

# Homework 10

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Q1

e)

The temporal cost of function `Graph::isClique()`, `Graph::CliqueExists()`, and `Graph::getMaxClique()` are  $O(n^2)$ ,  $O(n^2 * 2^n)$  and  $O(n^2 * 2^n)$ , separately.

Because the `isClique()` function just consists of two loops. The operator in the inner loop is constant. So the cost complexity is  $O(n^2)$ .

As for `CliqueExist()` and `getMaxClique()`, although I used the DFS to find all the subset of the input vector. The cost of this step is  $O(n^2)$ . The second step is to find the proper  $K$  or the `MaxClique`. The variable  $i$  in the outer loop varies from 0 to  $2^n - 1$ , in the inner loop, the `isClique()` is an  $n^2$  operator. So the total temporal cost is  $O(n^2 * 2^n)$ .

Q2

- a) NPC class of problem is a set of problems which meets two requirements: firstly, those problems are NP problems; secondly, all the NP problems in this class can be reduced to this NPC problem.
- b) The proof process can be divided into two step: firstly, we should prove the unknown problem is a NP problem; Then, prove a existed NPC problem can transform into this unknown problem here.
- c) Because if we find a polynomial solution for one NPC question, all the NP question will be solved. In this way, we can prove  $P = NP$ . In other word, we can earn 1 million dollar.
- d) The graph coloring problem is a classical NPC problem. It is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints, which is no two adjacent vertices share the same color. The chromatic number  $\chi(G)$  of a graph  $G$  is the minimal number of colors for which such an assignment is possible. It is a NPC problem.