上 海 交 通 大 学 试 卷

（2022~ 2023~1 Academic Year/Fall Semester）

Class No.________________________ Name in English or Pinyin: ________________________

Student ID No.________________ Name in Hanzi(if applicable): ________________________

VE215 Introduction to Circuits

Final Exam

12th December 08:00 – 09:40

The exam paper has 13 pages in total.

Pledge of Honor

The University of Michigan –Shanghai Jiao Tong University Joint Institute trusts its students to participate in examinations in an honorable and respectful manner, following a spirit of fairness and equality. Cheating, seeking unfair advantage, and disturbing the safe and harmonious environment of examinations are contrary to the ethical principles of students of the Joint Institute. The letter and spirit of the Honor Code shall guide the behavior of students, faculty and all members of the Joint Institute. Therefore, I hereby declare that

(i) I will neither give nor receive unauthorized aid during the present examination, nor will I conceal any violations of the Honor Code by others or myself.

(ii) I confirm that I have read and understood the rules and procedures for the examination set out by SJTU. I will follow them to the best of my ability.

(iii) I understand that violating the rules and procedures for examinations or the Honor Code will lead to administrative and/or academic sanctions.

Date:

Signature:

Please enter grades here:

Exercises No. 题号	Points 得分	Grader's Signature 流水批阅人签名
1		
2		
3		
4		
5		
Total 总分		

Instructions

You have **100 minutes** to complete this midterm. Please write your answers in this booklet. Remember to write neatly and clearly, so your answers can be fully understood.

- You **may not use** your electronic devices other than your calculator.
- **One A4 double-sided** cheating-sheet can be used.
- **Mathematics formula sheet** can be found at the end of this exam paper.
- **Chapter 9-14 except for Chapter 12** are covered in this exam.
- There are **5 questions** in total.
- Please **write your partial steps** and that will be counted.
- Please **manage your time properly**. If you encounter some questions and feel hard, please move on and go back after finishing all the rest.
- **No tolerance to cheating!** Any intentional violations of the SJTU exam rule of a vile nature will be assigned an "F" as course grade.

Fingers Crossed!

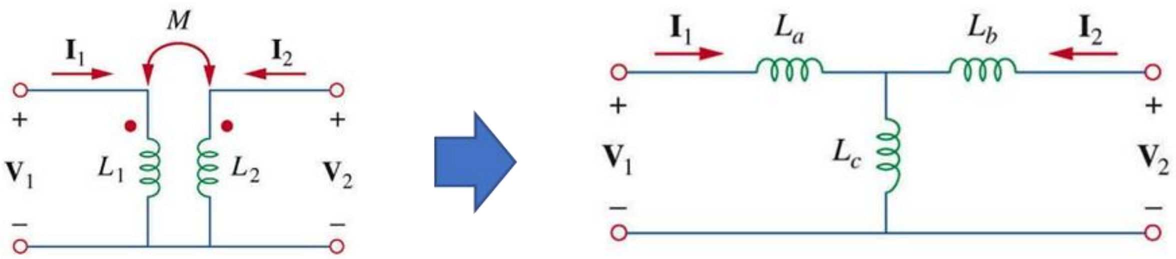
1. Please answer questions 1.1 – 1.5. [30 points]

1.1 The primary current to an **ideal transformer** rated at 3000/240 V is 10 A. Calculate (a) the turns ratio, (b) the kVA rating, and (c) the secondary current. [4 points]

Turns ratio = ____________________ kVA rating = ____________________ kVA

Secondary current = ______________________ A

1.2. A linear transformer can be converted into an equivalent circuit in T shape, as the figure below shows. Given $L_1 = j10$, $L_2 = j15$, and $M = j1$, calculate the **impedance** of three inductors in **T-equivalent circuit**. [4 points]



L_a = __________________; L_b = ________________; L_c = ___________________

1.3 When connected to a 120-V (rms), 60-Hz power line, a load absorbs 3 kW at a lagging power factor of 0.65. Find the **value of capacitance** necessary to raise the pf to 0.90 (lagging), in **parallel** with the load. [6 points]

C = ________________________________F

1.4 Please design a phase shifter consisting of one resistor and one capacitor. The output voltage (V_o) **leads** the input voltage (V_{in}) for **30°**. Also, please explain why the circuit cannot achieve **90° phase shift**. [8 points]

Design a phase shifter: (draw)

Note: notate the resistance and capacitor with its **impedance**, **respectively**.

Explain why the circuit cannot achieve 90° phase shift:

1.5 Two loads connected in parallel are respectively 3 kW at a pf of 0.85 **lagging** and 4.5 kW at a pf of 0.90 **leading**. Calculate the pf of the two loads. Find the complex power provided by the source. [8 points]

pf of the two loads = ________________, (value) [2]

________________ (choose one to write: lagging/leading) [2]

complex power **provided** by the source = __________________________ kVA [4]

(please use phasor form: $|S|\angle\varnothing$)

1.1 $n = \frac{V_2}{V_1} = \frac{240}{3000} = \frac{2}{25}$ (0.08) (wrong for $\frac{25}{2}$ (12.5))

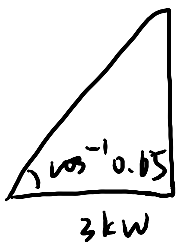
kVA rating $= V_1 I_1 = 3000 \times 10 = 30$ kVA

secondary current $= \frac{3000 \times 10}{240} = 125$ A

(one correct 2'
two 3'
all 4')

1.2 $L_a = L_1 - M = 9j$, $L_b = L_2 - M = 14j$, $L_c = M = 1j$ (Same as 1.1)

1.3



$S = 3 + 3.51j$ kVA $\quad \theta = \cos^{-1} 0.65 = 49.46°$

$S' = 3 + (3.51 - Q_C)j$ kVA $\quad \theta' = \cos^{-1} 0.9 = 25.84°$

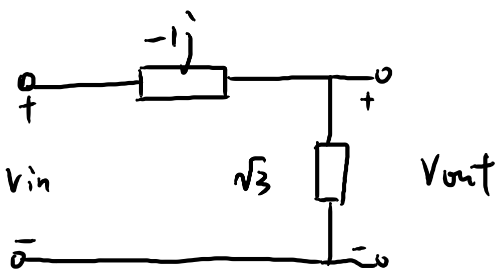
$\frac{3.51 - Q_C}{3} = \tan 25.84°$

$\Rightarrow Q_C = 2.06$ kVAR

$\Rightarrow \frac{120^2}{2.06k} = X_C = \frac{1}{wC}$

$\Rightarrow C = 3.8 \times 10^{-4}$ F $(\pm 0.05 \times 10^{-4}$ F$)$

1.4



proof: $V_{out} = \frac{\sqrt{3}}{\sqrt{3} - 1j} V_{in} = \frac{\sqrt{3}}{2} \angle 30° \, V_{in}$

i.e. $\angle V_{out} = \angle V_{in} + 30°$ (leads)

so as long as R terminal output, $\frac{R}{|X_C|} = \sqrt{3}$,

it's correct

notate Vin & Vout 2'

Z_R & Z_C 2'

if C terminals output, no points

if not one stage, no points

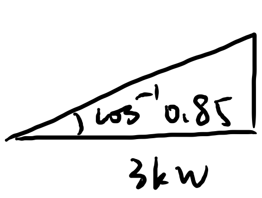
$\frac{V_o}{V_i} = \frac{R}{R - jX} = |Z| \angle Z$ 1'

if $\angle Z = 90°$, $R = 0$ or $X \to \infty$ ($C = 0$) } either 1'

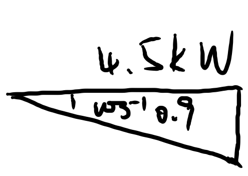
but both cause $|Z| = 0$

so no output voltage actually 2' (no duducing process, but having this, get full 4')

1.5



$S_1 = 3 + 1.86j$ kVA



$S_2 = 4.5 - 2.18j$ kVA

$S_{total} = 7.5 - 0.32j$

$pf = \cos(\tan^{-1}(\frac{0.32}{7.5})) = 0.999$ leading

S_{source} (provided) $= S_{total}$ (consumed) $= 7.5 - 0.32j = 7.51 \angle -2.44$ kVA

(not $S \angle \phi$, −2)

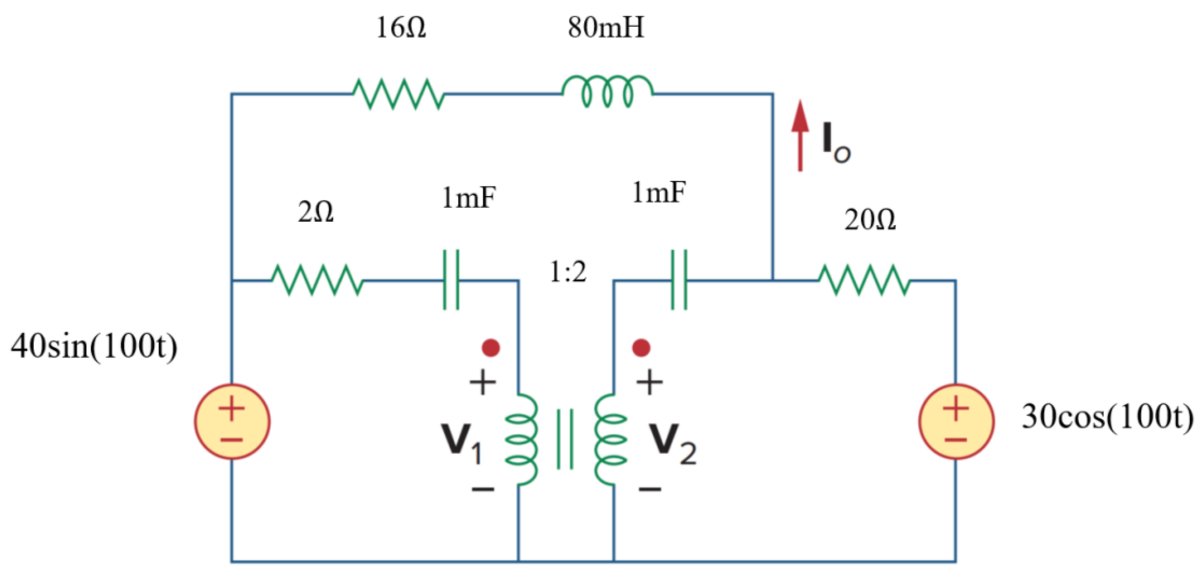
or (not kVA, −2)

2. Please analyze an **ideal transformer** circuit below. Assume all values in the question below are **RMS** values. [Total 16 points]

(1) Transform the circuit below to the phasor domain, e.g. L → jωL. [2 points]

(2) Please assign currents at the transformer terminals (any direction as you prefer). Determine polarities of turn ratio *n* of both voltage and current. [2 points]

(3) Please get values for V_1, V_2 and I_o respectively in the circuit. [12 points]



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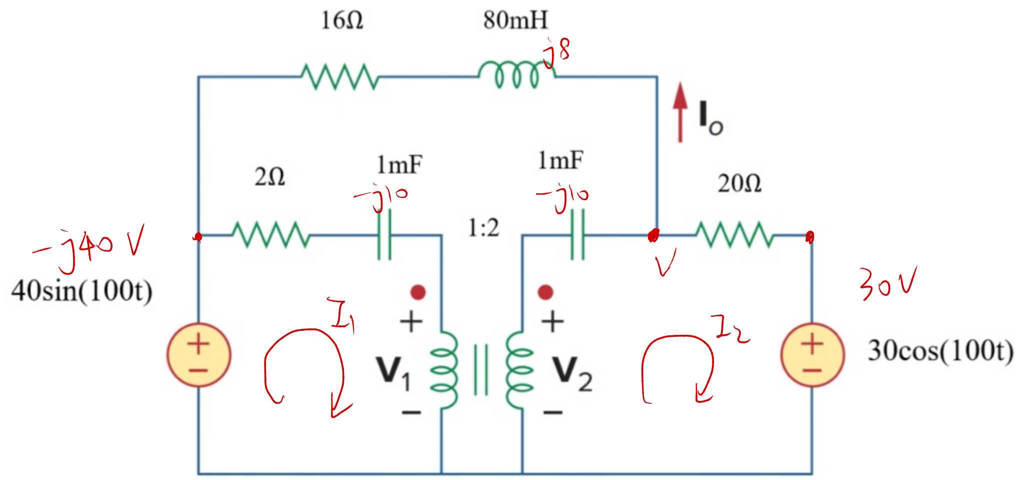


(1) $\omega = 100$

$80mH \to j8\ \Omega$ (1')

$1\ mF \to -j10\ \Omega$ (1')

(2) $\frac{V_1}{V_2} = \frac{1}{2}$ (1')

$\frac{I_1}{I_2} = \frac{2}{1}$ (depending on the assigned direction) (1')

(3)

$I_1 = \frac{-j40 - V_1}{2 - j10}$ (2')

$I_2 = \frac{V_2 - V}{-j10}$ (2')

$\frac{I_1}{I_2} = 2, \quad \frac{V_1}{V_2} = \frac{1}{2}$

$\frac{V - V_2}{-j10} + \frac{V + j40}{16 + j8} + \frac{V - 30}{20} = 0.$ (2')

$I_o = \frac{V + j40}{16 + j8}$ (2')

Any other partial procedure (1')

$\Rightarrow$

$V_1 = 7.56 - 15.80j$ V (1')

$= 17.52 \angle -64.43$ V

$V_2 = 15.12 - 31.60j$ V (1')

$= 35.03 \angle -64.43$ V

$I_o = 1.54 + 0.44j$ A (1')

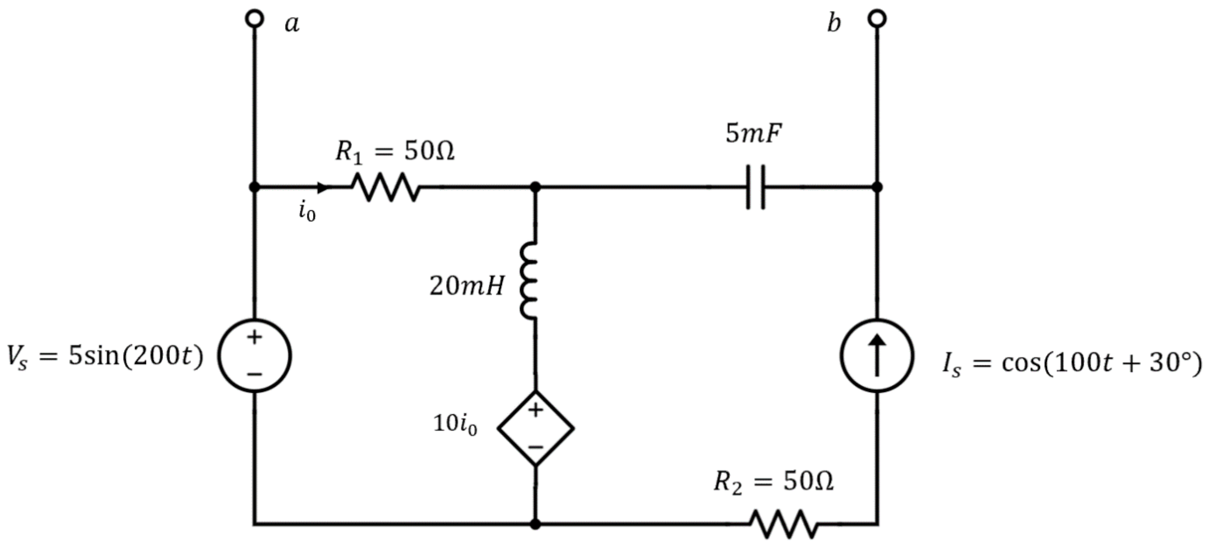
$= 1.60 \angle 15.95$ A

3. Below shows an **AC circuit** with an independent voltage and current source. [18 points]

(1) Please find the V_{TH} (Thevenin voltage) and Z_{TH} (Thevenin impedance) and draw Thevenin equivalent circuits between terminal a and b under $\boldsymbol{\omega = 100\, rad/s}$ and $\boldsymbol{\omega = 200\, rad/s}$. [12 points]

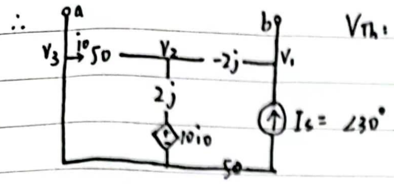
(2) Please draw a phasor diagram of the Thevenin equivalent impedance under (a) $\boldsymbol{\omega = 100\, rad/s}$ and (b) $\boldsymbol{\omega = 200\, rad/s}$. [6 points]

Hint: There should be 2 separate phasor diagrams in total. Please label all the angles, coordinates and magnitudes of the phasors on your phasor diagrams.



(1) 1' $w = 100\,rad/s$ $\quad \therefore Z_L = jwL = 2j,\ Z_C = \frac{1}{jwC} = -2j$

∴



V_{Th}: $\therefore \begin{cases} i_0 = \frac{V_3 - V_2}{50} \\ \frac{V_2 - V_3}{50} + \frac{V_2 - 10i_0}{2j} = I_s = \angle 30^\circ \\ V_1 - V_2 = -2jI_s = -2j \cdot \angle 30^\circ \end{cases}$

$\therefore V_1 = (2j+10)i_0,\ V_2 = (2j+10)i_0 + 2jI_s$

$V_3 = (2j+60)i_0 + 2jI_s$

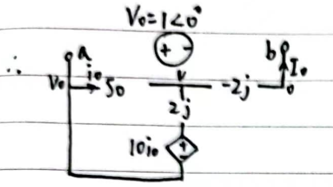
$\therefore V_{ab} = V_3 - V_1 = 50i_0 + 2jI_s$

$\therefore 50i_0 + 2j(i_0 + I_s) + 10i_0 = 0\ ,\ i_0 = -\frac{j}{30+j}I_s$

$\therefore V_{Th} = V_{ab} = (2j - \frac{50j}{30+j})I_s = -0.216 + 0.263j = 0.340\angle 129.40^\circ$

(3')

∴



Z_{Th}: $\begin{cases} V_0 - V = 50i_0 \\ \frac{V - V_0}{50} + \frac{V - (V_0 + 10i_0)}{2j} + \frac{V}{-2j} = 0 \end{cases}$

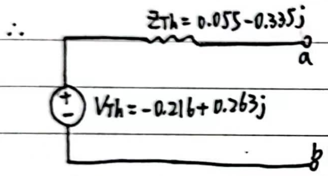
(Equation max 2')

$\therefore V = \frac{151 - 25j}{26}V_0$

$\therefore I_0 = \frac{V}{-2j} = \frac{151 - 25j}{-52j}V_0$

$\therefore Z_{Th} = \frac{V_0}{I_0} = \frac{-52j}{151 - 25j} = 0.055 - 0.335j = 0.340\angle -80.60^\circ$ (2')

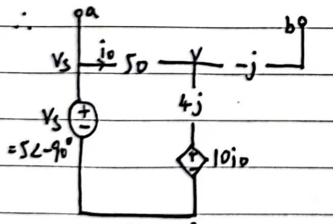
∴



(No Figure/Wrong −1)

2' $w = 200\,rad/s$ $\quad \therefore Z_L = jwL = 4j,\ Z_C = \frac{1}{jwC} = -j,\ V_S = 5\angle -90^\circ$

∴



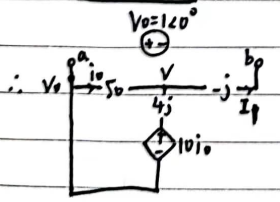
V_{Th}: $\therefore \begin{cases} V_S - V = 50i_0 \\ \frac{V - 10i_0}{4j} = i_0 \end{cases}$

$\therefore V_S = (60+4j)i_0,\ V = (10+4j)i_0$

$\therefore V_{Th} = V_S - V = 50i_0 = \frac{50}{60+4j}V_S = -0.277 - 4.148j$

$= 4.157\angle -93.81^\circ$ (3')

∴

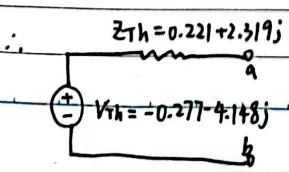


Z_{Th}: $\therefore \begin{cases} V_0 - V = 50i_0 \\ \frac{V - V_0}{50} + \frac{V - (V_0 + 10i_0)}{4j} + \frac{V}{-j} = 0 \end{cases}$

$\therefore V = -\frac{262 + 25j}{613}V_0 \quad \therefore I_0 = \frac{V}{-j} = \frac{262 + 25j}{613j}V_0$

$\therefore Z_{Th} = \frac{V_0}{I_0} = \frac{613j}{262 + 25j} = 0.221 + 2.319j = 2.329\angle 84.55^\circ$ (2')

∴



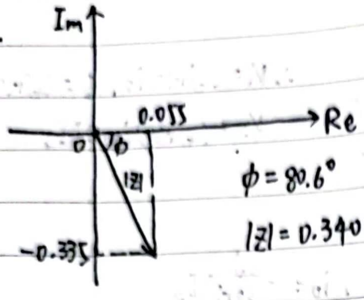
(1')

No.

Date

(2) 1' $\omega = 100\ rad/s$ $\therefore Z_{Th} = 0.340\angle \cancel{129.40} -80.60^\circ$

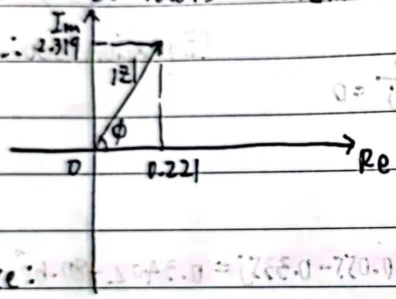
$\therefore$



$\phi = 80.6^\circ$

$|Z| = 0.340\ \Omega$

2' $\omega = 200\ rad/s$ $\therefore Z_{Th} = 2.329\angle 84.55^\circ$



$\therefore \phi = 84.55^\circ$

$|Z| = 2.329\ \Omega$

Notice:

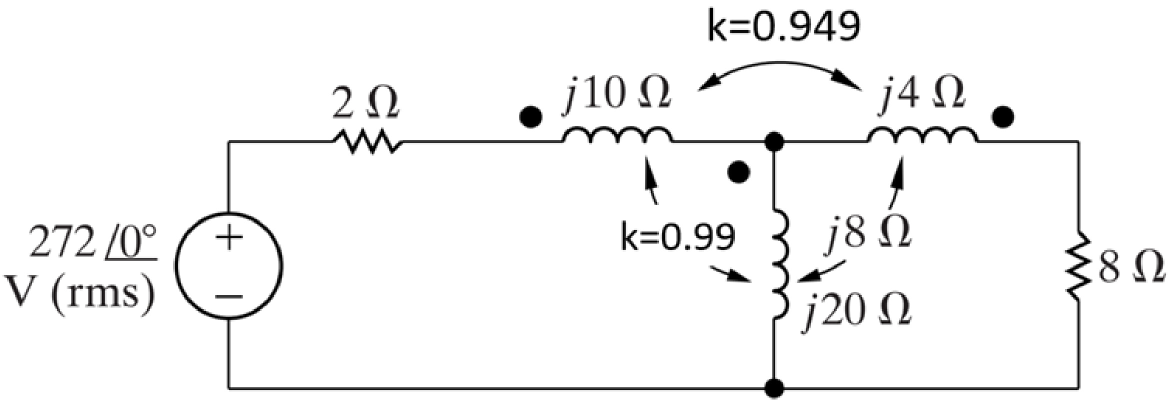
1. magnitude $|Z|$ 1'; coordinates (Re, Im) 1'; angle ϕ 1'

4. Please answer the following questions with the circuit shown below. [18 points]

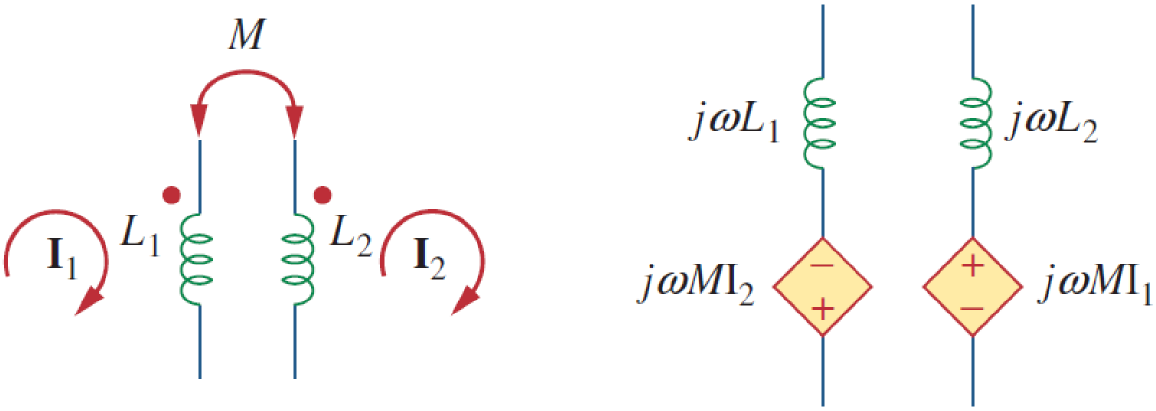
(1) Find the mutual inductance (assume **ω = 1000**) between j10 Ω, j4 Ω and j10 Ω, j20 Ω, are they tightly coupled or loosely coupled? [2 points]

(2) Please draw the equivalent circuit using the dependent voltage sources (example below). [8 points]

(3) Verify the conservation of average power in this circuit. [8 points]



Example: Magnetically coupled circuit and its equivalent circuit with dependent voltage sources.



4 (I)

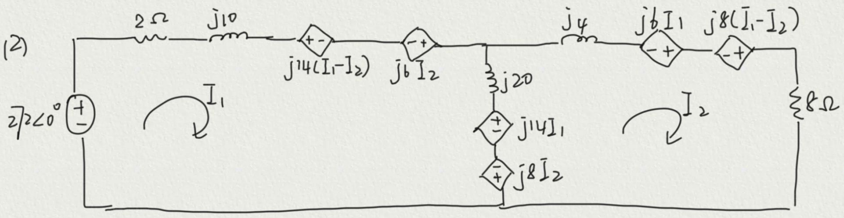
4

(1) j10, j4. $M = \dfrac{\sqrt{10\times 4}\times 0.949}{1000} = 6\ \text{mH}$ (0.5')

j10, j20. $M = \dfrac{\sqrt{10\times 20}\times 0.99}{1000} = 14\ \text{mH}$. (0.5')

They are both tightly coupled. (1')

(2)



All correct 8'

one dependent source wrong -1' : (start from 6')

(3)
$$\begin{cases} -272 + 2I_1 + j10I_1 + j20(I_1-I_2) + j14(I_1-I_2) - j6I_2 + j14I_1 - j8I_2 = 0 & (1') \\ j8I_2 - j14I_1 + j20(I_2-I_1) + j4I_2 - j6I_1 - j8(I_1-I_2) + 8I_2 = 0 & (1') \end{cases}$$

$$\begin{cases} I_1 = 20 - j4 & (1') \\ I_2 = 24 & (1') \end{cases}$$

$P_s = \mathrm{Re}(V_{rms} I_1^*) = 272\times 20 = 5440\ W$ (1')

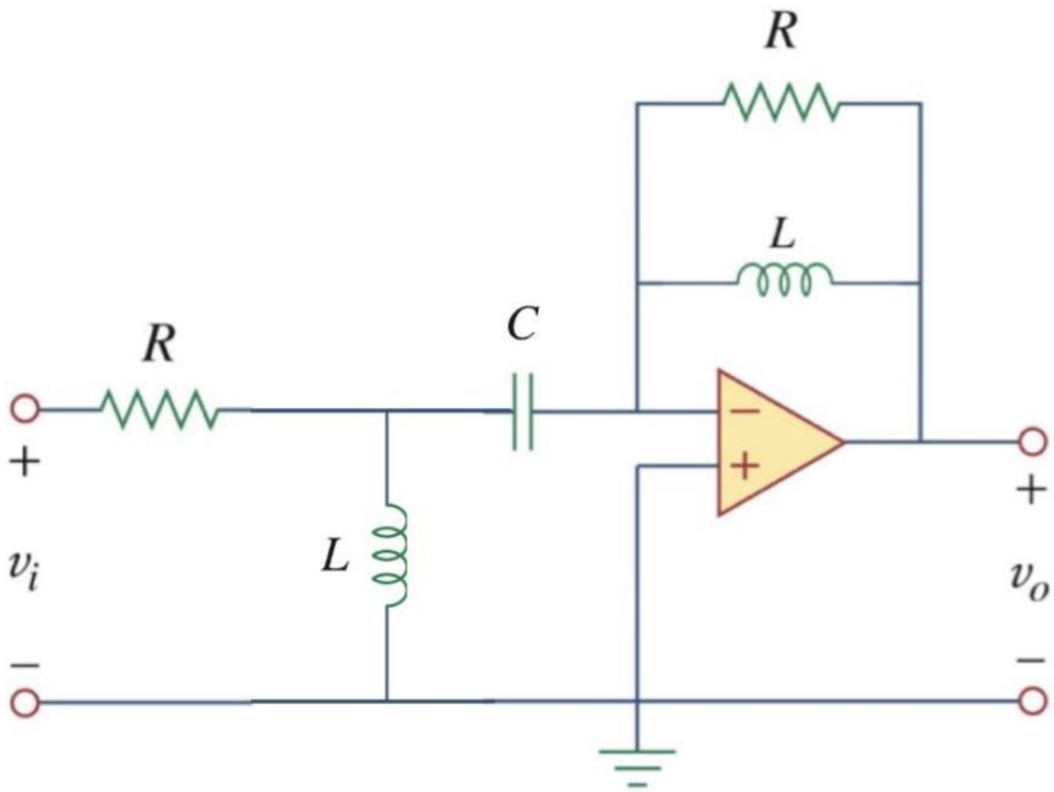
$P_{2\Omega} = |I_1|^2 \cdot R = 832W$ (1')

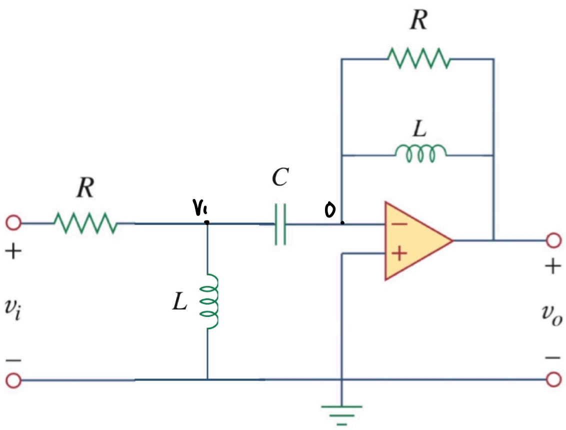
$P_{8\Omega} = |I_2|^2 \cdot R = 4608W$ (1')

No average power in inductence. $5440 = 832 + 4608$

$\Rightarrow$ Conservation of average powr. (1')

5. Please draw a **Bode plot** of transfer function $H(\omega) = V_o/V_i$. RLC have the following values: R = 60 Ω, L = 12 H, and C = 3 F. Please use the **straight-line approximation**. Bode plot template and guide are provided in the next page. [18 points]





by KCL: $\dfrac{V_i - V_1}{R} = \dfrac{V_1}{\frac{1}{j\omega C}} + \dfrac{V_1}{j\omega L}$ (2')

$\dfrac{V_1}{\frac{1}{j\omega C}} = \dfrac{-V_o}{R \| j\omega L}$ (2')

$R = 60\Omega$, $C = 3F$, $L = 12H$

$V_i = [R(j\omega C + \frac{1}{j\omega L}) + 1]V_1$

$V_o = -(R \| j\omega L) \cdot j\omega C V_1$

$$H(\omega) = \frac{-\frac{L^2C}{R} \cdot (j\omega)^3}{\left(1 + \frac{j\omega}{\frac{R}{L}}\right)\left[1 + 2\zeta\left(\frac{j\omega}{\frac{1}{\sqrt{LC}}}\right) + \left(\frac{j\omega}{\frac{1}{\sqrt{LC}}}\right)^2\right]} \qquad (4')$$

$\omega_n = \dfrac{1}{\sqrt{LC}}$, $\zeta = \dfrac{L}{2R\sqrt{LC}}$ $\Rightarrow$ $\omega_n = \frac{1}{6}$, $\zeta = \frac{1}{60}$

$H_{dB} = 20\log_{10}|(j\omega)^3| + 20\log_{10}\left|\frac{L^2C}{R}\right| - 20\log_{10}\left|1 + \frac{j\omega}{\frac{R}{L}}\right| - 20\log_{10}\left|1 + 2\zeta(j\omega/\omega_n) + (j\omega/\omega_n)^2\right|$ (1')

$\phi = -180° + 3\times 90° - \tan^{-1}(\omega/\frac{R}{L}) - \tan^{-1}\left(\dfrac{2\zeta\, \omega/\omega_n}{1 - \omega^2/\omega_n^2}\right)$ (1')

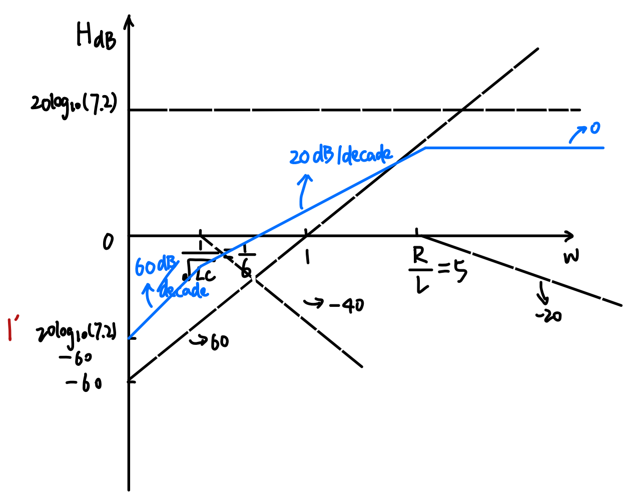
if equations are right and no plot: 2'

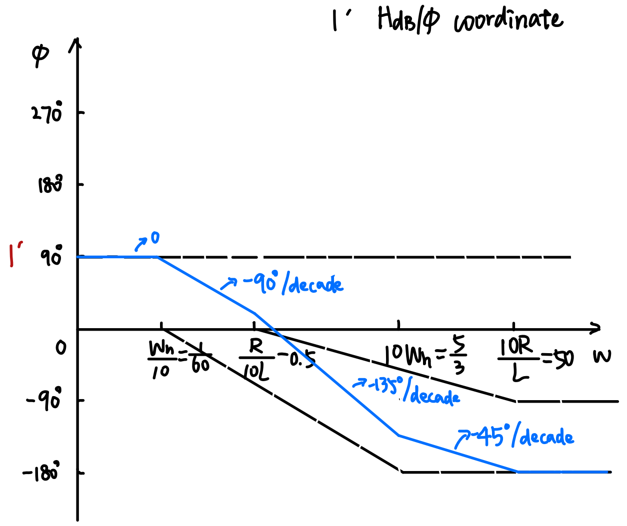
if have plot: 5' for each plot

2' shape (slope/amplitude)

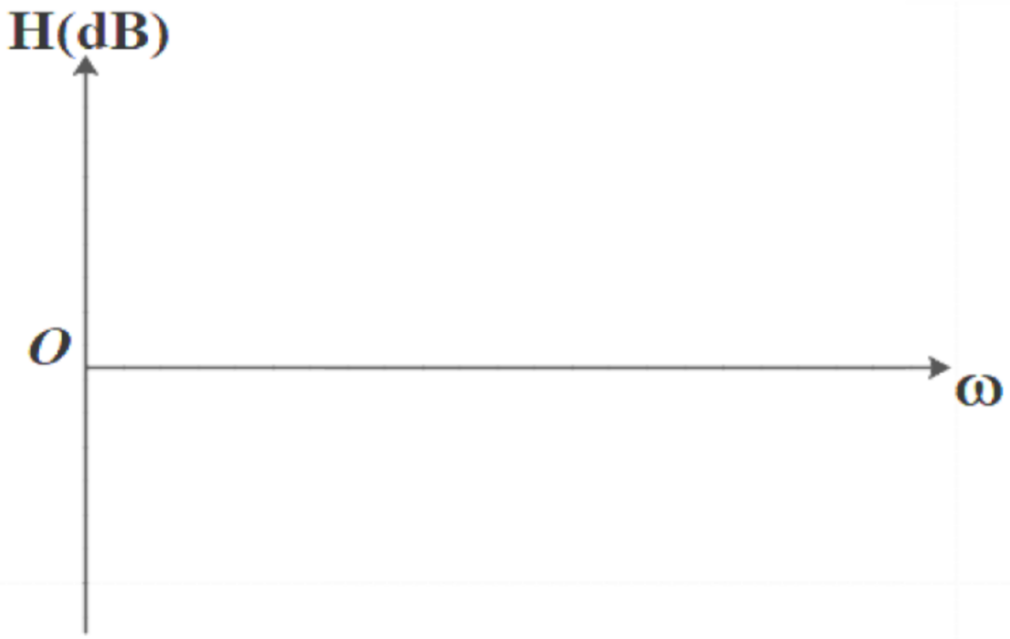
2' x-label

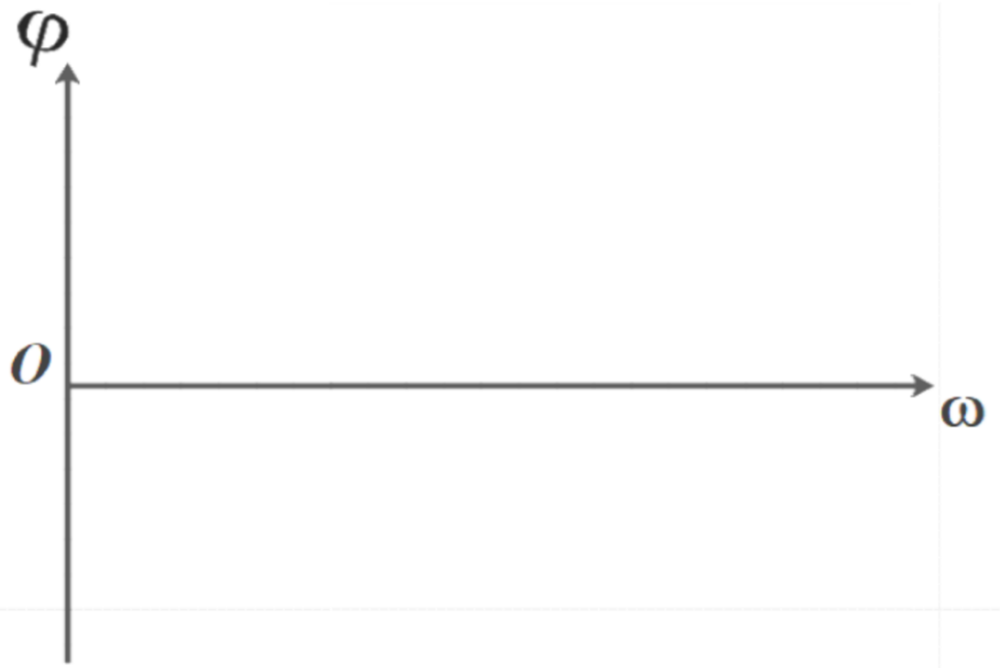
1' H_{dB}/ϕ coordinate



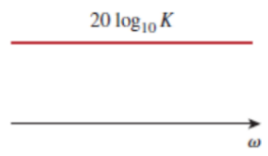
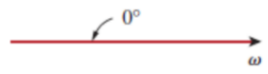
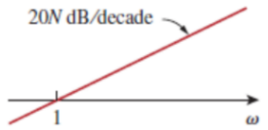
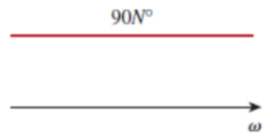
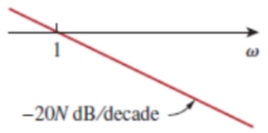
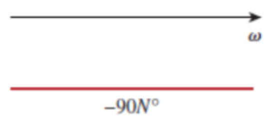
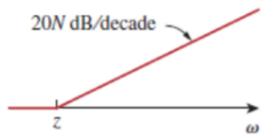
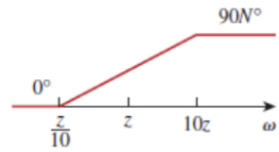
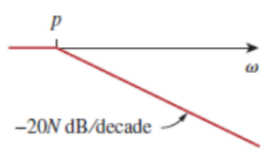
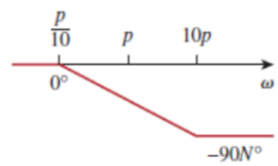
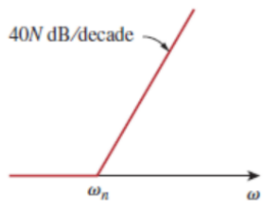
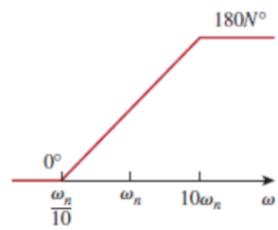
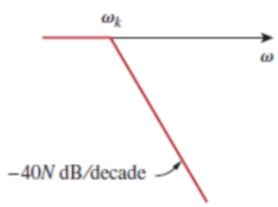
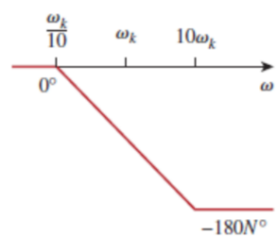


Bode Plot Template





Bode Plot Guide

Factor	Magnitude	Phase
K		
$(j\omega)^N$		
$\dfrac{1}{(j\omega)^N}$		
$\left(1+\dfrac{j\omega}{z}\right)^N$		
$\dfrac{1}{(1+j\omega/p)^N}$		
$\left[1+\dfrac{2j\omega\zeta}{\omega_n}+\left(\dfrac{j\omega}{\omega_n}\right)^2\right]^N$		
$\dfrac{1}{[1+2j\omega\zeta/\omega_k+(j\omega/\omega_k)^2]^N}$		

Mathematics Formula Sheet:

- **Phasor and Complex number**

$$j^2 = -1$$

$$\frac{a + jb}{c + jd} = \frac{(a + jb)(c - jd)}{(c + jd)(c - jd)} = \frac{(ac + bd) + j\,(bc - ad)}{c^2 + d^2}$$

$$A + jB = \sqrt{A^2 + B^2}\angle\left[\arctan\frac{B}{A}\right] = \sqrt{A^2 + B^2}\, e^{j\arctan\frac{B}{A}}$$

$$\frac{A\angle\theta_1}{B\angle\theta_2} = \frac{A}{B}\angle(\theta_1 - \theta_2), \quad A\angle\theta_1 \times B\angle\theta_2 = AB\angle(\theta_1 + \theta_2)$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Introduction to CASIO fx-991 calculator:

***Complex number calculation**: You may press "**menu**" button and select the "complex number" to do the complex number calculation.*

*For the unit imaginary "j", the button "**ENG**" stands for that.*

*For the phasor angle "∠", please press "**SHIFT**", and then press "**ENG**" button.*

Mention: It's invalid to input $\angle\theta$, like $\angle 60$! You must input a number before that, such as "1∠60" or "2∠60"

***Complex number and Phasor Transformation**: Please press "**SHIFT**" and then press "**menu**", you can select the "Complex" and then choose its form (" a+bi " or "r∠θ")*

***Angle unit**: Please press "**SHIFT**" and then press "**menu**", you can select the "angle unit" and then choose the unit of angle ("degrees" or "rad")*