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Part 1: Implementing policy gradient algorithms

Implementation

REINFORCE

1.参数化策略
在REINFORCE中，策略通过参数θ定义，表示在状态s下采取动作a的概率分布。θ是一个$nActions \times nStates$的向量，$\phi(s,a)$是$nActions \times nStates$维线性空间下的标准正交基：
$\pi_\theta(a|s) = \frac{exp(\theta_{a,s})}{\sum_{a'} exp(\theta_{a',s})}$

根据课件上softmax policy的gradient算法：
$\nabla_\theta log\pi(a|s) = \phi(s,a) - E_{a\sim\pi}[\phi(s,a)]$

2.生成轨迹
对于每个 episode：

- 从初始状态s_0开始。
- 根据当前策略π_θ选择动作a_t，执行动作后进入下一个状态s_{t+1}，并获得即时奖励r_t。
- 累计计算折扣奖励

3.策略梯度更新
$G_t \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} r_k$

$\theta \leftarrow \theta + \alpha\gamma^t G \nabla_\theta ln\pi_\theta(a_t|s_t)$

Actor-Critic

1.参数化策略（Parameterized Policy）
和上述REINFORCE相同，策略通过参数θ定义，表示在状态s下采取动作a的概率分布。θ是一个$nActions \times nStates$的向量，$\phi(s,a)$是$nActions \times nStates$维线性空间下的标准正交基：
$\pi_\theta(a|s) = \frac{exp(\theta_{a,s})}{\sum_{a'} exp(\theta_{a',s})}$

根据课件上softmax policy的gradient算法：
$\nabla_\theta log\pi(a|s) = \phi(s,a) - E_{a\sim\pi}[\phi(s,a)]$

2.更新规则
时间差分（TD）误差：
$\delta_t \leftarrow r_t + \gamma V_w(s_{t+1}) - V_w(s_t)$
Critic更新：

$w \leftarrow w + \beta\delta\nabla V(s,w)$
Actor更新：
$\theta \leftarrow \theta + \alpha I_t \delta \nabla_\theta ln\pi_\theta(a_t|s_t)$

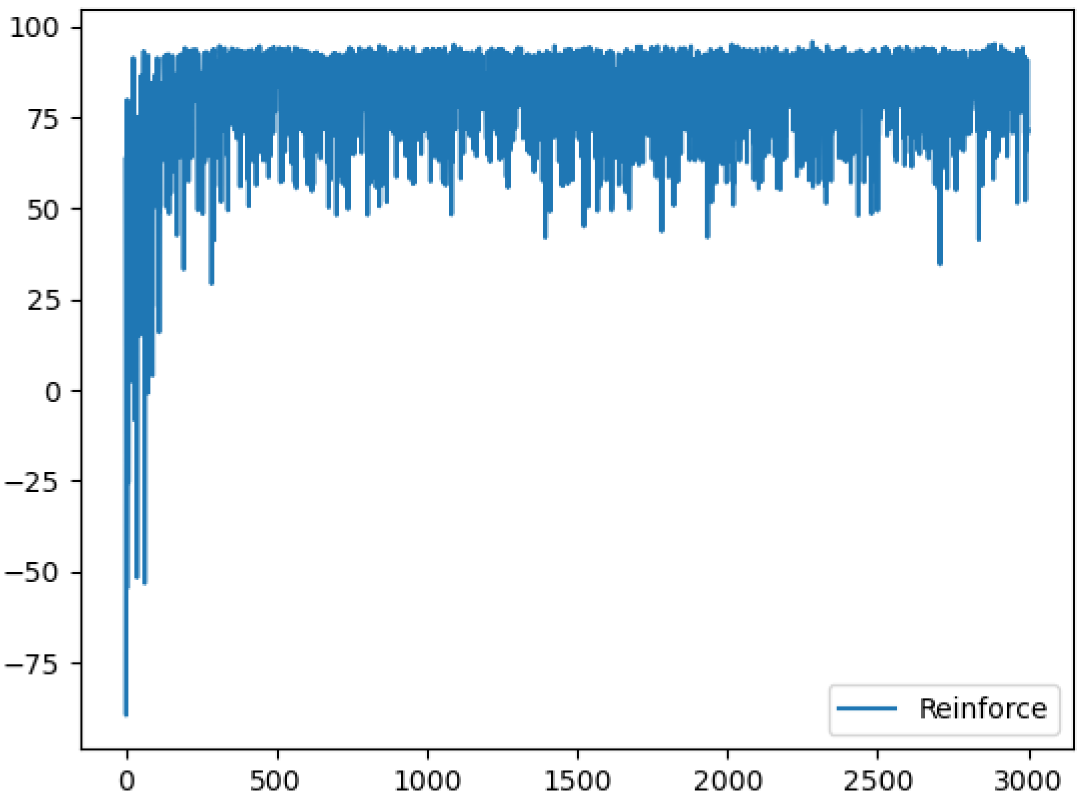
3.算法步骤

- 初始化策略参数θ和值函数参数w。
- 对于每个episode：
    - 从初始状态s_0开始，设置积累因子I=0
    - 生成轨迹：在每一步中，
        - 按照策略选择动作
        - 执行动作，观察即时奖励r_t和下一个状态s_{t+1}。
        - 计算TD误差δ_t
        - 更新Critic参数w和Actor参数θ
        - 累积折扣奖励

Evaluation

n_episode = 1000
n_trials = 10

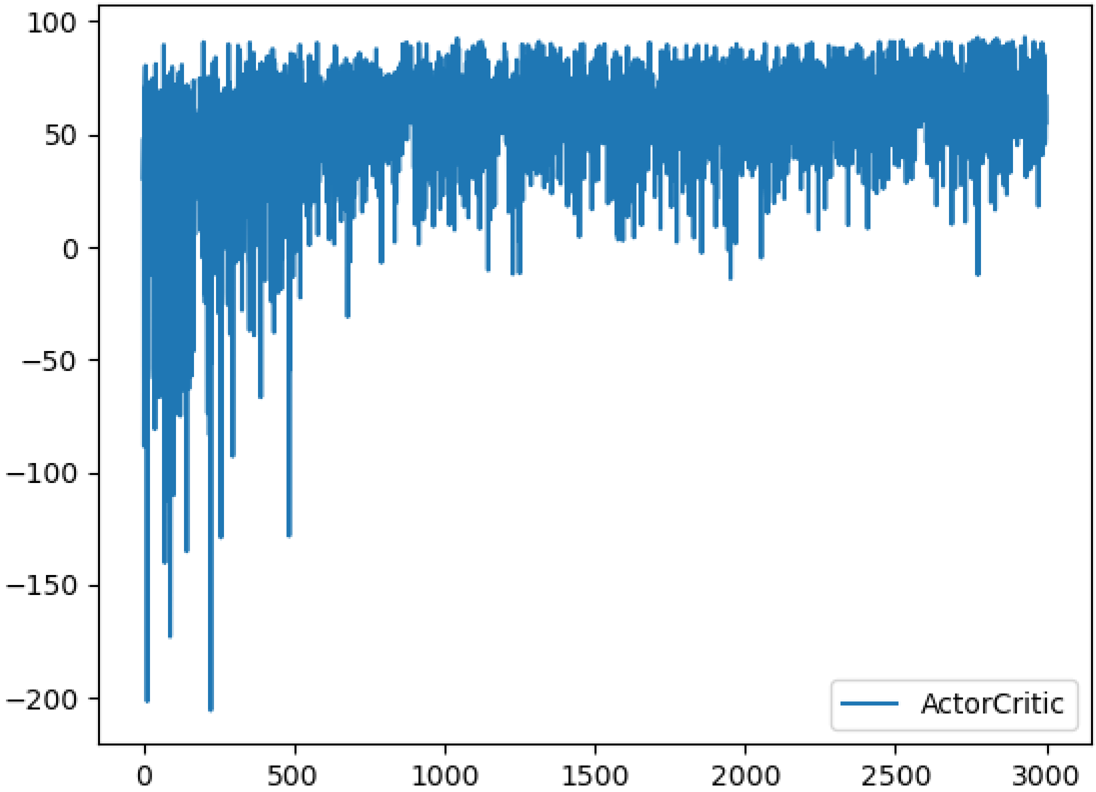
REINFORCE



Code

```
    def reinforce(self, theta, alpha=0.001, nEpisodes=3000):
        n_actions, n_states = self.mdp.R.shape
        if theta is None:
            theta = np.random.rand(n_states, n_actions)

        cum_rewards = np.zeros(nEpisodes)

        for episode in range(nEpisodes):
            state = 0
            trajectory = []
            done = False

            while not done:
                probs = self.policy(theta, state)
                action = np.random.choice(range(n_actions), p=probs)
                reward, next_state = self.sampleRewardAndNextState(state, action)
                trajectory.append((state, action, reward))
                cum_rewards[episode] += reward
                state = next_state

                if state == self.mdp.nStates - 1:
                    done = True

            for t, (s, a, r) in enumerate(trajectory):
                G_t = sum([(self.mdp.discount ** k) * reward for k, (_, _, reward) in
enumerate(trajectory[t:])])
                probs = self.policy(theta, s)
                grad_log_pi = -np.ones_like(probs) / self.mdp.nActions
                grad_log_pi[a] += 1
                theta[:, s] += alpha * (self.mdp.discount ** t) * G_t * grad_log_pi

        return cum_rewards, theta
```

Actor-Critic



Code

```
def actorCritic(self, theta, alpha=0.001, beta=0.01, nEpisodes=3000):
    n_actions, n_states = self.mdp.R.shape
    if theta is None:
        theta = np.random.rand(n_states, n_actions)
    w = np.zeros(n_states)

    cum_rewards = np.zeros(nEpisodes)

    for episode in range(nEpisodes):
        state = 0
        I = 1
        done = False

        while not done:
            probs = self.policy(theta, state)
            action = np.random.choice(range(n_actions), p=probs)
            reward, next_state = self.sampleRewardAndNextState(state, action)

            td_error = reward + self.mdp.discount * w[next_state] - w[state]
            w[state] += beta * td_error
            grad_log_pi = -np.ones_like(probs) / self.mdp.nActions
            grad_log_pi[action] += 1
            theta[:, state] += alpha * I * td_error * grad_log_pi
```

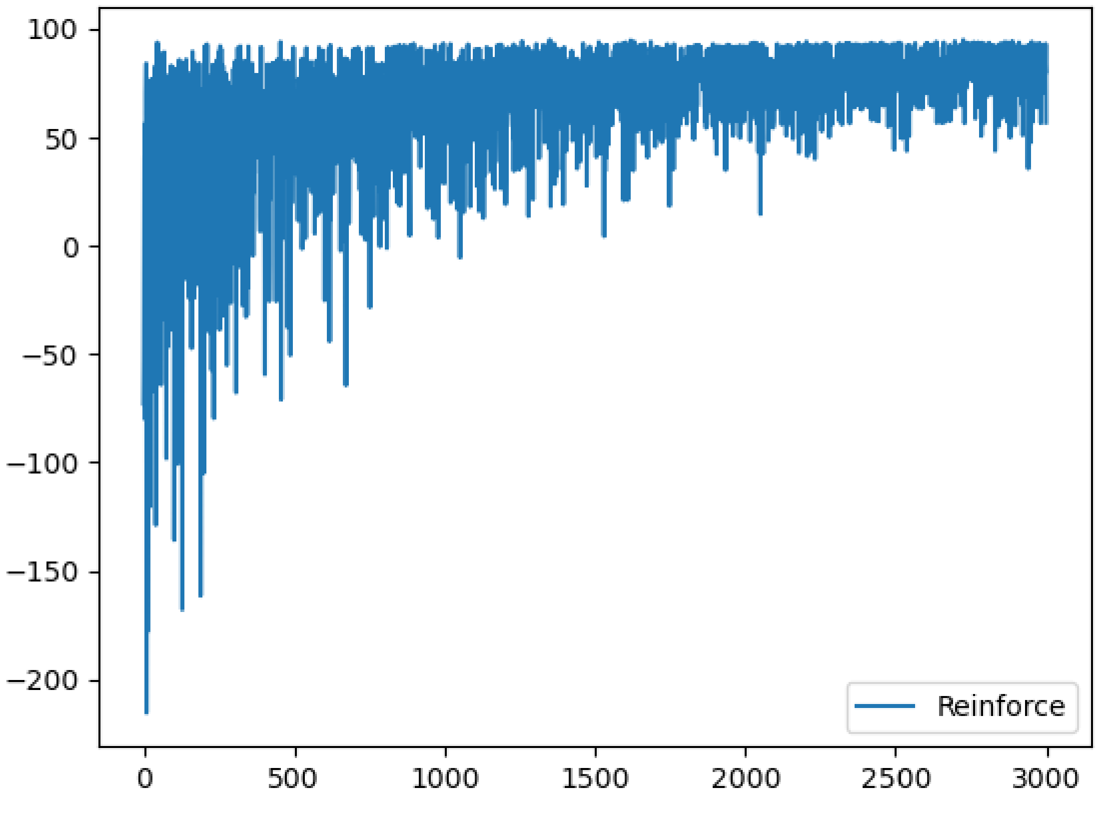
```
            cum_rewards[episode] += reward
            I = self.mdp.discount * I
            state = next_state

            if state == self.mdp.nStates - 1:
                done = True


        return cum_rewards, theta
```

Part 2: Fine-tuning and Comparison

learning rate tuning

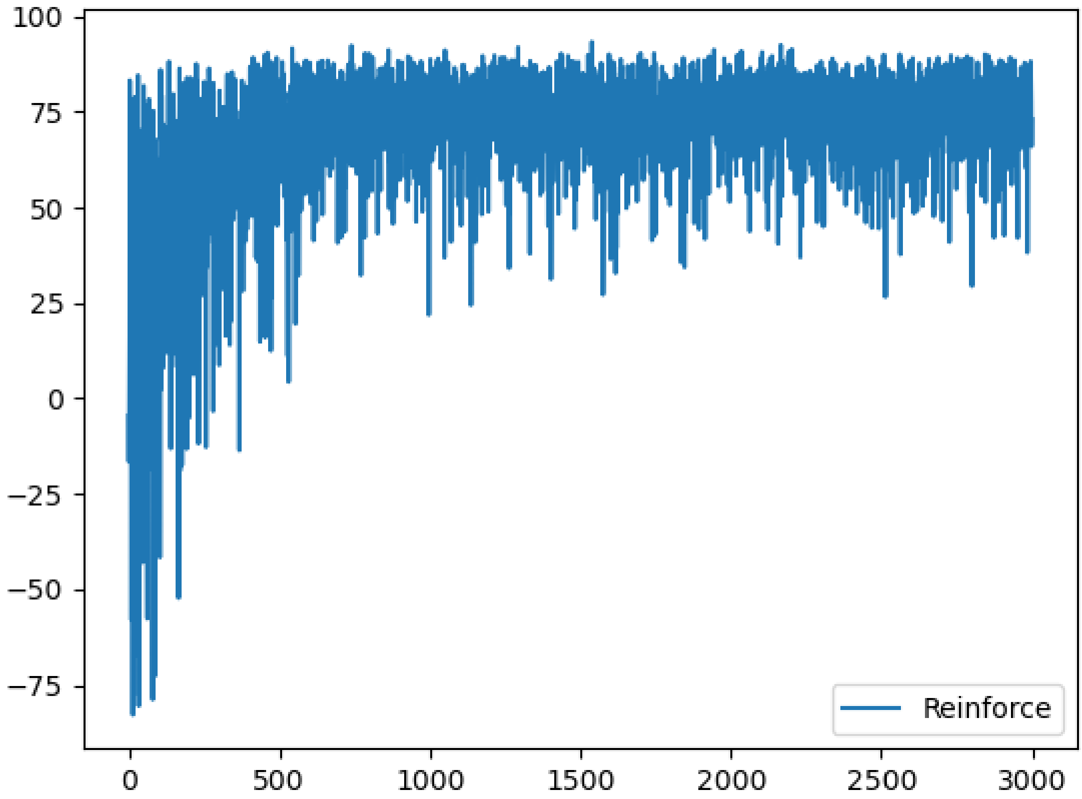
REINFORCE

$\alpha = 0.0001$:
大概在episode达到2000时收敛



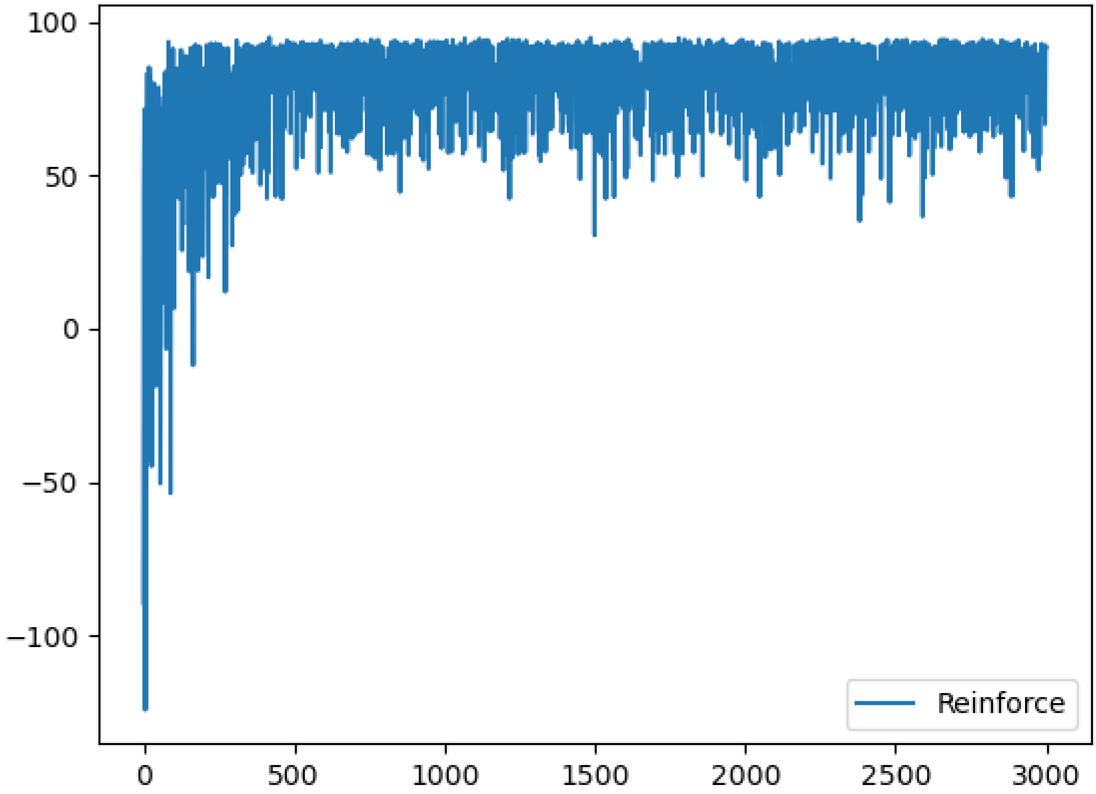
$\alpha = 0.0003$:

大概在episode达到1300时收敛



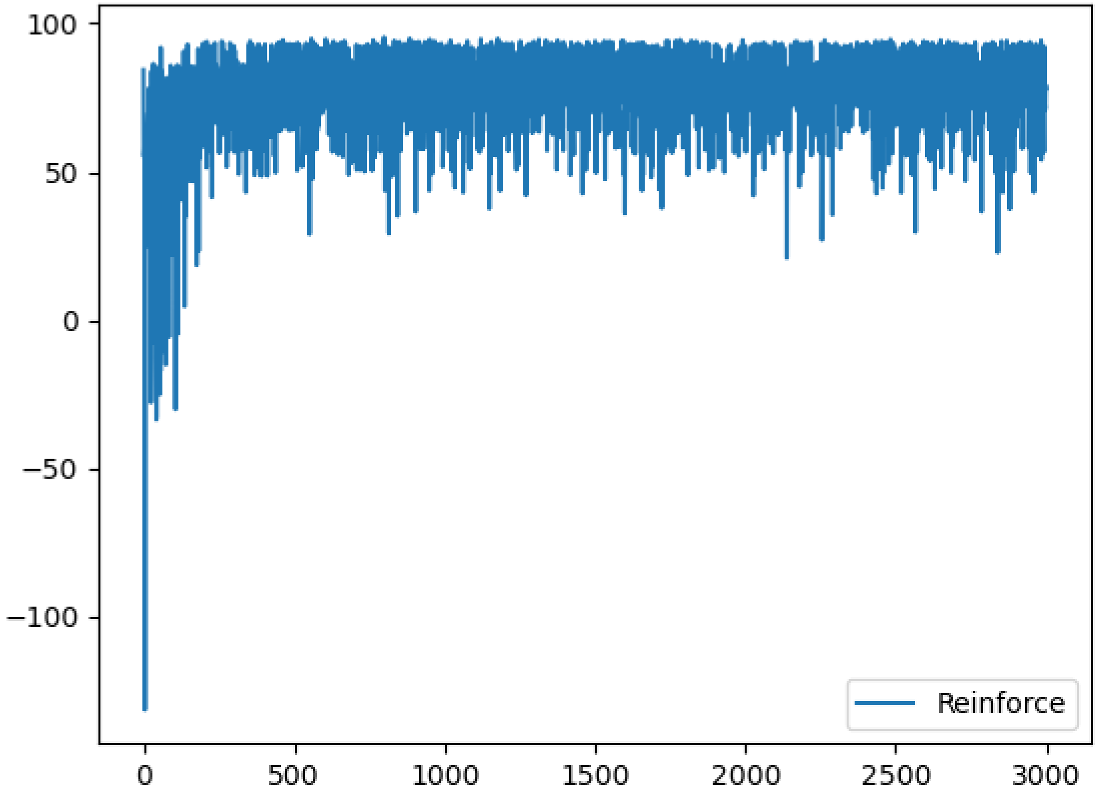
$\alpha = 0.0005$:

大概在episode达到900时收敛



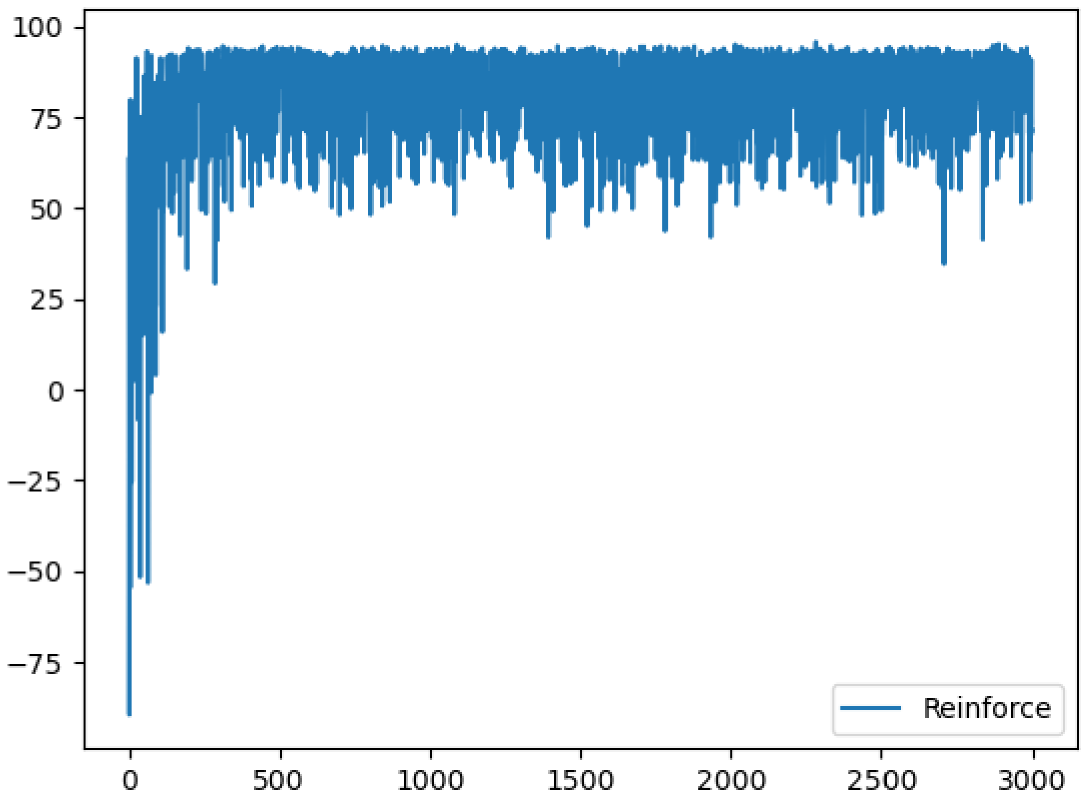
$\alpha = 0.0007$:

大概在episode达到500时收敛

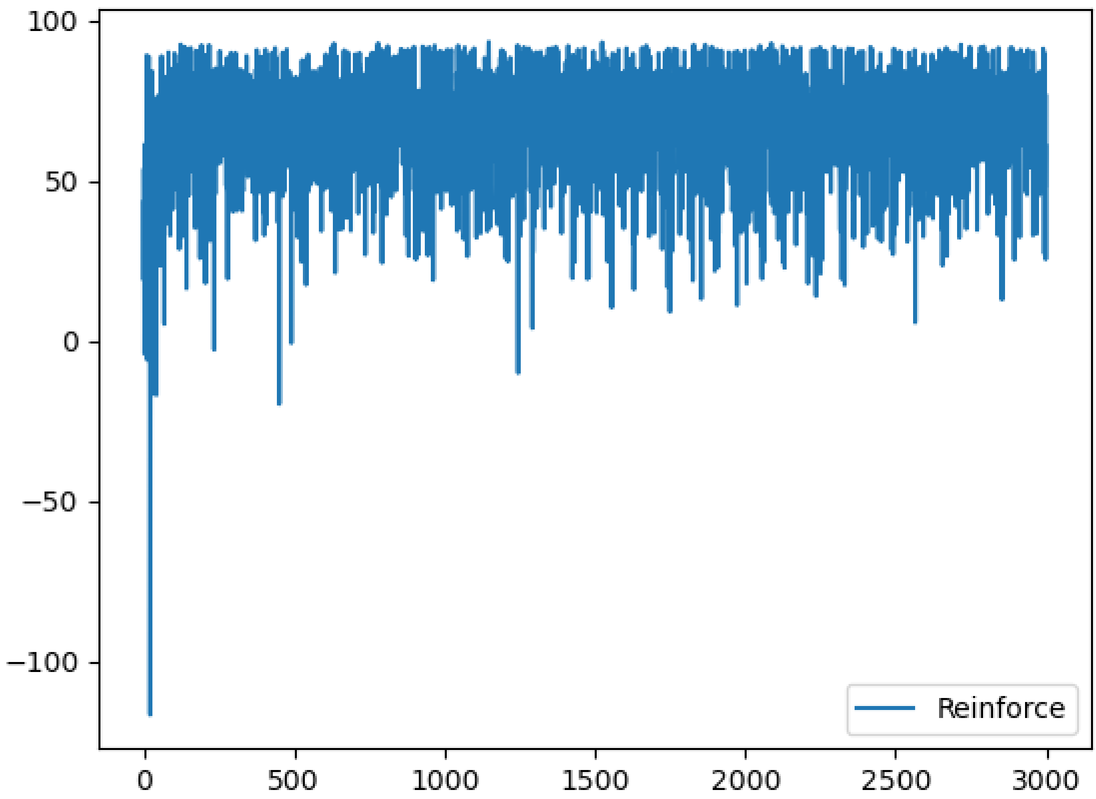


$\alpha = 0.001$:

大概在episode达到300时收敛



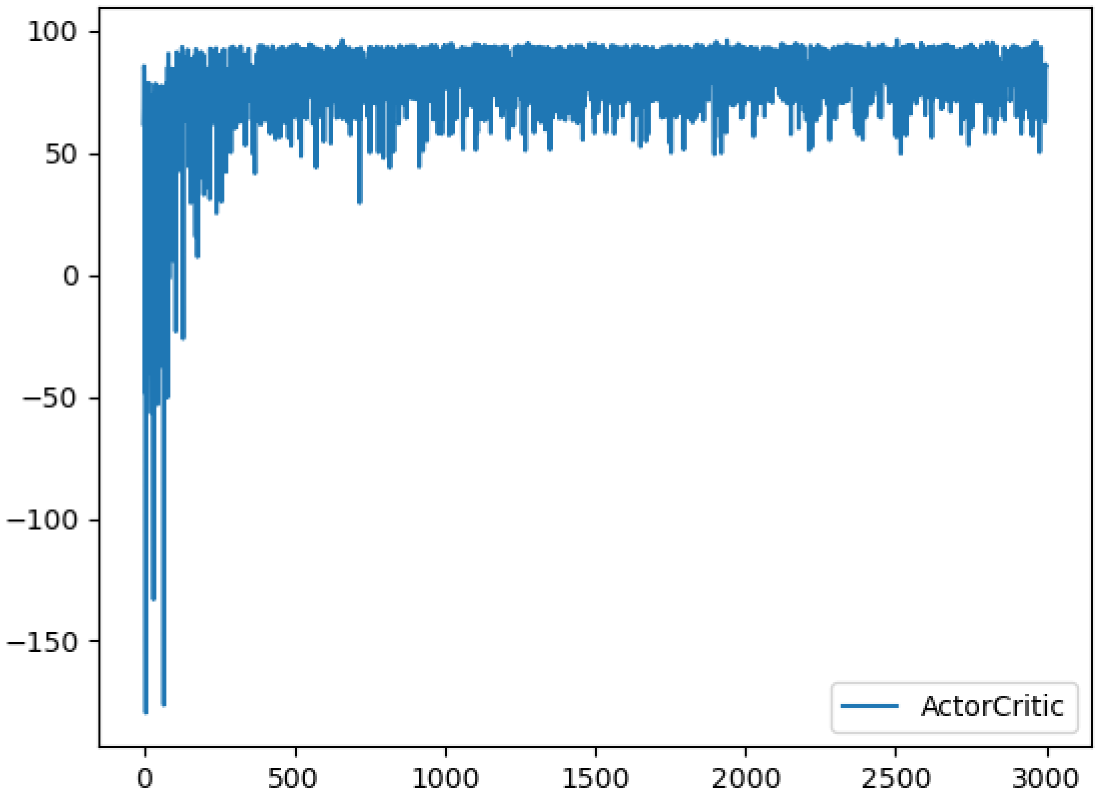
$\alpha = 0.003$:
大概在episode达到250时收敛



由上述可知在一定范围内α越大，收敛得越快。推荐选择α为0.001或0.0007，收敛速度较快，且较为稳定。

Actor-Critic

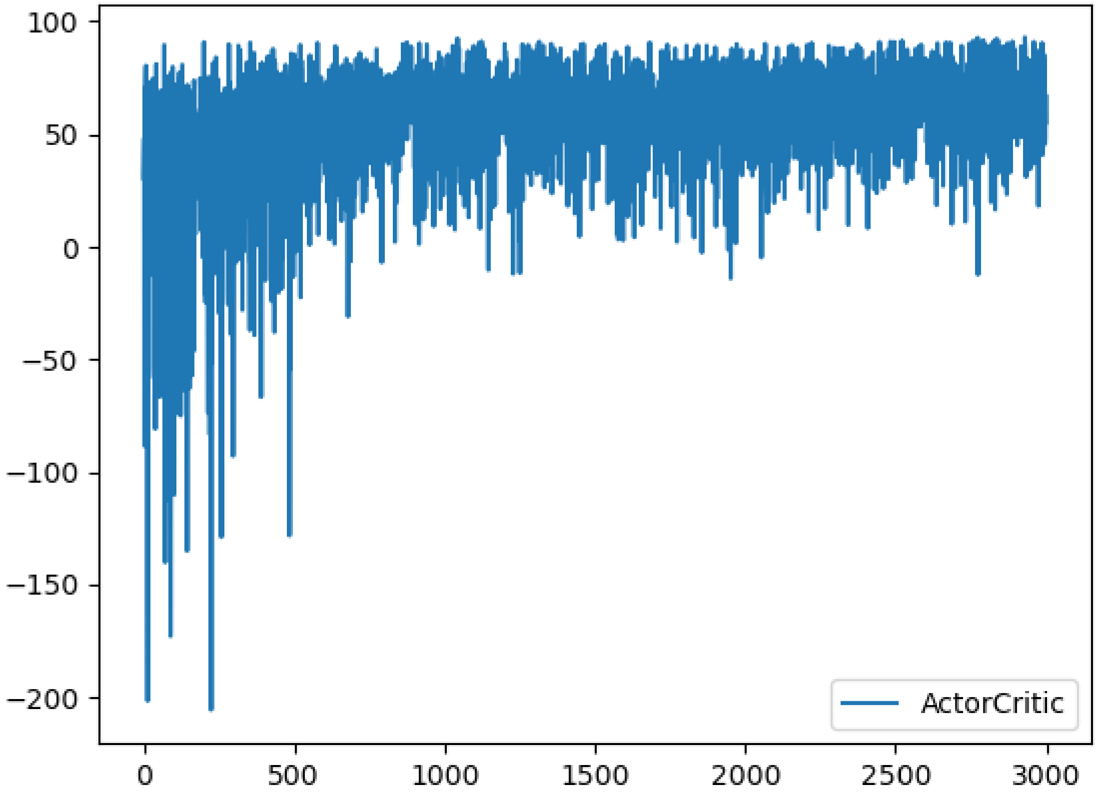
由一般的经验可知，Critic 的学习率一般比 Actor 大一到两个数量级，因为 Critic 的更新需要更快速跟踪状态值估计。
$\alpha = 0.001, \beta = 0.01$:
大概在episode达到800时收敛



100
50
0
−50
−100
−150
0
500
1000
1500
2000
2500
3000
ActorCritic

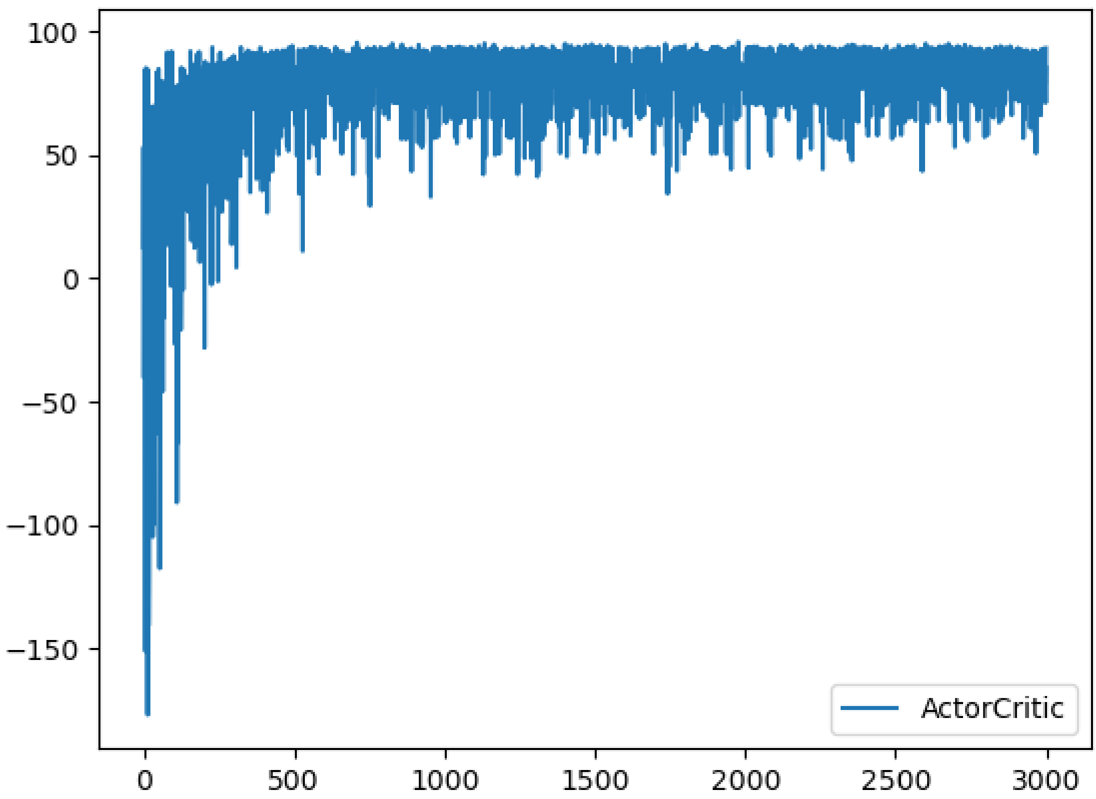
$\alpha = 0.001, \beta = 0.001$:

大概在episode达到1200时收敛



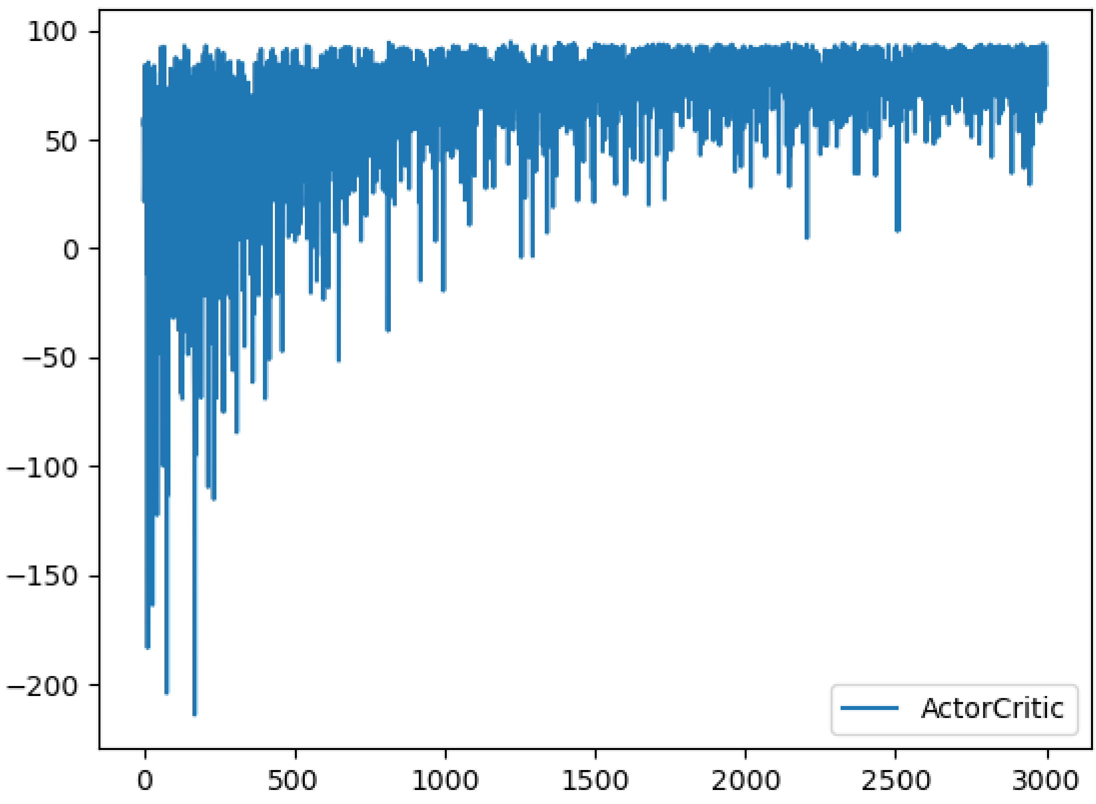
$\alpha = 0.001, \beta = 0.005$:

大概在episode达到1000时收敛



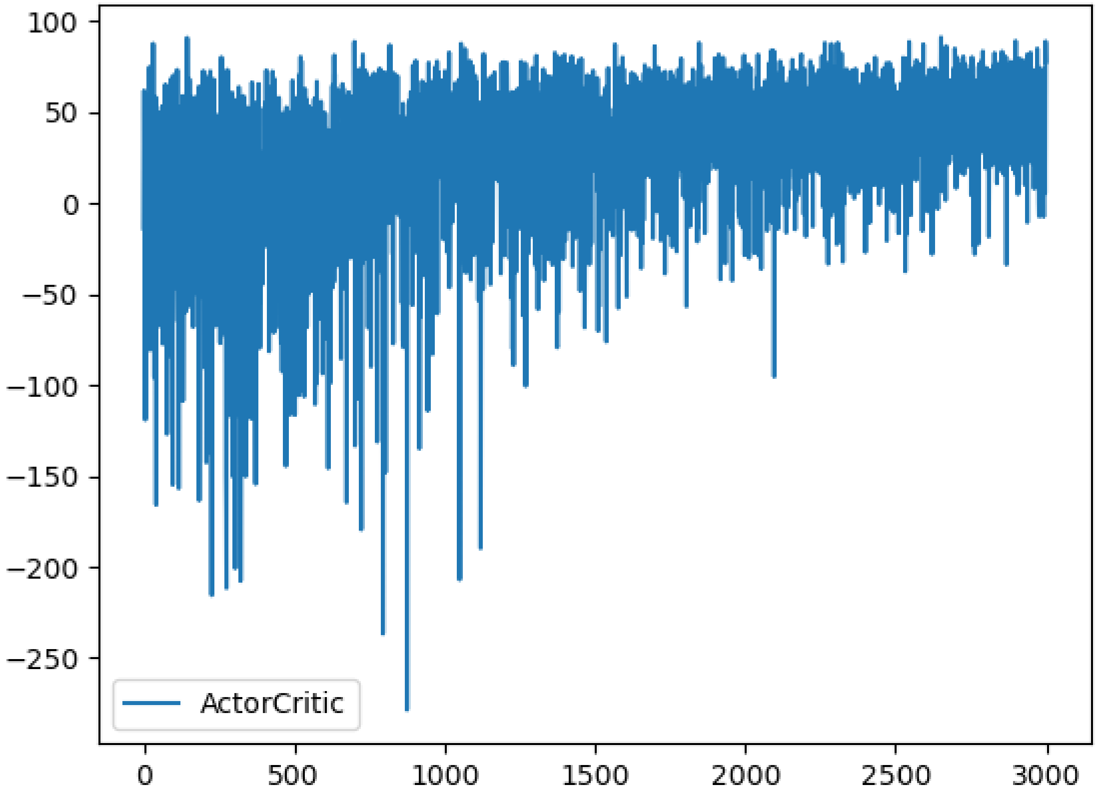
$\alpha = 0.0001, \beta = 0.01$:

大概在episode达到1700时收敛



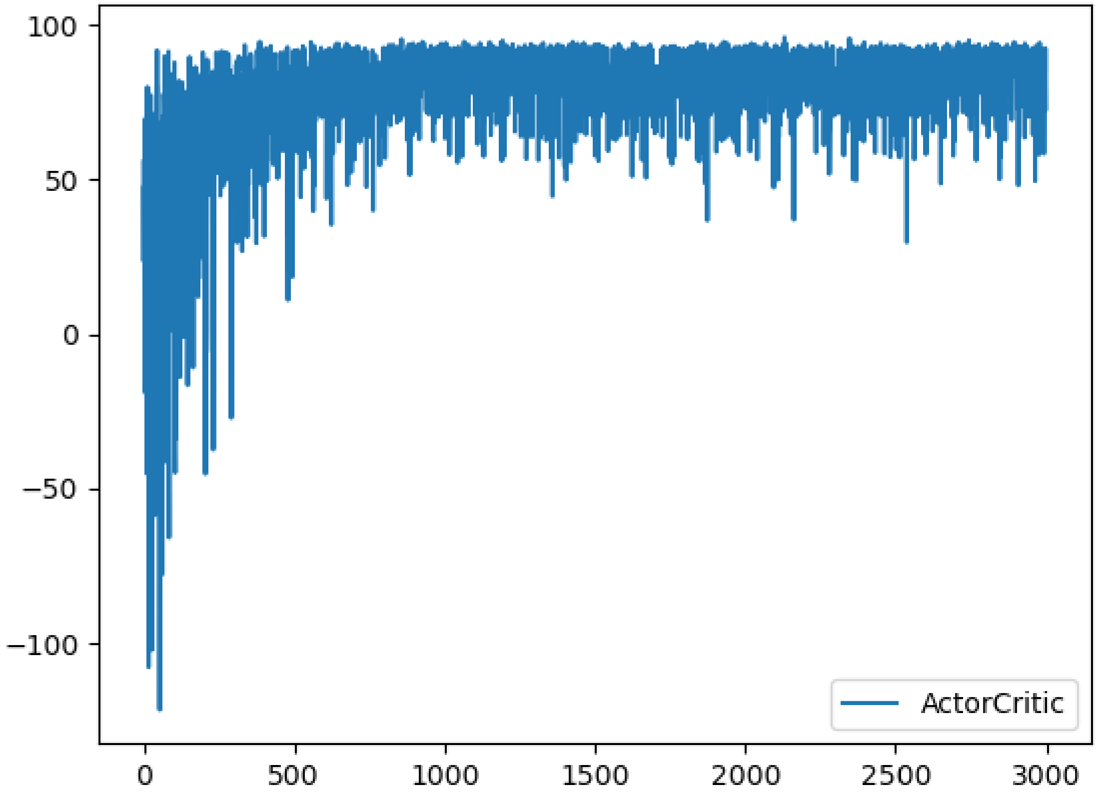
$\alpha = 0.0001, \beta = 0.001$:

大概在episode达到3000时收敛



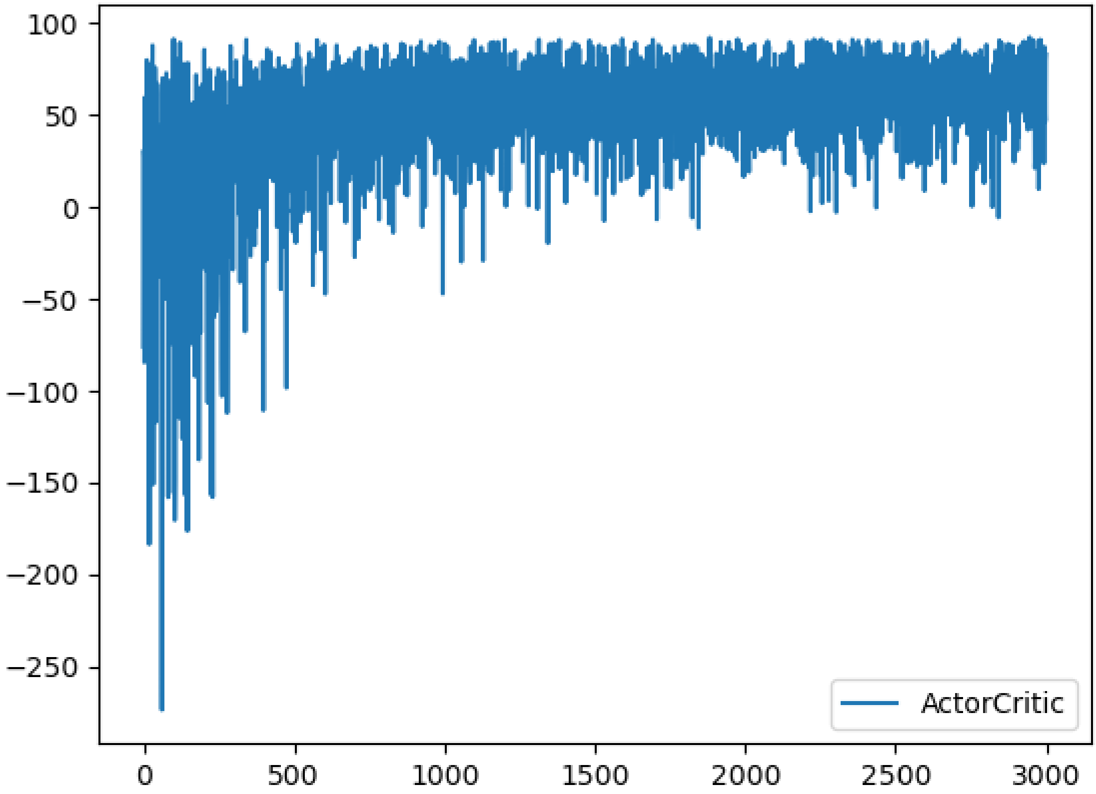
$\alpha = 0.0005, \beta = 0.01$:

大概在episode达到1100时收敛

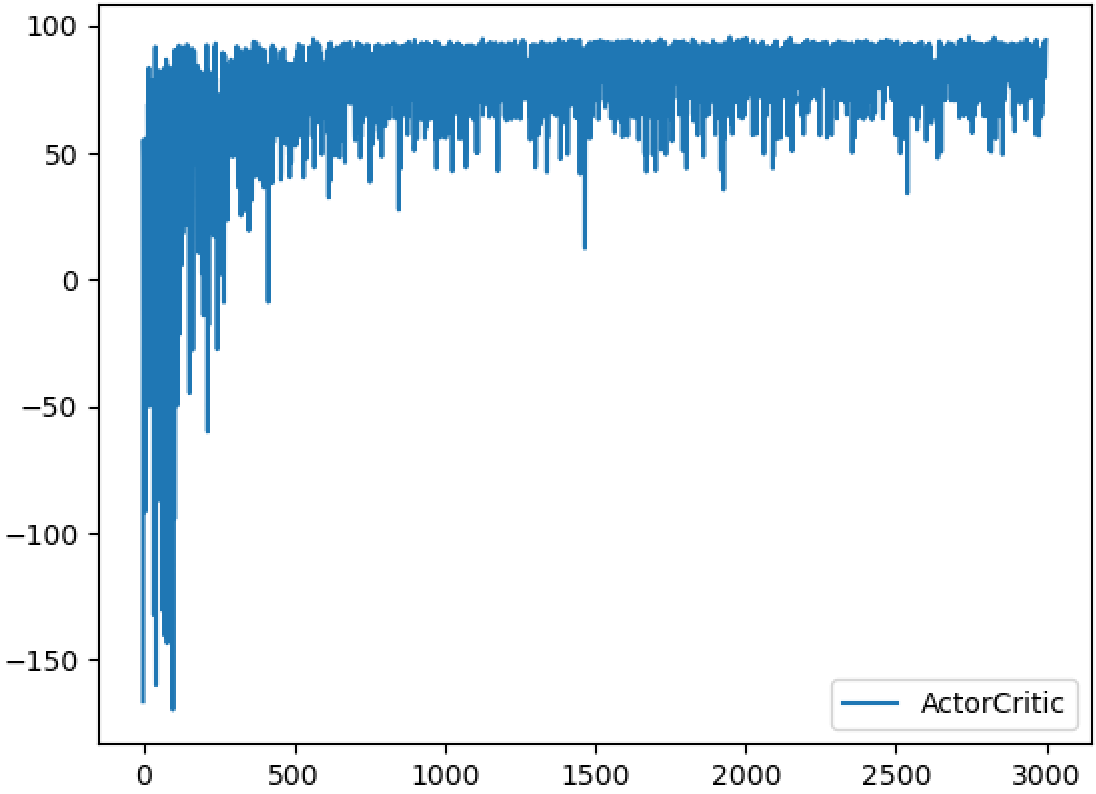


$\alpha = 0.0005, \beta = 0.001$:

大概在episode达到1700时收敛



$\alpha = 0.0005, \beta = 0.005$:
大概在episode达到1300时收敛



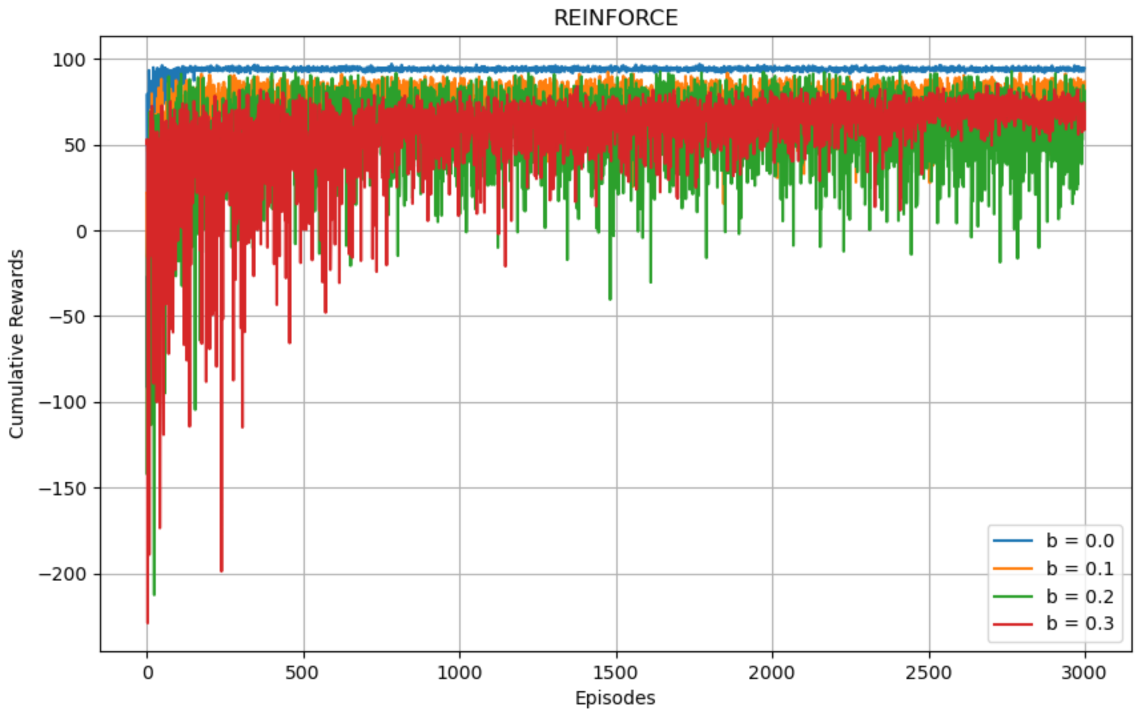
由上述可知当$\alpha = 0.001, \beta = 0.01$时曲线收敛较快。

affects

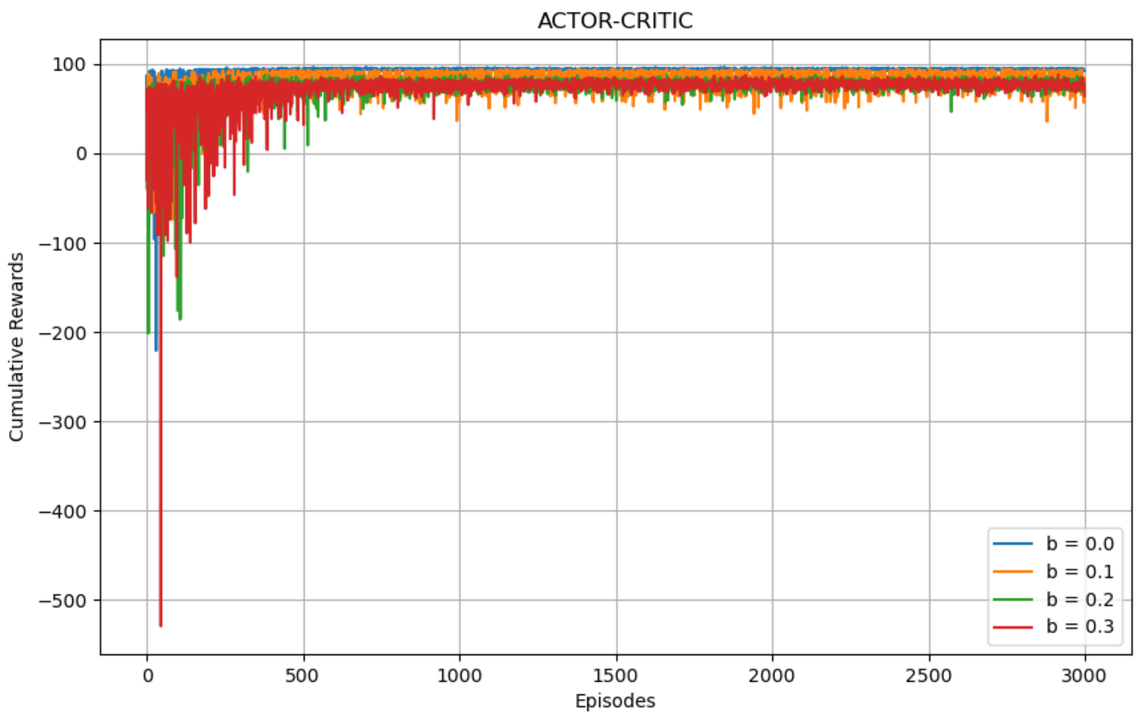
经过上述分析，学习率较小时曲线收敛速度较慢，但最后仍然可能收敛到全局最优。学习率较大时曲线收敛速度较快，但过大的学习率可能导致曲线剧烈波动，甚至不收敛。所以选择适合的学习率有利于算法的收敛速度和稳定性。

changing environment

REINFORCE



Actor-Critic

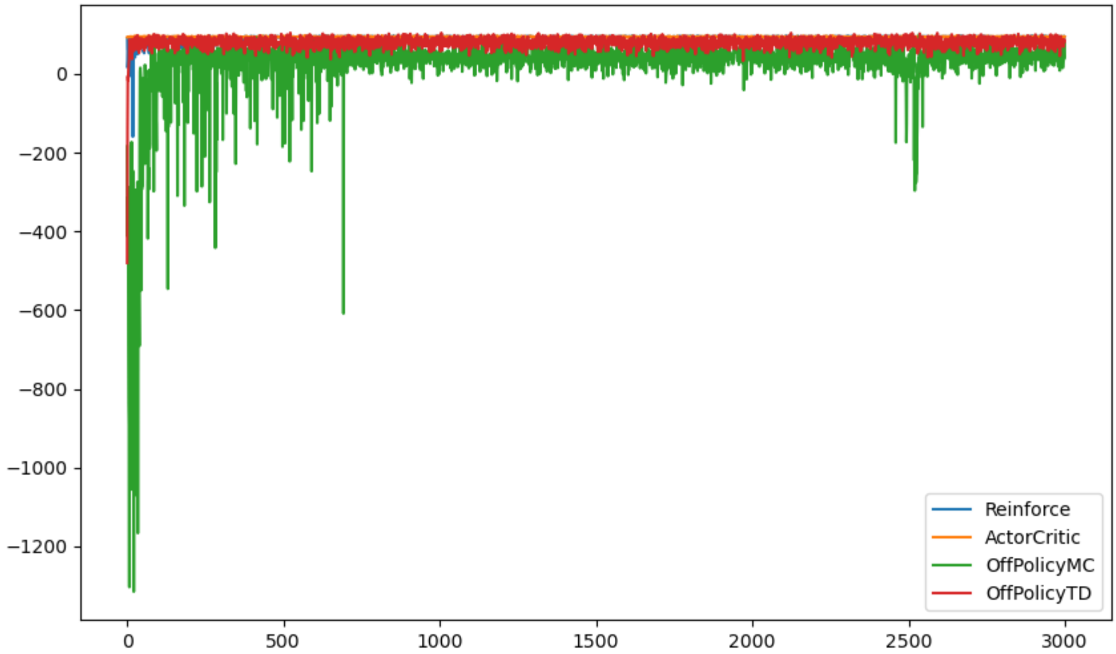


the impact of b

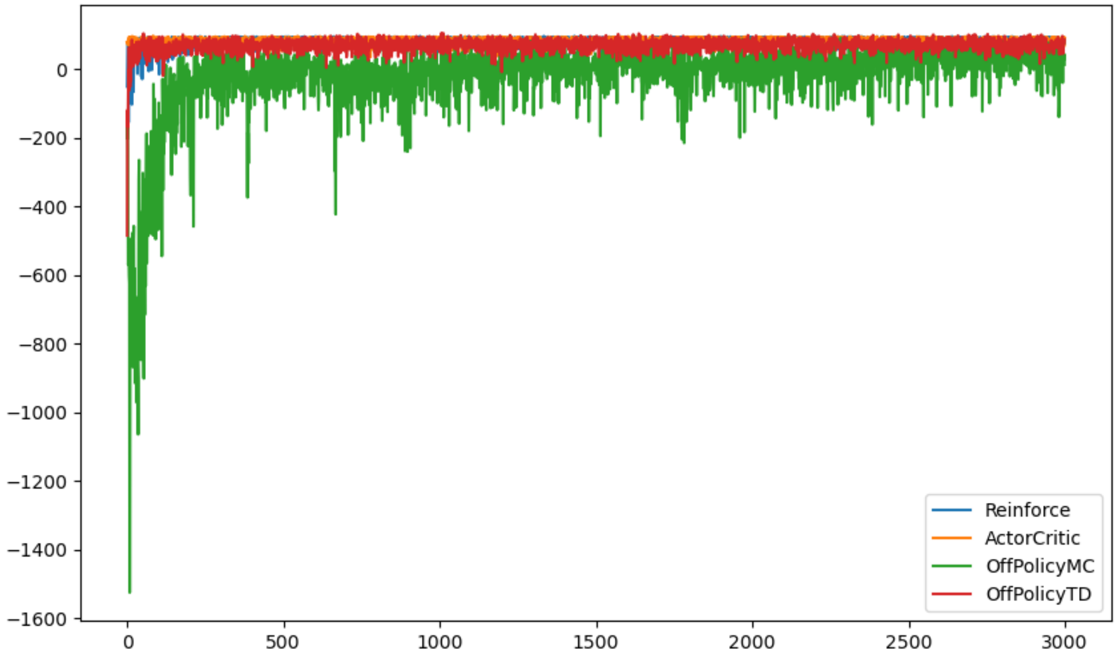
由上述图片可以看出，当b=0时，环境是确定性的，动作完全决定状态的转移，智能体可以稳定地利用学到的策略，所以训练期间累计奖励增长较快。而当b较大时，训练期间累计奖励增长更慢，且可能表现出较高的波动性。

comparison between value-based and policy-based algorithms

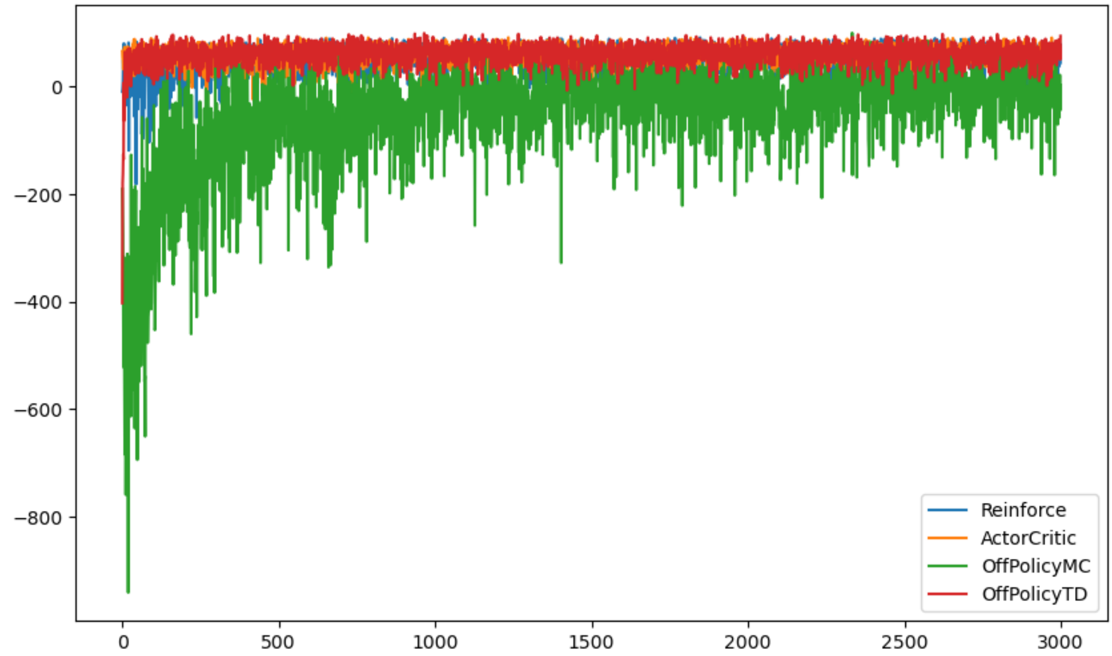
取b=0.0, 0.1, 0.2, 0.3, REINFORCE算法中的alpha取0.001, Actor-Critic算法中的alpha取0.001, beta取0.01, off-policy MC和Q-learning的epsilon均取0.1
b=0.0:



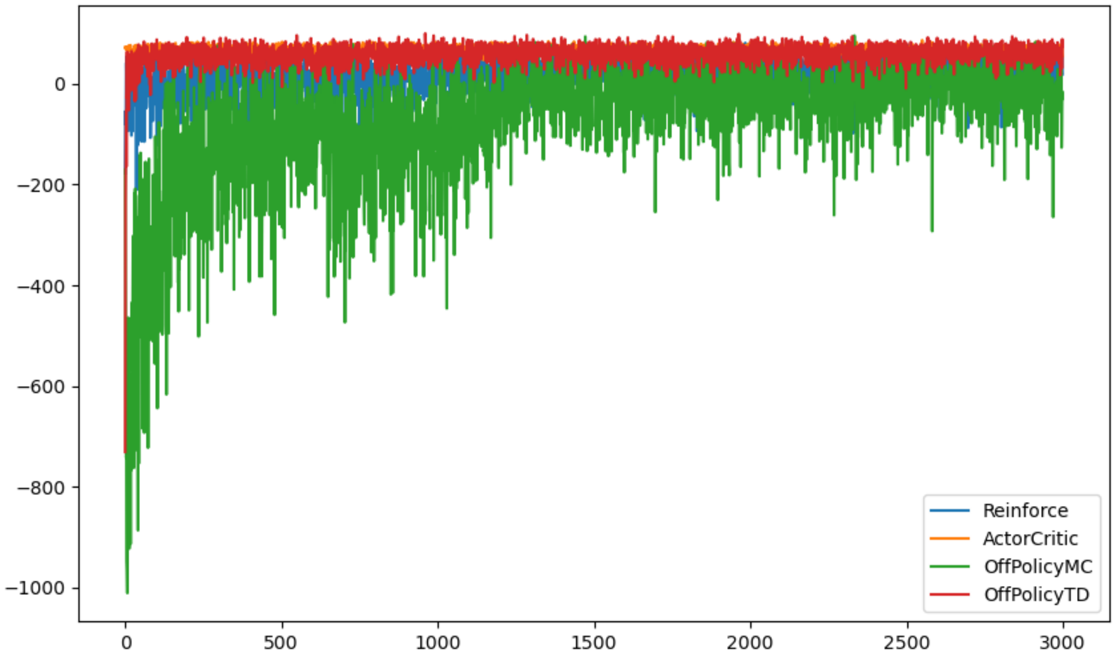
b=0.1:



b=0.2:



b=0.3:



由上述图片可以看出：
当b较小时（低随机性）：
Q-learning 和 Actor-Critic 表现最佳。因为环境更可预测，这两种算法可以较快地收敛到最优策略。
REINFORCE 和 Off-policy MC 方法可能因为梯度估计或采样方差的问题而略显不足。

当b较大时（高随机性）：
Actor-Critic 表现最好，因为它的双模块设计可以适应高随机性。
Q-learning 表现略有下降，主要是因为随机性的增加增大了Q(s,a)值的波动，可能当b值更大时表现得更明显。
REINFORCE 的性能显著下降，因为高随机性会进一步放大梯度估计的方差。
Off-policy MC 方法几乎无法有效收敛，重要性采样的方差问题显著。