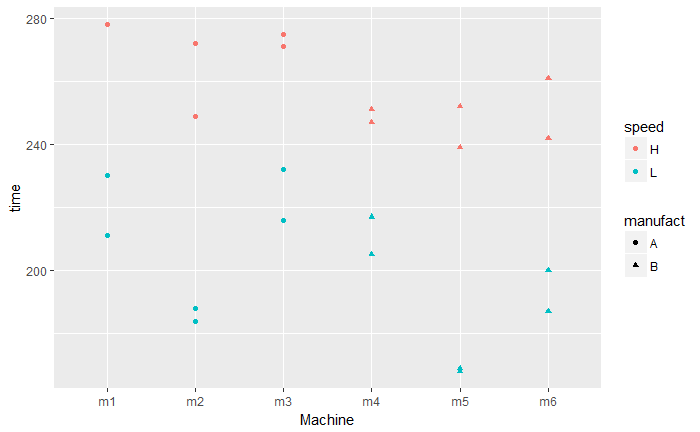
**STAT34700-HW4**

**Problem 3**

**(a)**



**Comments:**

From the plot of cutoff time of lawnmowers vs machine, we can see that the cutoff times for machine in high speed are longer than the cutoff times in low speed. In addition, the cutoff times for machines from manufacturer A are longer than the cutoff times from manufacturer B. For different machines, the cutoff times are also different.

**(b)**

**The fixed effects model:**

Fit the fixed effects model:

> fit\_fix = lm(time~., data = lawn)

**Output:**

> summary(fit\_fix)

Call:

lm(formula = time ~ ., data = lawn)

Residuals:

Min 1Q Median 3Q Max

-12.50 -8.50 -3.00 7.50 19.25

Coefficients: (1 not defined because of singularities)

Estimate Std. Error t value Pr(>|t|)

(Intercept) 278.750 6.042 46.138 < 2e-16 \*\*\*

manufactB -26.750 7.910 -3.382 0.00355 \*\*

machinem2 -26.000 7.910 -3.287 0.00435 \*\*

machinem3 -0.750 7.910 -0.095 0.92557

machinem4 7.500 7.910 0.948 0.35635

machinem5 -15.500 7.910 -1.959 0.06667 .

machinem6 NA NA NA NA

speedL -59.000 4.567 -12.919 3.23e-10 \*\*\*

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Signif. codes: 0 ?\*\*?0.001 ?\*?0.01 ??0.05 ??0.1 ??1

Residual standard error: 11.19 on 17 degrees of freedom

Multiple R-squared: 0.9251, Adjusted R-squared: 0.8986

F-statistic: 34.97 on 6 and 17 DF, p-value: 1.183e-08

From the output, the effect of machine level 6 cannot be estimated. The coefficient of machine6 cannot be defined because of singularity. It means that the column of machine level 6 can be expressed by the linear combination of other level columns. From the same manufacturer, different machines have some collinearity with each other. That’s why the coefficient of machine6 cannot be estimated.

**(c)**

**The mixed effects model:**

Fit the mixed effects model:

> fit\_c=lmer(time~manufact\*speed+(1|machine),data=lawn)

**Output:**

> summary(fit\_c)

Linear mixed model fit by REML ['lmerMod']

Formula: time ~ manufact \* speed + (1 | machine)

Data: lawn

REML criterion at convergence: 168.4

Scaled residuals:

Min 1Q Median 3Q Max

-1.0909 -0.6740 -0.1291 0.6661 1.5405

Random effects:

Groups Name Variance Std.Dev.

machine (Intercept) 145.2 12.05

Residual 132.3 11.50

Number of obs: 24, groups: machine, 6

Fixed effects:

Estimate Std. Error t value

(Intercept) 270.500 8.394 32.23

manufactB -21.833 11.871 -1.84

speedL -60.333 6.641 -9.09

manufactB:speedL 2.667 9.392 0.28

Correlation of Fixed Effects:

(Intr) mnfctB speedL

manufactB -0.707

speedL -0.396 0.280

mnfctB:spdL 0.280 -0.396 -0.707

> sqrt(145.2+132.3)

[1] 16.65833

If the same machine were tested at the same speed, the SD of the times observed should just be the SD of the residual term . From the output, we can get the SD of is .

If different machines were sampled from the same manufacturer and tested at the

same speed once only, the SD of the times observed should be

**(d)**

**Test the significance of interaction term:**

Fit the full model with interaction terms and the model without the interaction term.

> fit\_df = lmer(time~manufact\*speed+(1|machine), data=lawn, REML=FALSE)

> fit\_dn = lmer(time~manufact+speed+(1|machine), data=lawn, REML=FALSE)

We use the F-test to test the significance of interaction term:

**Output:**

> KRmodcomp(fit\_df, fit\_dn)

F-test with Kenward-Roger approximation; computing time: 0.10 sec.

large : time ~ manufact + speed + (1 | machine) + manufact:speed

small : time ~ manufact + speed + (1 | machine)

stat ndf ddf F.scaling p.value

Ftest 0.0806 1.0000 16.0000 1 0.7801

The null hypothesis : the interaction term is not significant in the mixed model.

The corresponding p-value is greater than the significance level , so we cannot reject the null hypothesis. Then we conclude that the interaction term is not significant in this mixed model.

**Test the significance of fixed effect *manufact*:**

Fit the model without the fixed effect *manufact*:

> fit\_ds = lmer(time~speed+(1|machine), data=lawn, REML=FALSE)

We use the F-test to test the significance of fixed effect *manufact*:

**Output:**

> KRmodcomp(fit\_dn, fit\_ds)

F-test with Kenward-Roger approximation; computing time: 0.03 sec.

large : time ~ manufact + speed + (1 | machine)

small : time ~ speed + (1 | machine)

stat ndf ddf F.scaling p.value

Ftest 3.5354 1.0000 4.0000 1 0.1332

The null hypothesis : the fixed effect *manufact* is not significant in the mixed model.

The corresponding p-value is greater than the significance level , so we cannot reject the null hypothesis. Then we conclude that the fixed effect *manufact* is not significant in this mixed model.

**Test the significance of fixed effect *speed*:**

Fit the model without the fixed effect *speed*:

> fit\_dm = lmer(time~manufact+(1|machine), data=lawn, REML=FALSE)

We use the F-test to test the significance of fixed effect *speed*:

**Output:**

> KRmodcomp(fit\_dn, fit\_dm)

F-test with Kenward-Roger approximation; computing time: 0.19 sec.

large : time ~ manufact + speed + (1 | machine)

small : time ~ manufact + (1 | machine)

stat ndf ddf F.scaling p.value

Ftest 166.89 1.00 17.00 1 3.229e-10 \*\*\*

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Signif. codes: 0 ?\*\*?0.001 ?\*?0.01 ??0.05 ??0.1 ??1

The null hypothesis : the fixed effect *speed* is not significant in the mixed model.

The corresponding p-value is smaller than the significance level , so we can reject the null hypothesis. Then we conclude that the fixed effect *speed* is significant in this mixed model.

**Final model:**

**(e)**

To check whether there is any variation between machines, we need to test the significance of the random effect *machine.*

We fit the mixed full model with speed as fixed effect and machine as random effect. Then we fit the fixed effect model with only predictor speed.

> fit\_e = lmer(time~speed+(1|machine), data=lawn)

> fit\_ef = lm(time~speed, data=lawn)

We use the likelihood ratio test to test the significance of the random effect machine.

**Output:**

> exactLRT(fit\_e, fit\_ef)

simulated finite sample distribution of LRT. (p-value based on 10000 simulated

values)

data:

LRT = 11.154, p-value = 4e-04

Signif. codes: 0 ?\*\*?0.001 ?\*?0.01 ??0.05 ??0.1 ??1

The null hypothesis : the random effect machine is not significant in this mixed model.

The corresponding p-value is smaller than the significance level , so we can reject the null hypothesis. Then we conclude that the random effect *machine* is significant in this mixed model.

**Conclusion:**

Therefore, we conclude that there is variation between machines.

**(f)**

**The nested model:**

Fit the mixed effects model:

> fit\_f = lmer(time~speed+(1|manufact)+(1|manufact:machine), data=lawn)

**Output:**

> summary(fit\_f)

Linear mixed model fit by REML ['lmerMod']

Formula: time ~ speed + (1 | manufact) + (1 | manufact:machine)

Data: lawn

REML criterion at convergence: 183.6

Scaled residuals:

Min 1Q Median 3Q Max

-1.0960 -0.6685 -0.0946 0.6668 1.5889

Random effects:

Groups Name Variance Std.Dev.

manufact:machine (Intercept) 147.0 12.12

manufact (Intercept) 150.7 12.28

Residual 125.1 11.19

Number of obs: 24, groups: manufact:machine, 6; manufact, 2

Fixed effects:

Estimate Std. Error t value

(Intercept) 259.583 10.501 24.72

speedL -59.000 4.567 -12.92

Correlation of Fixed Effects:

(Intr)

speedL -0.217

From the output, we find that the variability between machines is 147.0 and the variability between manufacturers is 150.7. These two variabilities are very close to each other.

**(g)**

Construct bootstrap confidence intervals for the terms of the previous model:

**Output:**

> confint(fit\_f, method="boot")

2.5 % 97.5 %

.sig01 0.000000 21.64731

.sig02 0.000000 32.37236

.sigma 7.337765 15.29026

(Intercept) 239.260877 279.88640

speedL -67.708176 -50.45319

The 95% CI for *machine* term = [0, 21.65]

The 95% CI for *manufact* term = [0, 32.37]

The 95% CI for residual term = [7.34, 15.29]

The 95% CI for intercept term = [239.26, 279.89]

The 95% CI for *speed* term = [-67.71, -50.45]

**Conclusion:**

The confidence intervals for random effects *machine* and *manufact* are all include 0. As a result, any of these two random effect terms can be insignificant. We might drop any random effect term but we are not sure which is best to be dropped. Hence, the variability can be ascribed solely to manufacturers or to machines. It is safest to conclude there is some variation in the cutoff time coming from both two sources.

**R Code**

library(faraway)

library(RLRsim)

library(ggplot2)

library(lme4)

library(pbkrtest)

data(lawn)

#a

ggplot(lawn, aes(y=time, x=machine, shape=manufact, col=speed))+geom\_point()+xlab("Machine")

#b

fit\_fix = lm(time~., data = lawn)

summary(fit\_fix)

#c

fit\_c=lmer(time~manufact\*speed+(1|machine),data=lawn)

summary(fit\_c)

sqrt(145.2+132.3)

#d

#test interaction term

fit\_df = lmer(time~manufact\*speed+(1|machine), data=lawn, REML=FALSE)

fit\_dn = lmer(time~manufact+speed+(1|machine), data=lawn, REML=FALSE)

KRmodcomp(fit\_df, fit\_dn)

#test speed and manufact

fit\_dm = lmer(time~manufact+(1|machine), data=lawn, REML=FALSE)

fit\_ds = lmer(time~speed+(1|machine), data=lawn, REML=FALSE)

KRmodcomp(fit\_dn, fit\_dm)

KRmodcomp(fit\_dn, fit\_ds)

#e

fit\_e = lmer(time~speed+(1|machine), data=lawn)

fit\_ef = lm(time~speed, data=lawn)

exactLRT(fit\_e, fit\_ef)

#f

fit\_f = lmer(time~speed+(1|manufact)+(1|manufact:machine), data=lawn)

summary(fit\_f)

#g

confint(fit\_f, method="boot")