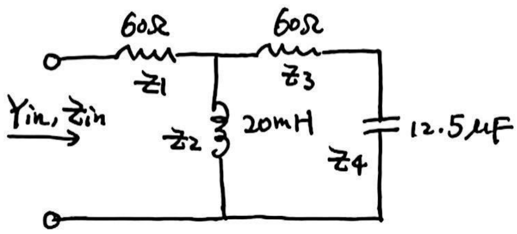
Exercise 5.1 (25%)

(a) (5%)



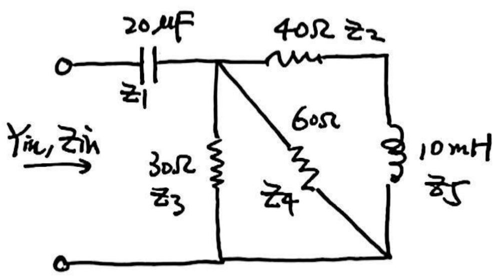
$Z_1 = 60\Omega$, $Z_2 = 20\times10^{-3}\times 50\,j = j\ \Omega$

$Z_3 = 60\Omega$, $Z_4 = -\frac{1}{12.5\times10^{-6}\times50}\,j = -1600\,j\ \Omega$

$Z_{in} = Z_1 + Z_2 \| (Z_3+Z_4) \doteq 60 + j\ \Omega$

So, $Y_{in} = \frac{1}{Z_{in}} = \frac{1}{60+j} \doteq \boxed{1.67\times10^{-2} - 2.78\times10^{-4}\,j\ (S)}$

(b) (10%)



$Z_1 = -\frac{1}{20\times10^{-6}\times50}\,j = -1000\,j\ \Omega$

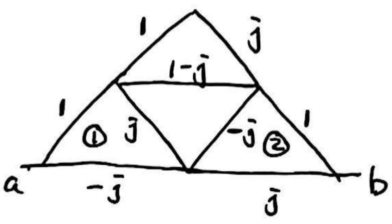
$Z_5 = 10\times10^{-3}\times50\,j = 0.5\,j\ \Omega$

$Z_3 = 30\Omega$, $Z_4 = 60\Omega$, $Z_2 = 40\Omega$.

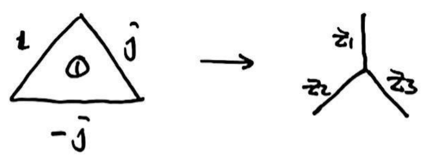
$Z_{in} = Z_1 + Z_3 \| Z_4 \| (Z_2 + Z_5) \doteq 13.33 - 999.94\,i\ \Omega$

So, $Y_{in} = \frac{1}{Z_{in}} \doteq \boxed{1.33\times10^{-5} + 9.999\times10^{-4}\,j\ \ (S)}$

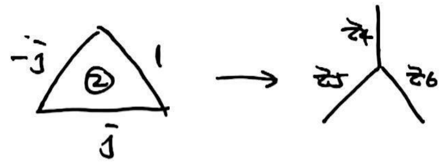
(c) (10%)



Use Y-Δ transformation:

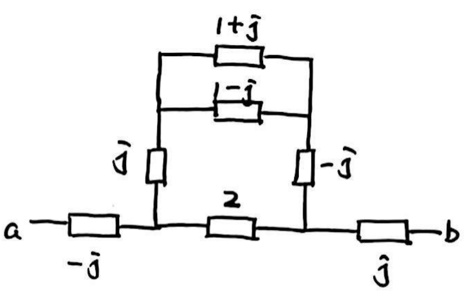


$$\begin{cases} Z_1 = \frac{1\times j}{1+j-j} = j\ \Omega \\ Z_2 = \frac{1\times(-j)}{1+j-j} = -j\ \Omega \\ Z_3 = \frac{j\times(-j)}{1+j-j} = 1\ \Omega \end{cases}$$



$$\begin{cases} Z_4 = \frac{1\times(-j)}{1+j-j} = -j\ \Omega \\ Z_5 = \frac{j\times(-j)}{1+j-j} = 1\ \Omega \\ Z_6 = \frac{1\times j}{1+j-j} = j\ \Omega \end{cases}$$

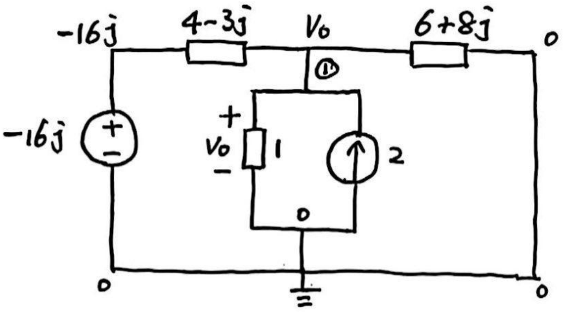
So, the equivalent circuit is:



$$\begin{aligned} Z_{ab} &= -j + j + 2 \| (j - j + (1-j)\|(1+j)) \\ &= 0 + 2 \| (0 + (1-j)\|(1+j)) \\ &= 2 \| 1 \\ &= \boxed{\frac{2}{3}\ (\Omega)} \end{aligned}$$

Exercise 5.2 (30%)

(a) (15%) we transform this circuit into Z-Domain: $\omega = 4$ rad/s.



$$\begin{cases} Z(\tfrac{1}{12}F) = -\dfrac{1}{\frac{1}{12}\times 4} j = -3j \\ Z(2H) = 2\times 4 j = 8j \end{cases}$$

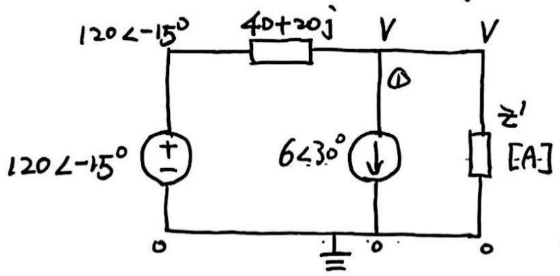
use node ①, we have:

$$V_o + \frac{V_o + 16j}{4-3j} + \frac{V_o}{6+8j} = 2 \Rightarrow V_o = \frac{468}{149} - \frac{328}{149} i \ V$$

transform back to time domain: $\tilde{V_o} = 3.836 \angle -35.025^\circ$,

$$\boxed{V_o = 3.836 \cos(4t - 35.025^\circ) \ (V)}$$

(b) (15%) It is already in Z-Domain.



$$Z' = 50 \| (-30j) = \frac{225}{17} - \frac{375}{17} i \ (\Omega) = [A]$$

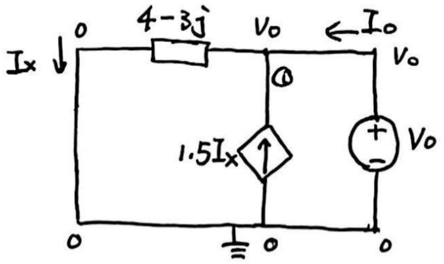
use node ①, we have:

$$\frac{V - 120\angle -15^\circ}{40+20j} + \frac{V}{[A]} + 6\angle 30^\circ = 0$$

$$\Rightarrow \boxed{V \doteq -111.49 - 54.47i \doteq 124.08 \angle -153.96^\circ \ (V)}$$

Exercise 5.3 (30%)

(a) (15%) First, we find Z_{Th}, shut off all independent sources:

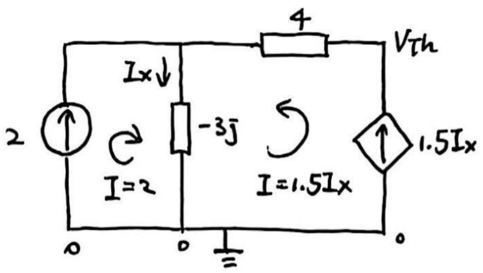


use node ①, we have:

$$\frac{V_o}{4-3j} = I_o + 1.5 I_x, \text{ where } \frac{V_o}{4-3j} = I_x$$

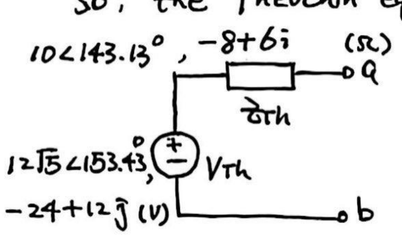
$$\Rightarrow \frac{V_o}{I_o} = -2\times(4-3j) = \boxed{-8+6i \doteq 10\angle 143.13^\circ = Z_{Th}}$$

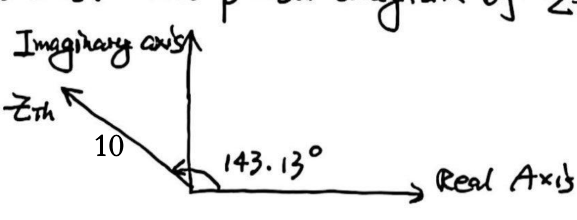
Second, we find V_{Th}, $\quad 2 + 1.5 I_x = I_x \Rightarrow I_x = -4$ (A)



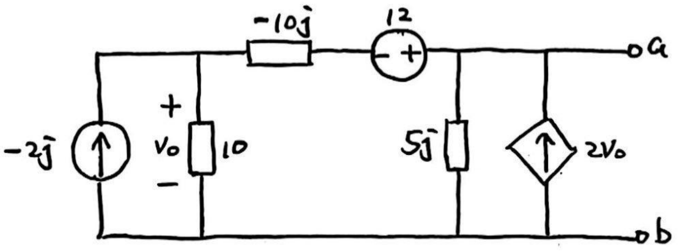
$$V_{Th} = 4\times 1.5 I_x - 3j \times I_x = \boxed{-24+12j \ (V) \doteq 12\sqrt{5}\angle 153.43^\circ} \ (V)$$

So, the Thevenin equivalent circuit at a-b is: and phasor diagram of Z_{Th} is:





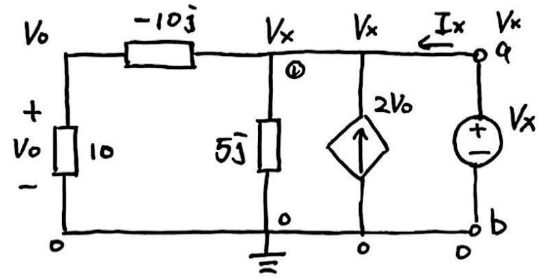
(b) (15%) First. we transform the circuit to Z-Domain. $\omega = 10$ (rad/s)



$$Z(10mF) = -\frac{1}{10\times10^{-3}\times10}\,j = -10j \ (\Omega)$$

$$Z(\tfrac{1}{2}H) = \tfrac{1}{2}\times10j = 5j \ (\Omega)$$

Then, we find Z_N. Shut off all independent sources.

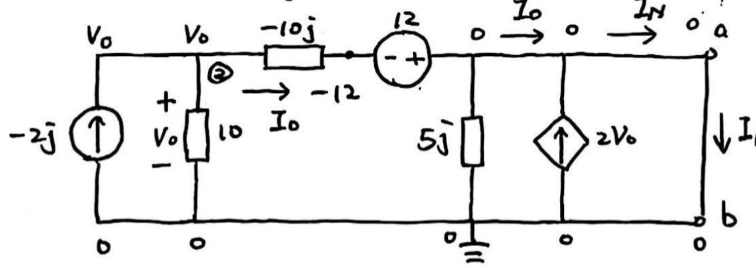


use node ①, we have:

$$\frac{V_x}{5j} + \frac{V_x - V_o}{-10j} = 2V_o + I_x \text{ , where } V_o = \frac{10}{10-10j}V_x$$

$$\Rightarrow Z_N = \frac{V_x}{I_x} = \boxed{-\frac{38}{89} + \frac{46}{89}i \doteq 0.67\angle 129.56^\circ} \ (\Omega)$$

Next, we find I_N:



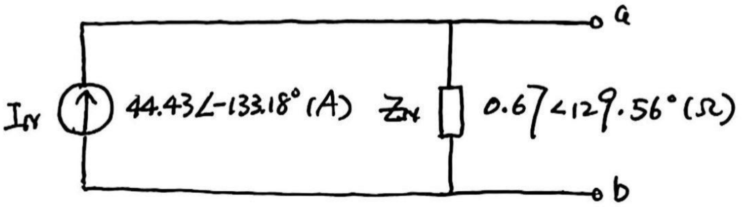
use node ②, we have:

$$\frac{V_o}{10} + \frac{V_o + 12}{-10j} = -2j \Rightarrow V_o = -16-16j \ (V)$$

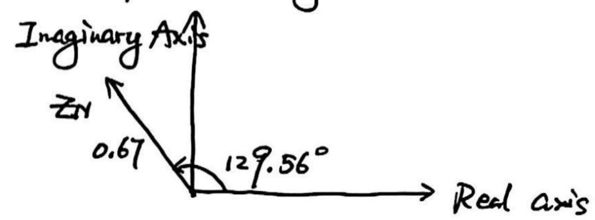
$$\Rightarrow I_N = I_o + 2V_o = \frac{V_o+12}{-10j} + 2V_o$$

$$= \boxed{-\frac{152}{5} - \frac{162}{5}j \doteq 44.43\angle -133.18^\circ} \ (A)$$

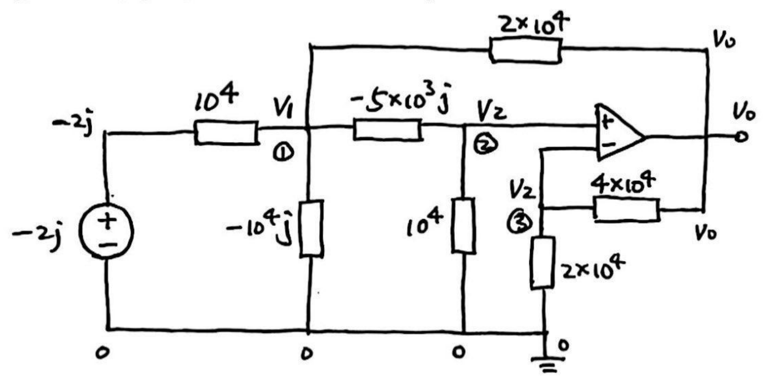
So, the Norton equivalent circuit is:



The phasor diagram of Z_N is:



Exercise 5.4 (15%) First. we transform the circuit to Z-Domain. $\omega = 400$ rad/s



$$Z(0.25\mu F) = -\frac{1}{0.25\times10^{-6}\times400}\,j = -10^4 j \ (\Omega)$$

$$Z(0.5\mu F) = -\frac{1}{0.5\times10^{-6}\times400}\,j = -5\times10^3 j \ (\Omega)$$

node ①: $\dfrac{V_1+2j}{10^4} + \dfrac{V_1}{-10^4 j} + \dfrac{V_1 - V_2}{-5\times10^3 j} + \dfrac{V_1 - V_o}{2\times10^4} = 0$

node ②: $\dfrac{V_2 - V_1}{-5\times10^3 j} + \dfrac{V_2}{10^4} = 0$

node ③: $\dfrac{V_2}{2\times10^4} + \dfrac{V_2 - V_o}{4\times10^4} = 0$

$$\Rightarrow \begin{cases} V_o = -\frac{24}{37} - \frac{144}{37}j \ (V) \\ V_1 = -\frac{32}{37} - \frac{44}{37}j \ (V) \\ V_2 = -\frac{8}{37} - \frac{48}{37}j \ (V) \end{cases}$$

Back to time Domain. $\tilde{V_o} \doteq 3.95\angle -99.46^\circ$, $\boxed{V_o = 3.95\cos(400t - 99.46^\circ) \ (V)}$