Construction of Quasi-Electromagnetic Field Theory and Its Analogy with Relativistic Field Theories

Abstract: Based on the fundamental frameworks of Lagrangian mechanics and relativistic field theory, this paper introduces the scalar field $\psi_{(r,t)}$ and quasi-magnetic field B to construct a quasi-electromagnetic field theory system. Firstly, the particle motion equation and energy conservation relation are derived from the Lagrangian; subsequently, the field evolution law analogous to Maxwell's equations is obtained through field assumptions and analogies; further, the scalar field $\psi_{(r,t)}$ is analogized with the Klein-Gordon field and Higgs field to reveal its relativistic evolution and mass generation mechanism; finally, with the help of gauge potential and field strength tensor, the tensor formulation of the quasi-electromagnetic field theory is completed, and an attempt is made to connect it with general relativity, providing a theoretical basis for the exploration of new field theory models. Among them, $\psi_{(r,t)}$ and B are functions of spacetime, and m=$(\frac{\psi(r,t)}{c})^2$ defines that mass originates from the wave function $\psi_{(r,t)}$, enabling a smooth analogy to the equations of general relativity.

Keywords: Quasi-Electromagnetic Field; Lagrangian Mechanics; Klein-Gordon Equation; Higgs Field; Tensor Form; General Relativity

1. Introduction

In modern physics, electromagnetic field theory (Maxwell's equations) is the core framework for describing electromagnetic interactions, while relativistic field theories (such as Klein-Gordon theory and the Higgs mechanism) further reveal the relativistic evolution of particles and the origin of mass. Inspired by this, this paper attempts to construct a "quasi-electromagnetic field theory": by introducing the scalar field $\psi_{(r,t)}$ and quasi-magnetic field B, it explores the laws of new field interactions and draws analogies with existing relativistic field theories, providing new ideas for expanding field theory research.

2. Definition of Basic Physical Quantities and Lagrangian

Main Perspectives:

- The scalar field ψ is used to describe points in spacetime, and the mass per unit volume is defined as: m=$(\frac{\psi(r,t)}{c})^2$
- $\varepsilon = \mathrm{i}\frac{1}{C}\frac{\psi}{h}$, $\mu = i\frac{1}{c}\frac{h}{\psi}$
- Lagrangian: L = T - V(r) - E(r,t) = $\frac{1}{2}m_0 r'^2 - V(r,t) - ({\psi_{(r,t)}}^2)$, $m_0 = (\frac{\psi_0}{c})^2$
- Equation: $\frac{d}{dt}\left(\frac{\partial L}{\partial r'}\right) - \frac{\partial L}{\partial r} = 0$
- Let $V(r,t) = B(r,t)^2$
- Energy conservation: $\frac{1}{2}m_0\dot{r}^2 + \psi_{(r,t)}^2 + V(r,t) = Constant$

  From the above, the following equation is derived: $B\left[\nabla \times B - \frac{1}{c}\frac{\partial B}{\partial t}\right] + \psi\left[\nabla \times \psi - \frac{1}{c}\frac{\partial \psi}{\partial t}\right] = 0$

  For the identity case: $B\nabla \times B - \psi\frac{1}{c}\frac{\partial \psi}{\partial t} = 0$

  Let: $\varepsilon = \frac{1}{C}\frac{\psi}{B}$, i.e., $\nabla \times B = \varepsilon\frac{\partial \psi}{\partial t}$

$$\psi \nabla \times \psi - \mathrm{B}\frac{1}{c}\frac{\partial B}{\partial t} = 0$$

Let: $\mu = -\frac{1}{c}\frac{B}{\psi}$, i.e., $\nabla \times \psi = -\mu \frac{\partial B}{\partial t}$

Furthermore, the Klein-Gordon equation is derived from this:

$$k^2 = \nabla(\nabla \cdot) + c\mu(\nabla \times \mu)\frac{ə}{ət}$$

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial^2 t} - k^2)\ \psi = 0$$

$$(\square - k^2)\psi = 0$$

Introduce the tensor notation:

$$F^{\mu v} = \begin{pmatrix} 0 & -\psi_x/c & -\psi_y/c & -\psi_z/c \\ \psi_x/c & 0 & -\mathrm{B}_z & \mathrm{B}_y \\ \psi_y/c & \mathrm{B}_z & 0 & -\mathrm{B}_x \\ \psi_z/c & -\mathrm{B}_y & \mathrm{B}_x & 0 \end{pmatrix}$$

$\nabla \times \psi = -\mu \frac{\partial B}{\partial t}$ corresponds to: $\partial_\mu^* F^{\mu v} = 0$

$\nabla \cdot \psi = 0$ corresponds to: $*F_{\mu v} = \frac{1}{2}\varepsilon^{\mu v\rho\sigma}F_{\rho\sigma}$

By connecting with general relativity, the following relationship is obtained:

$$\lambda = 2\left(\frac{B^2}{C^2} - c^2B^2\varepsilon^2\right) = 2\left(B^2 - \frac{\psi^2}{C^2}\right) = 2B^2\left(\frac{1}{C^2} - c^2\varepsilon^2\right)$$

$$G_{\mu v} = \boldsymbol{g}_{\mu v}\left(\boldsymbol{\lambda} - \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{R}\right)$$

3. Derivation Process

3.1 Using the equation: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$

Where: $L = T - v_{(r)} - E(\mathrm{r}, t) = \frac{1}{2}m_0\dot{r}^2 - \psi_{(r,t)}^2 - v_{(\mathrm{r})}$

Energy conservation: $\frac{1}{2}m_0\dot{r}^2 + \psi_{(r,t)}^2 + v_{(\mathrm{r})} = Constant$

Calculate the partial derivatives:

$$\frac{\partial L}{\partial \dot{r}} = m_0\dot{r}$$

Substitute into the Euler-Lagrange equation:

$$\frac{\partial L}{\partial r} = -2\psi\frac{\partial \psi}{\partial r} - \frac{\mathrm{d}v}{\mathrm{d}r}$$

Substitute into the Euler-Lagrange equation:

$$m_0\ddot{r} + 2\psi\frac{\partial \psi}{\partial r} + \frac{\mathrm{d}v}{\mathrm{d}r} = 0$$

Introduce the "wave function" assumption and energy conservation:

Assume that $\psi(r, t)$ is a field quantity coupled with particle motion, and the energy conservation equation is:

$$\frac{1}{2}m_0\dot{r}^2 + \psi_{(r,t)}^2 + v_{(\mathrm{r})} = E_{\mathrm{tot}}$$

(where E_{tot} is the total energy constant).

Take the derivative with respect to t: $m_0\dot{r}\ddot{r} = -2\psi\dot{\psi}$

Substitute $\ddot{r}$ into equation (1.1): $m_0\left(-\frac{2\psi\dot{\psi}}{m_0\dot{r}}\right) + 2\psi\frac{\partial\psi}{\partial r} + \frac{dv}{dr} = 0$

$$-\frac{\dot{\psi}}{\dot{r}} + \frac{\partial\psi}{\partial r} + \frac{1}{2\psi}\frac{dv}{dr} = 0$$

Let: $\dot{\psi} = -i\hbar\frac{\partial\psi}{\partial t}$ and $\frac{1}{2\psi}\frac{dv}{dr} = \frac{v_{(r)}}{\psi}$ (effective potential). Then, combined with the relationship between $\dot{r}$ and ψ, v in the energy conservation equation, it can be rearranged into a form similar to the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-i\hbar\frac{\partial}{\partial r} + v_{(r)}\right)\psi$$

3.2 Maxwell's Equations

Note: $B^2_{(r,t)} = v_{(r,t)}$

And: $B^2_{(r,t)} + \psi^2_{(r,t)} =$ Constant

Combine with: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$
Where: $L = T - v_{(r)} - E(\mathrm{r}, t) = \frac{1}{2}m_0\dot{r}^2 - \psi^2_{(r,t)} - v_{(\mathrm{r})}$

Energy conservation: $\frac{1}{2}m_0\dot{r}^2 + \psi^2_{(r,t)} + v_{(\mathrm{r})} = \mathit{Constant}$

Clarify the analogy between fields and electromagnetic quantities:

Assume that $\psi_{(r,t)}$ is analogous to the scalar potential ϕ, and B(r,t) is analogous to the magnetic field B, which satisfies $B^2_{(r,t)} + \psi^2_{(r,t)} = \mathit{Constant}$ (where V is the effective potential) and $B^2_{(r,t)} + \psi^2_{(r,t)} =$ Constant (analogous to electromagnetic energy conservation).

Take the derivative of the equation $B^2_{(r,t)} + \psi^2_{(r,t)} =$ Constant with respect to time t:

$$B_{(r,t)}\dot{B}_{(r,t)} + \psi_{(r\cdot t)}\dot{\psi}_{(r,t)} = 0$$

Substitute into the above to obtain the particle motion equation: $m_0\ddot{r} + 2\psi\frac{\partial\psi}{\partial r} + 2B\frac{\partial B}{\partial r} = 0$

Combine with the energy conservation equation: $m_0\dot{r}\ddot{r} + 2\psi\dot{\psi} + 2B\dot{B} = 0$

It is concluded that: $m_0\dot{r}\ddot{r} = 0$

Thus, the field evolution equation is derived: $\psi\frac{\partial\psi}{\partial r} + B\frac{\partial B}{\partial r} = 0$

This is consistent with the above equation, indicating that the spatial distribution of the field satisfies "vortex conservation" (analogous to $\nabla \cdot B = 0$ in Maxwell's equations, which reflects the divergence-free property of the magnetic field).

Further, combined with $\dot{B} = -\frac{\psi}{B}\dot{\psi}$, if $\dot{\psi}$ is assumed to be analogous to the "time-varying electric field" and B to the "magnetic field", a one-dimensional form similar to Maxwell's curl equation can be constructed: $\frac{\partial B}{\partial r} \propto -\frac{\dot{\psi}}{v_0}$

Finally, an equation similar to Maxwell's "relation between magnetic field curl and electric field time variation" is derived:

$$\frac{\partial B_{(r,t)}}{\partial r} = -\frac{1}{v_0}\frac{\partial \psi_{(r,t)}}{\partial t}$$

This equation is consistent with Maxwell's equation $\nabla \times B = \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$ in terms of the physical essence that "the time-varying field $\dot{\psi}$ excites the vortex field $\frac{\partial B_{(r,t)}}{\partial r}$ ", reflecting the coupling between the time variation of the field and the spatial gradient.

3.3 Klein-Gordon Equation

3.3.1 Using the equation: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$

Where: $L = T - v_{(r)} - E(\mathrm{r}, t) = \frac{1}{2}m_0\dot{r}^2 - \psi^2_{(r,t)} - v_{(\mathrm{r})}$

Energy conservation: $\frac{1}{2}m_0\dot{r}^2 + \psi^2_{(r,t)} + v_{(\mathrm{r})} = Constant$

3.3.2 Construct the Lagrangian Density of the Field

Assume that $\psi_{(r,t)}$ is a scalar field (analogous to the Klein-Gordon field ϕ). Define the Lagrangian density L as the Lagrangian per unit volume. Based on the given L = $\frac{1}{2}m_0\dot{r}^2 - \psi^2_{(r,t)} - v_{(\mathrm{r})}$, extend it to the field form (replace the particle kinetic energy with the field kinetic energy density): L = $\boldsymbol{\frac{1}{2}\left(\frac{\partial\psi}{\partial t}\right)^2 - \frac{1}{2}C^2\left(\frac{\partial\psi}{\partial r}\right)^2 - \frac{1}{2}\mu^2\psi^2 - v_{(r)}\psi^2}$
(Among them, $\boldsymbol{\frac{1}{2}\left(\frac{\partial\psi}{\partial t}\right)^2}$ is the kinetic energy density of the field, $\boldsymbol{\frac{1}{2}C^2\left(\frac{\partial\psi}{\partial r}\right)^2}$ is the gradient energy density of the field, $\boldsymbol{\frac{1}{2}\mu^2\psi^2}$ is the "mass term" of the field, and $\boldsymbol{v_{(r)}\psi^2}$ is the potential term corresponding to $\boldsymbol{v_{(r)}}$

The Euler-Lagrange equation of the field is: $\frac{\partial L}{\partial \psi} - \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\left(\frac{\partial\psi}{\partial t}\right)}\right) - \frac{\partial}{\partial r}\left(\frac{\partial L}{\partial\left(\frac{\partial\psi}{\partial r}\right)}\right) = 0$

After calculating the above partial derivatives and substituting them into L, the following is obtained:

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial r^2} + \left(\mu^2 + 2v_{(r)}\right)\psi = 0$$

This equation is consistent with the Klein-Gordon equation $\Box\phi - \mu^2\phi = 0$ in the structure of the "d'Alembert operator (time-space second derivative)" and "mass term/potential term", reflecting the unity of the "relativistic evolution of the scalar field" with the energy conservation and Lagrangian form in this study.

3.4 Higgs Field

3.4.1 Using the equation: ：$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$

Where: $L = T - v_{(r)} - E(\mathrm{r}, t) = \frac{1}{2}m_0\dot{r}^2 - \psi^2_{(r,t)} - v_{(\mathrm{r})}$

Energy conservation: $\frac{1}{2}m_0\dot{r}^2 + \psi^2_{(r,t)} + v_{(\mathrm{r})} = Constant$

3.4.2 Review the Core of the Higgs Mechanism

The Higgs field is a scalar field (spin-0), and its core function is to endow elementary particles with mass through "spontaneous symmetry breaking". The key elements include:

- Lagrangian structure: It contains the kinetic energy term of the field and the self-interaction potential term. The potential has a "Mexican hat" shape $V_{(\phi)} \propto \phi^{+}\phi - (\phi^{+}\phi)^2$, leading to a non-zero vacuum expectation value of the field $\langle\phi\rangle \neq 0$;

- Particle mass generation: Gauge bosons (e.g., W and Z bosons) couple with the Higgs field and gain mass by "absorbing" the longitudinal component of the Higgs field; fermions (e.g., quarks and leptons) also generate mass through the Yukawa coupling term with the Higgs field.

- Analogy between Fields and Potentials
- Analogize $\psi_{(r,t)}$with the Higgs field ϕ, and analyze the correspondence between the Lagrangian and energy conservation:
- The kinetic energy term of the Higgs field is $\frac{1}{2}\partial_{\mu}\phi + \partial^{\mu}\phi$ (in relativistic form). In this study, the particle kinetic energy term $\frac{1}{2}m\dot{r}^2$ can be analogized to the "effective kinetic energy of field-particle coupling", while $-\psi^2_{(r,t)}$ can correspond to the self-interaction potential term of the Higgs field (similar to the form of $(\phi^{+}\phi)^2$, reflecting the self-interaction of the field).

- External Potential and Vacuum Energy
- The energy conservation equation $\frac{1}{2}m_0\dot{r}^2 + \psi^2_{(r,t)} + v_{(\mathrm{r})} = \mathbf{Constant}$ can correspond to the energy conservation of the Higgs field (including vacuum energy):
- When the system energy is constant, the "vacuum expectation value" of $\psi_{(r,t)}$ (analogous to $\langle\phi\rangle$ is jointly determined by $V_{(r)}$ and the total energy. If $V_{(r)}$ satisfies a certain symmetry breaking condition (e.g., $v_{(r)} \propto -\psi_0^2$, where ψ_0 is a constant), $\psi_{(r,t)}$ will generate a non-zero vacuum expectation value, corresponding to "spontaneous symmetry breaking".

- Lagrangian Equation and Higgs Field Motion Equation
- When expanded, the Lagrangian equation $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$ becomes: $m_0\ddot{r} + 2\psi\frac{\partial\psi}{\partial r} + \frac{\mathrm{d}v}{\mathrm{d}r} = 0$
- The motion equation of the Higgs field is the Klein-Gordon equation: $\Box\phi - \mu^2\phi = 0$
- If $\psi_{(r,t)}$ is regarded as a scalar field and $v_{(\mathrm{r})}$ corresponds to the gradient term of the potential $\frac{\partial v}{\partial\psi}$, the equation in this study can be analogized to the motion equation of the Higgs field in the "radial + time" dimension, describing the coupling between the field's evolution, the external potential, and its self-interaction.

- Analogy of Mass Generation
- In the Higgs mechanism, particle mass originates from the "coupling between the field and Higgs particles". In this study, if m_0 is regarded as the "effective mass endowed by $\psi_{(r,t)}$ ", the term $\frac{1}{2}m_0\dot{r}^2$ in the energy conservation equation can correspond to the "kinetic energy acquired by particles due to coupling with ψ ". This is similar to the mass effect reflected in the kinetic energy term after fermions obtain mass from the Higgs field through Yukawa coupling.

Corresponding Terms

- $\psi_{(r,t)}$ ←→ ϕ

- -$\psi^2_{(r,t)}$ ←→Self-interaction potential term of the Higgs field

- Lagrangian equation $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{r}}\right)-\frac{\partial L}{\partial r}=0$ ←→Motion equation of the Higgs field (describing field evolution)
- Energy conservation equation $\frac{1}{2}m_0\dot{r}^2+\psi_{(r,t)}^2+v_{(\mathrm{r})}\ =\ Constant$ ←→
  Energy conservation of the Higgs field (including vacuum energy and symmetry breaking)

5. Expression in Tensor Form

a. Definition of Spacetime and Index Convention

In Minkowski spacetime, the metric $\eta_{\mu v}$= diag(-1, 1, 1, 1) is adopted (where μ,v = 0,1,2,3), corresponding to time t and space coordinates x, y, z. The Einstein summation convention (automatic summation over repeated indices) is followed.

b. $\psi=\psi(x^{\mu})\ (x^{\mu}\ =\ ct,x,y,z)$

c. $L=\frac{1}{2}m_0\eta^{\mu U}\partial_{\mu}\partial_{v}r-v_{(x^{\mu})}-\psi^2$

d.

The Euler-Lagrange equation for the field is: $\frac{\partial L}{\partial \phi}-\partial\mu\left(\frac{\partial L}{\partial\left(\partial_{\mu\phi}\right)}\right)=0$

Equation of Motion for r:

$\frac{\partial L}{\partial r}=0$,

$\frac{\partial L}{\partial(\partial\mu r)}=m_0\eta^{\mu v}\partial_v r$

This gives the wave equation (d'Alembert equation) for r: $\square r=0$**, where** $(\square\ =\ \partial_{\mu}\partial^{\mu}=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2)$

Equation of Motion for ψ**:**

$\frac{\partial L}{\partial \psi}=-2\psi$,

$\frac{\partial L}{\partial(\partial_{\mu}\psi)}=0$

e. Energy-Momentum

$\partial^{\mu}T_{\mu v}=0$

$T_{\mu v}\ =\ \frac{\partial l}{\partial(\partial^{\mu}\phi)}\partial_v\phi-\eta_{\mu v}L$

For the r-field, substituting the kinetic energy term of L yields:

$$T_{\mu v}\ =\ m_0\partial_{\mu}r\partial_{v}r-\eta_{\mu v}(\frac{1}{2}m_0\eta^{\rho\sigma}\partial_{\rho}\partial_{\sigma}r-v-\psi^2)$$

The conservation law $\partial^{\mu}T_{\mu v}=0$ **includes the conservation of energy (time component) and momentum (spatial components), which is consistent with the original energy conservation equation.**

Core Equations in Tensor Form:

- Lagrangian: $L = \frac{1}{2} m_0 \eta^{\mu v} \partial_\mu r \partial_v r \ - B^2 - \psi^2$
- Equation of motion (wave equation for r): $\Box r = 0$
- Energy-momentum conservation: $\partial^\mu T_{\mu v} = 0$

Introduce the gauge potential $A^\mu = (A^0, A)$ such that:

$$B = \nabla \times A$$

$$\psi = -A^0 - \mu \frac{\partial A}{\partial t}$$

Further, define the field strength tensor as follows: $F_{\mu v} = \partial_\mu A_v - \partial_v A_\mu$. From this, the dual tensor is obtained:

$F_{\mu v} = \frac{1}{2} \varepsilon^{\mu v \rho \sigma} F_{\rho \sigma}$ This leads to the equations: $\partial_\mu F^{\mu v} = 0$ and $\partial_\mu F^{\mu v} = 0$

6. Connection with General Relativity

a.The representation of $\psi_{(r,t)}$ only resembles that of an electromagnetic field; essentially, it describes spacetime. The field strength tensor can be introduced: $F_{\mu v} = \partial_\mu A_v - \partial_v A_\mu$

b.Lagrangian density: In Maxwell's theory, the Lagrangian density is $L = -\frac{1}{4\mu_0} F^{\mu v} F_{\mu v}$. Here, it is modified to:

$$L_{cov} = -\frac{1}{4\mu_{(r,t)}} F^{\mu v} F_{\mu v} \sqrt{-\boldsymbol{g}}$$

(where $\sqrt{-\boldsymbol{g}}$ is the spacetime volume element, ensuring the covariance of the equation after variation.)

Specifically, mass is determined by the spacetime structure. In Einstein's theory, spacetime and matter are analogous to a "stage" and "actors"; in our current framework, they are unified.

The definition m=$(\frac{\psi_{(r,t)}}{c})^2$ implies that mass "originates" from the wave function $\psi_{(r,t)}$. Both $\psi_{(r,t)}$ and $\boldsymbol{B}_{(r,t)}$ can be regarded as fields describing the quantum properties of spacetime (similar to how wave functions describe particle states in quantum mechanics, but extended here to spacetime itself).

The variation principle is given by: $\boldsymbol{\delta S = \delta \int L \sqrt{-g} d_x^4 = 0}$

The Ricci tensor is constructed by analogy with Einstein's field equations: $R_{\mu v} = F^{\alpha\beta} F_{\alpha\beta} \ g_{\mu v}$

Our proposition is that $F^{\alpha\beta} F_{\alpha\beta}$ is variable, and the permittivity of vacuum should be a function of spacetime: $\varepsilon = \varepsilon_{(r,t)}$. This leads to the consequence that the mass of an object, m=$(\frac{\psi_{(r,t)}}{c})^2$, varies at different spacetime points.

(This will be used in the next paper to explain the anomalous motion velocity of stars in the outer regions of the Milky Way.)

When $\mu = \mu_{(r)}$:

$$\nabla_\mu \left(\frac{1}{\mu_{(r)}} F^{\mu v} \right) = 0$$

Coupling with the Gravitational Field

Substitute $F^{\alpha\beta}F_{\alpha\beta}$ (including $\varepsilon_{(r,t)}$) into $R_{\mu\nu} = F^{\alpha\beta}F_{\alpha\beta}\ g_{\mu\nu}$ At this point, $F^{\alpha\beta}F_{\alpha\beta}$ varies with $\varepsilon_{(r,t)}$ across spacetime; consequently, the Ricci tensor $R_{\mu\nu}$ also varies with spacetime. This further describes how the electromagnetic property $\varepsilon_{(r,t)}$ curves spacetime.

Quasi-Electromagnetic Field Tensor $F_{\mu\nu}$:

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\psi_x/c & -\psi_y/c & -\psi_z/c \\ \psi_x/c & 0 & -\mathrm{B}_z & \mathrm{B}_y \\ \psi_y/c & \mathrm{B}_z & 0 & -\mathrm{B}_x \\ \psi_z/c & -\mathrm{B}_y & \mathrm{B}_x & 0 \end{pmatrix}$$

From this, the Ricci tensor becomes: $R_{\mu\nu} = F^{\alpha\beta}F_{\alpha\beta}\ g_{\mu\nu}\ = 2(B^2 - \frac{\psi^2}{c^2})\ g_{\mu\nu}$

$$\boldsymbol{\lambda}\ = 2(B^2 - \frac{\psi^2}{c^2})$$

The Einstein-like field equation is: $G_{\mu\nu} = \boldsymbol{g}_{\mu\nu}\left(\boldsymbol{\lambda} - \frac{1}{2}\boldsymbol{R}\right)$

8. Conclusion

Using the fundamental methods of Lagrangian mechanics, this paper constructs a quasi-electromagnetic field theory centered on the scalar field $\psi_{(r,t)}$ and quasi-magnetic field $B_{(r,t)}$ This theory not only derives the field evolution law analogous to Maxwell's equations but also achieves a natural analogy with relativistic field theories such as the Klein-Gordon equation and the Higgs mechanism. Finally, a covariant formulation is completed through tensor representation, and an attempt is made to connect it with general relativity.

This theory provides a new theoretical perspective for exploring novel field interactions and expanding the understanding of the origin of particle mass. In future research, its validity can be further verified by comparing it with experimental data (e.g., the influence of the quasi-electromagnetic field on particle motion).