EM algorithm

4/4 points (100%)

Quiz, 4 questions

✓ Congratulations! You passed!

Next Item



1/1 points

1.

Recall the derivation of EM algorithm. In our notation, X is observable data, Z is latent variable and θ is a vector of model parameters. We introduced q(Z) — an arbitrary distribution over the latent variable. Choose the correct expressions for the marginal log-likelihood $\log p(X\mid\theta)$:

$$\int q(Z) \log \frac{p(X,Z|\theta)}{q(Z)} dZ + \int q(Z) \log \frac{q(Z)}{p(Z|X,\theta)} dZ$$

Correct

$$\int q(Z) \log \frac{p(X,Z|\theta)}{q(Z)} dZ + \int q(Z) \log \frac{q(Z)}{p(Z|X,\theta)} dZ =$$

$$\int q(Z) \log p(X,Z|\theta) dZ - \int q(Z) \log q(Z) dZ +$$

$$+ \int q(Z) \log q(Z) dZ - \int q(Z) \log p(Z|X,\theta) dZ =$$

$$= \int q(Z) \log \frac{p(X,Z|\theta)}{p(Z|X,\theta)} dZ = \int q(Z) \log p(X|\theta) dZ = \log p(X|\theta)$$

Correct

Z is integrated out:

$$\log \int p(X, Z|\theta) dZ = \log p(X|\theta)$$

Correct

$$\mathbb{E}_{q(Z)} \log p(X, Z|\theta) - \mathbb{E}_{q(Z)} \log p(Z|X, \theta) =$$

EM algorithm=
$$\mathbb{E}_{q(Z)} \log \frac{p(X,Z|\theta)}{p(Z|X,\theta)} = \mathbb{E}_{q(Z)} \log p(X|\theta) = \log p(X|\theta)$$

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Quiz, 4 questions



 $\int q(Z)\log p(X|\theta)dZ$

Correct

 $\log p(X|\theta)$ does not depend on Z.

$$\int q(Z)\log p(X|\theta)dZ = \log p(X|\theta)$$



1/1 points

2.

In EM algorithm, we maximize variational lower bound $\mathcal{L}(q,\theta) = \log p(X|\theta) - \mathrm{KL}(q|lp) \text{ with respect to } q \text{ (E-step) and } \theta \text{ (M-step)}$ iteratively. Why is the maximization of lower bound on E-step equivalent to minimization of KL divergence?



Because uncomplete likelihood does not depend on q(Z)

Correct

Revise E-step details video

- Because we cannot maximize lower bound w.r.t. q(Z)
- Because posterior becomes tractable
- Because of Jensen's inequality



1/1 points

3.

Select correct statements about EM algorithm:



E-step can always be performed analytically

Un-selected is correct

EM algorithm

4/4 points (100%)

Quiz, 4 questions



M-step can always be performed analytically

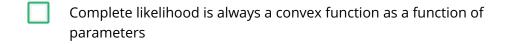
Un-selected is correct



EM algorithm always converges

Correct

Revise M-step details video



Un-selected is correct

EM algorithm always converges to a global optimum

Un-selected is correct



1/1 points

4.

Consider $p(x) = \mathcal{N}(\mu, \sigma_1)$ and $q(x) = \mathcal{N}(\mu, \sigma_2)$. Calculate KL divergence between these two gaussians $\mathrm{KL}(p||q)$ (hint: note that KL divergence is an expectation):



$$\log\frac{\sigma_2}{\sigma_1} - \frac{1}{2} + \frac{\sigma_1^2}{2\sigma_2^2}$$

Correct

$$KL(p||q) = \mathbb{E}_p \log \frac{\mathcal{N}(x|\mu,\sigma_1^2)}{\mathcal{N}(x|\mu,\sigma_2^2)} = \mathbb{E}_p \log \frac{\left(\sqrt{2\pi\sigma_1^2}\right)^{-1} \exp\left(-\frac{(x-\mu)^2}{2\sigma_1^2}\right)}{\left(\sqrt{2\pi\sigma_2^2}\right)^{-1} \exp\left(-\frac{(x-\mu)^2}{2\sigma_2^2}\right)} =$$

$$\text{EM algorithm} = \mathbb{E}_{p} \left[\log \frac{\sigma_{2}}{\sigma_{1}} + \log \frac{\exp \left(-\frac{(x-\mu)^{2}}{2\sigma_{1}^{2}} \right)}{\exp \left(-\frac{(x-\mu)^{2}}{2\sigma_{2}^{2}} \right)} \right] = \mathbb{E}_{p} \left[\log \frac{\sigma_{2}}{\sigma_{1}} - \frac{(x-\mu)^{2}}{2\sigma_{1}^{2}} + \frac{(x-\mu)^{2}}{2\sigma_{2}^{2}} \right] =$$
4/4 points (100%)

Quiz, 4 questions

$$= \log \frac{\sigma_2}{\sigma_1} - \frac{\mathbb{E}_p(x-\mu)^2}{2\sigma_1^2} + \frac{\mathbb{E}_p(x-\mu)^2}{2\sigma_2^2} = \log \frac{\sigma_2}{\sigma_1} - \frac{\sigma_1^2}{2\sigma_1^2} + \frac{\sigma_1^2}{2\sigma_2^2} = \log \frac{\sigma_2}{\sigma_1} - \frac{1}{2} + \frac{\sigma_1^2}{2\sigma_2^2}$$

- $\log \frac{\sigma_1}{\sigma_2} + \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2}$
- $\log \frac{\sigma_1}{\sigma_2} + \frac{\sigma_2^2}{\sigma_1^2}$
- $\log \frac{\sigma_2^2}{\sigma_1^2} \frac{\sigma_1^2}{2\sigma_2^2}$

