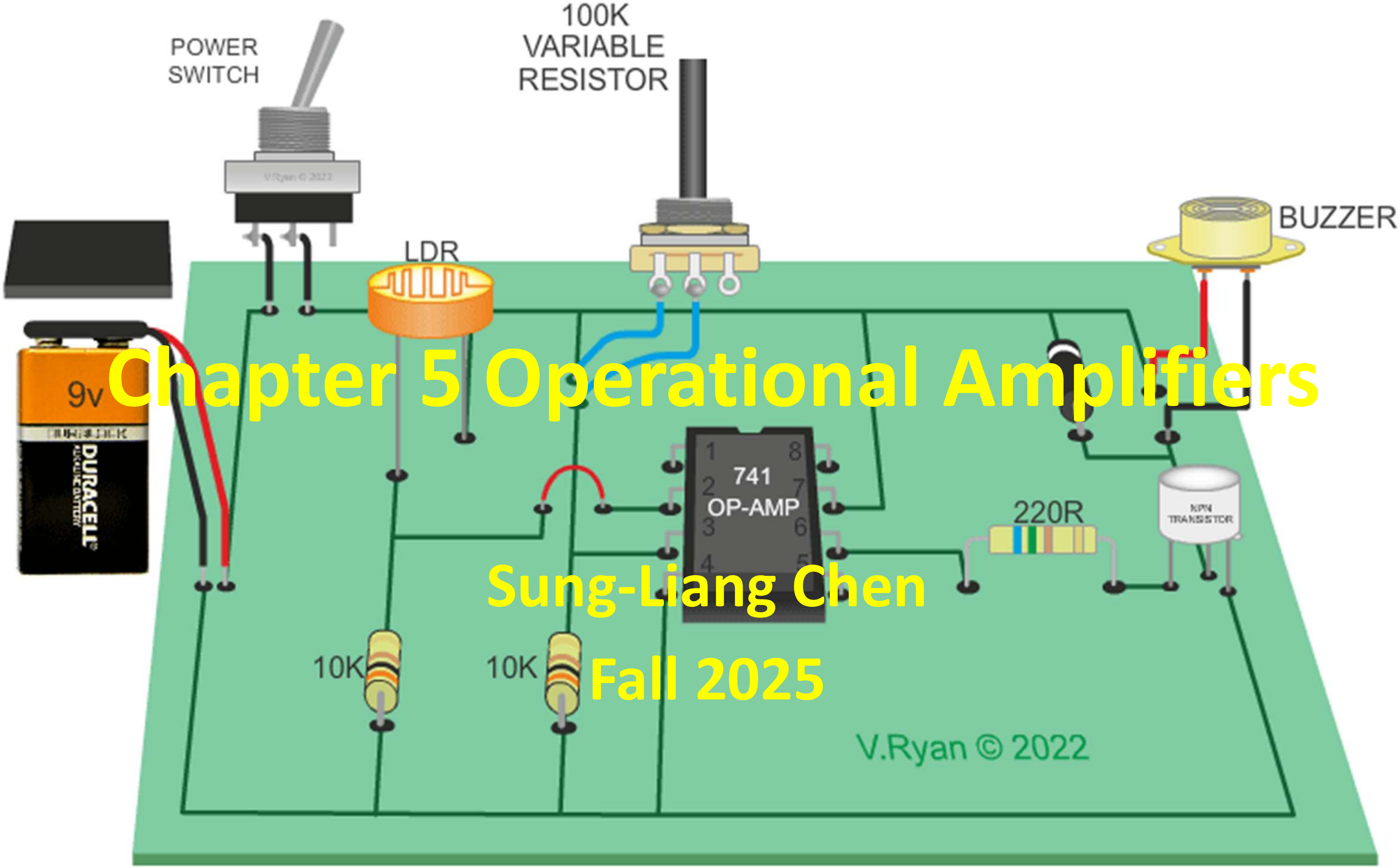

Chapter 5 Operational Amplifiers

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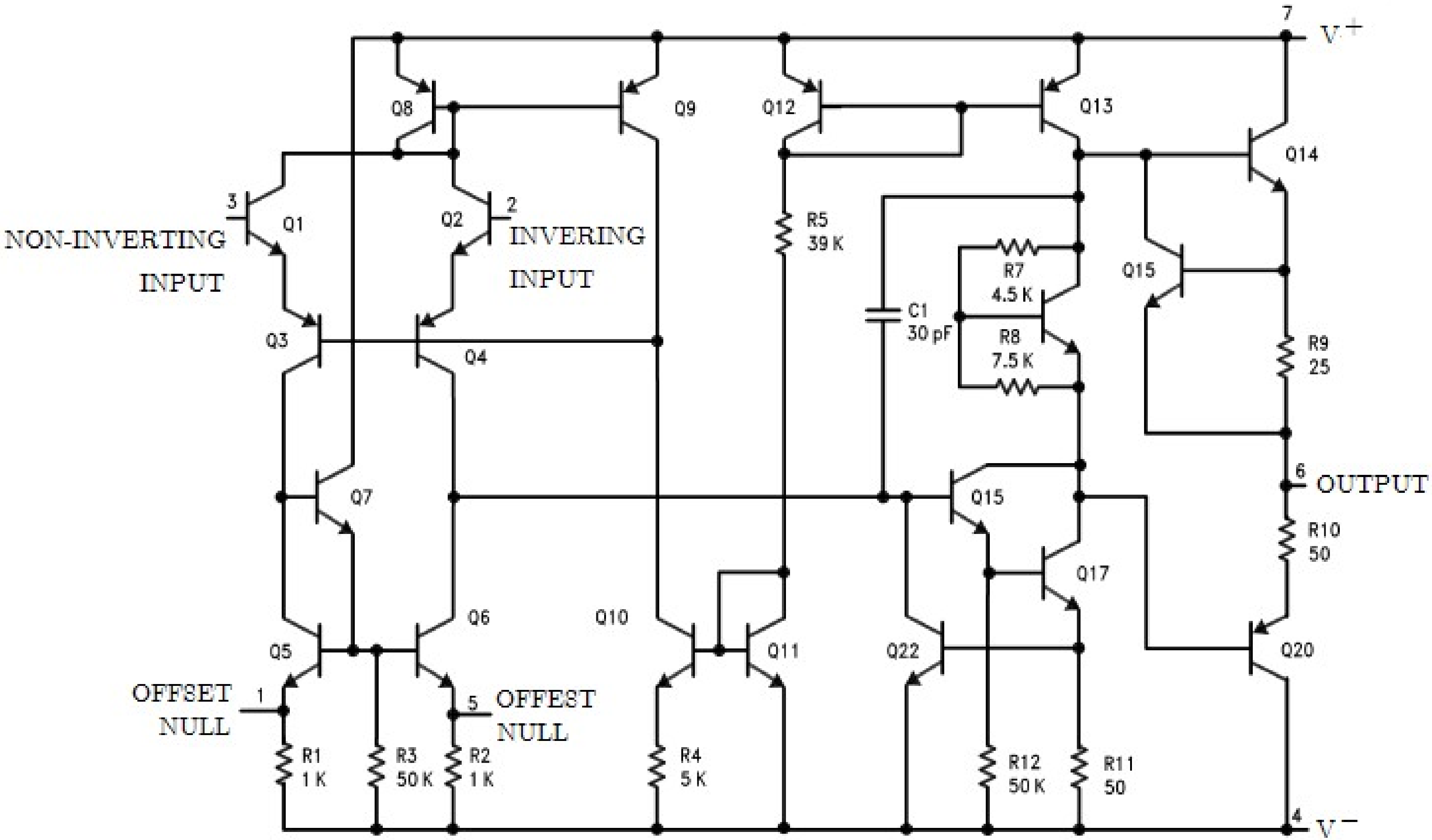
5.1 Introduction

- The *operational amplifier*, or *op amp*, is an electronic device that behaves like a voltage-controlled voltage source. An op amp circuit can perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.

5.2 Operational Amplifiers

- As shown on the next page, an op amp consists of a complex arrangement of resistors, transistors, capacitors, and diodes. A full discussion of what is inside the op amp is in Ve311.
- In Ve215, we treat the op amp as a circuit building block and simply study what takes place at its terminals.



Schematic diagram for LM741 op amp.

- Op amps are commercially available in integrated circuit packages in several forms. Figure 5.2(a) shows the pin configuration of a typical op amp. The circuit symbol for the op amp is the triangle in Fig. 5.2(b).

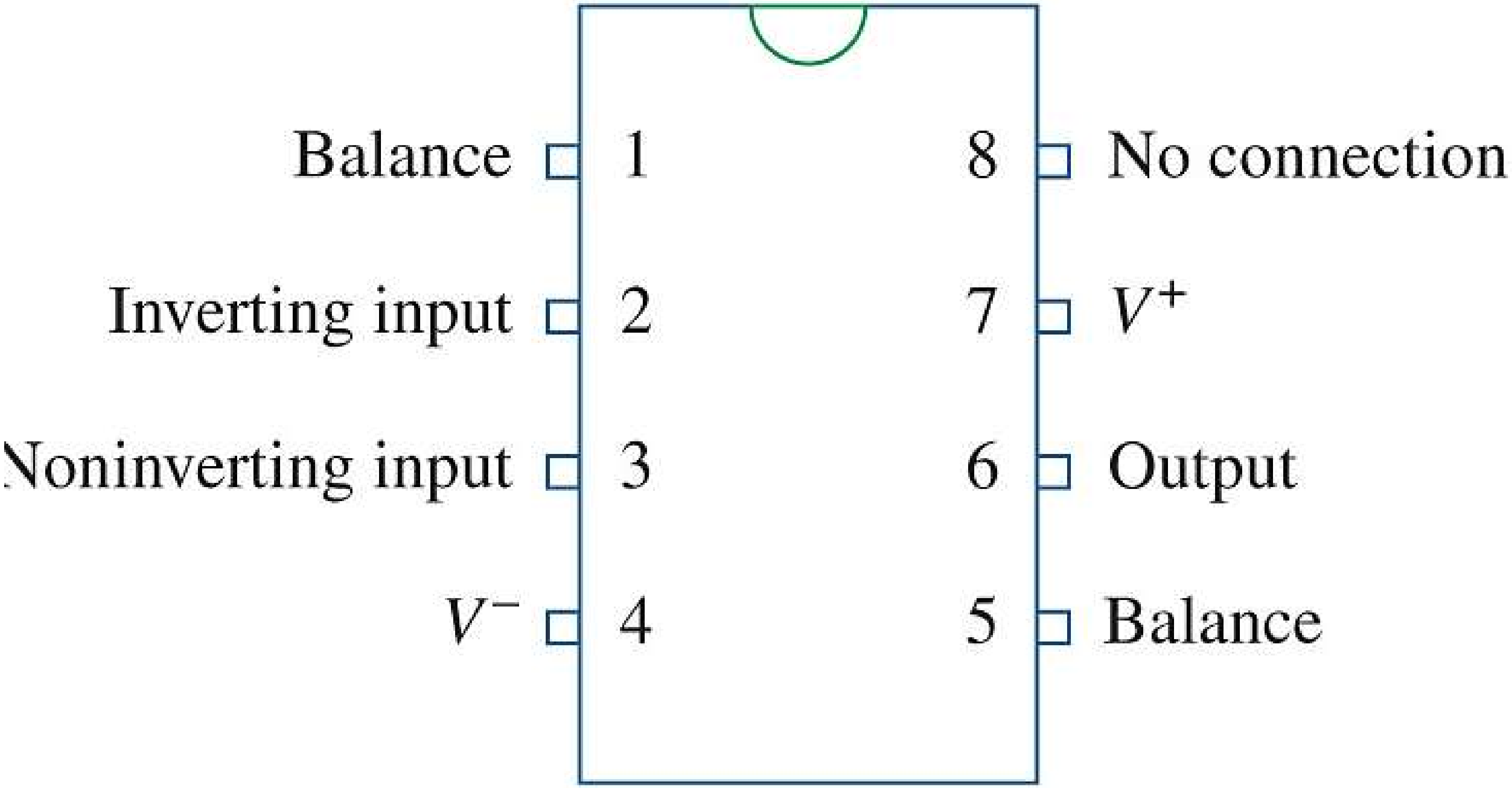


Figure 5.2(a) A typical op amp: pin configuration.

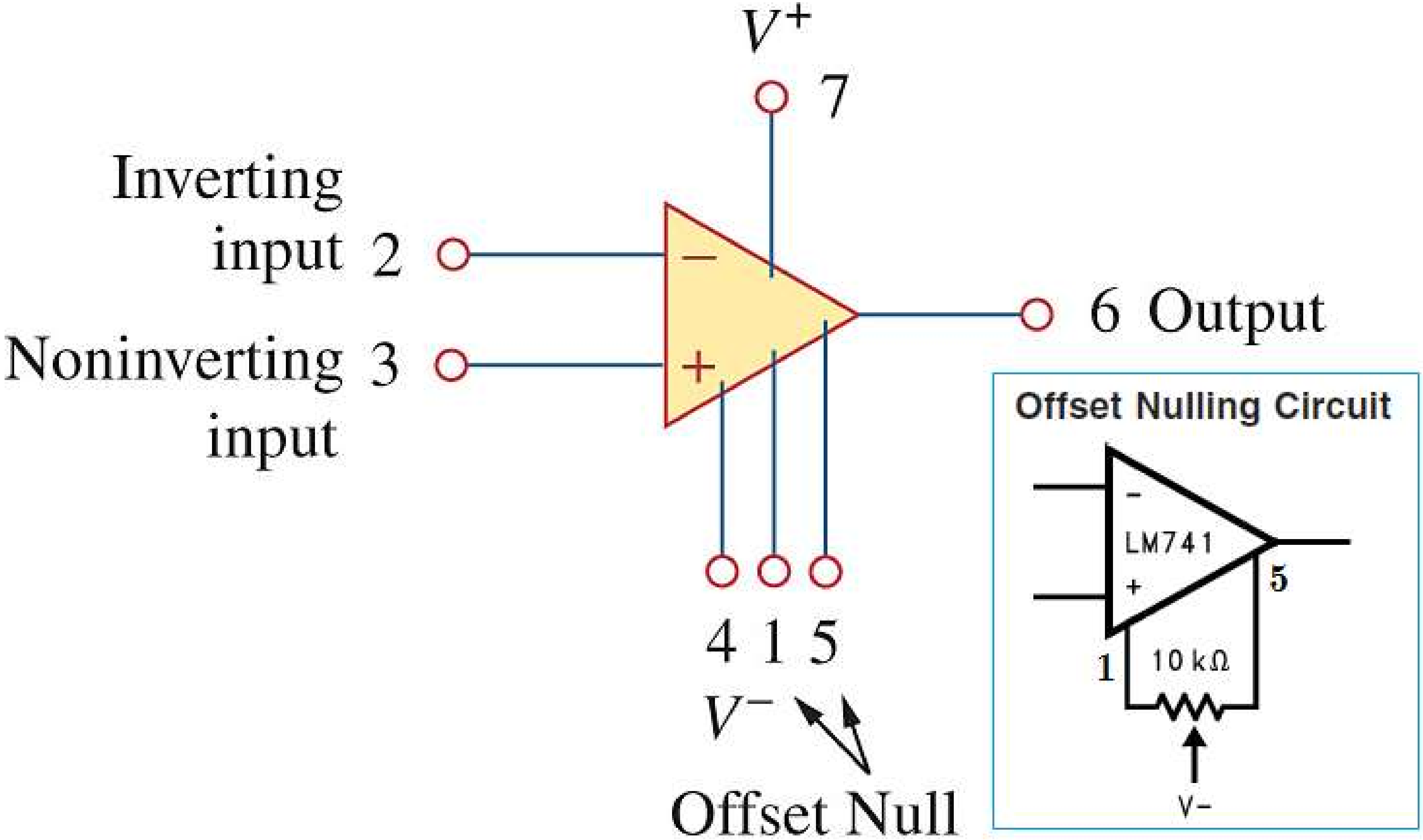


Figure 5.2(b) A typical op amp: circuit symbol.

About offset null

- By adjusting the pot we can null any offset error.
- Offset error
  - When the inputs are exactly equal but the output isn't exactly zero.
  - Due to random variation in manufacturing
  - Can be safely ignored in AC applications (since it's just a DC offset), but can be an issue in DC applications

    $v_o = Av_d + V_{os}$

    If $v_d = \sin(\omega t)$, then $v_o = A\sin(\omega t)+V_{os}$

    AC signal, $A\sin(\omega t)$, is still preserved

- As an active element, the op amp must be powered by one or two voltage supplies as typically shown in Fig. 5.3.

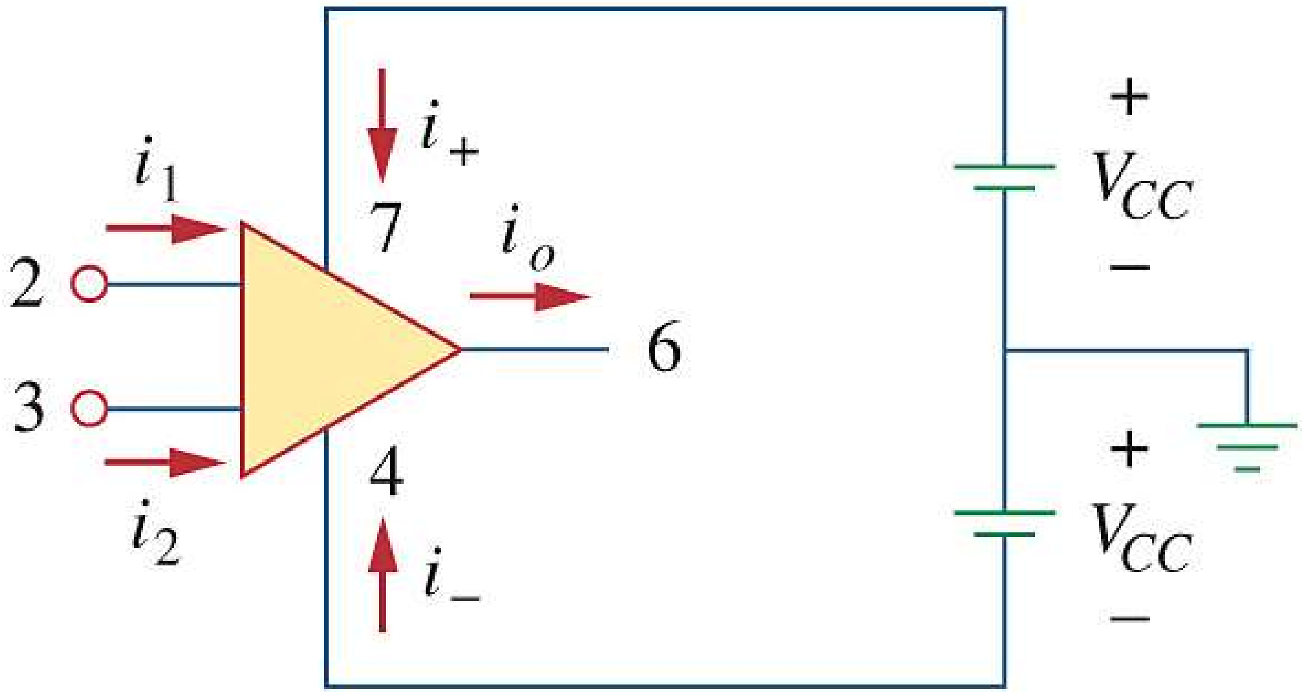
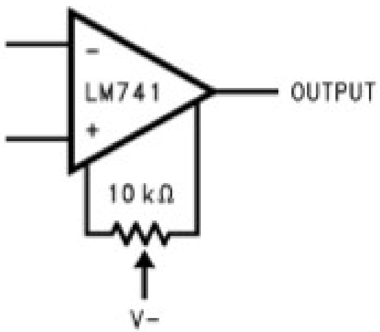


Figure 5.3 Powering the op amp.

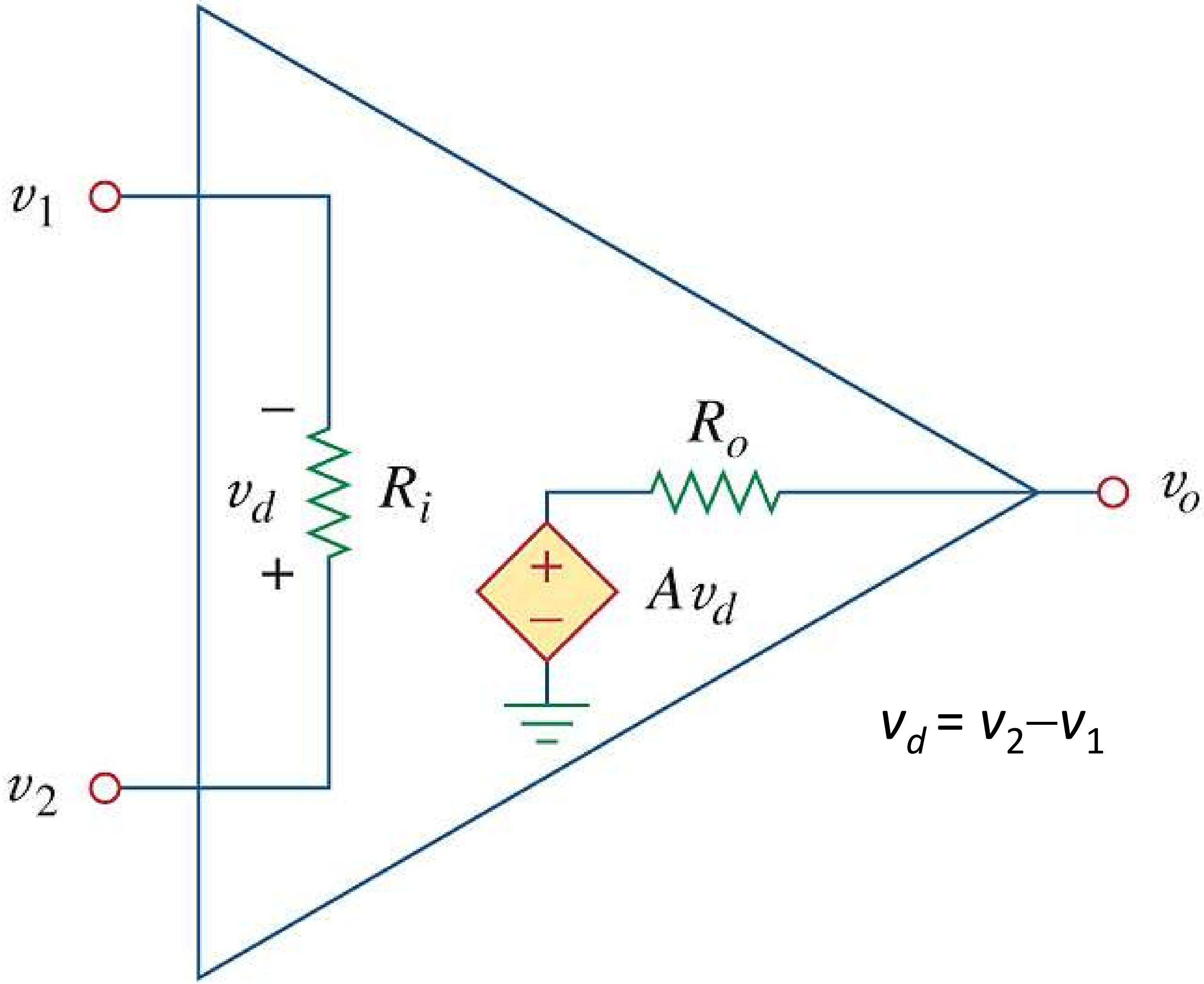
Although the power supplies are often ignored in op amp circuit diagrams for the sake of simplicity, the power supply currents must not be overlooked. By KCL,

$$i_o = i_1 + i_2 + i_+ + i_-$$

(This formular is valid even if pins 1 and 5 are used)



The equivalent circuit model of an op amp is shown in Fig. 5.4. $v_d = v_2 - v_1$ is called the differential input voltage and A is the *open-loop voltage gain*.



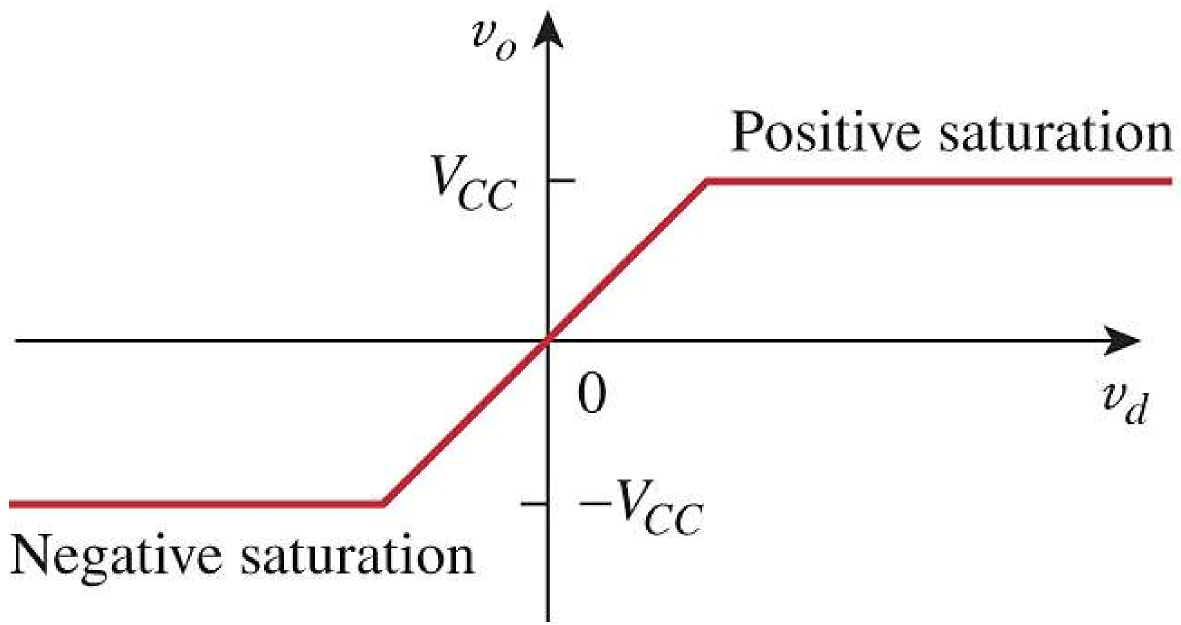
$v_d = v_2 - v_1$

Figure 5.4 The equivalent circuit of the nonideal op amp.

TABLE 5.1 Typical ranges for op amp parameters

Parameter	*Typical range*	*Ideal values*
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to 10^{13} Ω	∞
Output resistance, R_o	10 to 100 Ω	0
Supply voltage, V_{CC}	5 to 24 V	NA

A practical limitation of the op amp is that the magnitude of its output voltage cannot exceed $|V_{CC}|$. Figure 5.5 illustrates that the op amp can operate in three modes: (1) positive saturation, (2) linear region, (3) negative saturation.



5.3 Ideal Op Amp

An op amp is ideal if it has the following characteristics:

1. Infinite open-loop gain, $A = \infty$.
2. Infinite input resistance, $R_i = \infty$.
3. Zero output resistance, $R_o = 0$.

For circuit analysis, an ideal op amp is illustrated in Fig. 5.8.

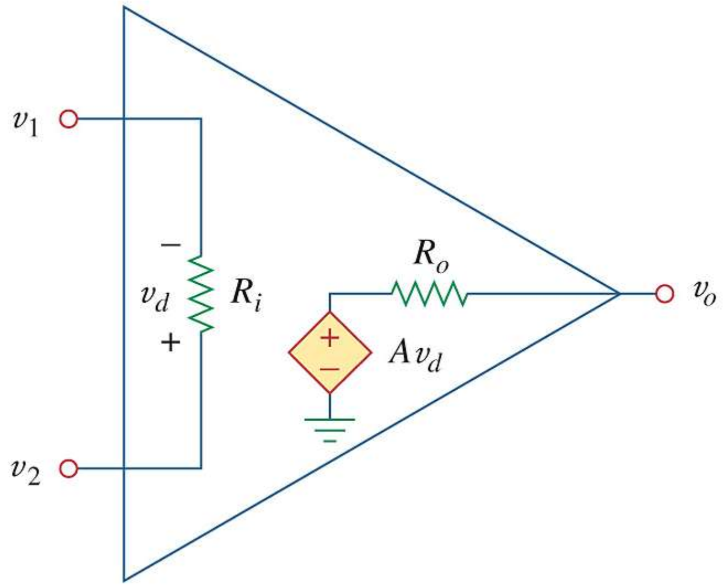
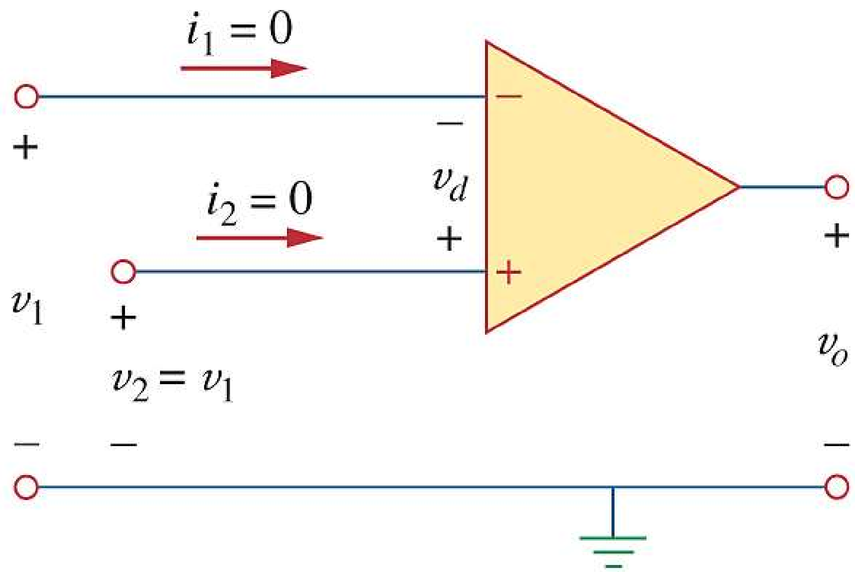


Figure 5.4 The equivalent circuit of the nonideal op amp.

Ideal



$A = \infty$

$R_i = \infty$

$R_o = 0$

Two important characteristics of the ideal op amp are

1. The current into both input terminals are zero due to $R_i = \infty$:

$i_1 = 0$ and $i_2 = 0$ (virtual open-circuit)

Proof :

$$v_d = v_2 - v_1 = i_2 R_i < \infty \Rightarrow i_2 = 0 \Rightarrow$$

$$i_1 = -i_2 = 0$$

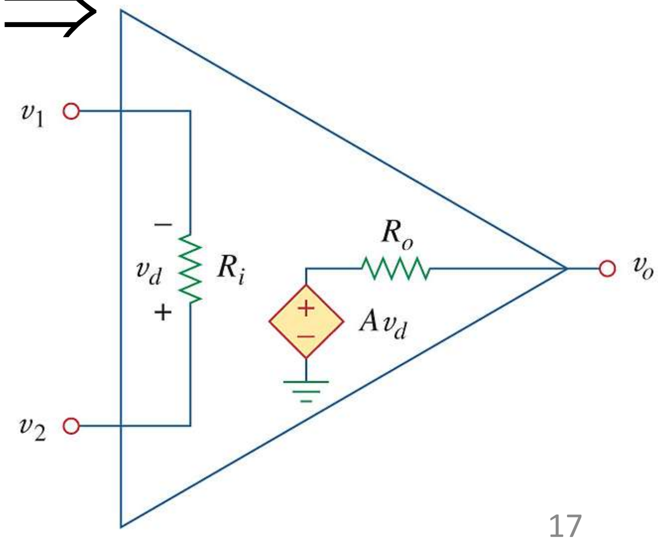


Figure 5.4 The equivalent circuit of the nonideal op amp.

2. When the op amp operates in the linear region, the voltage across the input terminals are zero due to $A = \infty$ and $R_o = 0$:

$v_d = v_2 - v_1 = 0$ (virtual short-circuit)

or $v_1 = v_2$

Proof :

$v_o = \underline{A}v_d < \underline{\infty} \Rightarrow v_d = 0$

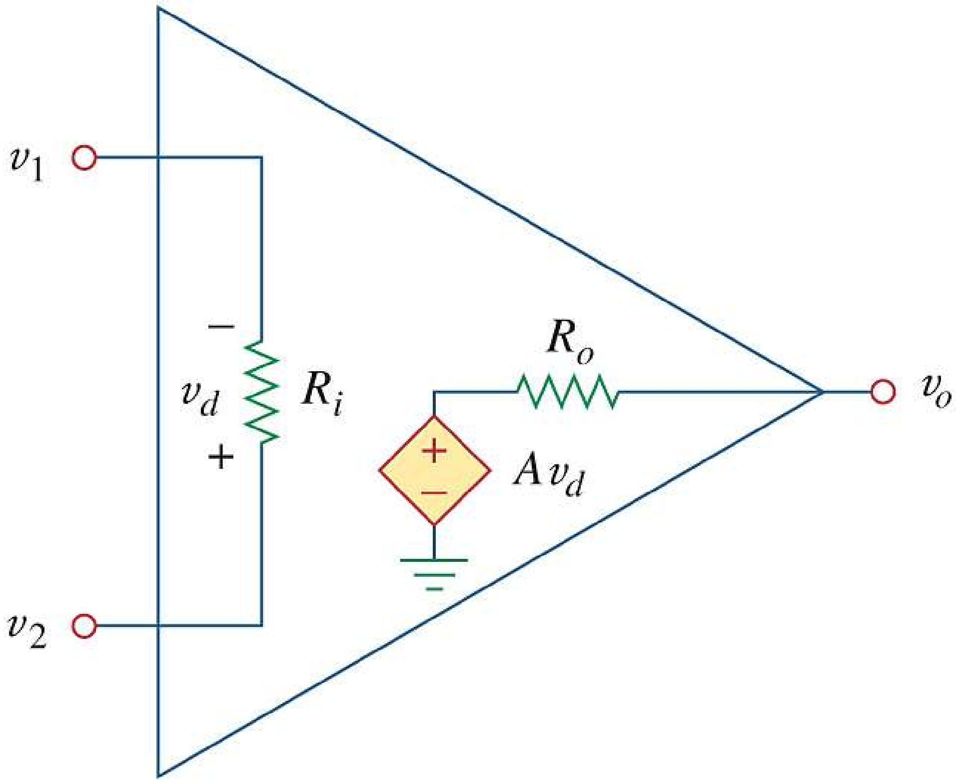
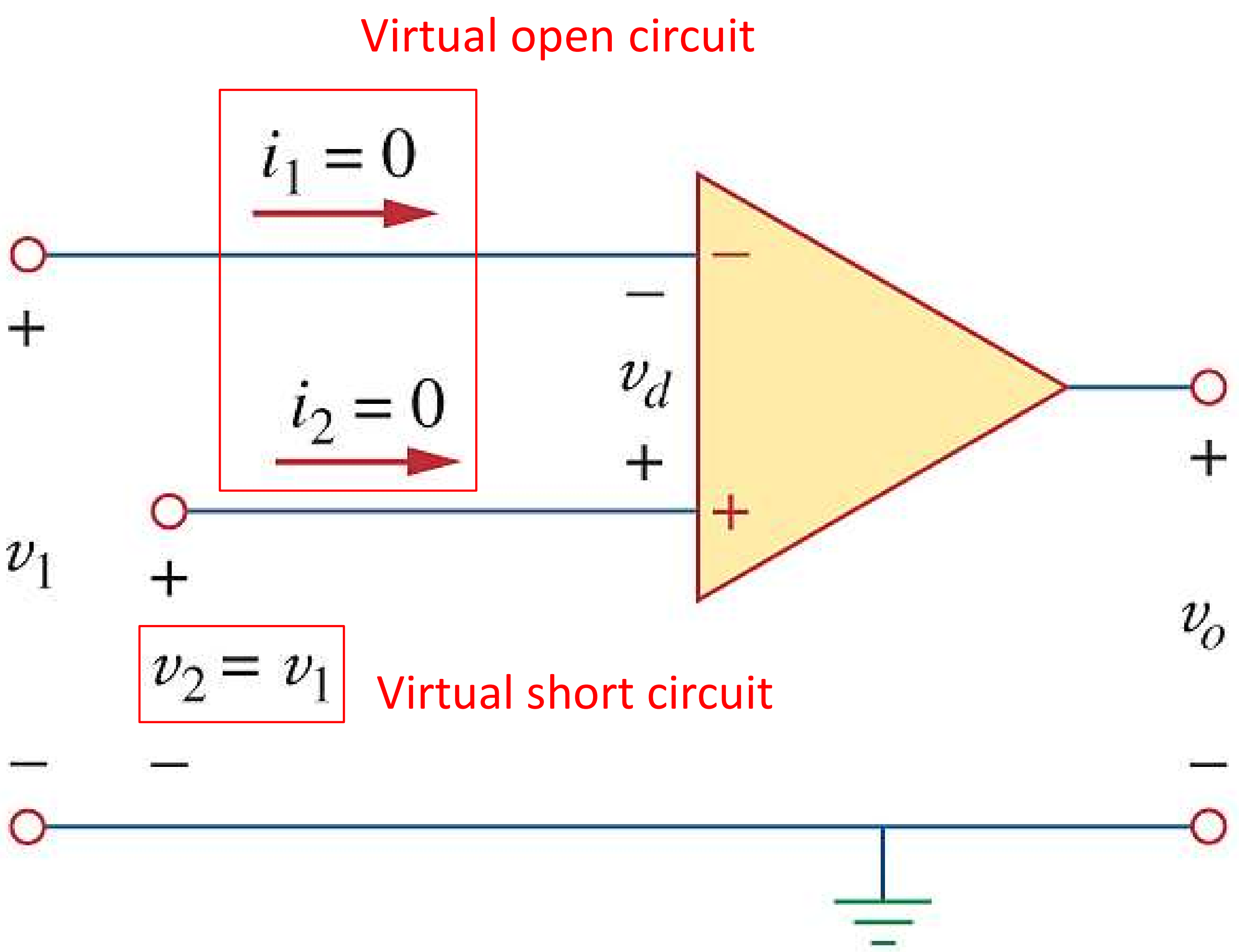


Figure 5.4 The equivalent circuit of the nonideal op amp.


Virtual open circuit
i1 = 0
i2 = 0
vd
v1
v2 = v1
Virtual short circuit
vo

5.4 Inverting Amplifier (Inverter)

An inverting amplifier reverses the polarity of the input signal while amplifying it.

The *closed-loop gain* of the inverting amplifier is

$$A_v = \frac{v_o}{v_i} = -\frac{R_f}{R_i}$$

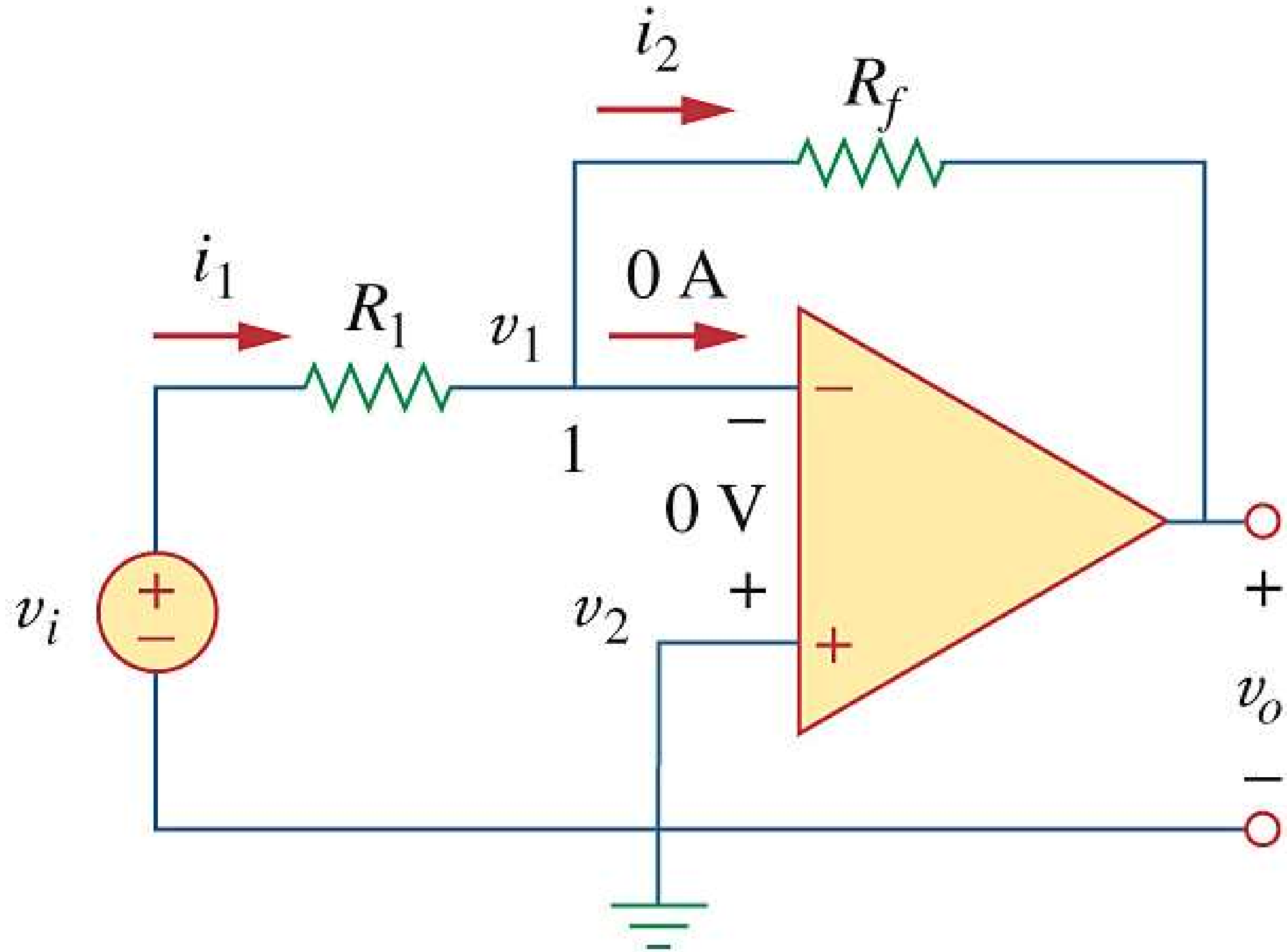


Figure 5.10 the inverting amplifier.

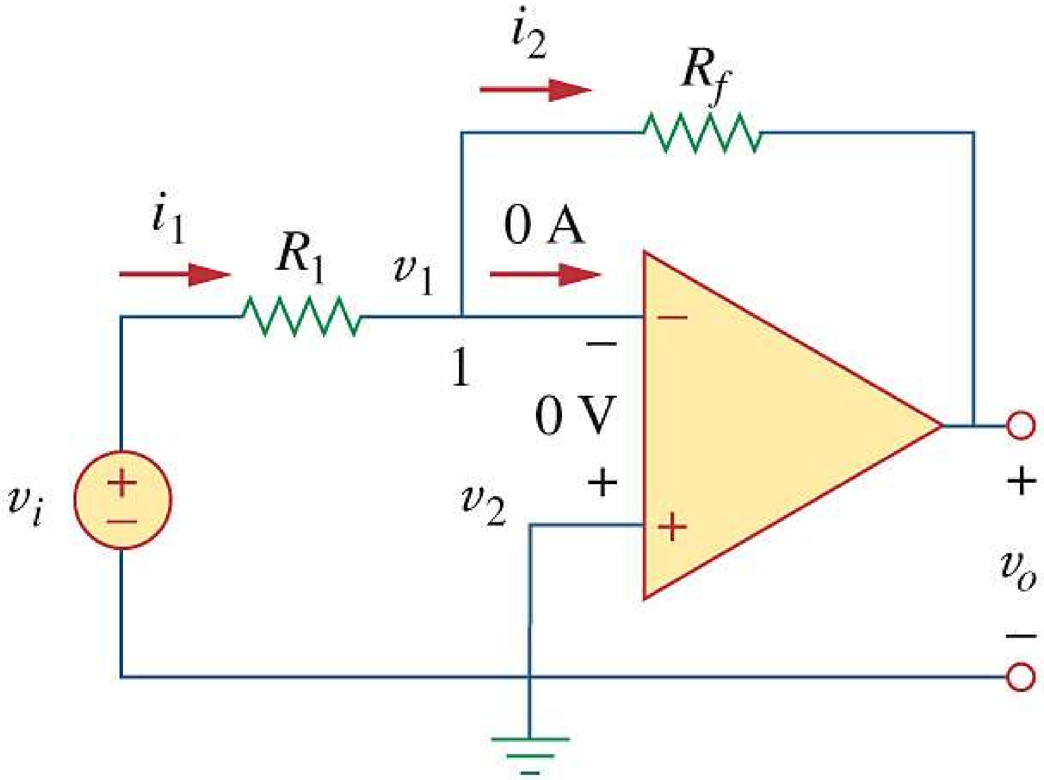


Figure 5.10 the inverting amplifier.

Proof :

$$i_1 = i_2 \Rightarrow \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

$$v_1 = v_2 = 0$$

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f} \Rightarrow v_o = -\frac{R_f}{R_1} v_i \Rightarrow A_v = \frac{v_o}{v_i} = -\frac{R_f}{R_1}$$

An equivalent circuit for the inverting amplifier is shown in Fig. 5.11.

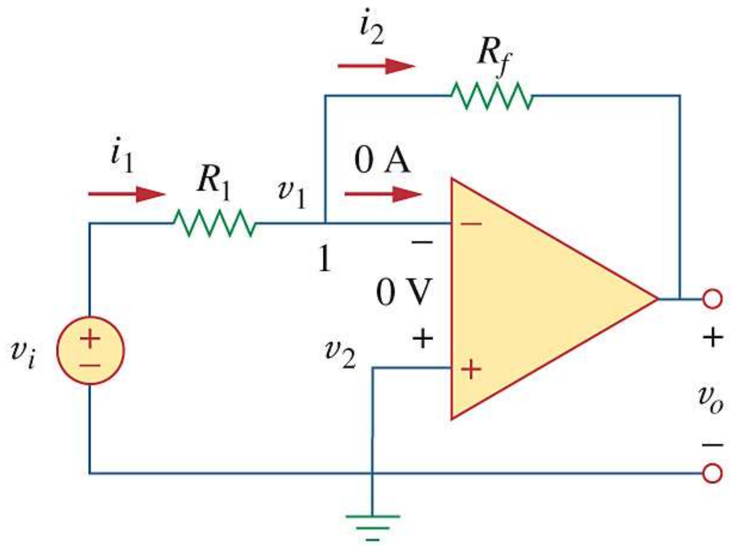


Figure 5.10 the inverting amplifier.

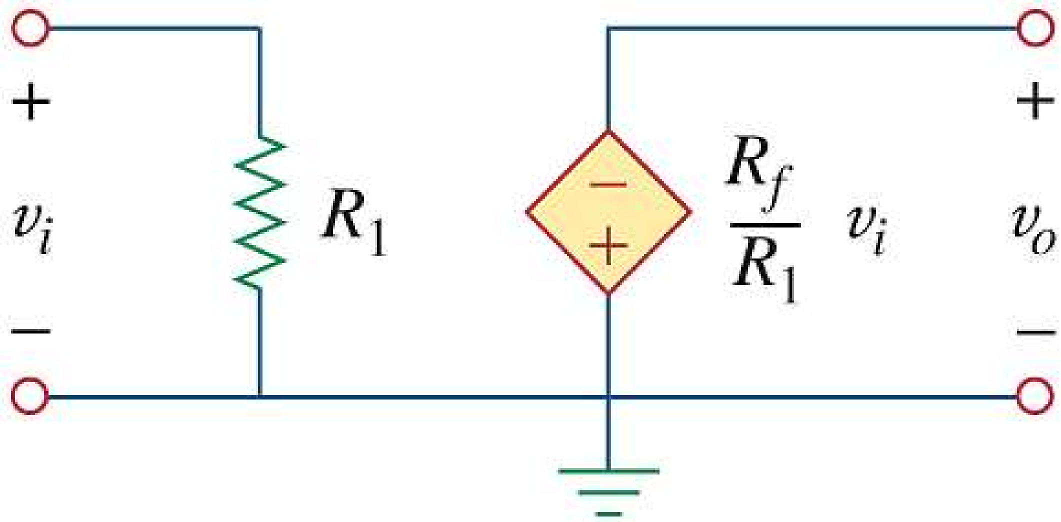


Figure 5.11 An equivalent circuit for the inverter in Fig. 5.10.

The concept of *feedback* is crucial to our understanding of op amp circuits. A *negative feedback* is achieved when the output is fed back to the inverting terminal of the op amp. As a result of the negative feedback, the op amp operates in the linear region.

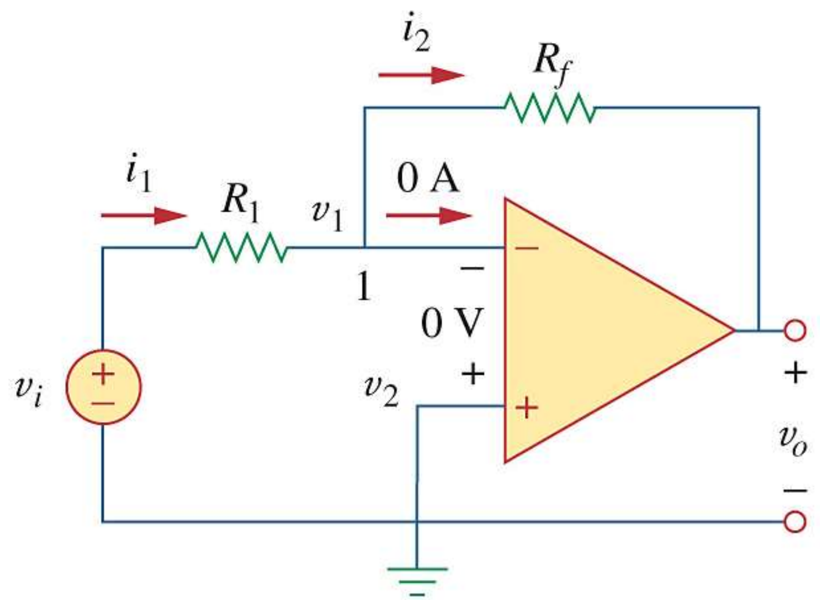


Figure 5.10 the inverting amplifier.

Example 5.3 Refer to the circuit in Fig. 5.12. If $v_i = 0.5$ V, calculate (a) the output voltage v_o, and (b) the current in the 10-kΩ resistor.

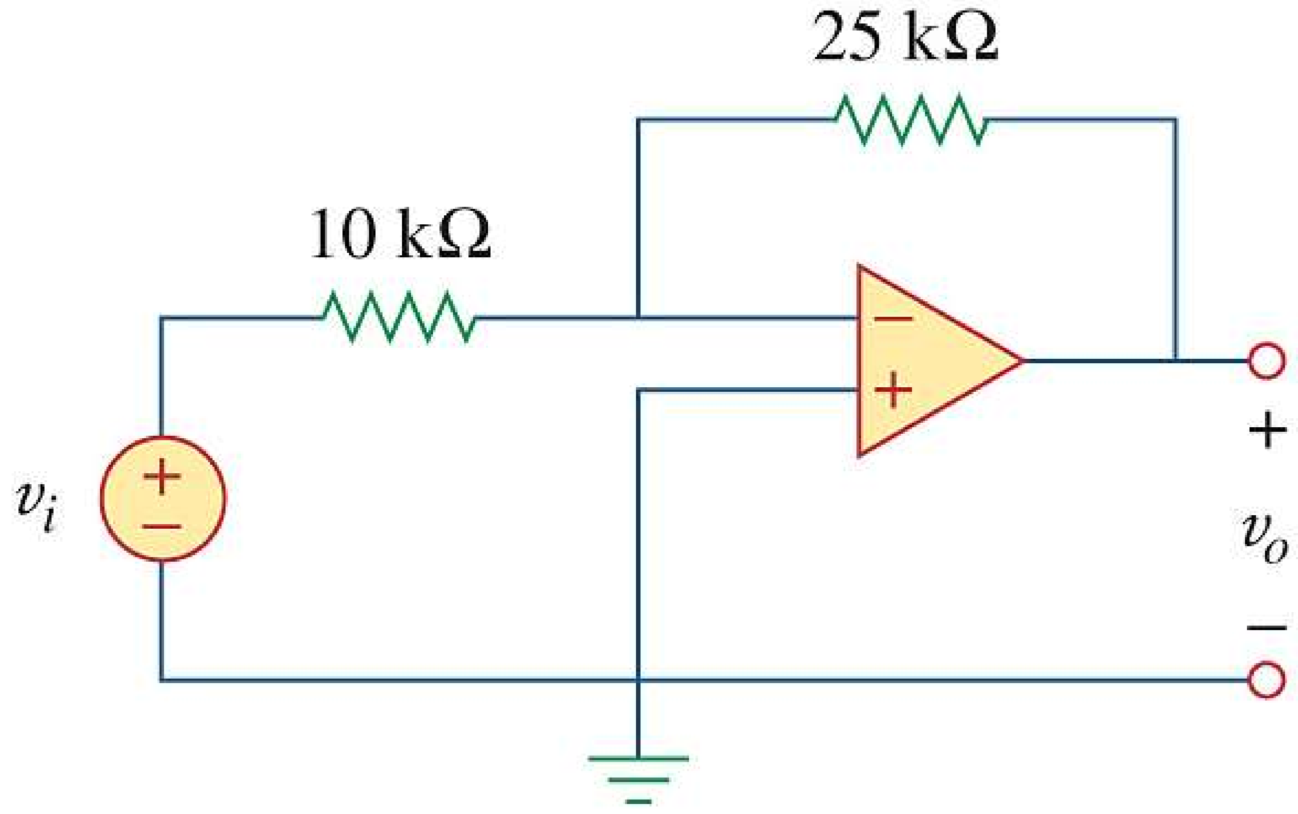


Figure 5.12

Solution :

$$v_o = -\frac{R_f}{R_1} v_i = -\frac{25}{10} \times 0.5 = -1.25 \text{ (V)}$$

$$i = \frac{v_i}{R_1} = \frac{0.5}{10 \times 10^3} = 5 \times 10^{-5} \text{ (A)} = 50\ \mu\text{A}$$

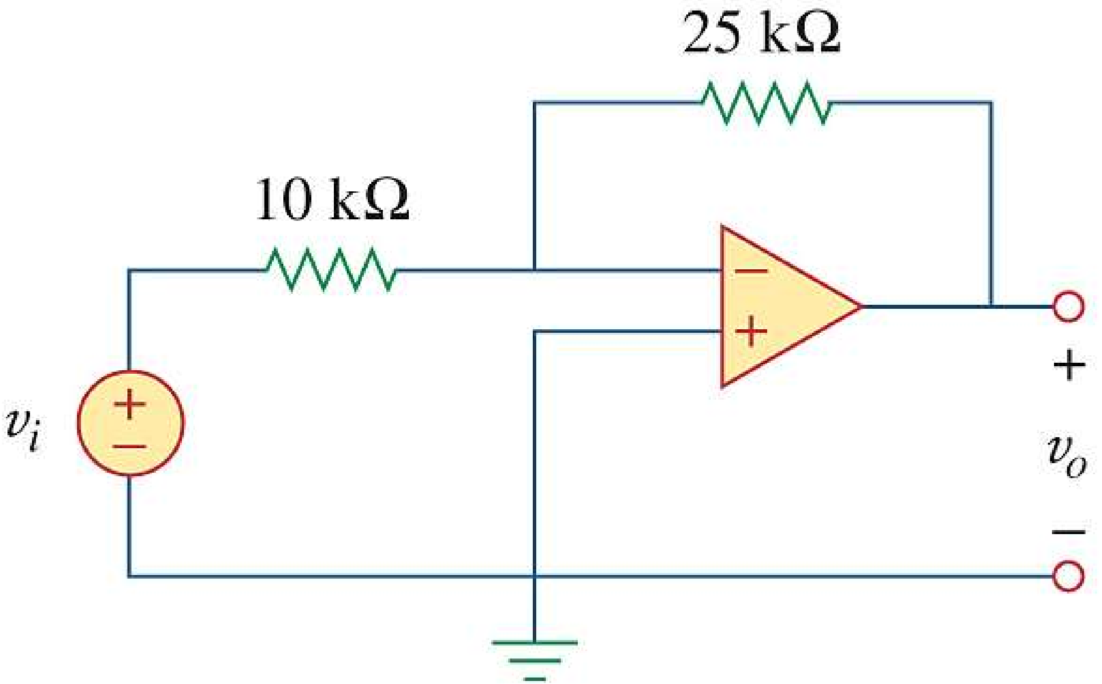


Figure 5.12

Example 5.4 Two kinds of current-to-voltage converters (also known as transresistance amplifier) are shown in Fig. 5.15.

(a) Show that for the converter in Fig. 5.15(a),

$$v_o / i_s = -R$$

(b) Show that for the converter in Fig. 5.15(b),

$$v_o / i_s = -R_1(1 + R_3 / R_1 + R_2 / R_1)$$

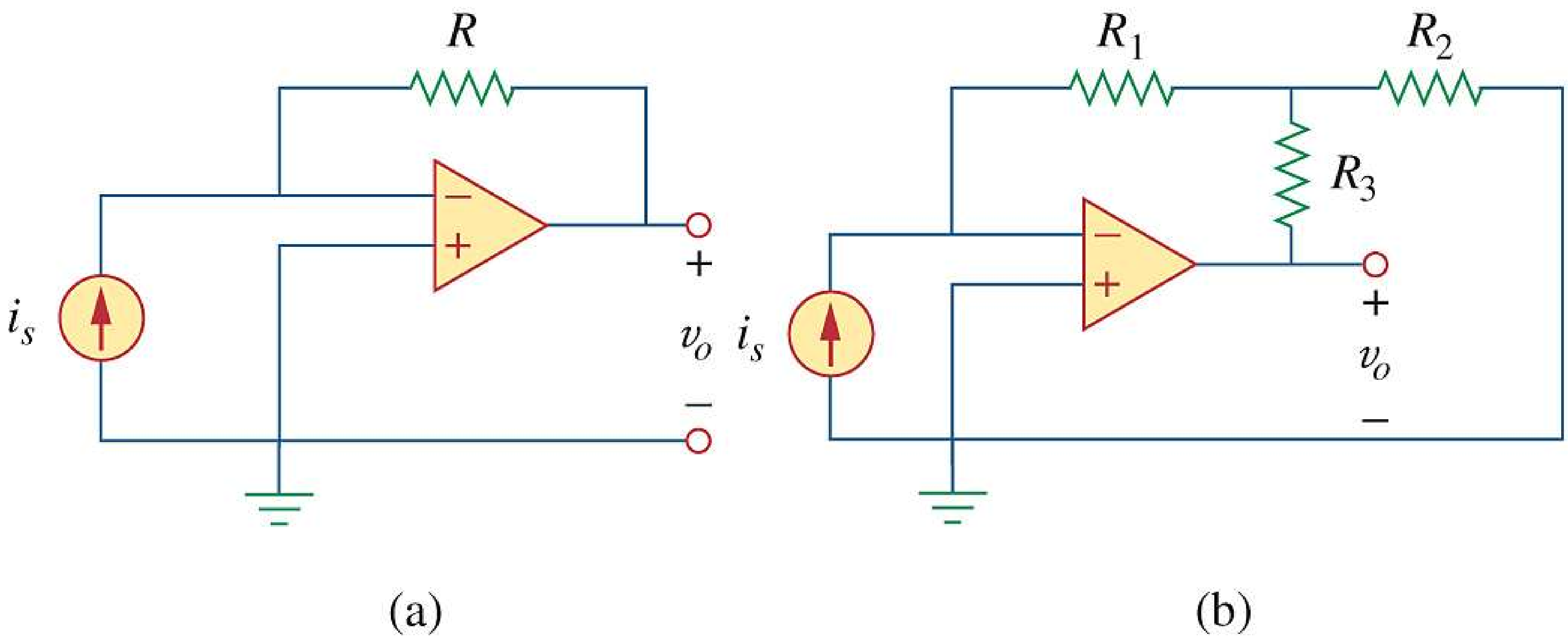


Figure 5.15

Proof :

(a)

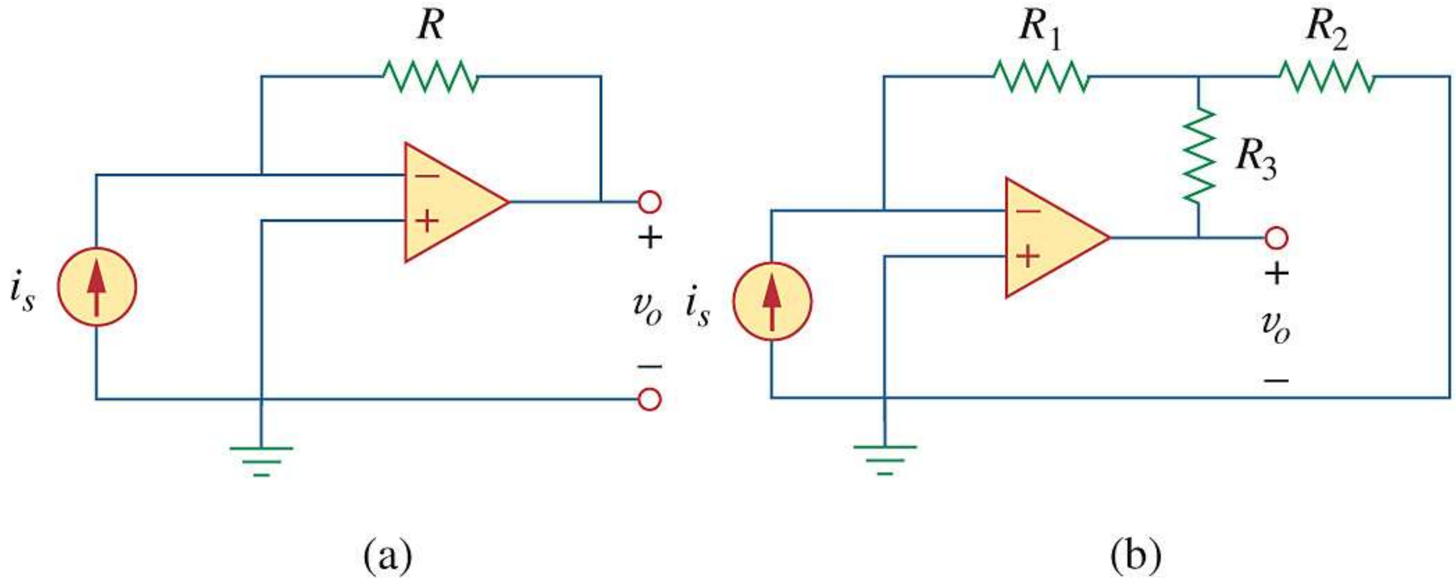
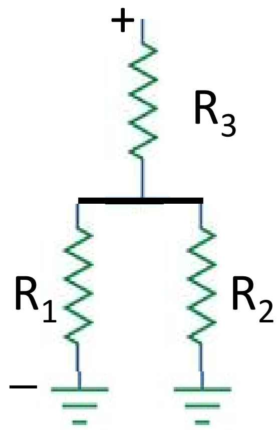


Figure 5.15

$$i_s = \frac{0 - v_o}{R} \Rightarrow \frac{v_o}{i_s} = -R$$

(b)



$$i_s = -\frac{R_2}{R_1 + R_2} \cdot \frac{v_o}{R_1 \| R_2 + R_3}$$

$$\frac{v_o}{i_s} = -\frac{R_1 + R_2}{R_2}\left(R_1 \| R_2 + R_3\right)$$

$$= -\frac{R_1 + R_2}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2} + R_3\right)$$

$$= -\left(R_1 + \frac{R_1 + R_2}{R_2} R_3\right)$$

$$= -R_1\left(1 + \frac{R_1 + R_2}{R_1 R_2} R_3\right)$$

$$= -R_1\left(1 + \frac{R_3}{R_2} + \frac{R_3}{R_1}\right)$$

5.4 Noninverting Amplifier

The noninverting amplifier is a circuit designed to provide positive voltage gain. The closed-loop gain for the noninverting amplifier is

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}$$

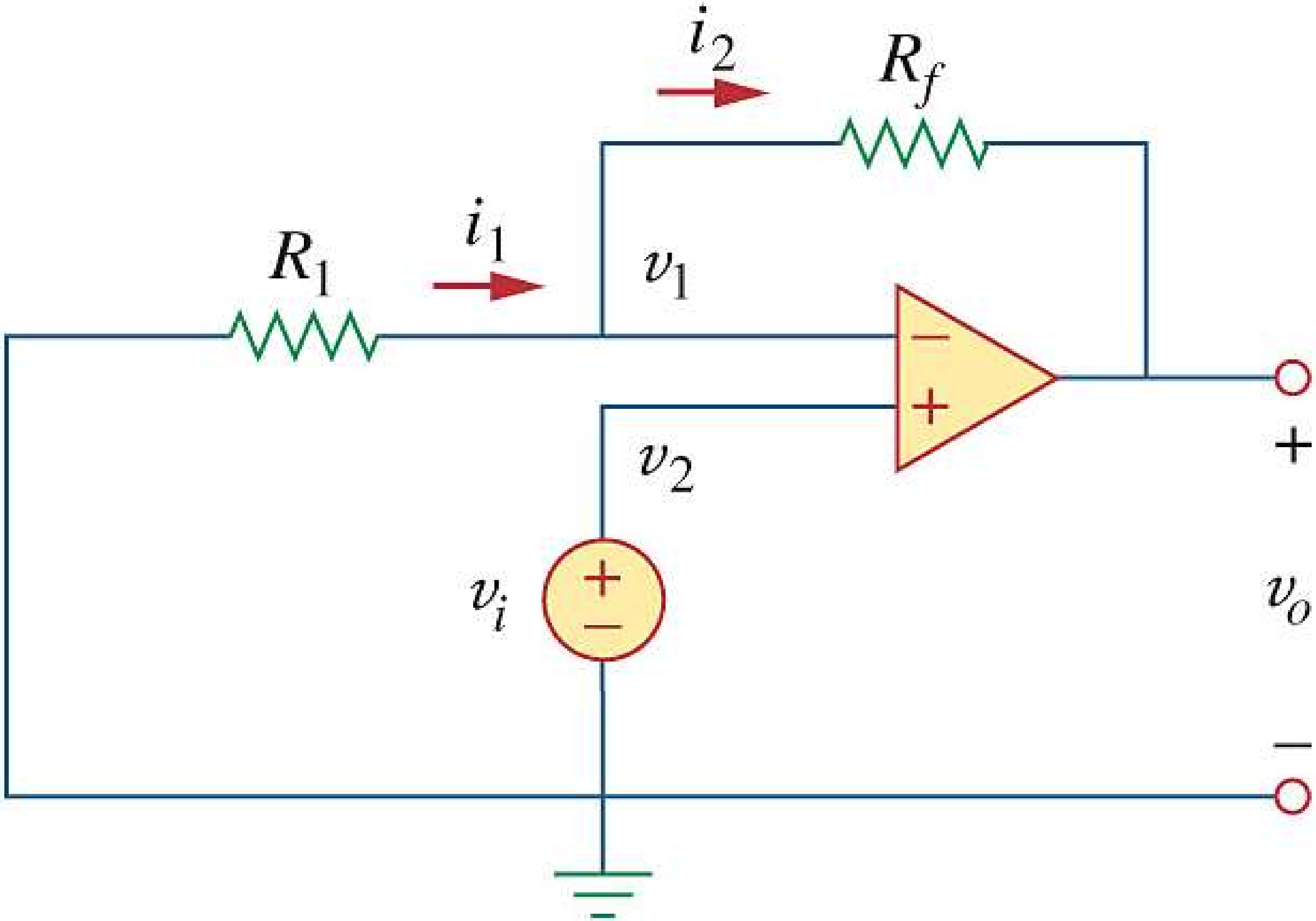


Figure 5.16 The noninverting amplifier.

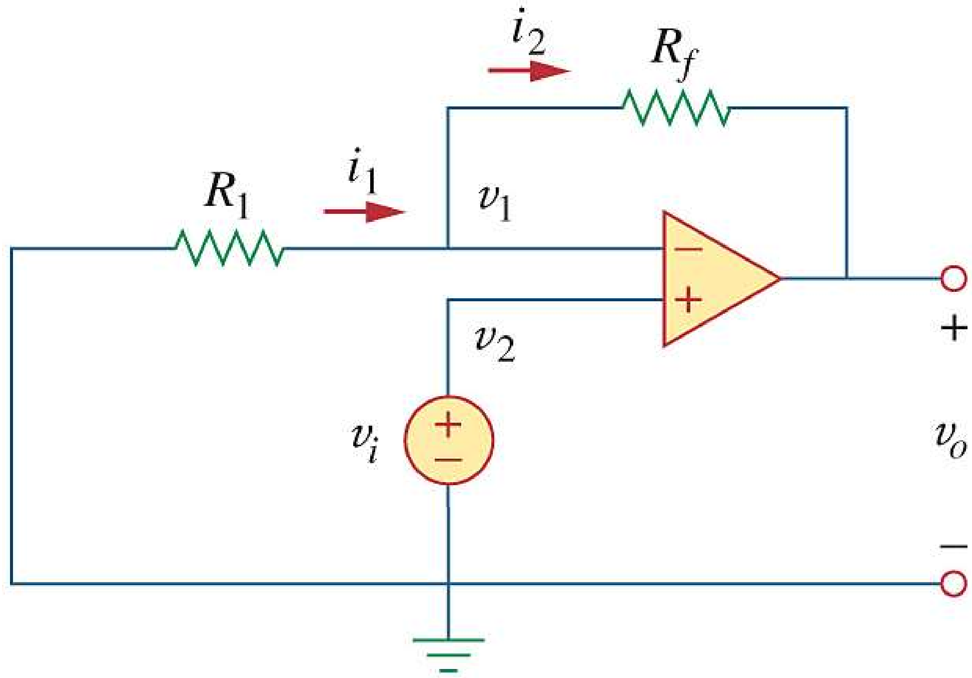


Figure 5.16 The noninverting amplifier.

Proof :

$$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \Rightarrow \frac{v_o}{v_1} = 1 + \frac{R_f}{R_1}$$

$$v_1 = v_2 = v_i$$

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}$$

Notice that if $R_f = 0$ or $R_1 = \infty$ or both, $A_v = 1$. The circuit in Fig. 5.16 becomes a *voltage follower* (or unity gain amplifier) because the output follows the input, i.e., $v_o = v_i$

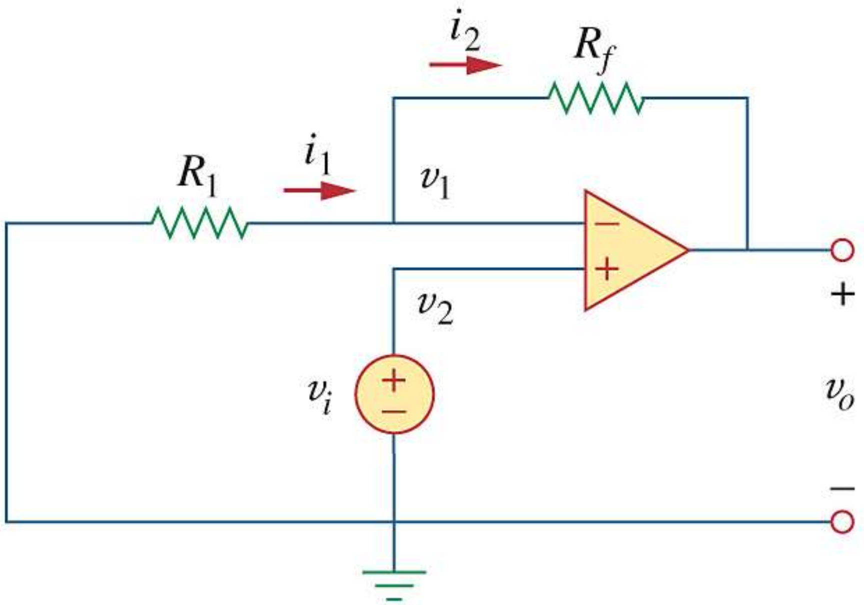


Figure 5.16 The noninverting amplifier.

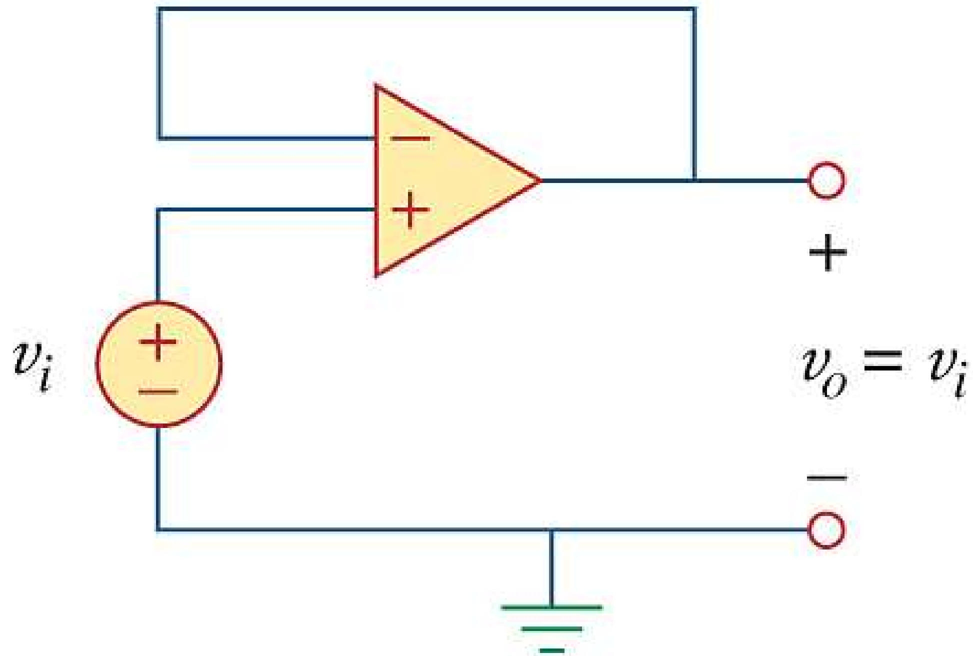


Figure 5.17 The voltage follower.

Such a circuit has a very high input resistance and a very low output resistance. It is therefore useful as an intermediate-stage (or buffer) to isolate two cascaded stages, as portrayed in Fig.5.18. The voltage follower minimizes interaction between the two stages and eliminates interstage loading.

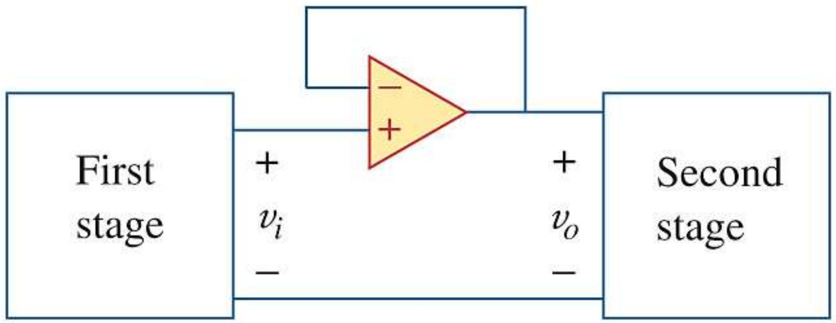


Figure 5.18 A voltage follower used to isolate two cascaded stages of a circuit.

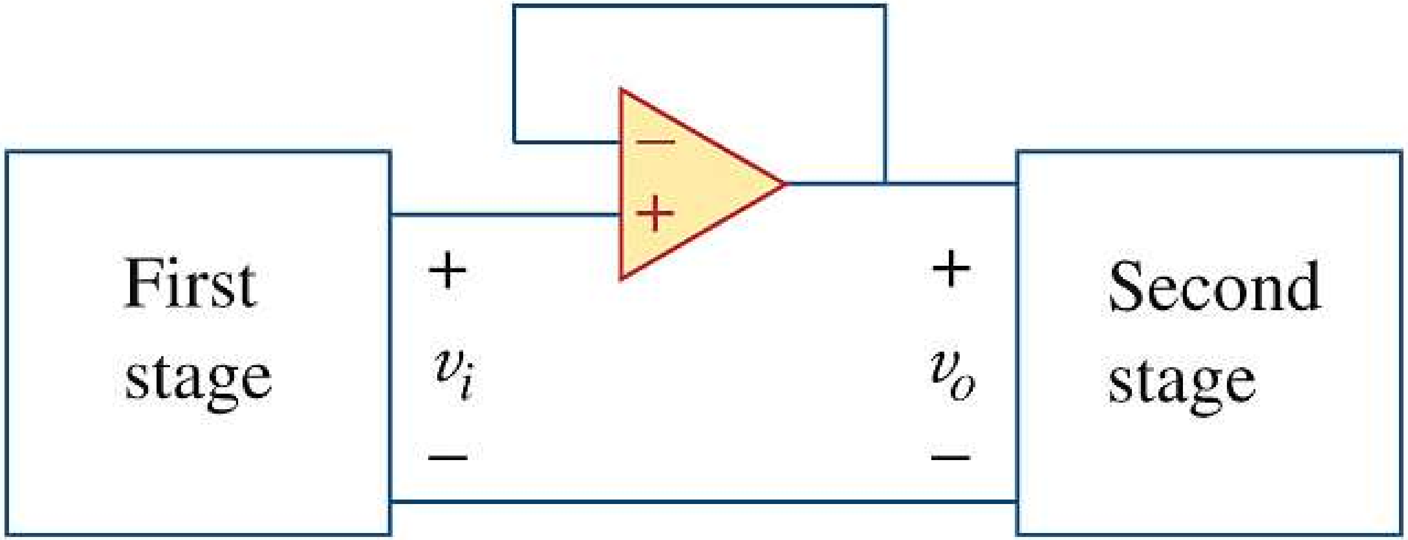


Figure 5.18 A voltage follower used to isolate two cascaded stages of a circuit.

5.6 Summing Amplifier (Adder)

A summing amplifier is an op amp circuit that combines more than one input and produces an output that is the weighted sum of the inputs, with all weights having the same sign. For example, in Fig. 5.21,

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$

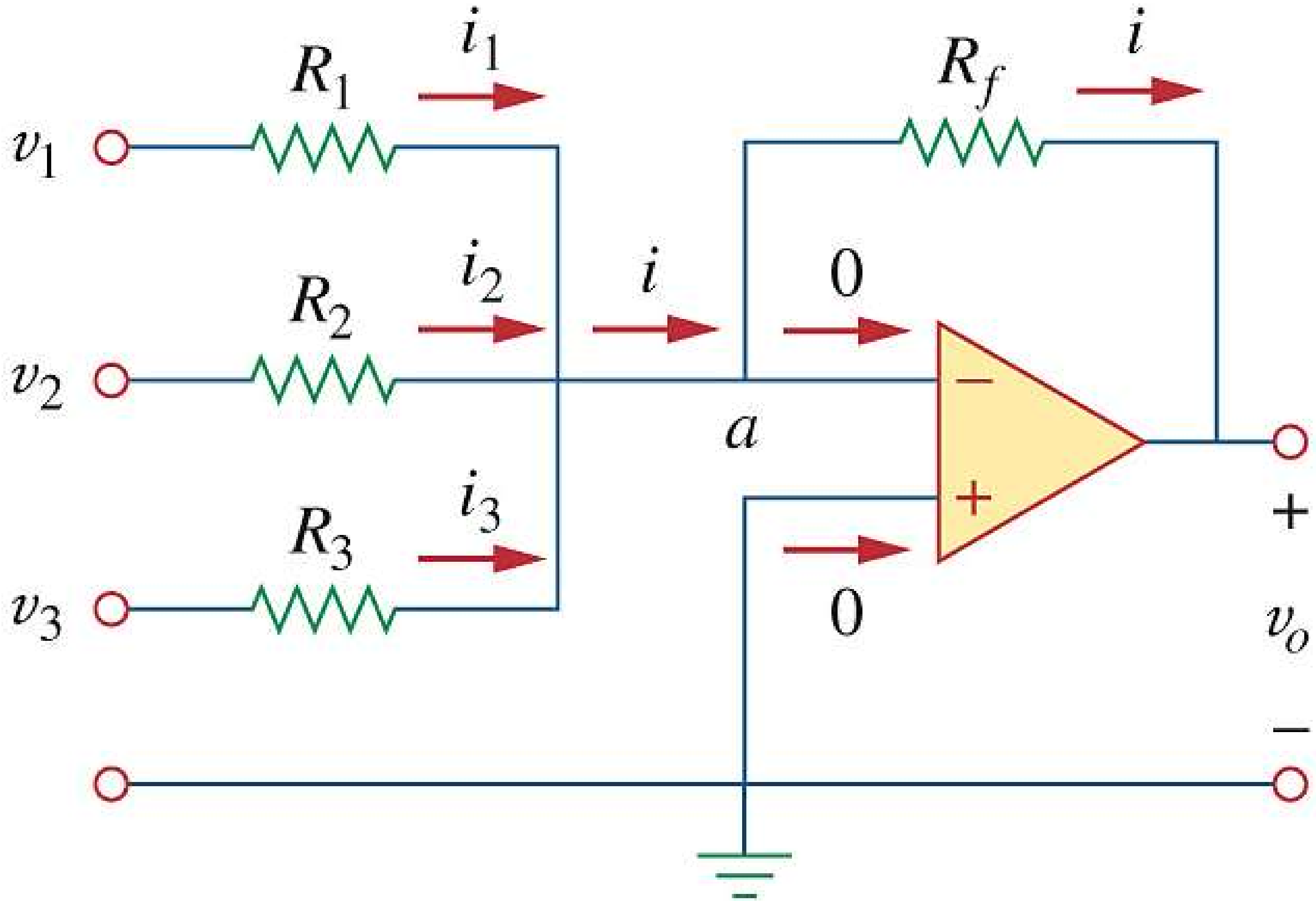


Figure 5.21 The summing amplifier.

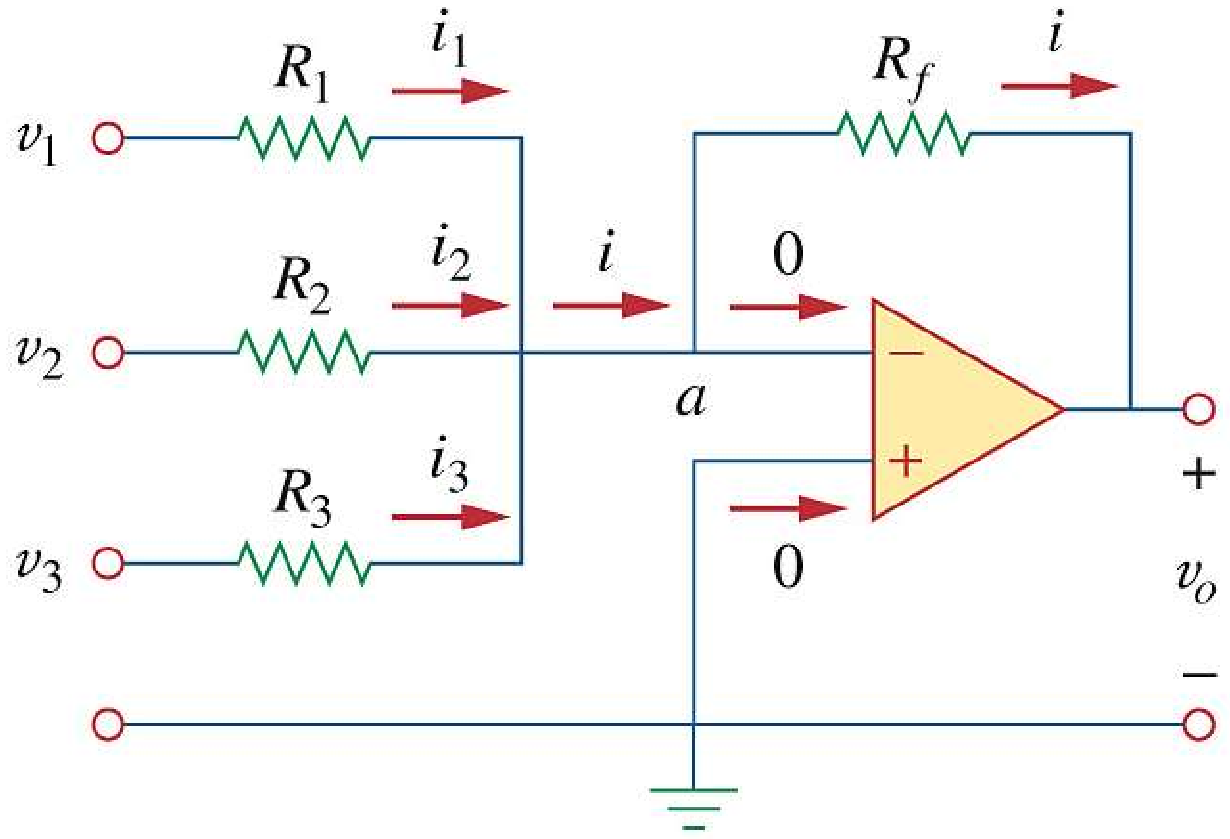


Figure 5.21 The summing amplifier.

Proof :

$$i = i_1 + i_2 + i_3 \Rightarrow$$

$$\frac{0 - v_o}{R_f} = \frac{v_1 - 0}{R_1} + \frac{v_2 - 0}{R_2} + \frac{v_3 - 0}{R_3}$$

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$

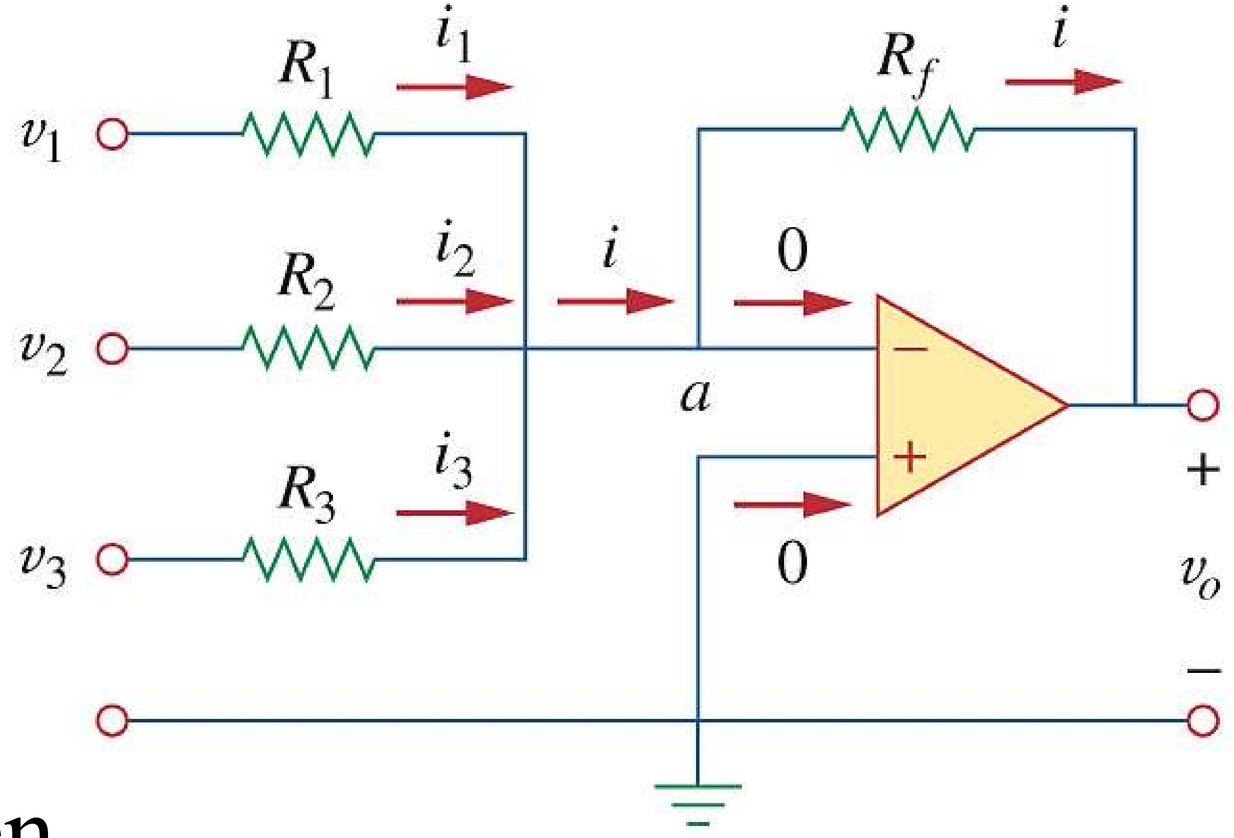


Figure 5.21 The summing amplifier.

If $R_3 = R_2 = R_1$, then

$$v_o = -\frac{R_f}{R_1}\left(v_1 + v_2 + v_3\right)$$

If $R_3 = R_2 = R_1 = R_f$, then

$$v_o = -\left(v_1 + v_2 + v_3\right)$$

Example 5.6 Calculate v_o and i_o in the op amp circuit in Fig. 5.22.

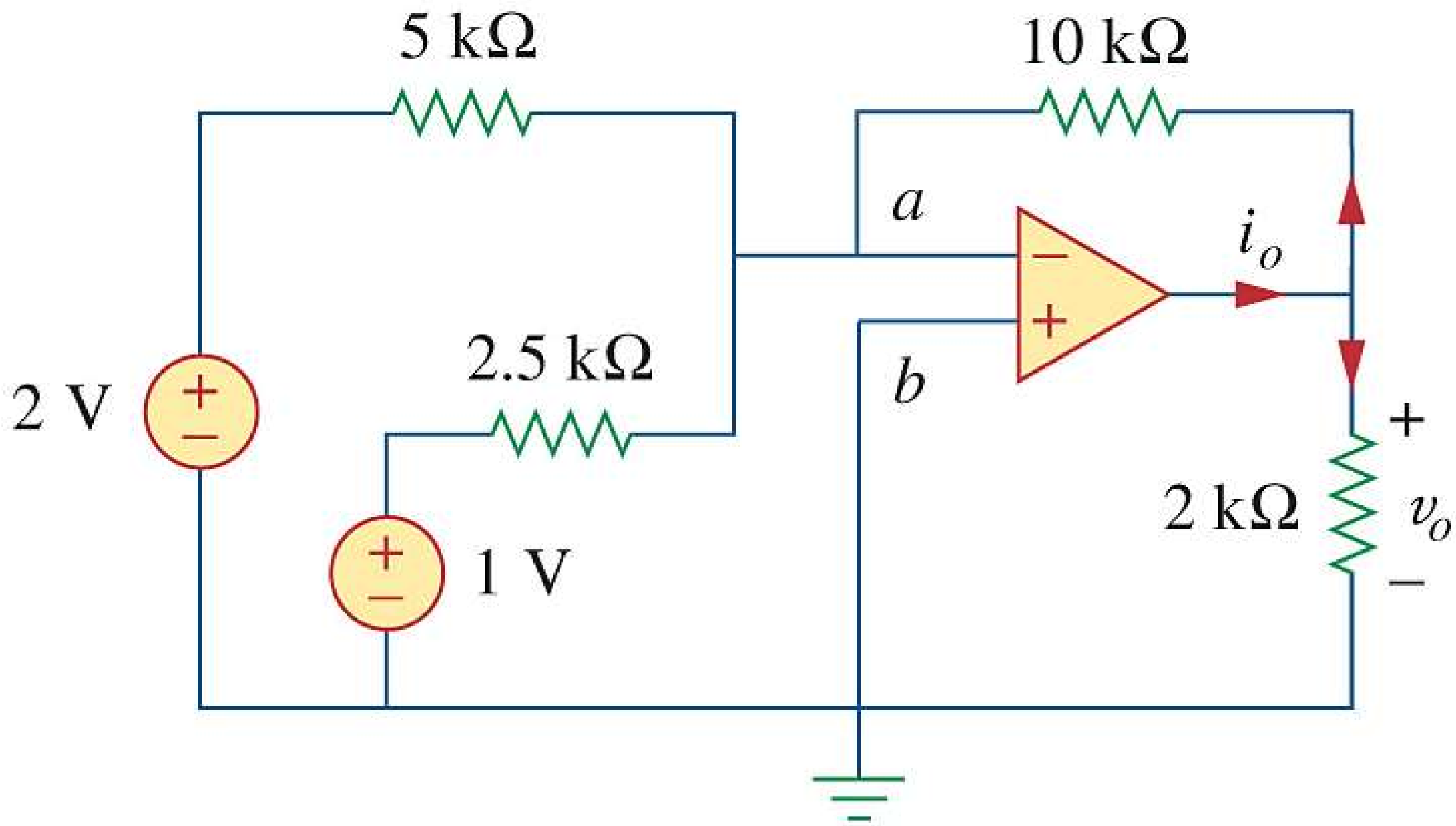


Figure 5.22

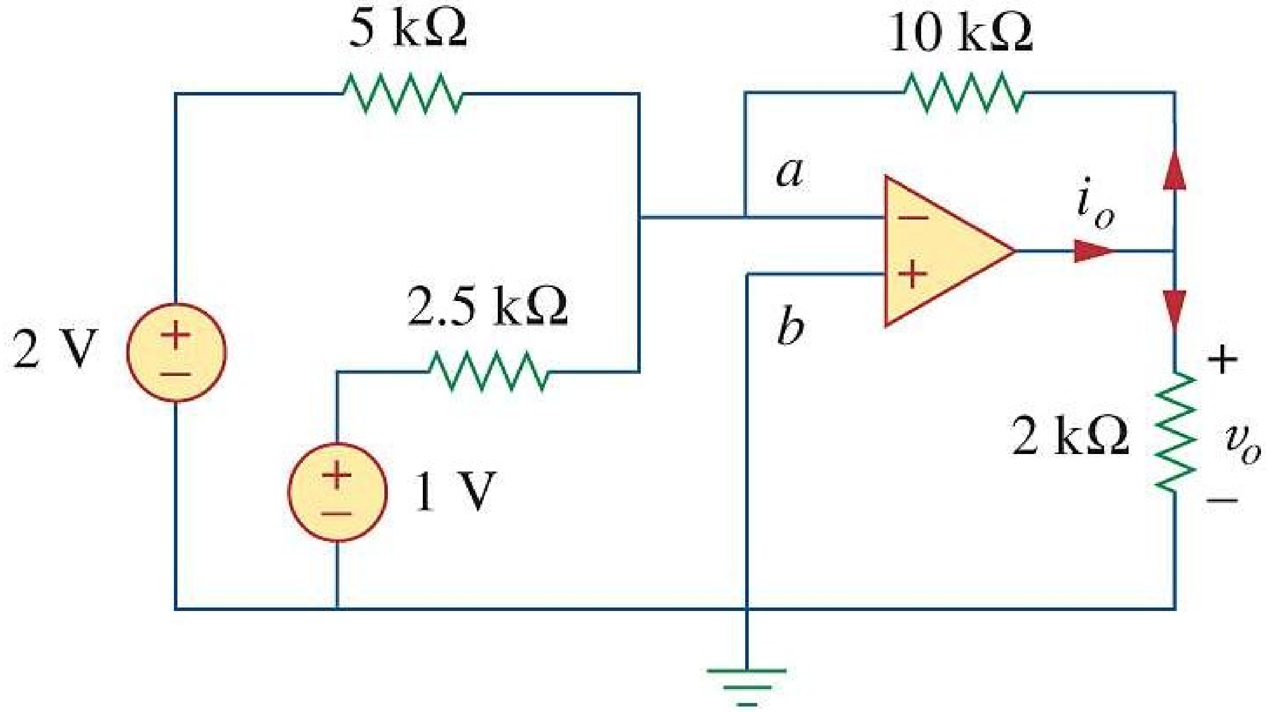


Figure 5.22

Solution :

$$v_o = -\left(\frac{R_f}{R_1} + \frac{R_f}{R_2}\right) = -\left(\frac{10}{5} \times 2 + \frac{10}{2.5} \times 1\right)$$

$$= -8 \text{ (V)}$$

$$i_o = \frac{v_o}{10 \parallel 2} = \frac{-8}{10 \times 2 / (10+2)} = -4.8 \text{ (mA)}$$

5.7 Difference Amplifier (Subtractor)

A difference (or differential) amplifier is an op amp circuit that combines two inputs and produces an output that is a weighted sum of the two inputs, with the two weights having different signs. For example, in Fig. 5.24,

$$v_o = \left(1 + R_2 / R_1\right) \frac{R_4 / R_3}{1 + R_4 / R_3} v_2 - \frac{R_2}{R_1} v_1$$

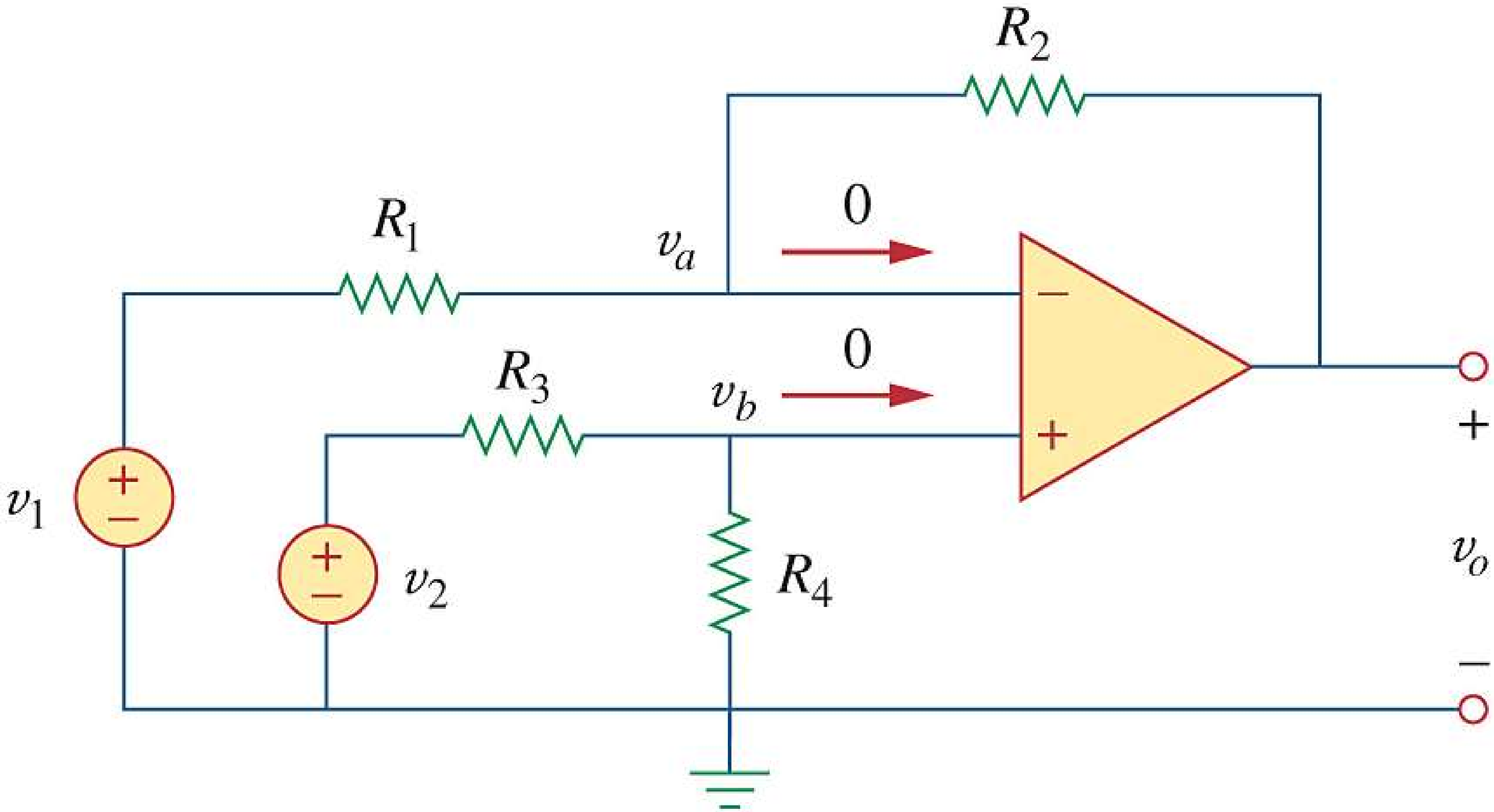


Figure 5.24 Difference amplifier.

Proof :

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \Rightarrow v_o = \left(1 + \frac{R_2}{R_1}\right) v_a - \frac{R_2}{R_1} v_1$$

$$v_a = v_b = \frac{R_4}{R_3 + R_4} v_2$$

$$v_o = \left(1 + \frac{R_2}{R_1}\right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

$$= \left(1 + R_2 / R_1\right) \frac{R_4 / R_3}{1 + R_4 / R_3} v_2 - \frac{R_2}{R_1} v_1$$

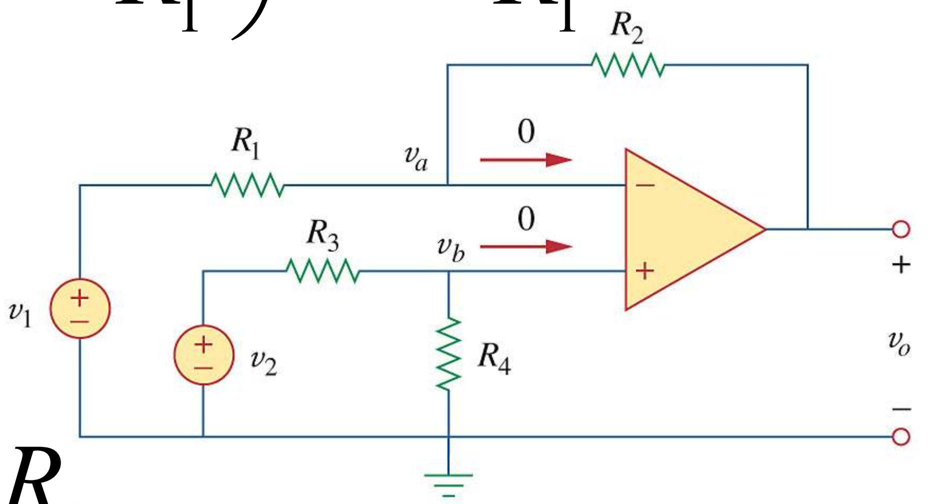


Figure 5.24 Difference amplifier.

If $R_4 / R_3 = R_2 / R_1$, then

$$v_o = \frac{R_2}{R_1}\left(v_2 - v_1\right)$$

If $R_2 = R_1$ and $R_4 = R_3$, then

$$v_o = v_2 - v_1$$

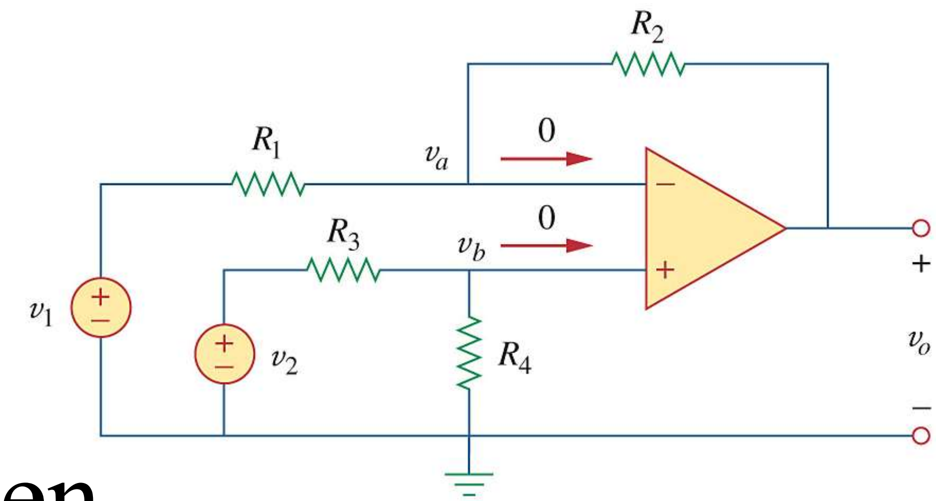


Figure 5.24 Difference amplifier.

5.8 Cascaded Op Amp Circuits

A cascade connection is a head-to-tail arrangement of two or more op amp circuits such that the output of one is the input of the next.

When op amp circuits are cascaded, each circuit in the string is called a *stage*.

Figure 4.28 displays the block diagram of a three-stage cascaded op amp circuits. The overall gain is the product of the gains of the individual stages:

$$A = \frac{v_o}{v_1} = \frac{v_2}{v_1} \cdot \frac{v_3}{v_2} \cdot \frac{v_o}{v_3} = A_1 A_2 A_3$$

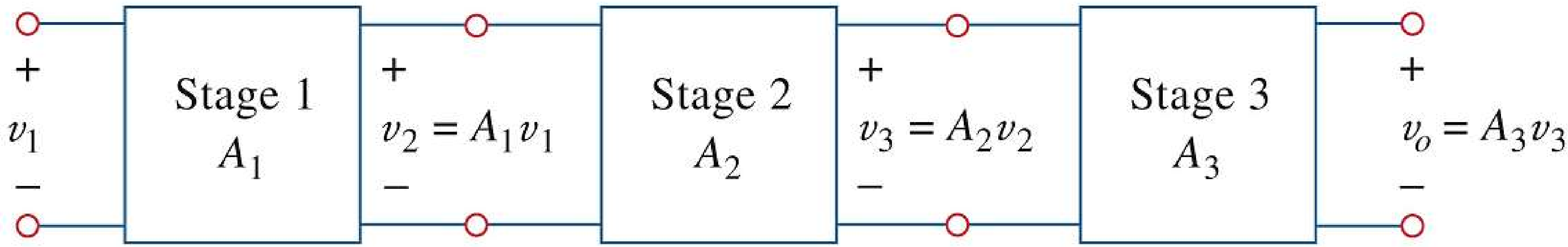


Figure 5.28 A three-stage cascaded connection.

Example 5.9 Find v_o and i_o in the circuit in Fig. 5.29.

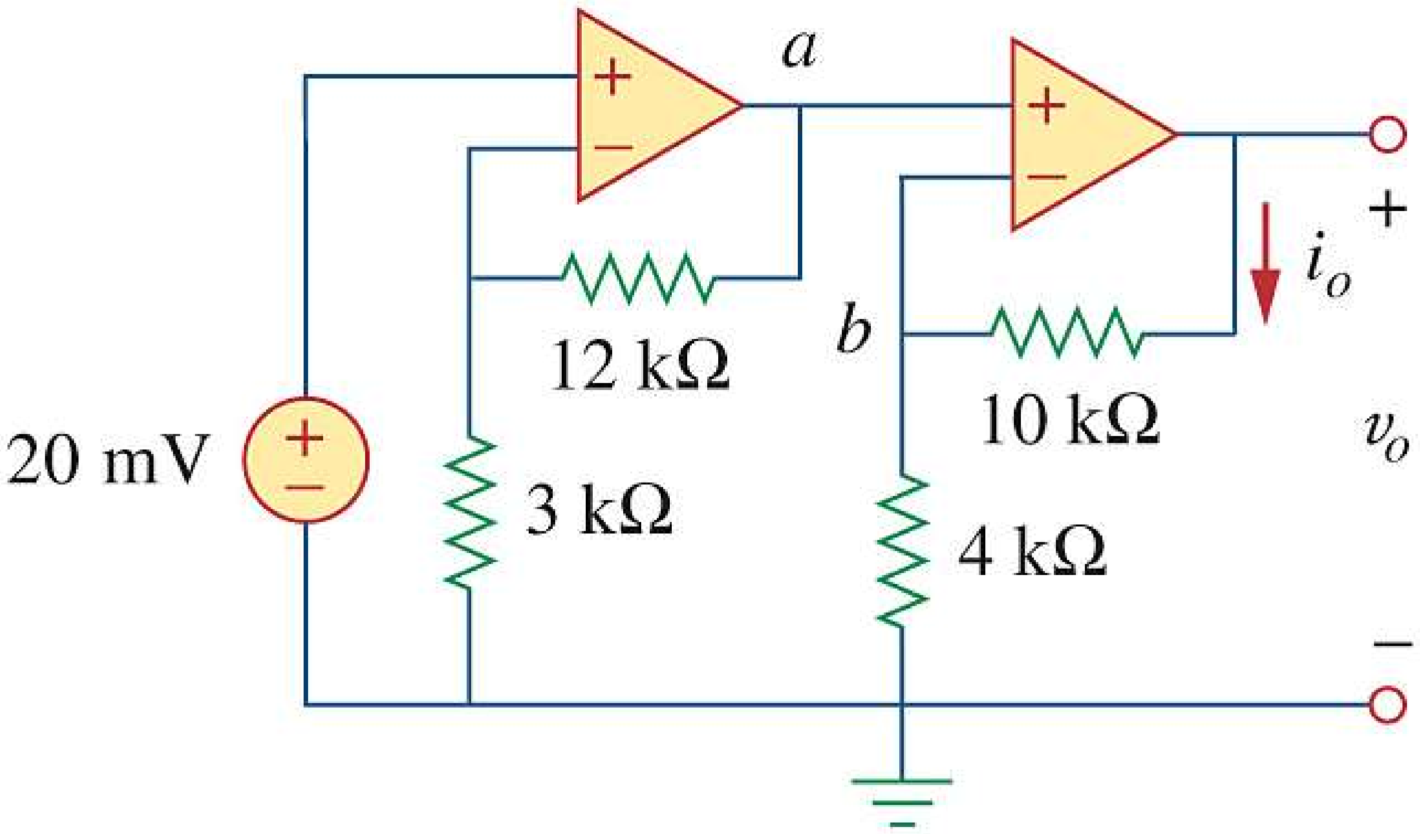


Figure 5.29

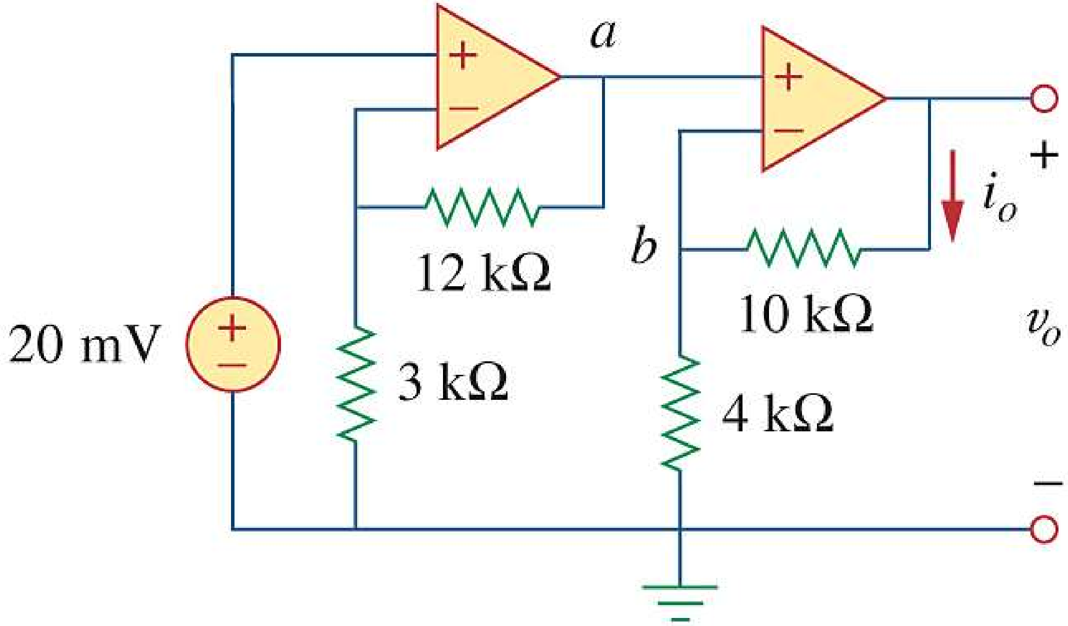


Figure 5.29

Solution :

$$v_a = \left(1+\frac{12}{3}\right)\times 20\times 10^{-3} = 0.1 \text{ (V)}$$

$$v_o = \left(1+\frac{10}{4}\right)v_a = \left(1+\frac{10}{4}\right)\times 0.1 = 0.35 \text{ (V)}$$

$$i_o = \frac{v_o}{10+4} = \frac{0.35}{14} = 0.025 \text{ (mA)} = 25\ \mu\text{A}$$

Example 5.10 If $v_1 = 1$ V and $v_2 = 2$ V, find v_o in the op amp circuit of Fig. 5.31.

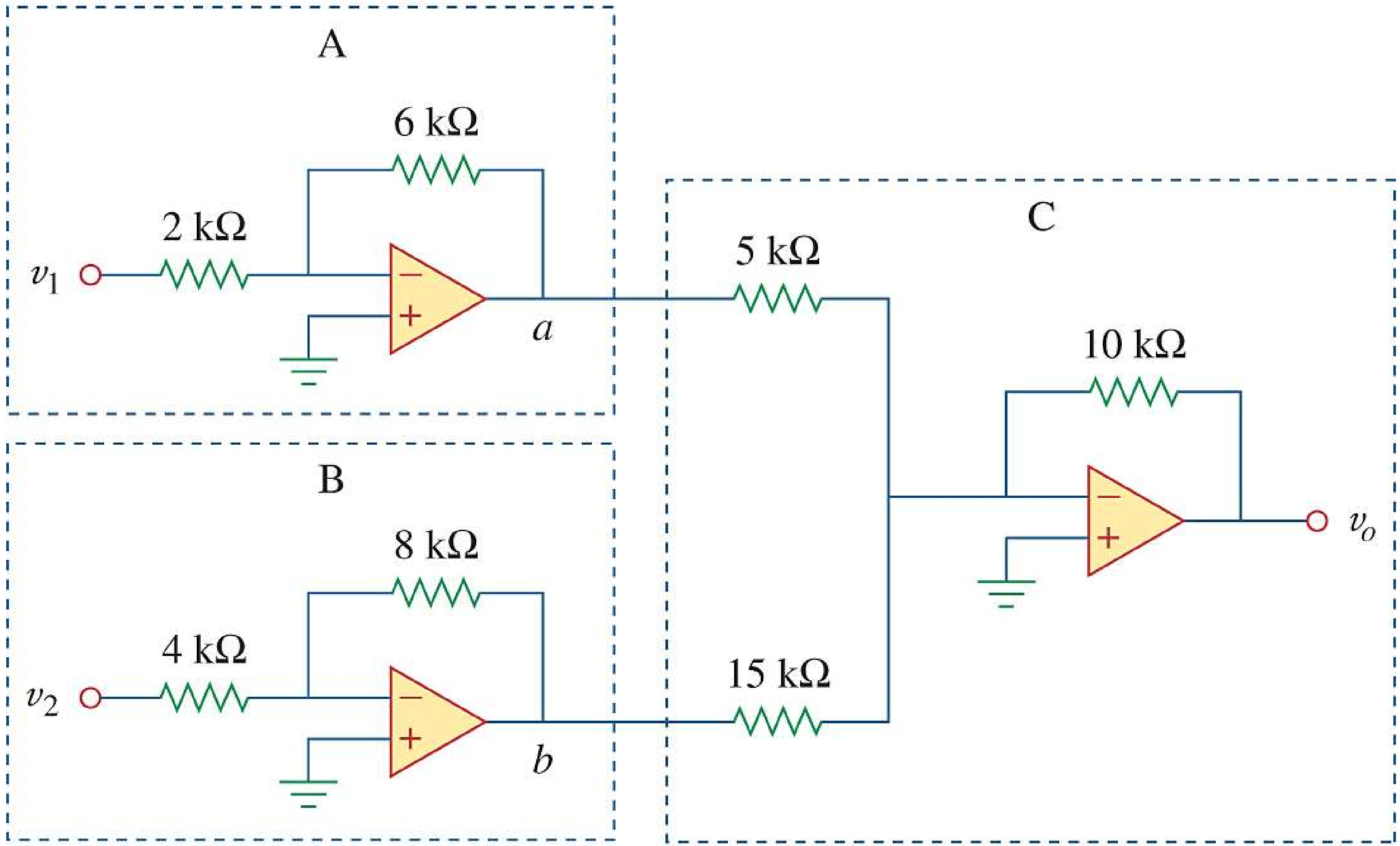


Figure 5.31

Solution :

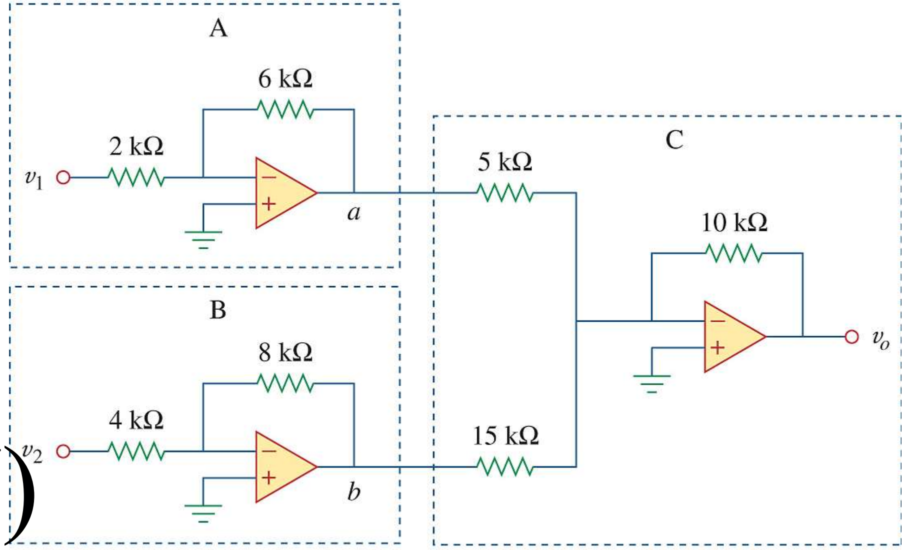


Figure 5.31

$$v_a = -\frac{6}{2}v_1 = -\frac{6}{2}\times 1 = -3\ (\text{V})$$

$$v_b = -\frac{8}{4}v_2 = -\frac{8}{4}\times 2 = -4\ (\text{V})$$

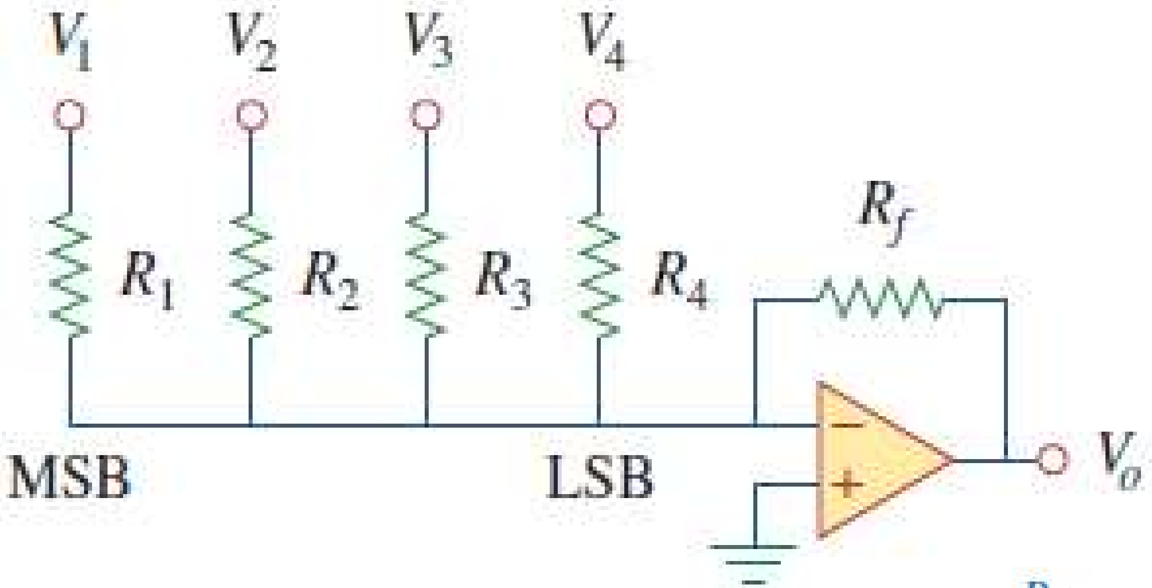
$$v_o = -\left(\frac{10}{5}v_a + \frac{10}{15}v_b\right)$$

$$= -\left(\frac{10}{5}\times(-3) + \frac{10}{15}\times(-4)\right) = \frac{26}{3} \approx 8.67\ (\text{V})$$

5.10.1 Digital-to-Analog Converter

- The digital-to-analog converter (DAC) transforms digital signals into analog form.
- Example: a four-bit DAC

- Realization is the binary weighted ladder



$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

  - an inverting summing amplifier
  - bits are weighted by descending value of R_f/R_n to produce 2 times difference for adjacent bits

– V_1: most significant bit (MSB)

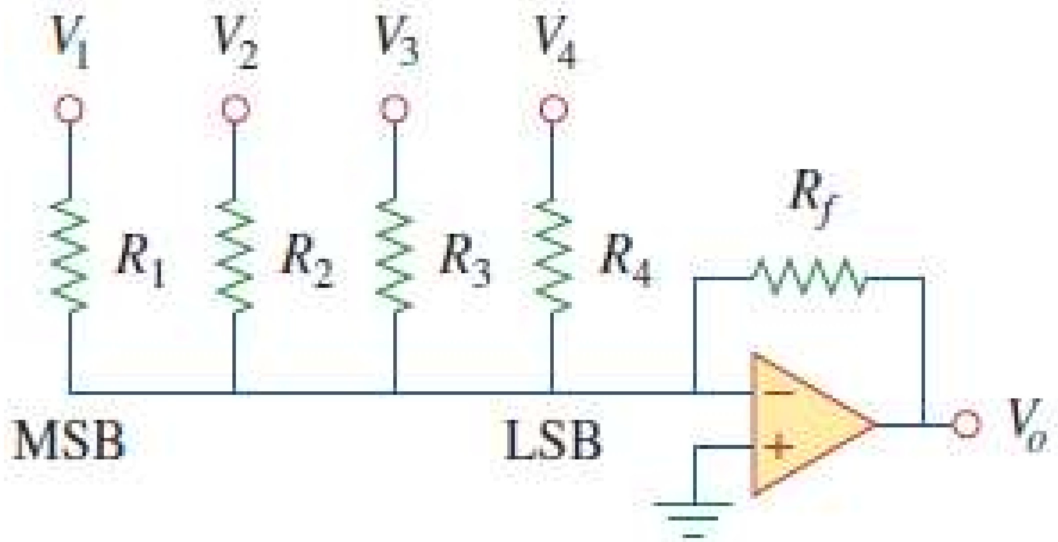
V_4: least significant bit (LSB)

– Assume only two voltage levels for V_1 to V_4: 0 and 1V

$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4 \Rightarrow \boxed{-V_o = k(2^3V_1+2^2V_2+2^1V_3+2^0V_4)}$$

Example 5.12

In the op amp circuit of Fig. 5.36(b), let $R_f = 10\ \text{k}\Omega$, $R_1 = 10\ \text{k}\Omega$, $R_2 = 20\ \text{k}\Omega$, $R_3 = 40\ \text{k}\Omega$, and $R_4 = 80\ \text{k}\Omega$. Obtain the analog output for binary inputs [0000], [0001], [0010], . . . , [1111].



- 1. Output voltage V_o

$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$
$$= V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

- 2. digital input $[V_1V_2V_3V_4]$ = [0000] produces an analog output of $-V_o = 0$ V

  [0001] → $-V_o = -0.125$ V

TABLE 5.2

Input and output values of the four-bit DAC.

Binary input $[V_1 V_2 V_3 V_4]$	Decimal value		Output $-V_o$
0000	0		0
0001	1		0.125
0010	2		0.25
0011	3		0.375
0100	4	÷ 0.125	0.5
0101	5		0.625
0110	6		0.75
0111	7		0.875
1000	8		1.0
1001	9		1.125
1010	10		1.25
1011	11	× 0.125	1.375
1100	12		1.5
1101	13		1.625
1110	14		1.75
1111	15		1.875

- Resolution: the smallest resolvable analog output
- **Question**: In practice, for 1V range, if you want to produce a resolution of 1 mV, roughly how many bits do you need?

5.10.2 Instrumentation Amplifiers

- One of the most useful and versatile op amp circuits is the instrumentation amplifier (IA), so called because of its widespread use in measurement systems.
- The IA is an extension of the **difference amplifier** in that it amplifies the difference between its inputs.

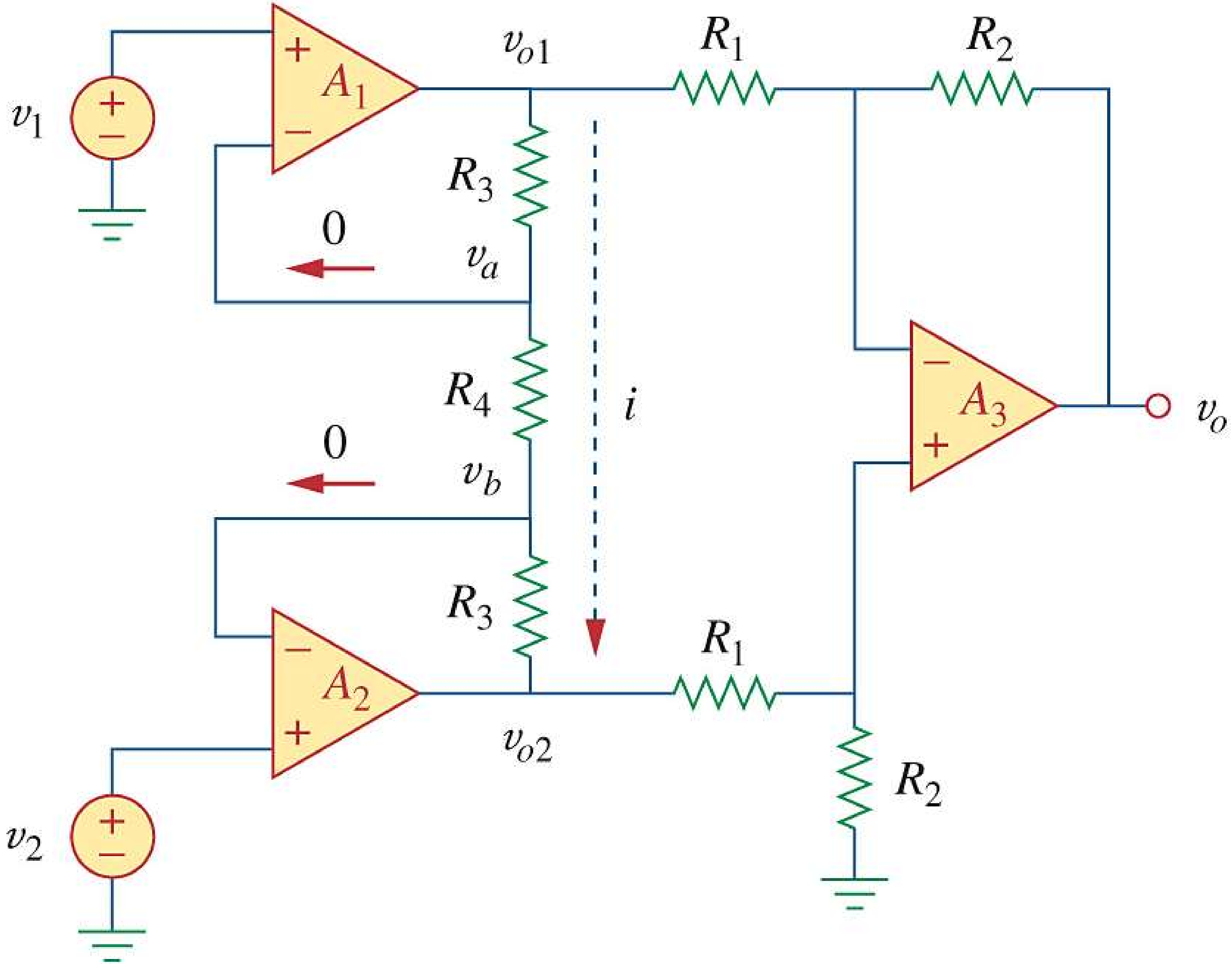


Figure 5.26 Instrumentation amplifier.

An IA circuit is shown in Fig. 5.26. Show that

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

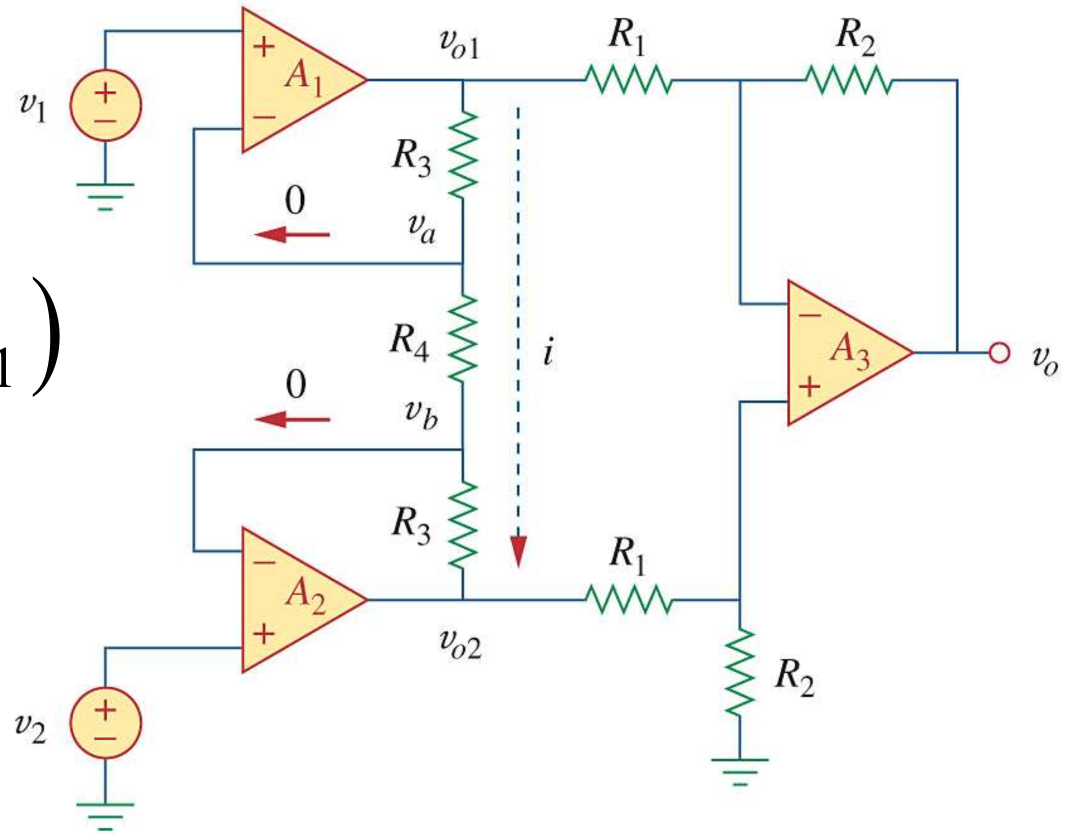


Figure 5.26 Instrumentation amplifier.

Proof :

$$\begin{cases} v_a = v_1, v_b = v_2 \\ i = \dfrac{v_a - v_b}{R_4} = \dfrac{v_{o1} - v_{o2}}{R_3 + R_4 + R_3} \end{cases}$$

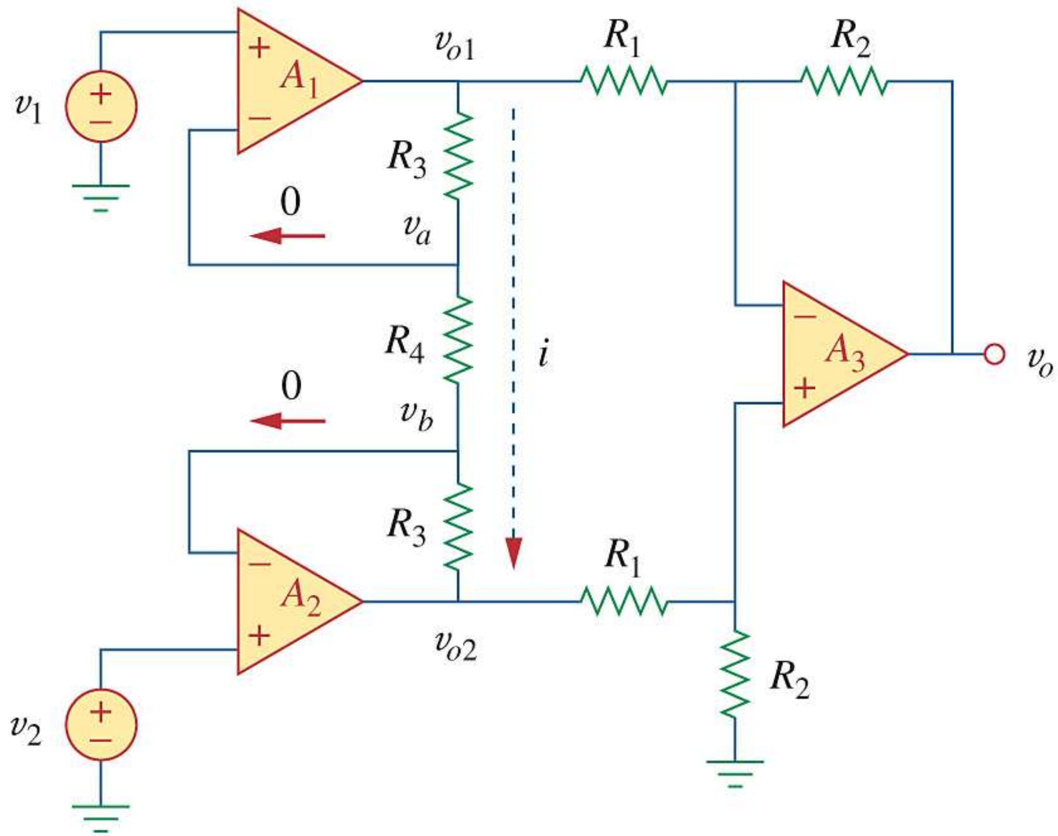


Figure 5.26 Instrumentation amplifier.

$$v_{o2} - v_{o1} = \left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1}) = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

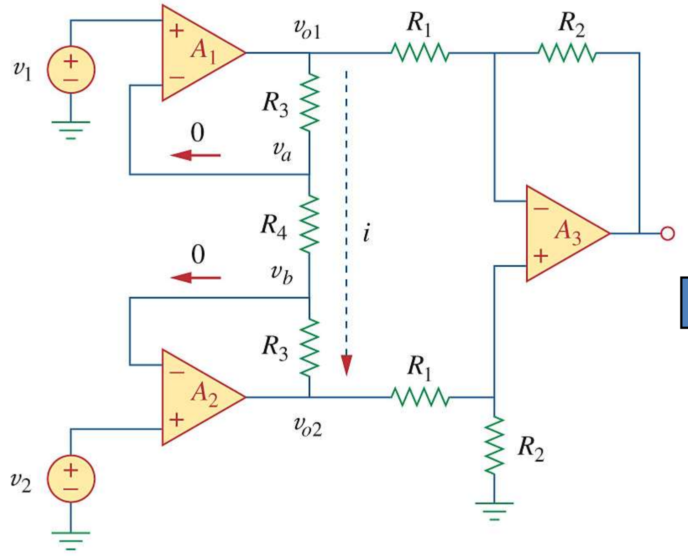


Figure 5.26 Instrumentation amplifier.

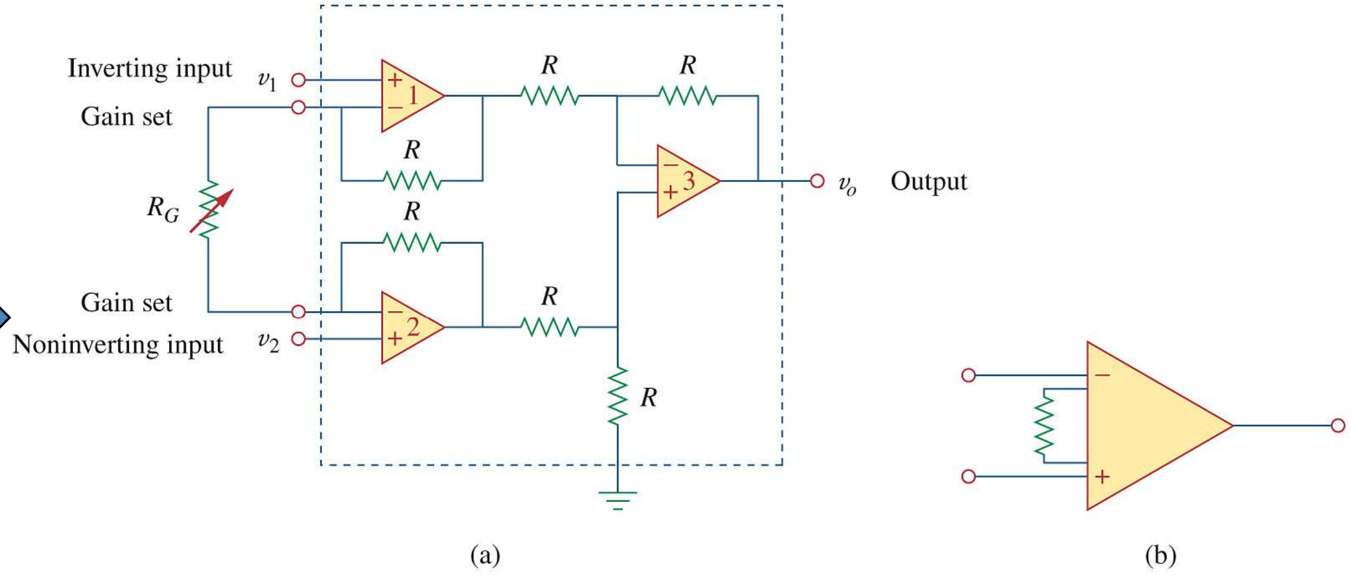


Figure 5.38 (a) The IA with an external resistance to adjust the gain, (b) schematic symbol.

In Fig. 5.38(a), $R_4 = R_G$, $R_3 = R_2 = R_1 = R$,

$$v_o = \frac{R_2}{R_1}\left(1+\frac{2R_3}{R_4}\right)(v_2 - v_1)$$

$$= \left(1+\frac{2R}{R_G}\right)(v_2 - v_1)$$

$$= A_v(v_2 - v_1)$$

The advantage over a difference amplifier:
Gain is adjustable by an external resistor

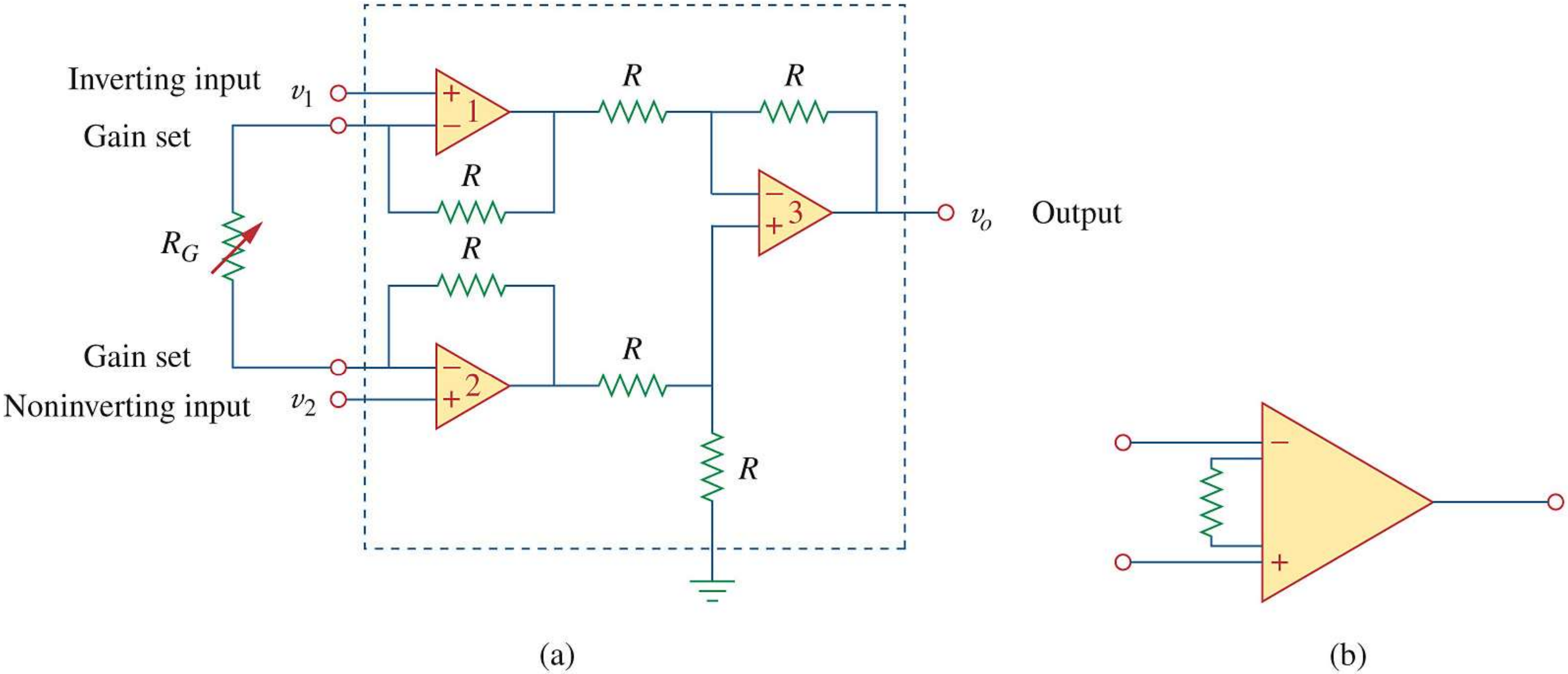


Figure 5.38 (a) The IA with an external resistance to adjust the gain, (b) schematic symbol.

As shown in Fig. 5.39, the IA amplifies small differential signals superimposed on larger common-mode signal.

$$\begin{cases} v_d = v_2 - v_1 \\ v_c = \dfrac{v_1 + v_2}{2} \end{cases} \Leftrightarrow \begin{cases} v_2 = \dfrac{v_d}{2} + v_c \\ v_1 = -\dfrac{v_d}{2} + v_c \end{cases}$$

$$v_o = A_v\left(v_2 - v_1\right) = A_v v_d$$

v_o is independent of v_c

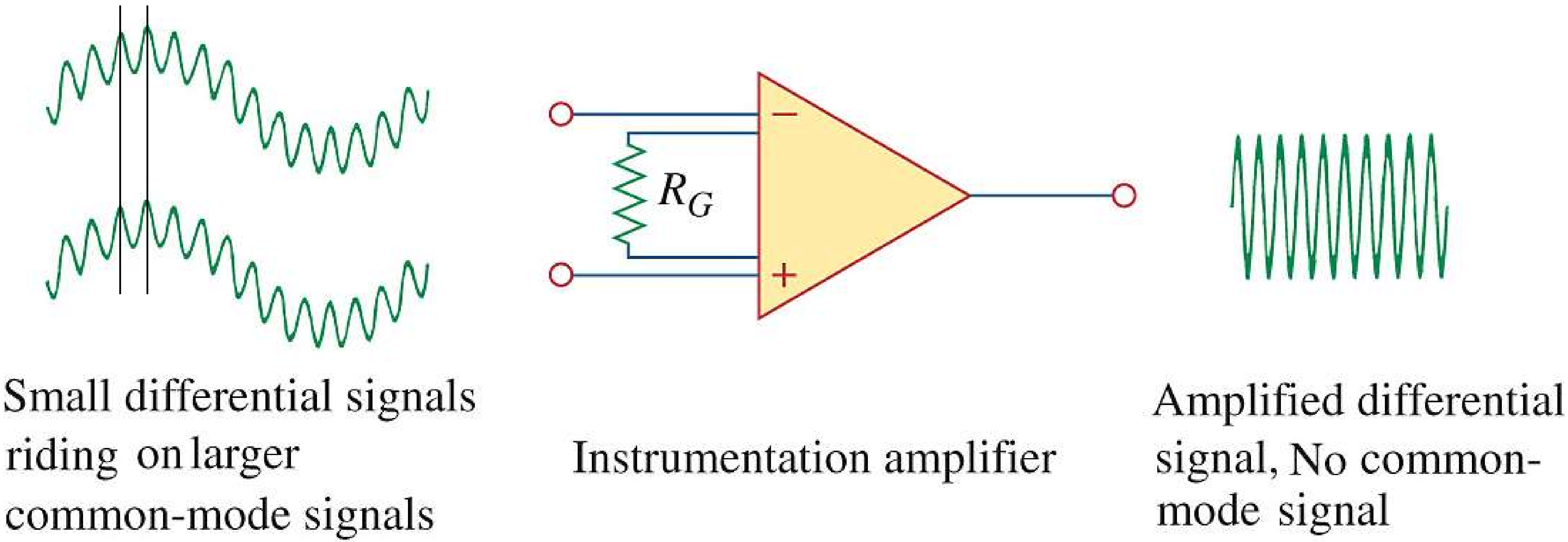


Figure 5.39 The IA rejects common voltages but amplifies differential voltages.

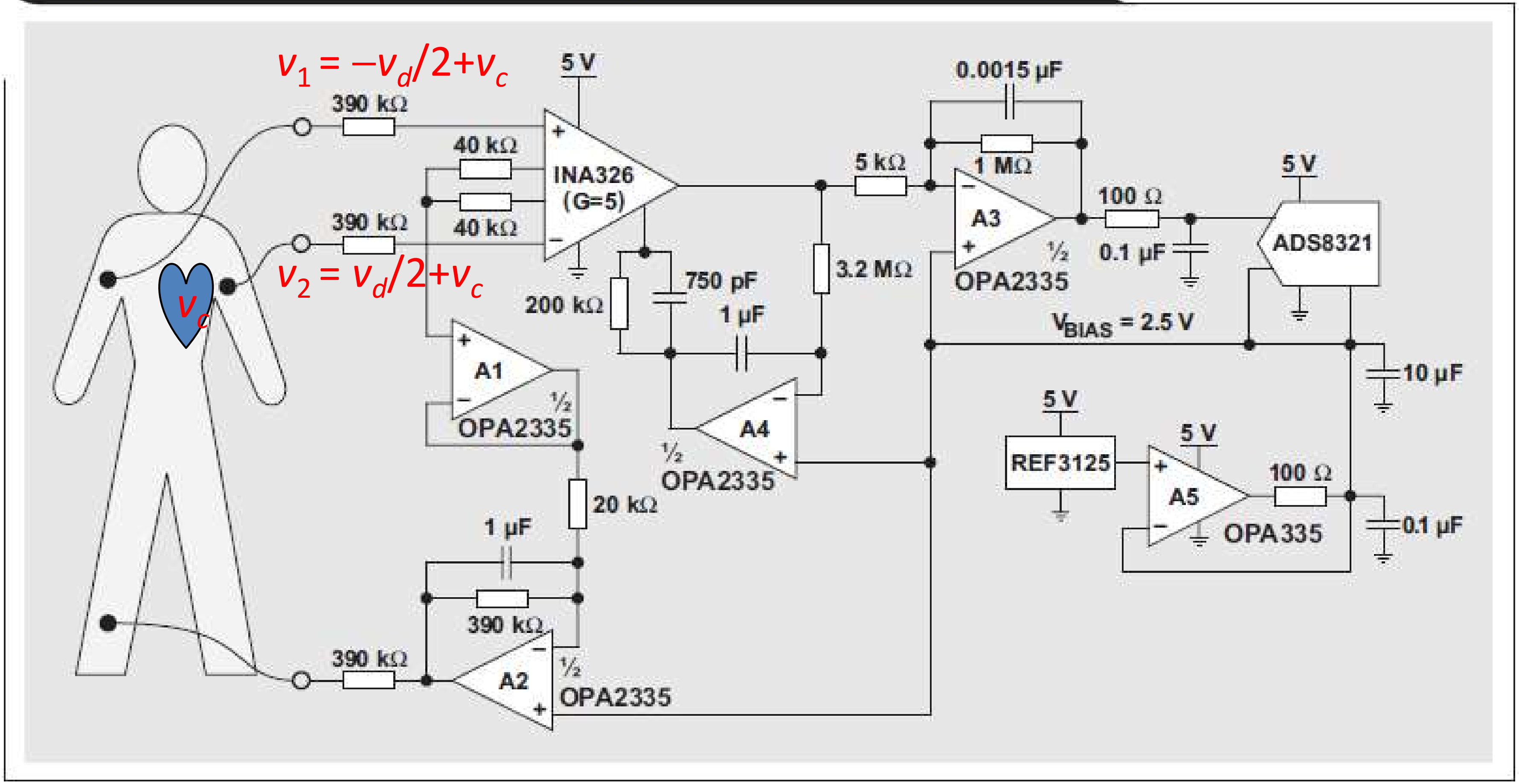
The IA has three major characteristics:

1. The voltage gain is adjusted by one external resistor R_G.

2. The input resistance is very high and does not vary as the gain is adjusted.

3. The output v_o depends on v_d, not on v_c.

Figure 5. High-precision analog front end of a portable ECG application



- **Electrocardiography** is the recording of the electrical activity of the heart. It can measure:
  - the rate and regularity of “heartbeats”
  - the “size and position of the chambers”
  - the presence of any “damage” to the heart
  - the effects of drugs or devices used to regulate the heart (drug evaluation…)

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