# Proof of the Collatz conjecture

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Number Theory

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## 1 Key-Words

Lothar Collatz, Ulam, Kakutani, Thwaites, Hasse, Syracuse, hailstone, wondrous, HOTPO, oneness. [1].

## 2 Introduction

The Collatz conjecture can be summarized as follows: take any positive integer n. If n is even, divide it by 2 to get n/2. If n is odd, multiply it by 3 and add 1 to obtain 3n+1. Repeat the process indefinitely. The conjecture is that no matter what number you start with, you will always eventually reach 1. [1]

## 3 Fundamental theorem rewritten

The fundamental theorem states the following [2]: for each natural number, there is a unique factorization:

$$n = p_1^{\alpha_1}.p_2^{\alpha_2} \cdots p_k^{\alpha_k} \tag{1}$$

where exponents $\alpha_i$ are positive integers and $p_1 < p_2 < \cdots p_i$ are primes.
For the purpose of this document, we rewrite the above formula as follows:

$$n = 2^{\beta} \cdot \prod p_i^{\alpha_i} \tag{2}$$

where exponents $\beta$ and $\alpha_i$ are positive integers that can also be 0 and $p_i$ are the odd primes (primes greater than 2).

## 4 Collatz conjecture rewritten

For the purpose of this document, we like to reformulate the Collatz conjecture as follows: take any positive integer n. n can be written as in formula (2). If $\beta > 0$ then make $\beta = 0$. If $\beta = 0$ (and at least one $\alpha_i > 0$)then multiply n by 3 and add 1 to obtain 3n+1. The new 3n+1 will again have a $\beta > 0$. Repeat this process indefinitely. The conjecture is that no matter what number you start with, you will always eventually reach a situation where $\beta > 0$ and all $\alpha_i = 0$. In the next step, n will eventually become 1 when we make $\beta = 0$.

## 5 Playing with some numbers

Let's do some simple arithmetic calculations while we refer to figure 1.
We start with 1. We multiply it with 4 and add 1. We get 5. We multiply 5 with 4 and add 1. We get 21. We continue indefinitely. We put all these numbers in row 1. We construct any number in row 2 by multiplying the number on the same column in row 1 with 3 and adding 1. We construct any number in row 3 by taking the logarithm with base 2 on the number of the same column in row 2. The result is the natural numbers multiplied with 2 (the even numbers). The alert reader will already realize that the numbers on the first row will lead us to the final number 1 when running the Collatz algorithm.
We could ask ourselves what numbers would lead us to the odd powers of 2 according to the conjecture? The answer is in figure 2. It seems that we have to start the same calculation as in figure 1 but this time, starting with 7/3. The first row gives us only fractions with 3 in the denominator and hence these are not natural numbers. So, the Collatz algorithm will not be able to generate the odd powers of 2 by itself. We would have to manually select such an odd power of 2 to run the algorithm. This would lead us immediately to the final 1.

## 6 Proof by description

We learned from the rewritten Collatz conjecture that the pure powers of 2 are one last step before n will reach 1. So, any number of the second row of figure (1) will lead to 1. Now the second row comes from row one. So, if during the Collatz iteration, at a certain point, after making $\beta = 0$ (in formula 2), we come to one of the numbers belonging to the first row of figure 1, then after 2 more steps, we will reach the final 1.
In the first row of figure 1, there is one number between $x$ and $4x + 1$. So, the probability $P(x)$ at x to find a winning number is:

$$P(x) = \frac{1}{4x + 1 - x} = \frac{1}{3x + 1} \tag{3}$$

This means that no matter how big the numbers get, there will always be a chance to accidentally reach a number that will lead us to 1. So, because of this higher than zero chance, each iteration will eventually end up to one of the numbers on the first row of figure 1 that will lead to the number 1.
This basically proves the Collatz conjecture.

# 7 References

1. Wikipedia. Collatz conjecture.

2. Springer ; Prime Numbers A computational perspective ; Richard Crandall and Carl Pomerance ; p1

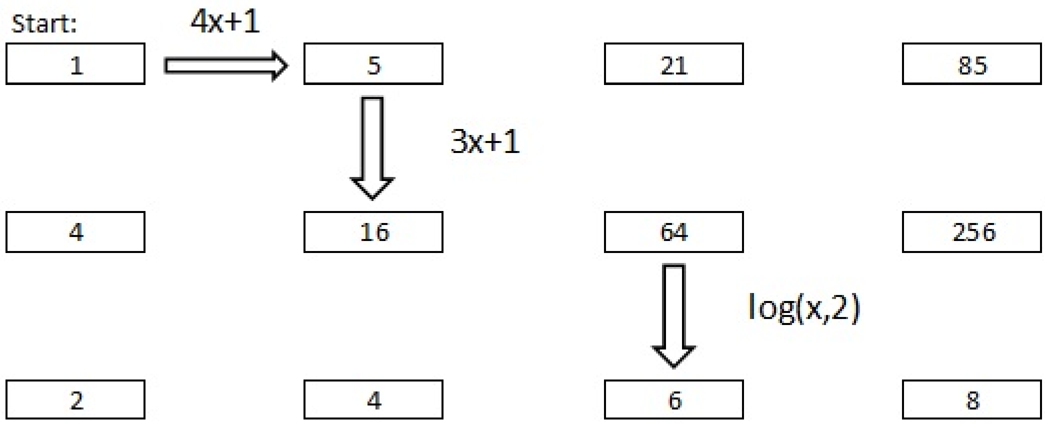


**Figure 1:** In this figure we demonstrate how to come to the even powers of 2 according to the Collatz conjecture. As the figures in row 1 are natural numbers, they can contribute to fulfilling the conjecture.

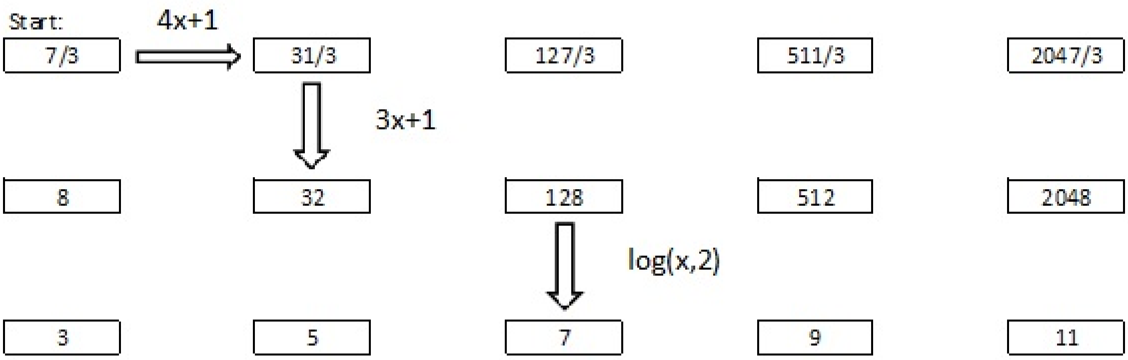


**Figure 2:** In this figure we demonstrate how to come to the odd powers of 2 according to the Collatz conjecture. As the figures in row 1 are fractions and hence no natural numbers, they can not contribute to fulfilling the conjecture.