

Joint Compressive Sensing in Wideband Cognitive Networks

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Abstract—In this paper, a distributed compressive spectrum sensing scheme in wideband cognitive radio networks is discussed. An AIC RF front-end sampling structure is proposed requiring only low rate ADCs and few storage units for spectrum sampling. Multiple CRs collect compressed samples through AICs and recover spectrum jointly. A novel joint sparsity model is defined in this scenario, along with a universal recovery algorithm based on S-OMP. Numerical simulations show this algorithm outperforms current existing algorithms under this model and works competently under other existing models.

Index Terms—Distributed compressive sensing, AIC, Wideband CR network, Joint sparsity, Spectrum recovery

I. INTRODUCTION

Cognitive Radio (CR) is emerging technology tackling the spectrum scarcity problem caused by current static spectrum allocation schemes in wireless communication networks. CRs can sense spectrum and learn from the environment, utilize unoccupied spectrum holes through Dynamic Spectrum Access, which can dramatically increase the throughput of the whole CR system. In wideband CR networks, main challenge for CRs is to sense and detect spectrum holes reliably without interfering with primary users (PU).

Several spectrum sensing mechanisms have been studied in CR networks. The most widely studied energy detection method, though simple, could only detect narrow-band spectrum, with relatively low detection reliability. Compressive sensing is a novel sampling technique that can sample sparse signals with sub-Nyquist rates and performs accurate recovery through l_1 minimization, which attains a considerable compression rate. Thus, compressive sensing could recover wideband spectrum precisely with much less samples, attaining higher detection probability than energy detection method. Also, recent emerging analog-to-information converters (AIC) techniques [7]-[9] guarantee hardware implementation of fast and efficient sampling of real time signals. Noting that the wideband signal is sparse enough with a remarkable low occupancy rate of the whole spectrum, [8]-[11] have studied compressive spectrum recovery schemes. In [8], [10], the spectrum is modelled as sparse in wavelet domain. A wavelet approach is proposed to detect the edges and make PSD estimations using compressive sensing. In this paper, we prefer a simple but efficient alternative compressive sample formulation by direct sampling of the spectrum.

Due to frequency selective fading, random shadowing and probable interference from other wireless facility, a single CR

couldn't avoid problems such as hidden terminals and high miss detection probability. Cooperative spectrum sensing [8]-[11] is proposed to overcome the uncertainty related with single detection. Those types of detection procedure rely on local decisions. In [12], [13], Joint Compressive Spectrum Recovery is considered, which supposes signals received at multiple CRs are highly correlated, known as Joint Sparsity [5]. It's important to notice that joint spectrum recovery requires much less samples for each CR comparing with separate recovery scheme, which further reduces power and bandwidth for communication and memory size for storage. However, uncertain channel fading and random shadowing would lead to varying signal strength at different CRs, even placed close to each other, deep shadowing can cause much weaker signal detected at related CRs, as shown in fig.1. Also, hidden terminal problems would probably lead to total loss of certain spectrum component at CRs. In this paper, An novel applicable model along with a universal recovery algorithm is proposed covering both existing joint sparse models and simple non-joint models.

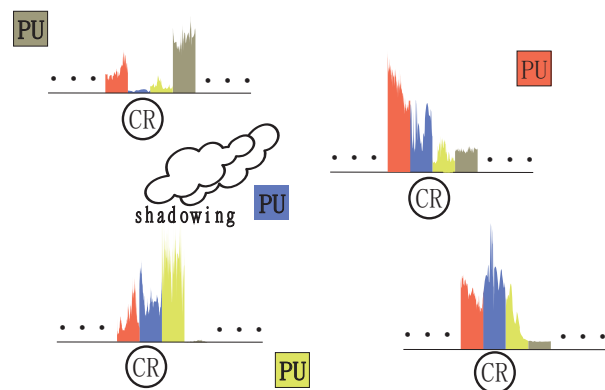


Fig. 1: illustration of GJSM in CR networks

The rest of this paper is organized as follows: In Section 2, compressive sensing backgrounds are introduced. In Section 3, the ADC based sampling architecture is discussed and compressive wideband sampling at single CRs is explored. In Section 4, After defining a new joint sparsity model in CR networks, recovery algorithm and simulation results are discussed here. Finally, we conclude our works in Section 5.

II. COMPRESSIVE SENSING BACKGROUNDS

Suppose a S -sparse signal $s \in R^N$ is to be sampled. Define $\text{supp}(v) = \{i|v_i \neq 0\}$, thus $|\text{supp}(s_f)| = S$. By compressive sensing, the sample has the form $y = \Phi s$, where the random measurement $M \times N$ matrix Φ projects s onto $y \in R^M$. Define the restricted isometry constant (RIC) δ_s to be the smallest positive constant such that

$$(1 - \delta_s) \|c\|_2^2 \leq \|\Phi_{T^c}\|_2^2 \leq (1 + \delta_s) \|c\|_2^2 \quad (1)$$

for all subsets T with $|T| \leq S$ and coefficient sequences $(c_j), j \in T$. One known result about RIC is that, when $\delta_{2s} + \delta_{3s} < 1$, the signal s can be exactly recovered by solving

$$\hat{s} = \arg \min \| \hat{s} \|_1 \quad \text{s.t. } y = \Phi s \quad (2)$$

For implementation issues, when the elements of Φ are chosen as i.i.d. random gaussian variables such that $\Phi_{i,j} \sim \mathcal{N}(0, 1/M)$, if

$$S \leq C \cdot M / \log(N/M) \quad (3)$$

then with probability $1 - O(e^{-\gamma^N})$, the signal can be exactly recovered.

When the measurement is corrupted with noise, such that $y = \Phi s + w$, $\|w\|_2 \leq \sigma$. The estimated signal \hat{s} can be recovered by solving

$$\hat{s} = \arg \min \| \hat{s} \|_1 \quad \text{s.t. } \|y - \Phi \hat{s}\|_2 \leq \sigma \quad (4)$$

When $\delta_{3s} + \delta_{4s} < 2$, C_s is some constant related to δ_{4s} , then we have

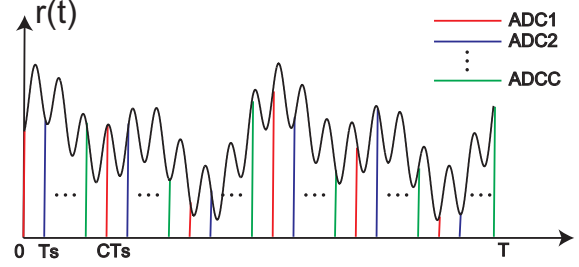
$$\| \hat{s} - s \|_2 \leq C_s \cdot \sigma \quad (5)$$

For the convex optimization problem in equation (2),(4), several types of Basic Pursuit (BP) and Orthogonal Matching Pursuit (OMP) algorithms can be applied for recovery. We propose a simple OMP based joint recovery algorithm in later sections.

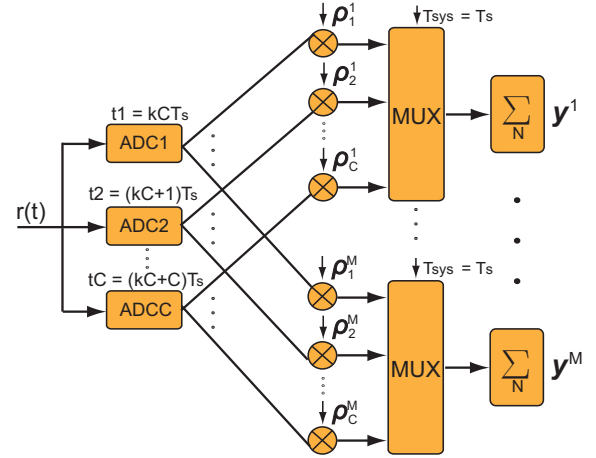
III. COMPRESSIVE SAMPLING AT SINGLE CR

In CR networks, the application of compressive sampling brings few measurements as well as higher detection rate. In this section, a low rate ADC based CR front-end architecture is firstly proposed, afterwards a compressive sensing formulation is discussed.

Consider a slot-segment model of wideband spectrum, where the whole spectrum is divided into K non-overlapping sub-bands. Suppose each sub-band requires C samples, totally $N = CK$ points at Nyquist rate should be sampled during each measurement procedure for exact spectrum recovery. In wideband cases, direct sampling requires high speed ADCs and produces huge data for storage and transmission. Recent emerging analog-to-information converter (AIC) techniques have made it possible to sample signals at sub-Nyquist rate. In this paper, we propose an novel AIC architecture for CR networks based on low rate ADCs. As shown in Fig.2, suppose during one measurement, a real-time signal $r(t)$ of finite-duration T is sampled. Suppose a N -point sample $r_t = [r(0), r(T_s), \dots, r(NT_s)]$ is required at Nyquist-Rate $1/T_s$,



(a) sampling sequence during one measurement period



(b) sampling structure synchronized by system clock $T_{sys} = T_s$

Fig. 2: AIC structure based on low rate ADCs

C parallel low speed ADCs operate at a sample rate $1/CT_s$. During one measurement period, each ADC starts conversion for K times at an interval of CT_s continuously. A $f = 1/T_s$ system clock controls the synchronization between C ADCs and switching sequences of multiplexers. The received signal $r(t)$ enters C ADCs simultaneously, each ADC data enters M sub-channels, multiplied by random weights ρ_c^m and accumulated N times. Noting that ρ_c^m changes to $\rho_c^m[k]$ at the k th conversion, produced by random number generators as i.i.d. gaussian variables, thus, after each measurement, the m th sub-channel outputs a compressed sample

$$y^m = \sum_{c=1}^C \sum_{k=1}^K \rho_c^m[k] r((cK + k)T_s) = \sum_{n=1}^N \mu^m[n] r_t[n] \quad (6)$$

where $\mu^m \in R^N$ consists of the rearranged weights $\rho_c^m[k]$, thus total M sub-channels will output

$$y = U r_t \quad (7)$$

where $y = [y^1, y^2, \dots, y^M]$ is the sampling vector and μ^m is the m th row of measurement matrix U . This AIC architecture is proposed because only $M \ll N$ memory units are required and each ADC sample at a low rate of $1/CT_s$ instead of the Nyquist rate $1/T_s$. Further, more ADCs could be integrated in one AIC to obtain higher spectrum resolution and further reduce sampling rate for each ADC.

Next, considering the signals are corrupted with noise in practise, we have

$$r_t = s_t + w_t \quad (8)$$

where s_t denotes signal before simultaneous ADCs. Suppose w_t is independent zero-mean white Gaussian noise with variance σ^2 , thus $w_t \sim \mathcal{N}(0, \sigma^2 I_N)$. After one measurement, the measurement vector becomes:

$$y = U r_t = U F_N s_f + \tilde{w} = \Phi s_f + \tilde{w} \quad (9)$$

where F_N denotes N-point orthogonal DFT matrix which is incoherent with matrix U , $\tilde{w} = U w_t$ denotes noise at the AIC output. s_f is sparse due to low occupance ratio across the whole sensing band, for simplicity, the subscript f is ignored in this paper. When ρ_c^m is chosen such that $\rho_c^m \sim \mathcal{N}(0, 1/M)$, it could be proved that $\Phi = U F_N$ also satisfies (1) with high probability.

Further, according to (1), the output noise level can be estimated as

$$\|\tilde{w}\|_2^2 \leq (1 + \delta_s) \|w_t\|_2^2 \quad (10)$$

These results can be immediately applied in compressive sampling (2), (3) and in joint recovery in next section.

IV. JOINT RECOVERY AT MULTIPLE CRS

In this section, we define a general joint sparse model for CR networks, and propose a universal recovery algorithm.

A. Model Definition

In wideband CR networks, signals received at each CR are highly spatial correlated, while shadowing and hidden terminal problems would cause signal strength detected at CRs varies significantly. This phenomenon indicates different support sets of s_j , for all $j = 1, 2, \dots, J$. Thus A general joint sparsity model (GJSM) is defined in this scenario.

Definition 1:

$$s_j = \Omega \theta_j + z_j, \quad j = 1, 2, \dots, J \quad (11)$$

We suppose each CR signal has the same partial common sparse support Ω and a proprietary term z_j . Define $\Lambda_c = \bigcap_j \text{supp}(s_j)$, for all $j = 1, 2, \dots, J$, then $\text{supp}(z_j) = \text{supp}(s_j) / \Lambda_c$. Ω is the sub-matrix of Identity matrix I_N , retaining columns indexed by Λ_c , θ_j denotes varying signal strength on this support set. Clearly, this model fits the scenario where frequency selective channel fading leads to different θ_j and hidden terminals contribute to existence of z_j .

We call this model general joint sparsity model, since when $\theta_j = \theta$, for all J CRs, this model degenerates to JSM1 [5]:

$$s_j = z_c + z_j, \quad j = 1, 2, \dots, J \quad (12)$$

z_c is the common component between all J signals. An γ weighted $l1$ algorithm [10] is proposed for recovery.

When $z_j = 0$, this model degenerates to JSM2 [5]:

$$s_j = \Omega \theta_j, \quad j = 1, 2, \dots, J \quad (13)$$

input : A $M \times N$ matrix Φ

$M \times 1$ samples $y_j, j = 1, 2, \dots, J$

A Discarding Threshold η

output: $N \times 1$ recovered signals $s_j, j = 1, 2, \dots, J$

initialization:

Residuals: $\varepsilon_j \leftarrow y_j$

Support sets: $\Lambda_j \leftarrow \emptyset$

Reference support set: $\Lambda_{ref} \leftarrow \emptyset$

1. Find the index ω that solve the optimization:

$$\omega = \arg \max_w \sum_{j=1}^J |\langle \varepsilon_j, \phi_\omega \rangle| \quad (14)$$

ϕ_ω denotes the ω th column of Φ .

2. **if** $\omega \notin \Lambda_{ref}$ **then**

$\Lambda_{ref} \leftarrow \Lambda_{ref} \cup \{\omega\}$

Calculate inner product terms $|\langle \varepsilon_j, \phi_\omega \rangle|$, suppose that ϕ_ω is related to the k th biggest term.

if $k > \eta N$ **then**

discard ω for the j th measurement.

else

$\Lambda_j \leftarrow \Lambda_j \cup \{\omega\}$

end

else

find the index ω separately that matches

$$\omega = \arg \max_w |\langle \varepsilon_j, \phi_\omega \rangle| \quad (15)$$

$\Lambda_j \leftarrow \Lambda_j \cup \{\omega\}$

end

3. Solve the least square problem to get new estimate of \hat{s}_{fj}

$$\hat{s}_{fj} = \arg \min \|y_j - \Phi_{\Lambda_j} \hat{s}_{fj}\|_2 \quad (16)$$

where Φ_{Λ_j} is of size $M \times N$ by setting $\phi_\omega = 0$, $\omega \notin \Lambda_j$, or equivalently,

$$\hat{s}_j = \Phi_{\Lambda_j}^\dagger y_j \quad (17)$$

4. Update residuals:

$$\varepsilon_j = y_j - \Phi_{\Lambda_j} \hat{s}_j \quad (18)$$

5. If stoping criterion is matched, iteration ends, else go to step 1.

Algorithm 1: SD-SOMP

Two efficient algorithms OSGA [5], SOMP [3], [5] are suggested for joint recovery under JSM2. Unfortunately, no existing joint recovery algorithm can be applied directly for our defined general model. We propose a novel joint algorithm which works well universally under different models.

B. Recovery Algorithm

In this subsection, we propose a novel joint recovery algorithm for the GJSM model. The algorithm consists of two major stages. In the first stage, like SOMP, one column is selected from the measurement matrix jointly at each iteration,

however, since not all signals share the same supports, a discarding step follows to make judgements separately. In the second stage, each measurement recovers signal separately using OMP [2], after joint selection failure in the previous stage. We call this method Select-Discard Simultaneous OMP (SD-SOMP). For simplicity, we assume all J CRs share the same measurement matrix Φ . According to earlier discussion, the stopping criterion can be made as $\|y_j - \Phi \hat{s}_j\|_2 \leq \alpha \sigma$ with some constant α or simply set a maximum iteration steps from estimated sparsity level.

C. Simulation Results

In this section, simulation results are provided to support the efficiency of our proposed SD-SOMP algorithm in wideband CR networks. The simulations are divided into two groups. The first group demonstrates superiority of SD-SOMP over existing joint recovery algorithms. The second validates the universality of this algorithm under different joint sparsity models.

Simulation 1: Efficiency of SD-SOMP

First, it's easy to see SOMP or OSGA algorithm cannot be applied under our joint sparsity model since these algorithm assume signals all share the same sparse supports. We perform two simulations to show γ weighted algorithm for JSM1 also deteriorates under GJSM. Suppose $N = 1024$ points signal to be recovered. $J = 5$ CR users are present during once measurement procedure. For simplicity, suppose all signals are 12 sparse, with $\text{SNR} = 30\text{dB}$. First assume all signals share a 11 sparse common component. Clearly, this fits the JSM1 model. The threshold η is set to 0.02. Fig.3 shows recovery probability vs. samples per CR. It can be seen γ weighted algorithm works better since it mostly utilize the joint sparsity level. However, when choosing the signals to have 11 common sparsity supports, which fits GJSM. SD-SOMP outperforms this algorithm, as shown in fig.4. Note all the other parameters remain the same as in fig.3. In fig.5, different SNR levels are compared, in high SNR level cases, recovery probability keeps relatively stable resulting from joint support selection. Clearly, more samples or measurements are required in low SNR cases.

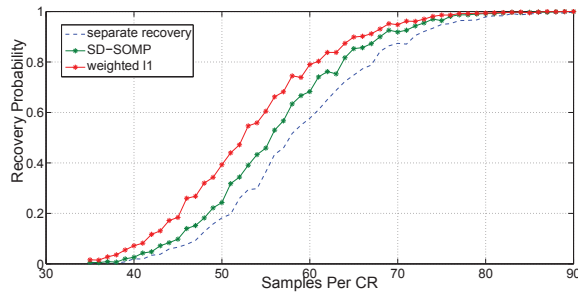


Fig. 3: Recovery probability vs. Samples per CR under JSM1

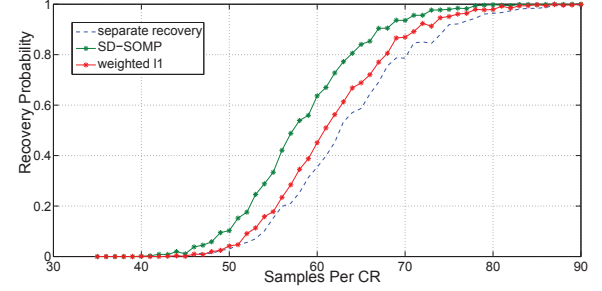


Fig. 4: Recovery probability vs. Samples per CR under GJSM

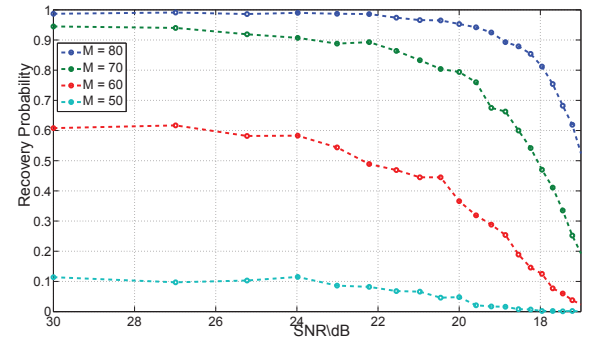


Fig. 5: Recovery probability vs. Different SNRs

Simulation 2: Universality of SD-SOMP

Here, we provide two simulations to demonstrate the universality of SD-SOMP, resulting from the choice of the discarding threshold η . Fig.6 shows recovery probability vs. discarding threshold under different joint sparsity levels. We set signal length to 1024 and each signal is 10 sparse with $\text{SNR} = 30\text{dB}$, each CR samples 80 points. 10 CRs are included. First observe that under lower thresholds, SD-SOMP outperforms separate recovery under all Joint Sparsity Levels. As the threshold increases, denoting looser discarding criterion, SD-SOMP will deteriorate caused by falsely selected sparse supports. Also, higher joint sparsity level deteriorate slower, further, under highest joint sparsity level, SD-SOMP always outperforms separate recovery. Noting that higher threshold contributes to faster convergence, there is a tradeoff between convergence speed and recovery probability. When joint sparsity level is high, threshold can be set higher, and vice versa. Fig.7 shows the universality of SD-SOMP under non-joint sparse cases, the signals are set to be totally uncorrelated with $\text{supp}(s_i) \cap \text{supp}(s_j) = \emptyset$, thus by decreasing the threshold, SD-SOMP degenerates to traditional separate algorithm.

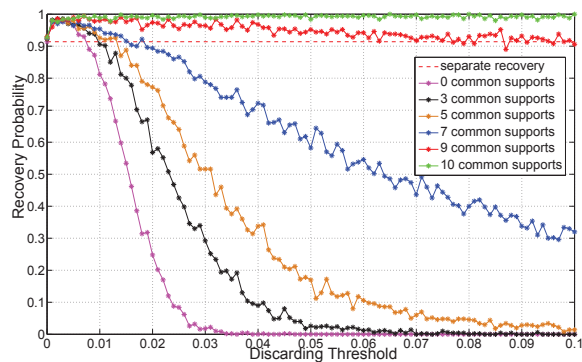


Fig. 6: Recovery probability vs. Discarding threshold under GJSM

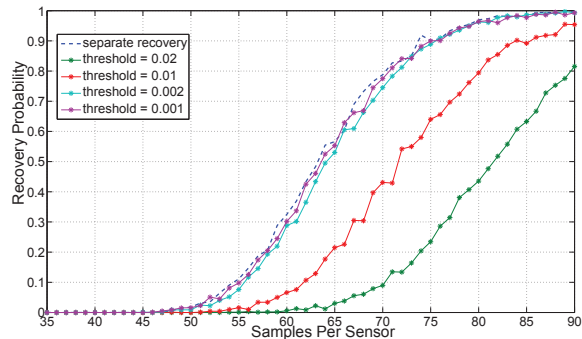


Fig. 7: Recovery probability vs. Samples per CR under non JSM

V. CONCLUSION

In this paper, we propose a novel AIC structure of CR front-end integrating low rate ADCs and few storage units. Then, we discuss compressive spectrum sampling under this scheme, considering noisy cases. Further, we explore a new joint sparsity model in CR networks and provide a universal SD-SOMP algorithm to perform joint spectrum reconstruction. Simulations show SD-SOMP is actually a robust recovery algorithm under different joint sparsity models.

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