ECE2150J Introduction to Circuits
Chapter 14. Frequency Response

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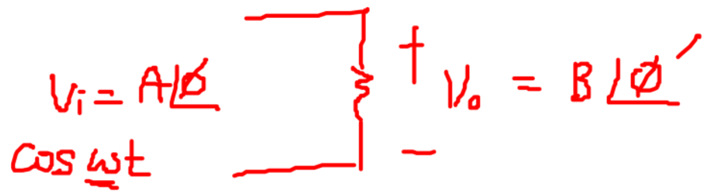
14.1 Introduction



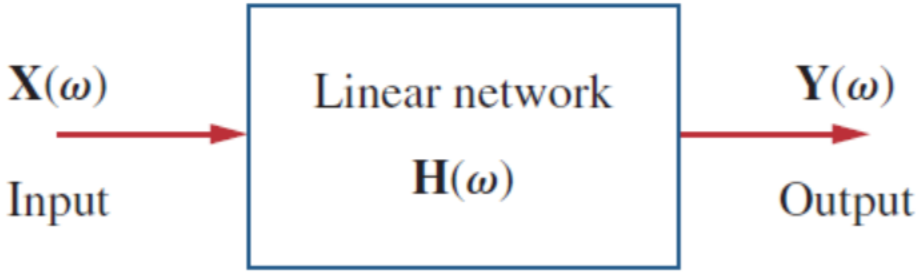
Frequency response: Amplitude and phase angle of the sinusoidal source remain constant and **vary the frequency** – f(ω).

Previously, we have learned how to find voltages and currents at **a constant frequency**.

The sinusoidal steady-state frequency responses of circuits are of significance in many applications, especially **in communications and control systems**.

14.2 Frequency Response

The frequency response of a circuit is the plot of the circuit's transfer function **H**(ω) versus ω.



$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

The transfer function **H**(ω) of a circuit is the frequency-dependent ratio of a phasor output **Y**(ω) (an element voltage or current) to a phasor input **X**(ω) (source voltage or current).

There are four possible transfer functions:

$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)}$$

We can obtain the frequency response of the circuit by **plotting the magnitude and phase of the transfer function as the frequency varies.**

The transfer function **H**(ω) can be expressed in terms of its numerator **N**(ω) polynomial and denominator **D**(ω) polynomial as

$$\mathbf{H}(\omega) = \frac{\boxed{\mathbf{N}(\omega)}}{\boxed{\mathbf{D}(\omega)}} \quad \begin{matrix} \rightarrow \text{zero} \\ \rightarrow \text{pole} \end{matrix}$$

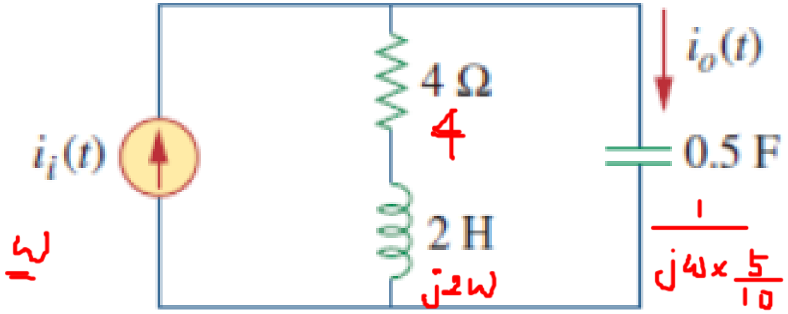
(i) zeros: values that result in a zero value of the function

(ii) Poles: values for which the function is infinite

Being a complex quantity, **H**(ω) has a **magnitude** and a **phase**: H∠Ø. The plot of H versus ω is called the **magnitude frequency response**. The plot of Ø versus ω is called the **phase frequency response**.

H(jω)=H(ω)∠ϕ(ω)
H(ω): magnitude frequency response
ϕ(ω): phase frequency response

Example 1. Find gain i_o/i_i, poles and zeros.



$= \frac{1}{j0.5\omega}$

$$\frac{i_o}{i_i} = \frac{(4+j2\omega) \times j0.5\omega}{4+j2\omega+\frac{1}{j0.5\omega} \quad \times j0.5\omega}$$

$$= \frac{j2\omega+(j\omega)^2}{j2\omega+(j\omega)^2+1} \Rightarrow \frac{s^2+2s}{s^2+2s+1}$$

$j\omega = s$

Example 14.2

For the circuit in Fig. 14.6, calculate the gain $\mathbf{I}_o(\omega)/\mathbf{I}_i(\omega)$ and its poles and zeros.

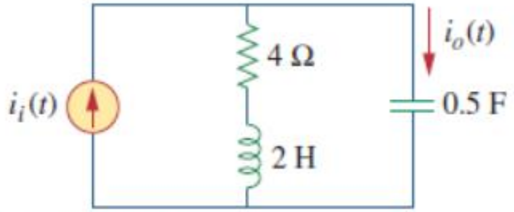


Figure 14.6
For Example 14.2.

Solution:

By current division,

$$\mathbf{I}_o(\omega) = \frac{4 + j2\omega}{4 + j2\omega + 1/j0.5\omega}\mathbf{I}_i(\omega)$$

or

$$\frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)} = \frac{j0.5\omega(4 + j2\omega)}{1 + j2\omega + (j\omega)^2} = \frac{s(s + 2)}{s^2 + 2s + 1}, \qquad s = j\omega$$

The zeros are at

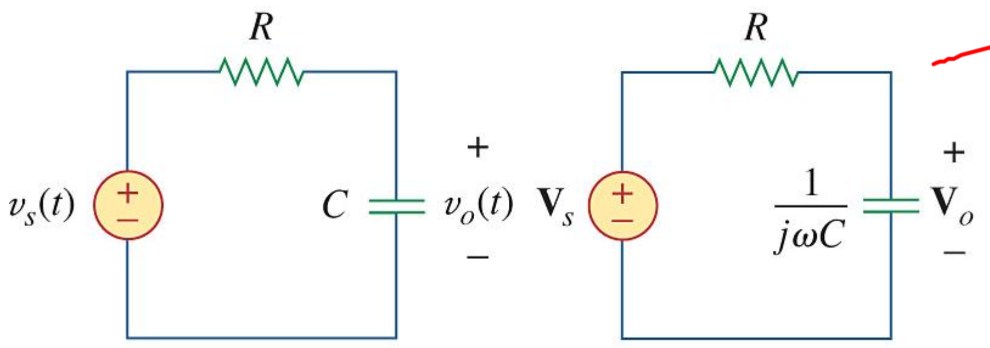
$$s(s + 2) = 0 \qquad \Rightarrow \qquad z_1 = 0, z_2 = -2$$

The poles are at

$$s^2 + 2s + 1 = (s + 1)^2 = 0$$

Thus, there is a repeated pole (or double pole) at $p = -1$.

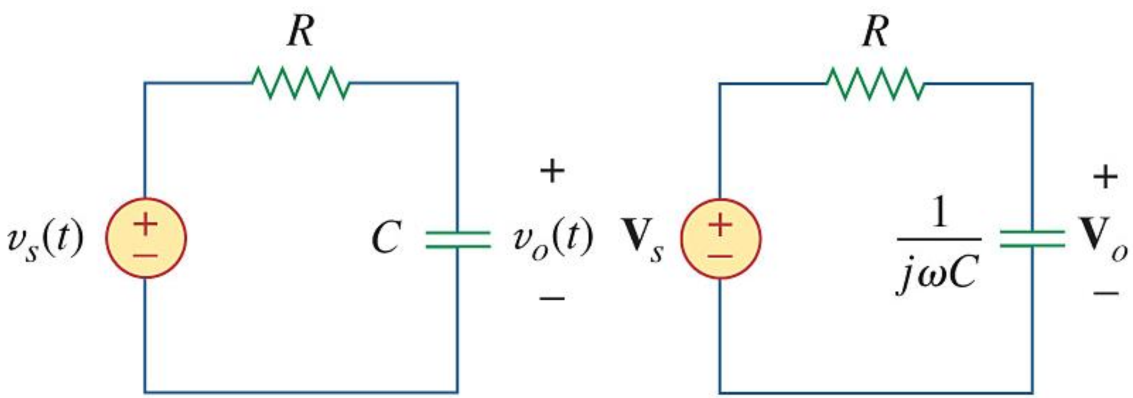
Example 14.1 For the *RC* circuit in Fig. 14.2(a), obtain the frequency response $\tilde{V}_o(\omega)/\tilde{V}_s(\omega)$. Let $v_s = V_m \cos \omega t$.



$$V_o = \frac{\frac{1}{j\omega C} \times j\omega C}{R + \frac{1}{j\omega C} \times j\omega C} \times V_s$$

$$= \frac{1}{j\omega RC + 1} V_s \qquad \frac{1}{RC} = \omega_o$$

$$= \frac{1}{\frac{j\omega}{\omega_o} + 1} \times V_s$$



Solution :

$$H(jW) = \frac{\tilde{V}_o(jW)}{\tilde{V}_s(jW)} = \frac{1/(jWC)}{R+1/(jWC)}$$

$$= \frac{1}{1+jWRC}$$

magnitude $$H = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}$$

phase $$\phi = -\tan^{-1}(\omega RC) = -\tan^{-1}(\omega/\omega_0)$$

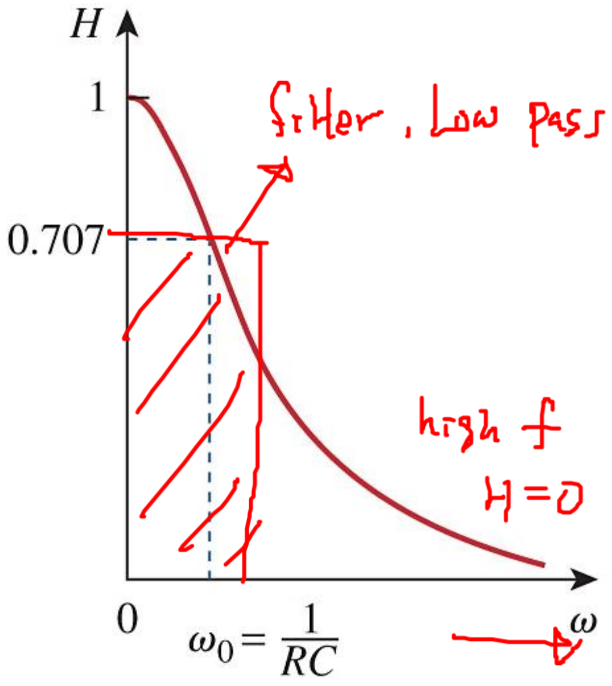
where $\omega_o = \dfrac{1}{RC}$

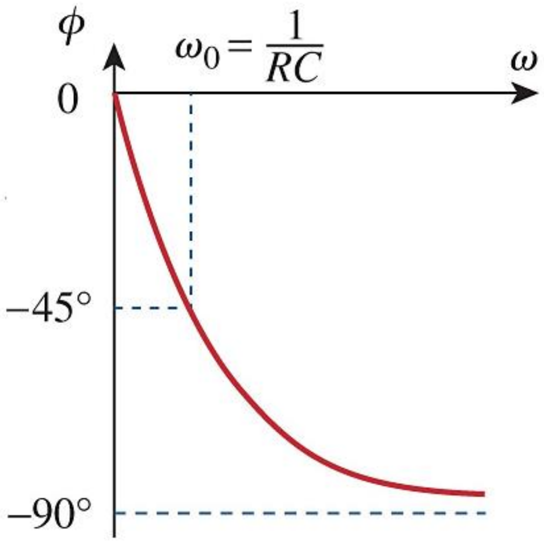
Finds some critical points to plot the responses

magnitude $\boxed{\dfrac{1}{\sqrt{1+(\omega/\omega_0)^2}}}$

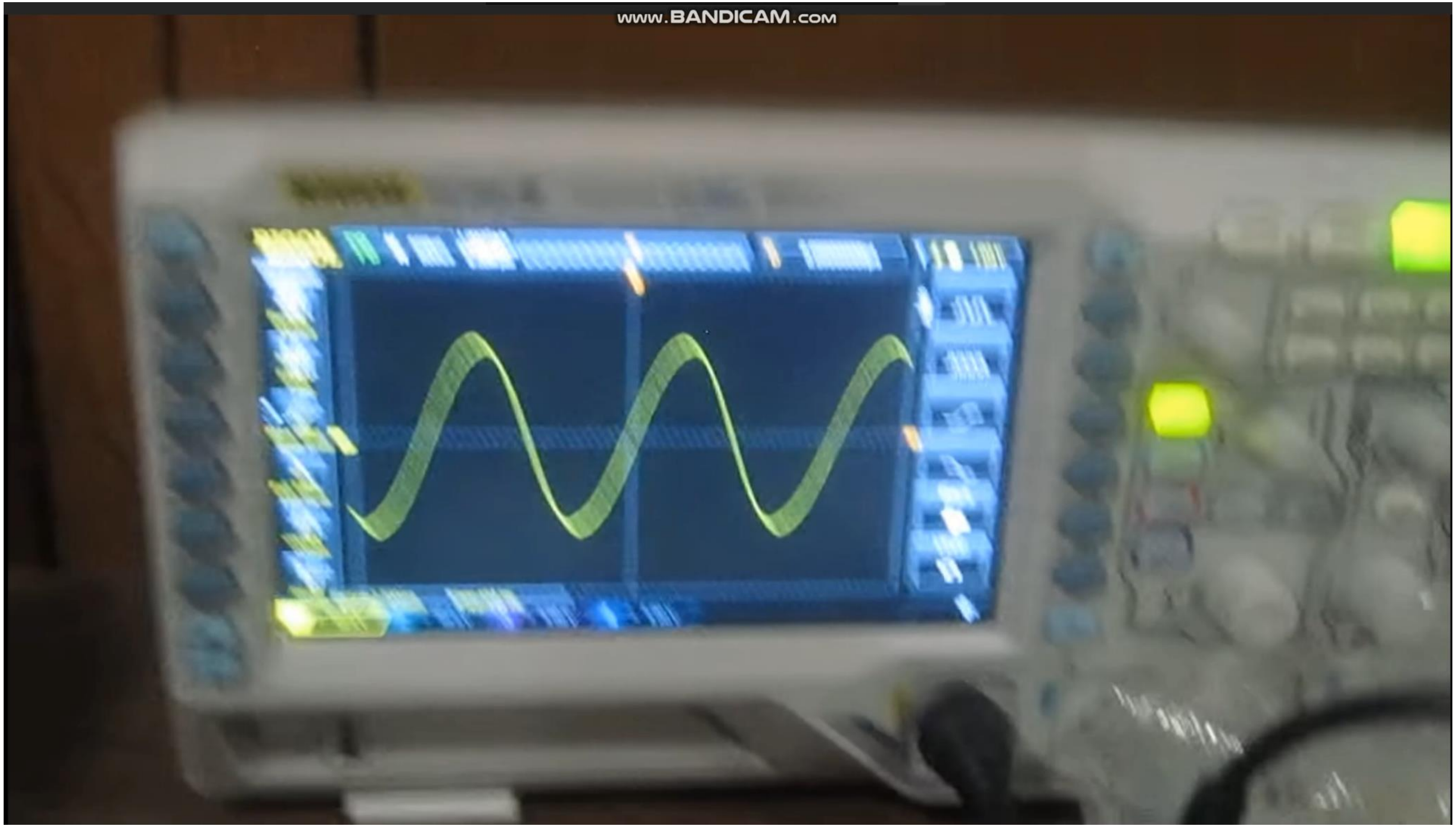
phase $\boxed{-\tan^{-1}(\omega/\omega_0)}$

ω/ω_0	H	ϕ
0	1	0
1	0.71	$-45°$
∞	0	$-90°$

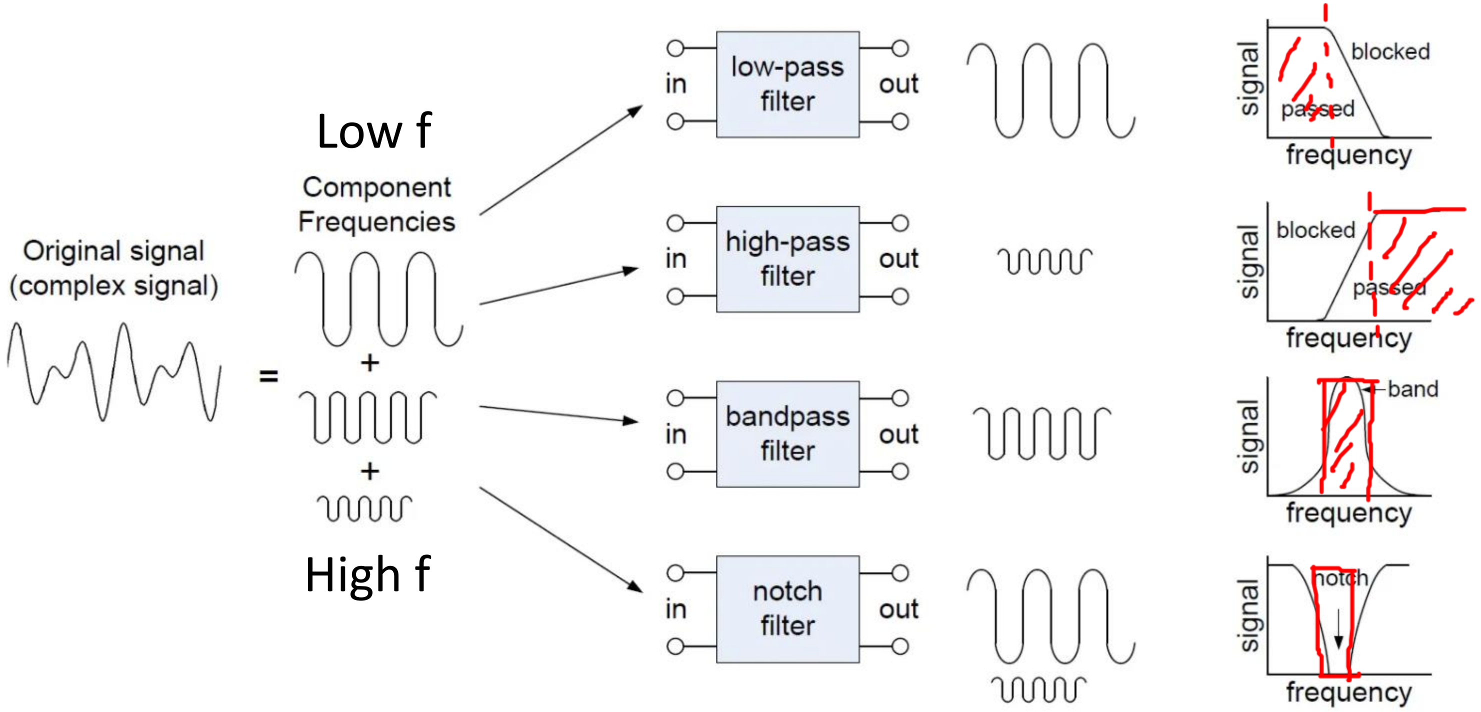




When frequency increases magnitude and phase decrease


www.BANDICAM.com

*Filters



https://www.allaboutcircuits.com/technical-articles/an-introduction-to-filters/

Terms "low" and "high" do not refer to any absolute values of frequency, but rather they are **relative values** with respect to the cutoff frequency.

14.3 The Decibel Scale

It is not always easy to get a quick plot of the magnitude and phase of the transfer function. A more **systematic way of obtaining the frequency response is to use Bode plots**.

To use Bode plots, we should take care of two important issues:

(i) Logarithms

(ii) Decibels

bel: the ratio of two levels of **power** or power gain G

$$G = \text{Number of bels} = \log_{10} \frac{P_2}{P_1}$$

10^{G_bel} = linear scale $= 10$

E.g., $10^2 = 100$

0 bel ⇔ 1 time

1 bel ⇔ 10 times

2 bels ⇔ 100 times

…

$\frac{1}{10}$ th

Decibel provides us with a unit of less magnitude. It is 1/10th of a bel and is given by

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1}$$

$10^{G\_dB/10}$ = linear scale $10^{\frac{1}{10}}$

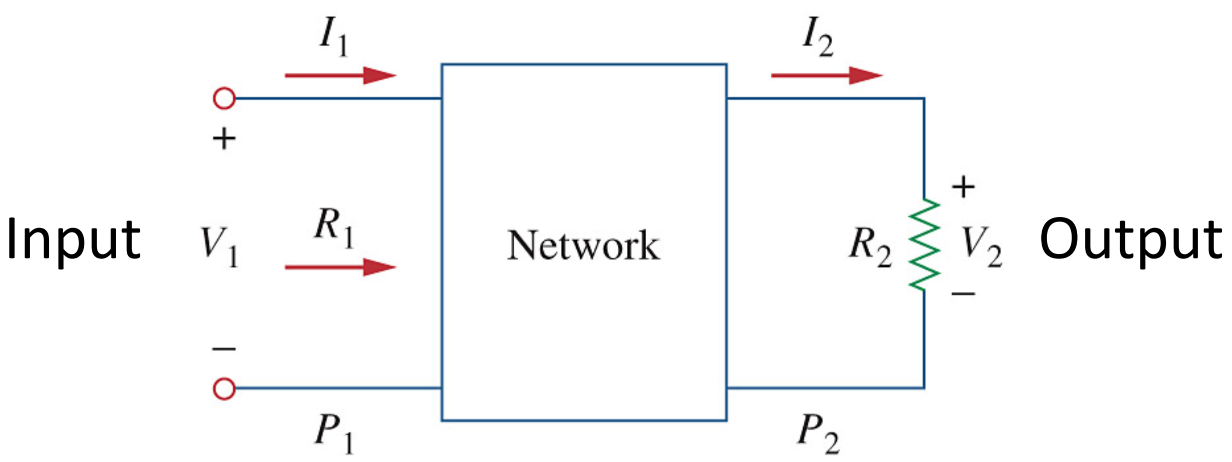
E.g., $10^1 = 10$

0 dB ⇔ 1 time

10 dB ⇔ 10 times

20 dB ⇔ 100 times

…



$$G_{\mathrm{dB}} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{V_2^2/R_2}{V_1^2/R_1}$$

$$= 20\log_{10}\frac{V_2}{V_1} - 10\log_{10}\frac{R_2}{R_1}$$

When $R_1 = R_2$

$$G_{dB} = 10\log_{10}\left(\frac{V_2}{V_1}\right)^2 = \boxed{20}\log_{10}\frac{V_2}{V_1}$$

Similarly, for $R_1 = R_2$,

$$G_{dB} = \boxed{20}\log_{10}\frac{I_2}{I_1}$$

Transfer functions in dB scale:

The dB value is a logarithmic measurement of the *ratio* of one variable to another of **the same type.** Therefore, it applies in expressing **the transfer function H with the same type** (dimensionless quantities).

For these transfer functions

$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$

$$G_{dB} = 10\log_{10}\left(\frac{V_2}{V_1}\right)^2 = 20\log_{10}\frac{V_2}{V_1}$$

$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$G_{dB} = 20\log_{10}\frac{I_2}{I_1}$$

Not for those transfer functions

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)}$$

Comparisons between power and amplitude in dB scale:

For power (amplitude^2)
$\mathbf{10^{G\_dB/10}}$ = linear scale
E.g.,
0 dB ⇔ P_2/P_1=1 time
10 dB ⇔ P_2/P_1=10 times
20 dB ⇔ P_2/P_1=100 times
…

For amplitude (voltage and current)
$\mathbf{10^{G\_dB/20}}$ = linear scale
E.g.,
0 dB ⇔ V_2/V_1=1 time
10 dB ⇔ $V_2/V_1=(10)^{0.5}$ time= 3.16 times
20 dB ⇔ V_2/V_1=10 times
…

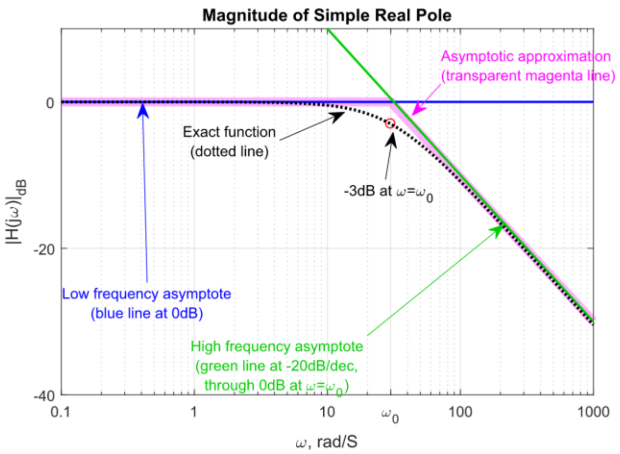
When we are talking about dB, we do not need to specify power or amplitude.

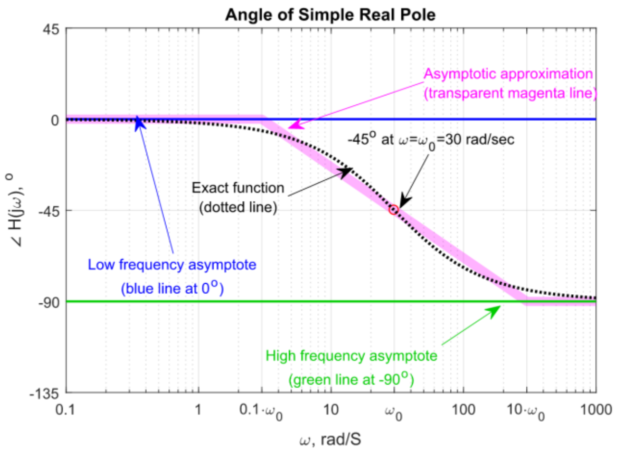
e.g.,
(i) 10 dB: 10 times in power; 3.16 times in amplitude
(ii) -3 dB: 0.5 times in power; approx. 0.7 times in amplitude

14.4 Bode Plots

A more systematic way of locating the important features of the magnitude and phase plots of the transfer function is required.

Bode plots are **semi-log plots** of the magnitude (in decibels) and phase (in degrees) of a transfer function *vs* frequency. ***Bode plots* have become the industry standard**.



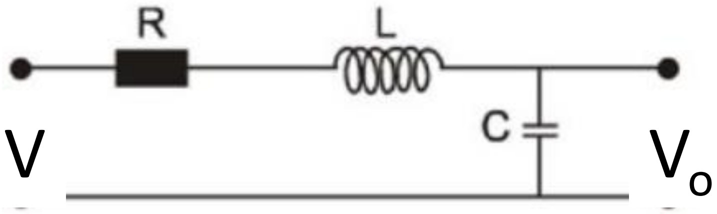


Useful website: https://lpsa.swarthmore.edu/Bode/Bode.html

Bode plot of the transfer function $\mathbf{H} = H\angle\emptyset$

- The **magnitude:** $H_{\mathrm{dB}} = 20\log_{10} H$
- **The phase:** $\angle \mathrm{H}(\omega)$

e.g. RLC system

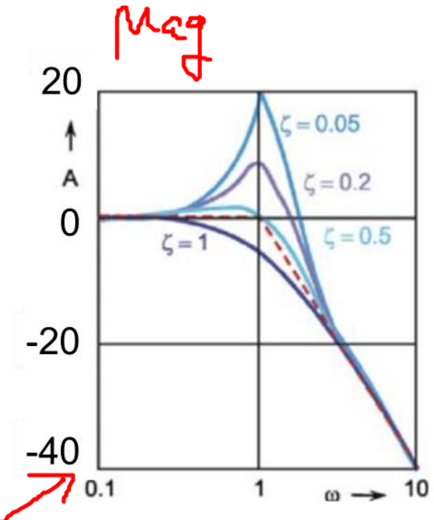


$$V_o = \frac{{}^{1}\!/_{j\omega C}\, V}{R + j\omega L + {}^{1}\!/_{j\omega C}}$$

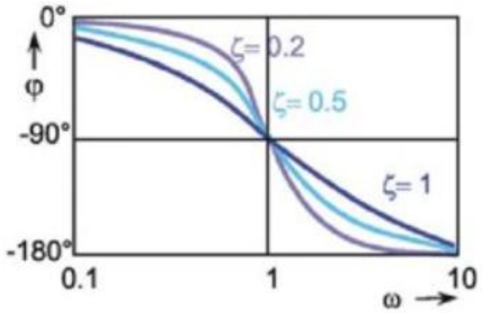
×jωC.

×jωC.

$$H(\omega) = \frac{{}^{1}\!/_{j\omega C}}{R + j\omega L + {}^{1}\!/_{j\omega C}} = \frac{1}{(j\omega)^2 + j\omega\, {}^{R}\!/_{L} + 1/LC}$$



$\emptyset$



Transfer function $\mathbf{H} = H\angle\emptyset$

- The **magnitude:** $H_{\mathrm{dB}} = 20\log_{10} H$
- **The phase:** $\angle \mathrm{H}(\omega)$

Standard form of H(ω)

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

H(ω) has **zeros** (z_1, z_2, …) and **poles** (p_1, p_2, …) and real and imaginary numbers

General form of H(ω) is

Complex conjugate

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)\boxed{[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]}\cdots}{(j\omega)\;(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

And, we can have **seven types of different factors**

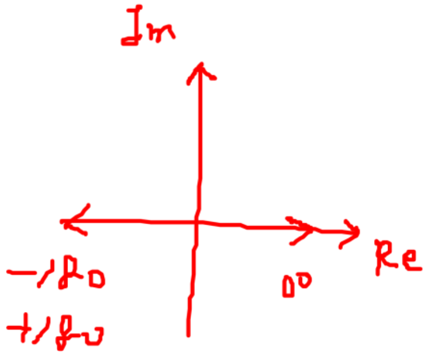
1. A gain K
2. A pole $(j\omega)^{-1}$ or zero $(j\omega)$ at the origin
3. A simple pole $1/(1+j\omega/p_1)$ or zero $(1+j\omega/z_1)$
4. A quadratic pole $1/[1+j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2$ or zero $[1+j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$

In constructing a Bode plot, **we plot each factor separately** and then **add them graphically**, i.e. factors can be considered one at a time and then combined additively.

(i) Gain K

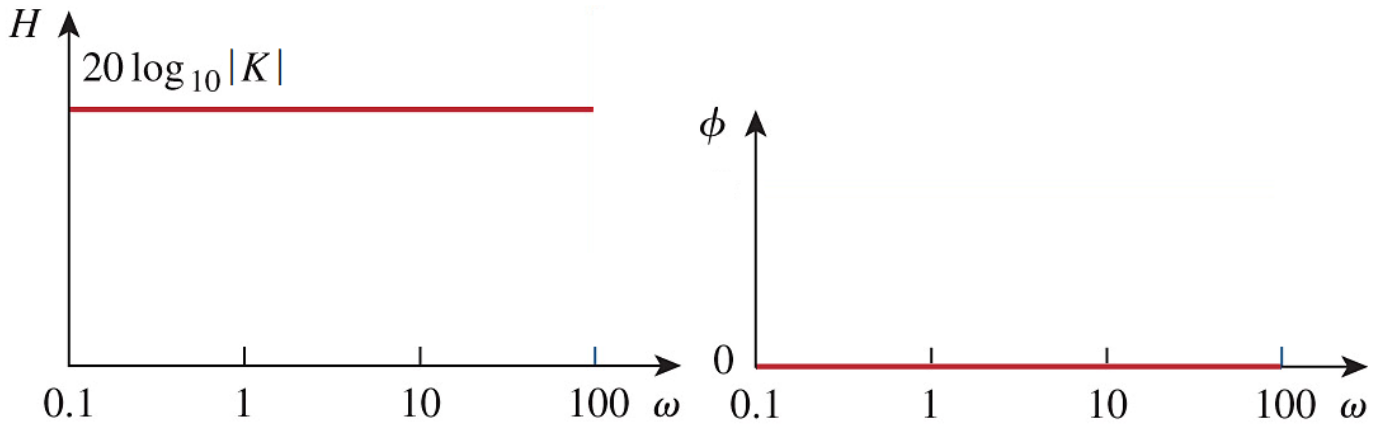
100 = 200.

10 = 20



$$H_{dB} = 20\log_{10}|K| \Leftarrow H_{\mathrm{dB}} = 20\log_{10} H$$

$$\phi = \begin{cases} 0^\circ, & K > 0 \\ \pm 180^\circ & K < 0 \end{cases}$$



(ii) Pole/zero at the origin

A pole $(j\omega)^{-1}$ or zero $(j\omega)$ at the origin

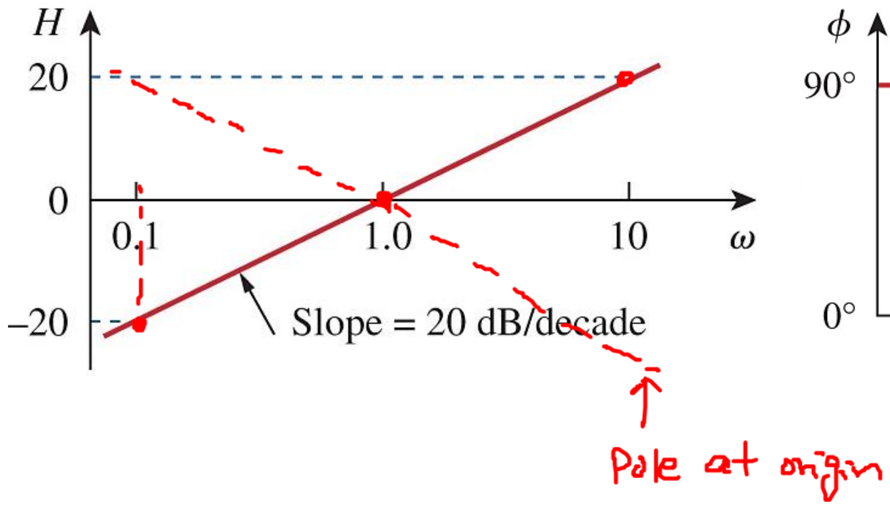
$\sqrt{w^2}$ 1

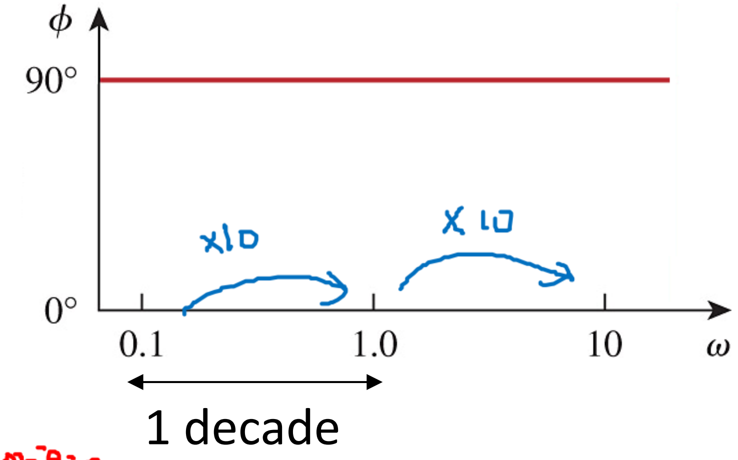
zero.

$$H_{dB} = 20\log_{10}|j\omega| = 20\log_{10}\omega \quad \Rightarrow$$

$$\phi = 90^\circ \quad \Leftarrow \quad \ln e^{j\phi}$$

i) ω = 0.1, H_{dB} = -20
ii) ω = 1, H_{dB} = 0
iii) ω = 10, H_{dB} = 20





The slope of the magnitude plot is **20 dB/decade**, where decade means a group or series of ten.

(ii) Pole/zero at the origin – other cases

- **$(j\omega)^{-1}$**

$-20\log_{10}|j\omega|$

$H_{dB}=20\log_{10}|(j\omega)^{-1}| = -20\log_{10}\omega$

Angle of $(j\omega)^{-1} = -90°$

i) $\omega = 0.1$, $H_{dB} = 20$
ii) $\omega = 1$, $H_{dB} = 0$
iii) $\omega = 10$, $H_{dB} = -20$

$N \times 20\log_{10}|j\omega|$ *

→ Slope is diff. shape same.

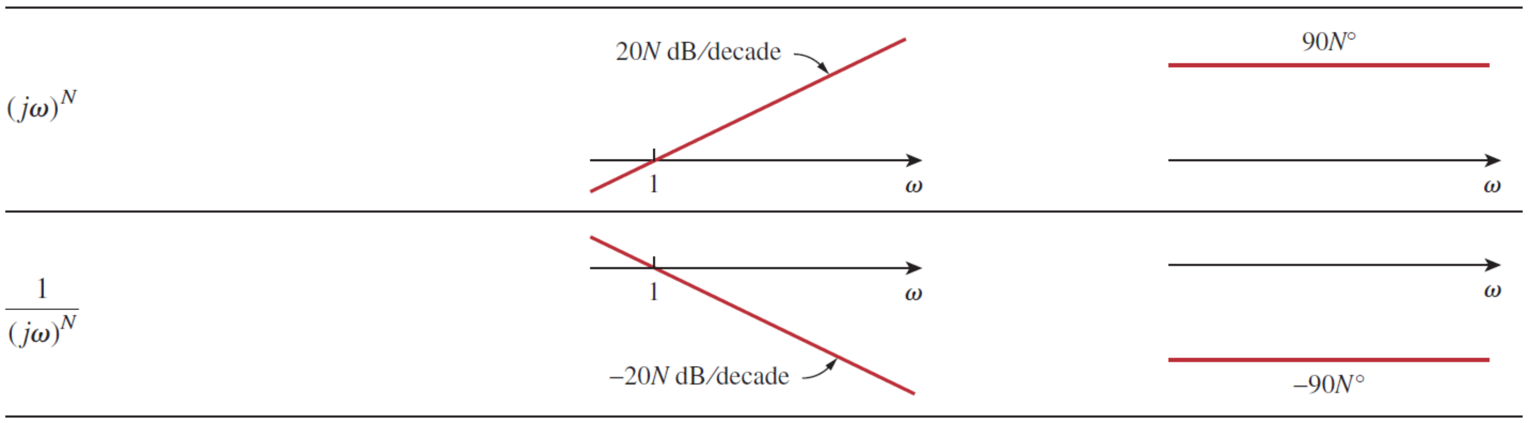
- **$(j\omega)^{n}$ or $(j\omega)^{-n}$ cases**

$-20N\log_{10}\omega$

$H_{dB}=20\log_{10}|(j\omega)^{N}| = 20N\log_{10}\omega$

Angle of $(j\omega)^{N}$ = j+j+... = $90N°$

i) $\omega = 0.1$, $H_{dB} = 20\cdot N$
ii) $\omega = 1$, $H_{dB} = 0$
iii) $\omega = 10$, $H_{dB} = -20\cdot N$



(iii) Simple pole/zero

A simple zero $(1+j\omega/z_1)$

$0.1z_1 \quad z_1 \quad 10z_1$

$\sqrt{1^2+\left(\frac{\omega}{z_1}\right)^2}$ $\quad\quad \sqrt{1^2+1^2}$

$$H_{dB} = 20\log_{10}\left|1+j\omega/z_1\right|$$

$a+jb \rightarrow \tan^{-1}\frac{b}{a}$

i) $\omega = 0.1z_1$, $H_{dB} = 20\log(1) =$ **0**

ii) $\omega = z_1$, $H_{dB} = 20\log(\sqrt{2}) = 3$ dB ≈ **0**

iii) $\omega = 10z_1$, $H_{dB} = 20\log10 =$ **20**

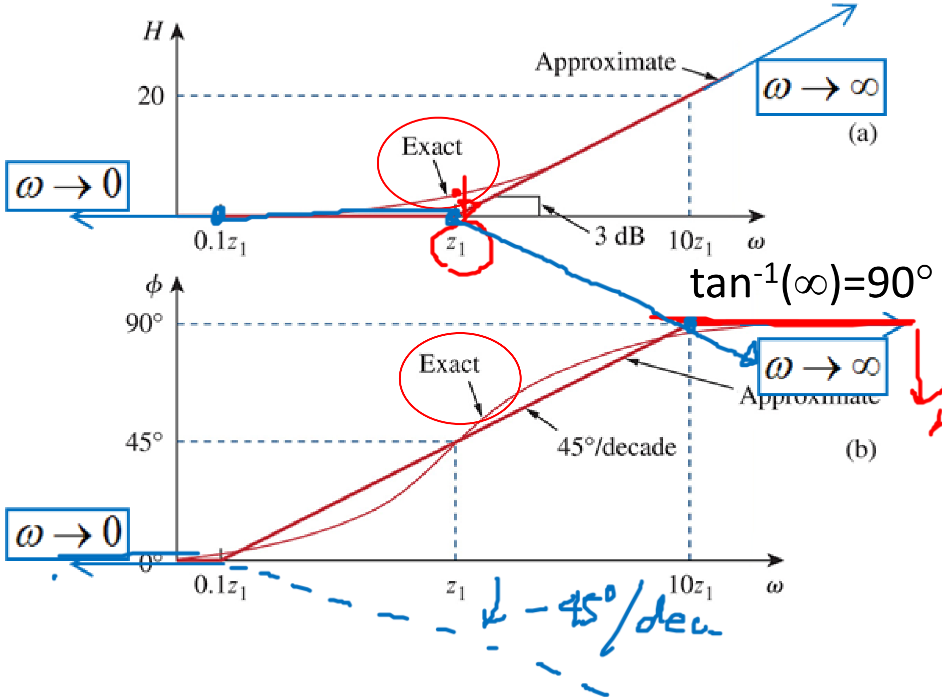
$$\phi = \tan^{-1}(\omega/z_1)$$

i) $\omega = 0.1z_1$, $\varnothing = 5.71 \approx$ **0**

ii) $\omega = z_1$, $\varnothing =$ **45°**

iii) $\omega = 10z_1$, $\varnothing = 84° \approx$ **90°**

$\tan^{-1} 0.1$



max 90°

Straight-line approximation

z_1: corner frequency or break frequency

(iii) Simple pole/zero – other cases

pole $1/(1+j\omega/p_1)$

$-20\log_{10}\sqrt{2}$

$$H_{dB} = 20\log_{10}\left|{}^{1}\!/\!{}_{(1+{}^{j\omega}/_{p_1})}\right| \Rightarrow$$

i) $\omega = 0.1p_1$, $H_{dB} = -20\log(1) = \mathbf{0}$

ii) $\omega = p_1$, $H_{dB} = 20\log(1/\sqrt{2}) = -3$ dB $\approx \mathbf{0}$

iii) $\omega = 10p_1$, $H_{dB} = -20\log 10 = \mathbf{-20}$ → -20dB/dec.

$-20 \times N$.

$\left(H\frac{j\omega}{p_1}\right)^{-1}$

Slope -20

$$\phi = -\tan^{-1}{}^{\omega}/_{p_1} \Rightarrow$$

i) $\omega = 0.1p_1$, $\emptyset = -5.71° \approx \mathbf{0°}$

ii) $\omega = p_1$, $\emptyset = \mathbf{-45°}$

iii) $\omega = 10p_1$, $\emptyset = -84° \approx \mathbf{-90°}$

In general, for $(1+j\omega/z_1)^N$, where N is an integer → **Multiply N to H_{dB} and Ø**

$\left(1+\frac{j\omega}{z}\right)^N$

$\frac{1}{(1+j\omega/p)^N}$

(iv) Quadratic pole/zero

1+something

Pole: $1/[1+j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2$

$-\left(\frac{w}{w_n}\right)^2$

w^4

$w_n.$

$$H_{dB} = -20\log_{10}\left|1+2\zeta_2(j\omega/\omega_n)+(j\omega/\omega_n)^2\right|$$

$$-20\log_{10}\left|\left[1-\frac{\omega^2}{\omega_n^2}\right]+j2\zeta_2\frac{\omega}{\omega_n}\right| = -20\log_{10}\sqrt{\left[1-\frac{\omega^2}{\omega_n^2}\right]^2+4\zeta_2{}^2\left(\frac{\omega}{\omega_n}\right)^2}$$

$\zeta < 0.707$

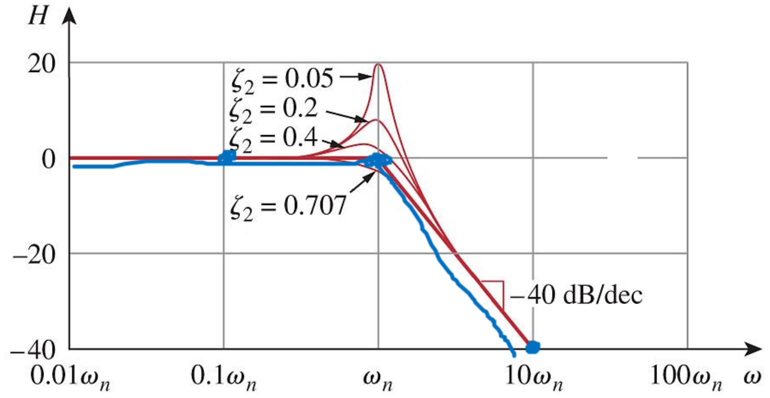
$\sqrt{4\zeta}$

i) $\omega = 0.1\omega_n$, $H_{dB} = -20\log_{10}1$ **= 0**

ii) $\omega = \omega_n$, H_{dB} = -20log($\sqrt{4}$) = -6 dB ≈ **0**

iii) $\omega = 10\omega_n \rightarrow \left[1-\frac{\omega^2}{\omega_n^2}\right] > j2\zeta_2\frac{\omega}{\omega_n} \rightarrow$

$$H_{dB} = -20\log_{10}\sqrt{\left[1-\frac{\omega^2}{\omega_n^2}\right]^2} = \mathbf{-40}\ \text{dB.}$$



(iv) Quadratic pole/zero

Pole: $1/[1+j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)]^2$

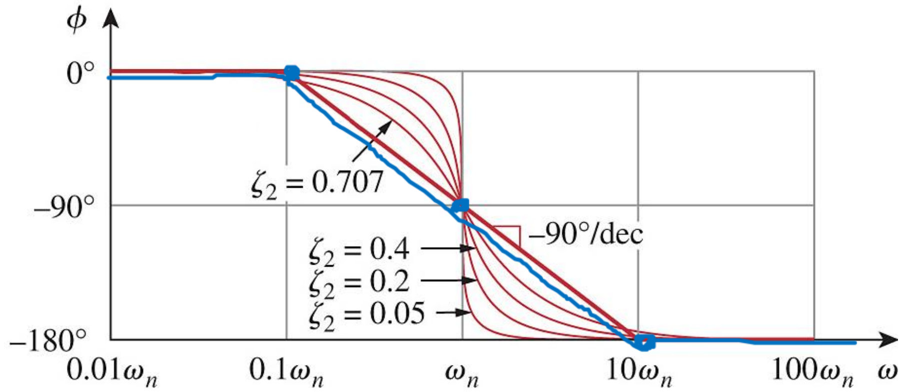
$-\frac{\omega^2}{\omega_n^2}$

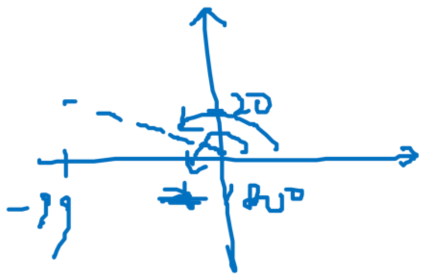
$$\phi = -\tan^{-1}\left(\frac{2\zeta_2\omega/\omega_n}{1-\omega^2/\omega_n^2}\right)$$

i) $\omega = 0.1\omega_n$, $\varnothing = -11.31° =$ **0 °**

ii) $\omega = \omega_n$, $\varnothing =$ **-90°**

iii) $\omega = 10\omega_n$ → $1/(-99+j20)$ → $\varnothing =$ **-180°**





(iv) Quadratic pole/zero – other cases

Zero $[1+j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$

i) $\omega = 0.1\omega_n$, $H_{dB} = 20\log_{10}1$ **= 0**
ii) $\omega = \omega_n$, $H_{dB} = 20\log(\sqrt{4}) = 6$ dB ≈ **0**
iii) $\omega = 10\omega_n \rightarrow \left[1-\frac{\omega^2}{\omega_n^2}\right] > j2\zeta_2\frac{\omega}{\omega_n} \rightarrow$

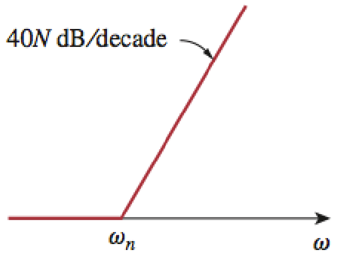
$$H_{dB} = 20\log_{10}\sqrt{\left[\left[1-\frac{\omega^2}{\omega_n^2}\right]\right]^2} = \mathbf{+40}$$

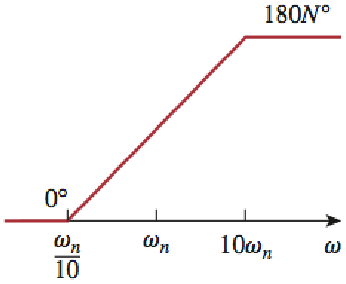
i) $\omega = 0.1\omega_n$, $\emptyset$ = **+**11.31°= **0 °**
ii) $\omega = \omega_n$, $\emptyset$ = **90°**
iii) $\omega = 10\omega_n \rightarrow$ **-99+j20** $\rightarrow \emptyset$ = **180°**

In general, for quadratic pole and zero to the power of N

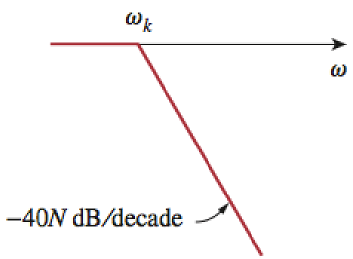
→ Multiply N to the output H_{dB} and Ø

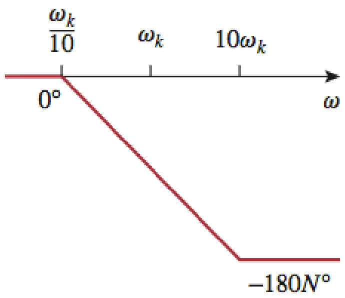
$$\left[1 + \frac{2j\omega\zeta}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right]^N$$





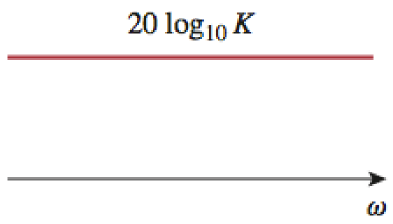
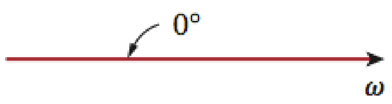
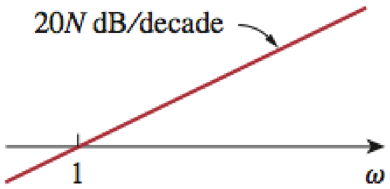
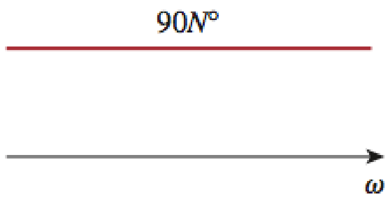
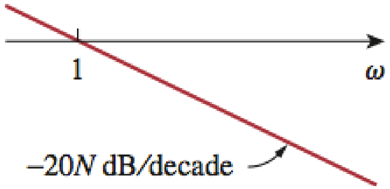
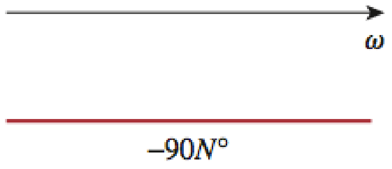
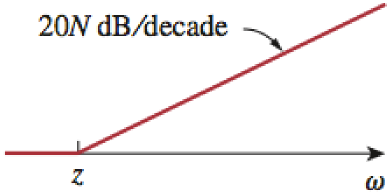
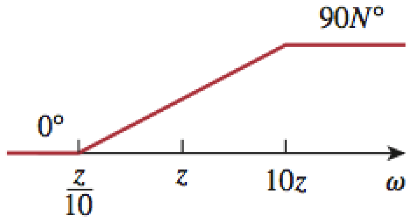
$$\frac{1}{[1 + 2j\omega\zeta/\omega_k + (j\omega/\omega_k)^2]^N}$$





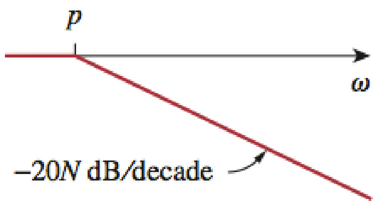
Summary

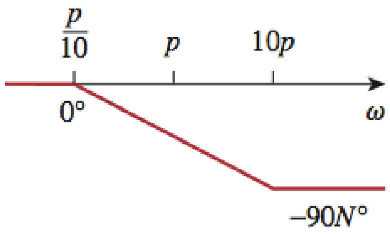
Zeros: upward turn
Poles: downward turn

Factor	Magnitude	Phase
K		
$(j\omega)^N$		
$\dfrac{1}{(j\omega)^N}$		
$\left(1+\dfrac{j\omega}{z}\right)^N$		

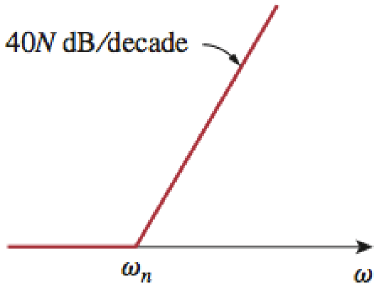
Zeros: upward turn
Poles: downward turn

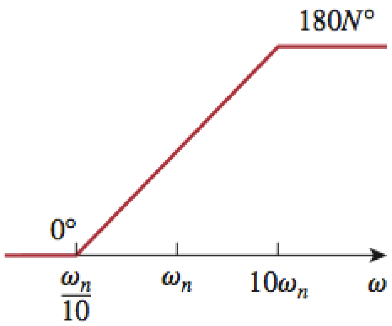
$$\frac{1}{(1 + j\omega/p)^N}$$



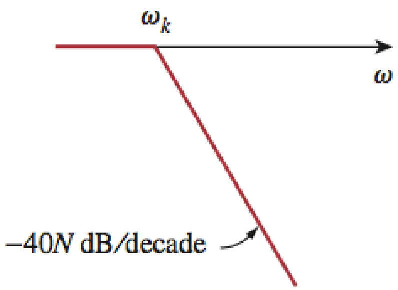


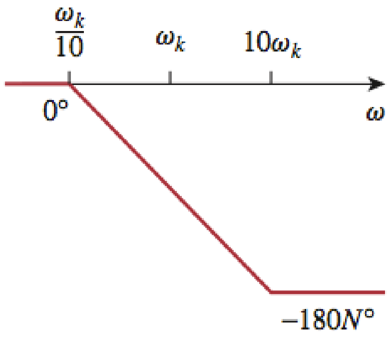
$$\left[1 + \frac{2j\omega\zeta}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right]^N$$





$$\frac{1}{[1 + 2j\omega\zeta/\omega_k + (j\omega/\omega_k)^2]^N}$$





Example 14.4 Obtain the Bode plots for

$$H(j\omega) = \frac{j\omega + 10}{j\omega(j\omega + 5)^2} \rightarrow \frac{10\left(1 + \frac{j\omega}{10}\right)}{25\, j\omega\left(1 + \frac{j\omega}{5}\right)^2} = 0.4\,\frac{\left(1 + \frac{j\omega}{10}\right)}{j\omega\left(1 + \frac{j\omega}{5}\right)^2}$$

Change the form

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

Magnitude

$$H(j\omega) = \frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$$

$$H_{dB} = 20\log_{10}\left|\frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}\right|$$

Four terms:

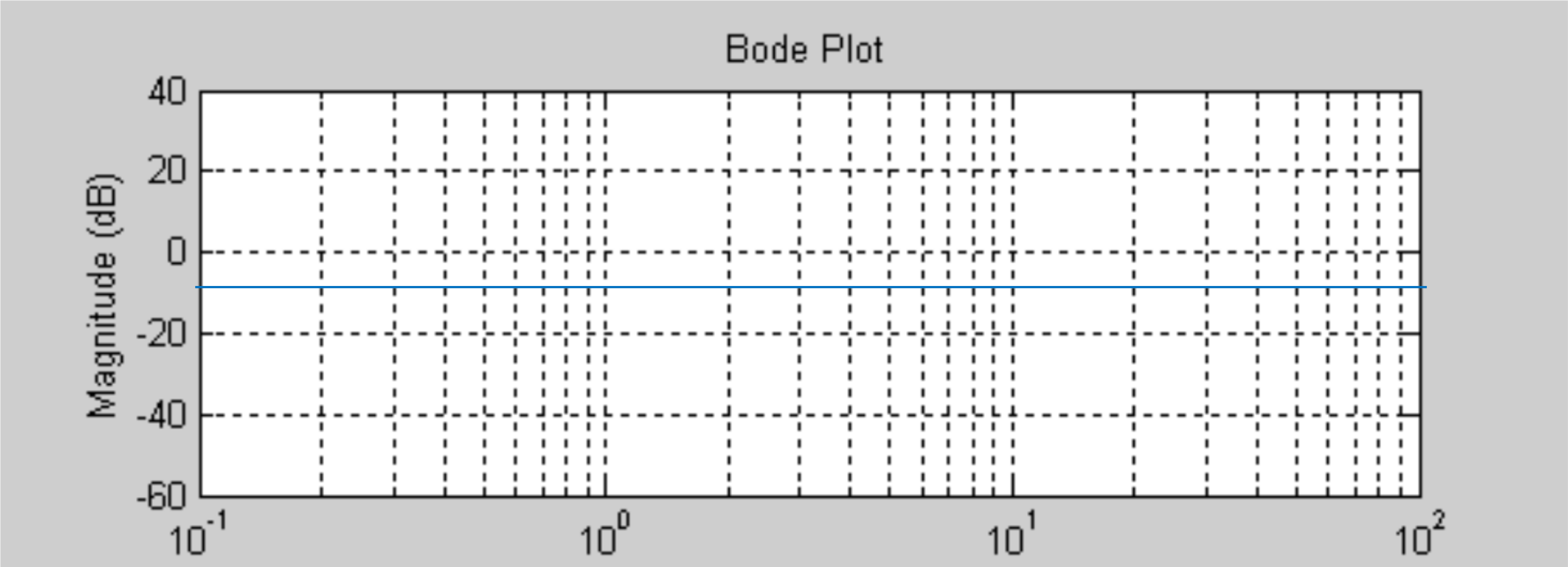
(i) 0.4

(ii) $(1+j\omega/10)$

(iii) $(j\omega)^{-1}$

(iv) $(1+j\omega/5)^{-2}$

(i) 0.4 → 20log0.4 = -8 dB



$$H(j\omega) = \frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$$

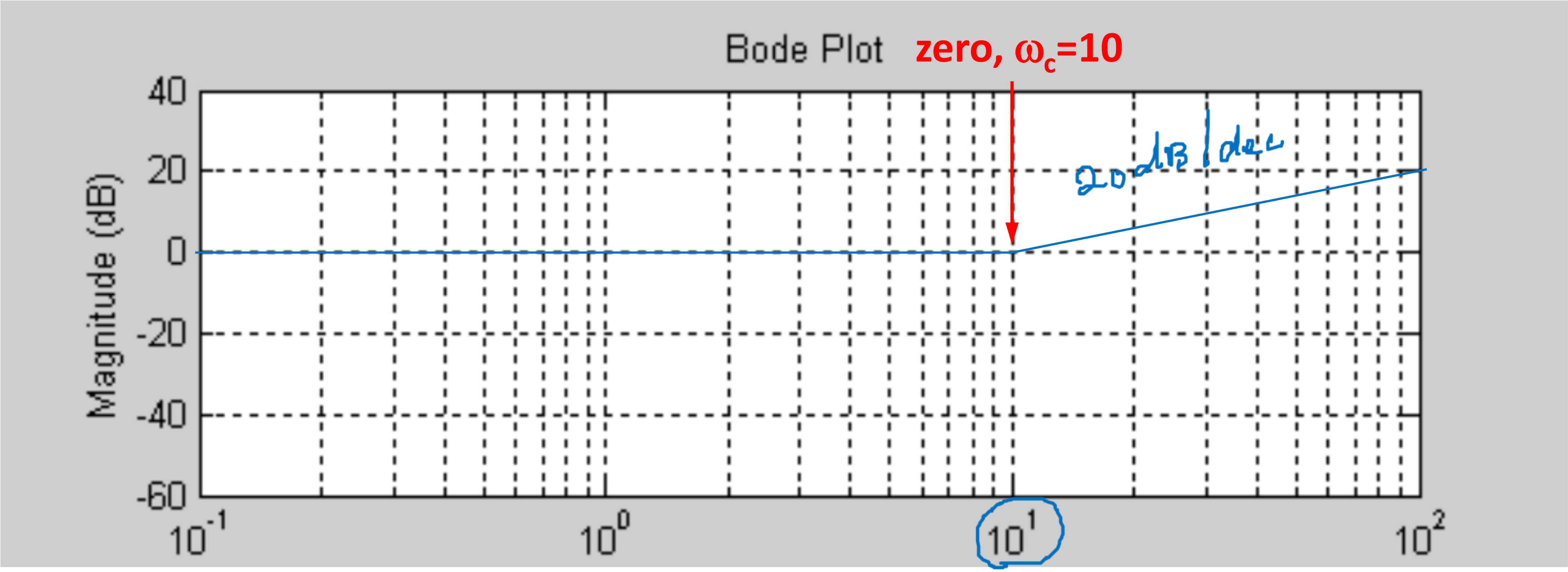
$$H_{dB} = 20\log_{10}\left|\frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}\right|$$

(i) $\omega = 1$, $H_{dB} = 0$ dB

(ii) $\omega = 10$, $H_{dB} = 3$ dB $= 0$

(iii) $\omega = 100$, $H_{dB} = 20$ dB

(ii) $(1+j\omega/10) \rightarrow$ 20log $|1+j\omega/10|$



$$H(j\omega) = \frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$$

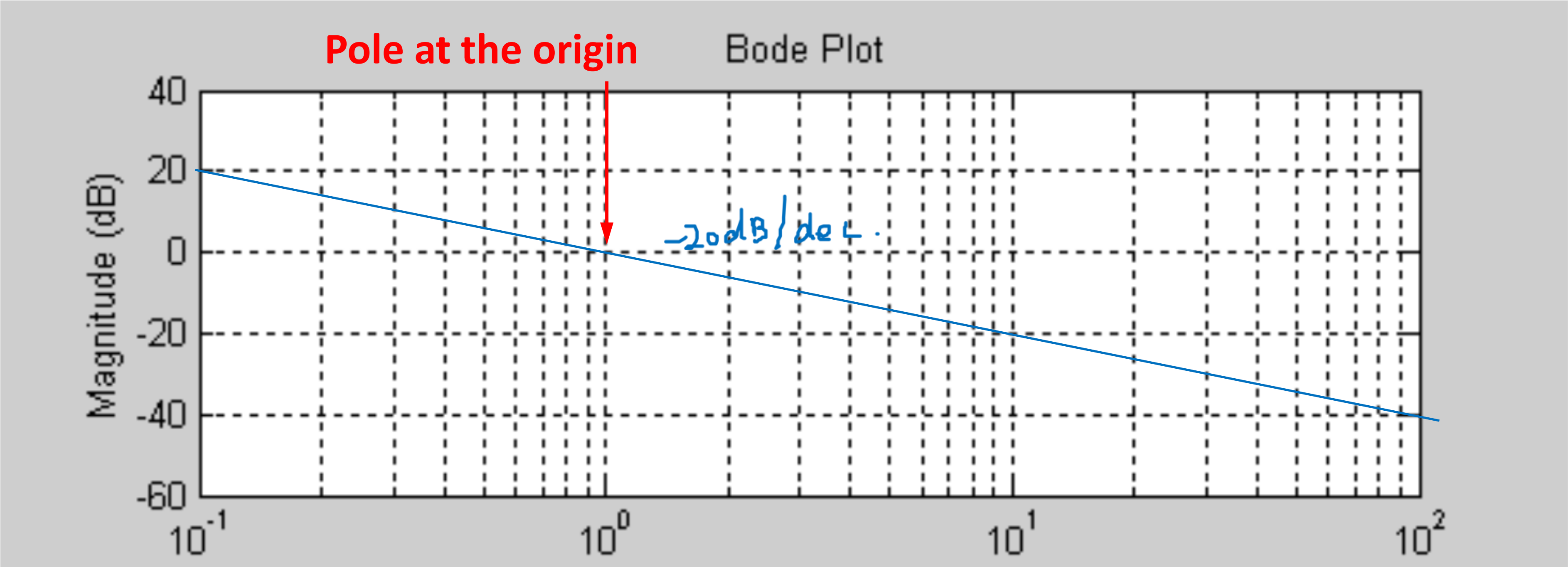
$$H_{dB} = 20\log_{10}\left|\frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}\right|$$

(i) $\omega = \mathbf{0.1}$, $H_{dB} = 20$ dB

(ii) $\omega = 10$, $H_{dB} = -20$ dB

(iii) $\omega = 100$, $H_{dB} = -40$ dB

(iii) $(j\omega)^{-1} \rightarrow -20\log|j\omega|$



$$H(j\omega) = \frac{0.4(1+j\omega/10)}{j\omega(1+j\omega/5)^2}$$

$$H_{dB} = 20\log_{10}\left|\frac{0.4(1+j\omega/10)}{j\omega(1+j\omega/5)^2}\right|$$

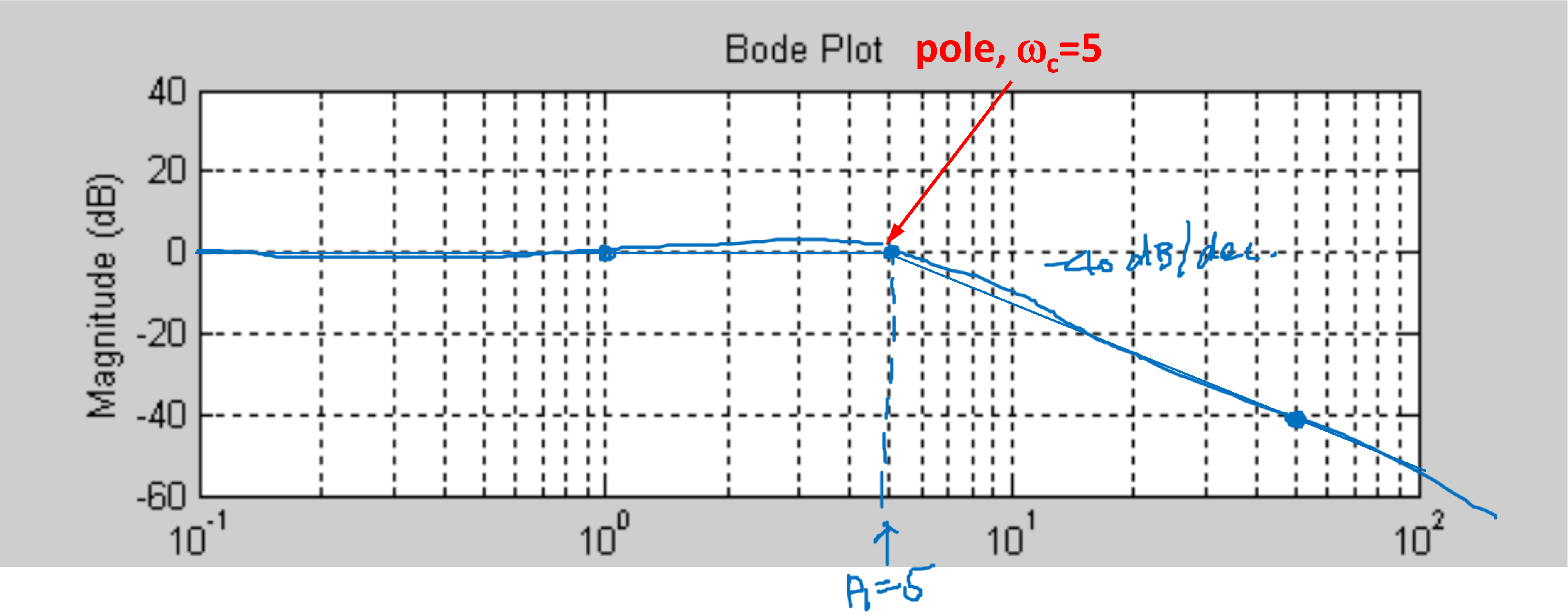
$-2\times 20\log(1+j\omega/5)$

(iv) $(1+j\omega/5)^{-2} \rightarrow -40\log|1+j\omega/5|$

(i) $\omega = 0.5$, $H_{dB} = 0$ dB

(ii) $\omega = 5$, $H_{dB} = -6$ dB $= 0$ dB

(iii) $\omega = 50$, $H_{dB} = -40$ dB

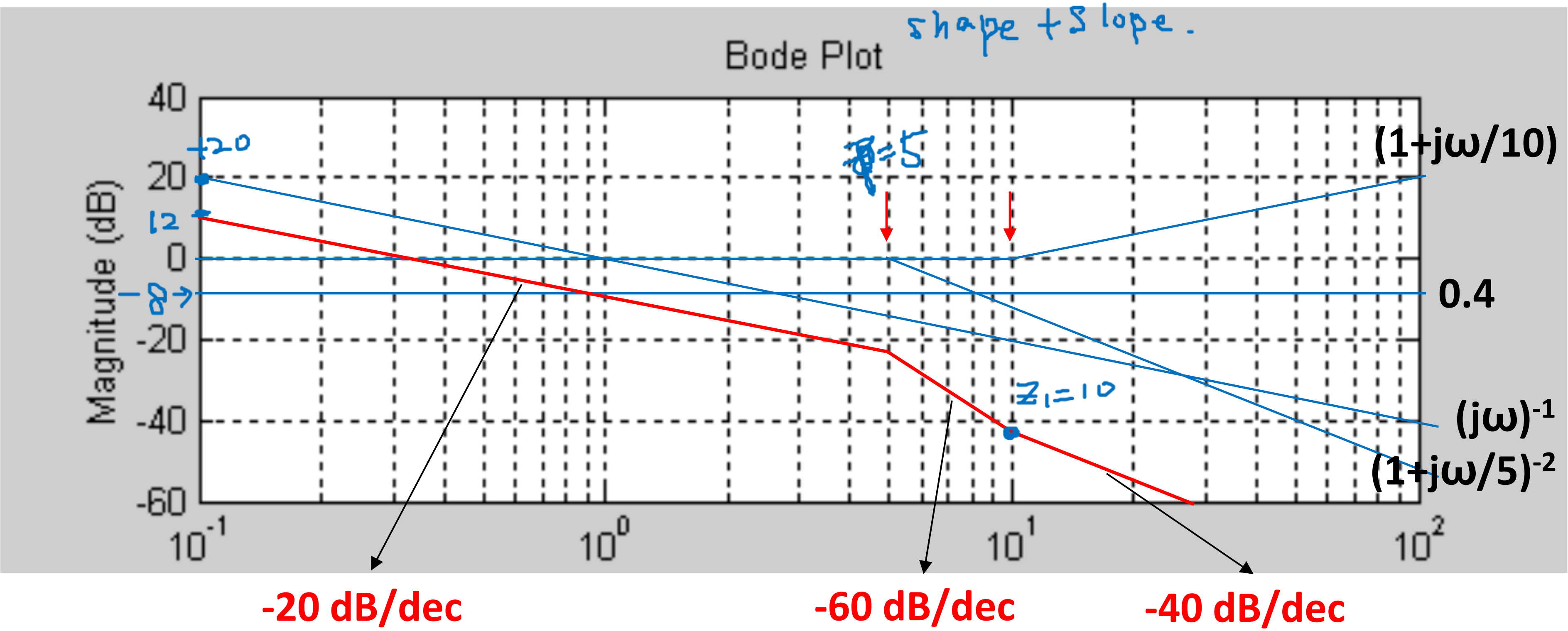


$$H(j\omega)=\frac{0.4(1+j\omega/10)}{j\omega(1+j\omega/5)^2}$$

$$H_{dB}=20\log_{10}\left|\frac{0.4(1+j\omega/10)}{j\omega(1+j\omega/5)^2}\right|$$

Draw all of the four terms

$$=20\log_{10}0.4+20\log_{10}|1+j\omega/10|\ -20\log_{10}|j\omega|-40\log_{10}|1+j\omega/5|$$



Phase

$$H(j\omega) = \frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$$

Four terms:

(i) 0.4 → 0°(ii) (1+jω/10)

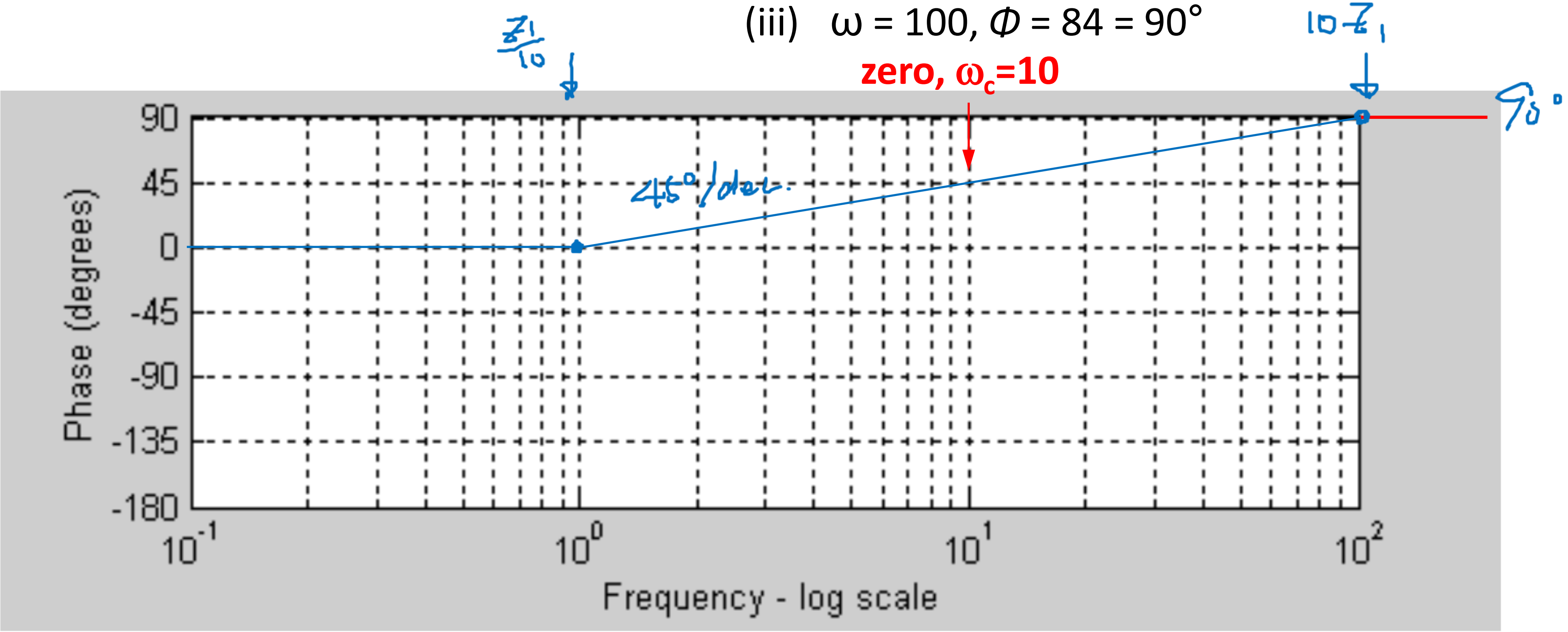
(iii) $(j\omega)^{-1}$ (iv) $(1+j\omega/5)^{-2}$

(ii) (1+jω/10) → $\angle\tan^{-1}(\omega/_{10})$

(i) $\omega = 1$, $\Phi = 5.7 = 0°$

(ii) $\omega = 10$, $\Phi = 45°$

(iii) $\omega = 100$, $\Phi = 84 = 90°$



Phase

$$H(j\omega)=\frac{0.4(1+j\omega/10)}{j\omega(1+j\omega/5)^2}$$

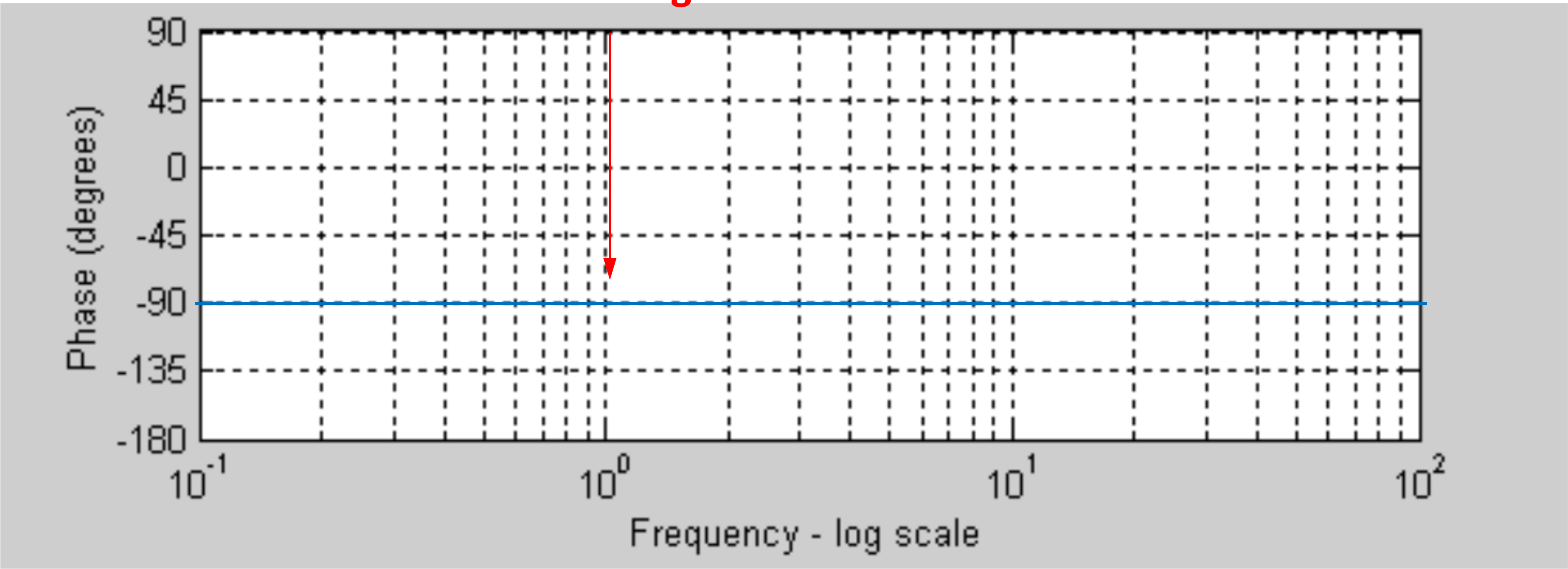
Four terms:

(i) 0.4 → 0°(ii) $(1+j\omega/10)$

(iii) $(j\omega)^{-1}$ (iv) $(1+j\omega/5)^{-2}$

(iii) $(j\omega)^{-1}$ → -90° ⇒ $-j\omega$ ⇒ $-j$

Pole at the origin



Phase

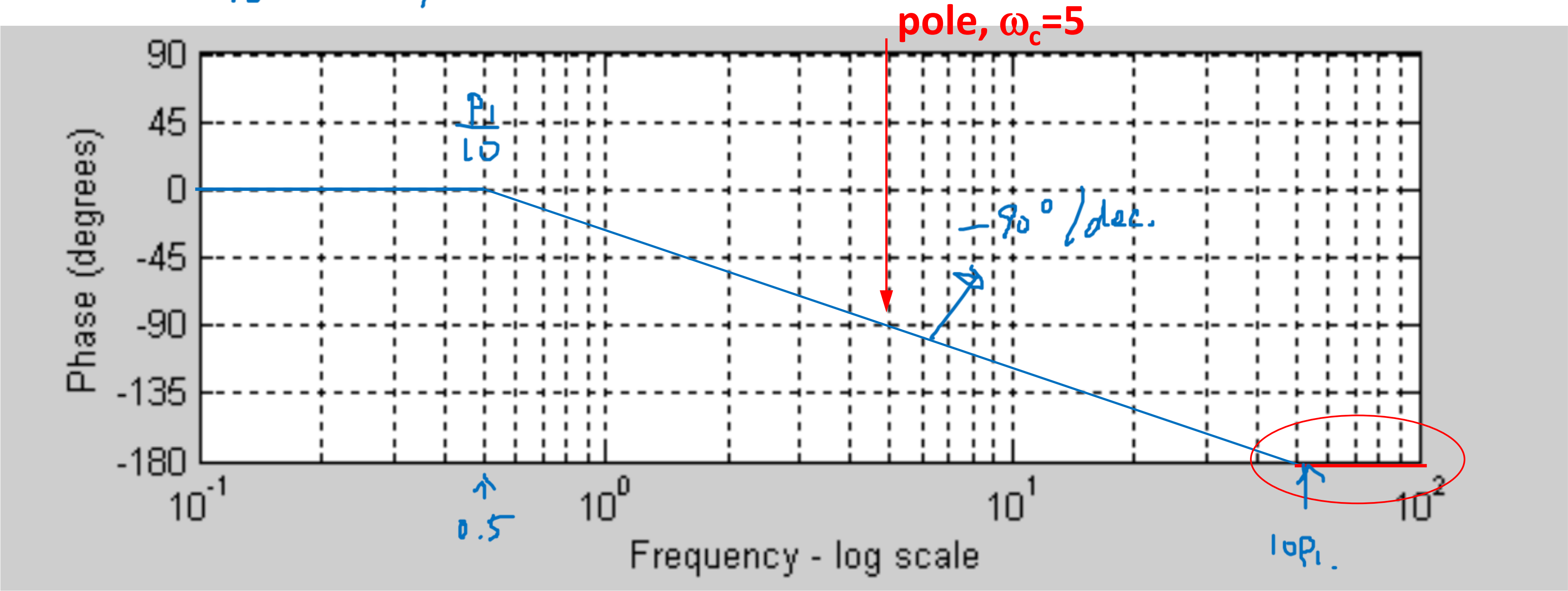
$$H(j\omega) = \frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$$

Four terms:
(i) 0.4 → 0°(ii) (1+jω/10)
(iii) $(j\omega)^{-1}$ (iv) $(1+j\omega/5)^{-2}$

(iv) $(1+j\omega/5)^{-2} \rightarrow -2 * \angle\tan^{-1}(\omega/5)$

$H: j\omega/5 \rightarrow -45°/\text{dec.} \times 2.$

(i) ω = 0.5, Φ = -2*5.7 = -11.42 = 0 °
(ii) ω = 5, Φ = -90°
(iii) ω = 50, Φ = -2*84 = -168 = -180°



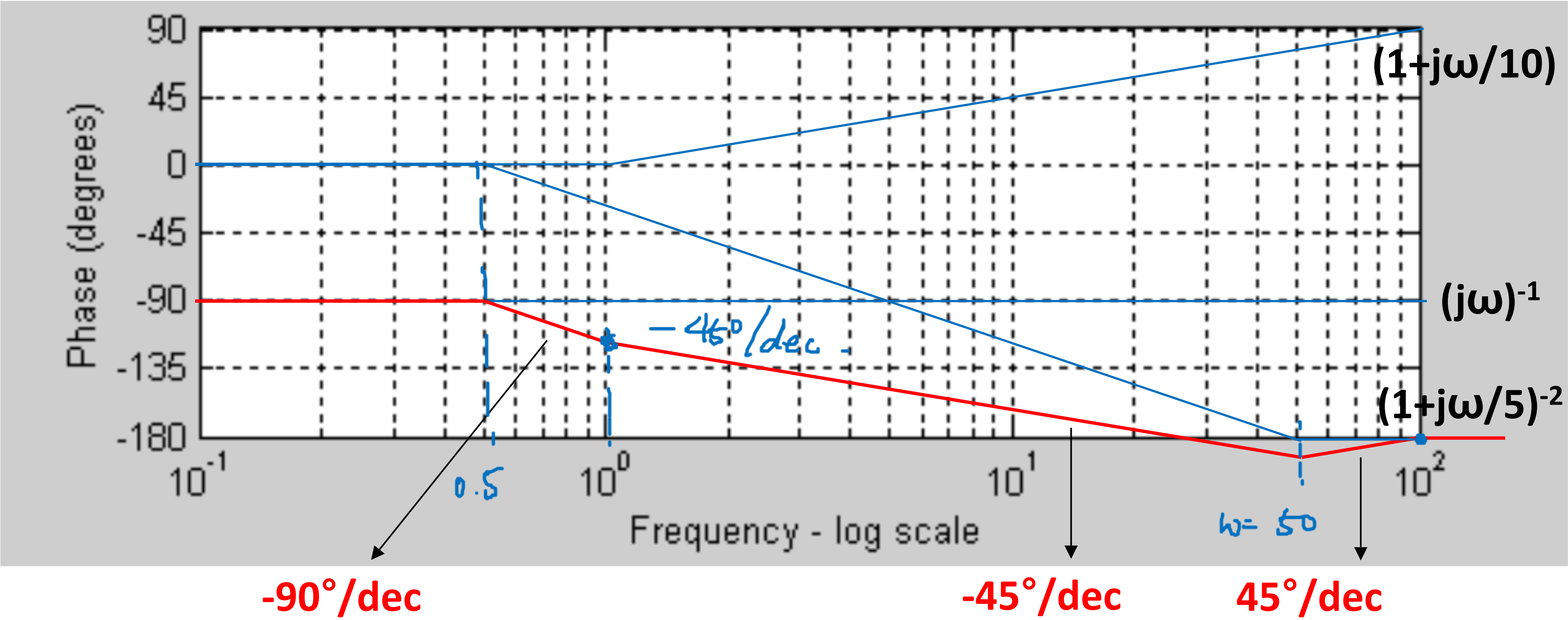
Phase

$$H(j\omega) = \frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$$

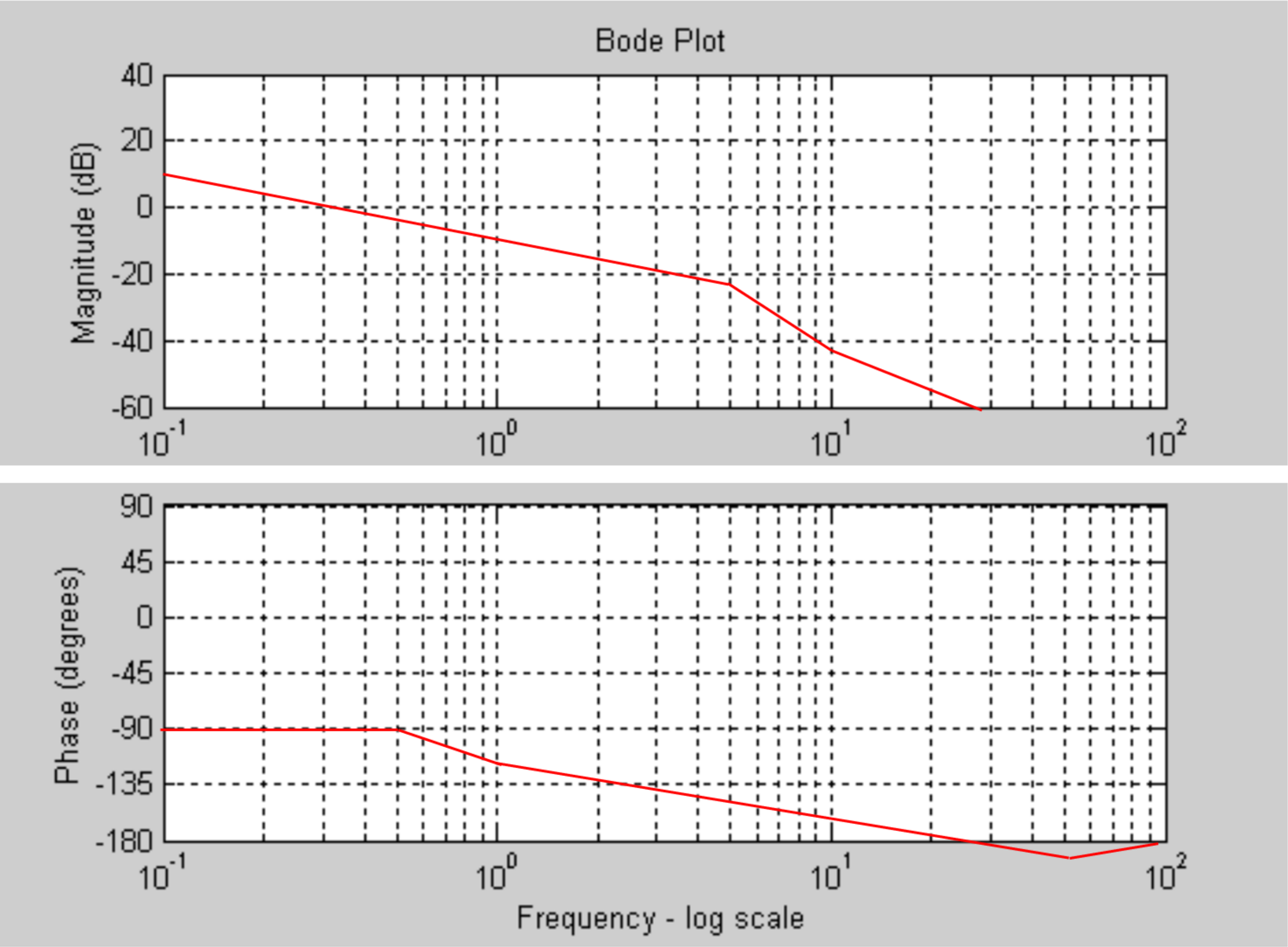
Four terms:
(i) 0.4 → 0°(ii) (1+jω/10)
(iii) $(j\omega)^{-1}$ (iv) $(1+j\omega/5)^{-2}$

Draw all of the four terms

$$\phi = 0° + \tan^{-1}(\omega/10) - 90° - 2\tan^{-1}(\omega/5)$$



Bode plot of the transfer function: $H(j\omega) = \dfrac{j\omega + 10}{j\omega(j\omega + 5)^2}$



Practice Problem 14.3 Draw the Bode plots for

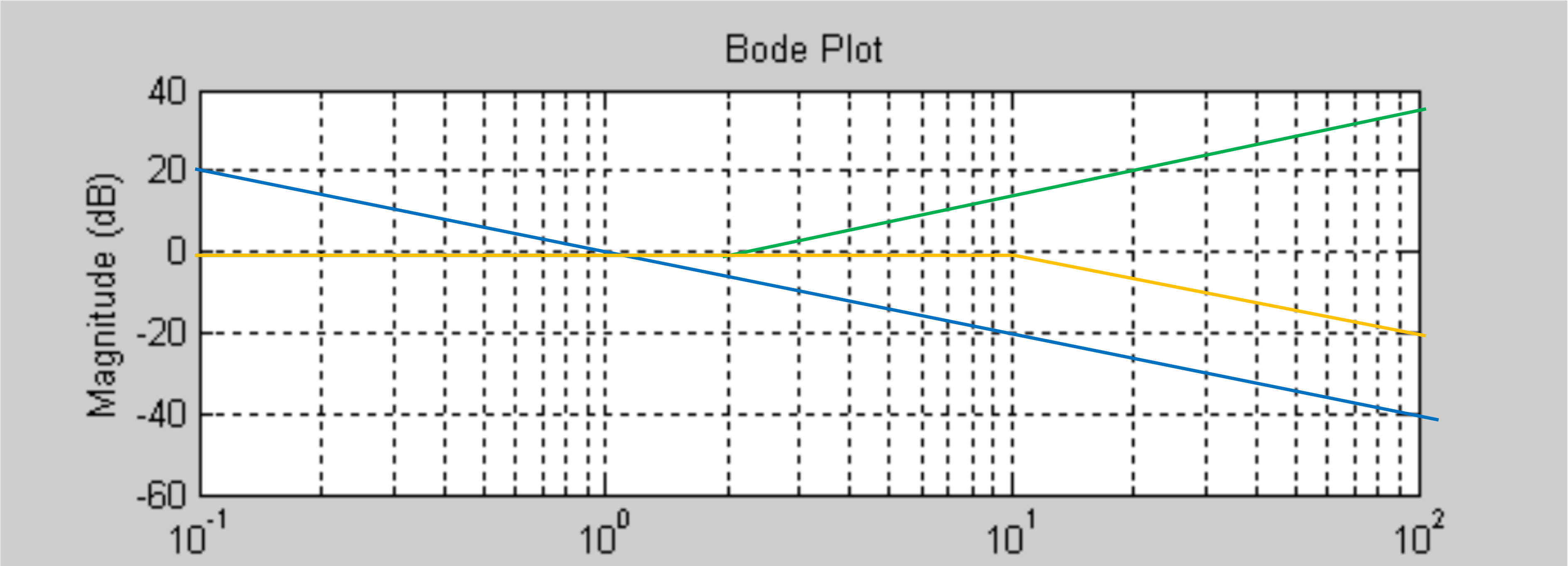
$$H(j\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)}$$

$$H_{dB} = 20\log_{10}\left|\frac{(1+j\omega/2)}{j\omega(1+j\omega/10)}\right| = 20\log_{10}|1+j\omega/2| - 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10|$$

$20\log_{10}|1+j\omega/2|$ **Green line**

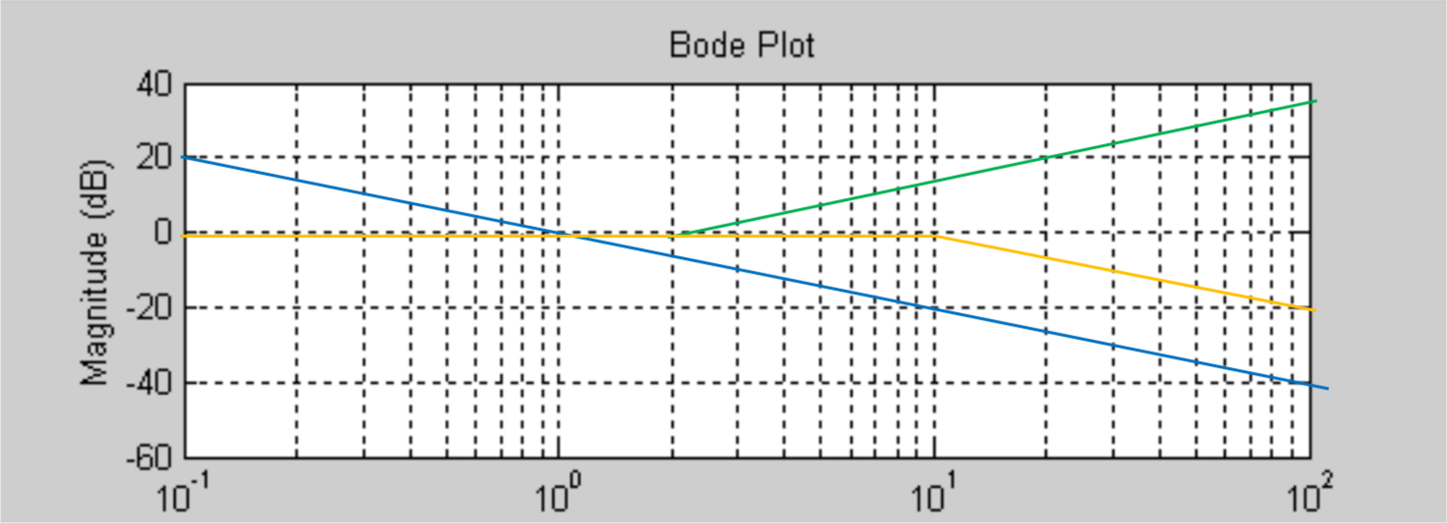
$-20\log_{10}|j\omega|$ **Blue line**

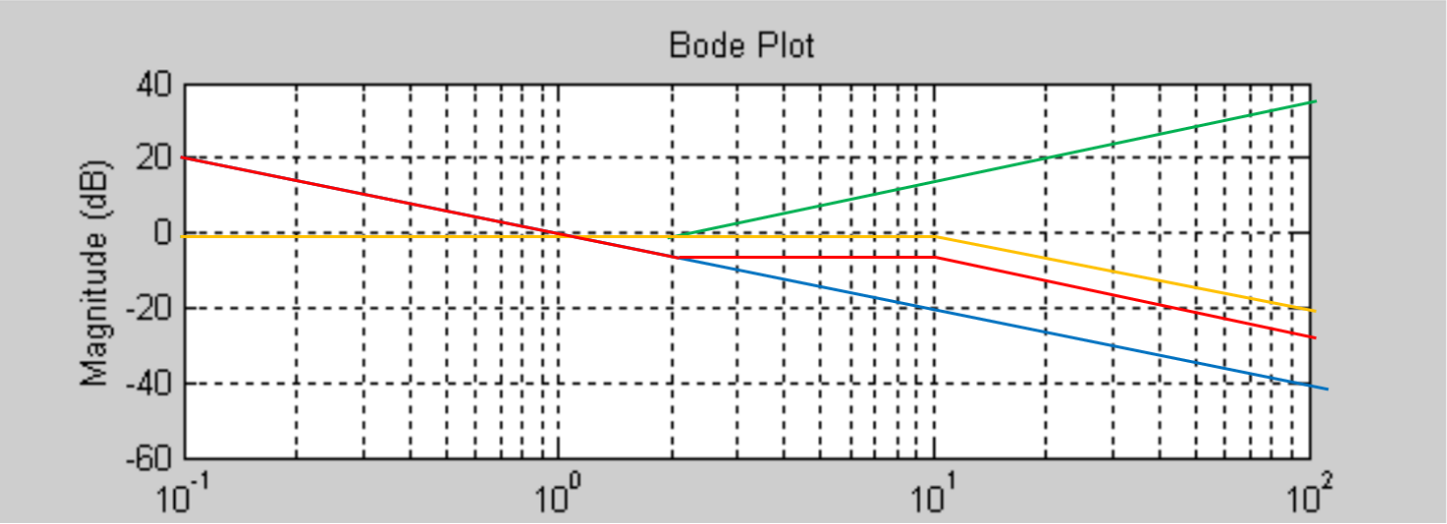
$-20\log_{10}|1+j\omega/10|$ **Yellow line**



$$H_{dB} = 20\log_{10}\left|\frac{(1+j\omega/2)}{j\omega(1+j\omega/10)}\right| = 20\log_{10}\left|1+j\omega/2\right| - 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1+j\omega/10\right|$$

Combined in red line

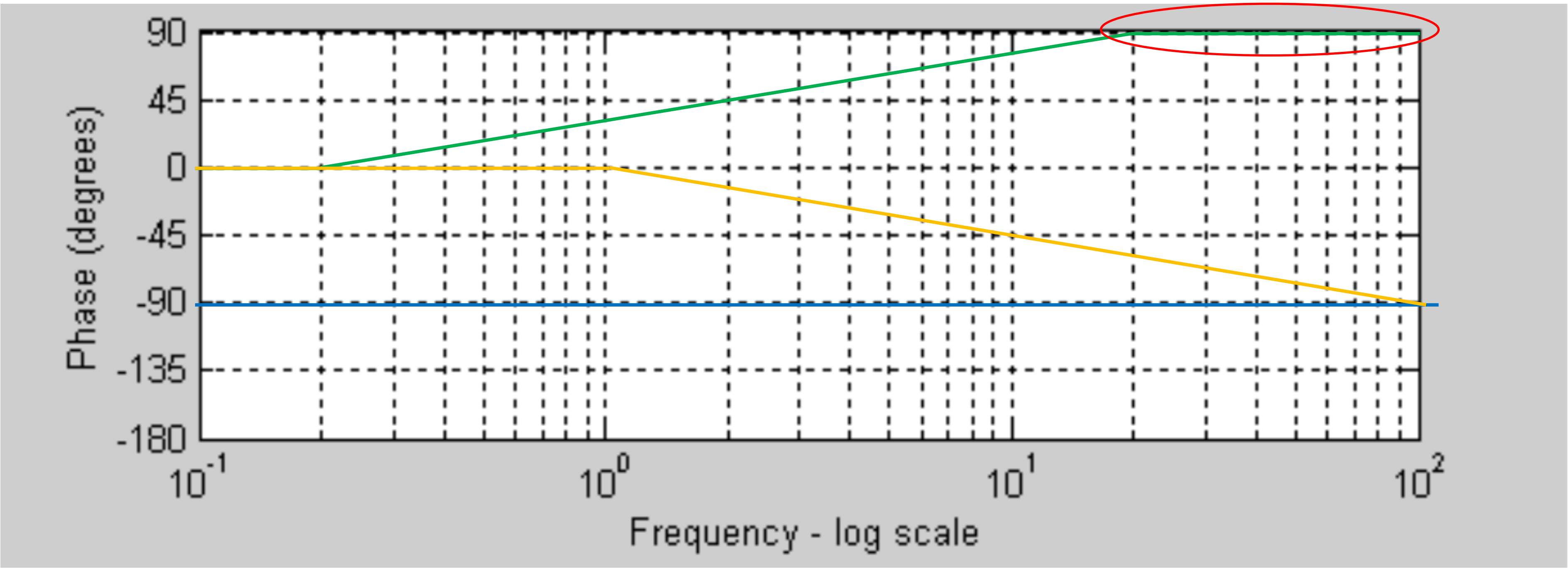




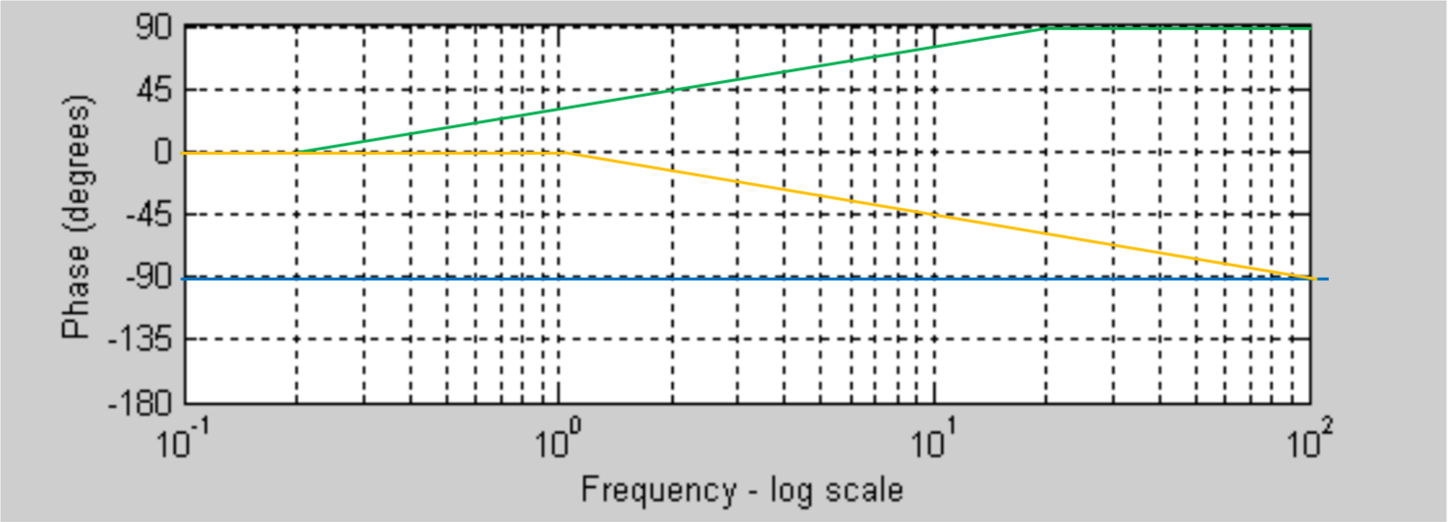
$$H(j\omega) = \frac{(1 + j\omega/2)}{j\omega(1 + j\omega/10)}$$

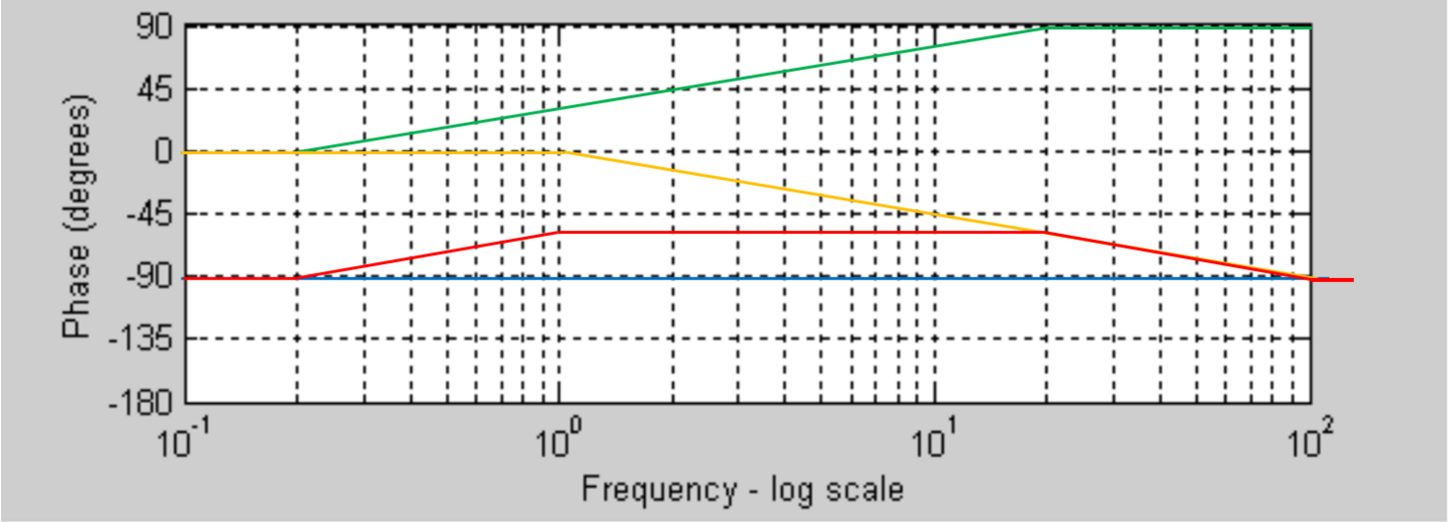
$$\phi = \tan^{-1}\left(\omega/2\right) - 90^\circ - \tan^{-1}(\omega/10)$$

Green line Blue line Yellow line



$$\phi = \tan^{-1}\left(\omega/2\right) - 90° - \tan^{-1}(\omega/10)$$ **Combined in red line**





$$H(j\omega) = \frac{(1 + j\omega/2)}{j\omega(1 + j\omega/10)}$$ Simple zero

$$H_{dB} = 20\log_{10}\left|\frac{(1 + j\omega/2)}{j\omega(1 + j\omega/10)}\right|$$

Pole at the origin Simple pole

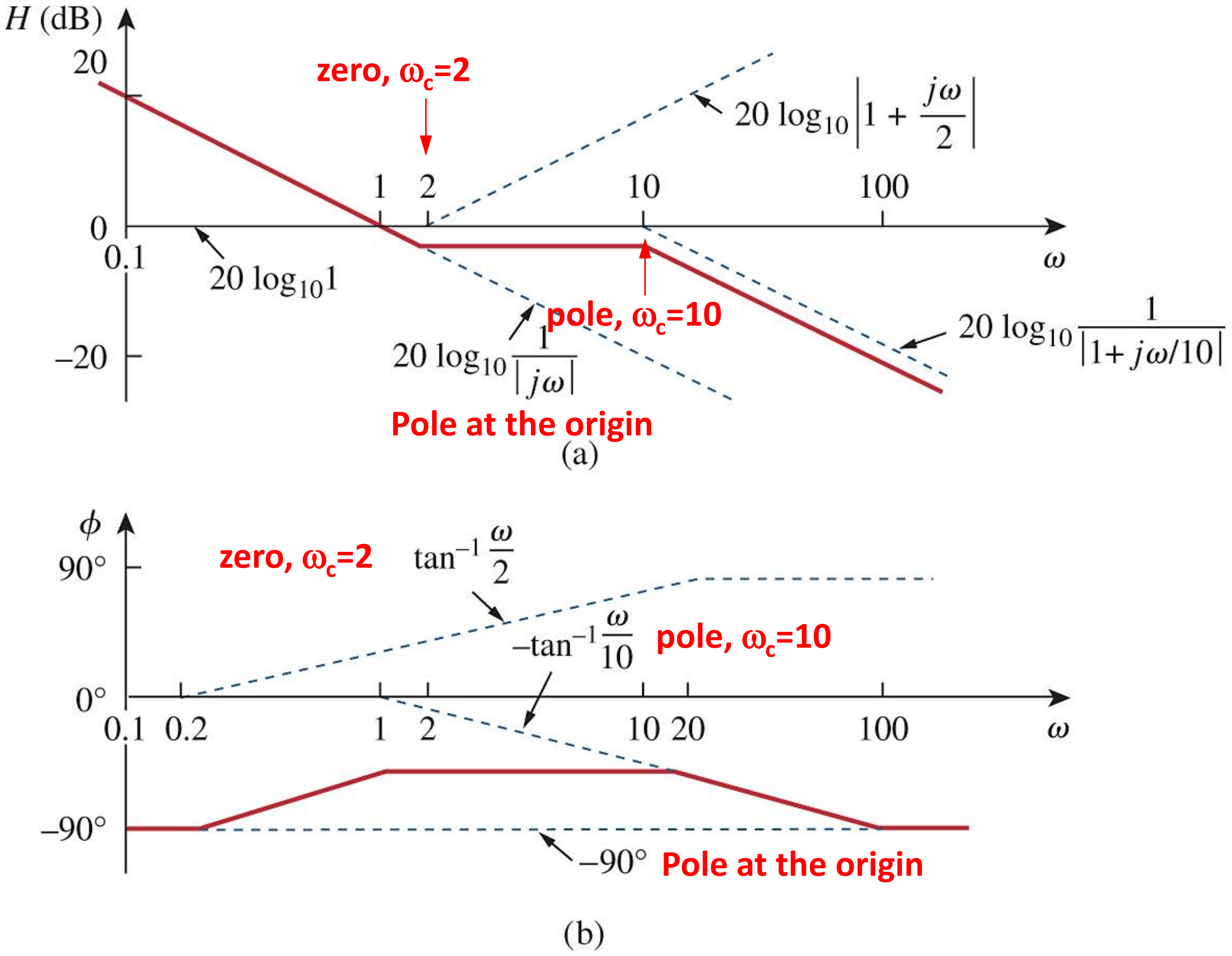
$$= 20\log_{10}|1 + j\omega/2| + 20\log_{10}\frac{1}{|j\omega|}$$

$$+20\log_{10}\frac{1}{|1 + j\omega/10|}$$

$$= 20\log_{10}|1 + j\omega/2| - 20\log_{10}|j\omega|$$

$$-20\log_{10}|1 + j\omega/10|$$

$$\phi = \tan^{-1}(\omega/2) - 90° - \tan^{-1}(\omega/10)$$



H (dB)
20
0
–20
0.1
1
2
10
100
ω
zero, ω_c=2
$20 \log_{10}\left|1+\frac{j\omega}{2}\right|$
$20 \log_{10} 1$
$20 \log_{10}\frac{1}{|j\omega|}$
pole, ω_c=10
$20 \log_{10}\frac{1}{|1+j\omega/10|}$
Pole at the origin
(a)
ϕ
90°
0°
–90°
zero, ω_c=2
$\tan^{-1}\frac{\omega}{2}$
$-\tan^{-1}\frac{\omega}{10}$
pole, ω_c=10
0.1
0.2
1
2
10
20
100
ω
–90°
Pole at the origin
(b)

Example 14.5 Draw the Bode plots for

$$H(j\omega) = \frac{j\omega + 1}{(j\omega)^2 + 12(j\omega) + 100} \Rightarrow 100\left(1 + \frac{12}{100} j\omega + \left(\frac{j\omega}{10}\right)^2\right)$$

$$\Downarrow$$

$$100\left(1 + 2 \times 0.6\left(\frac{j\omega}{10}\right) + \left(\frac{j\omega}{10}\right)^2\right)$$

$$\uparrow$$

$$\zeta .$$

$$H(j\omega) = \frac{1}{100} \frac{(1 + j\omega)}{\left[1 + 2 \times 0.6\left(\frac{j\omega}{10}\right) + \left(\frac{j\omega}{10}\right)^2\right]}$$

Example 14.5 Draw the Bode plots for

$$H(j\omega) = \frac{j\omega + 1}{(j\omega)^2 + 12(j\omega) + 100}$$

$$H(j\omega) = \frac{0.01(1 + j\omega/1)}{1 + 0.12(j\omega) + (j\omega/10)^2}$$

$$= \frac{0.01(1 + j\omega/1)}{1 + 2\times 0.6(j\omega/10) + (j\omega/10)^2}$$

$$H_{dB} = 20\log_{10}\left|\frac{0.01(1 + j\omega/1)}{1 + 2\times 0.6(j\omega/10) + (j\omega/10)^2}\right|$$

$$= 20\log_{10} 0.01 + 20\log_{10}\left|1 + j\omega/1\right|$$

$$-20\log_{10}\left|1 + 2\times 0.6(j\omega/10) + (j\omega/10)^2\right|$$

$$\phi = 0° + \tan^{-1}\left(\omega/1\right) - \tan^{-1}\left[\frac{2\times 0.6(\omega/10)}{1 - (\omega/10)^2}\right]$$

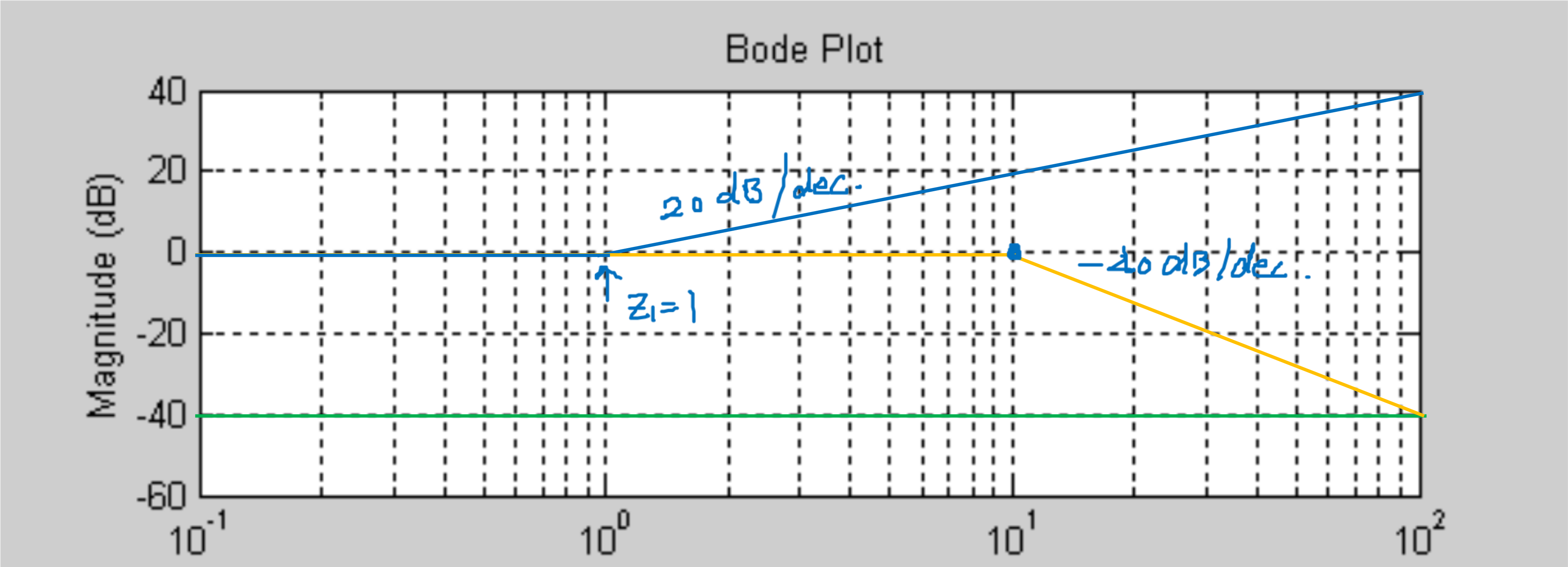
$$H_{dB} = 20\log_{10}\left|\frac{0.01(1+j\omega/1)}{1+2\times 0.6(j\omega/10)+(j\omega/10)^2}\right|$$

$$= 20\log_{10} 0.01$$ -40 dB Green line

$$+20\log_{10}\left|1+j\omega/1\right|$$ Blue line

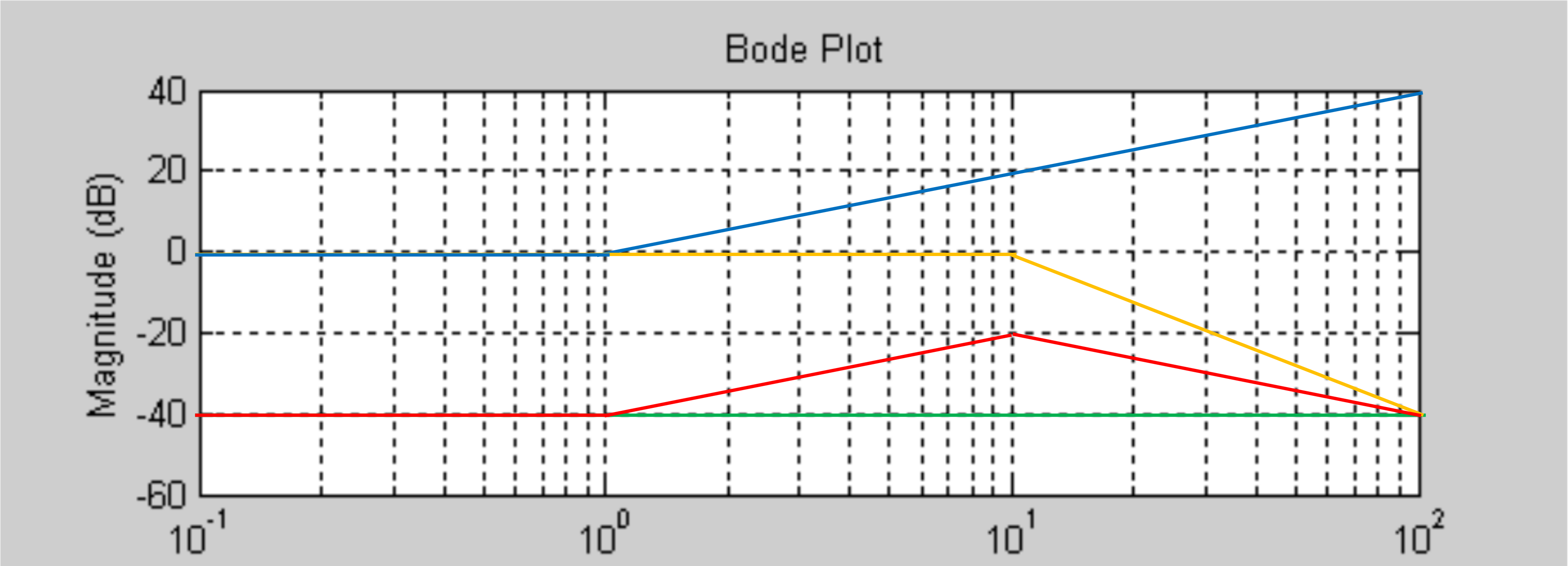
$$-20\log_{10}\left|1+2\times 0.6(j\omega/10)+(j\omega/10)^2\right|$$ Yellow line

$\omega_K = 10$.



$$H_{dB} = 20\log_{10}\left|\frac{0.01(1+j\omega/1)}{1+2\times 0.6(j\omega/10)+(j\omega/10)^2}\right|$$

Combined in red line



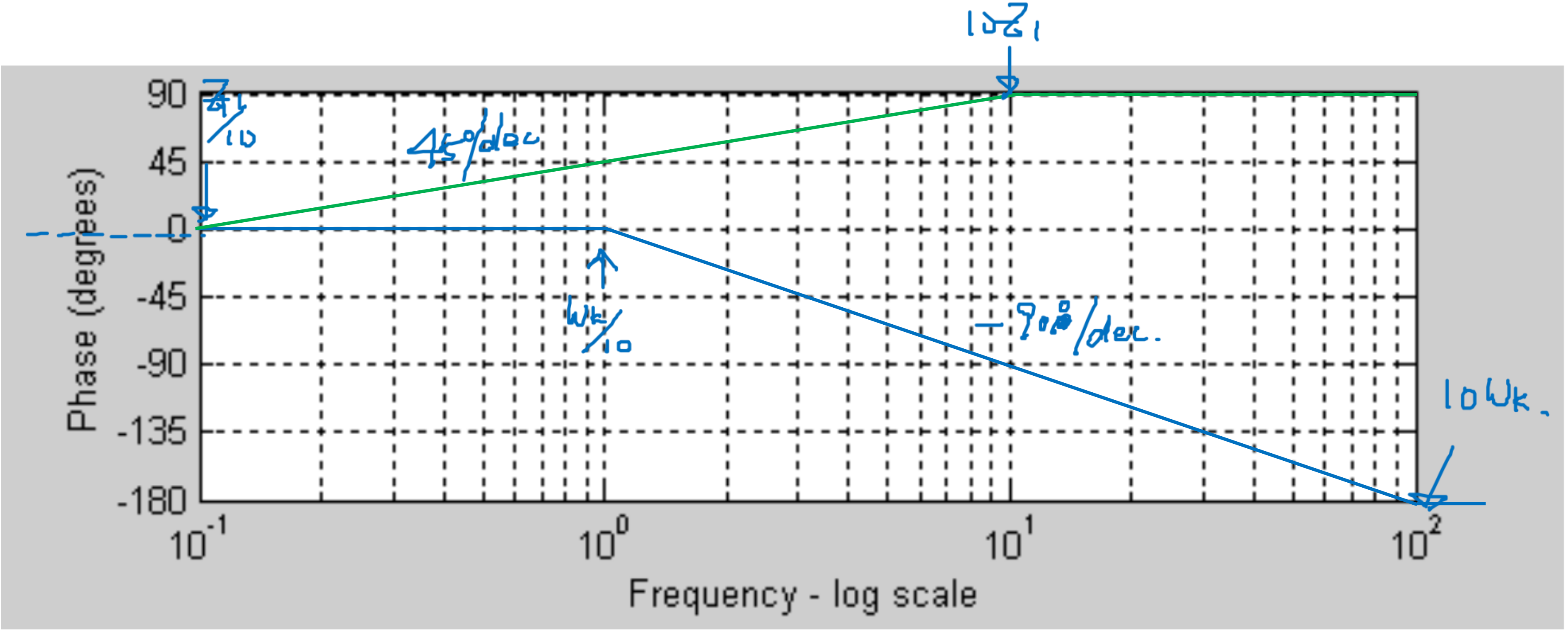
$$H(j\omega)=\frac{0.01(1+j\omega/1)}{1+0.12(j\omega)+(j\omega/10)^2}$$

$$=\frac{0.01(1+j\omega/1)}{1+2\times 0.6(j\omega/10)+(j\omega/10)^2}$$

$$\underbrace{\phi=0^\circ}_{\text{Const. } 0.01}+\tan^{-1}(\omega/1)-\tan^{-1}\left[\frac{2\times 0.6(\omega/10)}{1-(\omega/\underbrace{10}_{\omega_K=10})^2}\right]$$

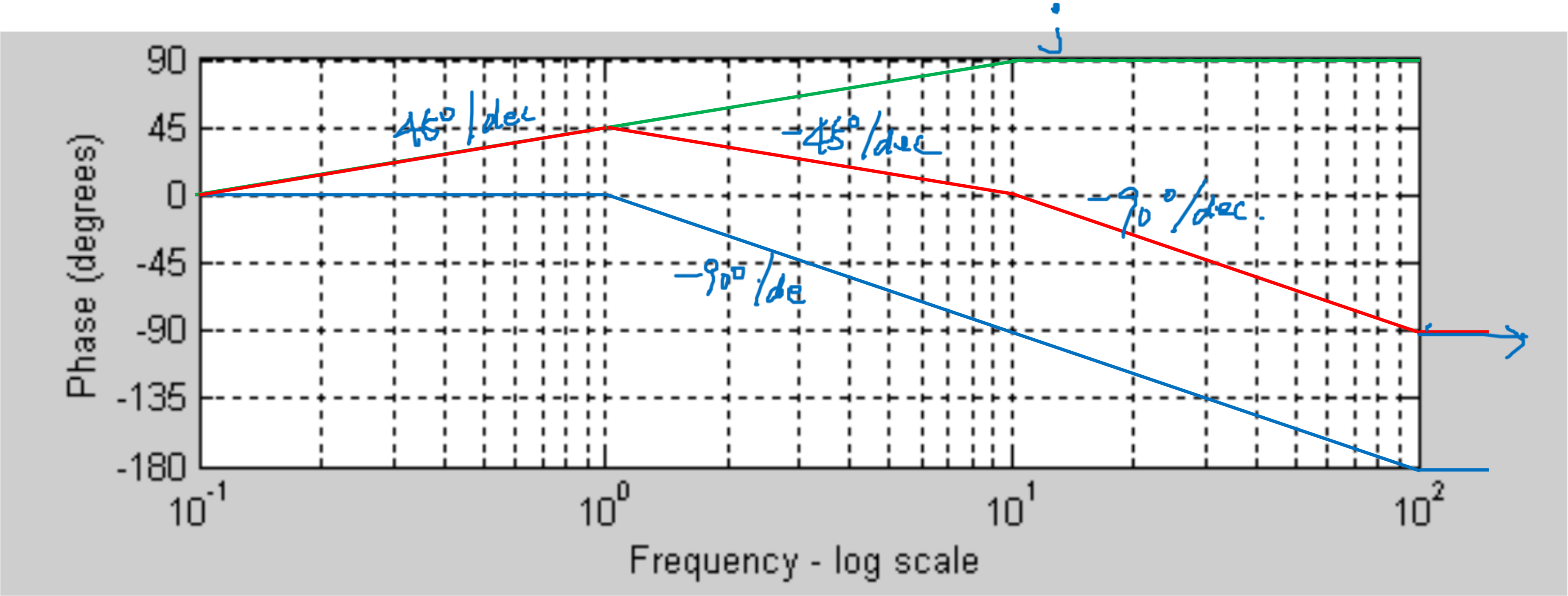
Green line

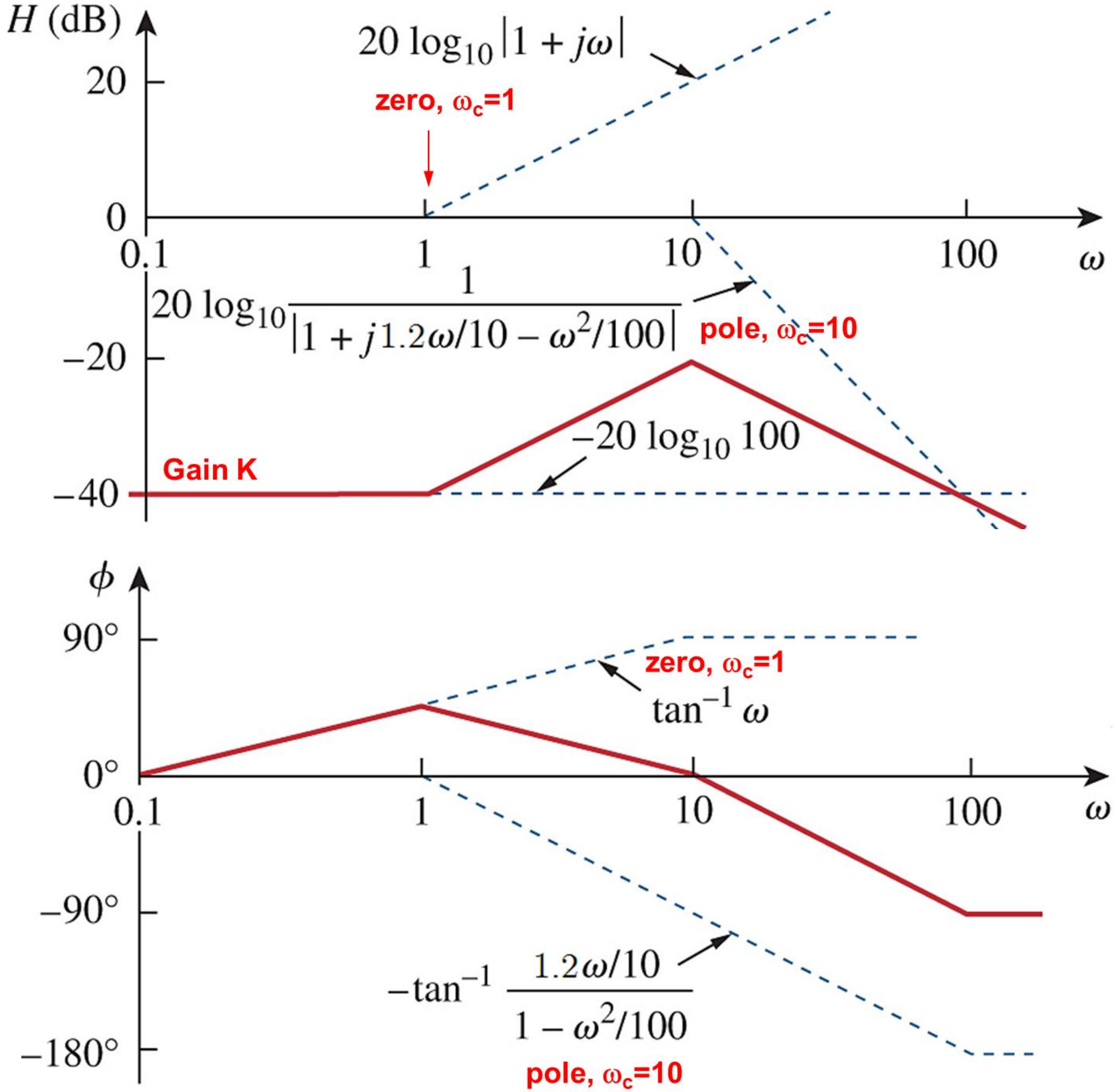
Blue line



$$\phi = 0^\circ + \tan^{-1}(\omega/1) - \tan^{-1}\left[\frac{2\times 0.6(\omega/10)}{1-(\omega/10)^2}\right]$$

Combined in red line



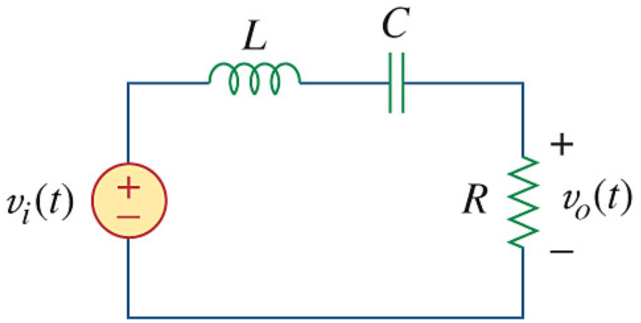


H (dB)
20
0
−20
−40
0.1
1
10
100
ω
$20\log_{10}|1 + j\omega|$
zero, ω_c=1
$20\log_{10}\frac{1}{|1 + j1.2\omega/10 - \omega^2/100|}$
pole, ω_c=10
$-20\log_{10}100$
Gain K
ϕ
90°
0°
−90°
−180°
zero, ω_c=1
$\tan^{-1}\omega$
$-\tan^{-1}\frac{1.2\omega/10}{1-\omega^2/100}$
pole, ω_c=10

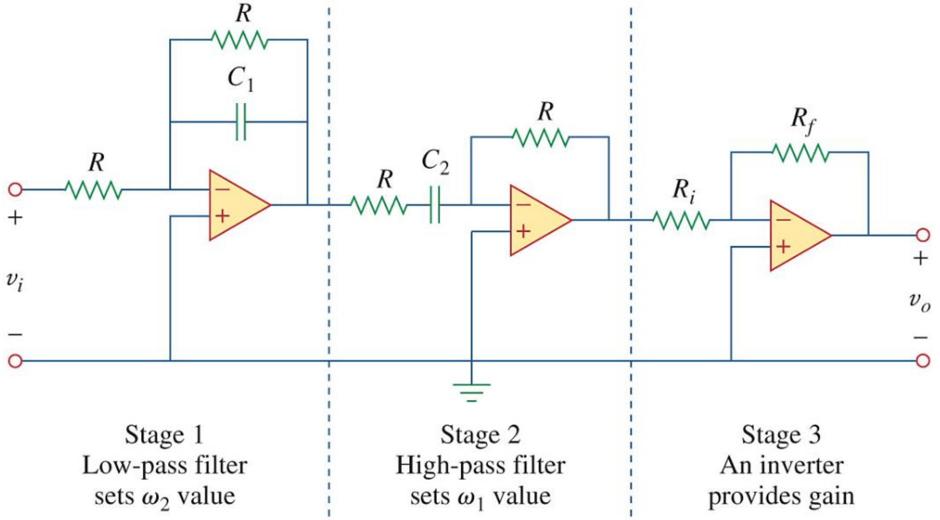
*Filters

A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others, i.e. a **frequency-selective device**.

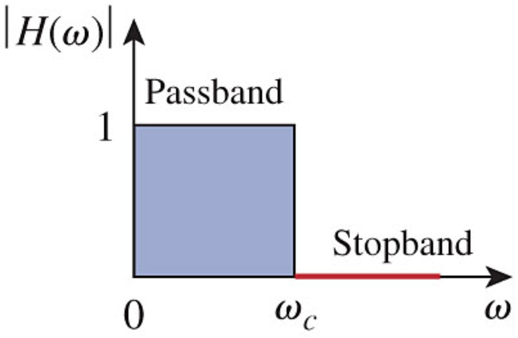
Passive filter: consists of only passive elements R, L, and C
Active filter: consists of active elements, such as, transistors and op amps in addition to RLC.

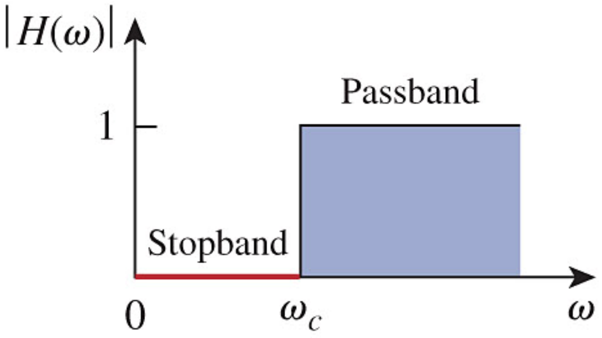


Passive bandpass filter

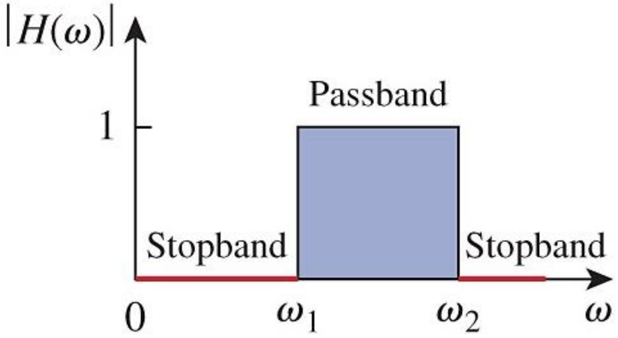


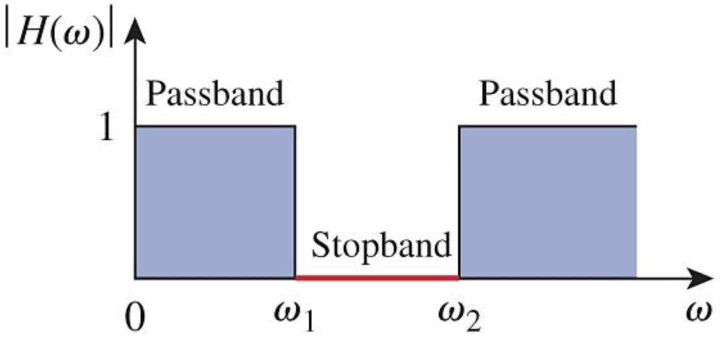
Active bandpass filter





Lowpass filter passes low frequencies and stops high frequencies.
Highpass filter passes high frequencies and rejects low frequencies.





Bandpass filter passes frequencies within a frequency band and blocks or attenuates frequencies outside the band.
Bandstop filter passes frequencies outside a frequency band and blocks or attenuates frequencies within the band.