VE215 2025SU Assignment 1

Due Date: 23:59, May.29th, 2025
In order to get full marks, you shall write all the intermediate steps of calculation or proof, unless otherwise indicated.

Exercise 1.1 (15%) The voltage v(unit:V) across a device and the current i (unit:A) through it are

$$v(t) = 10\cos\frac{\pi t}{2} + 10 \qquad i(t)= \begin{cases} 0 & t<0 \\ -10t^2+20t & 0 \le t < 1 \\ 10e^{-t+1} & 1 \le t < \ln 2 + 1 \\ 5 & t \ge \ln 2 + 1 \end{cases}$$

(a)(5%) Calculate the total charge in the device at t = 1.5 s.
(b)(5%) Calculate the power delivered to the element at $t = 1.5$ s.
(c)(5%) Calculate the energy delivered to the device between 3 and 5 s.

(a) $q=\int_{-\infty}^{1.5} i(t)dt = 0 + \int_0^1 \left(-10t^2+20t\right)dt + \int_1^{1.5} 10e^{-t+1} = 10.60C$
$((3'(\text{formula}) + 1'\text{ answer }) + 1'(\text{ unit }))$

(b) $p(t=1.5) = v(1.5)\cdot i(1.5) = \left(10\cos\frac{3}{4}\pi + 10\right)\cdot 10e^{-\frac{1}{2}} = 17.76$ W
$(3'(\text{formula}) + 1'(\text{ answer }) + 1'(\text{ unit }))$

(c) $W=\int_3^5 p(t)dt = \int_3^5 v(t)i(t)dt = \int_3^5 50\left(\cos\frac{\pi t}{2}+1\right)dt = 163.66J$
$((3'(\text{formula}) + 1'\text{ answer }) + 1'(\text{ unit }))$

Exercise 1.2 (25%) In the circuit below, all resistors have a resistance of R.
(a) (12%)Determine the number of branches, nodes, loops, and meshes. Write your answers directly.
(b) (13%)Calculate the equivalent resistance between the terminals.

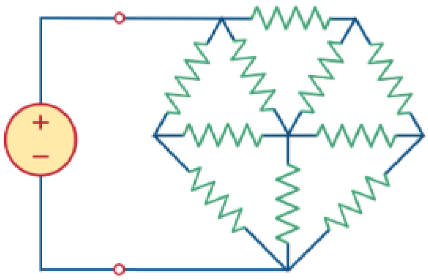

Figure 1: Exercise 1.2

(a) (12%) We can see directly from the circuit diagram that the numbers of branches, nodes and meshes are 11,6 and 6 respectively.
In order to count the number of loops, we need a more systematic approach. We can classify the loops by the number of areas they cover as follows:

- 6 loops covering 1 area: A, B, C, D, E, F;
- 7 loops covering 2 areas: AB, BC, CD, DE, EA, FE, FD;
- 8 loops covering 3 areas: ABC, BCD, CDE, DEA, EAB, FEA, FED, FDC;
- 9 loops covering 4 areas: ABCD, BCDE, CDEA, DEAB, EABC, FEAB, FDEA, FDEC, FDCB;
- 4 loops covering 5 areas: ABCDE, FDEAB, FDEAC, FDECB;
- 1 loop covering 6 areas: ABCDEF .

Hence, the number of loops is 35 .

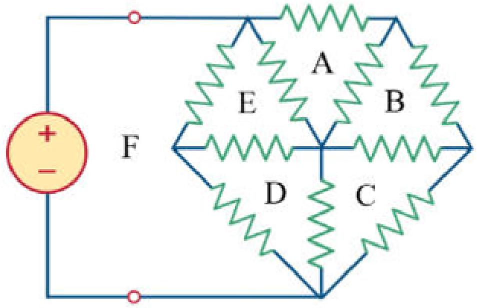

Figure 2: Exercise1.2(a)

(b)(13%)First, we apply $\Delta - Y$ transformation and simplify the circuit as shown, where

$$R_Y = \frac{R \cdot R}{R + R + R} = \frac{R}{3}$$

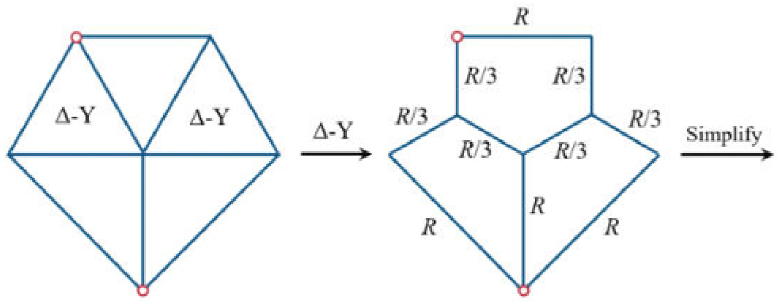

Figure 3: Exercise1.2(b)

Then, we apply another $\Delta - Y$ transformation and simplify the circuit as shown, where

$$R_1 = \frac{\frac{R}{3} \cdot \frac{4R}{3}}{\frac{R}{3} + \frac{4R}{3} + R} = \frac{R}{6}$$

$$R_2 = \frac{\frac{R}{3} \cdot R}{\frac{R}{3} + \frac{4R}{3} + R} = \frac{R}{8}$$

$$R_3 = \frac{\frac{4R}{3} \cdot R}{\frac{R}{3} + \frac{4R}{3} + R} = \frac{R}{2}$$

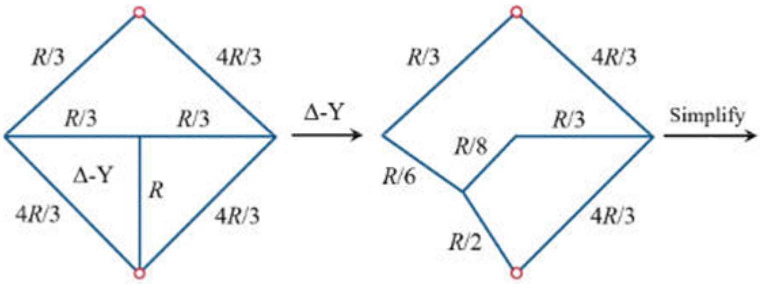

Figure 4: Exercise1.2(b2)

Finally, we apply $\Delta - Y$ transformation and simplify the circuit as shown, where

$$R_1 = \frac{\frac{R}{2} \cdot \frac{11R}{24}}{\frac{R}{2} + \frac{4R}{3} + \frac{11R}{24}} = \frac{R}{10}$$

$$R_2 = \frac{\frac{4R}{3} \cdot \frac{11R}{24}}{\frac{R}{2} + \frac{4R}{3} + \frac{11R}{24}} = \frac{4R}{15}$$

$$R_3 = \frac{\frac{4R}{3} \cdot \frac{R}{2}}{\frac{R}{2} + \frac{4R}{3} + \frac{11R}{24}} = \frac{16R}{55}$$

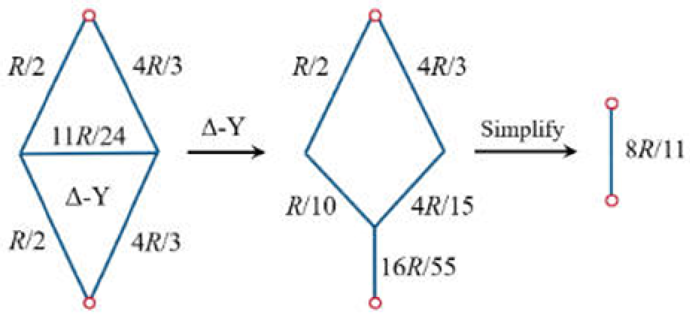

Figure 5: Exercise1.2(b3)

The equivalent resistance is then:

$$R_{eq} = \left(\frac{R}{2} + \frac{R}{10}\right) \parallel \left(\frac{4R}{3} + \frac{4R}{15}\right) + \frac{16R}{55} = \frac{8}{11}R$$

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Exercise 1.3(25%) Solve the following questions with the given methods.
(a) (10%) Calculate the unknown current I_x using mesh analysis.
(b) (15%) Calculate the unknown voltage V_x using nodal analysis.

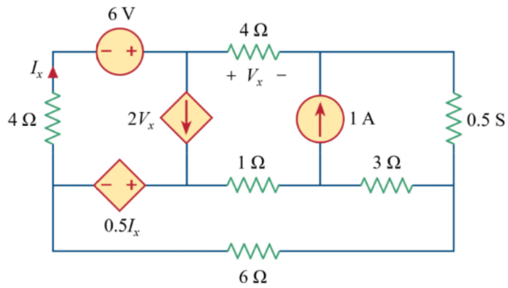

Figure 6: Exercise 1.3

Solution
(a) (**10**%) Assigning mesh currents as shown, we have

$$I_1 = I_x, \quad I_1 - I_2 = 2V_x, \quad I_3 - I_2 = 1, \quad V_x = 4I_2. \quad (2')$$

Applying KVL to supermesh $(1, 2, 3)$, we have

$$4I_1 - 6 + 4I_2 + 2I_3 + 3(I_3 - I_4) + (I_2 - I_4) + 0.5I_x = 0 \quad (2')$$

Applying KVL to mesh 4, we have

$$-0.5I_x + (I_4 - I_2) + 3(I_4 - I_3) + 6I_4 = 0. \quad (2')$$

Solving the equations, we obtain

$$I_1 = \frac{66}{157}\text{ A}, \quad I_2 = \frac{22}{471}\text{ A}, \quad I_3 = \frac{493}{471}\text{ A}, \quad I_4 = \frac{160}{471}\text{ A}. \quad (2')$$

Hence, the unknown current is

$$I_x = I_1 = \frac{66}{157}\text{ A} = 0.420\text{ A}. \quad (2')$$

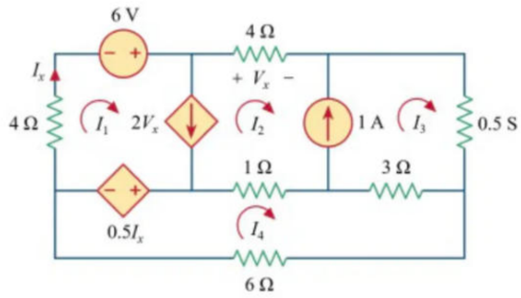

Figure 7: Exercise1.3(a)

(b) (15%) Assigning node voltages as shown, we have

$$V_x = V_1 + 6 - V_2, \quad I_x = \frac{0 - V_1}{4}. \quad (1')$$

Applying KCL to supernode ($V_1, V_1 + 6$), we have

$$\frac{0 - V_1}{4} + \frac{V_2 - (V_1 + 6)}{4} - 2V_x = 0 \quad (2')$$

Applying KCL to node V_2, we have

$$\frac{V_1 + 6 - V_2}{4} + 1 + \frac{V_4 - V_2}{2} = 0 \quad (2')$$

Applying KCL to node V_3, we have

$$\frac{0.5I_x - V_3}{1} - 1 + \frac{V_4 - V_3}{3} = 0 \quad (2')$$

Applying KCL to node V_4, we have

$$\frac{V_3 - V_4}{3} + \frac{V_2 - V_4}{2} + \frac{0 - V_4}{6} = 0 \quad (2')$$

Solving the equations, we obtain

$$V_1 = -\frac{264}{157}\text{ V}, \quad V_2 = \frac{1946}{471}\text{ V}, \quad V_3 = -\frac{13}{157}\text{ V}, \quad V_4 = \frac{320}{157}\text{ V}.$$

($4'$ ok if only V_1 and V_2 are obtained)

Hence, the unknown current is

$$V_x = V_1 + 6 - V_2 = \frac{88}{471}\text{ V} = 0.187\text{ V} \quad (2')$$

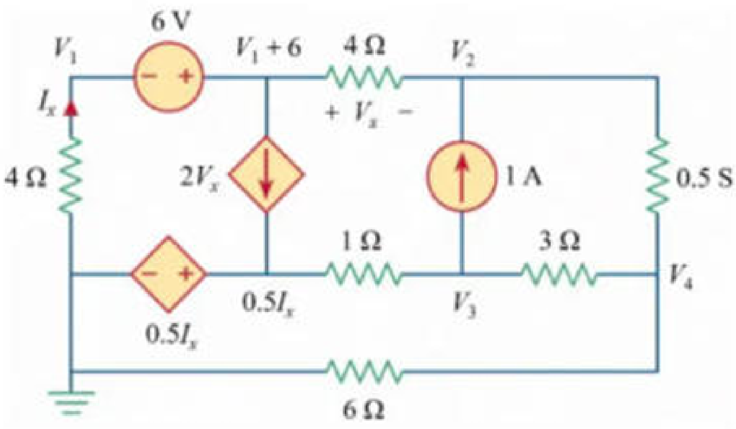

Figure 8: Exercise1.3(b)

Exercise 1.4(20%)Solve the following questions with the given methods.
(a)(10%)Using nodal analysis **by inspection**, find V_1, V_2, V_3 and V_0 in the following circuit.

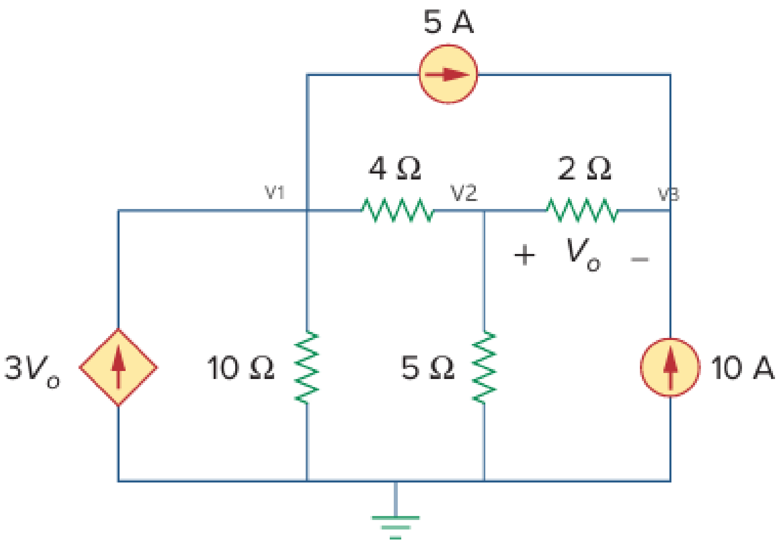

Figure 9: Exercise1.4(a)

Solution Apply nodal analysis by inspection:

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{5}+\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3V_0-5 \\ 0 \\ 10+5 \end{bmatrix}$$

Where $v_2 - v_3 = v_0$
Eliminate $V_0 \Rightarrow$

$$LHS + \begin{bmatrix} 0 & -3 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 10+5 \end{bmatrix}$$

which is

$$\begin{bmatrix} \frac{7}{20} & -\frac{13}{4} & 3 \\ -\frac{1}{4} & \frac{19}{20} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 15 \end{bmatrix}$$

$$\Delta = 0.0475 \quad \Delta_1 = -19.5 \quad \Delta_2 = -9.25 \quad \Delta_3 = -7.825$$

$$\Rightarrow v_1 = -410.526V, \quad v_2 = -194.737V, \quad v_3 = -164.737V, \quad V_0 = v_2 - v_3 = -30V$$

(b)(10%)Using mesh analysis **by inspection** to solve the mesh currents in the following circuit. **Solution**

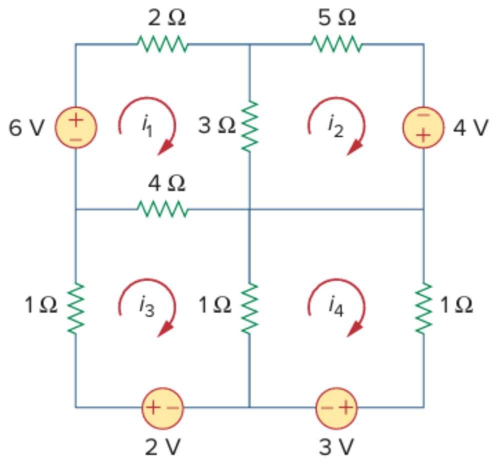

Figure 10: Exercise1.4(b)

$$\begin{bmatrix} 9 & -3 & -4 & 0 \\ -3 & 8 & 0 & 0 \\ -4 & 0 & 6 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \\ -3 \end{bmatrix}$$

$i_1 = \dfrac{692}{437}$ A ≈ 1.58 A $\quad i_2 = \dfrac{478}{437}$ A ≈ 1.09 A$i_3 = \dfrac{543}{437}$ A ≈ 1.24 A. $\quad i_4 = -\dfrac{384}{437}$ A ≈ -0.88 A

Exercise 1.5(15%)For the circuit below, find the node voltages v_1, v_2, v_3.

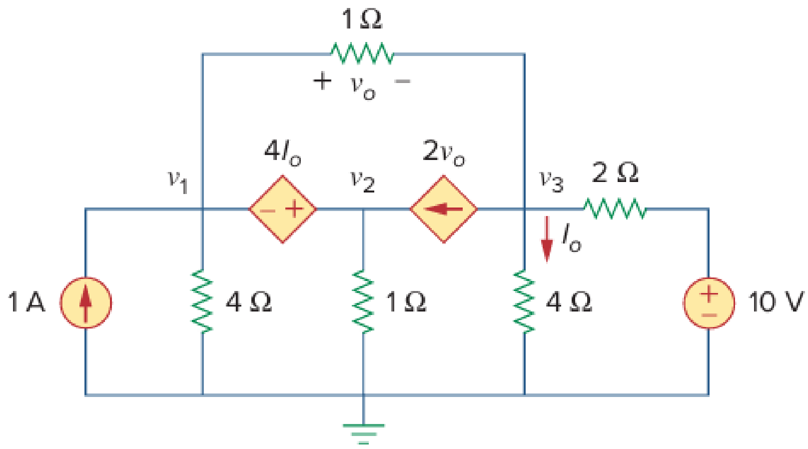

Figure 11: Exercise 1.5

Nodal analysis, supernode(v_1 and v_2):

$$1 - \frac{v_1}{4} - \frac{v_0}{1} + 2v_0 - \frac{v_2}{1} = 0$$

node 3:

$$-2v_0 + \frac{v_0}{1} - \frac{v_3}{4} + \frac{10 - v_3}{2} = 0$$

In supernode:

$$v_2 - v_1 = 4I_0.$$

And

$$v_0 = v_1 - v_3$$
$$I_0 = \frac{v_3}{4}$$

solve 5 equations

$$v_1 = 4.97 \text{ V}, v_2 = 4.85 \text{ V}, v_3 = -0.12 \text{ V}$$