



The mechanisms of labor division from the perspective of individual optimization

Lirong Zhu^a, Jiawei Chen^{a,b,*}, Zengru Di^a, Liujun Chen^a, Yan Liu^a,
H. Eugene Stanley^b

^a School of Systems Science, Beijing Normal University, Beijing, 100875, PR China

^b Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA

HIGHLIGHTS

- We present two evolutionary models on division of labor.
- The models are established from the perspective of individual optimization.
- We introduce information cost to the model.
- Some self-organizing mechanisms were found.

ARTICLE INFO

Article history:

Received 3 May 2017

Available online 16 July 2017

Keywords:

Division of labor

Multi-agent systems

Learning by doing

Master equation

ABSTRACT

Although the tools of complexity research have been applied to the phenomenon of labor division, its underlying mechanisms are still unclear. Researchers have used evolutionary models to study labor division in terms of global optimization, but focusing on individual optimization is a more realistic, real-world approach. We do this by first developing a multi-agent model that takes into account information-sharing and learning-by-doing and by using simulations to demonstrate the emergence of labor division. We then use a master equation method and find that the computational results are consistent with the results of the simulation. Finally we find that the core underlying mechanisms that cause labor division are learning-by-doing, information cost, and random fluctuation.

© 2017 Elsevier B.V. All rights reserved.

1. Introduction

The division of labor in real-world settings is ubiquitous. In a bee hive the worker bees work and the queen bee reproduces [1]. In an ant colony queens and males reproduce, workers build and expand the colony and gather and store food, and soldiers protect the colony [2,3]. The labor division in different groups varies, but in each case specialization enables the group to reproduce and survive [4]. Labor division is also pervasive in human society. Employees in an organization divide the work to be done according to function. The use of the division of labor by scientists carrying out research allows an increase in the level of specialization, reduces cost, and improves overall efficiency.

Most current research on the division of labor offers an experimental interpretation of the phenomenon as found in biological systems. Robinson found that labor division among insects is predominantly agent specialization and the plasticity of agent behavior [5]. Simola found that the genetic regulation of animal behavior plasticity is strongly affected by histone

* Corresponding author.

E-mail address: chenjiawei@bnu.edu.cn (J. Chen).

modification [6]. Lattorff studied labor division in insects from the perspective of genome sequencing [7]. Little research has focused on the self-organizing mechanism underlying the division of labor.

Duarte examined neural networks to study self-organization in labor division [8]. A model was used that focuses on worker specialization and the ratio of work performed by each task. Although the goal was to maximize network fitness by reducing idleness and achieving an optimal work ratio, the complexity of the neural network structure affects the degree of agent specialization, and its evolution mechanism remains unclear. Nakahashi used a mathematical model to investigate the effect of group size, resource sharing, and task allocation on labor division [9]. The model considers the impact of endogenous factors on labor division, but not the impact of the natural environment. Wu analyzed the emergence of labor division in a multi-agent system by the method of statistical physics [10]. Di et al. uses “Agent-based model” [11,12] to examine the genetic process of the survival of the fittest and finds that an increase in global income encourages the division of labor [13]. The model maximizes the global benefit and adjusts agent behavior according to global income rather than agent income. This multi-agent simulation contradicts the rational agent hypothesis in economics and is a bottom-up method that effectively explores system self-organization [14,15]. Agent-based model has been widely used in the fields of economy, finance, game and so on [16–19].

Using the complex adaptive system and fitness landscape theory [20], we here use an evolutionary model to understand the general principle of labor division. Using the Chai model as a basis [21], we expand it to include the information cost of transactions. We also introduce a feedback mechanism [22] to strengthen agent learning ability. From the perspective of the agent’s own income optimization, the community realizes labor division. At the same time, adjusting the information fee ratio changes agent structure and the total income of all agents.

Using multi-agent simulation we can uncover the evolution mechanism from the view of self-organization. We find that random fluctuations make the process of evolution unstable, and we use a master equation [23] to describe the evolution process mathematically. We describe the mechanism of interaction between agents and their environment quantitatively. The simulation results are consistent with the theoretical predictions.

The second section of the paper presents the multi-agent model of labor division and the simulation results. The third section analyzes the model using a master equation. The final section presents our conclusions.

2. Multi-agent model and the mechanisms of labor division

2.1. Environment and agents

Resources are distributed in a closed environment that consists of $L \times L$ lattices where each lattice is \vec{r} . Renewable resources in the environment are randomly distributed in the lattices and do not exceed the maximum capacity of any lattice. The resource of each lattice is $S_{\vec{r}}$ and the maximum capacity of the lattice is $M(\vec{r})$. N agents are randomly distributed in the lattices and each lattice can only contain one agent.

2.2. The behavior of agents

2.2.1. The decision-making of the agents

Agents make two kinds of decision: those pertaining to finding resources and those to exploiting resources. Here q_i is the decision-making of agent i and q_i as one of the values 0, 0.1, 0.2, ..., 0.9, 1. When q_i tends toward 1, agent i tends to find resources. Agent i marks the lattices in the visual field using the agent’s ID i and this provides resource information to all other agents whether or not a new lattice is marked by an agent. When other agents exploit resources in a lattice marked by agent i , they provide information fees to agent i . When q_i tends toward 0, agent i tends to exploit resources, i.e., agent i moves to a lattice where a maximum income can be obtained and resources exploited, but other agents pay the information fee to the individual who marked the lattice.

2.2.2. Learn-by-doing

Agents are more proficient in functions they repeatedly carry out, which is learning-by-doing [24,25] economics. Thus an agent adept at exploiting resources becomes increasingly efficient and the ability of an agent to discover new resources constantly grows. The mathematical model for learning-by-doing uses efficiency of exploitation $b(q_i)$ and visual field $v(q_i)$, the function of which is

$$b(q_i) = \frac{2}{1 + \exp(6q_i)} \quad (1)$$

and

$$v(q_i) = \frac{10}{1 + \exp(-7q_i + 7)}. \quad (2)$$

2.3. Evolution of agent decision-making

The evolution process of agent decision-making has M generations and each generation contains T moments. After the evolution of decision-making at moment t , the positions of agents and resources in the closed environment are updated. At the end of each generation, the decision-making of the agent is updated. The location of agent i is $L_i(m, t)$ and the lattice \vec{r} is marked as $F_{\vec{r}}(m, t)$. Initially the location of individual i is $L_i(1, 0)$ and the lattice is marked $F_{\vec{r}}(m, 0) = 0$. The evolutionary process of agent decision-making is as follows.

(1) In m generation at t moment, agents must make a decision:

When i finds resources, they randomly move to a lattice that contains no other agent and mark the unmarked lattice in the visual field. The new location of the agent and the process of marking the lattice in their visual field are

$$L_i(m, t) = L_i(m, t - 1) + \delta \quad (3)$$

and

$$F_{\vec{r}}(m, t) = \{i | |\vec{r} - L_i(m, t)| < v(q_i)\}, \quad (4)$$

where δ is the distance of the agent's random move.

When agent i exploits resources, they move to the lattice \vec{r}^* marked to obtain the maximum income to exploit those resources, and they pay the information fees to the agent who marked the lattice. The \vec{r}^* is

$$\vec{r}^* = \left\{ \vec{r} \mid \max_{F_{\vec{r}}(m, t) \neq 0} \begin{cases} b(q_i) \cdot S_{\vec{r}}(m, t) - c \cdot b(q_i) \cdot S_{\vec{r}}(m, t), & F_{\vec{r}}(m, t) \neq i \\ b(q_i) \cdot S_{\vec{r}}(m, t), & F_{\vec{r}}(m, t) = i \end{cases} \right\} \quad (5)$$

where c is the information fee ratio the agent exploiting the resource pays as a percentage of their income to the agent who marked the lattice, $S_{\vec{r}}(m, t)$ is the amount of resource in lattice \vec{r}^* in m generation at t moment. When agent i moves to lattice \vec{r}^* , the location of agent i is updated to $L_i(m, t) = \vec{r}^*$. The reduction in the exploited lattice resources is $E_{\vec{r}}(m, t)$

$$E_{\vec{r}}(m, t) = S_{\vec{r}}(m, t) \cdot b(q_i). \quad (6)$$

At the end of generation m at moment t , the system calculates the income of each agent. If i has searched for resources, their net income is now

$$\pi_i(m, t) = \left\{ \sum_{j=1, j \neq i}^N c \cdot b(q_j) \cdot S_{\vec{r}}(m, t) | L_j(m, t) = \vec{r}, F_{\vec{r}}(m, t) = i \right\}. \quad (7)$$

If agent i has exploited resources, their net income is now

$$\pi_i(m, t) = \begin{cases} b(q_i) \cdot (1 - c) \cdot S_{L_i(m, t)}(m, t) + \left\{ \sum_{j=1, j \neq i}^N c \cdot b(q_j) \cdot S_{\vec{r}}(m, t) | L_j(m, t) = \vec{r} \right\}, & F_{\vec{r}}(m, t) \neq i \\ b(q_i) \cdot S_{L_i(m, t)}(m, t) + \left\{ \sum_{j=1, j \neq i}^N c \cdot b(q_j) \cdot S_{\vec{r}}(m, t) | L_j(m, t) = \vec{r} \right\}, & F_{\vec{r}}(m, t) = i. \end{cases} \quad (8)$$

At the end of generation m at moment t , the resources in the environment grow according to a logistic curve, and the growth function is

$$S_{\vec{r}}(m, t + 1) = S_{\vec{r}}(m, t) + a \cdot S_{\vec{r}}(m, t) \cdot \left(1 - \frac{S_{\vec{r}}(m, t)}{M(\vec{r})} \right) - E_{\vec{r}}(m, t) \quad (9)$$

where a is the growth rate of the resource.

(2) When running T moments, the evolution of generation m is complete and each agent's decision-making is adjusted according to the total income in the m generation. Here the total income function of the agent in generation m is

$$\pi_i(m) = \sum_{t=1}^T \pi_i(m, t). \quad (10)$$

At the end of generation m , the agent's decision-making is updated and agent income strengthened. When an agent's decision-making strategy increases their income, they continue the strategy. We do not allow agent decision-making to be less than 0 or more than 1. If a strategy makes agent decision-making drop below 0, we adjust it to 0, and if it is more than 1, we adjust it to 1. The agent's decision-making strategy is

$$q_i(m + 1) = \begin{cases} q_i(m) + 2 \cdot (q_i(m) - q_i(m - 1)), & \pi_i(m) \geq \pi_i(m - 1) \\ q_i(m) - 2 \cdot (q_i(m) - q_i(m - 1)), & \pi_i(m) < \pi_i(m - 1). \end{cases} \quad (11)$$

Following the first generation of experiments, agent decision-making changes randomly. Random number X satisfies a uniform distribution. The updating of the agent decision-making strategy is

$$q_i(1) = \begin{cases} q_i(0) + 0.1, & q_i(0) \geq X, X \sim U(0, 1) \\ q_i(0) - 0.1, & q_i(0) < X, X \sim U(0, 1). \end{cases} \quad (12)$$

In the next generation of experiments, environmental resources are $S_r(m+1, 1) = S_r(m, T)$. Environmental resources are again marked by agents.

(3) Agent decision-making repeats for M generations in accordance with the above evolutionary rule.

2.4. The degree of labor division

In the simulation, if agent decision-making q is closer to 0, agent ability to exploit resources is stronger. If agent decision-making q is closer to 1, agent ability to find resources is stronger. We measure the degree of labor division using index η , where

$$\eta = \left(\sum_{q_i > 0.5} q_i + \sum_{q_i \leq 0.5} (1 - q_i) \right) / N. \quad (13)$$

2.5. Evolution results

Here the number of agents is $N = 30$, the growth rate of the resources $a = 0.3$, and the information fee ratio $c = 0.2$. The evolution process has $M = 200$ generations, and each generation m contains $T = 150$ moments. After the agent decision-making parameters evolve through 200 generations under different initial values, we get the final distribution shown in Fig. 1.

Fig. 1 shows that there are an increasing number of agents with decision-making that is close to 0 or 1 in the matrix color map showing the evolution. Different initial conditions generate labor divisions at the end of the evolution. The initial state of decision-making of the agent does not affect the creation of labor division. The randomness in the simulation process results in ultimate labor divisions that are not the same but that exhibit the same trend, i.e., that there is always a clear labor division.

2.6. The influence of information fee ratio

The information fee ratio is between 0 and 1 at intervals of 0.1. We assume that agent decision-making parameter q is randomly distributed. We repeat the experiment 10 times and calculate the index η of the degree of labor division—the average value of the total income for the 10 trials. The results are shown in Fig. 2.

The information fee ratio directly affects agent net income and also the degree of labor division and total revenue. When the information fee ratio is 0.2, the degree of labor division among the agents is the highest, and the total revenue of the system is the largest.

When the information fee ratio is smaller, because of reduced income fewer agents find resources. More agents exploit resources and discover that resources are limited, and this causes the income of agents exploiting resources to decline. Because of the increase in income when the information fee ratio is larger, agents find resources, the discovered resources are not exploited, and the income of all agents declines. Only when the information fee ratio ensures that the income of the agent who exploits resources balances the one who finds resources will the system will achieve a maximum of labor division and an optimization of total revenue.

2.7. Total revenue of the group

The decision-making parameters are randomly initialized and the information fee ratio is $c = 0.2$. Fig. 3 shows how the agent income optimization mechanism affects the total revenue.

We use a moving average method to deal with total revenue and find that agent optimization behavior causes the emergence of labor division and increases the total revenue of the group as it evolves. Because the simulation includes randomness, the total revenue fluctuates.

3. Master equation model

We use a mathematical model to explore and verify the results of the above simulation. We assume that the closed environment is composed of $n \times n$ discrete lattices in which all resources are evenly distributed, and the lattice unit resources are S_0 . The decision-making variable of group j is q_j , and agents with the same decision-making are homogeneous. The space of decision-making variables is divided into K , and P_j is the probability of decision j . Using the optimization of agent income, we establish a discrete non-autonomous dynamic model.

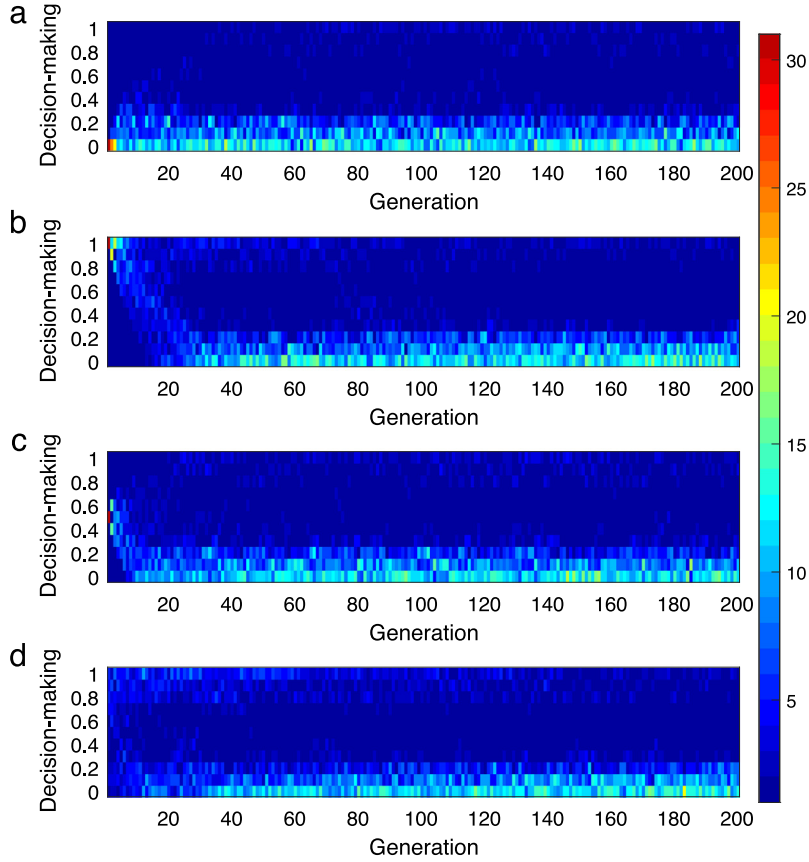


Fig. 1. The horizontal axis represents the simulation generation and the vertical axis represents agent' decision-making. The color bar represents the number of agents in different generations and different decision-making. Closer to the red, shows a greater number of agents. Before evolution, agent' decision-making has four different states: (a) $q = 0$; (b) $q = 1$; (c) $q = 0.5$; and (d) q is random distribution. Different initial conditions generate labor divisions at the end of the evolution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.1. Master equation

There are T evolution generations. In the evolution process between two generations, the agent decision-making variables will evolve toward maximum agent net income. In addition, the current decision-making state can only change to adjacent states, and the probability of state j is

$$P_j(t+1) = P_j(t) + w_t(j+1 \rightarrow j) \cdot P_{j+1}(t) + w_t(j-1 \rightarrow j) \cdot P_{j-1}(t) - w_t(j \rightarrow j-1) \cdot P_j(t) - w_t(j \rightarrow j+1) \cdot P_j(t) \quad (14)$$

For the decision-making state $j = 0$ and $j = K$, the master equation satisfies

$$P_0(t+1) = P_0(t) + w_t(1 \rightarrow 0)P_1(t) - w_t(0 \rightarrow 1)P_0(t). \quad (15)$$

$$P_K(t+1) = P_K(t) + w_t(K-1 \rightarrow K)P_{K-1}(t) - w_t(K \rightarrow K-1)P_K(t) \quad (16)$$

where $w_t(i \rightarrow j)$ is the transition probability of the decision-making state from i to j in generation t .

3.2. State transition probability

The transition probability of the decision-making state from i to j is in accordance with income in the different states [13],

$$w_t(i \rightarrow j) = \alpha \cdot [5 + 3 \cdot \text{sgn}(\pi_t(j) - \pi_t(i))], \quad (17)$$

where $\pi_t(j) - \pi_t(i)$ is the difference in income between decision-making state i and state j at generation t . The α value is a parameter related to the probability of mutation. The transition probability model increases the probability of moving to a state of increasing income.

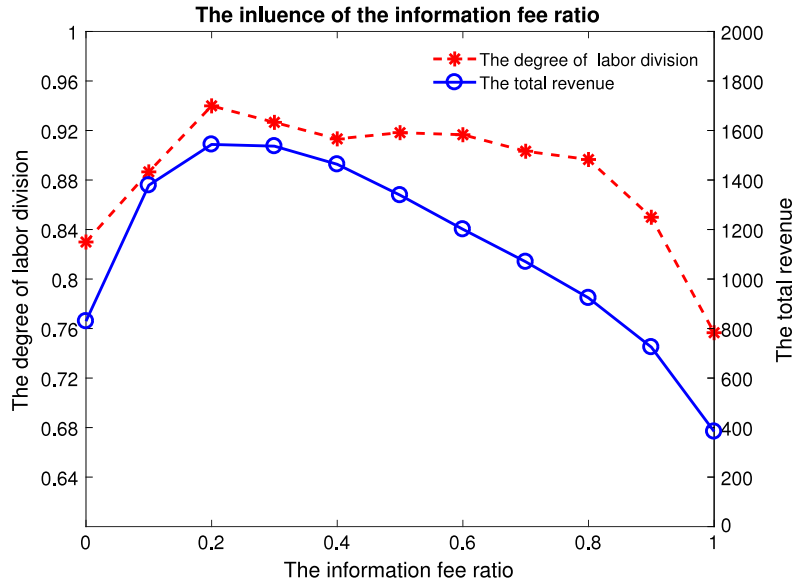


Fig. 2. The information fee ratio directly affects agent net income and also the degree of labor division and total revenue. When the information fee ratio is 0.2, the influence of labor division is the most obvious, and the total revenue is the largest. Very high or low information fee ratios lead to a decline in total revenue. Only when the information fee ratio ensures that the income of the agent who exploits resources balances the one who finds resources will the system will achieve a maximum of labor division and an optimization of total revenue.

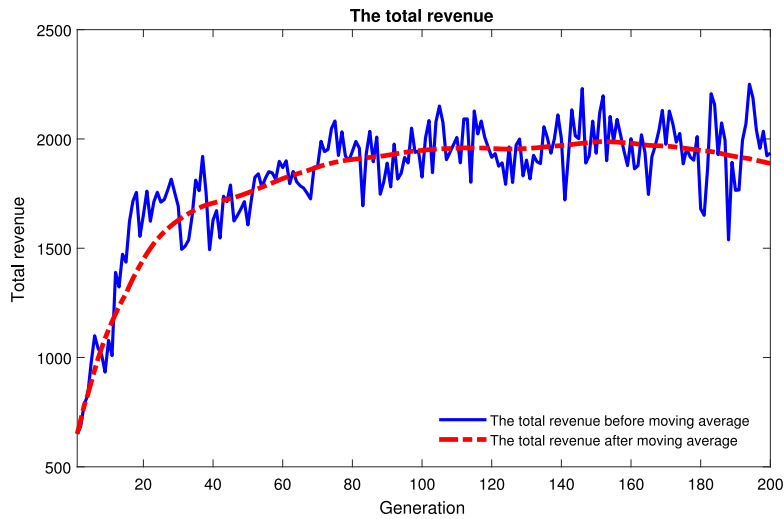


Fig. 3. The effect of the agent's income optimization mechanism on total revenue. The total revenue continues to increase with the evolution and tends to be stable after using a moving method.

3.3. Agent's income in different decision-making states

Agent net income is determined by agent income and information cost. Income includes the income from resource exploitation and information collection. Costs include the information cost. In generation t , the net income of the agent in decision-making state j is

$$\pi_j(t) = b(q_j) \cdot (\bar{v} \cdot N) \cdot S_0 \cdot (1 - c) + v(q_j) \cdot (\bar{b} \cdot N) \cdot S_0 \cdot c, \quad (18)$$

where c is the information fee ratio, $v(q_j)$ the visual field radius of the agent in decision state j , and $b(q_j)$ the exploitation ability of the agent in decision state j . The \bar{b} values is the average capacity of agents to exploit resources. The \bar{v} value is the average resource lattice found by a agent. The agent can learn-by-doing. If agents tend to exploit resources, their ability to exploit will grow. On the other hand, their ability to discover resources will also grow. Thus the model for the visual field

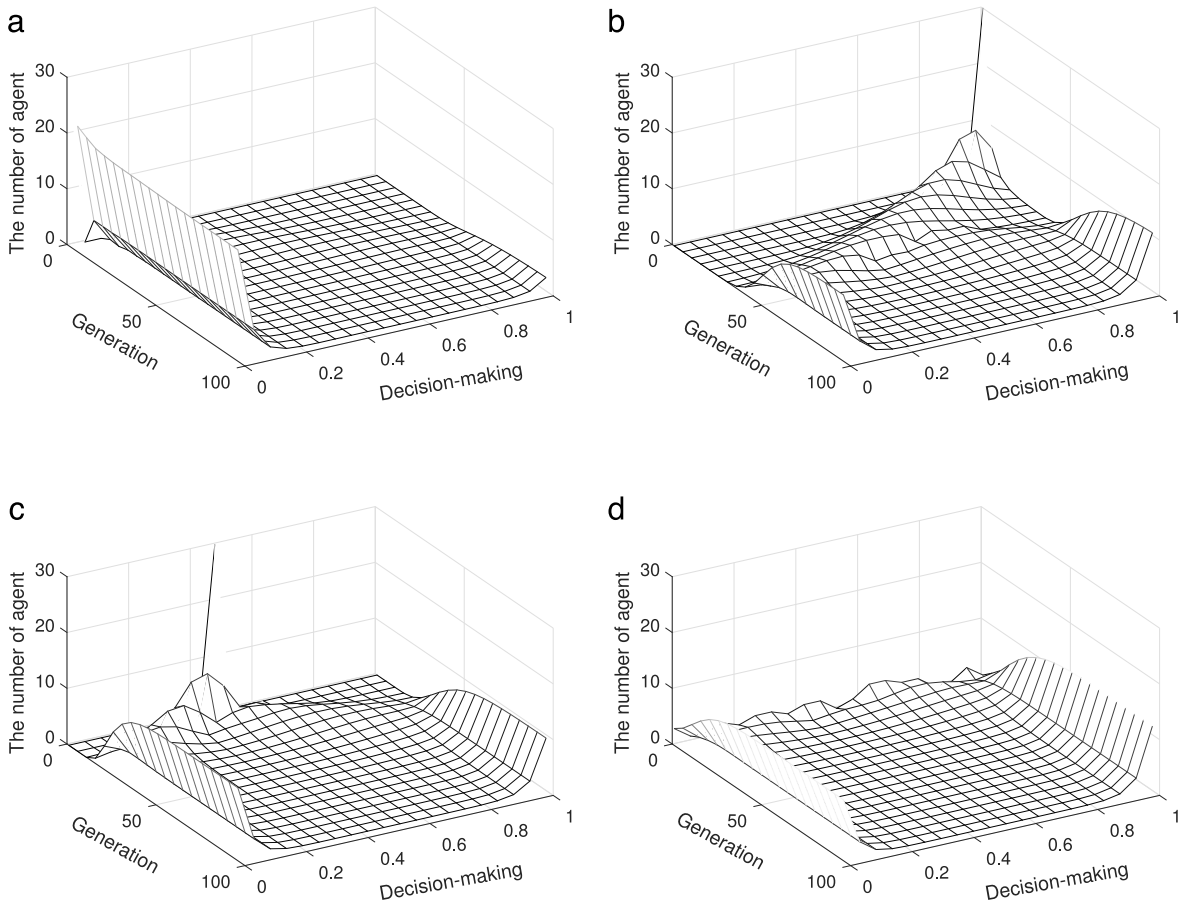


Fig. 4. The results of labor division: At the initial moment, the agent's decision-making has four different states: (a) $q = 0$; (b) $q = 1$; (c) $q = 0.5$; and (d) q is random distribution. The different initial distribution does not affect the emerging labor division. And the agent's decision-making tends to 0 or 1 with the evolution.

radius of the agent and the exploitation capacity of the agent are

$$b(q_j) = \frac{2}{1 + \exp(6q_j)} \quad (19)$$

and

$$v(q_j) = \frac{q_j^5}{0.5^5 + q_j^5}. \quad (20)$$

3.4. Model results

3.4.1. Evolution process and results of different initial distributions

In the theoretical model there are $N = 30$ agents, the probability interval of the agent's decision-making parameter is $K = 20$, the number of experiment generations is $T = 100$, and the information fee ratio is $c = 0.3$. We adjust the initial distribution of agents, apply it to four special cases, and show the evolution process and the final distribution results in Fig. 4. We compare the four graphs and find that the different initial distribution does not affect the emerging labor division.

3.4.2. Evolution results

The optimal income for the agent is strongly affected by the relationship between the total group revenue and the agent's income. In the theoretical model, the emerging labor division depends on information sharing and adaptive strategy adjustments. Fig. 5 shows the change of total income in different generations and the influence of the information fee ratio.

The total revenue is strongly affected by the information fee ratio. When the information fee ratio parameter is 0.3, the total revenue increases with an increase in evolution. When the information fee ratio is too large or too small the total

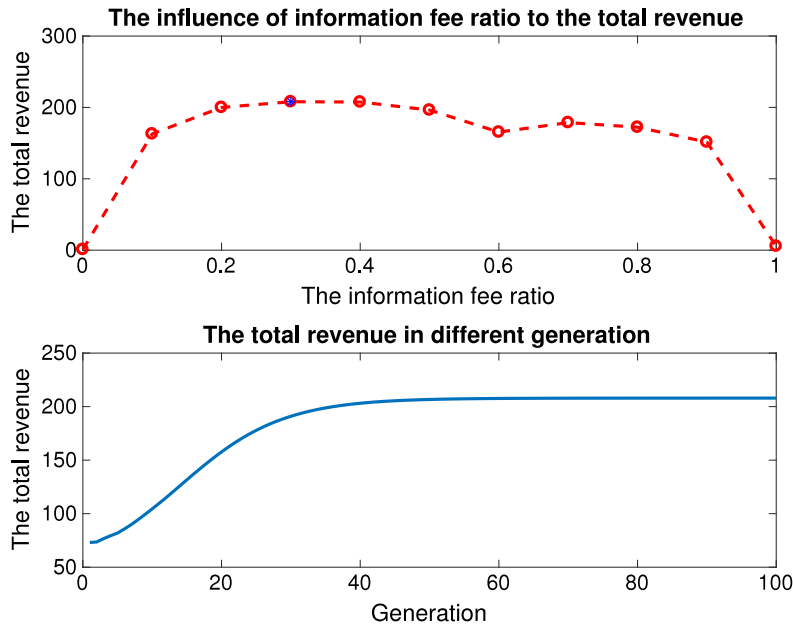


Fig. 5. First picture shows the impact of the information fee ratio on total revenue. We calculate the average of the total revenue in different information fee ratio. The total revenue reached the maximum when the information fee ratio parameter is 0.3. Second picture shows the change in total revenue in each evolution. When the information fee ratio parameter is 0.3, the total revenue increases with the evolution.

revenue decreases. When the information fee ratio is too low, the initiative to discover resources decreases because there are insufficient available resources. When the information fee ratio is too high, agents may choose to discover resources but, because there are fewer agents exploiting those resources, the income of the discovering agents decreases. Thus the information fee ratio directly affects agent income preference and this causes an imbalance in labor division. The information fee ratio is the only factor that can balance the labor division and maximize total revenue. The theoretical results quantifying this are similar to the simulation results.

4. Conclusion

We have modeled the division of labor as a self-organization process. Driven by individual income optimization, the system moves from disorder to order and a division of labor emerges. We do not set system-wide goals in the simulation process but apply a set of rules governing how agents interact with each other and with their environment. Agents seeking to increase their income is the driving factor in this self-organized evolution.

The rational for labor division is as follows.

- (1) Information sharing: Resource information is shared in a closed environment. This is a good approximation of the real-world information exchange process and makes the transaction rules between agents rational and realistic.
- (2) Learn-by-doing: This nonlinear mechanism enables agents to further improve what they are good at and – through the effect of positive feedback – become more strongly attached to doing it. Overall efficiency increases, which is an important factor in the emergence of labor division.
- (3) Random fluctuation: Stochastic fluctuation is a necessary condition for self-organized evolution and determines the state the agent arrives at in the final stage. In the process of evolution, a small change in the initial state of the system can strongly affect the final state. In our study the emergence of labor division does not depend on stochastic fluctuations, but the final outcome of labor division does depend on them.

Within the research field on labor division, we here make two contributions. (i) We introduce information cost as a factor in the formation of labor division and find that the level of information cost governs the aggregate income of the economy. This has significant implications for economic regulation and control. (ii) We find that labor division is caused by individual optimization not global optimization, but that stochastic characteristics can make the state of labor division unstable.

Acknowledgments

This work was supported by the National Social Science Fund of China under Grant No. 14BSH024 and State Scholarship Fund of China under Grant No. 201606045048 from China Scholarship Council (CSC).

References

- [1] G.V. Amdam, R.E. Page, The developmental genetics and physiology of honeybee societies, *Anim. Behav.* 79 (2010) 973–980.
- [2] A. Cornejo, A. Dornhaus, N. Lynch, R. Nagpal, Task allocation in ant colonies, in: *International Symposium on Distributed Computing*, Springer, Berlin, Heidelberg, 2014, pp. 46–60.
- [3] F. Manfredini, et al., Molecular and social regulation of worker division of labour in fire ants, *Mol. Ecol.* 23 (2014) 660–672.
- [4] C. Detrain, J.L. Deneubourg, Collective decision-making and foraging patterns in ants and honeybees, *Adv. Insect Physiol.* 35 (2008) 123–173.
- [5] G.E. Robinson, Regulation of division of labor in insect societies, *Ann. Rev. Entomol.* 37 (1992) 637–665.
- [6] D.F. Simola, et al., Epigenetic (re)programming of caste-specific behavior in the ant *Camponotus floridanus*, *Science* 351 (2016) aac6633.
- [7] H.M.G. Lattorff, R.F.A. Moritz, Genetic underpinnings of division of labor in the honeybee (*Apis mellifera*), *Trends Genet.* 29 (2013) 641–648.
- [8] A. Duarte, E. Scholtens, F.J. Weissing, Implications of behavioral architecture for the evolution of self-organized division of labor, *Plos Comput. Biol.* 8 (2012) e1002430.
- [9] W. Nakahashi, M.W. Feldman, Evolution of division of labor: emergence of different activities among group members, *J. Theoret. Biol.* 348 (2014) 65–79.
- [10] J. Wu, Z. Di, Z. Yang, Division of labor as the result of phase transition, *Physica A* 323 (2003) 663–676.
- [11] I. Kallel, A.M. Alimi, MAGAD-BFS: A learning method for Beta fuzzy systems based on a multi-agent genetic algorithm, *Soft Comput.* 10 (2006) 757–772.
- [12] J.H. Holland, *Hidden Order: How Adaptation Builds Complexity*, Perseus, Reading, MA, 1995.
- [13] Z. Di, J. Chen, Y. Wang, Z. Han, Emergence of specialization from global optimizing evolution in a multi-agent system, 2004, arXiv preprint nlin/0407005.
- [14] V.V. Isaeva, Self-organization in biological systems, *Biol. Bull.* 39 (2012) 110–118.
- [15] C. Fuchs, The self-organization of social movements, *Syst. Pract. Action Res.* 19 (2006) 101–137.
- [16] K. Liu, N. Lubbers, W. Klein, et al., The Effect of Growth On Equality in Models of the Economy, 2013, arXiv preprint arXiv:1305.0794.
- [17] H. Iyetomi, H. Aoyama, Y. Fujiwara, Y. Ikeda, W. Souma, Agent-based model approach to complex phenomena in real economy, *Progr. Theoret. Phys. Suppl.* 179 (2009) 123–133.
- [18] A. Chakraborti, I.M. Toke, M. Patriarca, F. Abergel, Econophysics review: II. Agent-based models, *Quant. Finance* 11 (2011) 1013–1041.
- [19] Y. Gao, S. Cai, L. Lv, B. Wang, Evolutionary model on market ecology of investors and investments, *Physica A* 392 (2013) 3385–3391.
- [20] G. Li, P. Ji, L.Y. Sun, W.B. Lee, Modeling and simulation of supply network evolution based on complex adaptive system and fitness landscape, *Comput. Ind. Eng.* 56 (2009) 839–853.
- [21] L. Chai, J. Chen, Z. Han, Z. Di, Y. Fan, Emergence of specialization from global optimizing evolution in a multi-agent system, in: *International Conference on Computational Science*, Springer, Berlin, Heidelberg, 2007, pp. 98–105.
- [22] M. Weng, Z. Wu, G. Qi, L. Zheng, Multi-agent-based workload control for make-to-order manufacturing, *Int. J. Prod. Res.* 46 (2008) 2197–2213.
- [23] J.O. Kephart, T. Hogg, B.A. Huberman, Dynamics of computational ecosystems: Implications for DAI, *Distrib. Artif. Intell.* 2 (1989) 79–96.
- [24] J.L. Simon, G. Steinmann, The economic implications of learning-by-doing for population size and growth, *Eur. Econ. Rev.* 26 (1984) 167–185.
- [25] K.J. Arrow, The economic implications of learning by doing, *Rev. Econom. Stud.* 29 (1962) 155–173.