VE215 Introduction to Circuits

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Chapter 10

Sinusoidal Steady-State Analysis

10.1 Introduction

In this chapter, we want to see how nodal analysis, mesh analysis, Thevenin's theorem, Norton's theorem, superposition, and source transformations are applied in analyzing ac circuits.

Steps to Analyze AC Circuits

1. Transform the circuit to the phasor domain.
2. Find the circuit output using nodal analysis, mesh analysis, superposition, etc.
3. Transform the resulting phasor to the time domain.

Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved.

Example 10.1 Find i_x in the circuit of Fig. 10.1 using nodal analysis.

Solution :

Step1

$$20\cos 4t \Rightarrow 20\angle 0^\circ \text{ V},\ \omega = 4 \text{ rad/s}$$

$$1 \text{ H} \Rightarrow j\omega L = j4\ \Omega$$

$$0.5 \text{ H} \Rightarrow j\omega L = j2\ \Omega$$

$$0.1 \text{ F} \Rightarrow \frac{1}{j\omega C} = -j2.5\ \Omega$$

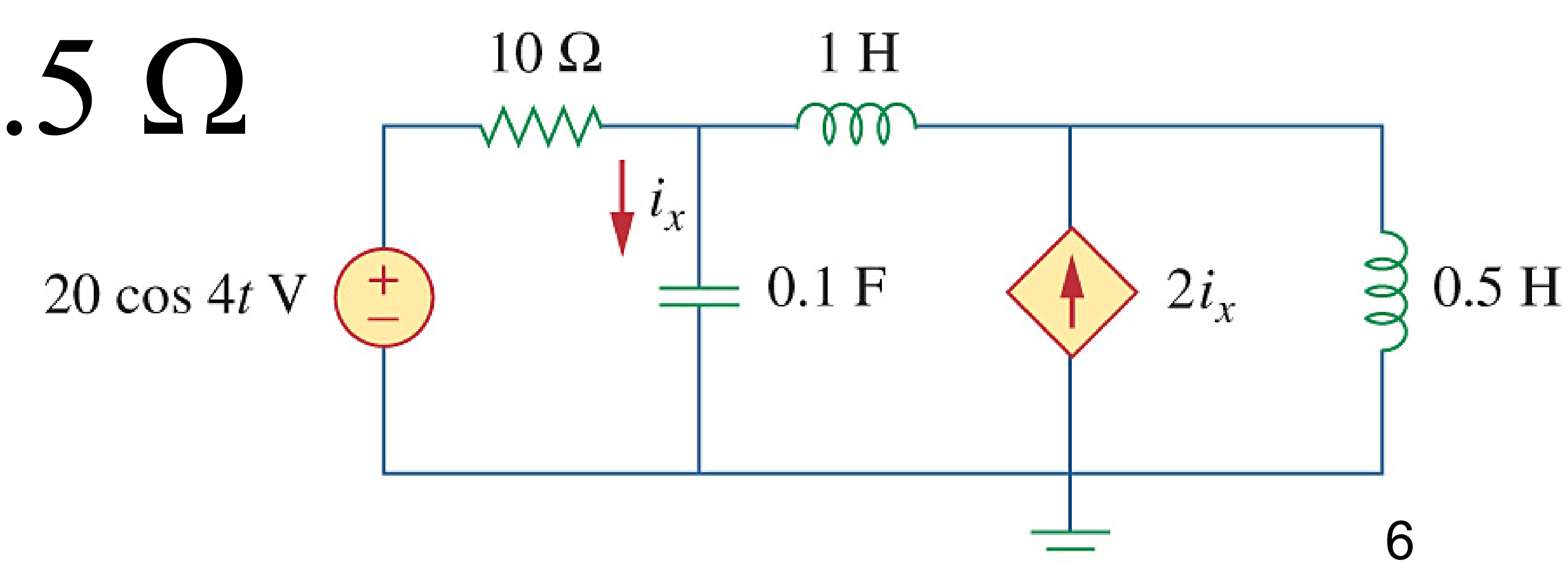


Figure 10.1

Step2

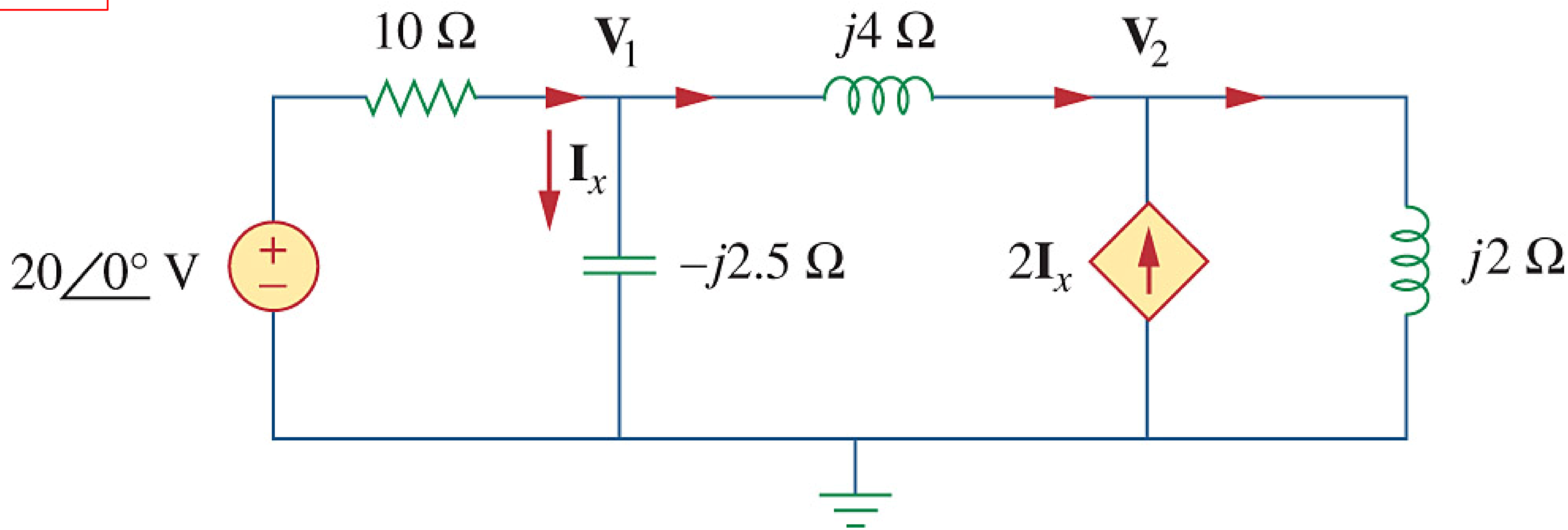


Figure 10.2 Frequency domain equivalent of the circuit in Fig. 10.1.

<u>Review of nodal analysis</u>:

1. Non-reference node and reference node
2. KCL equations for non-reference nodes
3. Solve simultaneous KCL equations

Or by inspection method: **GV=I**

Where G: conductance matrix

V: input vector

I: output vector

YV=I

$$\begin{array}{cc}\Sigma_i G_i & -G_{ij}\\ \multicolumn{2}{c}{\begin{bmatrix} \frac{1}{10}+\frac{1}{j4}+\frac{1}{-j2.5} & -\frac{1}{j4} \\ -\frac{1}{j4} & \frac{1}{j4}+\frac{1}{j2}\end{bmatrix}}\end{array}\begin{bmatrix}\tilde{V}_1\\ \tilde{V}_2\end{bmatrix} = \begin{array}{c}?\\ \begin{bmatrix}\frac{20\angle 0^\circ}{10}\\ 2\tilde{I}_x\end{bmatrix}\end{array}$$

$$\tilde{I}_x = \frac{\tilde{V}_1}{-j2.5}$$

Express independent parameter I_x in dependent sources in terms of V_1 or V_2

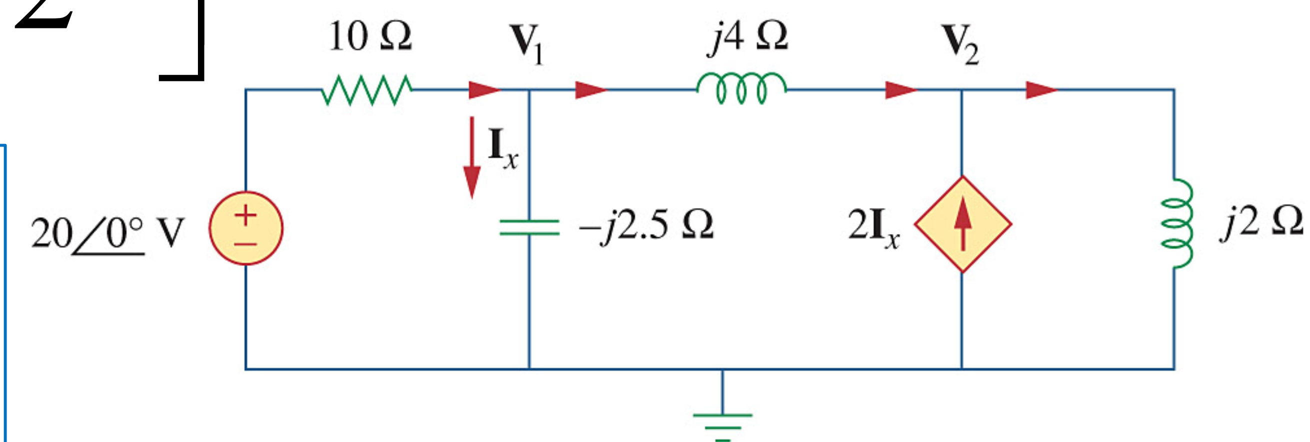


Figure 10.2 Frequency domain equivalent of the circuit in Fig. 10.1.

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{j4}+\frac{1}{-j2.5} & -\frac{1}{j4} \\ -\frac{1}{j4}-\frac{2}{-j2.5} & \frac{1}{j4}+\frac{1}{j2}\end{bmatrix}\begin{bmatrix}\tilde{V}_1\\ \tilde{V}_2\end{bmatrix} = \begin{bmatrix}\frac{20\angle 0^\circ}{10}\\ 0\end{bmatrix}$$

$$\begin{bmatrix} 2+j3 & j5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2+j3 & j5 \\ 11 & 15 \end{vmatrix} = 30 - j10$$

$$\Delta_1 = \begin{vmatrix} 40 & j5 \\ 0 & 15 \end{vmatrix} = 600$$

$$\tilde{V}_1 = \frac{\Delta_1}{\Delta} = \frac{600}{30 - j10} = \frac{60}{3 - j}$$

$$\approx \frac{60}{3.1623\angle -18.43^\circ}$$

$$\approx 18.9735\angle 18.43^\circ \text{ (V)}$$

$$\tilde{I}_x = \frac{\tilde{V}_1}{-j2.5} = \frac{18.9735\angle 18.43^\circ}{2.5\angle -90^\circ}$$

$$\approx 7.59\angle 108.43^\circ \text{ (A)}$$

Step3

$$i_x = 7.59\cos(4t + 108.43^\circ) \text{ (A)}$$

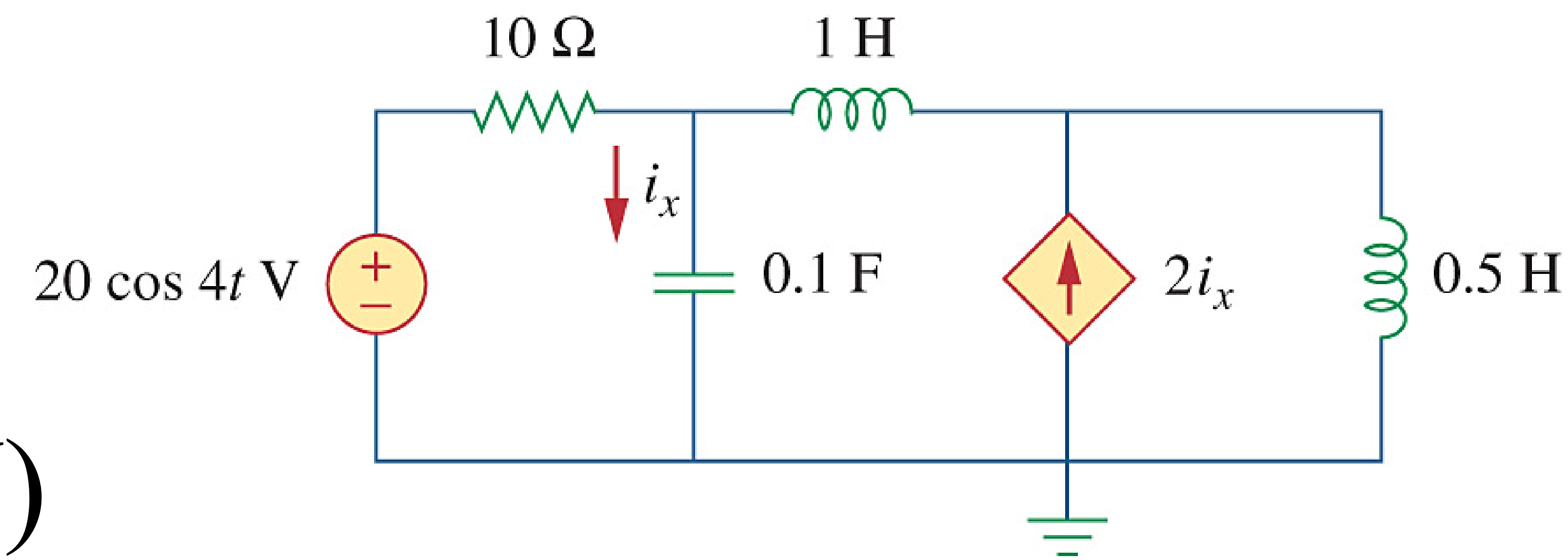


Figure 10.1

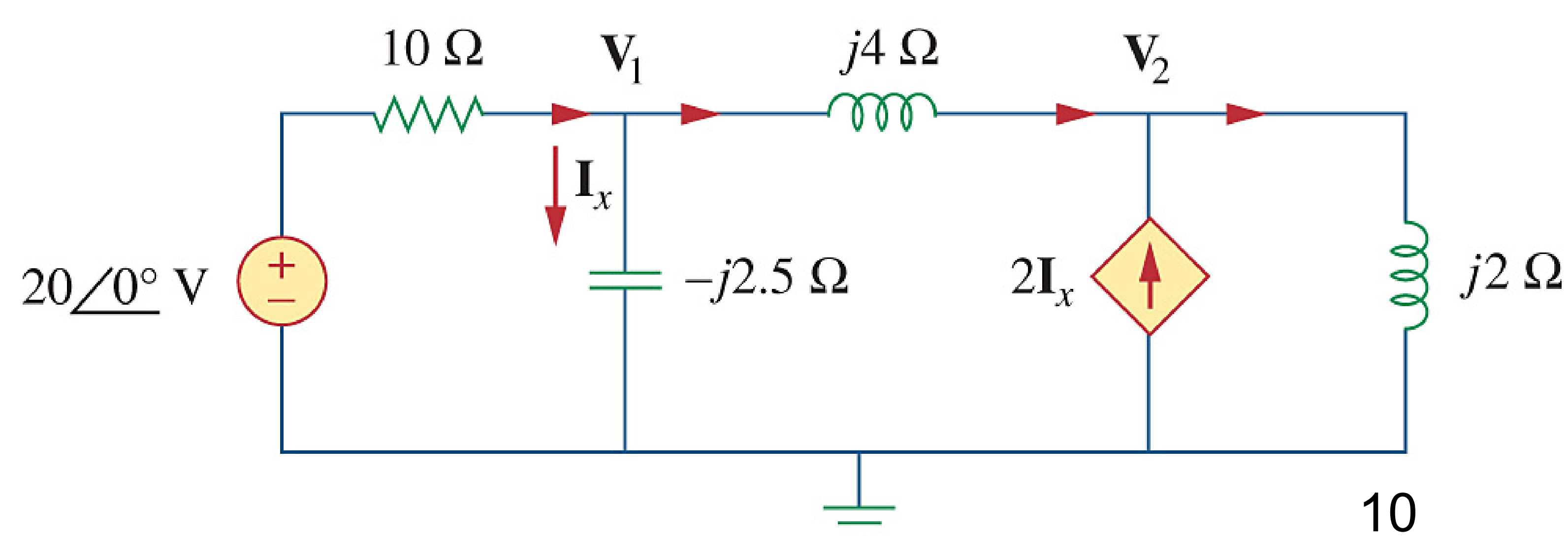


Figure 10.2 Frequency domain equivalent of the circuit in Fig. 10.1.

10.3 Mesh Analysis

<u>Review of mesh analysis</u>:

1. Mesh currents
   -deal with the case with current source first
2. KVL equations for meshes
3. Solve simultaneous KVL equations

Or by inspection method: **RI=V**

Where R: resistance matrix
       I: input vector
       V: output vector

$\boxed{\mathbf{ZI=V}}$

10.3 Mesh Analysis

Example 10.3 Determine current $\dot{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.

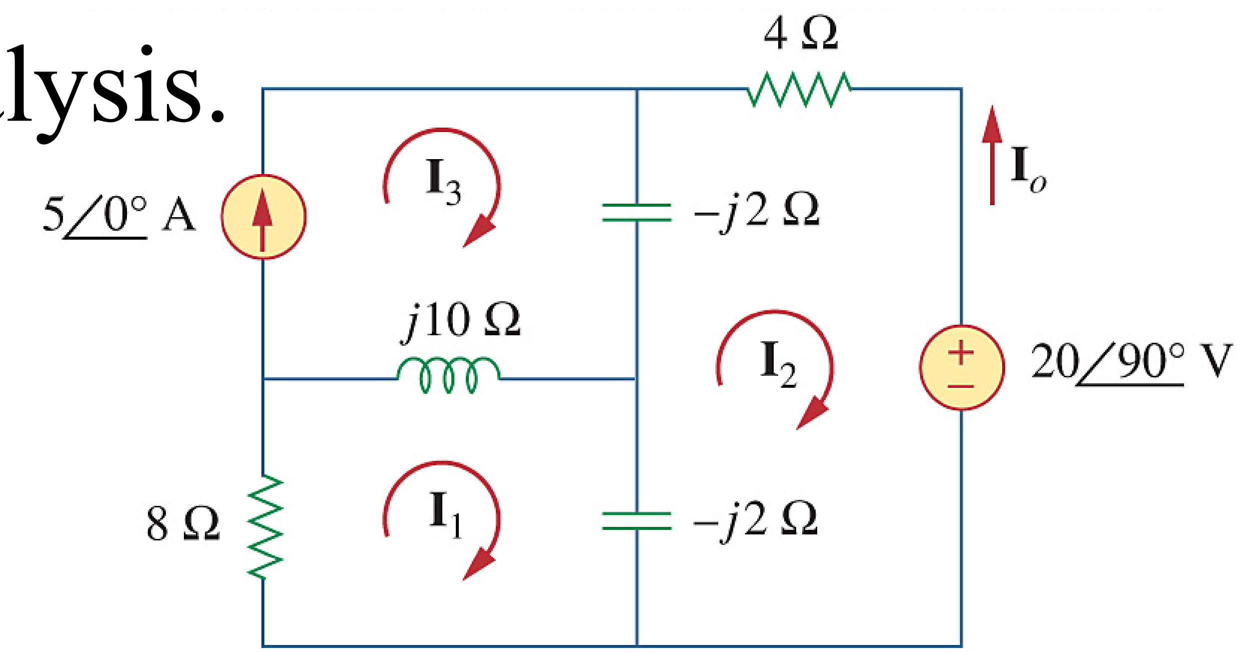


Figure 10.7

Solution :

$$\tilde{I}_3 = 5\angle 0° = 5 \text{ (A)}$$

$$20\angle 90° = j20$$

$$\begin{array}{c} \begin{array}{ccc} \Sigma_i R_i & -R_{ij} & \end{array} \\ \begin{bmatrix} 8+j10-j2 & j2 & -j10 \\ j2 & 4-j2-j2 & j2 \end{bmatrix} \end{array} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ -j20 \end{bmatrix}$$

I_3 is known, so just neglect 3rd row

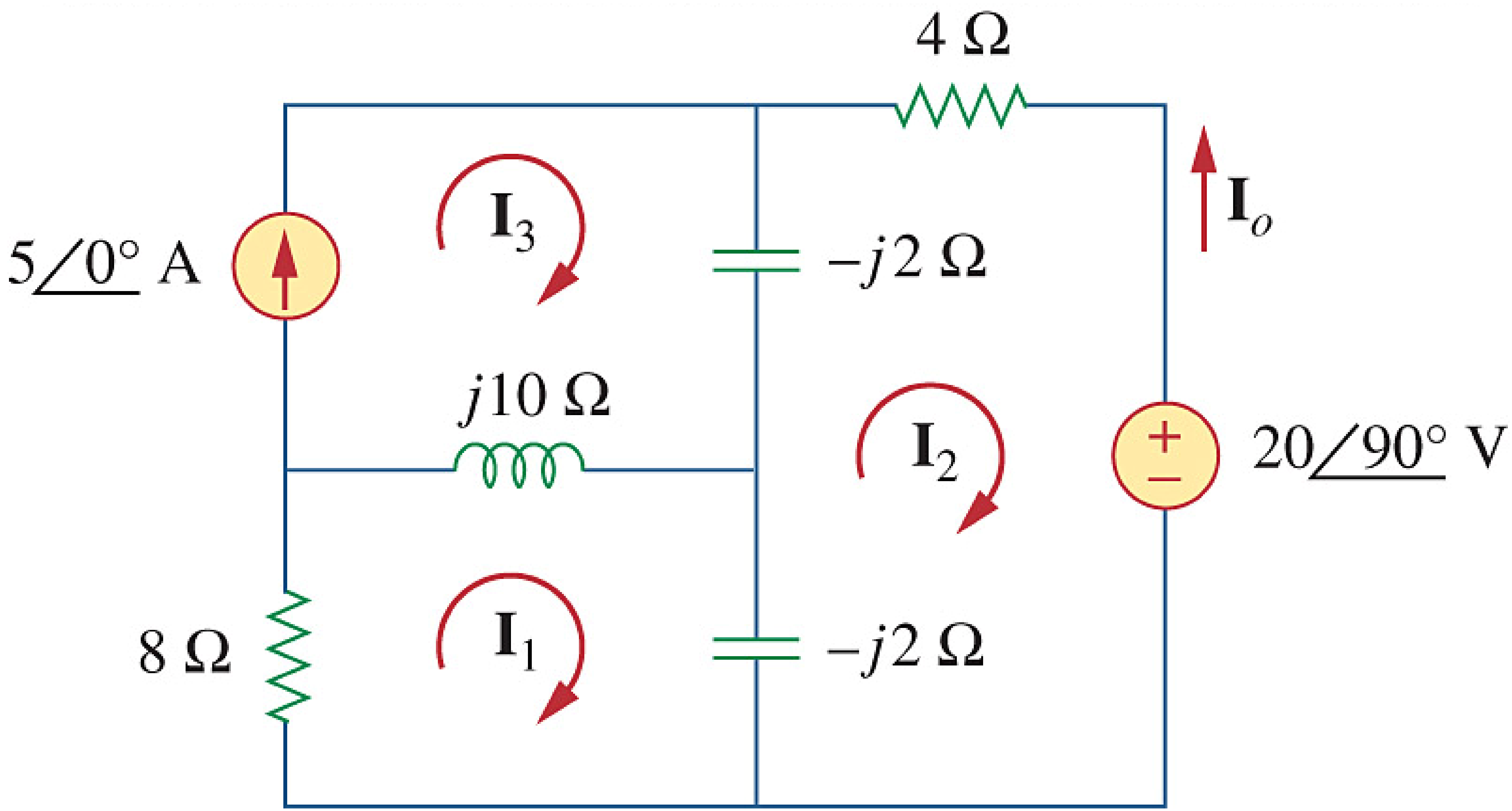

4 Ω
I_o
$5\angle 0°$ A
I_3
$-j2\ \Omega$
$j10\ \Omega$
I_2
$20\angle 90°$ V
+
−
8 Ω
I_1
$-j2\ \Omega$

Figure 10.7

$$\begin{bmatrix} 8+j8 & j2 & -j10 \\ j2 & 4-j4 & j2 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ -j20 \end{bmatrix}$$

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix} = 68$$

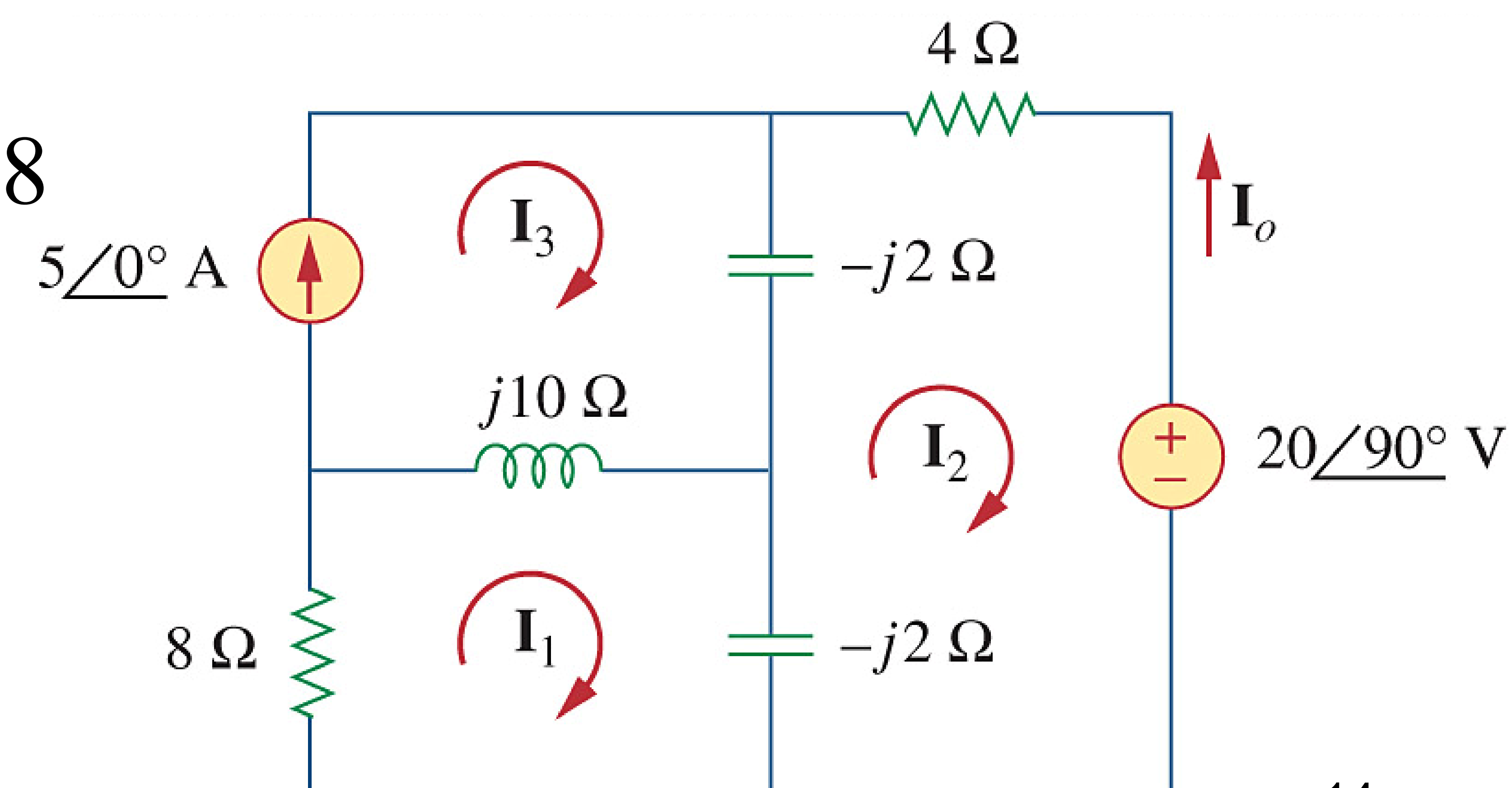


Figure 10.7

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240$$

$$\approx 416.1730\angle -35.22°$$

$$\tilde{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.1730\angle -35.22°}{68}$$

$$\approx 6.12\angle -35.22° \text{ (A)}$$

$$\tilde{I}_o = -\tilde{I}_2 = 6.12\angle 144.78° \text{ (A)}$$

Only step 2 is needed in this problem.

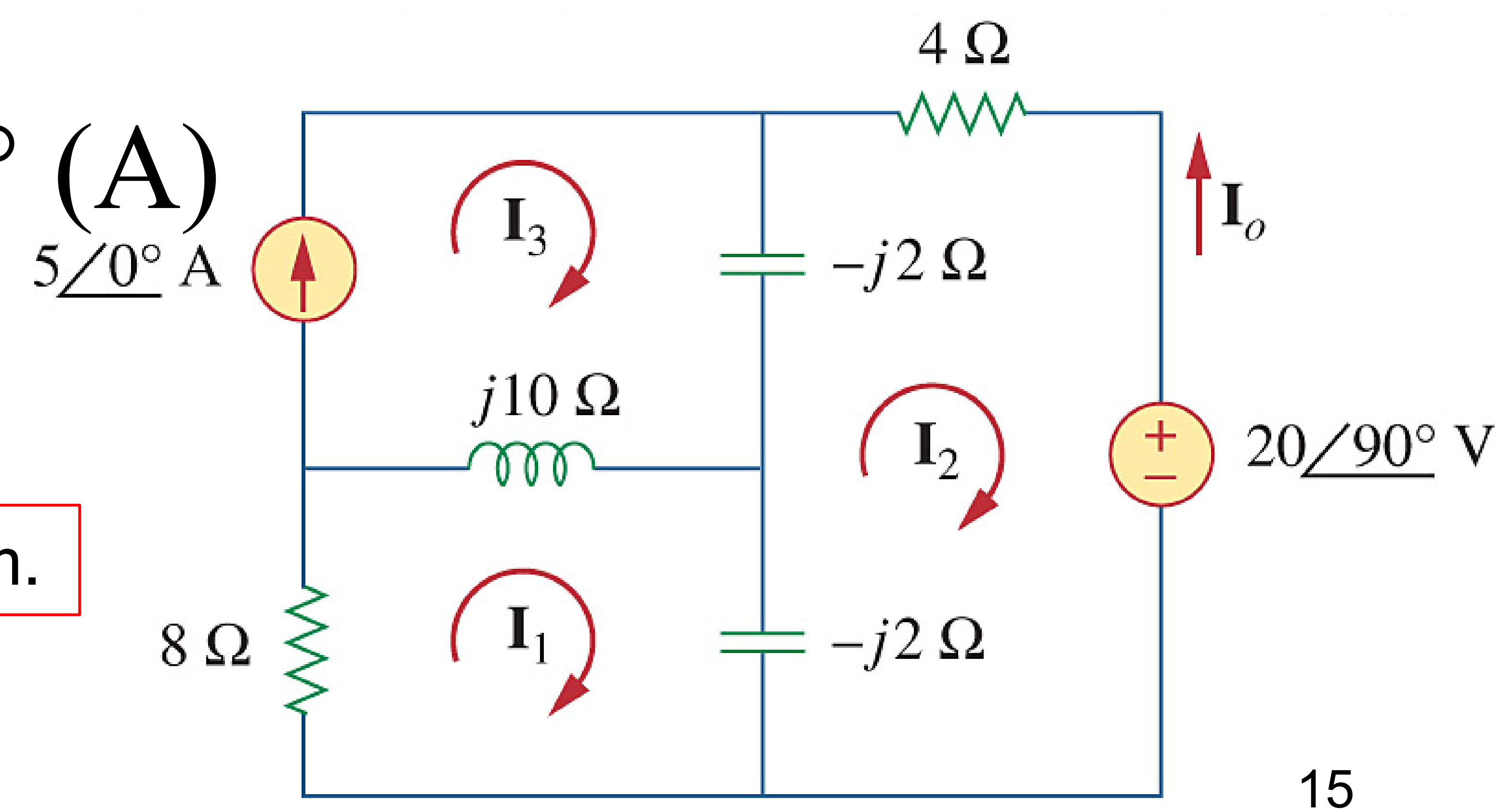


Figure 10.7

10.4 Superposition Theorem

The superposition theorem becomes important if the circuit has sources operating at *different* frequencies. In this case, since the impedances depend on frequency, we must have a different frequency domain circuit for each frequency. The total response must be obtained by adding the individual responses in the *time* domain.

Review of superposition

• Superposition principle is based on additivity.

• It states that whenever a linear system is excited, or driven, by more than one independent source, the total response is the sum of the individual responses.

•The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately and adding algebraically all the contributions to find the total contribution.

Example 10.6 Find v_o of the circuit of Fig. 10.13 using the superposition theorem.

Solution :

$v_o = v_1 + v_2 + v_3$

where v_1 is due to the 5-V dc source, v_2 is due to the $10\cos 2t$ V voltage source, and v_3 is due to the $2\sin 5t$ current source.

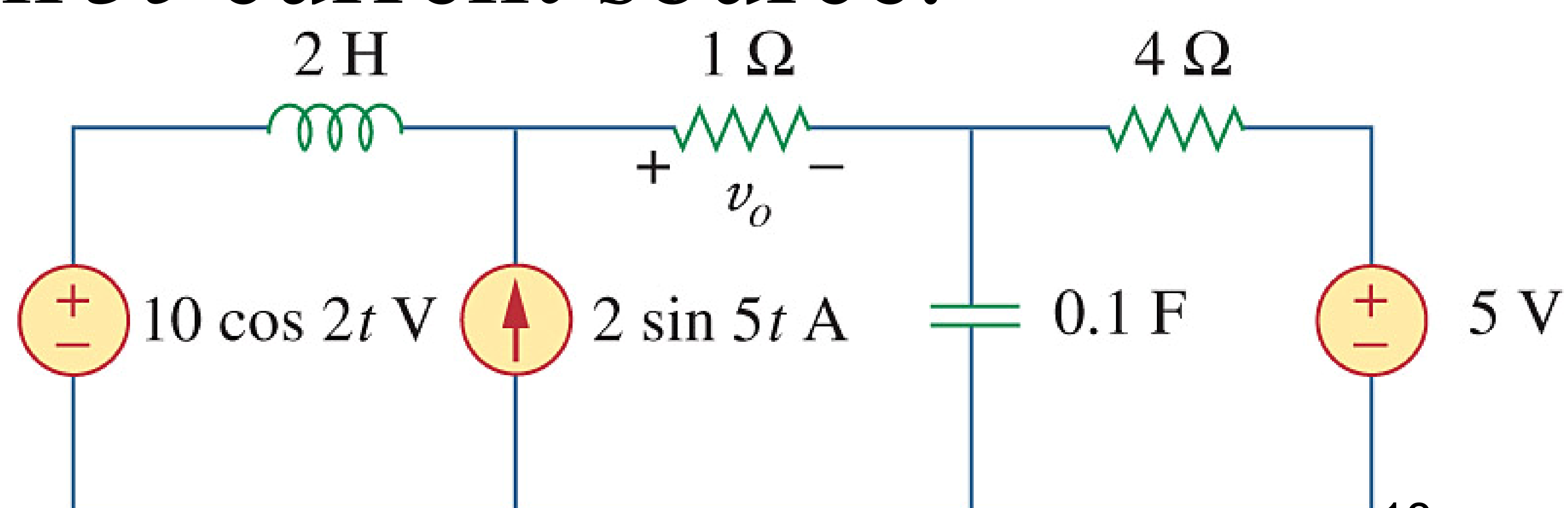


Figure 10.13

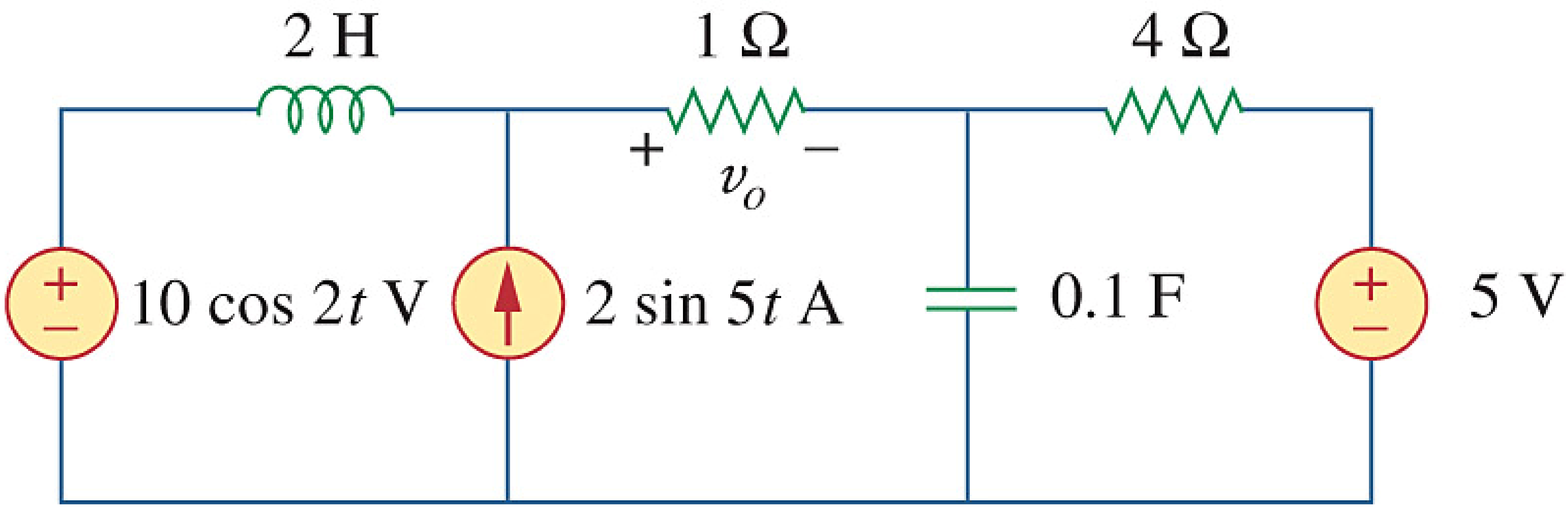


Figure 10.13

(1) From 5V source

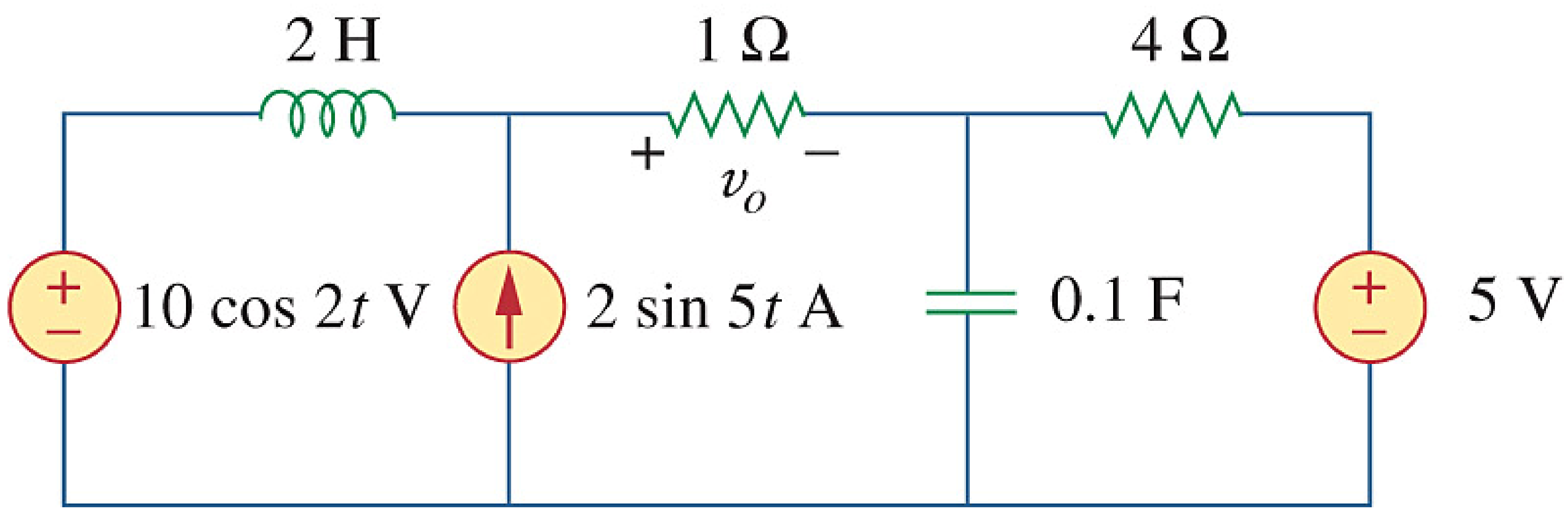


Figure 10.13

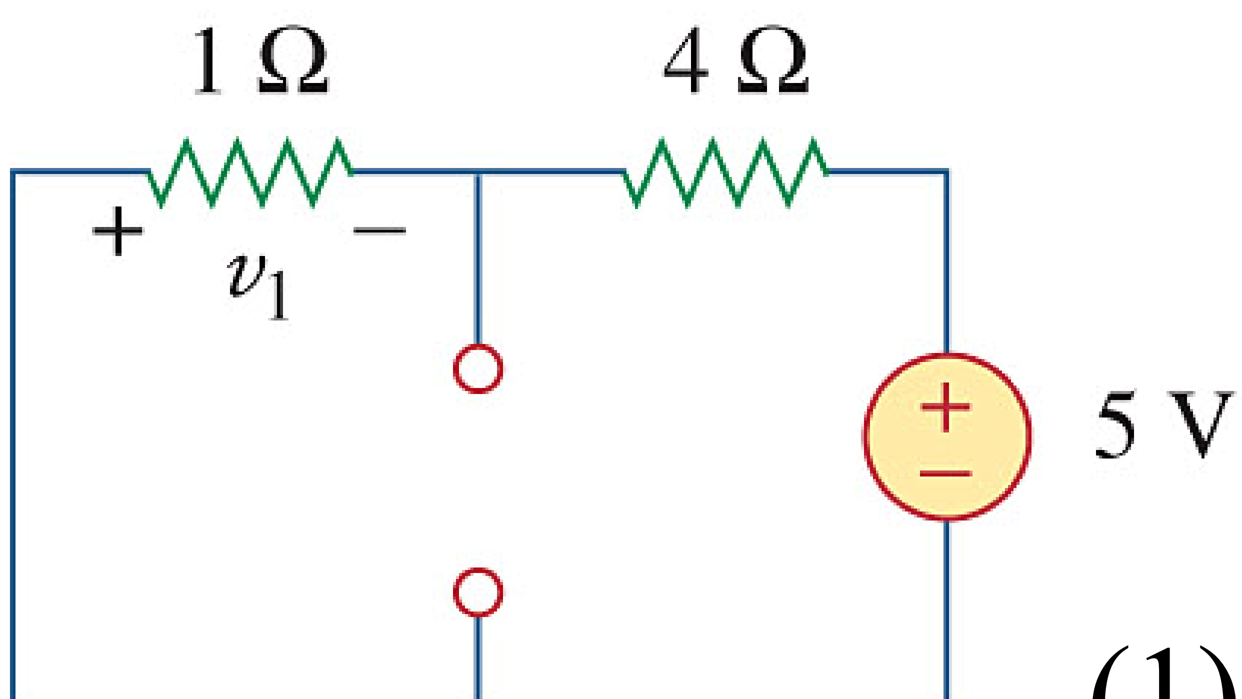


Figure 10.14(a)

$$(1)\ v_1 = -5 \times \frac{1}{1+4} = -1\ (\mathrm{V})$$

(2) From 10cos(2t) V source

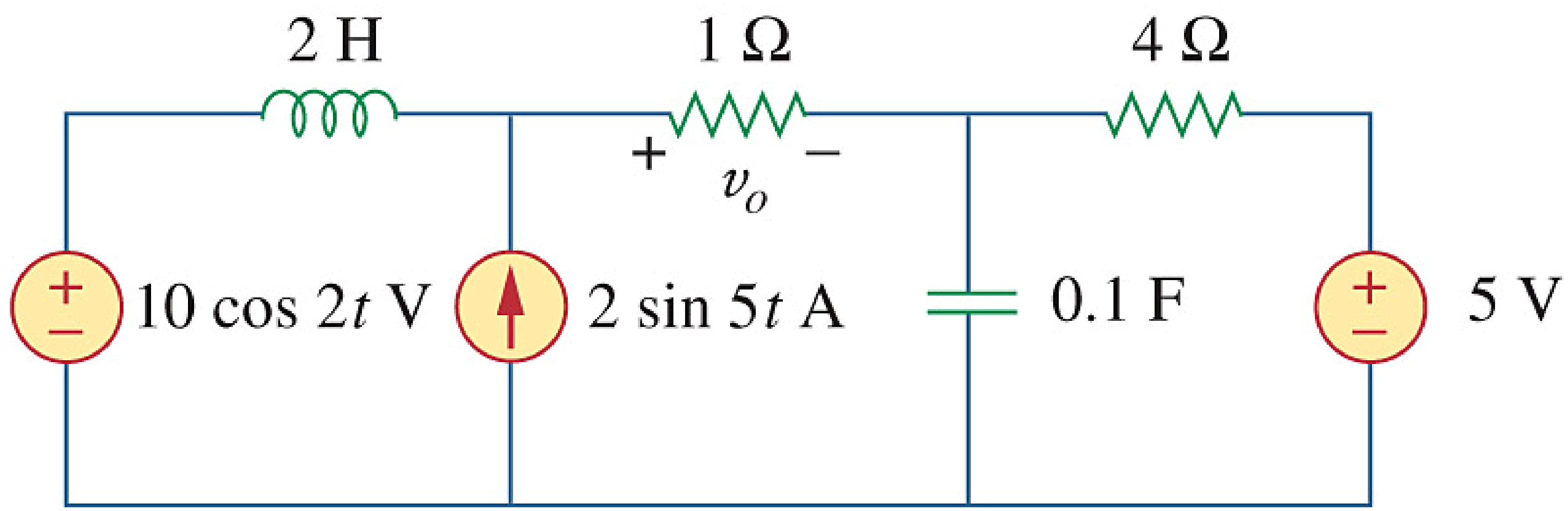


Figure 10.13

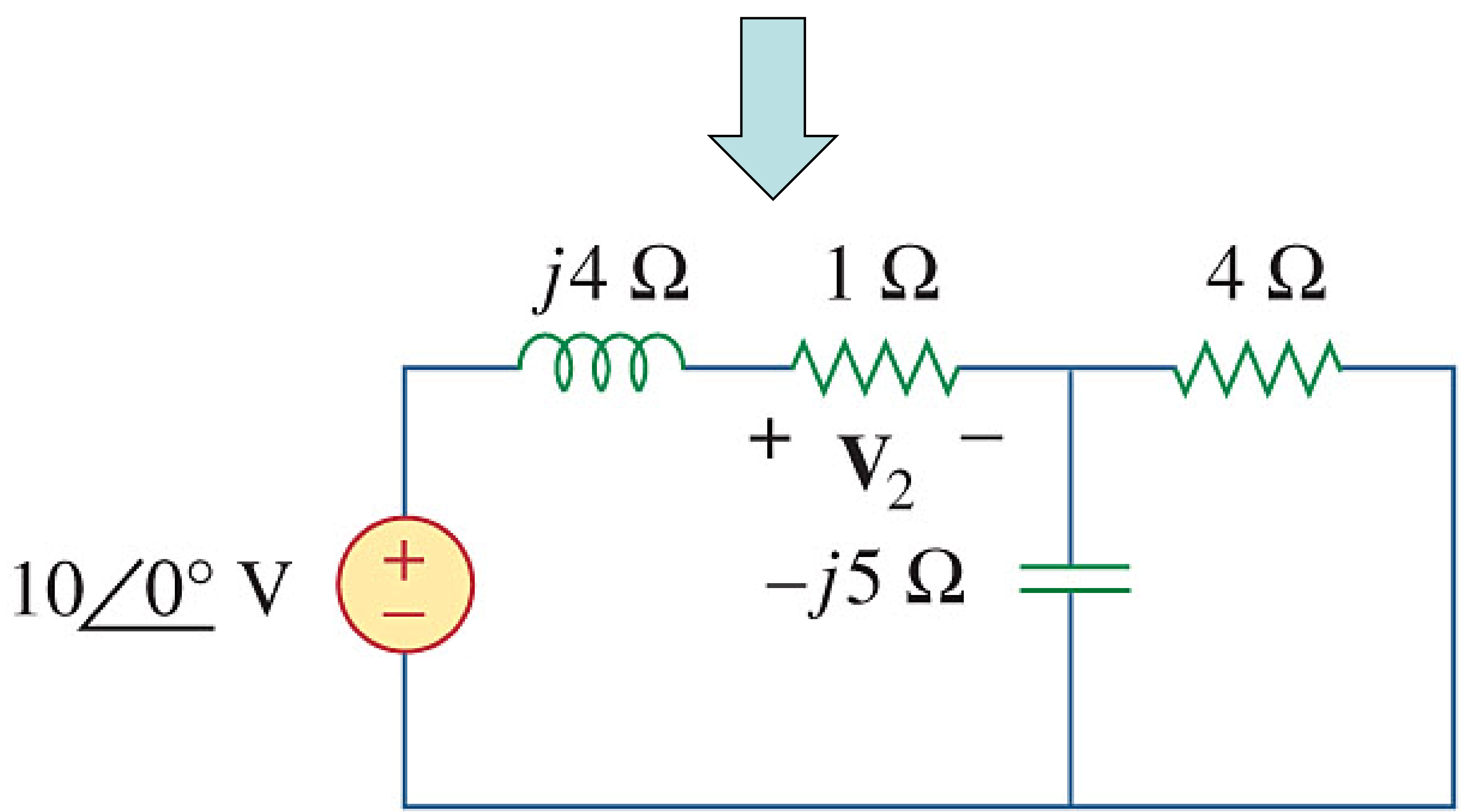


Figure 10.14(b)

Impedance is based on ω=2 rad/s

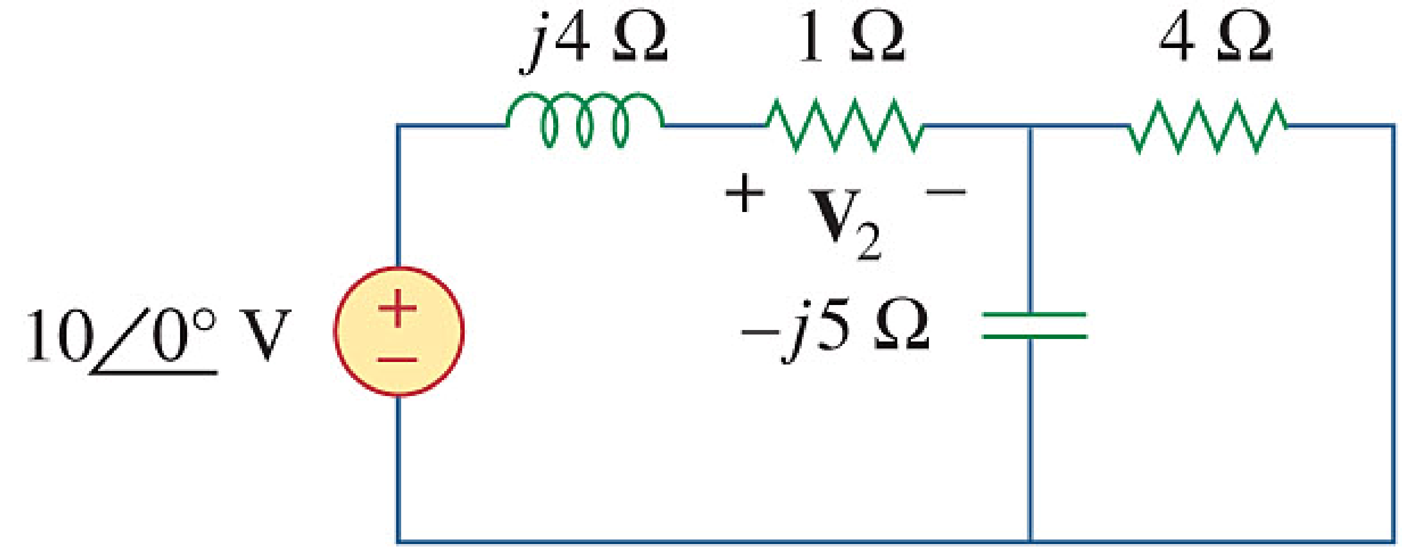


Figure 10.14(b)

$(2)\ v_2$

Step1

$$10\cos 2t\ \text{V} \Rightarrow 10\angle 0^\circ\ \text{V},\ \omega = 2\ \text{rad/s}$$

$$2\ \text{H} \Rightarrow j\omega L = j4\ (\Omega)$$

$$0.1\ \text{F} \Rightarrow 1/(j\omega C) = -j5\ (\Omega)$$

Step2

$$\tilde{V}_2 = 10\angle 0^\circ \frac{1}{j4 + 1 + (-j5) \parallel 4}$$

Voltage division

$$(-j5) \parallel 4 = \frac{(-j5)\times 4}{(-j5)+4} \approx \frac{20\angle -90^\circ}{6.4031\angle -51.34^\circ}$$

$$\approx 3.1235\angle -38.66° \approx 2.4390 - j1.9512$$

$$\tilde{V}_2 = 10\angle 0° \frac{1}{j4 + 1 + (2.4390 - j1.9512)}$$

$$= \frac{10\angle 0°}{3.4390 + j2.0488} = \frac{10\angle 0°}{4.0030\angle 30.78°}$$

$$\approx 2.50\angle -30.78° \text{ (V)}$$

Step3

$$v_2 = 2.50\cos(2t - 30.78°) \text{ (V)}$$

Complex number calculation:
+,–: rectangular form
×,÷: polar form

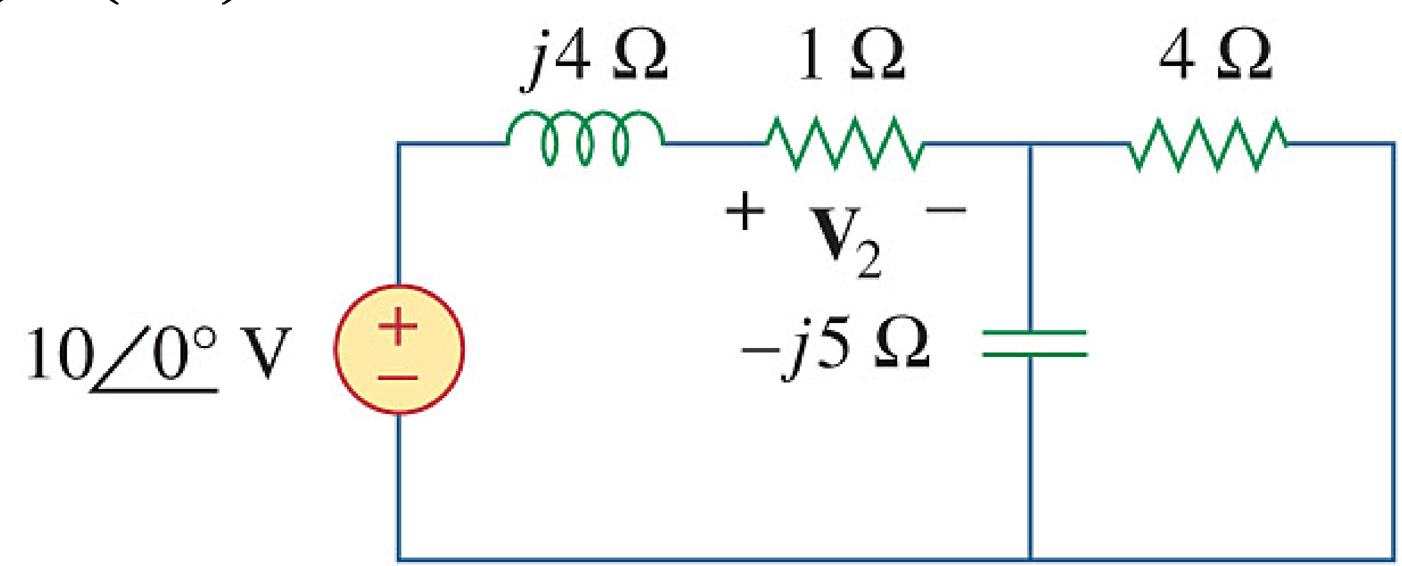


Figure 10.14(b)

(3) From 2sin(5t) V source

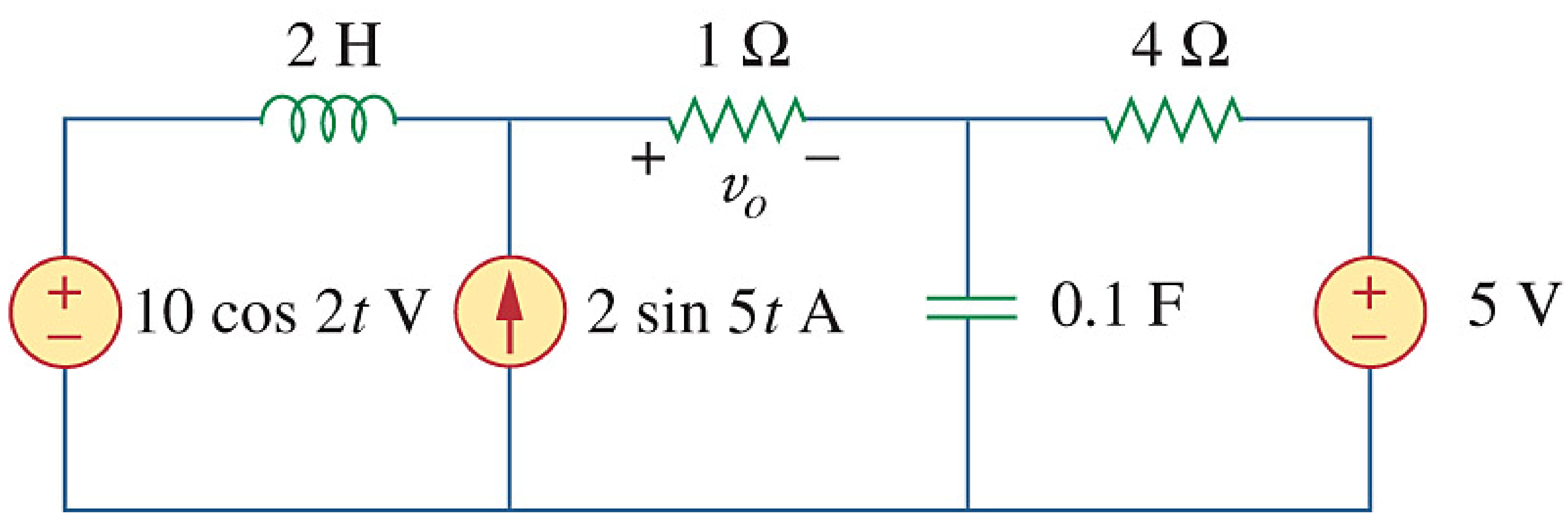


Figure 10.13

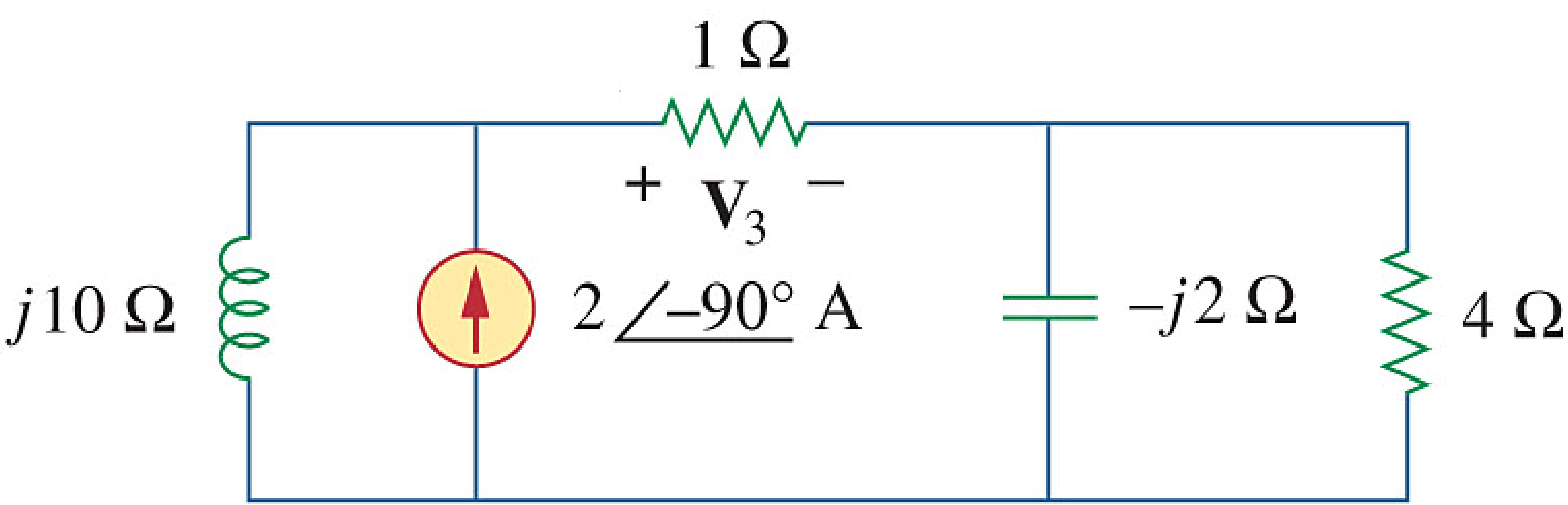


Figure 10.14(c)

Impedance is based on ω=5 rad/s

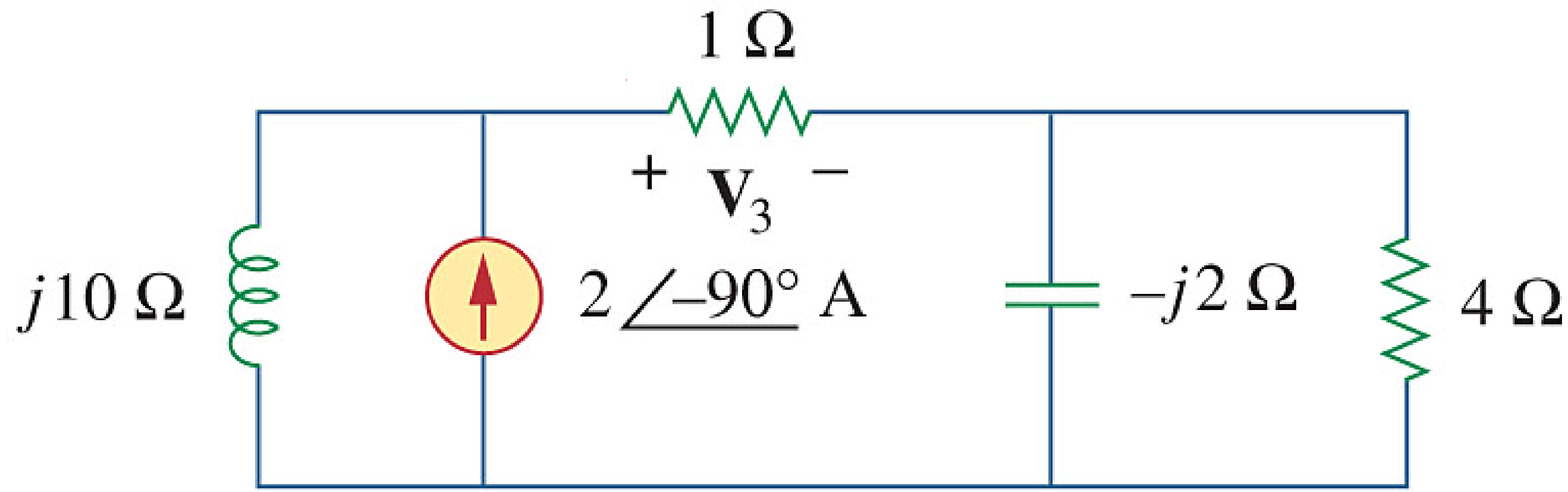


Figure 10.14(c)

$$(3)\ v_3$$

Step1

$$2\sin 5t\ \text{A} \Rightarrow 2\angle\underline{-90^\circ}\ \text{A},\ \omega = 5\ \text{rad/s}$$

sin(5t) = cos(5t–90°)

$$2\ \text{H} \Rightarrow\ j\omega L = j10\ (\Omega)$$

$$0.1\ \text{F} \Rightarrow 1/(j\omega C) = -j2\ (\Omega)$$

Step2

$$\tilde{V}_3 = (2\angle -90^\circ)\times(j10)\times\frac{1}{j10+1+(-j2)\parallel 4}$$

$$(-j2)\parallel 4 = \frac{(-j2)\times 4}{(-j2)+4} = 0.8 - j1.6$$

$$\tilde{V}_3 = (2\angle -90°)\times(j10)\times\frac{1}{j10+1+(0.8-j1.6)}$$

$$= \frac{20}{1.8+j8.4} \approx \frac{20}{8.5907\angle 77.91°}$$

$$\approx 2.33\angle -77.91° \text{ (V)}$$

Step3

$$v_3 = 2.33\cos(5t-77.91°) \text{ V}$$

$$\boxed{\begin{aligned} v &= -1+2.50\cos(2t-30.78°) \\ &+2.33\cos(5t-77.91°) \text{ (V)} \end{aligned}}$$

If **I**=$2\angle 0°$ is used,
$\mathbf{V}_3=2.33\angle 12.09°$
$v_3=2.33\sin(5t+12.09°)$
$=2.33\cos(5t-77.91°)$

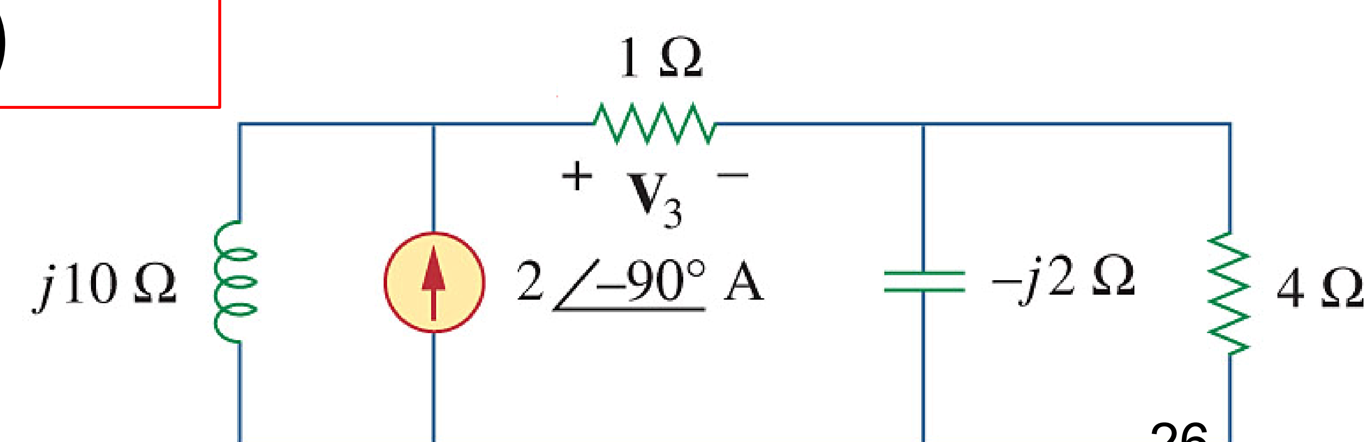


Figure 10.14(c)

10.5 Source Transformation

As Fig. 10.16 shows, source transformation in the frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance, or vice versa.

Review of source transformation

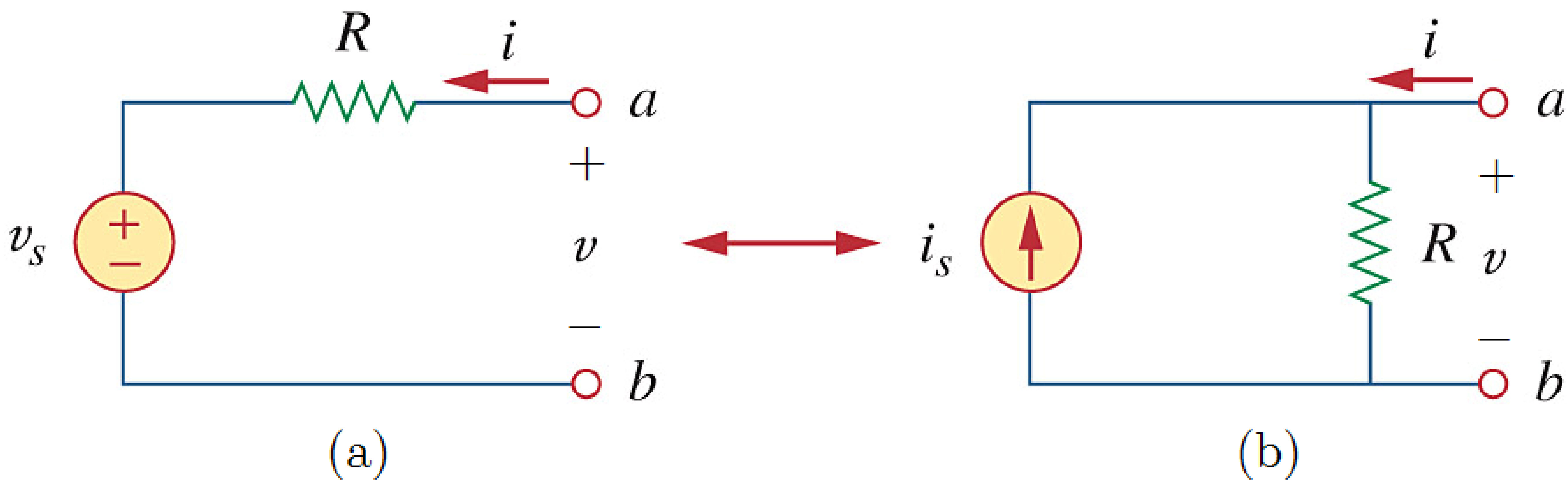
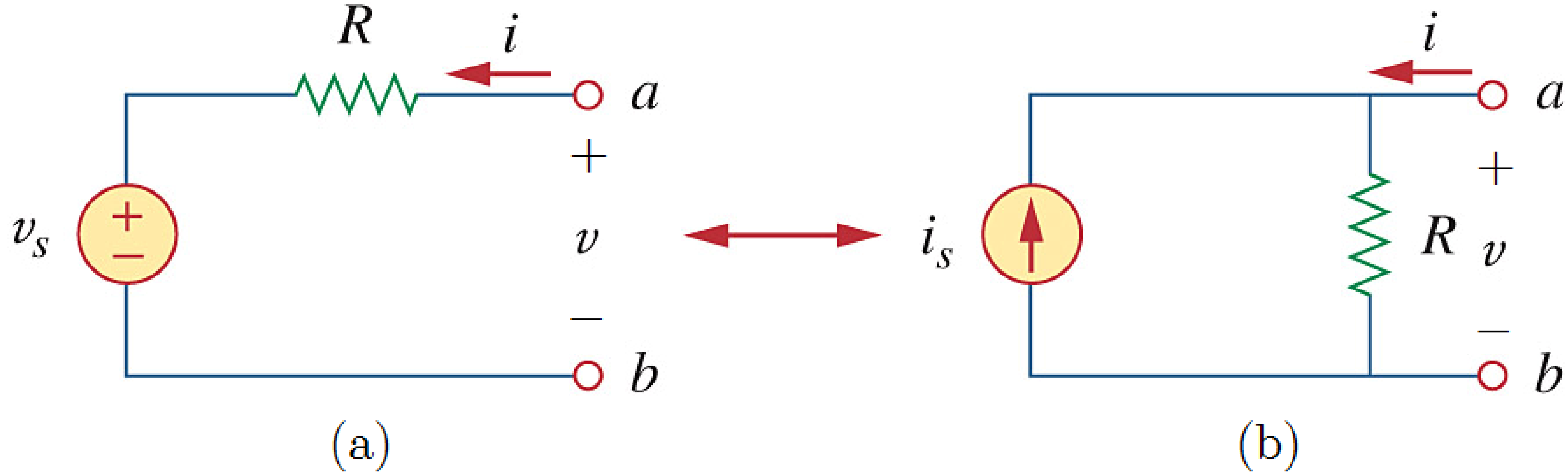


Figure 4.15 Transformation of independent sources .



Proof

Figure 4.15 Transformation of independent sources .

Two circuits are said to be equivalent if they have the same i - v relation. — Terminal voltage and current

For circuit in Fig. 4.15(a), $v = iR + v_s$

For circuit in Fig. 4.15(b), $v = iR + i_s R$

Obviously, the two i - v relations are identical provided that $v_s = i_s R$ or $i_s = v_s / R$.

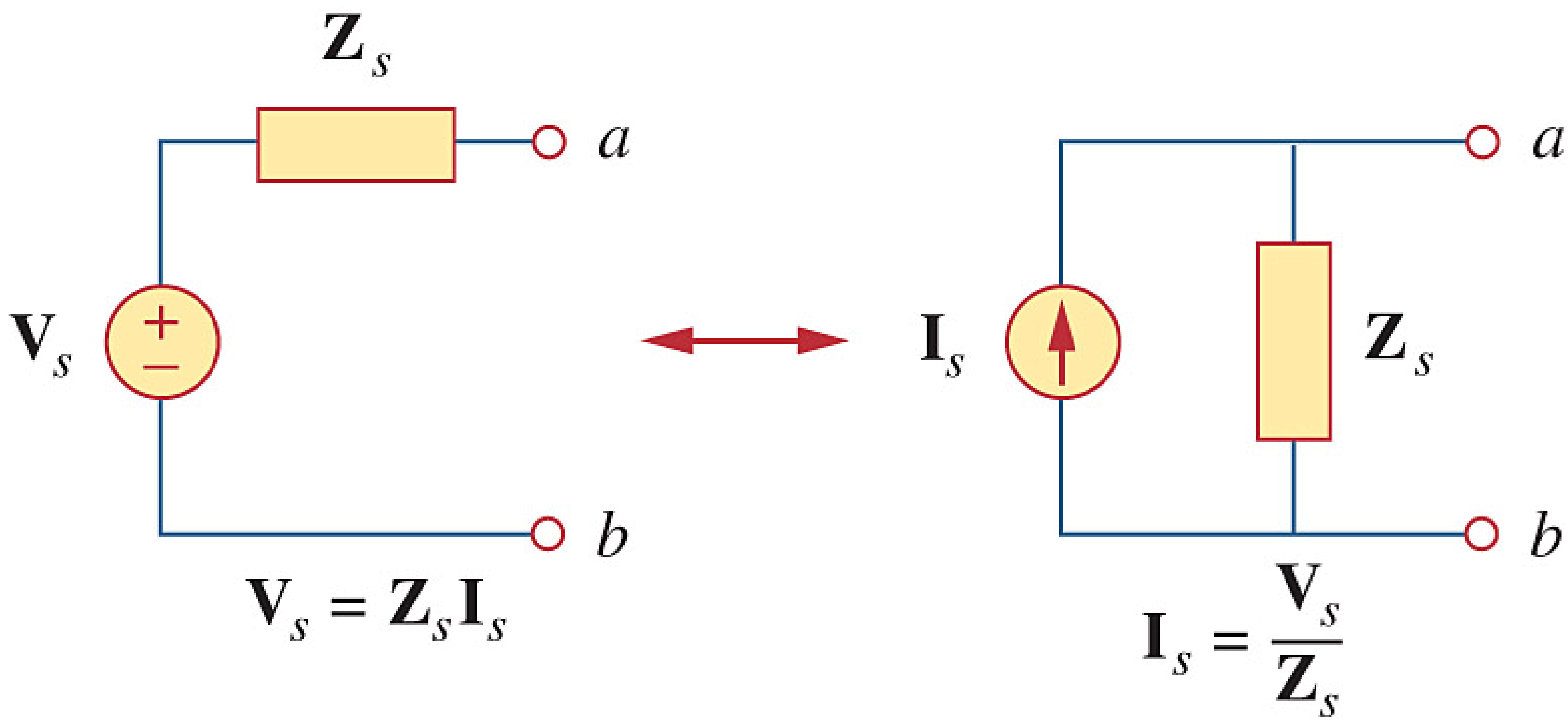


Figure 10.16 Source transformation.

Practice Problem 10.7 Find $\dot{I}_o$ in the circuit of Fig. 10.19 using the concept of source transformation.

Solution :

$$\begin{cases} \tilde{I}_1 = 8\angle 90^\circ \text{ A} \\ Z_1 = 4 - j3\ \Omega \end{cases} \Rightarrow \begin{cases} \tilde{V}_1 = 24 + j32\ \text{(V)} \\ Z_1 = 4 - j3\ \Omega \end{cases}$$

$$\tilde{V}_1 = \tilde{I}_1 Z_1 = 8\angle 90^\circ \times (4 - j3) = j8 \times (4 - j3)$$

$$= 24 + j32\ \text{(V)}$$

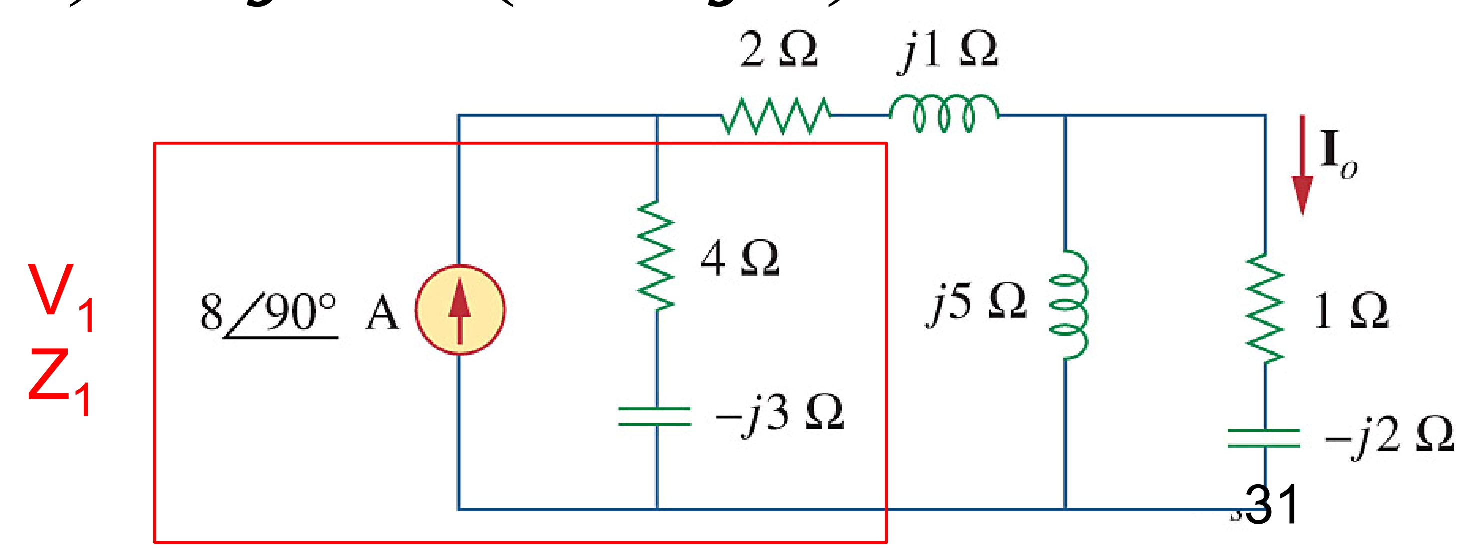


Figure 10.19

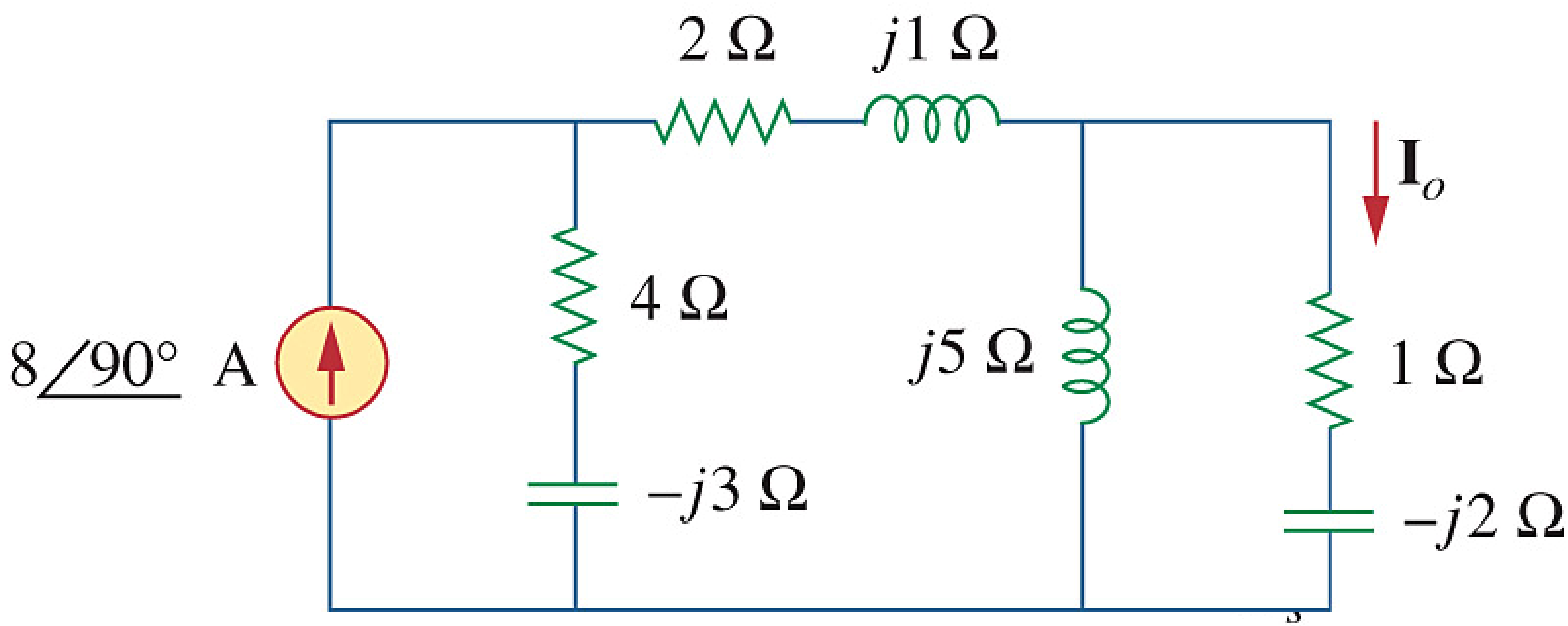


Figure 10.19

$$Z_2 = Z_1 + (2 + j1) = (4 - j3) + (2 + j1)$$

$$= 6 - j2 \ (\Omega)$$

$$\begin{cases} \tilde{V}_1 = 24 + j32 \text{ V} \\ Z_2 = 6 - j2 \ \Omega \end{cases} \Rightarrow \begin{cases} \tilde{I}_2 = 6.3245\angle 71.56^\circ \text{ A} \\ Z_2 = 6 - j2 \ \Omega \end{cases}$$

$$\tilde{I}_2 = \frac{\tilde{V}_1}{Z_2} = \frac{24 + j32}{6 - j2} = \frac{40\angle 53.13^\circ}{6.3246\angle -18.43^\circ}$$

$$\approx 6.3245\angle 71.56^\circ \ (\text{A})$$

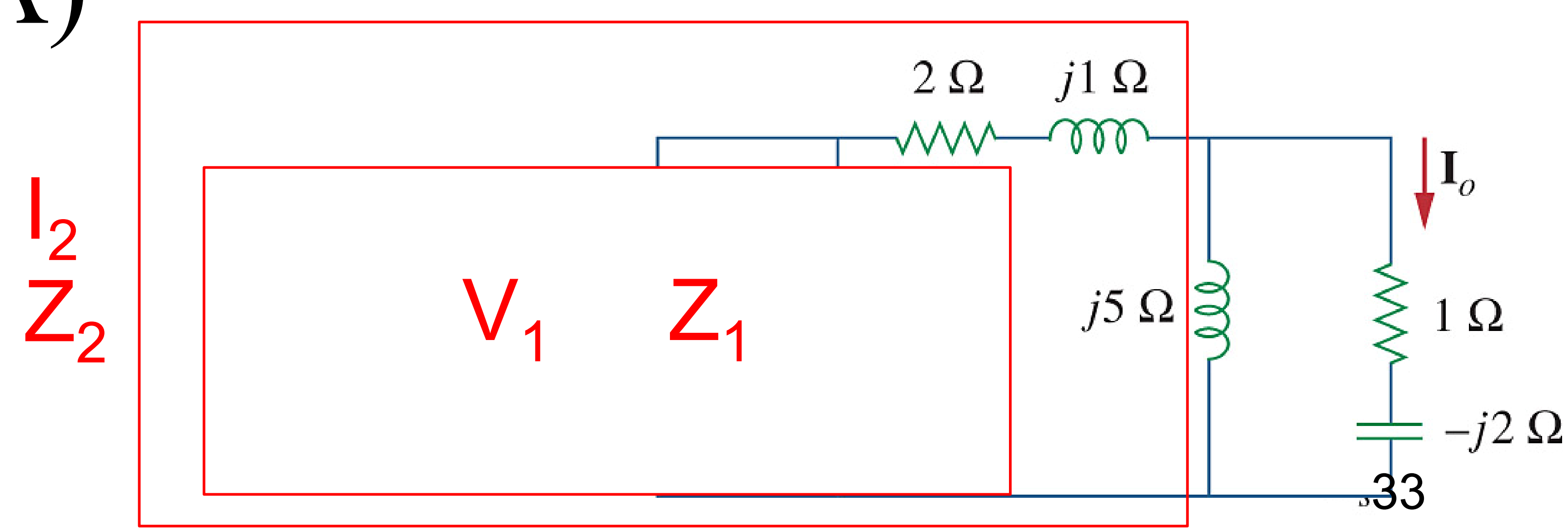


Figure 10.19

$$\tilde{I}_o = \tilde{I}_2 \frac{1/(1-j2)}{1/(1-j2)+1/(j5)+1/(6-j2)}$$

Current division

$I_o = I_2 \times (Y_1/(Y_1+Y_2+Y_3))$

$$\frac{1/(1-j2)}{1/(1-j2)+1/(j5)+1/(6-j2)}$$

$$= \frac{1}{1+(1-j2)/(j5)+(1-j2)/(6-j2)}$$

$$= \frac{1}{1+(-2-j)/5+(1-j)/4}$$

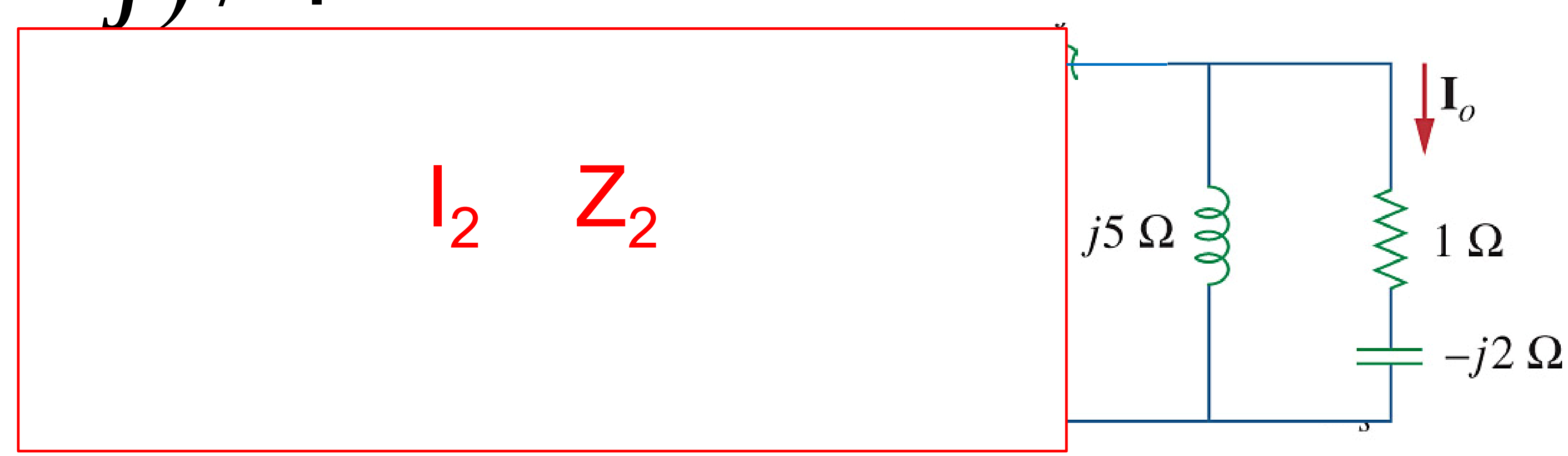


Figure 10.19

$$= \frac{20}{17 - j9}$$

$$\approx 1.0398\angle 27.90°$$

$$\tilde{I}_o = \tilde{I}_2 \times 1.0398\angle 27.90°$$

$$= 6.3245\angle 71.56° \times 1.0398\angle 27.90°$$

$$\approx 6.58\angle 99.46° \text{ (A)}$$

10.6 Thevenin and Norton Equivalent Circuits

The frequency domain version of a Thevenin equivalent circuit is depicted in Fig. 10.20. The Norton equivalent circuit is illustrated in Fig. 10.21.

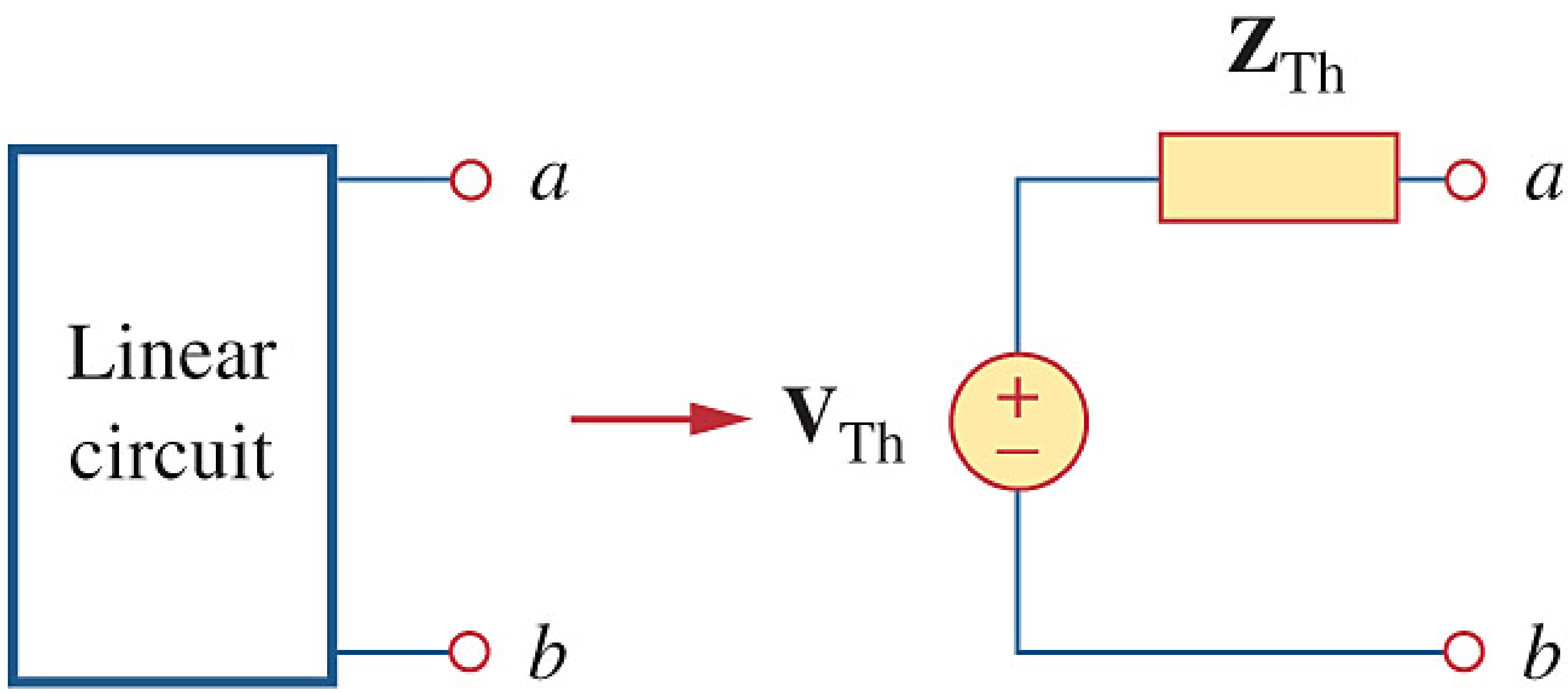


Figure 10.20 Thevenin equivalent.

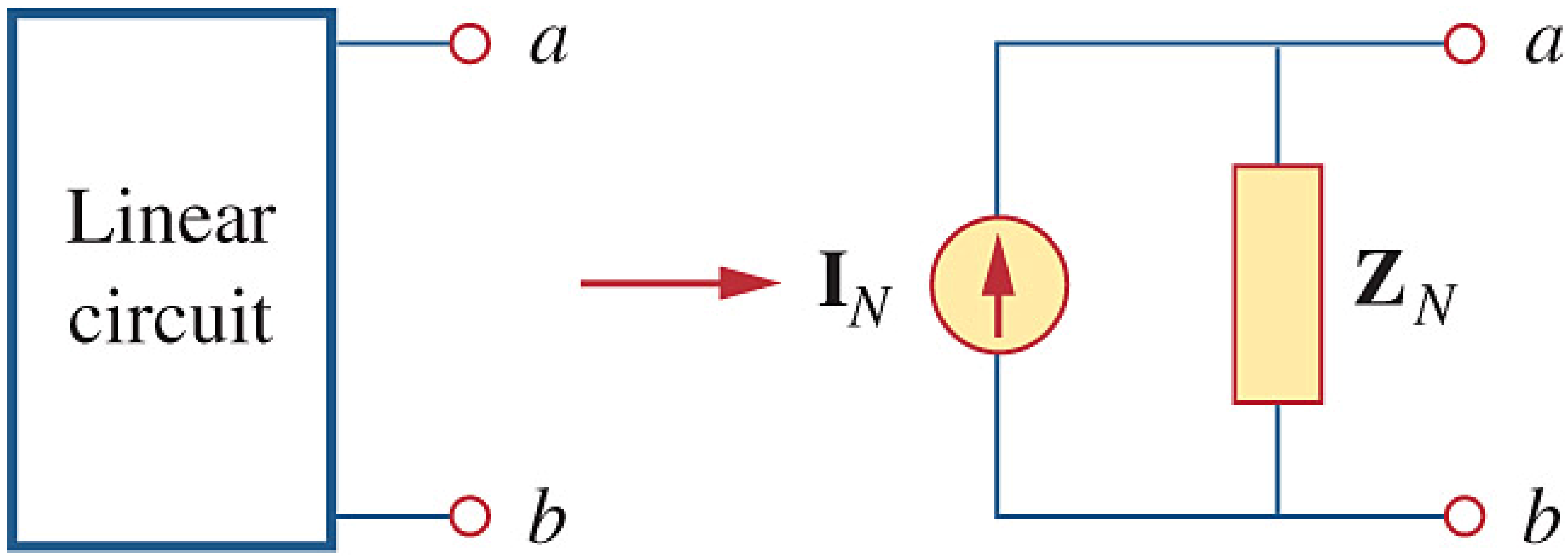
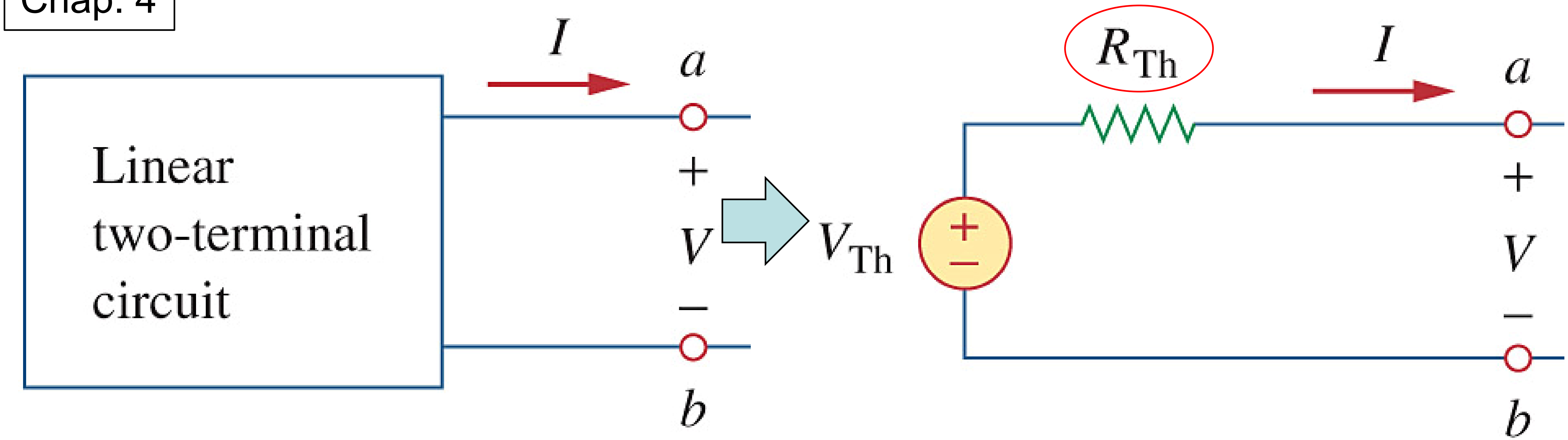


Figure 10.21 Norton equivalent.

Chap. 4



Chap. 10

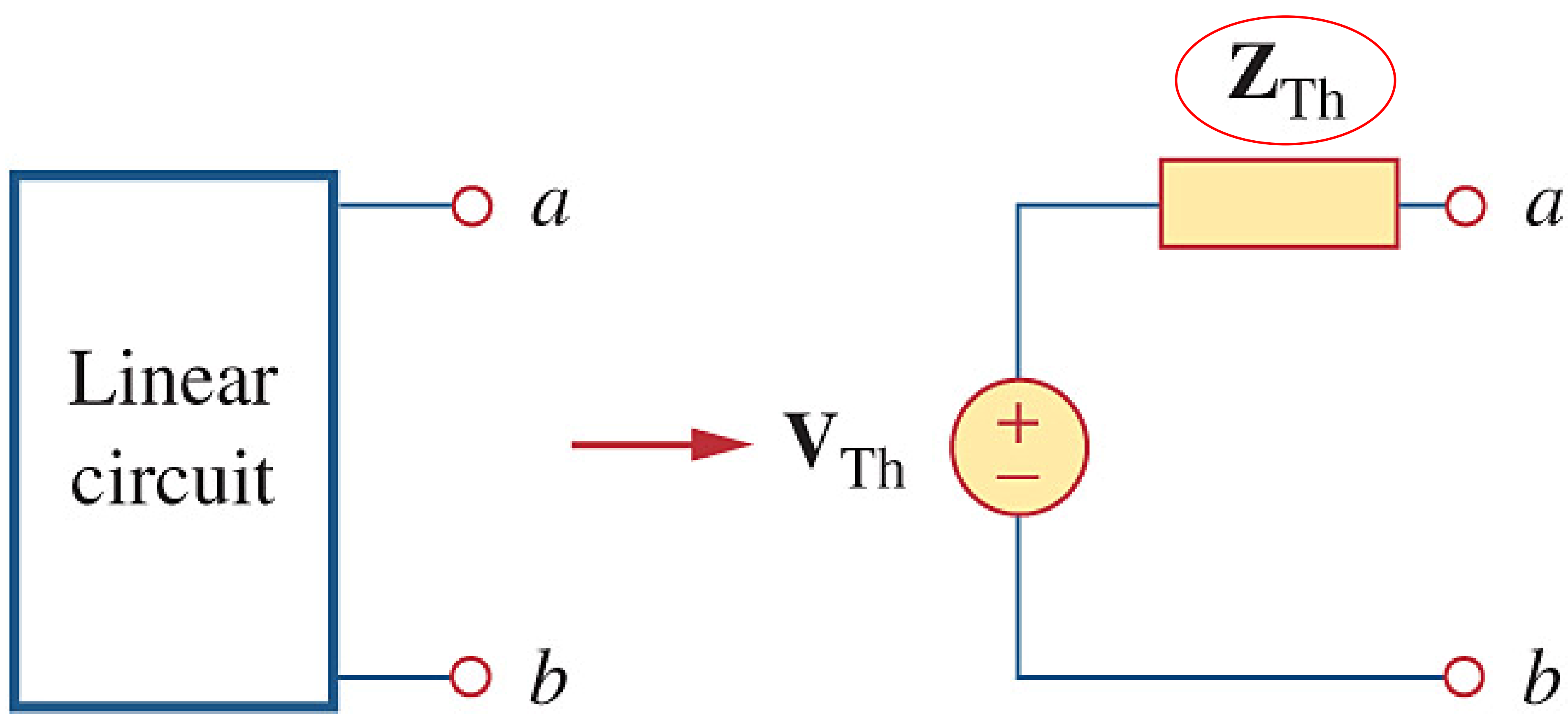
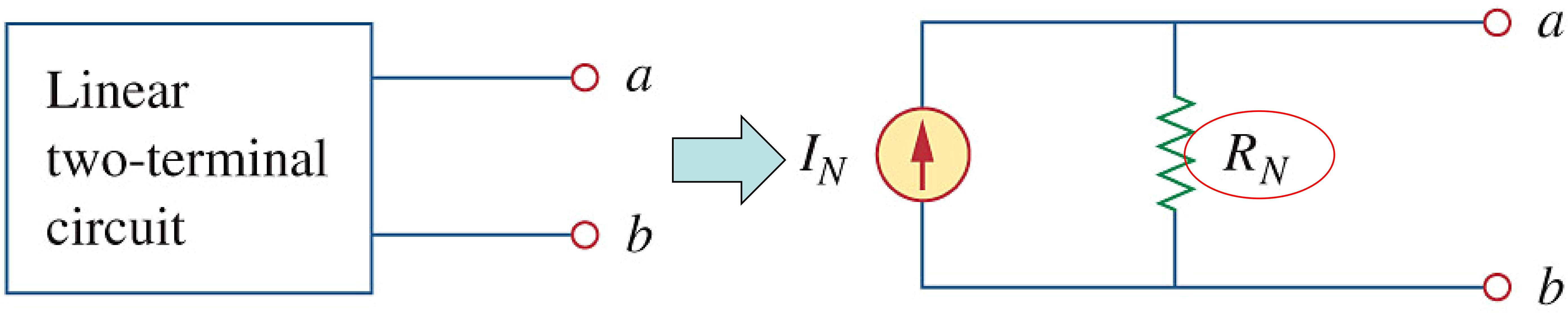


Figure 10.20 Thevenin equivalent.

Chap. 4



Chap. 10

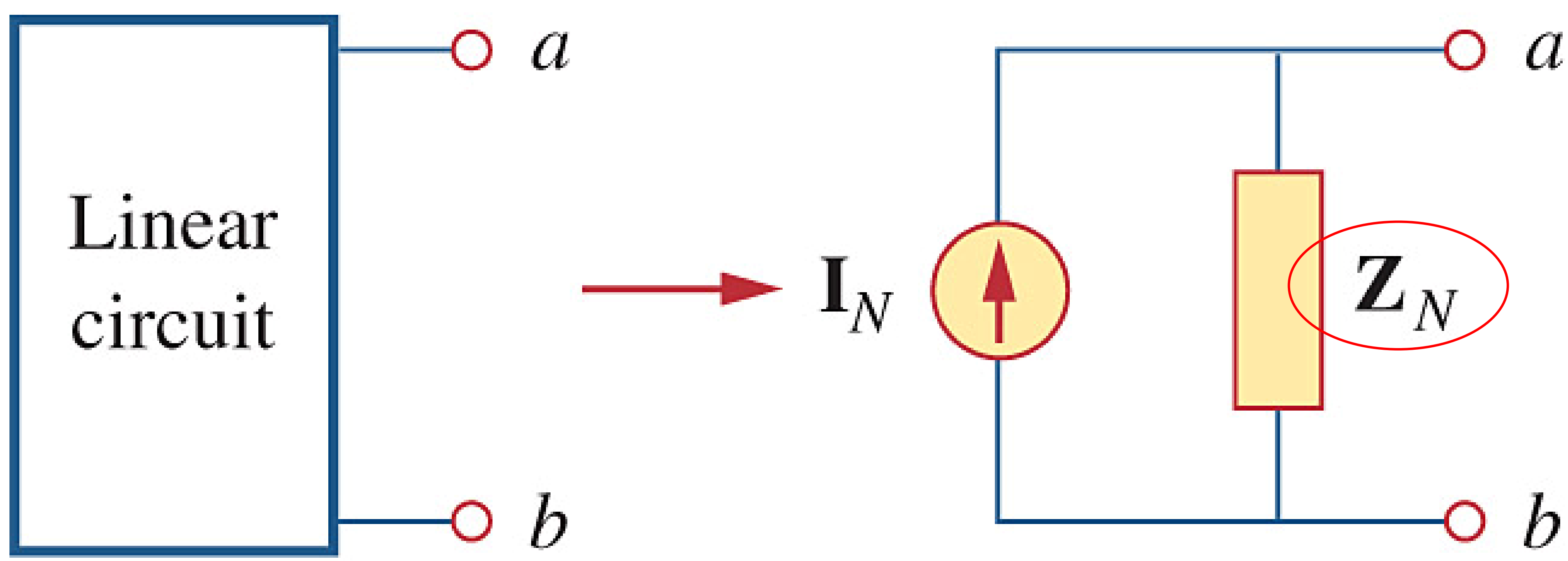


Figure 10.21 Norton equivalent.

Practice Problem 10.10 Determine the Norton equivalent of the circuit in Fig. 10.30 as seen from terminals *a*-*b*. Use the equivalent to find $\tilde{I}_o$.

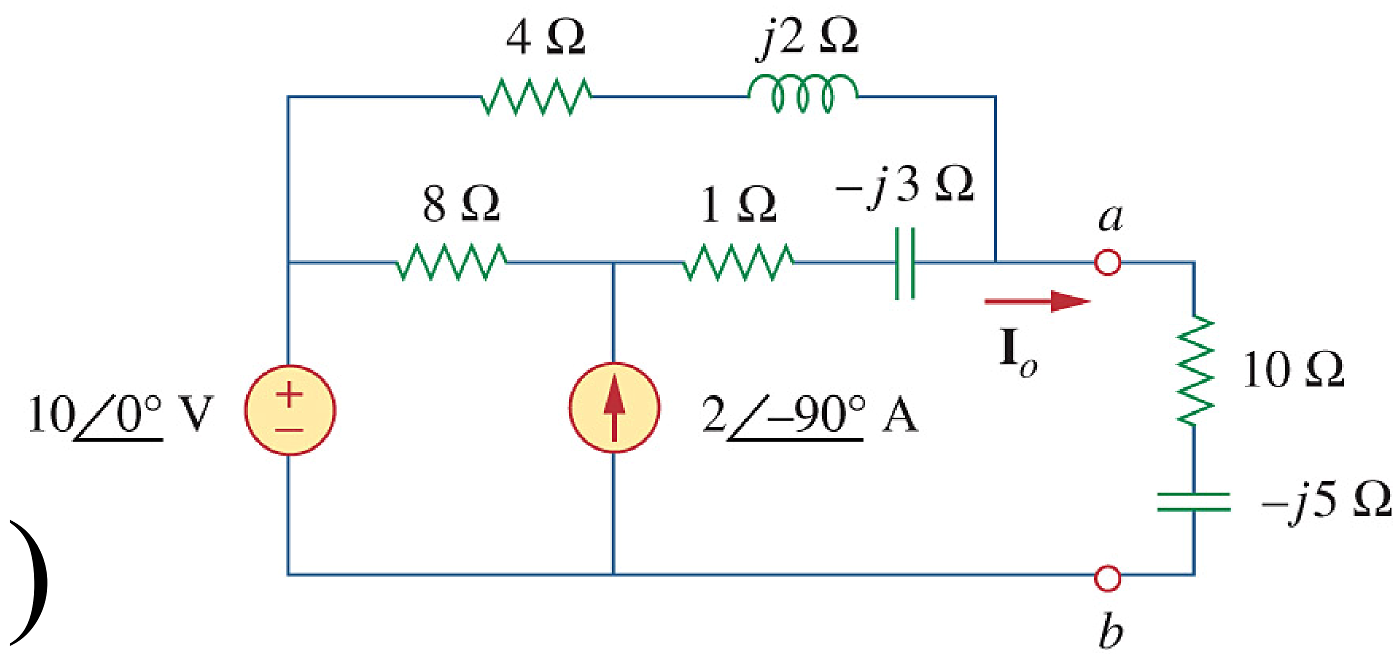


Figure 10.30

Solution :

$$Z_N = (4 + j2) \,||\, (8 + 1 - j3)$$

$$= \frac{(4 + j2)(9 - j3)}{(4 + j2) + (9 - j3)} = \frac{6(7 + j)}{13 - j} = \frac{6(9 + j2)}{17}$$

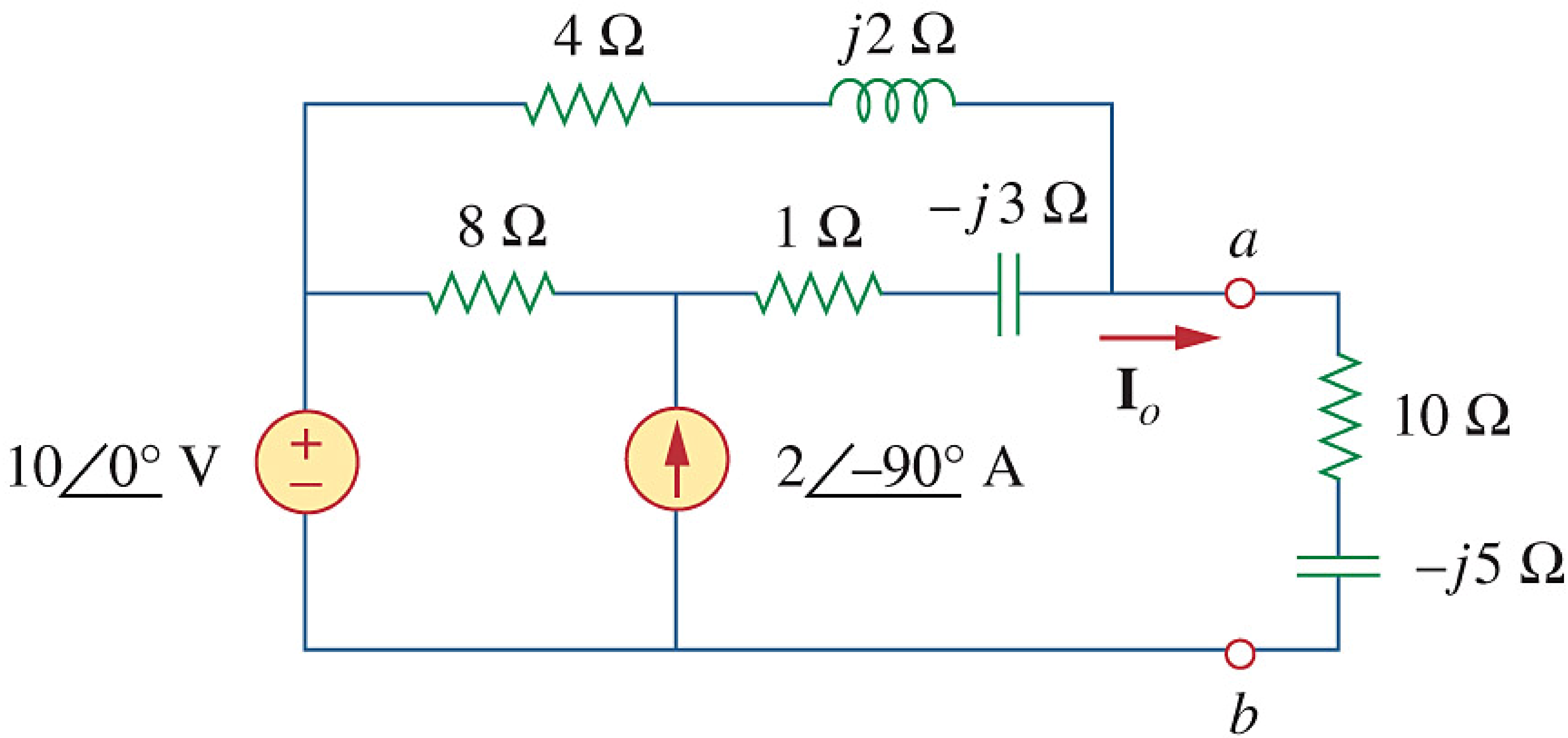


Figure 10.30

$$\approx 3.1765 + j0.7059\ (\Omega)$$

Use superposition to find $\dot{I}_N$:

Recall: $I_N = I_{SC}$

$$\tilde{I}_N = \tilde{I}'_N + \tilde{I}''_N$$

$$\tilde{I}'_N = \frac{10\angle 0^\circ}{(4 + j2) \parallel (8 + 1 - j3)}$$

$$= \frac{10}{6(7 + j)/(13 - j)} = \frac{9 - j2}{3}$$

$$\approx 3 - j0.6667\ (\text{A})$$

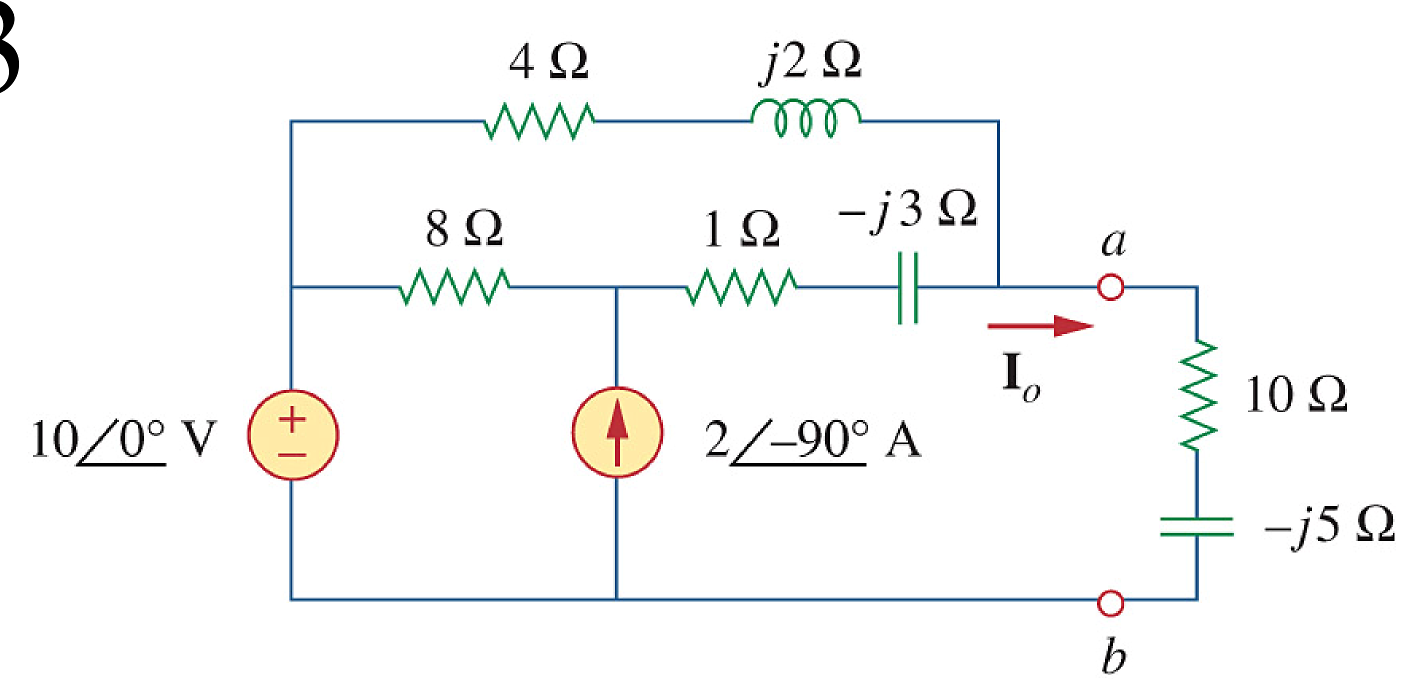


Figure 10.30

Current division

$$\tilde{I}''_N = 2\angle -90^\circ \times \frac{8}{8+1-j3}$$

$$\approx 0.5333 - j1.6 \text{ (A)}$$

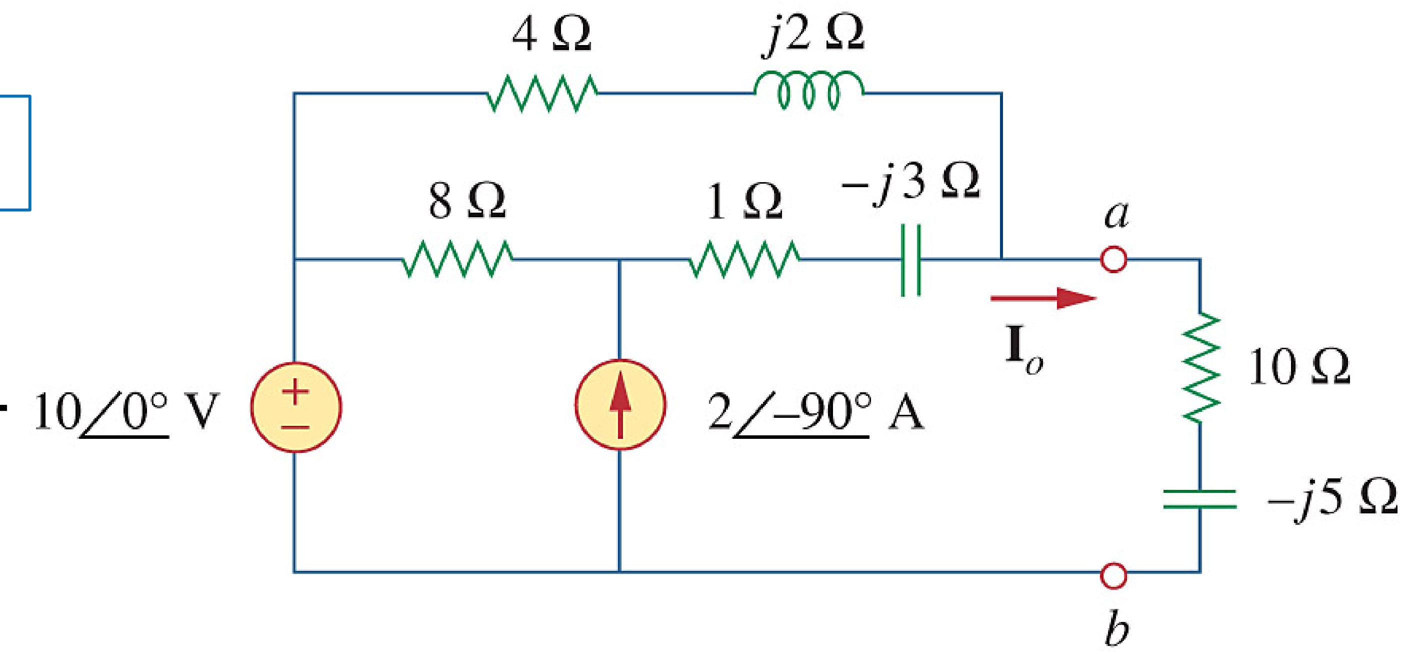


Figure 10.30

$$\tilde{I}_N = 3.5333 - j2.2667$$

$$\approx 4.1979\angle -32.68^\circ \text{ (A)}$$

$$\tilde{I}_o = \tilde{I}_N \frac{Z_N}{Z_N + (10 - j5)} = 4.1979\angle -32.68^\circ \times$$

$$\frac{3.1765 + j0.7059}{(3.1765 + j0.7059) + (10 - j5)}$$

About current division

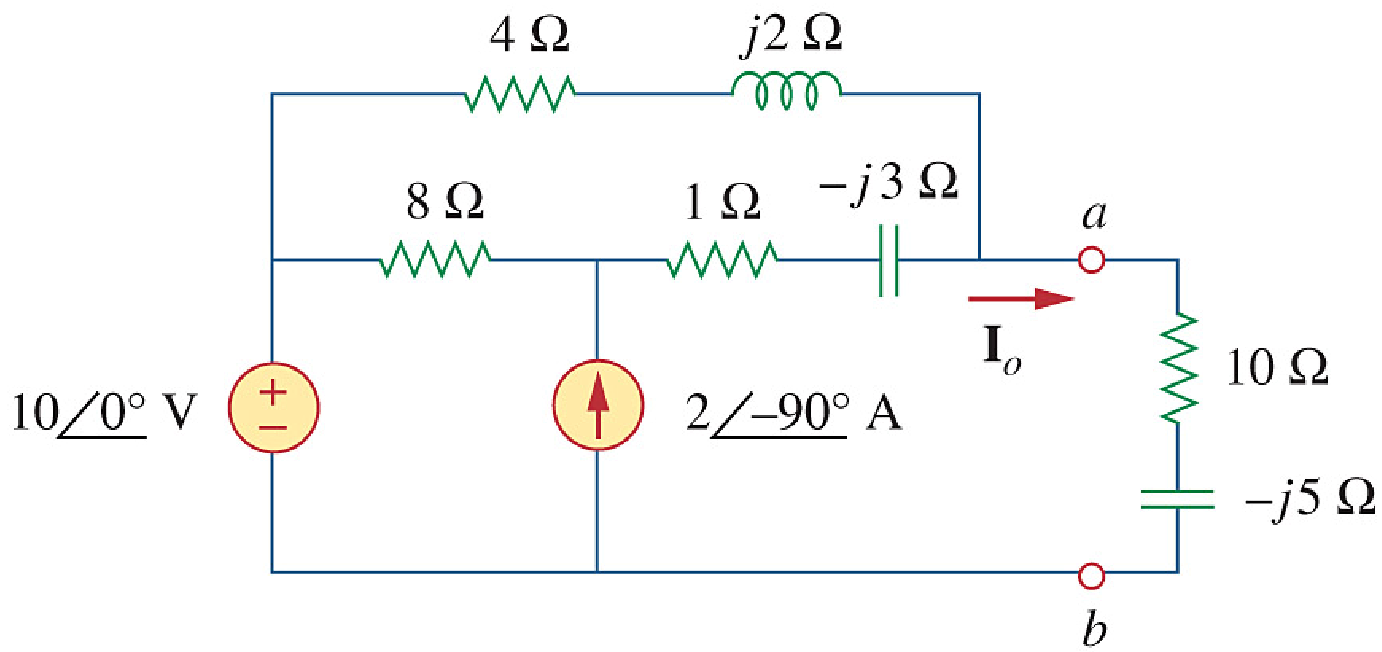


Figure 10.30

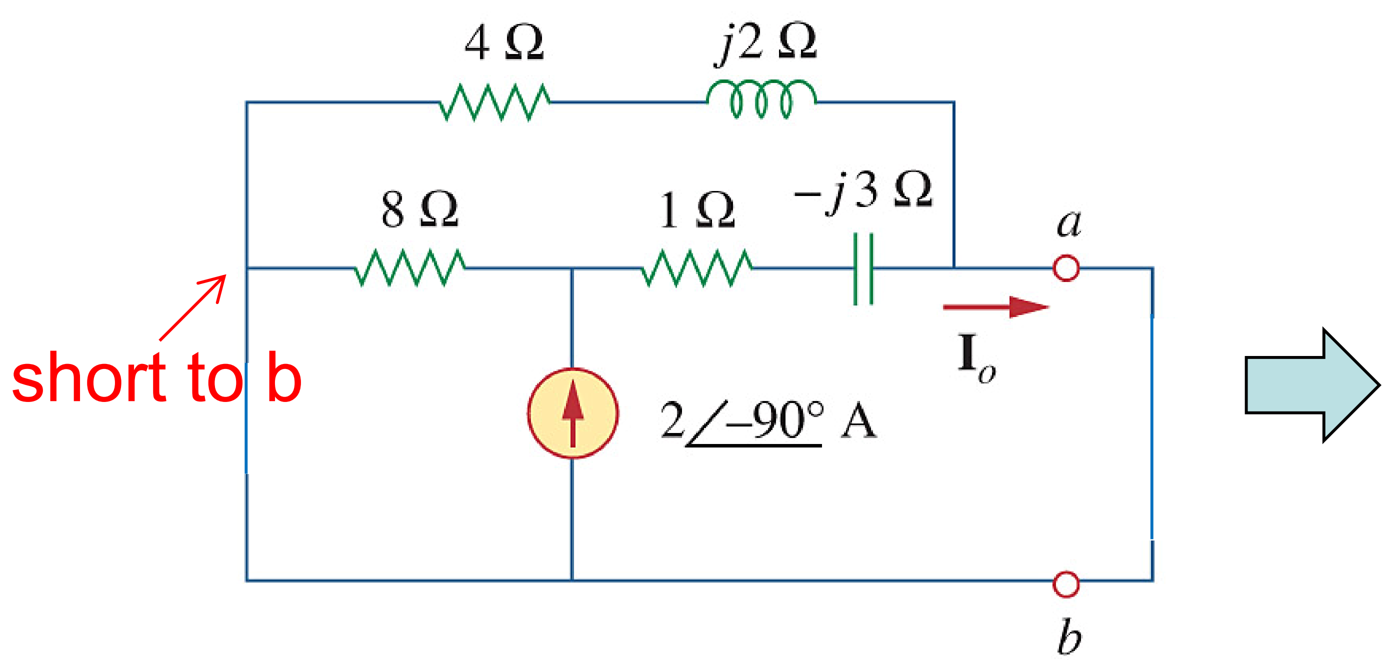


Figure 10.30

$$= 4.1979\angle -32.68^\circ \times \frac{3.1765 + j0.7059}{13.1765 - j4.2941}$$

$$\approx 4.1979\angle -32.68^\circ \times \frac{3.2540\angle 12.53^\circ}{13.8586\angle -18.05^\circ}$$

$$\approx 0.99\angle -2.10^\circ \text{ (A)}$$

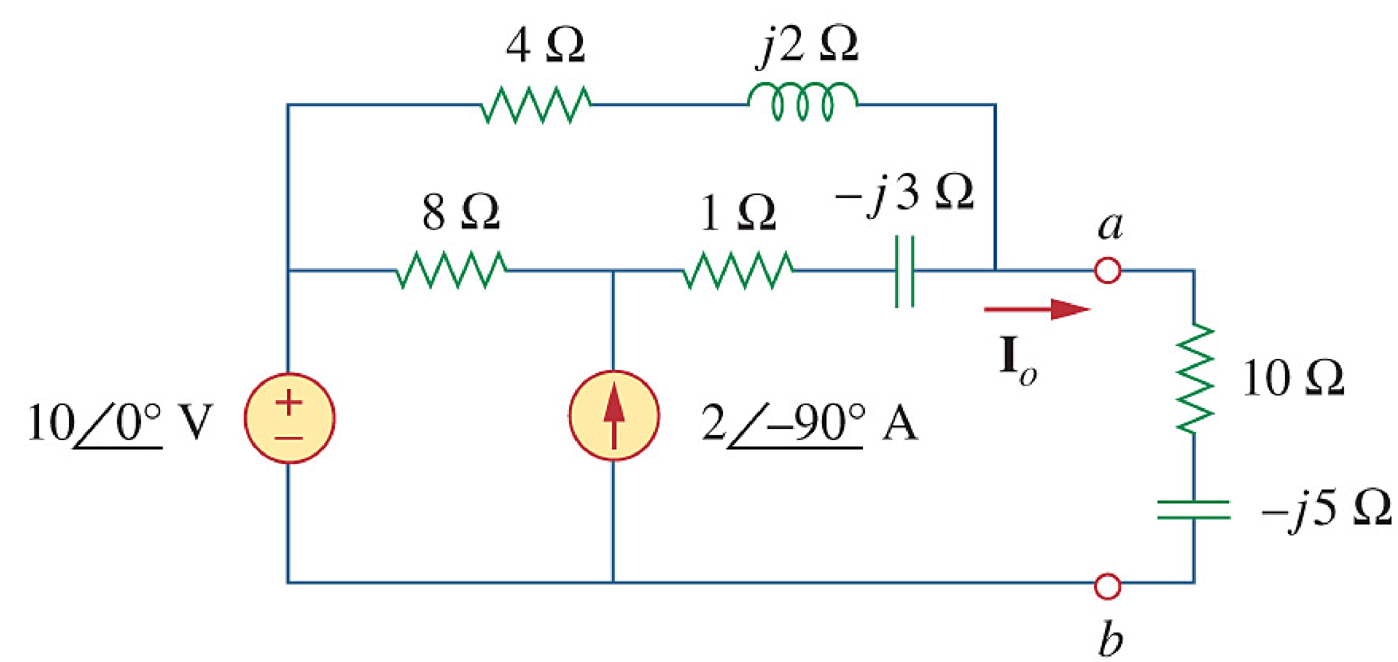


Figure 10.30

10.7 Op Amp AC Circuits

The three steps stated in Section 10.1 also apply to op amp circuits, as long as the op amp is in the linear region.

Practice Problem 10.11 Find v_o and i_o in the op amp circuit of Fig. 10.32. Let $v_s = 4\cos 5000t$ V.

Solution :

Step1

$$4\cos 5000t \text{ V} \Rightarrow 4\angle 0° \text{ V},\ \omega = 5000 \text{ rad/s}$$

$$10 \text{ nF} \Rightarrow \frac{1}{j\omega C} = \frac{1}{j5000\times 10\times 10^{-9}}$$

$$= -j2\times 10^4\ (\Omega) = -j20\ (\text{k}\Omega)$$

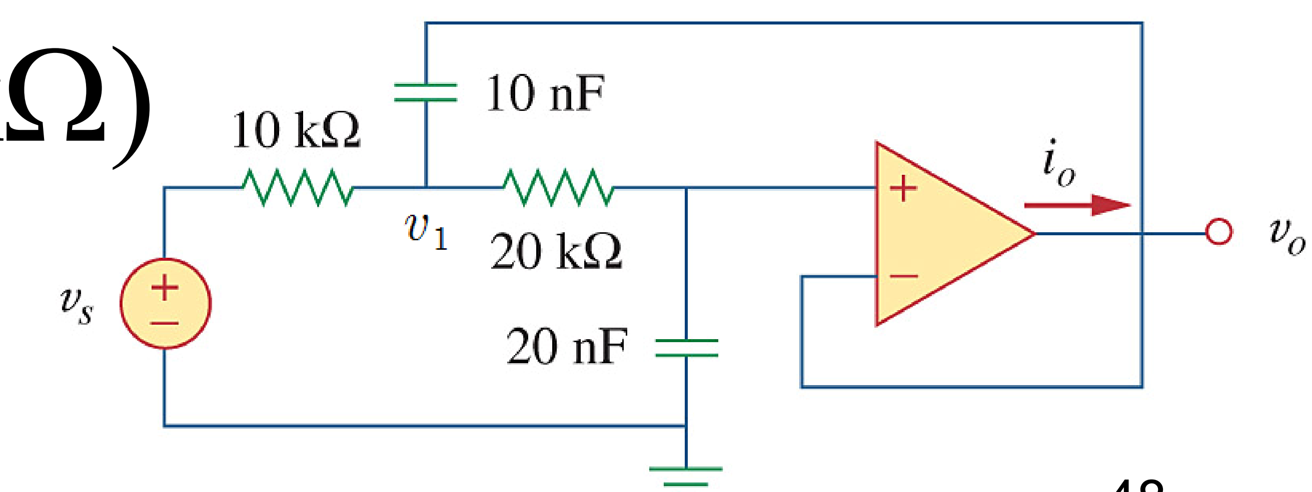


Figure 10.32

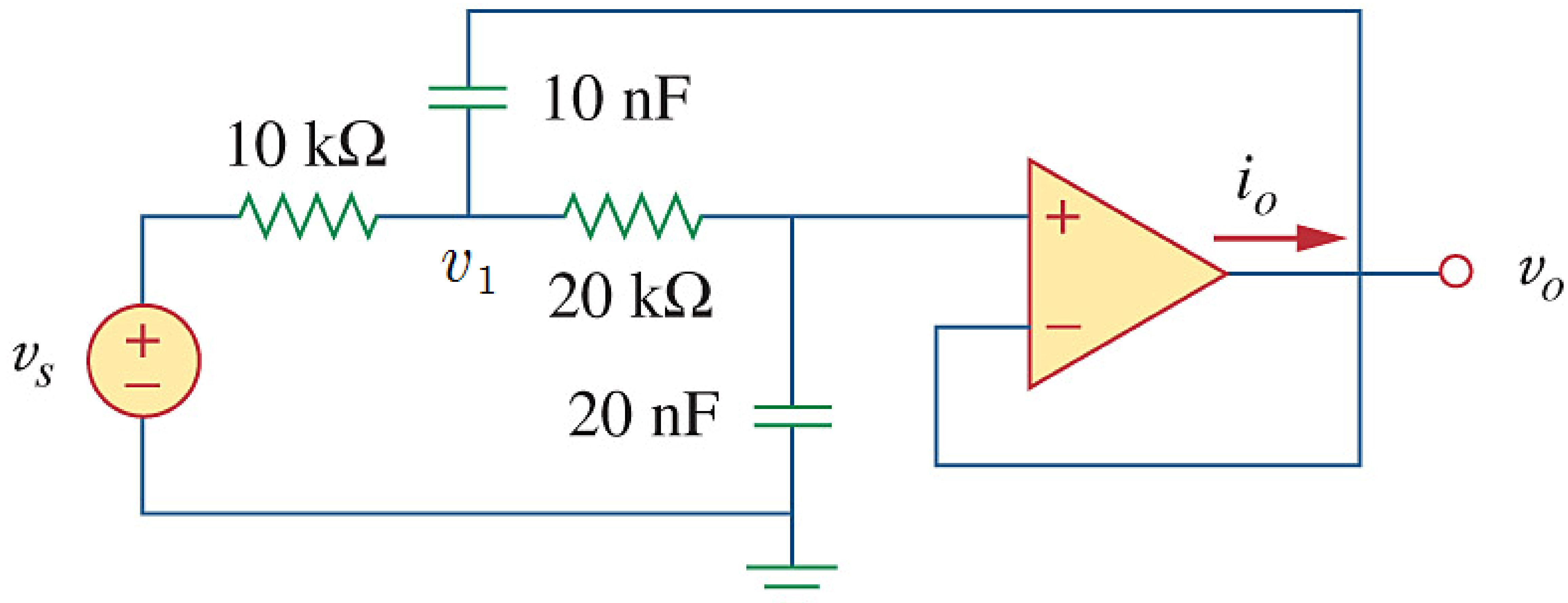


Figure 10.32

$$20\ \text{nF} \Rightarrow \frac{1}{j\omega C} = \frac{1}{j5000 \times 20 \times 10^{-9}}$$

$$= -j1 \times 10^4\ (\Omega) = -j10\ (\text{k}\Omega)$$

Step2

$$\begin{cases} \dfrac{\tilde{V}_s - \tilde{V}_1}{10} = \dfrac{\tilde{V}_1 - \tilde{V}_o}{-j20} + \dfrac{\tilde{V}_o}{-j10} \\ \tilde{V}_1 = \tilde{V}_o \dfrac{20 - j10}{-j10} = \tilde{V}_o (1 + j2) \end{cases}$$

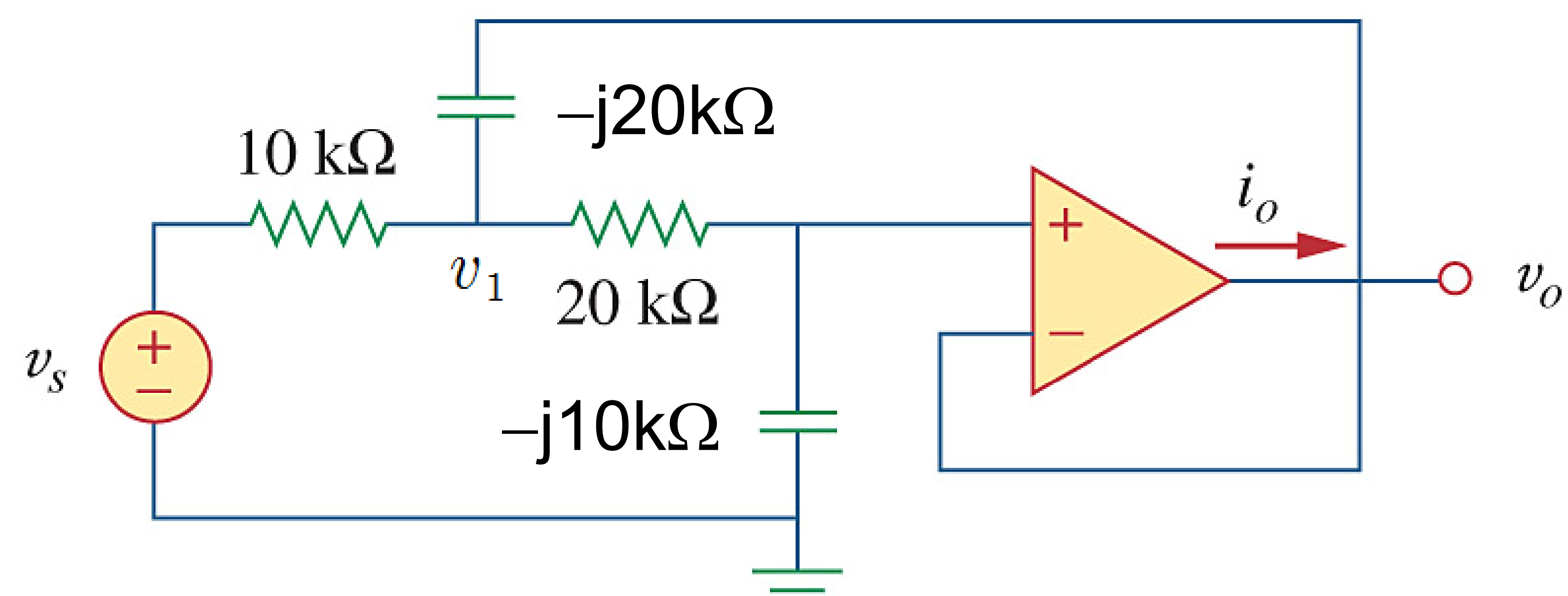


Figure 10.32

$$\begin{cases} \dfrac{\tilde{V}_s - \tilde{V}_1}{10} = \dfrac{\tilde{V}_1 - \tilde{V}_o}{-j20} + \dfrac{\tilde{V}_o}{-j10} \\ \tilde{V}_1 = \tilde{V}_o \dfrac{20 - j10}{-j10} = \tilde{V}_o(1 + j2) \end{cases}$$

$$-j2(\tilde{V}_s - \tilde{V}_1) = (\tilde{V}_1 - \tilde{V}_o) + 2\tilde{V}_o$$

$$-j2\tilde{V}_s = \tilde{V}_1(1 - j2) + \tilde{V}_o$$

$$-j2\tilde{V}_s = \tilde{V}_o(1 + j2)(1 - j2) + \tilde{V}_o = 6\tilde{V}_o$$

$$\tilde{V}_o = \frac{-j2\tilde{V}_s}{6} = \frac{-j2 \times 4\angle 0^\circ}{6}$$

$$\approx 1.3333\angle -90^\circ \text{ (V)}$$

Step3

$$v_o = 1.33\cos(5000t - 90^\circ)$$

$$= 1.33\sin 5000t \text{ (V)}$$

cos(θ–90°)=sin(θ)

$$\tilde{I}_o = \frac{\tilde{V}_o - \tilde{V}_1}{-j20} = \frac{\tilde{V}_o - \tilde{V}_o(1+j2)}{-j20} = 0.1\tilde{V}_o$$

$$= 0.1 \times 1.3333\angle -90^\circ \approx 0.13\angle -90^\circ \text{ (mA)}$$

Step3

$$i_o = 0.13\cos(5000t - 90^\circ)$$

$$= 0.13\sin 5000t \text{ (mA)}$$

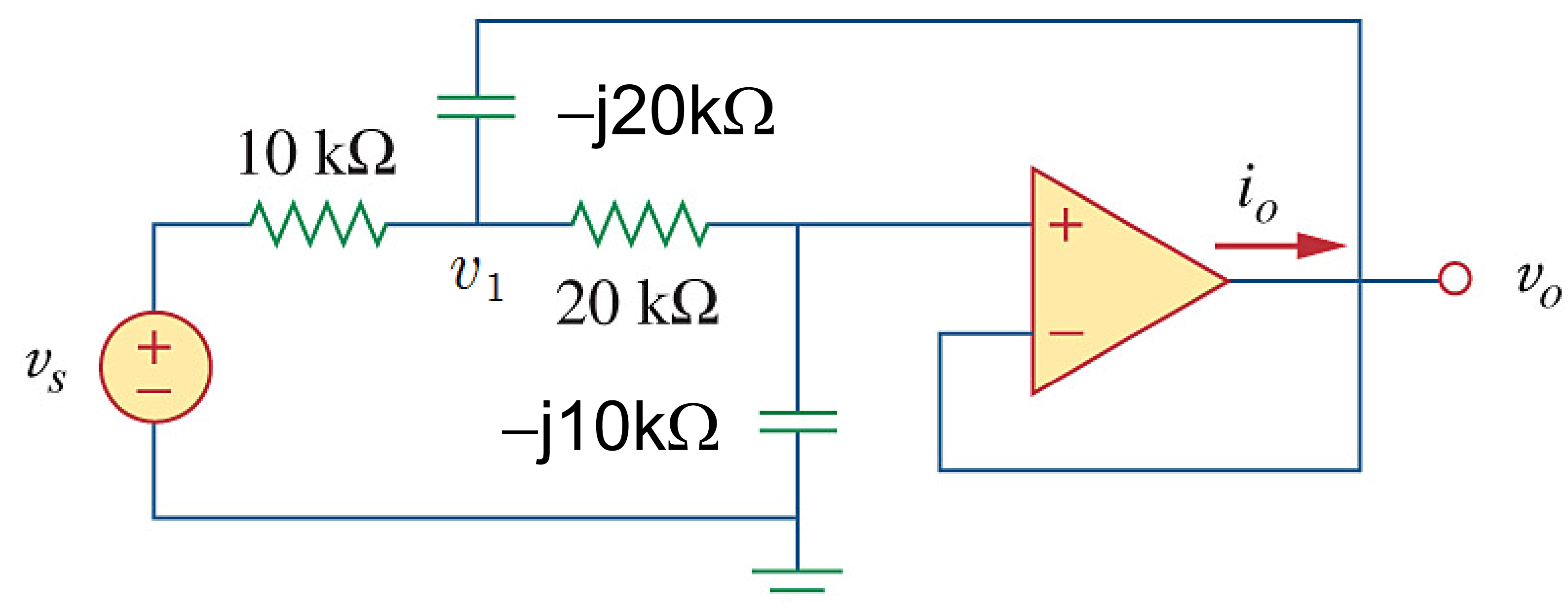


Figure 10.32

10.9 Applications

We study two practical ac circuits: the capacitance multiplier and the sine wave oscillator.

Capacitance Multiplier The circuit in Fig. 10.41 is used in integrated-circuit technology to produce a large capacitance.

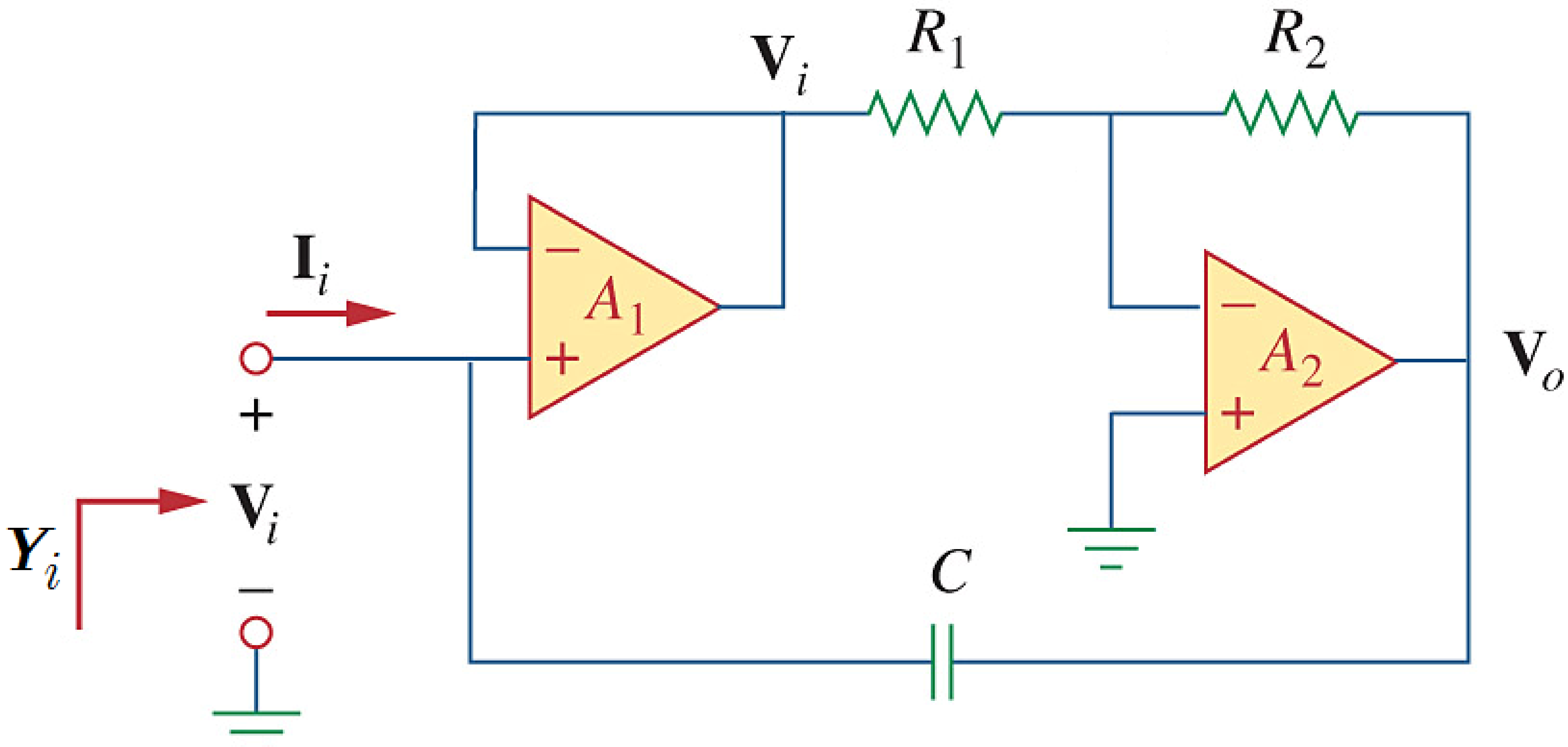


Figure 10.41 Capacitance multiplier.

$$\begin{cases} Y_i = \dfrac{\tilde{I}_i}{\tilde{V}_i} = \dfrac{j\omega C(\tilde{V}_i - \tilde{V}_o)}{\tilde{V}_i} = \dfrac{j\omega C(\tilde{V}_i - \tilde{V}_o)}{\tilde{V}_i} \\ \dfrac{\tilde{V}_o}{\tilde{V}_i} = -\dfrac{R_2}{R_1} \end{cases}$$

Upper loop: Inverting amplifier

$$Y_i = j\omega C\left(1 + \frac{R_2}{R_1}\right)$$

$$C_{eq} = C\left(1 + \frac{R_2}{R_1}\right)$$

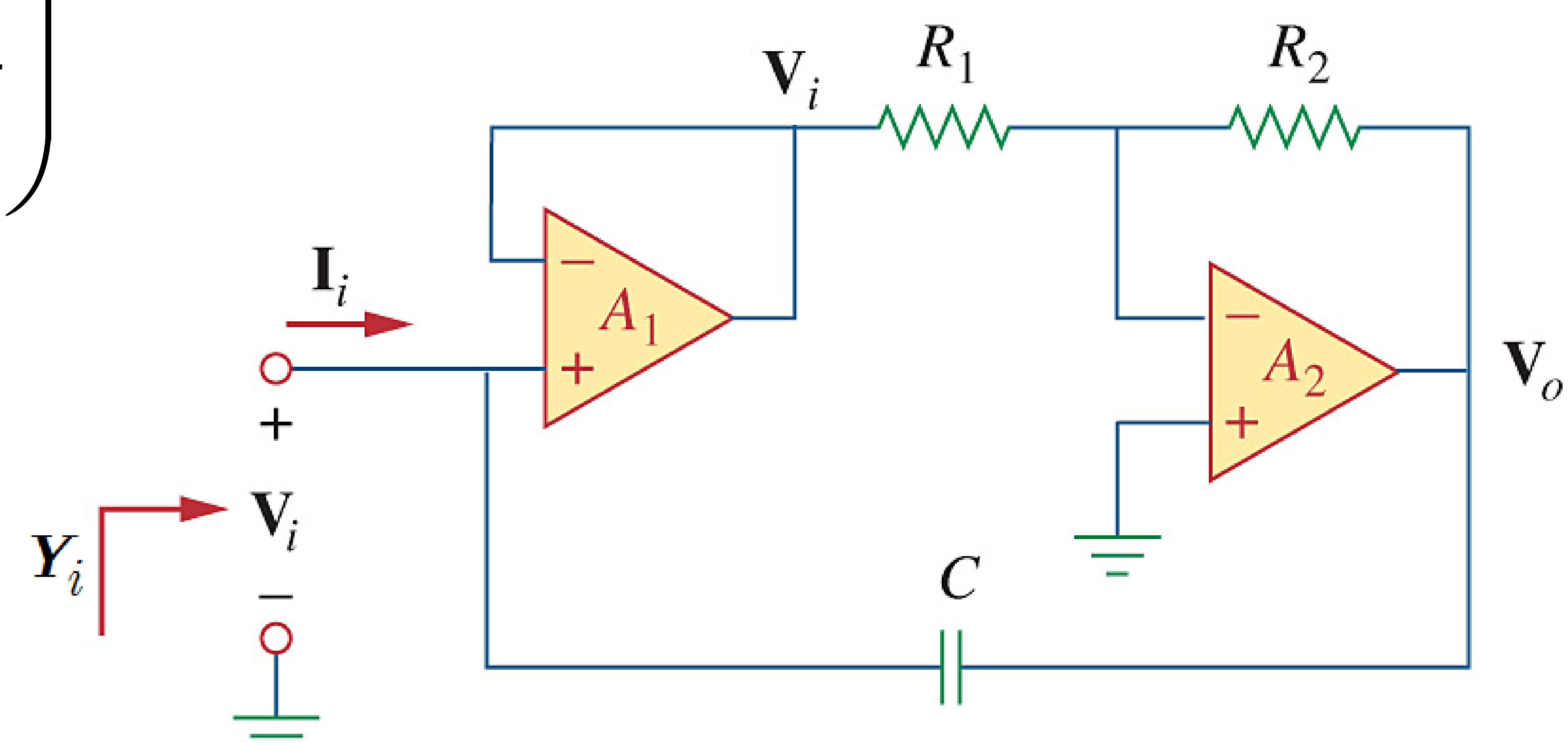


Figure 10.41 Capacitance multiplier.

An *oscillator* is a circuit that produces an ac waveform as output when powered by a dc supply.

In order for sine wave oscillators to sustain oscillations, they must meet the *Barkhausen criteria* :

1. Unity gain (or greater)
2. Zero phase shift

1. The overall gain of the oscillator must be unity or greater.

2. The overall phase shift must be zero.

The *Wien-bridge oscillator* is widely used for generating sinusoids in the frequency range below 1 MHz. As shown in Fig. 10.42, the oscillator consists of an amplifier and a frequency-selective network.

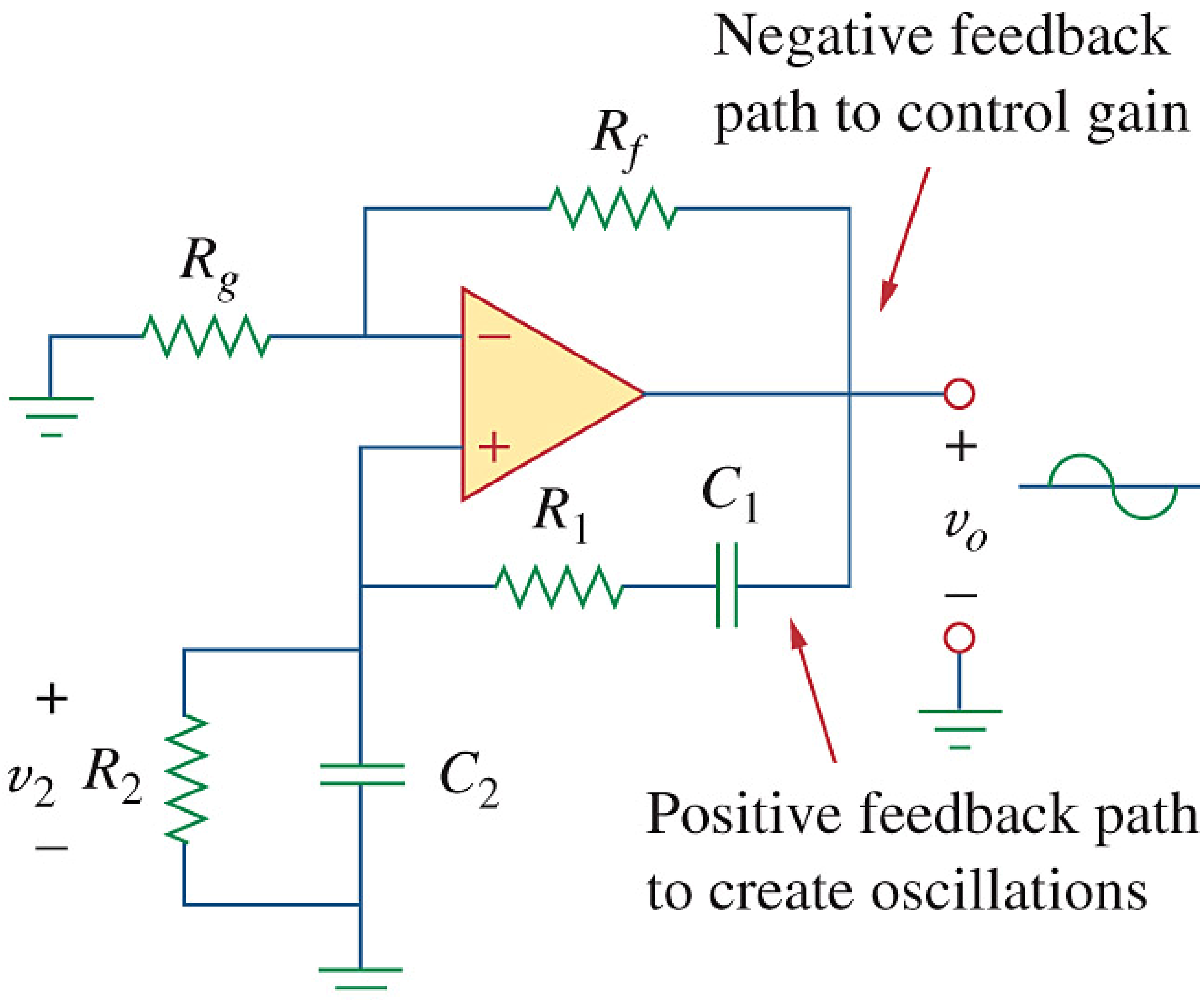


Figure 10.42 Wien-bridge oscillator.

$$\frac{\tilde{V}_2}{\tilde{V}_o}=\frac{R_2 \parallel \left(\frac{1}{j\omega C_2}\right)}{R_2 \parallel \left(\frac{1}{j\omega C_2}\right)+\left(R_1+\frac{1}{j\omega C_1}\right)}$$

$$=\frac{R_2/(1+j\omega R_2 C_2)}{R_2/(1+j\omega R_2 C_2)+(1+j\omega R_1 C_1)/(j\omega C_1)}$$

$$=\frac{j\omega R_2 C_1}{j\omega R_2 C_1+(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$$

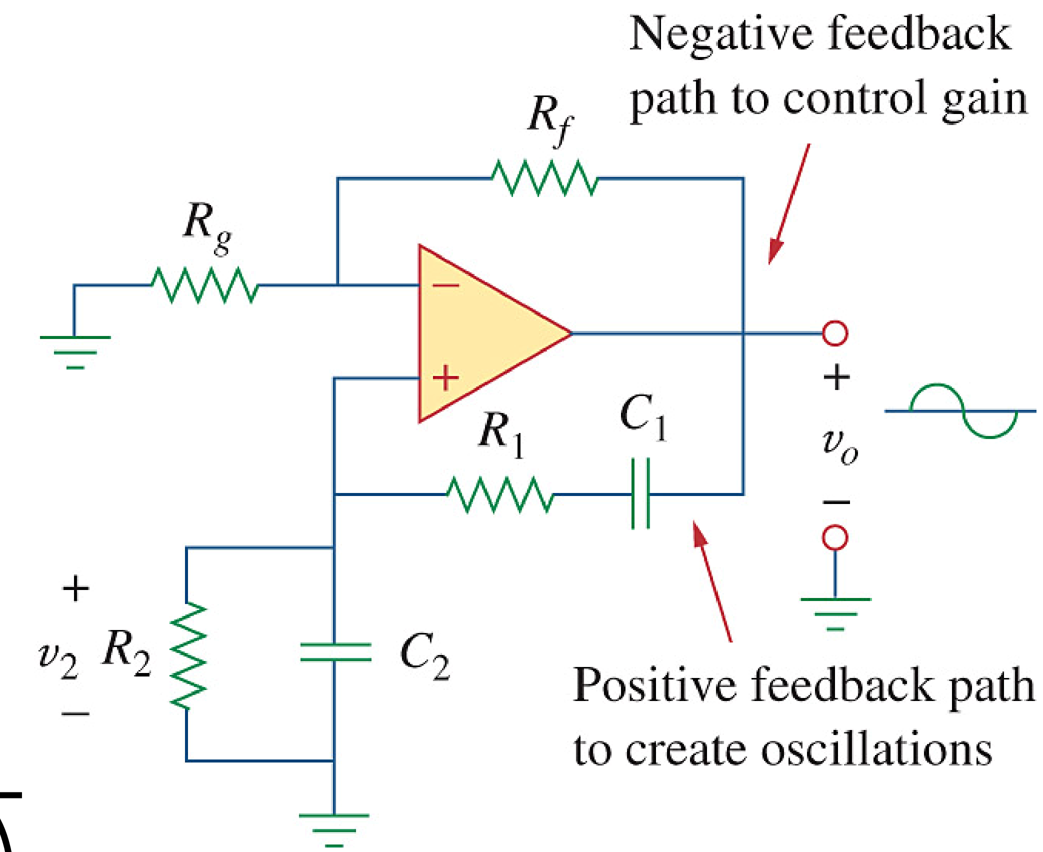


Figure 10.42 Wien-bridge oscillator.

$$= \frac{\omega R_2 C_1}{\omega(R_2C_1 + R_1C_1 + R_2C_2) + j(\omega^2 R_1R_2C_1C_2 - 1)}$$

To satisfy the second Barkhausen criterion, $\tilde{V}_2 / \tilde{V}_o$ must be real. Setting the imaginary part equal to zero gives the oscillation frequency as

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

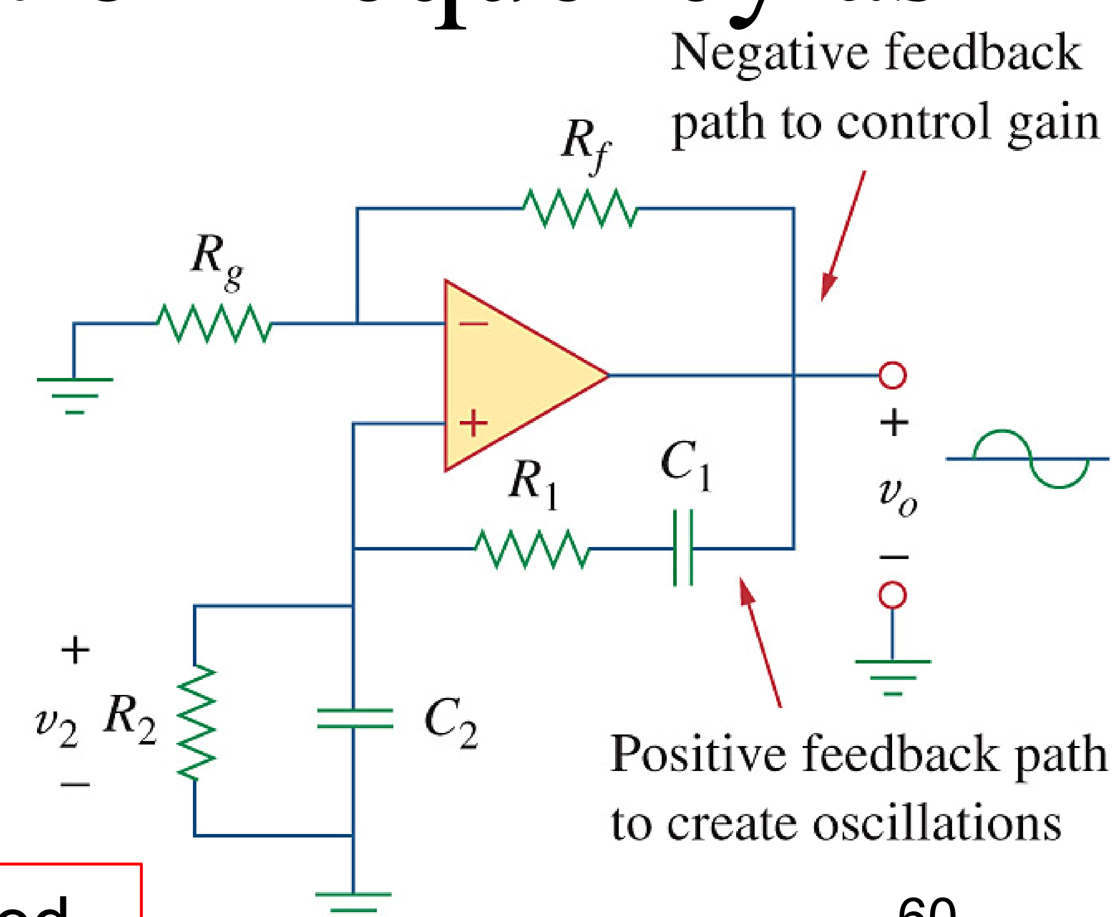


Figure 10.42 Wien-bridge oscillator.

$\boxed{\text{Zero phase shift} \Leftrightarrow \mathbf{V_2/V_o} \text{ is real} \Leftrightarrow \text{Only } \omega_0 \text{ is satisfied}}$

In most practical applications, $R_1 = R_2 = R$ and $C_1 = C_2 = C$, so that

$$\omega_0 = \frac{1}{RC} \text{ and } \frac{\tilde{V}_2}{\tilde{V}_o} = \frac{1}{3}$$

Thus, in order to satisfy the first Barkhausen criterion, the amplifier must provide a gain of 3 or greater, i.e.,

$$\frac{\tilde{V}_o}{\tilde{V}_2} = 1 + \frac{R_f}{R_g} = 3 \Rightarrow R_f = 2R_g$$

For $R_1=R_2$, $C_1=C_2$

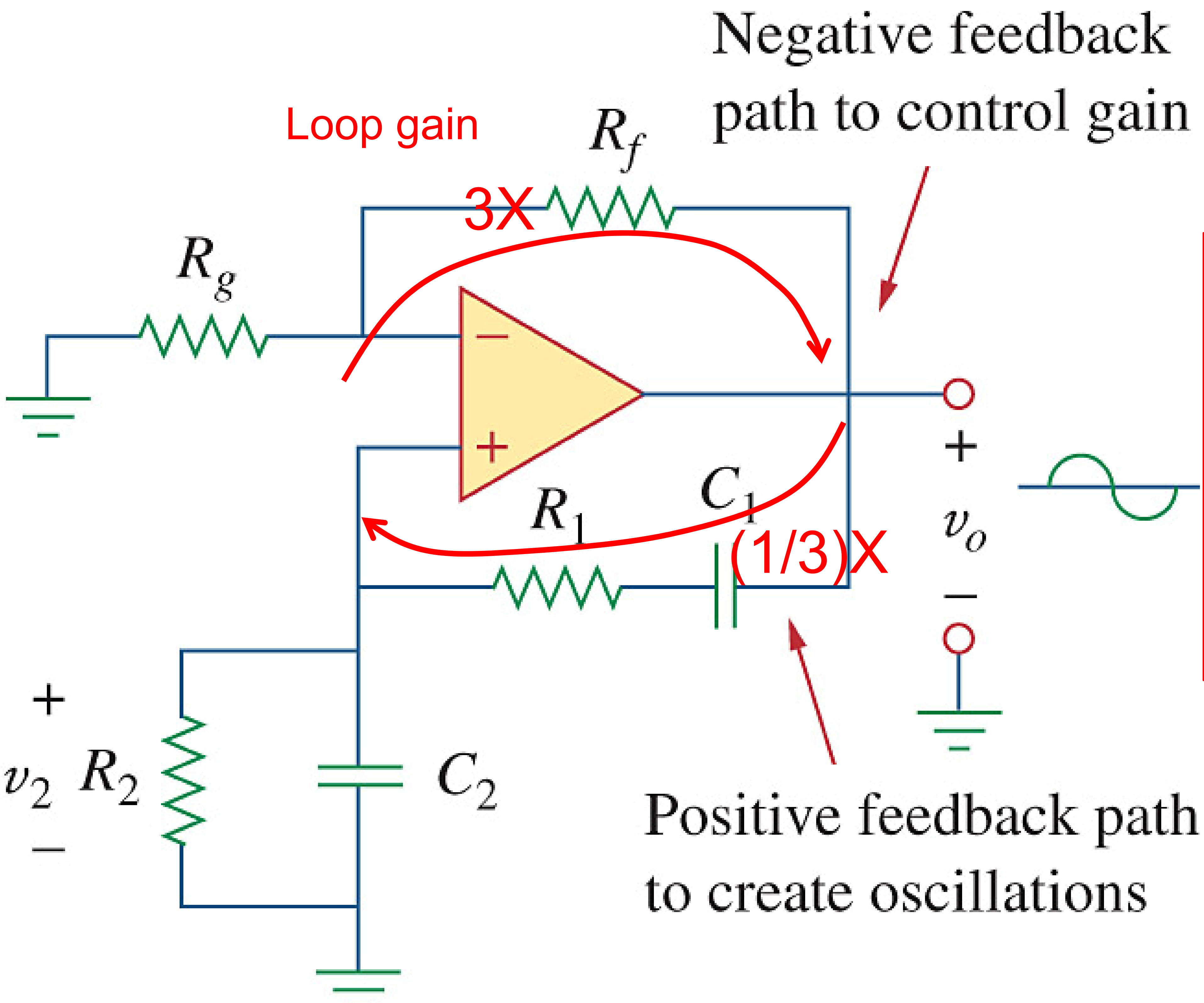


Figure 10.42 Wien-bridge oscillator.

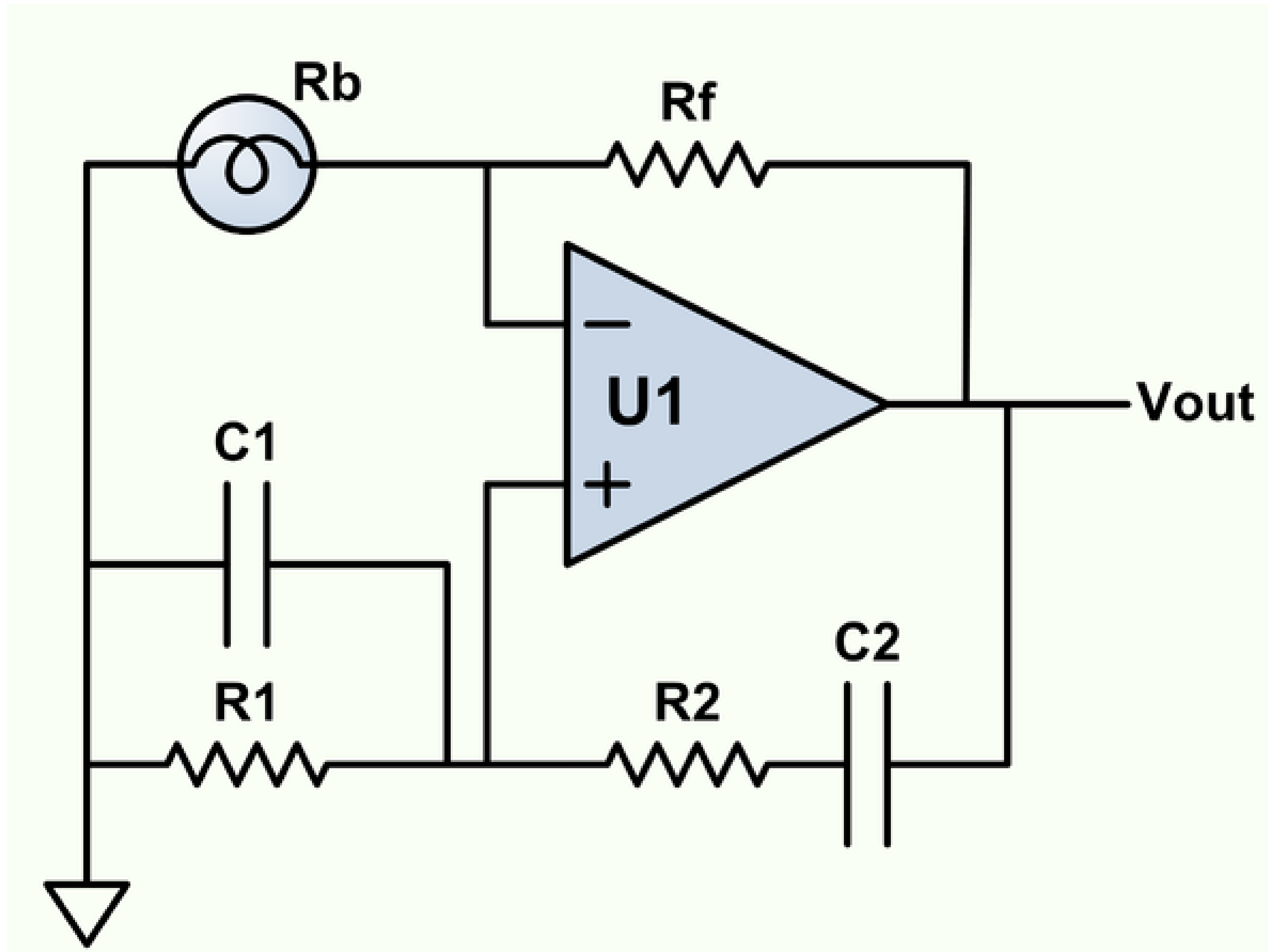
-The positive feedback: to produce <u>zero phase shift</u>, it imposes a constraint of $v_2/v_o=1/3$

-The negative feedback: provide a gain of 3 to satisfy this constraint!

When Barkhausen criteria are satisfied

- In practice, the loop gain is initially larger than unity. **Random *noise* is present in all circuits, and some of that noise will be near the desired frequency.** A loop gain greater than one allows the amplitude of frequency to increase exponentially each time around the loop. With a loop gain greater than one, the oscillator will start. E.g., $V_o^n = V_o \times 1.1^n$, n: # of loops; gain=1.1
- Nonlinear elements can be used for amplitude stabilization (E.g., gain=1.1 for lower V and gain=1 for higher V).

https://en.wikipedia.org/wiki/Wien_bridge_oscillator


Rb
Rf
U1
Vout
C1
R1
R2
C2