Low Complexity Outage Optimal Distributed Channel Allocation for Vehicle-to-Vehicle Communications

Bo Bai, *Student Member, IEEE*, Wei Chen, *Member, IEEE*,
Khaled Ben Letaief, *Fellow, IEEE*, and Zhigang Cao, *Senior Member, IEEE*

***Abstract*—Due to the potential of enhancing traffic safety, protecting environment, and enabling new applications, vehicular communications, especially vehicle-to-vehicle (V2V) communications, has recently been receiving much attention. Because of both safety and non-safety real-time applications, V2V communications has QoS requirements on rate, latency, and reliability. How to appropriately design channel allocation is therefore a key MAC/PHY layer issue in vehicular communications. The QoS requirements of real-time V2V communications can be met by achieving a low outage probability and high outage capacity. In this paper, we first formulate the subchannel allocation in V2V communications into a maximum matching problem on random bipartite graphs. A distributed shuffling based Hopcroft-Karp (DSHK) algorithm will then be proposed to solve this problem with a sub-linear complexity of $\mathcal{O}(N^{2/3})$, where N is the number of subchannels. By studying the maximum matching generated by the DSHK algorithm on random bipartite graphs, the outage probabilities are derived in the high (two near vehicles) and low (two far away vehicles) SNR regimes, respectively. It is then demonstrated that the proposed method has a similar outage performance as the scenario of two communicating vehicles occupying N subchannels. By solving high degree algebraic equations, the outage capacity can be obtained to determine the maximum traffic rate given an outage probability constraint. It is also shown that the proposed scheme can take an advantage of small signaling overhead with only one-bit channel state information broadcasting for each subchannel.**

***Index Terms*—Vehicle-to-vehicle communications, Outage probability, Outage capacity, Random bipartite graph, Maximum matching**

I. INTRODUCTION

VEHICULAR communications has recently been receiving much attention because of its potential to improve traffic safety, reduce traffic congestion and fuel consumption, while enabling new applications such as wireless mobile ad hoc networks, relay-based cellular networks, and intelligent transportation systems. In contrast to conventional communication systems, vehicle-to-vehicle (V2V) communications has no central control node, but more QoS requirements on rate, latency, and reliability due to both safety and non-safety real-time applications [1]. Therefore, how to appropriately design the channel allocation scheme for vehicular communications is a key MAC/PHY layer issue. In this case, the channel allocation algorithm should be efficient to guarantee the rate, latency, reliability, and fairness requirements with a low computation complexity and small signaling overhead while being amenable to adaptation in response to environment changes.

There are many works on V2V communications. Luo and Hubaux in [2] surveyed the V2V communications with respect to key enabling technologies ranging from physical radio frequency to group communication primitives and security issues. The major classes of applications and the types of services in V2V communication networks are presented in [3], [4]. In these works, some existing networking protocols were also analyzed from the physical to the transport layers, as well as security aspects. On the other hand, there are many works on dynamic channel allocation for conventional communication systems. The topics include multi-user diversity, scheduling, cross-layer design, and fairness [5]. Rhee and Cioffi [6] studied the topic of increasing the capacity of OFDMA systems by dynamic subchannel allocation. Wong *et al.*[7] were the first to consider multi-user diversity in OFDMA systems. Ma [8] exploited the use of multi-user diversity to maximize the transmission rate with proportional fairness guarantee for downlink OFDMA systems. Song and Li [9], [10] used an utility function to balance the total transmission rate and fairness in OFDMA networks. In our work [11], the maximum matching method is used as a subchannel allocation scheme to improve the transmission reliability. There are also some specialized works on dynamic channel allocation in V2V communications. In [12], Eskafi *et al.* proposed two centralized dynamic channel allocation methods, which are based on a cellular mobile framework. In a recent work [1], Bi *et al.* used the method of token ring to allocate channels to vehicles while guaranteeing QoS. The underlying principle behind these works is the multi-user diversity, which usually relies on a central control node, rate adaption, and perfect channel state information (CSI). However, a V2V communication system may not easily have a central control node and perfect CSI

Manuscript received 5 January 2010; revised 7 May 2010 and 12 July 2010. This work is supported in part by the RGC/NSFC under Grant No. N_HKUST622/06, and NSFC under Grant No. 60832008 and No. 60902001.

B. Bai, W. Chen, and Z. Cao are with the State Key Laboratory on Microwave and Digital Communications in Department of Electronic Engineering, Tsinghua National Laboratory for Information Science and Technology (TNList), Tsinghua University, Beijing 100084, China (e-mail: bb04@mails.tsinghua.edu.cn, {wchen, czg-dee}@tsinghua.edu.cn). B. Bai is also with the Hong Kong University of Science & Technology where this work has been done.

K. B. Letaief is with the Center for Wireless Information Technology in Department of Electronic & Computer Engineering, the Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong (e-mail: eekhaled@ece.ust.hk).

Digital Object Identifier 10.1109/JSAC.2011.110116.

due to its dynamic and distributed infrastructure as well as protocol complexity constrains. In addition, the rate adaption may not be applicable for some delay constrained traffic or low complexity transceivers. Hence, in V2V communications, it is not trivial to apply the multi-user diversity based dynamic channel allocation mechanism to guarantee QoS parameters for safety and non-safety real-time applications with a low computation complexity and small signaling overhead.

In this paper, the V2V communication system consists of multiple vehicles in which any two vehicles can communicate with each other. It uses the method of frequency-division multiplexing (FDM) with each frequency band considered as a subchannel, to assure that all the transmitter and receiver (Tx-Rx) pairs in the set of vehicles can communicate at the same time. The key issue here is to design a low complexity subchannel allocation scheme so as to fulfill the QoS requirements for both safety and non-safety real-time applications under the constraint of severe and complicated channel fading [13], [14]. In this paper, the QoS requirements considered includes the latency, target transmission rate, outage probability, and fairness. In V2V communications, when the channel is in deep fading, the Tx cannot wait until the channel gets better in order to obey the latency requirement. On the other hand, if it transmits immediately, the rate requirement will be violated. In this case, if the fading subchannel cannot support the required rate, the communication of Tx-Rx pair will be in an outage state. Thus, the outage probability and outage capacity are two natural metrics for evaluating the performance of V2V communications, where the outage probability is defined as the probability that the channel capacity is smaller than a target transmission rate [15]. In this context, the outage probability of a subchannel will be determined by the statistical model of V2V communication channels, which can be found in [16], [17], [18]. Since all the Tx-Rx pairs in V2V communications share a limited number of subchannels, a Tx-Rx pair will be in outage if it fails to get a non-outage subchannel. Another important performance metric, known as outage capacity, determines the maximum allowable rate for a delay constrained traffic at a given reliability requirement [19]. The outage formulation enjoys at least three advantages: 1) for safety applications (the latency tolerance may be smaller than the coherence time), it can guarantee a high reliability; 2) for non-safety real-time applications (the latency tolerance may be larger than the coherence time), it can guarantee high average transmission rate; and 3) the broadcast CSI of a subchannel is only one bit to denote the outage state.

This paper will propose a fair outage optimal distributed subchannel allocation scheme in order to minimize the outage probability for each Tx-Rx pair with a low computation complexity and small signaling overhead. The random bipartite graph based methodology is proposed to formulate the subchannel allocation problem in V2V communications. In the random bipartite graph model, one partition class of vertices are used to denote Tx-Rx pairs and the other partition class of vertices are used to denote subchannels. The edge between a subchannel vertex and a Tx-Rx pair vertex occurs if and only if the subchannel is not in outage. Otherwise, there is no edge between them. A maximum matching on the random bipartite graph allows the largest amount of Tx-Rx pairs to get the non-outage subchannels, so as to minimize the outage probability. By using the distributed shuffling based Hopcroft-Karp (DSHK) algorithm to generate a maximum matching with identical probability, the same channel allocation result with fairness assurance is obtained in a distributive way on every Tx-Rx pair. Based on the parallel implementation the DSHK algorithm, the computation complexity is only $\mathcal{O}(N^{2/3})$, where N is the number of subchannels. The signaling overhead for each Tx-Rx pair is only the CSI broadcasting of N bits, where each bit denotes the outage state of a subchannel for the specific Tx-Rx pair. By studying the structural properties of the maximum matching generated by the DSHK algorithm, the second-order approximation of the outage probability in the high SNR regime (i.e., the case of two near vehicles) is obtained. Then, by studying every sample with few edges in the random bipartite graph, the approximation of the outage probability in the low SNR regime (i.e., the case of two far away vehicles) is also obtained. It can be seen that the outage probability obtained by the DSHK algorithm has an order N which is the same as the scenario of only one Tx-Rx pair with N subchannels. Thus, each Tx-Rx pair can obtain the full frequency diversity gain. The ϵ-outage capacity C_ϵ is also obtained by solving high degree equations. The key problem herein is the Abel–Ruffini theorem which implies that there is no general solutions in radicals to polynomial equations of degree five or higher [20]. Fortunately, our results on outage probabilities indicate that the obtained polynomial equations with a general form have the degree of four at most, and the equations of higher degrees have special solvable forms.

The rest of this paper is organized as follows. Section II presents the system model and the random bipartite graph formulation. Section III presents the DSHK algorithm for V2V communication systems. The outage probability analysis for high and low SNR regimes are illustrated in Section IV. Section V calculates the outage capacity by solving algebraic equations. Some numerical results which confirm our theoretical results are shown in Section VI. Finally, Section VII concludes the paper.

II. System Model and Problem Formulation

A. Channel Model and Outage Formulation

The V2V communication system considered, as shown in Fig. 1, has M Tx-Rx pairs sharing N subchannels where $N \geq M \geq 2$. Each Tx communicates with the corresponding Rx through the subchannel allocated to them. The set $\mathcal{U}$[1], defined as $\{u_m, m = 1, \ldots, M\}$, contains all the Tx-Rx pairs, where u_m is the mth Tx-Rx pair. The subchannel set $\mathcal{S}$ is given by $\{s_n, n = 1, \ldots, N\}$, where s_n is the nth subchannel. The wireless channel for any Tx-Rx pair $u_m \in \mathcal{U}$ is assumed to undergo frequency-selective fading. Let $h_{m,n}$, $n = 1, \ldots, N$ denote the channel gains for Tx-Rx pair u_m in the frequency domain, which are random but constant during coherence time T_c. Then, all the channel gain vectors $\boldsymbol{h}_m = [h_{m,1}, \ldots, h_{m,N}]^{\mathrm{T}}$, $u_m \in \mathcal{U}$ are independent with the identical distribution of $\mathcal{CN}(0, 1)$, i.e., Rayleigh fading [21].

[1]The script symbol $\mathcal{X}$ is used to denote a set, whose cardinality is denoted by $|\mathcal{X}|$.

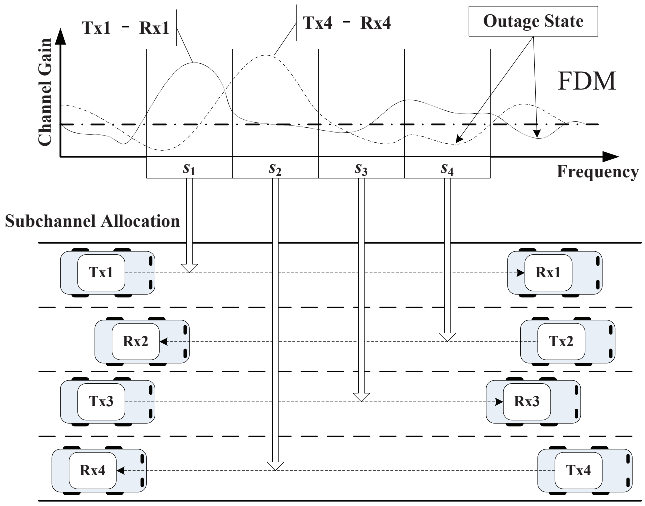


Fig. 1. A V2V communication system with four Tx-Rx pairs communicating over frequency-selective fading channels. Each Rx estimates the channel and broadcasts one bit CSI per subchannel, then each Tx calculates the subchannel allocation in a distributive way to minimize the outage probability.

An example for the channel gains in the frequency domain is shown in Fig. 1. Due to the deep fading, the capacity of each subchannel may be smaller than the required rate, i.e., the subchannel may be in outage state. In this case, the outage optimal dynamic subchannel allocation can guarantee a high reliability for the safety applications. For the non-safety real-time applications, this framework can guarantee a high transmission rate. Besides, the outage formulation only needs to broadcast one bit CSI per subchannel to denote the outage state. This fact makes the framework amenable to fast adaptation in response to environment changes.

Assume that the transmission rate required is $R\,\mathrm{bit/s/Hz}$ for any Tx-Rx pair. If the capacity of subchannel $s_n \in \mathcal{S}$ for the Tx-Rx pair $u_m \in \mathcal{U}$ is smaller than R, the Tx-Rx pair u_m cannot transmit any information with an arbitrary small error probability over this subchannel [19], [22]. This is referred to as the subchannel outage, and the outage probability of a subchannel is defined as

$$p_{\rm s}\left(R\right) \triangleq \Pr\left\{\log\left(1+\left|h_{m,n}\right|^2 \mathsf{SNR}\right) < R\right\}. \qquad (1)$$

Throughout this paper, the logarithm function $\log\left(\cdot\right)$ has a base 2, and R will be omitted for $p_{\rm s}\left(R\right)$. For convenience, we define the non-outage probability of a subchannel as

$$q_{\rm s} = 1 - p_{\rm s}. \qquad (2)$$

According to the Rayleigh fading model, the outage probability of a subchannel is given by

$$p_{\rm s} = 1 - \exp\left(-\frac{2^R - 1}{\mathsf{SNR}}\right). \qquad (3)$$

In fact, the V2V communications may have different statistical channel models [16], [17], [18], but the proposed method in the following is still applicable. The only difference is the exact form of the outage probability of each subchannel. In practice, the channel estimation in Rx only needs to determine whether $\left|h_{m,n}\right|^2$ is larger than $\frac{2^R-1}{\mathsf{SNR}}$ or not for each subchannel. Thus, the CSI broadcasting is only N bits per Tx-Rx pair.

After obtaining the CSI, each Tx will calculate the subchannel allocation in a distributive way to minimize the outage probability. As a result, a Tx-Rx pair will be in outage if it fails to get a non-outage subchannel. Since fairness requires that each Tx-Rx pair has an identical opportunity to access the subchannel, the outage probability for a Tx-Rx pair can be defined formally as follows.

Definition 1: For a given target rate R, the outage probability for a Tx-Rx pair, denoted by $P_{\rm u}$, is defined as

$$P_{\rm u} = \Pr\left\{\log\left(1+\left|h_{m,n}\right|^2 \mathsf{SNR}\right) < R \,\middle|\, \left\{s_n \rightarrow u_m\right\}\right\}, \qquad (4)$$

where the event $\left\{s_n \rightarrow u_m\right\}$ denotes the subchannel $s_n \in \mathcal{S}$ is allocated to the Tx-Rx pair $u_m \in \mathcal{U}$.

Another important performance metric closely related to the outage probability is the ϵ-outage capacity, which can be used to determine the maximum allowable rate of real-time applications at a given reliability requirement. Similar to [19], the ϵ-outage capacity is defined as follows.

Definition 2: The ϵ-outage capacity, denoted by C_ϵ, is the largest transmission rate such that the outage probability $P_{\rm u}$ is less than ϵ. That is,

$$C_\epsilon \triangleq \sup_{P_{\rm u} < \epsilon} R. \qquad (5)$$

B. Random Bipartite Graph Model

According to one-bit CSI being broadcast, the subchannel allocation problem can be formulated into a random bipartite graph model in Tx. In a random bipartite graph, the vertices are fixed, but every edge occurs according to some kind of probability measure. Let $\mathcal{V}$ denote the vertex set, then $\mathcal{V}$ can be divided into two partition classes such that every edge has its ends in different partition class. Here, one partition class of $\mathcal{V}$ is the set $\mathcal{U}$, in which one vertex denotes one Tx-Rx pair. The other partition class is the set $\mathcal{S}$, in which one vertex denotes one subchannel. Clearly, $\mathcal{V} = \mathcal{U} \cup \mathcal{S}$ and $\mathcal{U} \cap \mathcal{S} = \emptyset$. From Eq. (3), the subchannel s_n is in outage with probability $p_{\rm s}$ for the Tx-Rx pair u_m. In other words, the probability that the Tx-Rx pair u_m can transmit at the target rate R on subchannel s_n is $q_{\rm s} = 1 - p_{\rm s}$. An intuitive description of the random bipartite graph model for V2V communications is as follows: if s_n is not in outage to u_m, join u_m and s_n with an edge. Otherwise, there is no edge between them. Thus, for the Tx-Rx pair u_m, there will be a subset of $\mathcal{S}$, denoted by $\mathcal{N}\left(u_m\right)$, which contains all the vertices having an edge incident with u_m. So $\mathcal{N}\left(u_m\right)$ is the set of all the non-outage subchannels for the Tx-Rx pair u_m. The probability space of the random bipartite graph is denoted by $\mathscr{G}\left\{\mathcal{K}_{M,N}; q_{\rm s}\right\}$, where $\mathcal{K}_{M,N}$ denotes a complete bipartite graph [23]. A sample in $\mathscr{G}\left\{\mathcal{K}_{M,N}; q_{\rm s}\right\}$ is a deterministic bipartite graph, denoted by $\mathcal{G}\left(\mathcal{U} \cup \mathcal{S}, \mathcal{E}\right)$, where $\mathcal{E}$ is the set of edges. Clearly, the number of all samples in the probability space $\mathscr{G}\left\{\mathcal{K}_{M,N}; q_{\rm s}\right\}$ is 2^{MN}. In practical V2V communications, the same $\mathcal{G}\left(\mathcal{U} \cup \mathcal{S}, \mathcal{E}\right)$ can be constructed in each Tx by receiving the broadcast CSI by each Rx. Fig. 2 shows a sample of $\mathscr{G}\left\{\mathcal{K}_{4,5}; q_{\rm s}\right\}$, where $\mathcal{N}\left(u_1\right) = \emptyset$, $\mathcal{N}\left(u_2\right) = \left\{s_3\right\}$, $\mathcal{N}\left(u_3\right) = \left\{s_3\right\}$, and $\mathcal{N}\left(u_4\right) = \left\{s_1, s_4, s_5\right\}$.

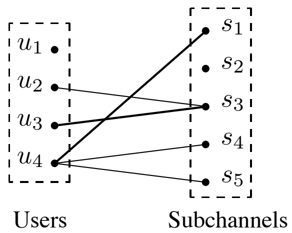


Fig. 2. A sample of $\mathscr{G}\{\mathcal{K}_{4,5}; q_{\rm s}\}$, where the thick lines denote the matching edges, i.e., $\mathcal{M} = \{u_3 s_3, u_4 s_1\}$.

III. Distributed Subchannel Allocation Scheme

In this section, a low complexity distributed subchannel allocation algorithm, known as DSHK, is presented in order to minimize the number of outage Tx-Rx pairs in V2V communications.

According to one-bit CSI being broadcast, each Tx vehicle will generate the sample bipartite graph $\mathcal{G}(\mathcal{U} \cup \mathcal{S}, \mathcal{E}) \in \mathscr{G}\{\mathcal{K}_{M,N}; q_{\rm s}\}$. Then, they allocate one subchannel in $\mathcal{N}(u_m)$ to the Tx-Rx pair u_m in a distributive way. However, if $\mathcal{N}(u_m)$ is $\emptyset$ or every $s_n \in \mathcal{N}(u_m)$ has been allocated to other Tx-Rx pairs, this Tx-Rx pair will be in outage. The edge set created by this kind of allocation is a maximum matching, denoted by $\mathcal{M}$. In graph theory, a matching is a subset of $\mathcal{E}$, and every edge in $\mathcal{M}$ has no common ends. A matching $\mathcal{M}$ saturates a vertex u_m (or s_n), and u_m (or s_n) is said to be $\mathcal{M}$-saturated, if some edge of $\mathcal{M}$ is incident with u_m or s_n. Otherwise, u_m or s_n is $\mathcal{M}$-unsaturated. By generating a maximum matching on $\mathcal{G}(\mathcal{U} \cup \mathcal{S}, \mathcal{E})$, we can allow the largest amount of Tx-Rx pairs to get non-outage subchannels, i.e., minimize the outage probability. In addition, $\mathcal{M}$ must be generated with identical probability to guarantee fairness among Tx-Rx pairs. Therefore, the Tx-Rx pair is in outage if and only if it is $\mathcal{M}$-unsaturated. Fig. 2 shows an example of the maximum matching, where $\mathcal{M} = \{u_3 s_3, u_4 s_1\}$.

The Hopcroft-Karp algorithm can be used to generate a maximum matching on $\mathcal{G}(\mathcal{U} \cup \mathcal{S}, \mathcal{E})$. Based on this idea, the subchannel allocation scheme is shown in Algorithm 1, which will be executed in a distributive way on every Tx. Step 1 constructs the bipartite graph according to the received CSI. Step 3 can guarantee fairness among Tx-Rx pairs by executing the same shuffle operation on each Tx. Step 5–10 generates a maximum matching by the parallel implementation of Hopcroft-Karp algorithm. Thus, Algorithm 1 will be referred to as the distributed shuffling based Hopcroft-Karp algorithm.

In fact, the steps of BFS, DFS and AO have corresponding parallel implementations, and the detail can be found in [24]. With the parallel implementations, the computation complexity of the DSHK algorithm is only $\mathcal{O}(N^{2/3})$. Moreover, the only signaling needed by the DSHK algorithm is N bit CSI per Tx-Rx pair. So Rx only needs to do simple channel estimation to determine if a subchannel is in outage. As mentioned in the introduction, multi-user diversity is hard to be applied to V2V communications. The detailed comparison can be found in Table I.

Remark 1: When the highway traffic density is high, there must be more Tx-Rx pairs in the V2V communication systems. However, the proposed DSHK algorithm only depends

Algorithm 1 DSHK Algorithm

1: According to the received CSI, construct $\mathcal{G}(\mathcal{U} \cup \mathcal{S}, \mathcal{E})$.
2: Delete every $u \in \mathcal{U}$ with $\deg(u) = 0$, and every $s \in \mathcal{S}$ with $\deg(s) = 0$.
3: Shuffle all the vertices in $\mathcal{U}$ with the same rule.
4: Let $\mathcal{M} \leftarrow \emptyset$.
5: **repeat**
6: Construct a digraph $\overrightarrow{\mathcal{G}}$ from $\mathcal{G}(\mathcal{U} \cup \mathcal{S}, \mathcal{E})$ and $\mathcal{M}$.
7: Execute a breadth-first searching (BFS) on $\overrightarrow{\mathcal{G}}$ to find the length $l(\mathcal{M})$ of the shortest $\mathcal{M}$-augmenting path: start from all the $\mathcal{M}$-unsaturated vertices in $\tilde{\mathcal{U}}$ and stop at the first encountered $\mathcal{M}$-unsaturated vertex in $\mathcal{S}$.
8: Execute a depth-first searching (DFS) to find the maximal set $\mathscr{Q}$ of the shortest $\mathcal{M}$-augmenting paths $\mathcal{Q}_i, i \in \mathcal{I}$: $\mathscr{Q} = \{\mathcal{Q} | l(\mathcal{M}) = |\mathcal{Q}|, \mathcal{V}(\mathcal{Q}_i) \bigcap \mathcal{V}(\mathcal{Q}_j) = \emptyset, \forall i \neq j \in \mathcal{I}\}$, where $\mathcal{V}(\mathcal{Q}_i)$ is the vertex set incident with edges in $\mathcal{Q}_i$.
9: Augmenting operation (AO): $\mathcal{M} \leftarrow \mathcal{M} \oplus \left(\bigoplus_{i \in \mathcal{I}} \mathcal{Q}_i\right)$, where $\mathcal{Q}_i \in \mathscr{Q}$ and $\oplus$ is the symmetric difference of two sets.
10: **until** There is no $\mathcal{M}$-augmenting path.
11: Output $\mathcal{M}$ then stop.

on the number of sub-channel rather than the number of Tx-Rx pairs. Therefore, the computation complexity of the DSHK algorithm does not grow as the highway traffic density increases. The high-speed mobility is another key issue in V2V communications, which causes that the topology of the V2V communication system will change from time to time. The proposed DSHK algorithm, however, only need one bit channel state information. The CSI estimation is required by any coherent demodulator in the Rx. Hence, it is only needed the Rx to compare the average SNR with a threshold and feedback one bit for each subchannel to denote if the specific subchannel is in outage. The DSHK algorithm is a distributed algorithm, and also enjoys a low computation complexity of $\mathcal{O}(N^{2/3})$. Therefore, each Tx can generate the bipartite graph sample immediately, as long as the channel state and/or the network topology have changed. Then, each Tx will run DSHK algorithm independently to find out the outage optimal subchannel allocation with a sub-linear computation complexity. As a result, the DSHK algorithm can be applied to V2V communication systems with the high-speed mobility.

IV. Outage Probability Analysis

In this section, the second-order approximation of the outage probability in the high SNR regime is first obtained by studying the structural properties of the maximum matching generated by the DSHK algorithm. The approximation of the outage probability in the low SNR regime is also obtained by studying every sample with few edges in the random bipartite graph.

A. Outage Probability in the High SNR Regime

In V2V communications, the distance between two vehicles changes at all time. This subsection will analyze the case of short distances between two vehicles, i.e., the high SNR

TABLE I
DSHK ALGORITHM AND MULTI-USER DIVERSITY COMPARISON

	DSHK Algorithm	**Multi-User Diversity**
Performance Metric	Outage probability/capacity	Sum capacity
Channel Model	Frequency selective fast/slow fading	Fast fading
Transceiver Design	Fixed modulation: Transmit at a target rate	Adaptive modulation: Transmit at the instantaneous channel capacity
Scheduling Objective	Minimize the number of outage Tx-Rx pairs	Maximize the sum ergodic capacity of the system
Scheduling Method	Allocate a non-outage subchannel to a Tx-Rx pair	Allocate a subchannel to the best Tx-Rx pair
CSI Needed	1 bit per subchannel to identify outage	Perfect

regime. The case of long distances will be analyzed in the next subsection.

Random variables $\xi_1, \dots, \xi_M$ are introduced by taking $\xi_m = 1$ if Tx-Rx pair u_m is in outage after subchannel allocation. Otherwise, $\xi_m = 0$. Define $\Xi = \sum_{m=1}^{M} \xi_m$, then

$$\mathbb{E}\{\Xi\} = \sum_{m=1}^{M} \mathbb{E}\{\xi_m\} = MP_{\mathrm{u}}.$$

Thus, the outage probability for any Tx-Rx pair can be given by

$$P_{\mathrm{u}} = \frac{\mathbb{E}\{\Xi\}}{M}. \tag{6}$$

As a result, analyzing the outage probability of the DSHK algorithm based subchannel allocation is equivalent to analyzing the distribution of Ξ, which will be addressed in the following two lemmas.

Lemma 1: For a bipartite graph $\mathcal{G}(\mathcal{U} \cup \mathcal{S}, \mathcal{E})$ with $|\mathcal{U}| = M$, $|\mathcal{S}| = N$ and $3 \leq M \leq N$, if $|\mathcal{E}| = (M-1)N - 1$ and $\mathcal{M} \subseteq \mathcal{E}$ is a maximum matching, then there is at most one $\mathcal{M}$-unsaturated vertex in $\mathcal{U}$. If $M < N$, this unsaturated vertex in $\mathcal{U}$ appears if and only if this vertex is isolated. If $M = N$, this unsaturated vertex appears if and only if this vertex or any other vertex in $\mathcal{S}$ is isolated.

Proof: See Appendix A. ■

Lemma 1 shows the structure of the maximum matching under which at most one Tx-Rx pair is in outage. Note that Lemma 1 holds when $M \geq 3$. The following lemma shows the properties of maximum matching for $M = 2$.

Lemma 2: For a bipartite graph $\mathcal{G}(\mathcal{U} \cup \mathcal{S}, \mathcal{E})$ with $|\mathcal{U}| = M$, $|\mathcal{S}| = N$ and $2 = M \leq N$, if $|\mathcal{E}| = N-1$ and $\mathcal{M} \subseteq \mathcal{E}$ is a maximum matching, then there is at most one $\mathcal{M}$-unsaturated vertex in $\mathcal{U}$. If $N \neq 3$, this unsaturated vertex in $\mathcal{U}$ appears if and only if this vertex and any other vertex in $\mathcal{S}$ are isolated. If $N = 3$, this outage vertex also appears when any two vertices in $\mathcal{S}$ are isolated.

Proof: See Appendix B. ■

According to these lemmas, the second-order approximation of the outage probability in the high SNR regime for V2V communications can be summarized in the following theorem.

Theorem 1: For a V2V communication system with M Tx-Rx pairs and N subchannels, the outage probability in the high SNR regime satisfies

$$P_{\mathrm{u}} = a_1 p_{\mathrm{s}}^{N} + a_2 p_{\mathrm{s}}^{N+1} + \mathsf{O}\left(p_{\mathrm{s}}^{N+1}\right). \tag{7}$$

$$a_1 = \begin{cases} 2, & M = N; \\ 1, & M < N. \end{cases} \tag{8}$$

$$a_2 = \begin{cases} -2, & M = 2,\ N = 2; \\ \frac{3}{2}, & M = 2,\ N = 3; \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Here, the symbol $\mathsf{O}(\cdot)$ denotes the high order infinitesimal.

Proof: From Eq. (6), the outage probability satisfies

$$P_{\mathrm{u}} = a_1 p_{\mathrm{s}}^{N} + a_2 p_{\mathrm{s}}^{N+1} + \mathsf{O}\left(p_{\mathrm{s}}^{N+1}\right),$$

and a_1 has been obtained in our previous work [11]. According to the properties of the maximum matching generated by the DSHK algorithm, $\Pr\{\Xi\}$ is composed by terms with the form of $p_{\mathrm{s}}^{n} q_{\mathrm{s}}^{MN-n}$, $n = N, \dots, MN$. The term $a_2 p_{\mathrm{s}}^{N+1}$ can be generated under two situations: 1) When $p_{\mathrm{s}}^{N} q_{\mathrm{s}}^{(M-1)N} = p_{\mathrm{s}}^{N}(1-p_{\mathrm{s}})^{(M-1)N}$, i.e., there are N edges in outage, which means only $(M-1)N$ edges would appear; or 2) $p_{\mathrm{s}}^{N+1} q_{\mathrm{s}}^{(M-1)N-1} = p_{\mathrm{s}}^{N+1}(1-p_{\mathrm{s}})^{(M-1)N-1}$, i.e., there are $N+1$ edges in outage, which means that only $(M-1)N-1$ edges would appear.

Note that a_2 can be divided into two parts α and β, which are generated by two cases as stated above. By considering Eq. (6), we have

$$\alpha = -a_1 M \begin{pmatrix} MN - N \\ 1 \end{pmatrix} = -a_1 M (M-1) N.$$

If $3 \leq M < N$, according to Lemma 1, there is one Tx-Rx pair in outage only when this Tx-Rx pair is an isolated vertex in $\mathcal{U}$. The occurrence number of this event is given by

$$\beta = \binom{M}{1}\binom{M-1}{1}\binom{N}{1} = M(M-1)N.$$

By recalling Eq. (6) and Eq. (8), we have

$$a_2 = \frac{\alpha + \beta}{M} = 0.$$

If $3 \leq M = N$, according to Lemma 1, there is one Tx-Rx pair in outage only when this Tx-Rx pair is an isolated vertex in $\mathcal{U}$ or one subchannel is an isolated vertex in $\mathcal{S}$. The occurrence number of this event is given by

$$\begin{aligned}\beta =& \binom{M}{1}\binom{N}{1}\left[\binom{M-1}{1} + \binom{N-1}{1}\right] \\ =& 2M(M-1)M.\end{aligned}$$

Consider Eq. (6) and Eq. (8), then we have

$$a_2 = \frac{\alpha + \beta}{M} = 0.$$

If $M = 2$ and $N \geq 4$, according to Lemma 2, there is one Tx-Rx pair in outage only when this Tx-Rx pair in $\mathcal{U}$ and any other vertex in $\mathcal{S}$ are isolated. The occurrence number of this event is given by

$$\beta = \binom{M}{1}\binom{N}{1} = MN = 2N.$$

Consider Eq. (6) and Eq. (8), then we have

$$a_2 = \frac{\alpha + \beta}{M} = 0.$$

If $M = 2$ and $N = 3$, according to Lemma 2, there is one Tx-Rx pair in outage only when this vertex in $\mathcal{U}$ and any other vertex in $\mathcal{S}$ are isolated or any two vertices in $\mathcal{S}$ are isolated. The occurrence number of this event is given by

$$\beta = \binom{M}{1}\binom{N}{1} + \binom{N}{2} = MN + \frac{N(N-1)}{2} = 9.$$

Consider Eq. (6) and Eq. (8), we have

$$a_2 = \frac{\alpha + \beta}{M} = \frac{3}{2}.$$

Finally, it is easy to verify that $P_{\mathrm{u}} = 2p_{\mathrm{s}}^2 - 2p_{\mathrm{s}}^3 + p_{\mathrm{s}}^4$, when $M = 2$ and $N = 2$. ■

The theorem indicates that the coefficient of the second order approximation term is zero in most cases. This result guarantees that the outage performance of DSHK algorithm is similar to the scenario with one Tx-Rx pair and N subchannels. Specifically, each Tx-Rx pair can achieve the full diversity gain N by using the DSHK algorithm.

Remark 2: Through this theorem, we can get a second-order approximation of P_{u} when $\mathsf{SNR} \to \infty$. According to Eqs. (3) (7) (9), we have

$$(\log_{10} P_{\mathrm{u}})_{\mathrm{H}}^2 \approx \eta_{\mathrm{H}} + \zeta_{\mathrm{H}} - \frac{N}{10}\mathsf{SNR}(\mathsf{dB}), \quad \mathsf{SNR} \to \infty, \tag{10}$$

where, $(\log_{10} P_{\mathrm{u}})_{\mathrm{H}}^2$ is the second order approximation of $\log_{10} P_{\mathrm{u}}$ in the high SNR regime, and η_{H}, ζ_{H} are given by

$$\eta_{\mathrm{H}} = \begin{cases} N\log_{10}\left(2^R - 1\right), & M < N; \\ N\log_{10}\left(2^R - 1\right) + \log_{10} 2, & M = N. \end{cases} \tag{11}$$

$$\zeta_{\mathrm{H}} = \begin{cases} -\frac{2^R-1}{\ln 10}\frac{1}{\mathsf{SNR}}, & M = 2,\, N = 2; \\ \frac{3\left(2^R-1\right)}{2\ln 10}\frac{1}{\mathsf{SNR}}, & M = 2,\, N = 3; \\ 0, & \text{otherwise}. \end{cases} \tag{12}$$

B. Outage Probability in the Low SNR Regime

A long distance between two vehicles will lead to the low operating SNR regime, which will be analyzed in this subsection. According to Eq. (3), we have $p_{\mathrm{s}} \to 1$ as $\mathsf{SNR} \to 0$, which indicates that almost all the subchannels are in outage. Thus, the outage state caused by two Tx-Rx pairs competing for one subchannel will nearly not happen. As a result, the outage state in the low SNR regime is mainly determined by the subchannel outage. This intuitive thinking is proved to be true by the following theorem.

Theorem 2: For a V2V communication system with M Tx-Rx pairs and N subchannels, the outage probability in the low SNR regime satisfies

$$P_{\mathrm{u}} = p_{\mathrm{s}}^N + \mathsf{O}\left(q_{\mathrm{s}}\right), \tag{13}$$

where $q_{\mathrm{s}} = 1 - p_{\mathrm{s}}$.

Proof: Consider a sample with few edges in the random bipartite graph $\mathscr{G}\left(\mathcal{K}_{M,N}; q_s\right)$. For a Tx-Rx pair u_m in the set $\mathcal{U}$, there are two cases that make u_m outage: 1) There is no subchannel in $\mathcal{S}$ which is non-outage for u_m; and 2) There are other Tx-Rx pairs competing for the same subchannel s_n with u_m and it is not saturated by the maximum matching.

In the first case, i.e., $\mathcal{N}\left(u_m\right) = \emptyset$ on $\mathcal{G}\left(\mathcal{U} \cup \mathcal{S}, \mathcal{E}\right)$, the probability of this event occurrence is given by p_s^N.

In the second case, there will be at least two edges in the bipartite graph $\mathcal{G}\left(\mathcal{U} \cup \mathcal{S}, \mathcal{E}\right)$. One is $u_m s_n$, and the other one is $u_{m'} s_n$. Assuming that there are only two edges in the sample of this random bipartite graph, i.e., $\mathcal{E} = \{u_m s_n, u_{m'} s_n\}$. In the maximum matching, u_m or $u_{m'}$ is chosen with equal probability. The outage probability of u_m is then given by

$$\frac{1}{2}\binom{M-1}{1}\binom{N}{1} q_{\mathrm{s}}^2 p_{\mathrm{s}}^{MN-2} = \mathsf{O}\left(q_{\mathrm{s}}\right), \quad \mathsf{SNR} \to 0.$$

The last equation follows the fact that

$$\lim_{\mathsf{SNR} \to 0} q_{\mathrm{s}} = \lim_{\mathsf{SNR} \to 0} \exp\left(-\frac{2^R - 1}{\mathsf{SNR}}\right) = 0.$$

Thus, if there are more than two edges in this random bipartite graph, the outage probability of u_m must have a factor q_{s}^x with $x \geq 3$. Hence, the Tx-Rx pair conflicts in subchannel allocation do not affect the Tx-Rx pair outage probability in the low SNR regime. That is,

$$P_{\mathrm{u}} = p_{\mathrm{s}}^N + \mathsf{O}\left(q_{\mathrm{s}}\right).$$

■

By taking a V2V communication system with $M = 2$ and $N = 3$ as an example, where R is set to 1, we have

$$(\lg P_{\mathrm{u}})_{\mathrm{L}} = 3\lg p_{\mathrm{s}} \approx -\frac{3}{\ln 10}\exp\left(-\frac{1}{\mathsf{SNR}}\right),$$

where $(\lg P_{\mathrm{u}})_{\mathrm{L}}$ is the approximation value of $\lg P_{\mathrm{u}}$ in the low SNR regime. Fig. 3 compares the numerical result and the approximation curve in the low SNR regime. It shows that the theoretical curve is very close to the simulation one at low SNR.

V. Outage Capacity Analysis

The ϵ-outage capacity provides another performance metric for V2V communications, which can be used to determine the maximum allowable traffic rate for real-time applications at a given reliability requirement. According to Definition 2, the ϵ-outage capacity can be obtained by solving $P_{\mathrm{u}} = \epsilon$. The equations derived from the approximations of the outage probability are algebraic equations of high degree. The Abel–Ruffini theorem (also known as Abel's impossibility theorem) states that there is no general solution in radicals to polynomial equations of degree five or higher [20]. Galois theory also shows that for every $n > 4$, there exist polynomials of degree

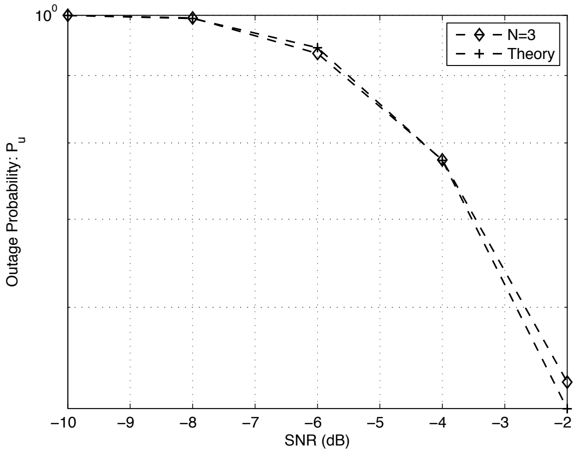


Fig. 3. The outage probability and its approximation in the low SNR regime for a V2V communication system with $M = 2$ and $N = 3$.

n which are not solvable by radicals [25]. Fortunately, the obtained equations with general forms only have the degree of four, and the equations of higher degrees have special solvable forms. So the ϵ-outage capacity of V2V communications is summarized in the following theorem.

Theorem 3: For a V2V communication system with M Tx-Rx pairs and N subchannels, the approximation for the ϵ-outage capacity C_ϵ for each Tx-Rx pair is given by

$$C_\epsilon = B_{\mathrm{s}} \log\left(1 + \mathsf{SNR} \ln \frac{1}{1-\zeta}\right), \tag{14}$$

where B_{s} is the bandwidth of a subchannel, and ζ is given by the following formulas.

1) $M = 2$, $N = 2$,

$$\zeta = \frac{1}{2}\left[1 - \mu + \sqrt{(1-\mu)^2 - 4\rho\left(1 - \frac{1}{\mu}\right)}\right], \tag{15}$$

where $\mu = \sqrt{2\rho - 1}$, and ρ is given by

$$\rho = \frac{1}{3} + \sqrt[3]{\sqrt{\left(\frac{\epsilon}{12} + \frac{1}{27}\right)^2 + \left(\frac{\epsilon}{3} - \frac{1}{9}\right)^3} + \frac{\epsilon}{12} + \frac{1}{27}} - \sqrt[3]{\sqrt{\left(\frac{\epsilon}{12} + \frac{1}{27}\right)^2 + \left(\frac{\epsilon}{3} - \frac{1}{9}\right)^3} - \frac{\epsilon}{12} - \frac{1}{27}}. \tag{16}$$

2) $M = 2$, $N = 3$ with $\mathsf{SNR} > 1$,

$$\zeta = -\frac{1}{6}\left[1 + \mu - \sqrt{(1+\mu)^2 - 36\rho\left(1 + \frac{1}{\mu}\right)}\right], \tag{17}$$

where $\mu = \sqrt{18\rho + 1}$, and ρ is given by

$$\rho = \frac{1}{3}\sqrt[3]{\frac{\epsilon}{2}\sqrt{32\epsilon + 1} - \frac{\epsilon}{2}} - \frac{1}{3}\sqrt[3]{\frac{\epsilon}{2}\sqrt{32\epsilon + 1} + \frac{\epsilon}{2}}. \tag{18}$$

3) $M \geq 3$, $N \geq 3$ with $\mathsf{SNR} > 1$,

$$\zeta = \begin{cases} \left(\frac{\epsilon}{2}\right)^{\frac{1}{N}}, & M = N; \\ \epsilon^{\frac{1}{N}}, & M < N. \end{cases} \tag{19}$$

4) $M \geq 2$, $N \geq 3$ with $\mathsf{SNR} < 1$,

$$\zeta = \epsilon^{\frac{1}{N}}. \tag{20}$$

Proof: From the approximation of the outage probability in the high SNR regime, Eq. (7), and also that in the low SNR regime, Eq. (13), the polynomial equations can be obtained.

Case I: $M = 2$, $N = 2$. From Eq. (7), the following equation holds,

$$\zeta^4 - 2\zeta^3 + 2\zeta^2 - \epsilon = 0. \tag{21}$$

From the radical of quartic equations, Eq. (21) has the same solutions as the following two quadratic equations,

$$\begin{cases} \zeta^2 - (1-\mu)\zeta + \rho\left(1 - \frac{1}{\mu}\right) = 0, \\ \zeta^2 - (1+\mu)\zeta + \rho\left(1 + \frac{1}{\mu}\right) = 0; \end{cases} \tag{22}$$

where $\mu = \sqrt{2\rho - 1}$, and ρ is one of the real roots of the following cubic equation,

$$\rho^3 - \rho^2 + \epsilon\rho - \frac{\epsilon}{2} = 0.$$

By substituting ρ with $\eta + \frac{1}{3}$, we have

$$\eta^3 + \left(\epsilon - \frac{1}{3}\right)\eta - \frac{\epsilon}{6} - \frac{2}{27} = 0.$$

According to the judgment formula of a cubic equation, we have

$$\Delta = \left(-\frac{\epsilon}{12} - \frac{1}{27}\right)^2 + \left(\frac{\epsilon}{3} - \frac{1}{9}\right)^3 > 0.$$

Thus, this cubic equation has one real root and two complex roots. By Cardano's formula [20], η is given by

$$\eta = \sqrt[3]{\sqrt{\left(\frac{\epsilon}{12} + \frac{1}{27}\right)^2 + \left(\frac{\epsilon}{3} - \frac{1}{9}\right)^3} + \frac{\epsilon}{12} + \frac{1}{27}} - \sqrt[3]{\sqrt{\left(\frac{\epsilon}{12} + \frac{1}{27}\right)^2 + \left(\frac{\epsilon}{3} - \frac{1}{9}\right)^3} - \frac{\epsilon}{12} - \frac{1}{27}}.$$

Then, Eq. (16) follows by $\rho = \eta + \frac{1}{3}$, and it is easily seen that $\frac{1}{2} < \rho \leq 1$. Next, by solving Eq. (22), we have

$$\begin{cases} \zeta_{1,2} = \frac{1}{2}\left[1 - \mu \pm \sqrt{(1-\mu)^2 - 4\rho\left(1 - \frac{1}{\mu}\right)}\right]; \\ \zeta_{3,4} = \frac{1}{2}\left[1 + \mu \pm \sqrt{(1+\mu)^2 - 4\rho\left(1 + \frac{1}{\mu}\right)}\right]. \end{cases}$$

Since $\zeta \in [0, 1]$ and $\rho \in \left(\frac{1}{2}, 1\right]$, then ζ follows Eq. (15).

Case II: $M = 2$, $N = 3$ with $\mathsf{SNR} > 1$. From Eq. (7), the following equation holds,

$$\zeta^4 + \frac{2}{3}\zeta^3 - \frac{2}{3}\epsilon = 0. \tag{23}$$

By using the same method as in Case I, ζ and ρ follow Eq. (17) and Eq. (18), respectively.

Case: $M \geq 3$, $N \geq 3$ with $\mathsf{SNR} > 1$. From Eq. (7), the following equation holds,

$$\begin{cases} 2\zeta^N - \epsilon = 0, & M = N; \\ \zeta^N - \epsilon = 0, & M < N. \end{cases}$$

Thus, Eq. (19) holds.

Case: $M \geq 2$, $N \geq 3$ with $\mathsf{SNR} < 1$. From Eq. (13), the following equation holds,

$$\zeta^N - \epsilon = 0.$$

Thus, Eq. (20) holds.

According to Eq. (3) and by letting $p_{\mathrm{s}} = \zeta$, we have

$$C_\epsilon = B_{\mathrm{s}} \log\left(1 + \mathsf{SNR} \ln \frac{1}{1-\zeta}\right).$$

■

Usually, the normalized outage capacity is used as the outage performance measure, which is defined by

$$C_{\mathrm{norm}} = \frac{C_\epsilon}{C_{\mathrm{awgn}}},$$

where $C_{\mathrm{awgn}} = B_{\mathrm{s}} \log(1 + \mathsf{SNR})$. It is easy to verify that

$$\lim_{\mathsf{SNR}\to\infty} C_{\mathrm{norm}} = 1,$$

and

$$\lim_{\mathsf{SNR}\to 0} C_{\mathrm{norm}} = 1.$$

Remark 3: From Eq. (5), it is not difficult to verify that C_{norm} may exceed 1 under some conditions. That is,

$$\begin{cases} C_{\mathrm{norm}} \geq 1, & \zeta \geq 1 - e^{-1}; \\ C_{\mathrm{norm}} < 1, & \zeta < 1 - e^{-1}. \end{cases} \tag{24}$$

Note that $e = 2.71828\ldots$ in Eq. (24). The phenomenon of $C_{\mathrm{norm}} \geq 1$ is due to the fact that more subchannels will provide higher reliability.

Fig. 4 compares the ϵ-outage capacity between the fixed allocation scheme and the DSHK algorithm in a V2V communication system with $M = 4$ and $N = 8$. In the fixed allocation scheme, each Tx-Rx pair has two fixed subchannels, and the Tx-Rx pair will be in outage if and only if two subchannels are both in outage. It is shown that all the curves of the DSHK algorithm are higher than those of the fixed allocation scheme. Note that C_ϵ of the DSHK algorithm when $P_{\mathrm{u}} = 0.1$ is larger than 1 and approaches 1 as SNR increases. From Eq. (24), we get $\zeta = 0.1^{\frac{1}{8}} > 1 - e^{-1}$.

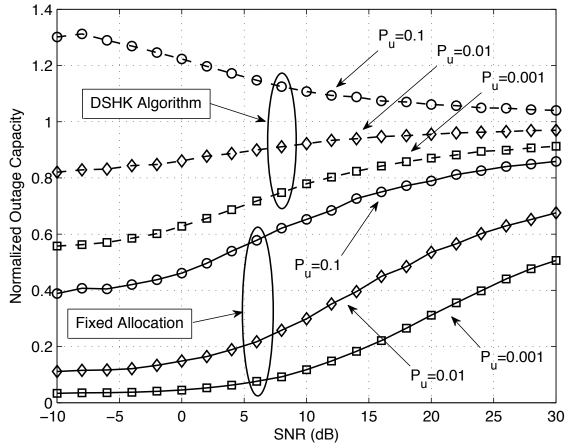


Fig. 4. The ϵ-outage capacity comparison between the DSHK algorithm and the fixed allocation scheme for a V2V communication system with $M = 4$ and $N = 8$.

VI. Numerical Results

In this section, some numerical results are given to validate the theoretical derivations. MATLAB is chosen as the simulation tool, since it has been widely used in performance evaluation for wireless communications. For simplicity, the parameter R is set to 1. At each SNR value, Eq. (3) is used to calculate the subchannel outage probability p_{s}. Then, we generate a large amount of samples of $\mathscr{G}(\mathcal{K}_{M,N}, q_{\mathrm{s}})$ and search for a maximum matching $\mathcal{M}$ by the DSHK algorithm. The average outage number of each Tx-Rx pair can then be obtained. Accordingly, P_{u} can be calculated via Eq. (6). The theoretical approximations are calculated by Eqs. (7) (9) (13).

The outage probability P_{u} for a V2V communication system with $M = 2, 3$ and $N = M, \ldots, M+4$ are shown in Figs. 5–6, respectively. In these figures, we show the complete outage performance for the full SNR range, where the lowest SNR is $-10\,\mathrm{dB}$. In the high SNR regime, Eq. (7) is used to calculate the outage probability of a Tx-Rx pair, while in the low SNR regime, Eq. (13) is used. It can be seen that the theoretical approximation curves and the simulation results are identical in the high SNR regime, because p_{s} in Eq. (7) tends to zero as SNR goes to infinity. Note that the approximation curves have a gap between $-2\,\mathrm{dB}$ and $0\,\mathrm{dB}$, because the low SNR regime and the high SNR regime are distinguished at $0\,\mathrm{dB}$. The curves calculated by theoretical formulas are close to the simulation ones even when SNR is near $0\,\mathrm{dB}$. For $M = N = 2$, the accurate formula is used to calculate the outage probability, so that the two curves are identical for any SNR in Fig. 5. In Fig. 6, we also show that the diversity gain achieved by the DSHK algorithm is N, which is illustrated at $10\,\mathrm{dB}$. By comparing all the curves in Figs. 5–6, one can find that the SNR needed to get the same outage probability become small as the subchannel number gets larger. For example in Fig. 5, let the target outage probability be 10^{-6}, then if $N = 2$ the SNR needed is $30\,\mathrm{dB}$, but if $N = 6$ the SNR needed is only $10\,\mathrm{dB}$. There is a $20\,\mathrm{dB}$ gain for 4 more degrees of freedom in the frequency domain. Note that this gain has nothing to do with the number of Tx-Rx pairs if $M \leq N$.

To verify the theoretical formulas for the ϵ-outage capacity, some numerical results will further be shown. The binary search based ordinal optimization method is used to calculate C_ϵ [26], [27]. First, fix a desired outage probability P_{u}, and let $C_{\mathrm{norm}} = C_{\mathrm{norm}}^{\max} = 2$ and $C_{\mathrm{norm}}^{\min} = 0$. If we set $R = C_{\mathrm{norm}} \cdot C_{\mathrm{awgn}}$ and give a SNR value, p_{s} can be obtained by Eq. (3). Then, we generate samples of $\mathscr{G}(\mathcal{K}_{M,N}, q_{\mathrm{s}})$ and get the simulated outage probability $\hat{P}_{\mathrm{u}}$. If $|\hat{P}_{\mathrm{u}} - P_{\mathrm{u}}| < \delta$, C_{norm} is seen as the correct value. However, if $|\hat{P}_{\mathrm{u}} - P_{\mathrm{u}}| \geq \delta$, $C_{\mathrm{norm}}^{\max}$ and $C_{\mathrm{norm}}^{\min}$ will be changed by the following rules:

1) if $\hat{P}_{\mathrm{u}} - P_{\mathrm{u}} \geq \delta$, set $C_{\mathrm{norm}}^{\max} = C_{\mathrm{norm}}$ and go to 3);
2) if $\hat{P}_{\mathrm{u}} - P_{\mathrm{u}} \leq -\delta$, set $C_{\mathrm{norm}}^{\min} = C_{\mathrm{norm}}$;
3) set $C_{\mathrm{norm}} = \frac{1}{2}(C_{\mathrm{norm}}^{\max} + C_{\mathrm{norm}}^{\min})$.

After the correct C_{norm} is obtained, the SNR value will be changed to a new value. Then, the normalized ϵ-outage capacity at any SNR value can be obtained. In our simulations, δ is set to $\frac{1}{1000} P_{\mathrm{u}}$.

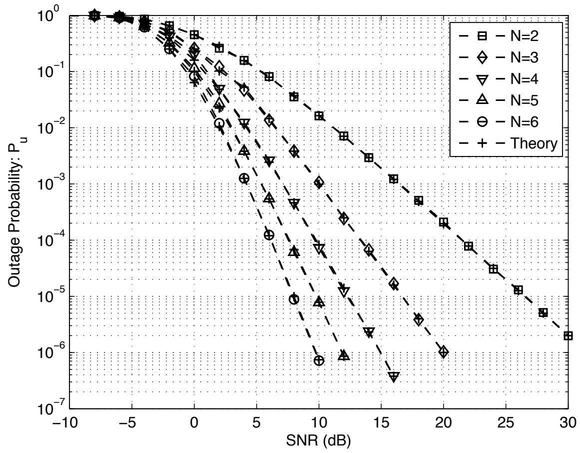


Fig. 5. The outage probability P_{u} of a V2V communication system with $M = 2$ and $N = 2, \ldots, 6$.

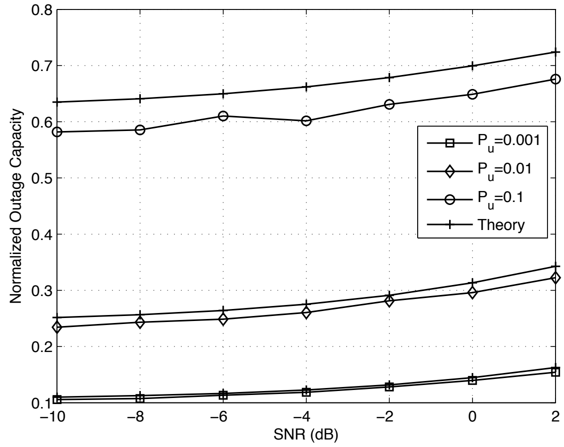


Fig. 7. The ϵ-outage capacity C_ϵ in the low SNR regime of a V2V communication system with $M = 2$ and $N = 3$.

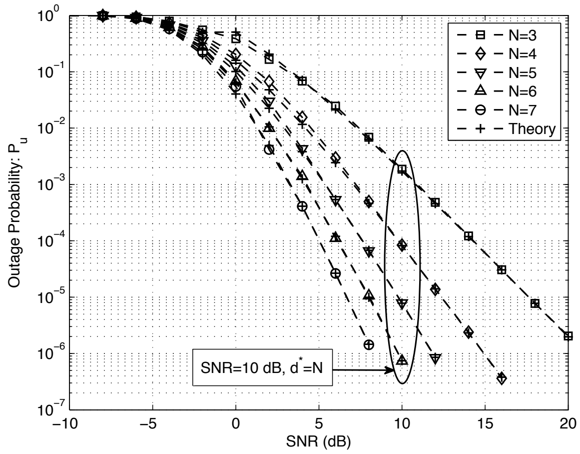


Fig. 6. The outage probability P_{u} of a V2V communication system with $M = 3$ and $N = 3, \ldots, 7$.

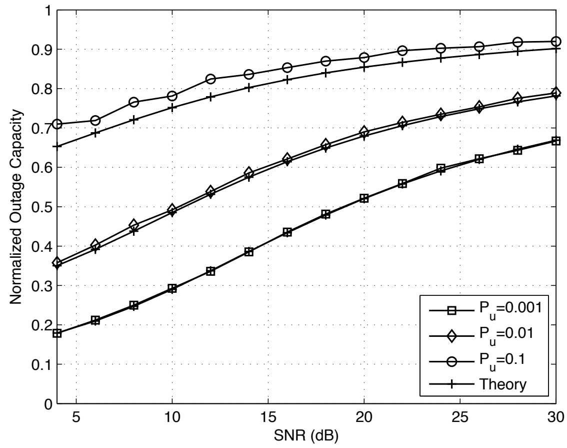


Fig. 8. The ϵ-outage capacity C_ϵ in the high SNR regime of a V2V communication system with $M = 2$ and $N = 3$.

Fig. 7 and Fig. 8 show the numerical results for a V2V communication system with $M = 2$ and $N = 3$ in the low and high SNR regimes, respectively. From these figures, one can observe that the three curves all approach to 1 as desired, and C_ϵ increases as P_{u} increases. In addition, the fading impact is more significant in the low SNR regime. It is also shown that the difference between the simulation curves and the theoretical ones increases as P_{u} increases, because the higher order term have a significant impact as P_{u} increases. However, two curves of $P_{\mathrm{u}} = 10^{-3}$ are identical, so that the proposed theoretical results can be used in practical systems. In the low SNR regime, shown in Fig. 7, the theoretical curve is higher than the simulation one, since the outage events caused by multiple Tx-Rx pairs compete for one subchannel is neglected. In the high SNR regime, shown in Fig. 8, the theoretical curve is lower than the simulation one, since the negative higher order term is neglected. Next, Fig. 9 show the simulation results of a V2V communication system with $M = 3$ and $N = 3$. It also has an excellent accuracy.

The theoretical and simulation results can be used for system design. For example, we can use the results to determine how many subchannels do we need to accommodate the outage probability requirement. According to the value of SNR, a proper equation in the high or low SNR regime can be chosen. By solving the inequality that the outage probability is smaller than the given value, the minimum needed number of subchannels will be obtained. This can also be found in the figures showing the outage probability. Our formulas can also be used to calculate how many Tx-Rx pairs can be supported by a V2V communication system with a given outage probability P_{u}. This, in fact, is a connection admission control mechanism. If a Tx-Rx pair wants to communicate with each other, the Rx estimates and broadcasts the CSI, then Tx calculates the outage probability P'_{u} assuming to communicate. The Tx-Rx pair will be admitted, if and only if P'_{u} is smaller than the requirement. Another important question is how to determine the maximum allowable traffic rate for real-time applications at a given reliability requirement. With the result of ϵ-outage capacity, the traffic rate cannot exceed C_ϵ, if ϵ outage probability is required. On the other hand, this result is also useful to determine how many subchannels do we need

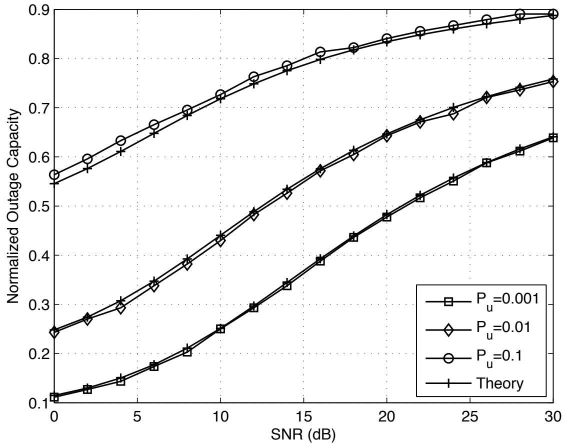


Fig. 9. The ϵ-outage capacity C_ϵ in the high SNR regime of a V2V communication system with $M = 3$ and $N = 3$.

to accommodate the requirement of Tx-Rx pair transmission rate.

VII. Conclusion

In this paper, we focused on the subchannel allocation problem, which is very important in V2V communications. Based on the outage formulation, we designed a random bipartite graph model for this problem. By using the maximum matching method on random bipartite graphs, the DSHK algorithm has been proposed to solve this problem with a sub-linear complexity of $\mathcal{O}(N^{2/3})$, where N is the number of subchannels. Based on the properties of maximum matching generated by the DSHK algorithm, the outage probabilities are then derived in the high (near vehicles) and low (far vehicles) SNR regimes, respectively. It was then found that the coefficient of the second-order term in the outage probability is zero in the high SNR regime except for the scenario of two Tx-Rx pairs with two or three subchannels. Thus, the proposed method has a similar outage performance as the scenario of one Tx-Rx pair with N subchannels. By using the proposed scheme, each Rx can only need to estimate the outage status of each subchannel and broadcast N bit to indicate its channel states, thereby having a low signaling overhead. The outage capacity has also been derived to determine the maximum allowable coding rate of real-time traffic in V2V communications. Therefore, this paper proposed a low complexity distributed subchannel allocation algorithm to accommodate the QoS requirements in V2V communications. Moreover, the random bipartite graph and maximum matching based framework can be used as a powerful tool for performance evaluation of existing and upcoming V2V communication systems.

Appendix A
Proof of Lemma 1

Consider vertices in $\mathcal{U}$ first. Without loss of generality, we assume that $\mathcal{N}(u_1) \geq \cdots \geq \mathcal{N}(u_M)$, and satisfies

$$\sum_{m=1}^{M} \mathcal{N}(u_m) = (M-1)N - 1. \tag{25}$$

According to Hall's theorem [28], if there is one unsaturated vertex, there must be a minimum value K satisfying $\sum_{m=K+1}^{M} \mathcal{N}(u_m) < M - K$, where $0 \leq K < M$. From Eq. (25), we have

$$\sum_{m=1}^{K} \mathcal{N}(u_m) > (M-1)N - 1 - M + K.$$

It is known that $\mathcal{K}_{K,N}$ has KN edges, thus $KN > (M-1)N - 1 - M + K$. By solving this inequality, we get

$$K > (M-1) - \frac{2}{N-1}.$$

Since $N \geq M \geq 3$ and by considering the condition on K, we have $K = M - 1$. Then, we have $\mathcal{N}(u_M) = 0$. Thus, $\mathcal{G}(\mathcal{U}' \cup \mathcal{S}, \mathcal{E})$ with $\mathcal{U}' = \mathcal{U} \setminus \{u_M\}$ is a complete bipartite graph, i.e., $\mathcal{K}_{M-1,N}$, then there is only one unsaturated vertex in $\mathcal{U}$.

Next, consider vertex set $\mathcal{S}$, then there are two cases that need to be considered.

Case I: $M = N$. In this case, we have the same result as stated before, i.e., only s_N is in outage. Thus, $\mathcal{N}(\mathcal{U}) = M-1$. However, if any one vertex in $\mathcal{U}$ is deleted, a complete bipartite graph $\mathcal{K}_{M-1,M-1}$ or a bipartite graph has one less edge than $\mathcal{K}_{M-1,M-1}$ can be obtained. Since $M \geq 3$, we have

$$(M-1)M - 1 \geq (M-1)(M-1) + 1.$$

According to Lemma 3 in [11], there is no unsaturated vertex in $\mathcal{U}$ except the deleted one. As a result, there is only one unsaturated vertex in $\mathcal{U}$.

Case II: $M < N$. Assume $\mathcal{N}(s_1) \geq \cdots \geq \mathcal{N}(s_N)$, and execute the same process. Then, $K = N - 1$ is obtained due to $M \geq 3$, i.e., only one vertex in $\mathcal{S}$ is isolated. Consider the bipartite graph $\mathcal{G}(\mathcal{U} \cup \mathcal{S}', \mathcal{E})$ with $\mathcal{S}' = \mathcal{S} \setminus \{s_N\}$, then we have

$$(M-1)N - 1 \geq (M-1)(N-1) + 1.$$

From Lemma 3 in [11], there is no unsaturated vertex in $\mathcal{U}$.

Appendix B
Proof of Lemma 2

Consider vertices in $\mathcal{U}$ and assume that $\mathcal{N}(u_1) \geq \mathcal{N}(u_2)$. It is easy to verify that the lemma holds for $N = 2$. Consider the situation of $N = 3$, which means that $|\mathcal{E}| = N - 1 = M$. According to Lemma 5 in [11], if $|\mathcal{N}(u_1)| = 2$ and $|\mathcal{N}(u_2)| = 0$, then u_2 is in outage. This is the case when one vertex in $\mathcal{U}$ and one vertex in $\mathcal{S}$ are isolated. Another situation is $\mathcal{N}(u_1) = \mathcal{N}(u_2)$, which means only one subchannel is non-outage to both Tx-Rx pairs. However, this situation does not occur when $N \geq 4$. Since at this case $|\mathcal{E}| = N - 1 > M$, then at least one equation holds between $\mathcal{N}(u_1) > \mathcal{N}(u_2) \geq 1$ and $\mathcal{N}(u_1) \geq \mathcal{N}(u_2) \geq 2$. According to Hall's theorem, there is a matching that can make u_1 and u_2 saturated.

References

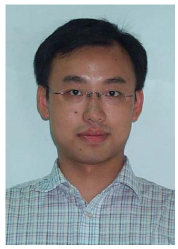
[1] Y. Bi, K. H. Liu, L. X. Cai, X. Shen, and H. Zhao, "A multi-channel token ring protocol for QoS provisioning in inter-vehicle communications," *IEEE Trans. Wireless Commun.*, to appear.

[2] J. Luo and J.-P. Hubaux, "A survey of research in inter-vehicle communications," in *Embedded Security in Cars – Securing Current and Future Automotive IT Applications.* New York, USA: Springer-Verlag, 2006, vol. 2, pp. 111–122.

[3] M. L. Sichitiu and M. Kihl, "Inter-vehicle communication systems: a survey," *IEEE Commun. Surveys. & Tutorials*, vol. 10, no. 2, pp. 88–105, 2008.
[4] T. L. Willke, P. Tientrakool, and N. F. Maxemchuk, "A survey of inter-vehicle communication protocols and their applications," *IEEE Commun. Surveys. & Tutorials*, vol. 11, no. 2, pp. 3–20, 2009.
[5] K. B. Letaief and Y. J. A. Zhang, "Dynamic multiuser resource allocation and adaptation for wireless systems," *IEEE Wireless Commun. Mag.*, vol. 13, no. 4, pp. 38–47, 2006.
[6] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel," in *IEEE Proc. Vehic. Technol. Conf. 2000*, Tokyo, Japan, May 2000.
[7] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, 1999.
[8] Y. Ma, "Rate-maximization for downlink OFDMA with proportional fairness," *IEEE Trans. Veh. Technol.*, vol. 57, no. 5, pp. 3267–3274, 2008.
[9] G. Song and Y. G. Li, "Cross-layer optimization for OFDM wireless networks — part I: theoretical framework," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 614–624, 2005.
[10] —— , "Cross-layer optimization for OFDM wireless networks — part II: algorithm development," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 625–634, 2005.
[11] B. Bai, W. Chen, Z. Cao, and K. B. Letaief, "Max-matching diversity in OFDMA systems," *IEEE Trans. Commun.*, vol. 58, no. 4, pp. 1161–1171, 2010.
[12] F. Eskafi, K. Petty, and P. Varaiya, "Dynamic channel allocation for vehicle-to-vehicle communications inautomated highway systems," in *IEEE Conf. on Intel. Trans. System*, Boston: USA, 9–12 Nov. 1997, pp. 58–63.
[13] C.-X. Wang, X. Cheng, and D. I. Laurenson, "Vehicle-to-vehicle channel modeling and measurements: recent advances and future challenges," *IEEE Commun. Mag.*, vol. 47, no. 11, pp. 96–103, 2009.
[14] C.-X. Wang, X. Hong, X. Ge, X. Cheng, G. Zhang, and J. S. Thompson, "Cooperative MIMO channel models: a survey," *IEEE Commun. Mag.*, vol. 48, no. 2, pp. 80–87, 2010.
[15] L. Ozarow, S. Shamai, and A. Wyner, "Information theoretic considerations for celluar mobile radio," *IEEE Trans. Veh. Technol.*, vol. 43, no. 2, pp. 359–378, 1994.
[16] A. S. Akki and F. Haber, "A statistical model for mobile-to-mobile land communication channel," *IEEE Trans. Veh. Technol.*, vol. 35, no. 1, pp. 2–10, 1986.
[17] L. Cheng, B. Henty, D. Stancil, F. Bai, and P. Mudalige, "Mobile vehicle-to-vehicle narrowband channel measurement and characterization of the 5.9 ghz dedicated short range communication (DSRC) frequency band," *IEEE J. Sel. Areas in Commun.*, vol. 25, no. 8, pp. 1501–1516, 2007.
[18] A. G. Zajić, G. L. Stüber, T. G. Pratt, and S. T. Nguyen, "Wide-band MIMO mobile-to-mobile channels: statistical modeling with experimental verification," *IEEE Trans. Veh. Technol.*, to appear.
[19] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. New York, USA: Cambridge University Press, 2005.
[20] E. Dehn, *Algebraic Equations: An Introduction to the Theories of Lagrange and Galois*. New York: USA: Columbia University Press, 1930.
[21] X. Cheng, C.-X. Wang, D. I. Laurenson, S. Salous, and A. V. Vasilakos, "An adaptive geometry-based stochastic model for non-isotropic MIMO mobile-to-mobile channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4824–4835, 2009.
[22] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: information-theoretic and communications aspects," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2619–2692, 1998.
[23] B. Bollobás, *Random Graphs*, 2nd ed. New York, USA: Cambridge University Press, 2001.
[24] M. Karpiński and W. Rytter, *Fast Parallel Algorithms for Graph Matching Problems*. New York, USA: Oxford University Press, 1998.
[25] H. M. Edwards, *Galois Theory (Galois' original paper with extensive background and commentary)*. New York, USA: Springer-Verlag, 1984.
[26] Q.-S. Jia, Y.-C. Ho, and Q.-C. Zhao, "Comparison of selection rules for ordinal optimization," *Mathematical and Computer Modelling*, vol. 43, pp. 1150–1171, 2006.
[27] T. H. Cormen, C. E. Leiserson, R. L. Rivestv, and C. Stein, *Introduction to Algorithm*, 2nd ed. Cambridge, USA: The MIT Press, 2001.
[28] R. Diestel, *Graph Theory*, 3rd ed. New York, USA: Springer-Verlag, 2005.

Bo Bai (S'09) received his BS degrees in Department of Communication Engineering with the highest honors from Xidian University in 2004, Xi'an China. He also obtained the honor of "Outstanding Graduates of Shaanxi Province". Now he is a Ph.D. candidate in Department of Electronic Engineering in Tsinghua University, Beijing China. From Apr. 2009, he is a visiting research staff in Department of Electronic and Computer Engineering, the Hong Kong University of Science & Technology.

His research interests include hot topics in wireless communications, information theory, random graph, and combinatorics. He has served as a TPC member for IEEE ICC '10 and a Session Chair for IEEE Globecom '08. He has also served as reviewers for IEEE Communication Letters, IET Communications, and many international conferences, including IEEE ICC, Globecom, and WCNC. He received the Student Travel Grant at IEEE Globecom '09.



Wei Chen (S'03-M'07) received his BS and PhD degrees in Electronic Engineering (both with the highest honors) from Tsinghua University, Beijing, China, in 2002, and 2007, respectively. From May 2005 to March 2007, he was a visiting research staff in Department of Electronic and Computer Engineering, the Hong Kong University of Science & Technology. Since July 2007, he has been with Department of Electronic Engineering, Tsinghua University, where he is currently an Assistant Professor. He visited the Chinese University of Hong Kong on October 2007, the Hong Kong University of Science & Technology on September 2008 and September 2009, and University of Southampton from June to September 2010.

His research interests are in the broad areas of wireless communications, information theory and applied optimizations. He has served as TPC members for a number of major international conferences, including IEEE ICC, Globecom, and WCNC. He served as student travel grant chair of ICC '08, co-chair of cognitive and cooperative network workshop 2008, TPC co-chair of wireless communication symposium in ICC '10, and TPC co-chair of VTC, Spring 2011. He received the 2009 IEEE Marconi Prize Paper Award, the Best Paper Award at IEEE ICC 2006 and the Best Paper Award at IEEE IWCLD 2007.

Khaled Ben Letaief (S'85-M'86-SM'97-F'03) received the BS degree with distinction in Electrical Engineering from Purdue University at West Lafayette, Indiana, USA, in December 1984. He received the MS and Ph.D. Degrees in Electrical Engineering from Purdue University, in August 1986, and May 1990, respectively. From January 1985 and as a Graduate Instructor in the School of Electrical Engineering at Purdue University, he has taught courses in communications and electronics.

From 1990 to 1993, he was a faculty member at the University of Melbourne, Australia. Since 1993, he has been with the Hong Kong University of Science & Technology, where he is currently the Dean of Engineering. He is also Chair Professor of Electronic and Computer Engineering as well as the Director of the Hong Kong Telecom Institute of Information Technology and that of the Wireless IC System Design Center. His current research interests include wireless and mobile networks, Broadband wireless access, OFDM, Cooperative networks, Cognitive radio, MIMO, and Beyond 3G systems. In these areas, he has over 400 journal and conference papers and given invited keynote talks as well as courses all over the world. He has also 3 granted patents and 10 pending US patents. Dr. Letaief served as consultants for different organizations and is the founding Editor-in-Chief of the *IEEE Transactions on Wireless Communications*. He has served on the editorial board of other prestigious journals including the *IEEE Journal on Selected Areas in Communications - Wireless Series* (as Editor-in-Chief). He has been involved in organizing a number of major international conferences and events. These include serving as the Co-Technical Program Chair of the 2004 *IEEE International Conference on Communications, Circuits and Systems*, ICCCS'04; General Co-Chair of the 2007 *IEEE Wireless Communications and Networking Conference*, WCNC'07; Technical Program Co-Chair of the 2008 *IEEE International Conference on Communication*, ICC'08, and Vice General Chair of the 2010 *IEEE, ICC*'10. He served as an elected member of the IEEE Communications Society Board of Governors, and IEEE Distinguished lecturer. He also served as the Chair of the IEEE Communications Society Technical Committee on Wireless Communications, Chair of the Steering Committee of the *IEEE Transactions on Wireless Communications*, and Chair of the 2008 IEEE Technical Activities/Member and Geographic Activities Visits Program. He is a member of the IEEE Communications Society and IEEE Vehicular Technology Society Fellow Evaluation Committees as well as member of the IEEE Technical Activities Board/PSPB Products & Services Committee. He is the recipient of many distinguished awards including the Michael G. Gale Medal for Distinguished Teaching (Highest university-wide teaching award at HKUST); 2007 IEEE Communications Society Publications Exemplary Award; and 9 Best Paper Awards with the latest prestigious 2009 IEEE Marconi Prize Paper Award in Wireless Communications. Dr. Letaief is a Fellow of IEEE and the Vice-President Elect for Conferences of the IEEE Communications Society.

Zhigang Cao (M'84-SM'85) graduated with golden medal from the Department of Radio Electronics at Tsinghua University, Beijing in 1962. He was a visiting scholar at Stanford University from 1984 to 1986. Since 1962, He has been with Tsinghua University, where he is currently a professor of Electronic Engineering Department, deputy director of the National Key Laboratory on Microwave and Digital Communications. He has published 6 books and more than 450 papers on Communication and Signal Processing fields, and held 20 patents. He has won the first prize of the Best Engineering Textbook Award in China. He won 10 awards from National Science Congress, Ministry of Electronic Industry, State Education Commission, and Ministry of Education, etc. He has also won special grant issued by the Chinese Government for his outstanding contributions to education and research. His current research interests include mobile and satellite communications, as well as, speech signal processing. Prof. Cao is a fellow of Chinese Institute of Communications (CIC) and a vice chairman of Academic Committee of CIC. He is also a senior member of IEEE, IEICE overseas member, senior member of Chinese Institute of Electronics, member of the board of directors of Beijing Communication institute. His PhD students obtained 2003 best PhD dissertation award among all universities in China and 2005 best PhD dissertation award nomination. He also serves as vice chief editor of Chinese Journal of Electronics, editor of ACTA Electronica Sinica, as well as, Journal of Astronautics.