EM algorithm

Quiz, 4 questions

4/4 points (100%)

Congratulations! You passed!

Next Item

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points

1.
Recall the derivation of EM algorithm. In our notation, X is observable data, Z is latent variable and θ is a vector of model parameters. We introduced $q(Z)$ — an arbitrary distribution over the latent variable. Choose the correct expressions for the marginal log-likelihood $\log p(X \mid \theta)$:

- [ ] $\int q(Z) \log \frac{p(X,Z|\theta)}{q(Z)}\, dZ + \int q(Z) \log \frac{q(Z)}{p(Z|X,\theta)}\, dZ$

Correct

$$\int q(Z) \log \frac{p(X,Z|\theta)}{q(Z)}\, dZ + \int q(Z) \log \frac{q(Z)}{p(Z|X,\theta)}\, dZ =$$

$$\int q(Z) \log p(X,Z|\theta) dZ - \int q(Z) \log q(Z) dZ +$$

$$+ \int q(Z) \log q(Z) dZ - \int q(Z) \log p(Z|X,\theta) dZ =$$

$$= \int q(Z) \log \frac{p(X,Z|\theta)}{p(Z|X,\theta}\, dZ = \int q(Z) \log p(X|\theta) dZ = \log p(X|\theta)$$

- [ ] $\log \int p(X,Z|\theta) dZ$

Correct

Z is integrated out:

$$\log \int p(X,Z|\theta) dZ = \log p(X|\theta)$$

- [ ] $\mathbb{E}_{q(Z)} \log p(X,Z|\theta) - \mathbb{E}_{q(Z)} \log p(Z|X,\theta)$

Correct

$$\mathbb{E}_{q(Z)} \log p(X, Z|\theta) - \mathbb{E}_{q(Z)} \log p(Z|X, \theta) =$$

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$$= \mathbb{E}_{q(Z)} \log \frac{p(X,Z|\theta)}{p(Z|X,\theta)} = \mathbb{E}_{q(Z)} \log p(X|\theta) = \log p(X|\theta)$$

4/4 points (100%)

Quiz, 4 questions

- [ ] $\int q(Z) \log p(X|\theta) dZ$

Correct

$\log p(X|\theta)$ does not depend on Z.

$$\int q(Z) \log p(X|\theta) dZ = \log p(X|\theta)$$

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1 / 1 points

2.

In EM algorithm, we maximize variational lower bound $\mathcal{L}(q, \theta) = \log p(X|\theta) - \mathrm{KL}(q||p)$ with respect to q (E-step) and θ (M-step) iteratively. Why is the maximization of lower bound on E-step equivalent to minimization of KL divergence?

- ( ) Because uncomplete likelihood does not depend on $q(Z)$

Correct

Revise E-step details video

- ( ) Because we cannot maximize lower bound w.r.t. $q(Z)$
- ( ) Because posterior becomes tractable
- ( ) Because of Jensen's inequality

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1 / 1 points

3.

Select correct statements about EM algorithm:

- [ ] E-step can always be performed analytically

Un-selected is correct

EM algorithm

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- [ ] M-step can always be performed analytically

Un-selected is correct

- [ ] EM algorithm always converges

Correct

Revise M-step details video

- [ ] Complete likelihood is always a convex function as a function of parameters

Un-selected is correct

- [ ] EM algorithm always converges to a global optimum

Un-selected is correct

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1 / 1 points

4.

Consider $p(x) = \mathcal{N}(\mu, \sigma_1)$ and $q(x) = \mathcal{N}(\mu, \sigma_2)$. Calculate KL divergence between these two gaussians KL$(p||q)$ (hint: note that KL divergence is an expectation):

- $\log \frac{\sigma_2}{\sigma_1} - \frac{1}{2} + \frac{\sigma_1^2}{2\sigma_2^2}$

Correct

$$KL(p||q) = \mathbb{E}_p \log \frac{\mathcal{N}(x|\mu,\sigma_1^2)}{\mathcal{N}(x|\mu,\sigma_2^2)} = \mathbb{E}_p \log \frac{\left(\sqrt{2\pi\sigma_1^2}\right)^{-1} \exp\left(-\frac{(x-\mu)^2}{2\sigma_1^2}\right)}{\left(\sqrt{2\pi\sigma_2^2}\right)^{-1} \exp\left(-\frac{(x-\mu)^2}{2\sigma_2^2}\right)} =$$

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$$= \mathbb{E}_p \Big[\log \frac{\sigma_2}{\sigma_1} + \log \frac{\exp\left(-\frac{(x-\mu)^2}{2\sigma_1^2}\right)}{\exp\left(-\frac{(x-\mu)^2}{2\sigma_2^2}\right)} \Big] = \mathbb{E}_p \Big[\log \frac{\sigma_2}{\sigma_1} - \frac{(x-\mu)^2}{2\sigma_1^2} + \frac{(x-\mu)^2}{2\sigma_2^2} \Big] =$$

$$= \log \frac{\sigma_2}{\sigma_1} - \frac{\mathbb{E}_p(x-\mu)^2}{2\sigma_1^2} + \frac{\mathbb{E}_p(x-\mu)^2}{2\sigma_2^2} = \log \frac{\sigma_2}{\sigma_1} - \frac{\sigma_1^2}{2\sigma_1^2} + \frac{\sigma_1^2}{2\sigma_2^2} = \log \frac{\sigma_2}{\sigma_1} - \frac{1}{2} + \frac{\sigma_1^2}{2\sigma_2^2}$$

- $\log \frac{\sigma_1}{\sigma_2} + \frac{\sigma_1^2-\sigma_2^2}{\sigma_1^2+\sigma_2^2}$
- $\log \frac{\sigma_1}{\sigma_2} + \frac{\sigma_2^2}{\sigma_1^2}$
- $\log \frac{\sigma_2^2}{\sigma_1^2} - \frac{\sigma_1^2}{2\sigma_2^2}$