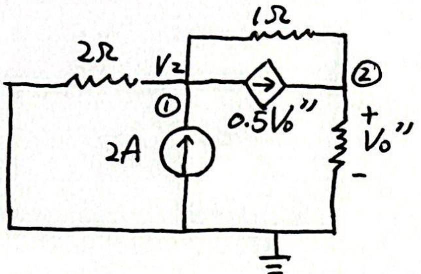
Exercise 2.1 (15%)

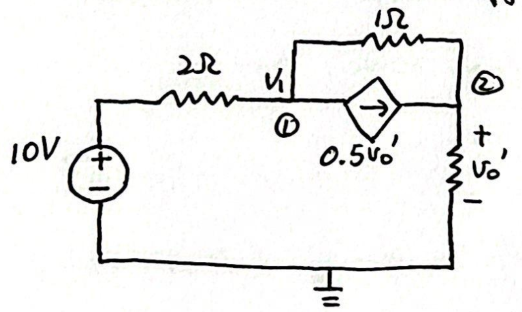
First, we shut off the 10V independent Voltage Source:



$$\begin{cases} \frac{V_2}{2} + 0.5V_0'' + \frac{V_2 - V_0''}{1} = 2 & \text{node ①} \\ \frac{V_0''}{4} + \frac{V_0'' - V_2}{1} = 0.5V_0'' & \text{node ②} \end{cases}$$

$$\Rightarrow \begin{cases} -0.5V_0'' + 1.5V_2 = 2 \\ 0.75V_0'' - V_2 = 0 \end{cases} \Rightarrow \begin{cases} V_0'' = 3.2V \\ V_2 = 2.4V \end{cases}$$

Second, we shut off the 2A independent Current Source:



$$\begin{cases} \frac{V_1 - 10}{2} + 0.5V_0' + \frac{V_1 - V_0'}{1} = 0 & \text{node ①} \\ \frac{V_0'}{4} + \frac{V_0' - V_1}{1} = 0.5V_0' & \text{node ②} \end{cases}$$

$$\Rightarrow \begin{cases} -0.5V_0' + 1.5V_1 = 5 \\ 0.75V_0' - V_1 = 0 \end{cases} \Rightarrow \begin{cases} V_0' = 8V \\ V_1 = 6V \end{cases}$$

So, by superposition Thm. $V_0 = V_0' + V_0'' = 8 + 3.2 = \boxed{11.2V}$

Exercise 2.2 (15%)

First, we transform the independent current source 3A parallel with 10Ω to an independent voltage source $V = 3 \times 10 = 30V$, also, transform the independent voltage source 15V series with 3Ω to an independent current source $I = \frac{15}{3} = 5A$: show in FIG1.

Next, we combine two parallel resistor 3Ω and 6Ω equivalent to $\frac{6 \times 3}{6+3} = 2Ω$, show in FIG2

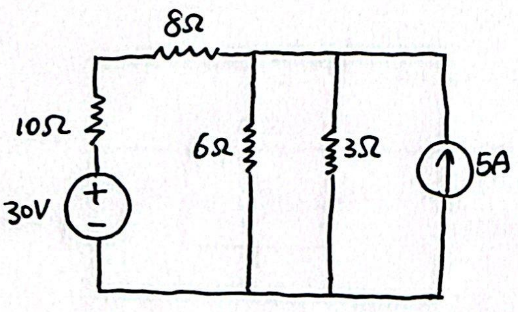


FIG1

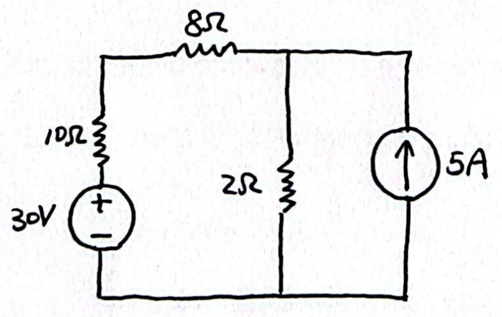


FIG2

Then, we transform the independent current source 5A parallel with 2Ω in FIG2 to an independent voltage source $V = 5\times2 = 10V$. Show in FIG3.

Combine two series resistors 2Ω and 10Ω, two voltage sources 10V and 30V in FIG3. Show in FIG4.

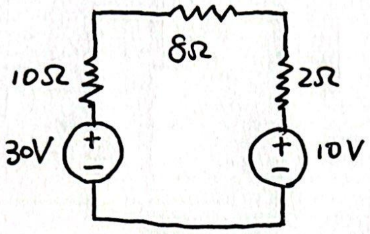


FIG3

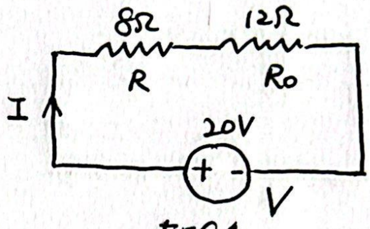


FIG4

Finally, the current absorbed by 8Ω resistor is $I = \frac{V}{R+R_0} = \frac{20}{8+12} = \boxed{1A}$

the power absorbed by 8Ω resistor is $P = VI = I^2R = \boxed{8w}$

Exercise 2.3 (25%)

(a) (20%) First, we find R_{Th}. Shut off all independent source: show FIG5

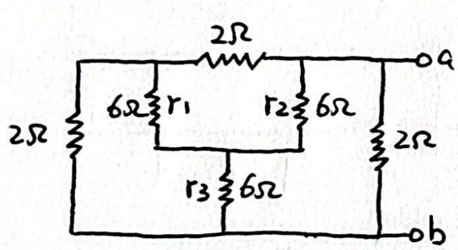


FIG5

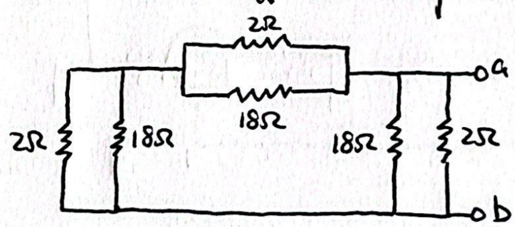


FIG6

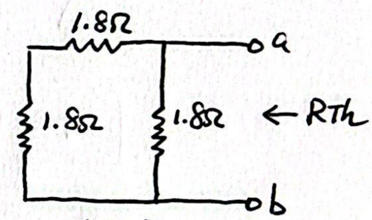


FIG7

using Y-Δ transformation, turn Y (r_1, r_2, r_3) into Δ (R_1, R_2, R_3):

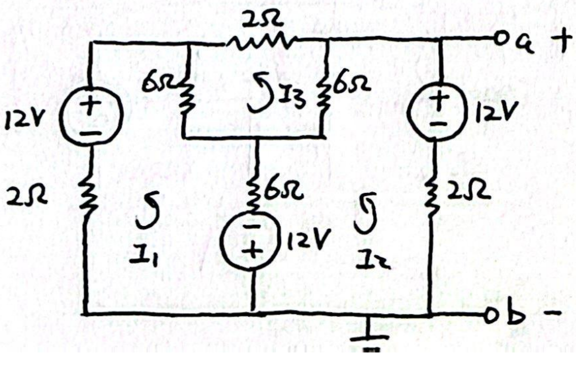
$R_1 = \frac{r_1r_2 + r_2r_3 + r_3r_1}{r_1} = 18\Omega$. Similarly, $R_2 = R_3 = 18\Omega$ (symmetry)

the circuit is equivalent to FIG6. Combine parallel resistor. FIG7

$2\Omega \parallel 18\Omega = \frac{2\times18}{2+18} = 1.8\Omega$

So. $R_{Th} = 1.8 \parallel (1.8+1.8) = 1.8 \parallel 3.6 = \frac{1.8\times3.6}{1.8+3.6} = 1.2\Omega$

Next, we find V_{Th}, let a-b become a open circuit, $V_{ab} = V_{Th}$.



mesh analysis by inspection:

$$\begin{bmatrix} 2+6+6 & -6 & -6 \\ -6 & 2+6+6 & -6 \\ -6 & -6 & 2+6+6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -24 \\ 24 \\ 0 \end{bmatrix}$$

$\Delta = 800$

$\Delta_1 = -960$

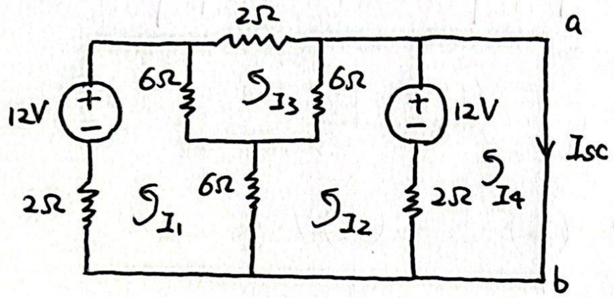
$\Delta_2 = 960$

$\Delta_3 = 0$

So, $I_1 = \frac{\Delta 1}{\Delta} = -1.2A$. $I_2 = \frac{\Delta 2}{\Delta} = 1.2A$. $I_3 = \frac{\Delta 3}{\Delta} = 0A$.

$\frac{V_{ab}-12}{2} = -I_2$, so, $V_{Th} = V_{ab} = 12 - 2I_2 = 9.6V$. Thevenin equivalent circuit is shown in FIG8.

Then, we find I_N, let a-b become an short circuit. ($R_N = R_{Th} = 1.2\Omega$).



mesh analysis by inspection:

$$\begin{bmatrix} 2+6+6 & -6 & -6 & 0 \\ -6 & 2+6+6 & -6 & -2 \\ -6 & -6 & 2+6+6 & 0 \\ 0 & -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -24 \\ 24 \\ 0 \\ -12 \end{bmatrix}$$

$\Delta = 960$
$\Delta_1 = -3456$
$\Delta_2 = -1920$
$\Delta_3 = -2304$
$\Delta_4 = -7680$

So, $I_1 = \frac{\Delta 1}{\Delta} = -3.6A$. $I_2 = \frac{\Delta 2}{\Delta} = -2A$, $I_3 = \frac{\Delta 3}{\Delta} = -2.4A$. $I_4 = \frac{\Delta 4}{\Delta} = -8A$.

$I_N = I_{sc} = -I_4 = 8A$. Norton equivalent circuit is shown in FIG9.

Also, R_N and I_N can be derived from R_{Th} and V_{Th} (Source transformation):

$R_N = R_{Th}$. $I_N = \frac{V_{Th}}{R_{Th}}$.

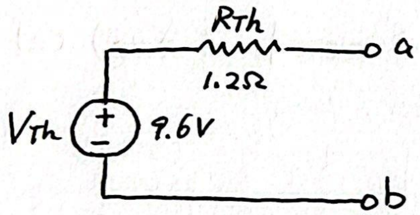


FIG8

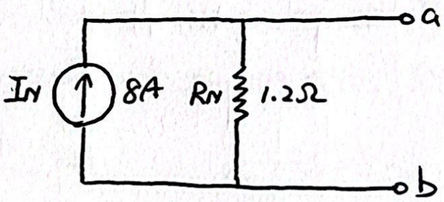
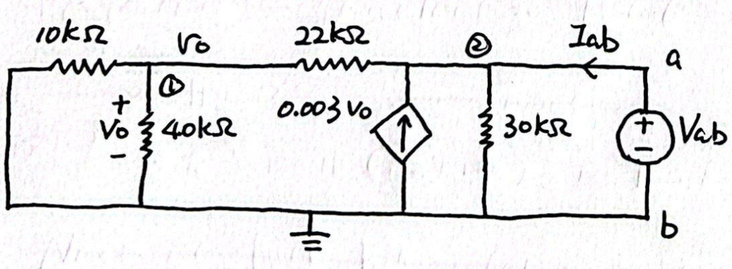


FIG9.

(b) (5%) $P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{9.6^2}{4\times 1.2} = \boxed{19.2W}$ when $R = R_{Th} = 1.2\Omega$

Exercise 2.4 (20%).

We first use Thevenin's Thm to find the equivalent circuit: for R_{Th}:



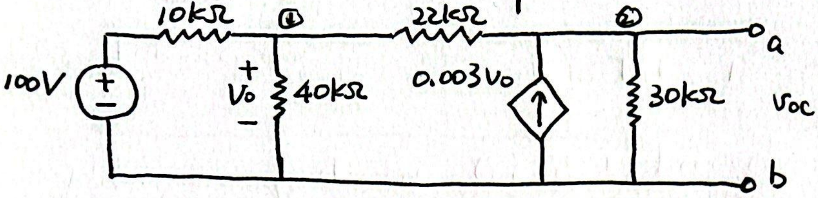
we connet an independent voltage source V_{ab} between a and b, the current is I_{ab}, then $R_{Th} = \frac{V_{ab}}{I_{ab}}$

$$\begin{cases} \frac{V_o}{40} + \frac{V_o}{10} + \frac{V_o - V_{ab}}{22} = 0, \text{ node ①} & \Rightarrow V_o = \frac{4}{15} V_{ab} \\ \frac{V_{ab}}{30} + \frac{V_{ab} - V_o}{22} = 3V_o + 1000 I_{ab} \quad \text{node ②} \end{cases}$$

$$\Rightarrow 1000\times I_{ab} = \left(\frac{1}{30} + \frac{1-\frac{4}{15}}{22} - 3\times\frac{4}{15}\right) V_{ab} = -\frac{11}{15} V_{ab}.$$

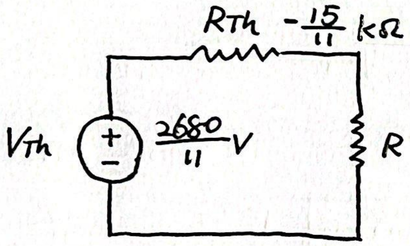
$$\Rightarrow R_{Th} = \frac{V_{ab}}{I_{ab}} = -\frac{15}{11}\times 10^3\,\Omega = -\frac{15}{11}\,k\Omega \doteq -1.364\,k\Omega$$

Next, we make ab an open circuit, $V_{Th} = V_{oc}$:



$$\begin{cases} \frac{V_o}{40} + \frac{V_o - 100}{10} + \frac{V_o - V_{oc}}{22} = 0, & \text{node ①} \\ \frac{V_{oc} - V_o}{22} + \frac{V_{oc}}{30} = 3V_o, & \text{node ②} \end{cases} \Rightarrow \begin{cases} \frac{15}{88} V_o - \frac{1}{22} V_{oc} = 10 \\ -\frac{67}{22} V_o + \frac{13}{165} V_{oc} = 0 \end{cases} \Rightarrow \begin{cases} V_o = -\frac{208}{33} V \\ V_{oc} = -\frac{2680}{11} V \end{cases}$$

So, the equivalent circuit is:



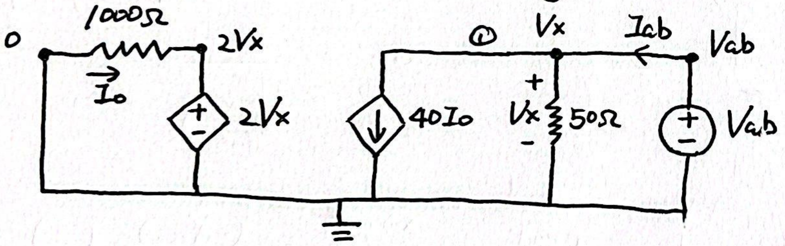
$$P = I^2 R = \frac{V_{Th}^2 R}{(R + R_{Th})^2}$$

when $R = -R_{Th} = \frac{15}{11} k\Omega \doteq 1.364 k\Omega$,

$P \to \infty$. so. P_{max} $\boxed{\text{doesn't not exist.}}$

Exercise 2.5 (25%)

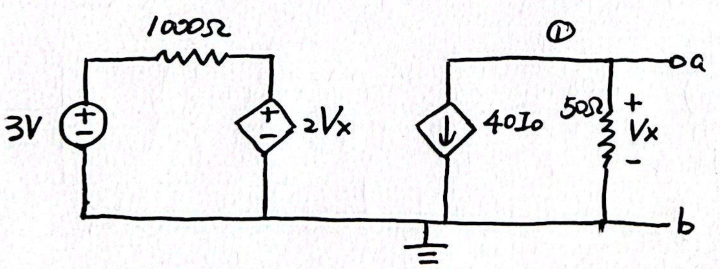
(a) (20%) First, we derive R_{Th} by shutting off all independent source and add voltage V_{ab} source between a-b. $R_{Th} = \frac{V_{ab}}{I_{ab}}$



$$\begin{cases} V_{ab} = V_x \\ \frac{0 - 2V_x}{1000} = I_o \\ \frac{V_x}{50} + 40 I_o = I_{ab}. & \text{node ①} \end{cases}$$

$\Rightarrow \frac{V_{ab}}{50} - \frac{40 V_{ab}}{500} = I_{ab}$. $-\frac{3}{50} V_{ab} = I_{ab}$. $R_{ab} = -\frac{50}{3}\Omega = R_{Th}$

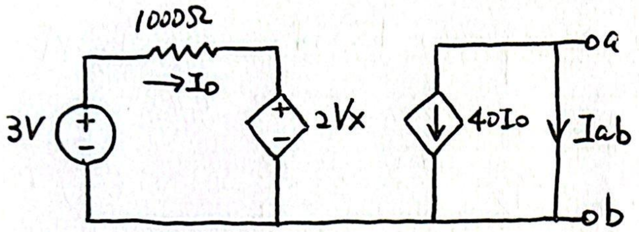
Next, we derive V_{Th} by make a-b open circuit. $V_{Th} = V_{oc} = V_x$



$$\begin{cases} I_o = \frac{3 - 2V_x}{1000} \\ \frac{V_x}{50} = -40 I_o, & \text{node ①} \end{cases}$$

$\Rightarrow V_{Th} = V_x = V_{oc} = 2V$

So, the Thevenin equivalent circuit is shown in FIG 10, then, we derive I_N by make a-b a short circuit:



In this case, $V_x = V_{ab} = 0$, $I_N = I_{ab}$.

$$\begin{cases} I_o = \frac{3}{1000} \\ I_{ab} = -40I_o = -40\times\frac{3}{1000} = -0.12A = I_N. \end{cases}$$

So, the Norton equivalent circuit is shown in FIG 11.

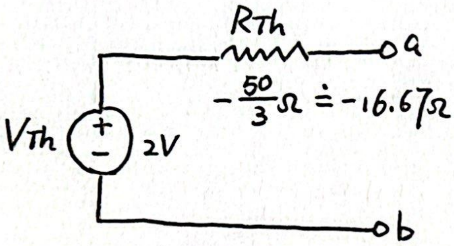


FIG 10

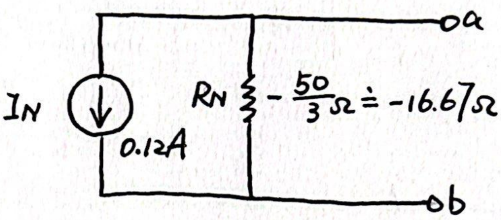


FIG 11

(b) (5%). First, calculate the I using Norton equivalent circuit.

$$I = \frac{V}{R} = \frac{2}{10-\frac{50}{3}} = -0.3A.$$

Next, calculate the power $P = VI = I^2R = 0.3^2 \times \cancel{\frac{50}{3}}^{\,10} = \boxed{0.9\,w}$