CS5340 - Lab4 Report

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Part 1: Importance Sampling

In part 1 we use Importance Sampling with given target conditional distribution $p(X) = \prod_{p_u}(x_u|x_{\pi_u})$ and proposal distribution $q(X) = \prod_{q_u}(x_u|x_{\pi_u})$, to estimate a conditional probability distribution $p(X_F|X_E)$ where $X = X_F \cup X_E$ is the set of all variables.

- After modifying the graph structure as well as the proposal distribution $q(X)$ with evidence, we first generate a sample $x^{(l)}$ of all variables X in **topological order**, by sampling each variable x_u from $q_u(x_u|x_{\pi_u})$ given its parents x_{π_u}.
- Given samples $x^{(1)}, x^{(2)}, \ldots, x^{(L)}$, we can estimate the conditional probability $p(X_F|X_E)$ by computing the importance weights w_l with

$$w_l = \frac{\tilde{p}(x^{(l)})}{q(x^{(l)})} = \frac{\prod_{p_u} p_u(x_u^{(l)}|x_{\pi_u}^{(l)})}{\prod_{q_u} q_u(x_u^{(l)}|x_{\pi_u}^{(l)})}$$

$$p(X_F|X_E) \approx \frac{\sum_{l=1}^{L} w_l \mathbb{I}(x_F^{(l)} = x_F)}{\sum_{l=1}^{L} w_l}$$

  where $\mathbb{I}(x_F^{(l)} = x_F)$ is the indicator function that is 1 if $x_F^{(l)} = x_F$ and 0 otherwise.

Part 2: Gibbs Sampling

In part 2 we use Gibbs Sampling to estimate a similar $p(X_F|X_E)$ where $X = X_F \cup X_E$ is the set of all variables, given conditional probabilities $q_u(x_u|X - \{x_u\})$ for each variable x_u.

- we first update the given conditional probabilities $q_u(x_u|X - \{x_u\})$ to $q_u(x_u|MB(x_u))$ where $MB(x_u)$ is the Markov Blanket of x_u.
- we then define a Gibbs Sampling procedure to sample x_u from $q_u(x_u|MB(x_u))$ given the current values of all variables in Markov Blanket $MB(x_u)$.
- After initial burn-in stage, we generate samples $x^{(1)}, x^{(2)}, \ldots, x^{(L)}$ by running the Gibbs Sampling procedure for L iterations. We can estimate the conditional probability $p(X_F|X_E)$ by computing the empirical distribution of X_F given X_E from the samples.

$$p(X_F|X_E) \approx \frac{1}{L}\sum_{l=1}^{L} \mathbb{I}(x_F^{(l)} = x_F)$$

Case	1	2	3	4	5
Importance Sampling	7.97s	4.34s	20.70s	14.93s	3.85s
Gibbs Sampling	11.82s	7.38s	28.34s	20.40s	-

Table 1: Timings on M1-pro chip @ 3.22GHz