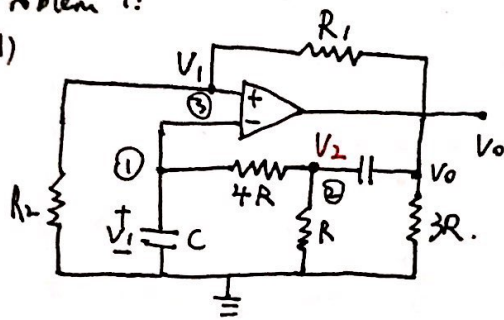VE215 Quiz 5 Solution.

Problem 1:

(1)

Assume V_2.

From the property of op-amp, we know $V_+ = V_- = V_1$.

Then we can apply KCL considering node ① and node ②

$Z_c = \dfrac{1}{j\omega C}$

$$\begin{cases} \dfrac{V_1}{\frac{1}{j\omega C}} = \dfrac{V_2 - V_1}{4R} \Rightarrow V_1 = \dfrac{V_2 - V_1}{4Rj\omega C} \Rightarrow (4Rw cj + 1)V_1 = V_2 \quad \text{Formula A} \\[4mm] \dfrac{V_2 - V_1}{4R} = \dfrac{V_0 - V_2}{\frac{1}{j\omega C}} + \dfrac{-V_2}{R} \quad \text{Formula B.} \end{cases}$$

We plug Formula A into Formula B.

$$\dfrac{4Rj c\omega V_1}{4R} = \dfrac{V_0 - (4Rj c\omega + 1)V_1}{\frac{1}{j\omega C}} - \dfrac{(4Rj c\omega + 1)V_1}{R}$$

$\Rightarrow j c\omega V_1 = j\omega C V_0 + 4Rc^2\omega^2 V_1 - j\omega C V_1 - 4j c\omega V_1 - \dfrac{1}{R}V_1$

$\Rightarrow 6 j c\omega V_1 - 4Rc^2\omega^2 V_1 + \dfrac{1}{R}V_1 = j\omega C V_0$

Then $\dfrac{V_0}{V_1} = \dfrac{6j c\omega - 4Rc^2\omega^2 + \frac{1}{R}}{j\omega C} = 6 + 4Rj c\omega + \dfrac{1}{Rj\omega C} = 6 + 4Rj c\omega - \dfrac{1}{Rc\omega}j$

To make this real, $4Rc\omega - \dfrac{1}{Rc\omega} = 0 \Rightarrow 4R^2c^2\omega^2 = 1 \Rightarrow \boxed{\omega = \dfrac{1}{2RC}}$
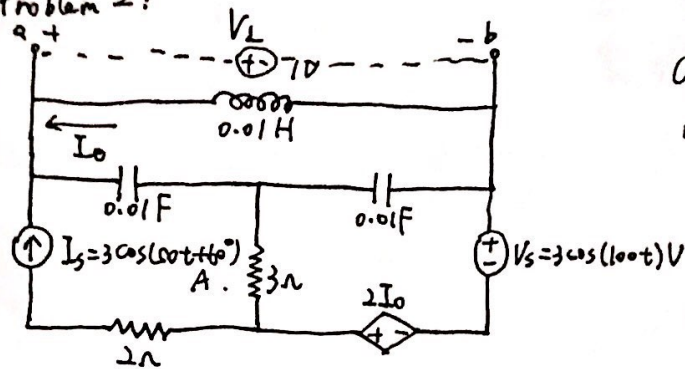
Here $\dfrac{V_0}{V_1} = 6$, which is real

And then we also apply KCL to node ②.

$\dfrac{V_1 - 0}{R_2} + \dfrac{V_1 - V_0}{R_1} = 0$

$\Rightarrow V_1 R_1 + (V_1 - V_0)R_2 = 0 \Rightarrow V_1(R_1 + R_2) = V_0 R_2 \Rightarrow \dfrac{V_0}{V_1} = \dfrac{R_1 + R_2}{R_2} = 6$

$\Rightarrow \dfrac{V_0}{V_1} = 1 + \dfrac{R_1}{R_2} = 6 \Rightarrow \boxed{\dfrac{R_1}{R_2} = 5}$
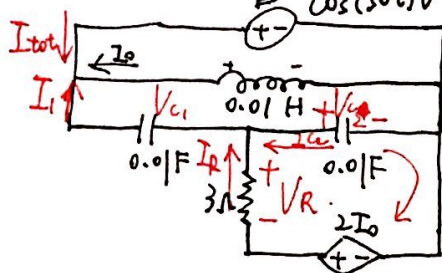
Problem 2:

Clearly, there are two independent sources of two different frequencies.

1. First, we find thevenin resistance when all independent sources are turned off. Since there is a dependent source.

So we assume a, b are connected to ac source of $\omega = 50$.

$\cos(50t)V = 1\angle 0° \quad \omega = 50$.

$$I_0 = -\frac{1\angle 0°}{Z_L} = -\frac{1\angle 0°}{j \cdot 50 \cdot 0.01} = 2j \, A$$

So voltage of dependent source $= 2I_0 = 4j \, V$

So by KVL

$V_R + V_{C_2} = 0 \Rightarrow V_{C_2} = -V_R = 12j \, V.$

Then current through C_2 is $I_{C_2} = \frac{12j}{\frac{1}{j\omega C}} = \frac{12j}{\frac{1}{j \cdot 50 \cdot 0.01}} = 6 \, A$

By KCL, $I_1 = I_R + I_{C_2} = (4j - 6) \, A$

Then $I_{tot} + I_0 + I_1 = 0$. (KCL)

$I_{tot} = -I_0 - I_1 = (6 - 6j) \, A$

Then Z_{TH} (under $\omega = 50$) $= \frac{1\angle 0°}{6 - 6j} = \frac{1}{12} + \frac{1}{12}j \, \Omega$.

And RE's V_{TH} is calculated by turning off V_s and turning on I_s

Phasor: $3\angle 60°$

Then the graph can be simplified to.

$\cos(50t)V = 1\angle 0°$

assume b side is connected to gnd

Assume node voltage V_1.

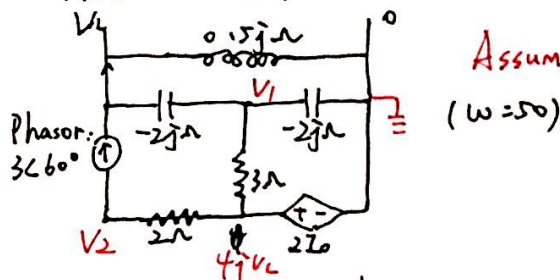then $\dfrac{1 - V_1}{-2j} = \dfrac{V_1}{-2j} + \dfrac{V_1 - 4j}{3} \Rightarrow (6 - 2j)V_1 = 3 + 8$

$3 - 3V_1 = 3V_1 - 8 - 2j V_1 \Rightarrow V_1 = \dfrac{11}{6 - 2j}$

$I_1 = \dfrac{1 - V_1}{-2j} = \left(\dfrac{11}{40} - \dfrac{13}{40}j\right) A$

Then $I + I_0 = I_1$ $\frac{11}{40} - \frac{13}{40}j$ A

$\Rightarrow I = I_1 - I_0 = \frac{\cancel{11}}{\cancel{40}} + \frac{\cancel{}}{\cancel{}} - 2j = \frac{\cancel{}}{\cancel{}}$ A.

Then $Z_{TH} = \frac{1\angle 0°}{\cancel{}} = \frac{\cancel{}}{(\frac{44}{877} + \frac{372}{877}j)}$ Ω.   (under $\omega = 50$)

$\frac{11}{40} - \frac{93}{40}j$

Then we need to calculate V_{TH} by turning off V_s and turning on I_s.

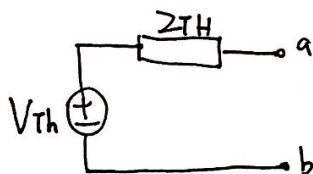Assume node voltages V_1 and V_2

($\omega = 50$)

Then by KCL,

$$\begin{cases} \dfrac{V_L - 0}{0.5j} + \dfrac{V_L - V_1}{-2j} = 3\angle 6°° \\[2mm] \dfrac{V_1 - V_L}{-2j} + \dfrac{V_1 - 0}{-2j} + \dfrac{V_1 - 4jV_L}{3} = 0. \\[2mm] \dfrac{V_2 - 4jV_L}{2} = -3\angle 6°° \end{cases} \Rightarrow \begin{cases} -1.5jV_L - 0.5j V_1 = \dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}j \quad ① \\[2mm] (\frac{1}{3} + j)V_1 - (0.5j + \frac{4}{3}j)V_L = 0 \quad ② \\[2mm] \frac{1}{2}V_2 - 2j V_L = -\dfrac{3}{2} - \dfrac{3\sqrt{3}}{2}j \quad ③ \end{cases}$$

$\overset{by②}{\Rightarrow} V_1 = \dfrac{0.5j + \frac{4}{3}j}{\frac{1}{3} + j} V_L$

$\overset{by①}{\Rightarrow} -1.5j V_L - 0.5j \cdot \dfrac{0.5j + \frac{4}{3}j}{\frac{1}{3} + j} V_L = \dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}j \Rightarrow V_L = \dfrac{-1.02\cancel{7} + 0.767j}{\cancel{}} V = \dfrac{1.282 \angle -36.75°}{\cancel{}} V$

When $\omega = 50$ (I_s)

$V_{Th} = 1.282 \angle -36.75°$.

$Z_{Th} = \dfrac{\cancel{}}{\cancel{}}$ Ω.

$(\frac{44}{877} + \frac{372}{877}j)$ Ω.

II. For another independent source V_s

Z_{Th} is calculated by the same way but $\omega = 100$.

$I_0 = -\dfrac{1\angle 0°}{Z_L} = -\dfrac{1\angle 0°}{j \cdot 100 \times 0.01} = j$ A.

voltage of dependent source $= 2I_0 = 2j$ V.

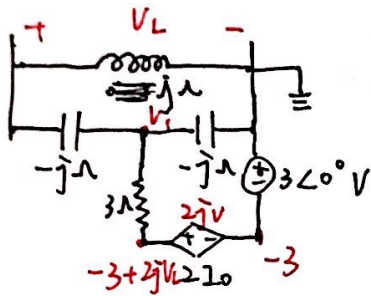$Z_C = \dfrac{1}{j\omega C} = \dfrac{1}{j \cdot 100 \cdot 0.01} = -j$ Ω

$\dfrac{1 - V_1}{-\cancel{}j} = \dfrac{V_1}{-j} + \dfrac{V_1 - 2j}{3}$

$\Rightarrow 3 - 3V_1 = 3V_1 - 2 - jV_1 \Rightarrow (6-j)V_1 = 5 \Rightarrow V_1 = \dfrac{5}{6-j}$.

$I_1 = \dfrac{1 - V_1}{-j} = (\frac{5}{37} + \frac{7}{37}j)$ A

$I = I_1 - I_0 = (\frac{5}{37} - \frac{30}{37}j)$ A

$Z_{TH} (\omega = 100) = (0.2 + 1.2j)$ Ω
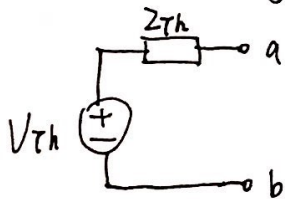
Then we also need to calculate V_{TH}.

Assume node voltage V_1

$+$ V_L $-$

$j\wedge$

V_1

$-j\wedge$

$-j\wedge$ $3\angle 0° V$

3Λ $2\tilde{j}V$

$-3+2jV_1 \geq 2_0$ -3

Then by KCL,

$$\begin{cases} \dfrac{V_L-0}{j} + \dfrac{V_L-U_1}{-j} = 0. \\ \dfrac{V_1-V_L}{-j} + \dfrac{V_1-0}{-j} + \dfrac{V_1-(-3+2jV_L)}{3} = 0 \end{cases} \Rightarrow \begin{cases} V_1 = 0 \\ -jV_L + \dfrac{3-2jV_L}{3} = 0 \end{cases}$$

$$\Rightarrow V_L = \frac{1}{j+\frac{2}{3}j} = -0.6j = 0.6\angle -90° V.$$

Z_{Th}

a

V_{Th}

b

$Z_{Th} = (0.2 + 1.2j)\Lambda$

$V_{Th} = 0.6\angle -90° V.$