Midterm Exam #2

(2013/12/5–2013/12/12)

Notes:

- 4 problems (choose either Problem 3.1 or 3.2, not both), 100 points.
- Take-home, open to everything.
- Cross-talk among peer classmates is prohibited (except for programming skill).
- Submit your written/printed answers to TA by 12/12 (Thursday), 11:00 am.
- Matlab codes "CFT.m" and "ICFT.m" are provided to facilitate (inverse) Fourier transform for continuous functions..

1) (10%) Let $a(t)=\sqrt{I(t)}\times e^{j\phi(t)}$ and $A(\omega)=|A(\omega)|\times e^{j\psi(\omega)}=F\{a(t)\}$ represent the complex pulse envelopes in time and frequency domains, respectively. Prove that: $\psi(\omega)=0$, $\Rightarrow I(t)$ is even, $\phi(t)$ is odd. Describe the physical meanings.

2) Referring to Problem 2 in HW4, where the permitted small signal gain g_0 lies between 0.25 and 0.344.

2A) (5%) Define the normalized small signal gain as $\tilde{g}_0=a\times g_0+b$, such that $\tilde{g}_0=0$ (1) when $g_0=0.25$ (0.344). What are the constant coefficients *a* and *b*?

2B) (20%) Plot the steady-state pulse width Δt (FWHM) versus $\tilde{g}_0$ for $\tilde{g}_0=$0-1. (*Hint*: Solve the steady-state amplitude a_0 numerically, then use the fixed relation between a_0 and Δt.)

3.1) (25%) Assume a double-peak pulse $I(t)=e^{-[(t+T/2)/t_p]^2}+e^{-[(t-T/2)/t_p]^2}$, where t_p and T represent the individual pulse width (FWHM is $\Delta t=2\sqrt{\ln 2}t_p$) and peak separation, respectively. The intensity autocorrelation function $G_2(\tau)\propto\langle I(t)I(t-\tau)\rangle$ is expected to have a single main lobe then two side lobes as T gradually increases. What is the "boundary peak separation T_b" (normalized to Δt) above which the resulting $G_2(\tau)$ starts to have side lobes? Discuss your result. (*Hint*: The slope of any peak is zero.)

3.2) (25%) Let the complex spectral envelope be $A(\omega)=e^{-(\omega/\Delta_\omega)^2}\cdot e^{-j\gamma(\omega/\Delta_\omega)^3}$, where Δ_ω and γ represent the bandwidth (non-angular frequency FWHM is $\Delta\nu=\sqrt{\ln 2/(2\pi^2)}\Delta_\omega$) and normalized cubic phase strength, respectively. The temporal intensity $I(t)$ is expected to have asymmetric oscillating tails as γ gradually increases (page 7, Lesson 6 slides). What are the γ-values $\gamma_{0.1}$, $\gamma_{0.5}$ where 10% and 50% pulse energies belong to the tails

(90% and 50% energies belong to the main lobe), respectively? Plot the two corresponding $G_2(\tau)$ curves (τ is normalized to the FWHM of $I(t)$ at γ=0, i.e. $2.35/\Delta_\omega$). Discuss your result.

4) (40%) Implement the iterative Fourier transform algorithm to retrieve a PG-FROG trace, where the gate function is the intensity of the signal pulse:

$$g(t) = |a(t)|^2 .$$

The data are stored in "PGFROGdata.mat", which contains:

f (1×256): non-angular frequencies (detuning), $[-4 \sim +4]$ (unit: THz).

tau (1×129): time delays, roughly $[-8 \sim +8]$ (unit: ps).

I_PGFROG (256×129): PG-FROG trace.

You can visualize the trace in Matlab program by typing the following commands:

```
load PGFROGdata;      contourf(tau,f,I_PGFROG);        axis([-4 4 –2 2])
```

A figure like Fig. 1 will be illustrated.

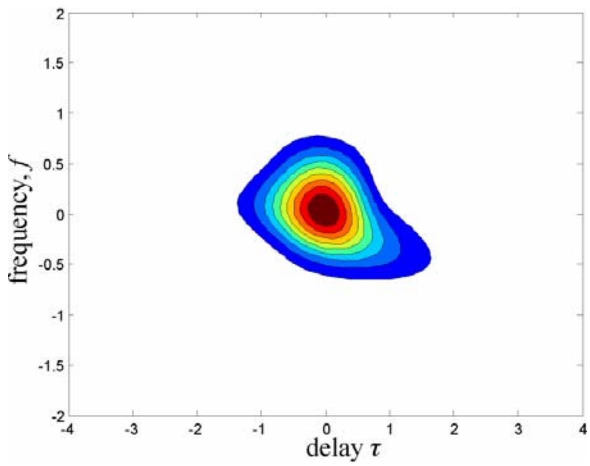

Fig. 1. PG-FROG trace.

Try your best to retrieve the pulse numerically. Pay attention to the practical issues discussed in class. Iterate the loop by ~150 times or as many as you wish until your rms FROG error $G^{(k)}$ defined in page 20 of Lesson 6 slides is low enough or converged.

Keeping track of the result corresponding to the lowest FROG error, and sketch the following figures for it:

4A) rms FROG error $G^{(k)}$ (in dB, i.e. $10\log_{10}\left(G^{(k)}\right)$) versus the number of iterations. Denote the minimum FROG error that you have achieved.

4B) The retrieved FROG trace $I_{FROG}(f,\tau)$ for $\tau =$[-4,4] (ps), $f =$[-2,2] (THz).

4C) Temporal intensity $|a(t)|^2$ for $t =$[-4,4] (ps). Calculate the FWHM Δt.

4D) Spectral intensity $|A(f)|^2$ and phase $\psi(f)$ in the same figure for $f =$[-2,2] (THz). Calculate the FWHM $\Delta\nu$.