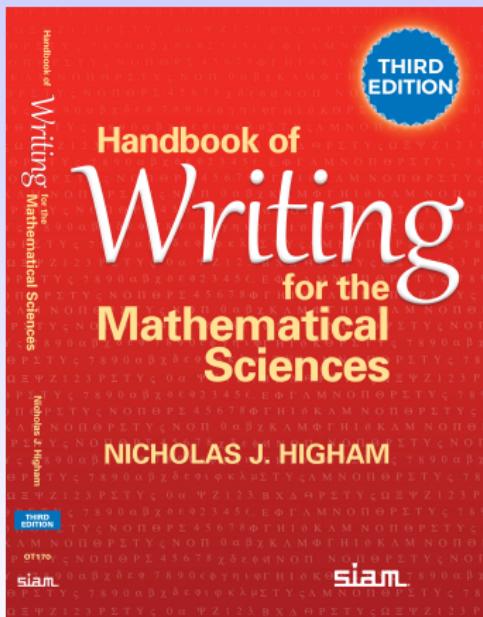
Slides available at <https://bit.ly/polyu21>

Department of Applied Mathematics,
The Hong Kong Polytechnic University



Handbook of Writing, 3rd edition, 2020

For more information about any of the topics discussed here see



- Book website.
- SIAM Bookstore.

Outline

Writing

Mathematical Writing

English Usage

Workflow

Writing a Paper

Why is it Important to Write Well?

- Promote your research (globally).
- Persuade the reader to agree with your views.
- Improve chance of your papers being published . . .
- . . . and then read.
- Demonstrate your communication skills.
- Feeling of satisfaction from job done well.

Most worthwhile careers involve writing!

What is Difficult?

- Need something to say.
- Hard to get started.
- Need to repeatedly improve drafts.
- Must convince reader of value of your work.

Less difficult:

- English grammar.
- Maths-specific writing.
- L^AT_EX.

Begin Sentences with Subjects and Verbs

R. P. Clark:

- Make meaning early, then let weaker elements branch to the right.
- Don't separate the subject too far from its verb.

Example

“Motivated by the works of Feireisl-Rocca-Schimperna [10] and Feireisl-Fremond-Rocca-Schimperna [6], where the non-isothermal model with the penalty term $f(d)$ is considered, and those of Lin-Lin-Wang [17], Hong [18], and Hong-Xin [19], where the isothermal model with term $|\nabla d|^2 d$ is considered, we consider the non-isothermal model, that is, systems (1.1)–(1.4) and (1.8)–(1.11), and we are going to prove the global existence of weak solutions.”

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Adjectives

Original: The numerical experiments show that the algorithm produces very accurate results.

Better: The numerical experiments show that the algorithm produces results with errors at the level of the unit roundoff.

Minimize use of adjectives and adverbs.

See **Cluttered writing: Adjectives and adverbs in academia** by A. Okulicz-Kozaryn (2013).

Revising

- “Begin by cutting out the big stuff. Brevity comes from selection, not compression” (R. P. Clark).
- Editor Maxwell Perkins reduced one four-page passage of Thomas Wolfe’s about his uncle to six words:
‘Henry, the oldest, was now thirty’.
- Wait a few hours or days between writing and revising.
- Revise by hand on a printout?

Recommended Reading on Writing

- Roy Peter Clark, **Writing Tools: 55 Essential Strategies For Every Writer**, Little, Brown and Company, 2016.
- Stephen King, **On Writing Well: A Memoir of the Craft**, xv+367, Hodder and Stoughton, 2000
- William Zinsser, **On Writing Well: An Informal Guide to Writing Nonfiction**, xiii+288, HarperCollins, 1990

Exercise

Original: The editors strongly encourage authors to deposit, in a permanent repository or as supplementary materials, any software and data that allow the results of published articles to be reproduced, and every effort should be made to include sufficient information in manuscripts to enable this.

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Better: Authors should include sufficient information in the manuscript to enable the results of the manuscript to be reproduced. To this end, authors should deposit in a permanent repository, or as supplementary materials, relevant software and data.

Exercise

Original: Combining the previous two inequalities together, we obtain . . .

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Original: The maximal value grows larger for larger m .

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Better: The maximal value grows with m .

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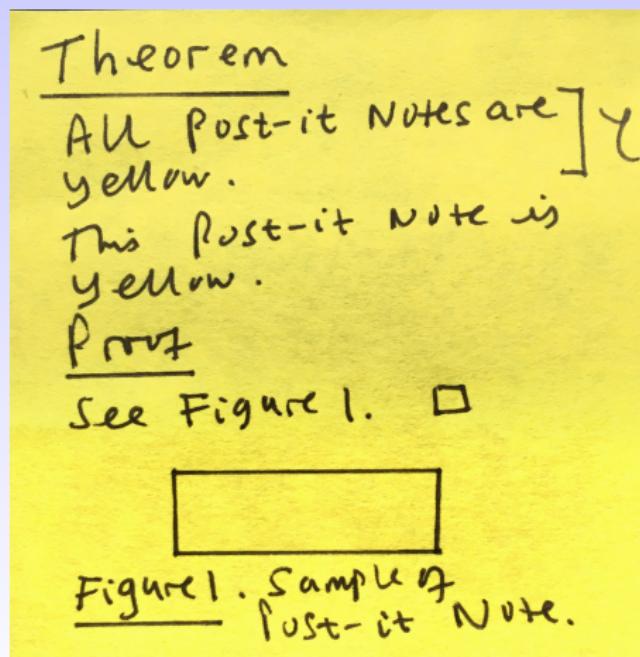
Writing a Paper

Summarize Results in a Theorem

Be precise and do not overstate your claims.

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Fine-Tuning a Theorem

Word as simply and clearly as possible.

Theorem

Let $A \in \mathbb{C}^{n \times n}$ have no eigenvalues on \mathbb{R}^- . Then

$$\operatorname{acos} A + \operatorname{asin} A = \frac{\pi}{2} I.$$

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Dos and Don'ts (1)

Bad:

$$x^2 + bx - c = 0$$

$$(x + b/2)^2 = c + b^2/4$$

$$x + b/2 = \sqrt{c + b^2/4}$$

$$x = -b/2 + \sqrt{c + b^2/4}.$$

Much better: We have

$$0 = x^2 + bx - c = (x + b/2)^2 - c - b^2/4.$$

Rearranging and taking the positive square root gives

$$x = -b/2 + \sqrt{c + b^2/4}.$$

Dos and Don'ts (2)

Watch for notational synonyms.

Original: If $\lambda_i \neq \lambda_j$ for all $i \neq j \dots$

Alternative: If the eigenvalues are distinct \dots

Original:

$$A = X \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} X^{-1}$$

Alternative: $A = X \text{diag}(\lambda_i) X^{-1}$.

Dos and Don'ts (3)

- Punctuate all equations. E.g.

We consider the initial value problem

$$y'(t) = 2y(t), \quad y(0) = 1,$$

over the range $0 \leq t \leq 1$.

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- Don't start a sentence with a symbol.

Bad: A is an ill-conditioned matrix.

Good: The matrix A is ill conditioned.

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over the range $0 \leq t \leq 1$.

- Don't start a sentence with a symbol.

Bad: A is an ill-conditioned matrix.

Good: The matrix A is ill conditioned.

- Separate symbols by punctuation marks or words.

Bad: If $x = 1$ $f(x) = 0$.

Fair: If $x = 1$, $f(x) = 0$.

Good: If $x = 1$ then $f(x) = 0$.

Dos and Don'ts (4)

Don't put “big” equations in-line.

Bad: We find that $f(x) = \frac{1}{1-x} - \frac{1}{1+x}$.

Good: We find that $f(x) = 1/(1-x) - 1/(1+x)$.

Good: We find that

$$f(x) = \frac{1}{1-x} - \frac{1}{1+x}.$$

Example

From a textbook on linear algebra:

Define a square matrix E_{ij} by

$$[E_{i,j}]_{kl} = \begin{cases} 0, & k \neq i, k \neq j, l \neq k, \\ 1, & k \neq i, k \neq j, l = k, \\ 0, & k = i, l \neq j, \\ 1, & k = i, l = j, \\ 0, & k = j, l \neq i, \\ 1, & k = j, l = i. \end{cases}$$

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Better:

Let $E_{i,j}$ be the matrix obtained by swapping the i th and j th rows of the identity matrix.

Notation

Desiderata:

- Clarity
- Conciseness
- Readability
- Aesthetics

Example:

Let $\widehat{H}_k = Q_k^H \widetilde{H}_k Q_k$, partition $X = [X_1, X_2]$, and let $\mathcal{X} = \text{range}(X_1)$. Let U^* denote the nearest orthonormal matrix to X_1 in the 2-norm.

Roman Versus Italic

Compare

$$\frac{d}{dx} f(x), \quad \int_C \frac{e^z}{z} dz = 2\pi i, \quad \frac{d}{dx} f(x), \quad \int_C \frac{e^z}{z} dz = 2\pi i.$$

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- ISO standard for mathematical typesetting (2009) requires mathematical constants whose values do not change to be written in roman.
- Hence `\mathrm{d} x` instead of `dx`.
- L^AT_EX does not have an upright π !

Study Other Writers

Learn by studying the work of writers you admire.

- What makes their papers stand out?
- What vocabulary do they use?
- How do they weave together text and mathematics?
- How do they tell the story?
- What stops their writing from being boring?

Outline

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Active versus Passive

Active: X did Y.

Passive: Y was done by X.

- Active voice is more lively and often more concise.
- Passive voice is more vague and indirect.

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Passive: Estimation of the error was achieved by application of the Peano kernel formula.

Active: We estimated the error using the Peano kernel formula.

Active versus Passive

Active: X did Y.

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- Active voice is more lively and often more concise.
- Passive voice is more vague and indirect.

Passive: Estimation of the error was achieved by application of the Peano kernel formula.

Active: We estimated the error using the Peano kernel formula.

Passive: The specimen was accidentally strained during mounting.

Active: I dropped the specimen on the floor.

Passive Zombies



[←](#) **Tweet**

 **Rebecca Johnson**
@johnsonr

I finally learned how to teach my guys to ID the passive voice. If you can insert "by zombies" after the verb, you have passive voice.

8:26 PM · Oct 18, 2012 · Twitter for iPhone

3.3K Retweets **1.8K Likes**

Zombie Nouns

- Nouns, made from adjectives or verbs, that end in **-ance, -ence, -end, -ity, -ment, -sion, -tion**.
- “*An abstract noun that obscures the agent of the action it denotes*” (Garner 2016).

Original: Minimization of the integral shows that x_* is positive.

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Better: By minimizing the integral we find that x_* is positive.

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Better: By minimizing the integral we find that x_* is positive.

Original: Method A displays much better performance than Method B.

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Better: By minimizing the integral we find that x_* is positive.

Original: Method A displays much better performance than Method B.

Better: Method A performs much better than Method B.

Naked This

Consider the sentence

This raises the question of whether the method always converges.

“This” could refer to

- most recent noun in previous sentence,
- subject of previous sentence,
- idea of previous sentence,
- something else.

Qualify “this” with the appropriate noun: “this result”, “this argument”, . . . to avoid ambiguity.

Even better: **rewrite** to avoid the overused “this” completely.

Watch Out for These Mistakes

- supersede (*incorrect*: supercede)
- loose instead of lose
- MATLAB (*incorrect*: Matlab).
- Do not write put apostrophe on trademark.
Bad: MATLAB's `expm` function.
Good: The MATLAB `expm` function.
- **Its** is a possessive ("its solution"),
It's is a contraction of **it is** ("it's clear that").

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1. INTRODUCTION. In 1966 Kac [7] asked the question “Can you hear the shape of a drum?”, that is, if you know the frequencies at which a drum vibrates, can you determine its shape? Mathematically this corresponds to the following problem. If u is the displacement of a membrane D from its mean position, then u satisfies

For Non-Native Writers (1)

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Right: For small θ , Assumption 1.2 is no longer satisfied.
- **Bad:** We remind that f is discontinuous.
Right: We remind the reader that f is discontinuous.
Right: We recall that f is discontinuous.

For Non-Native Writers (2)

- See Laurette Tuckerman's **Anglais pour scientifiques français** (English for French scientists).
- See 15-page chapter *When English Is a Foreign Language* in **Handbook of Writing**.
- Read good expository technical English writing, e.g. SIAM News, Scientific American.

For Non-Native Writers (3)

Take a look at this book, available [here](#).

Leonardo da Vinci programme
European Commission

Writing in English

A Practical Handbook for Scientific and Technical
Writers

A Pilot Project

Project Partners

Zuzana Svobodova, Technical University Brno, Czech Republic
Heidrun Katzorke and Ursula Jaekel, Technische Universität Chemnitz, Germany
Stefania Dugovicova and Mike Scoggin, Comenius University, Bratislava, Slovakia
Peter Treacher, ELT Centre, University of Essex, England

Lively Writing (1)

Try *not* to begin a sentence with

- It is ...
- It can be ...
- There is ...
- There are ...
- Also ...

Never use the word “thing”—be more specific.

Lively Writing (2)

Bad: It can be seen in Table 4 that the residual is monotonically decreasing.

Good: Table 4 shows that the residual decreases monotonically.

Lively Writing (2)

Bad: It can be seen in Table 4 that the residual is monotonically decreasing.

Good: Table 4 shows that the residual decreases monotonically.

Bad: There are several possibilities that can be used to break ties.

Good: Ties can be broken in several ways.

Outline

Writing

Mathematical Writing

English Usage

Workflow

Writing a Paper

Workflow

Tools, techniques for whole research process, esp. typesetting, numerical experiments.

See

- chapter **Workflow** in **Handbook of Writing for the Mathematical Sciences, 3rd ed**,
- chapter **Workflow** in **The Princeton Companion to Applied Mathematics**.

Overleaf

LATEX in a browser and collaborative writing

<https://www.overleaf.com>.

- ✓ No need to install LATEX.
- ✓ Continuously compiles and previews.
- ✓ Editor has completion.
- ✓ Git integration.
- ✓ Lots of templates.
- ✓ Collaboration (one collaborator in free version).
- ✗ Can go down for maintenance or otherwise.
- ✗ Does not have latest TEX Live.

Plain Text is King

- **Maximize** use of plain text.
- **Minimize** use of proprietary formats such as .doc, .docx, .xls, .xlsx.
- For writing can use markup languages:
 - **Markdown ([cheatsheet](#))**.
 - Emacs **Org mode ([cheatsheet](#))**.
 - **L^AT_EX**.

Pandoc (1)

Tool to convert files between different formats, including from Markdown and Org to L^AT_EX, Word (docx), or html.

Pandoc a universal document converter PayPal - The safer, easier way to pay online!

[About](#)

[Installing](#)

[Getting started](#)

[Demos](#) ▾

[Documentation](#) ▾

[Help](#)

[Extras](#)

[Releases](#)

About pandoc

If you need to convert files from one markup format into another, pandoc is your swiss-army knife. Pandoc can convert between the following formats:

(← = conversion from; → = conversion to; ↔ = conversion from and to)

Lightweight markup formats

- ↔ Markdown (including CommonMark and GitHub-flavored Markdown)
- ↔ reStructuredText
- AsciiDoc
- ↔ Emacs Org-Mode
- ↔ Emacs Muse
- Textile
- ↔ txt2tags

HTML formats

- ↔ (X)HTML 4
- ↔ HTML5

Word processor formats

- ↔ Microsoft Word docx
- ↔ OpenOffice/LibreOffice ODT
- OpenDocument XML
- Microsoft PowerPoint

Interactive notebook formats

- ↔ Jupyter notebook (ipynb)

Page layout formats

- InDesign ICML

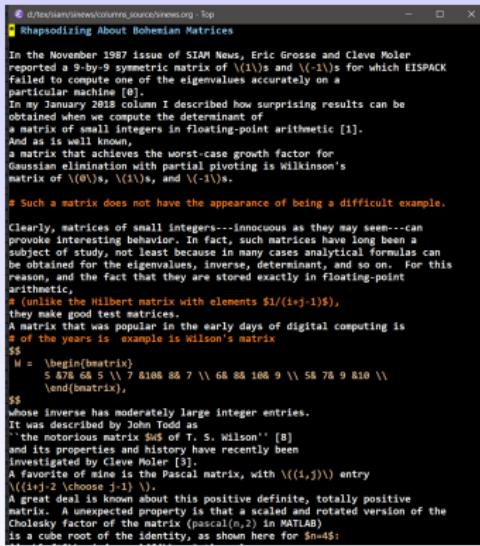
Wiki markup formats

- ↔ MediaWiki markup

Pandoc

PDF and Word output can be produced with

```
pandoc -s org_example.org -o org_example_pandoc.pdf
pandoc -s org_example.org -o org_example_pandoc.docx
```

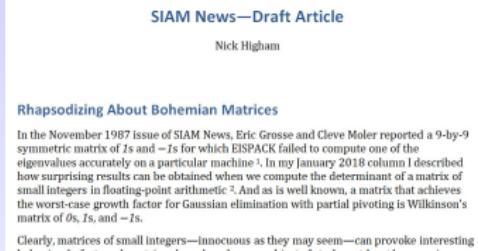


```
d:\test\mian\view\columns_sources\news.org -Top
# Rhapsodizing About Bohemian Matrices

In the November 1987 issue of SIAM News, Eric Grosse and Cleve Moler reported a 9-by-9 symmetric matrix of  $\sqrt{1}$ 's and  $-\sqrt{1}$ 's for which EISPACK failed to compute one of the eigenvalues accurately on a particular machine [8]. In my January 2018 column I described how surprising results can be obtained when we compute the determinant of a matrix of small integers in floating-point arithmetic [1]. And as is well known, a matrix that achieves the worst-case growth factor for Gaussian elimination with partial pivoting is Wilkinson's matrix of  $\sqrt{0}$ 's,  $\sqrt{1}$ 's, and  $\sqrt{-1}$ 's. # Such a matrix does not have the appearance of being a difficult example. Clearly, matrices of small integers—innocuous as they may seem—can provoke interesting behavior. In fact, such matrices have long been a subject of study, not least because in many cases analytical formulas can be obtained for the eigenvalues, inverse, determinant, and so on. For this reason, and the fact that they are stored exactly in floating-point arithmetic, # (unlike the Hilbert matrix with elements  $1/(i+j-1)$ ), they make good test matrices. A matrix that was popular in the early days of digital computing is # of the years is example is Wilson's matrix
$#
M = \begin{pmatrix} 5 & 7 & 6 & 5 \\ 7 & 10 & 8 & 7 \\ 6 & 8 & 10 & 9 \\ 5 & 7 & 9 & 10 \end{pmatrix},
$#
whose inverse has moderately large integer entries. It was described by John Todd as "the notorious matrix  $\mathbb{M}$  of T. S. Wilson" [8] and its properties and history have recently been investigated by Cleve Moler [3]. A favorite of mine is the Pascal matrix, with  $\binom{(i,j)}{j}$  entry  $\binom{(i+j-2)}{j-1}$ . A great deal is known about this positive definite, totally positive matrix. An unexpected property is that a scaled and rotated version of the Cholesky factor of the matrix (pascal(n,2) in MATLAB) is a cube root of the identity, as shown here for  $n=4$ :
```

SIAM News—Draft Article

Nick Higham



Rhapsodizing About Bohemian Matrices

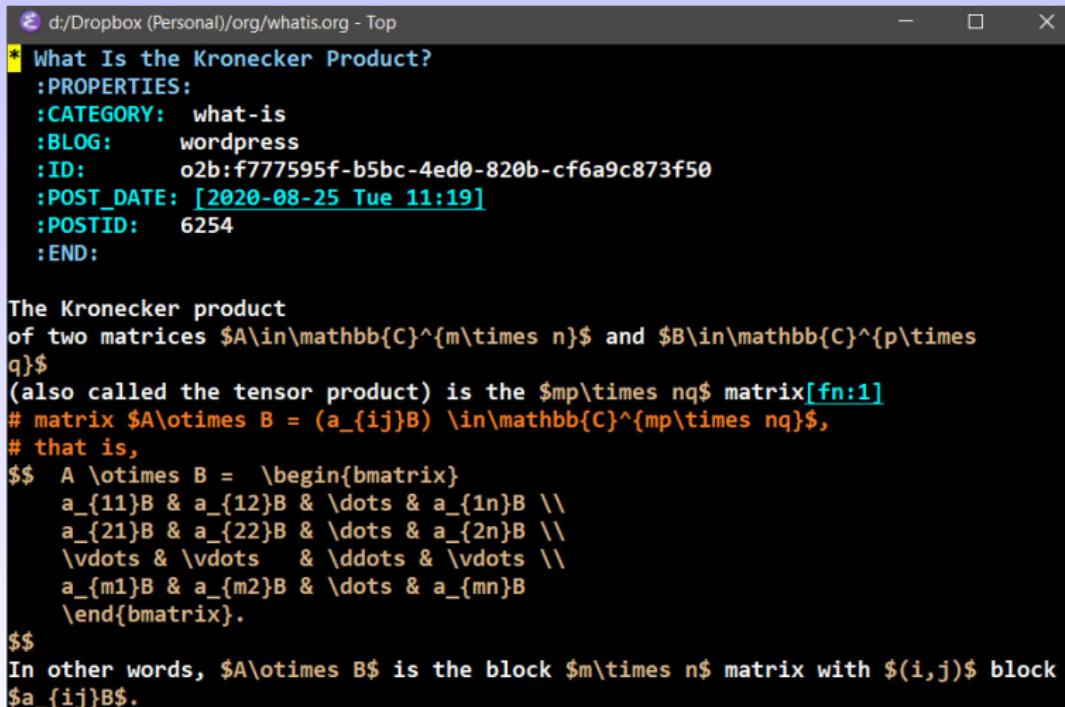
In the November 1987 issue of SIAM News, Eric Grosse and Cleve Moler reported a 9-by-9 symmetric matrix of $\sqrt{1}$ s and $-\sqrt{1}$ s for which EISPACK failed to compute one of the eigenvalues accurately on a particular machine¹. In my January 2018 column I described how surprising results can be obtained when we compute the determinant of a matrix of small integers in floating-point arithmetic². And as is well known, a matrix that achieves the worst-case growth factor for Gaussian elimination with partial pivoting is Wilkinson's matrix of 0s, 1s, and $-\sqrt{1}$ s.

Clearly, matrices of small integers—innocuous as they may seem—can provoke interesting behavior. In fact, such matrices have long been a subject of study, not least because in many cases analytical formulas can be obtained for the eigenvalues, inverse, determinant, and so on. For this reason, and the fact that they are stored exactly in floating-point arithmetic, they make good test matrices. A matrix that was popular in the early days of digital computing is

$$\mathbb{M} = \begin{bmatrix} 5 & 7 & 6 & 5 \\ 7 & 10 & 8 & 7 \\ 6 & 8 & 10 & 9 \\ 5 & 7 & 9 & 10 \end{bmatrix},$$

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Emacs Org Source



The screenshot shows a window titled "d:/Dropbox (Personal)/org/whatis.org - Top" containing Org mode source code. The code defines a document structure and content related to the Kronecker product.

```
* What Is the Kronecker Product?
:PROPERTIES:
:CATEGORY: what-is
:BLOG: wordpress
:ID: o2b:f777595f-b5bc-4ed0-820b-cf6a9c873f50
:POST_DATE: [2020-08-25 Tue 11:19]
:POSTID: 6254
:END:

The Kronecker product
of two matrices $A \in \mathbb{C}^{m \times n}$ and $B \in \mathbb{C}^{p \times q}$ 
(also called the tensor product) is the $mp \times np$ matrix[fn:1]
# matrix $A \otimes B = (a_{ij}B) \in \mathbb{C}^{mp \times np}$,
# that is,
$$ A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{bmatrix}.
$$
In other words, $A \otimes B$ is the block $m \times n$ matrix with $(i,j)$ block $a_{ij}B$.
```

PDF Output

What is the Kronecker Product?

Nicholas J. Higham*

August 25, 2020

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$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{bmatrix}.$$

In other words, $A \otimes B$ is the block $m \times n$ matrix with (i, j) block $a_{ij}B$. For example,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} & a_{12}b_{11} & a_{12}b_{12} & a_{12}b_{13} \\ a_{11}b_{21} & a_{11}b_{22} & a_{11}b_{23} & a_{12}b_{21} & a_{12}b_{22} & a_{12}b_{23} \\ a_{21}b_{11} & a_{21}b_{12} & a_{21}b_{13} & a_{22}b_{11} & a_{22}b_{12} & a_{22}b_{13} \\ a_{21}b_{21} & a_{21}b_{22} & a_{21}b_{23} & a_{22}b_{21} & a_{22}b_{22} & a_{22}b_{23} \end{bmatrix}.$$

Notice that the entries of $A \otimes B$ comprise every possible product $a_{ij}b_{rs}$, which is not the case for the usual matrix product AB when it is defined. Indeed if A and B are $n \times n$

Blog Output

Nick Higham

Applied mathematics, software and workflow.



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← What Is the Gerstenhaber Problem?

What Is the Sylvester Equation? →

What is the Kronecker Product?

Posted on [August 25, 2020](#)

The Kronecker product of two matrices $A \in \mathbb{C}^{m \times n}$ and $B \in \mathbb{C}^{p \times q}$ (also called the tensor product) is the $mp \times nq$ matrix⁴

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{bmatrix}.$$

In other words, $A \otimes B$ is the block $m \times n$ matrix with (i, j) block $a_{ij}B$. For example,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} & a_{12}b_{11} & a_{12}b_{12} & a_{12}b_{13} \\ a_{11}b_{21} & a_{11}b_{22} & a_{11}b_{23} & a_{12}b_{21} & a_{12}b_{22} & a_{12}b_{23} \\ a_{21}b_{11} & a_{21}b_{12} & a_{21}b_{13} & a_{22}b_{11} & a_{22}b_{12} & a_{22}b_{13} \\ a_{21}b_{21} & a_{21}b_{22} & a_{21}b_{23} & a_{22}b_{21} & a_{22}b_{22} & a_{22}b_{23} \\ a_{31}b_{11} & a_{31}b_{12} & a_{31}b_{13} & a_{32}b_{11} & a_{32}b_{12} & a_{32}b_{13} \\ a_{31}b_{21} & a_{31}b_{22} & a_{31}b_{23} & a_{32}b_{21} & a_{32}b_{22} & a_{32}b_{23} \end{bmatrix}.$$

Nick Higham

How to Write Scientific Papers

HTML Output

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Notice that the entries of $A \otimes B$ comprise every possible product $a_{ij}b_{rs}$, which is not the case for the usual matrix product AB when it is defined. Indeed if A and B are $n \times n$ then

Examples of Useful L^AT_EX Macros

```
\def\alg{algorithm}
\def\nbyn{n\times n}
\def\DS{\displaystyle}
\def\py{polynomial}
\def\resp{respectively}
\def\sq{square root}
\def\alpha{\alpha}
```

- Use `\newcommand` instead of `\def`.
- Do not overwrite existing T_EX macros:

```
\def\l{\lambda}
```

LATEX (1)

- Start new paragraph by leaving blank line, *not* by using `\\"`.
But don't leave blank line after displayed equation or other environment except to start new paragraph.
- Always *type mathematics in math mode* (inside \$ signs), to produce “ $y = f(x)$ ” instead of “y = f(x)”, and “the dimension n ” instead of “the dimension n”.
- *Mathematical functions should be in roman font*. For standard functions, type as `\sin`, `\exp`, `\max`, etc.

To define your own:

```
\usepackage{amsmath}
```

```
...
```

```
\DeclareMathOperator{\diag}{diag}
```

LATEX (2)

Bad:

```
$$U(\bar{z}) = U(-z) =  
\begin{cases} -U(z), & z \in D, \\\  
-U(z)-1, & \text{otherwise}. \\ \end{cases}  
\end{cases}$$
```

Good:

```
$$  
U(\bar{z}) = U(-z) =  
    \begin{cases} -U(z), & z \in D, \\\  
-U(z)-1, & \text{otherwise}. \\ \end{cases}  
\end{cases}$$
```

LATEX Tips

See my LATEX Tips document in my GitHub repository

<https://github.com/higham/latex-tips>

The document also exists in the form of one of my

LATEX blog posts 

Mathpix Snipping Tool

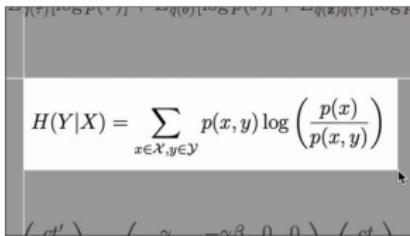
<https://mathpix.com/>

DESKTOP

Go from PDF to LaTeX document in 2 steps

1 Start
by clicking the screenshot button

Or use the keyboard shortcut:
Ctrl + ⌘ + M on Mac
Ctrl + Alt + M on Windows & Linux



2 Paste
the LaTeX into any compatible editor

After you've selected your desired format in the result page, just paste into your document!



```
# Snip demo
Simply paste the LaTeX already copied to your clipboard.

You can use any editor that supports LaTeX syntax!
$ 
$ p(x, y) | \mathbb{P}(x, y)
\mathbb{P}(x, y) | \ln \frac{p(x)}{p(x, y)}
\frac{p(x)}{p(x, y)} | \right)
$ 
$
```

$$H(Y|X) = \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} p(x, y) \log \left(\frac{p(x)}{p(x, y)} \right)$$

Outline

Writing

Mathematical Writing

English Usage

Workflow

Writing a Paper

Abstract, Introduction, Conclusions

Abstract

- Summarize the contents of the paper.
- Help reader decide whether to read the whole paper.
- Write it from scratch.

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Introduction

- Introduce problem, summarize the results (usually).
- Keep short and get quickly to the point.
- Write it last.

Abstract, Introduction, Conclusions

Abstract

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Introduction

- Introduce problem, summarize the results (usually).
- Keep short and get quickly to the point.
- Write it last.

Conclusions

- Is it necessary?
- Point out limitations and further work.
- Write in past tense.

Exercise

Original:

Abstract. We consider the $n \times n$ Hankel matrix H whose entries are defined by $H_{ij} = 1/s_{i+j}$, where $s_k = (k - 1)!$ and prove that H is invertible for all $n \in \mathbb{N}$ by providing an explicit formula for its inverse matrix.

Exercise

Original:

Abstract. We consider the $n \times n$ Hankel matrix H whose entries are defined by $H_{ij} = 1/s_{i+j}$, where $s_k = (k - 1)!$ and prove that H is invertible for all $n \in \mathbb{N}$ by providing an explicit formula for its inverse matrix.

Better:

Abstract. We derive an explicit formula for inverse of the $n \times n$ Hankel matrix whose (i, j) element is $1/(i + j - 1)!$.

Table or Graph?

- Table better for small sets of numbers.
- Graph better for large data sets & to show trends.
- Tables:
 - Keep simple.
 - Minimum number of rules.
 - Like quantities to be compared better in columns than rows.
- Graphs:
 - Ensure readable.
 - Iterate on design.

See **Ten Simple Rules for Better Figures** (2014).

Table (original)

n	Mflops			
	128	256	512	1024
Method 1 ($n_b = 1$)	95	162	231	283
Method 2 ($n_b = 1$)	114	211	289	330
k variant ($n_b = 1$)	114	157	178	191
Method 1B ($n_b = 64$)	125	246	348	405
Method 2C ($n_b = 64$)	129	269	378	428
k variant ($n_b = 64$)	148	263	344	383

Table (with **booktabs**)

		<i>n</i>			
		128	256	512	1024
Unblocked:	Method 1	95	162	231	283
	Method 2	114	211	289	330
	<i>k</i> variant	114	157	178	191
Blocked: $(n_b = 64)$	Method 1B	125	246	348	405
	Method 2C	129	269	378	428
	<i>k</i> variant	148	263	344	383

Indexing

Index your book or thesis using L^AT_EX and MakeIndex.

- Doing so will reveal gaps and inconsistencies,
- It's fun to do.
- Very useful for examiners.
- Allow plenty of time.
- Don't start indexing too early or too late.

SIAM NEWS BLOG

Programs and Publications | November 26, 2014

Print

A Call for Better Indexes

By Nicholas Higham

 An index is an important component of a book. It provides a view with a much smaller granularity than a table of contents, reveals what is not present as well as what is, and by abstracting concepts can lead the reader to unexpected content.

Most academic books in mathematics are typeset in L^AT_EX, which has an excellent system for indexing. By inserting `\index` commands in the source code, and running the `MakeIndex` program as part of the L^AT_EX sequence, an author can iteratively build up an index during the late stages of the writing process, safe in the knowledge that the automatically generated page locators will be correct.

Indices. Photo credit: Lentigradphoto.

b

Strabić (2016)

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- pencil without a common left kernel, *see* matrix, pencil, without a common left kernel
- permuted Lagrangian graph
 - representation, *see* Lagrangian subspace, Riccati representation, permuted
- permuted Riccati matrix, *see* matrix, Riccati, permuted
- permuted Riccati representation
 - (basis), *see* Lagrangian subspace, Riccati representation, permuted
- principal pivot transform (PPT), *see* symmetric principal pivot transform (symmetric PPT), definition

Proofreading Test (1)

- One of the best known methods for determining all the eigenvalues and eigenvectors of (2.1) was developed by the nineteenth century mathematician Jacobi.

Proofreading Test (1)

- One of the best unknown methods for determining all the eigenvalues and eigenvectors of (2.1) was developed by the nineteenth century mathematician Jacobi.
- It would be hard to underestimate the importance of optimization to scientific computing.

Proofreading Test (2)

- **Acknowledgements.** I thank the anonymous referees, particularly Dr. J. R. Ockendon, for numerous suggestions and for the source of references.

Proofreading Test (2)

- **Acknowledgements.** I thank the anonymous referees, particularly Dr. J. R. Ockendon, for numerous suggestions and for the source of references.
- The ability of physics to explain similar phenomena over scales of centimeters to 10^{18} centimeters is why it is such a powerful science.

Proofreading Test (2)

- **Acknowledgements.** I thank the anonymous referees, particularly Dr. J. R. Ockendon, for numerous suggestions and for the source of references.
- The ability of physics to explain similar phenomena over scales of centimeters to 10^{18} centimeters is why it is such a powerful science.
- All these approaches lead to nontrivial loss of accuracy when larger CNN models were trained for ILSVRC classification tasks (Zhou et al. (2016)).

Proofreading Test (3)

the balanced oscillator gain function defined by

$$k(z) \equiv \frac{2}{z} \int_0^1 \tanh(9z) \sin(2\pi t) \sin(2\pi t) dt \quad (17.2)$$

is not a canonical special function either, but Boyd and Visser have derived power se-

Proofreading Test (3)

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Should be

$$k(z) \equiv \frac{2}{z} \int_0^1 \tanh(z \sin(2\pi t)) \sin(2\pi t) dt.$$

Proofreading Test (4)

WITH chancellor George Osborne pinning his hopes on private investment in infrastructure, the call from the West Midlands transport authority to nationalise the M6 toll road, overpriced and merely forcing traffic on to the congested main M6, could hardly have come at a worse time.

The toll road, completed under the private finance initiative ten years ago, epitomises most of what is wrong with PFI. The high tolls that render it a failure are demanded by sky-high interest payments that the operator, Midland Expressway Ltd, pays at rates up to 12 percent. Through a series of holding companies, these end up with a parent company in Bermuda that is managed by the Australian Macquarie group.

Levied on borrowings of more than £700m, the interest wipes out all taxable profits so, contrary to the assumptions made when PFI contracts are agreed, this one makes no corporate tax contribution at all. It's not so much the "Birmingham Northern relief road" promised as a tax relief road to Bermuda.

Proofreading Test (4)

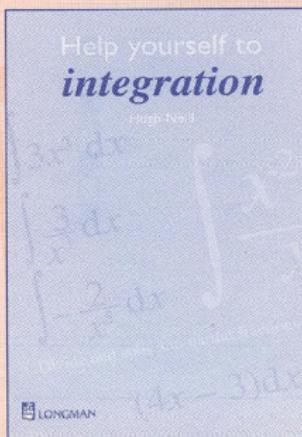
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In Conclusion

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Three things to remember:

- Avoid zombie nouns.
- Revise to shorten, minimize adjective and adverbs, and simplify notation.
- Exploit plain text (pandoc, markdown, Org mode, . . .).

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