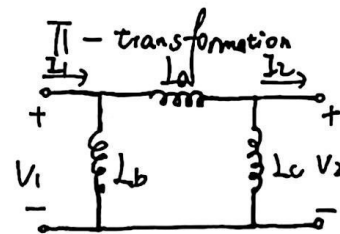
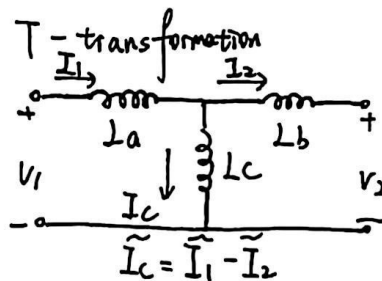
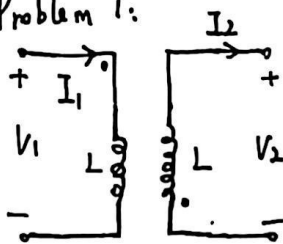


Problem 1:



For T-transformation,

$$\begin{cases} \tilde{V}_1 = \tilde{I}_1 j\omega L_a + (\tilde{I}_1 - \tilde{I}_2) j\omega L_c \\ \tilde{V}_2 = -\tilde{I}_2 j\omega L_b + (\tilde{I}_1 - \tilde{I}_2) j\omega L_c \end{cases}$$

$$\begin{pmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{pmatrix} = \begin{pmatrix} j\omega(L_a + L_c) & -j\omega L_c \\ j\omega L_c & -j\omega(L_b + L_c) \end{pmatrix} \begin{pmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{pmatrix}$$

$$\Rightarrow \begin{cases} L_a + L_c = L \\ L_b + L_c = L \\ L_c = -M \end{cases} \Rightarrow \begin{cases} L_a = L + M \\ L_b = L + M \\ L_c = -M \end{cases}$$

For the original circuit,

$$\begin{cases} \tilde{V}_1 = j\omega L \tilde{I}_1 + j\omega M \tilde{I}_2 \\ \tilde{V}_2 = -j\omega L \tilde{I}_2 - j\omega M \tilde{I}_1 \end{cases}$$

(Note: the direction of current and the position of dot)

Convert to matrix form.

$$\begin{aligned} \tilde{I}_1 &= \frac{L}{j\omega(L^2 - M^2)} \tilde{V}_1 + \frac{M}{j\omega(L^2 - M^2)} \tilde{V}_2 \\ \tilde{I}_2 &= \frac{-M}{j\omega(L^2 - M^2)} \tilde{V}_1 + \frac{-L}{j\omega(L^2 - M^2)} \tilde{V}_2 \end{aligned}$$

For Π -transformation, it's better to use KCL first.

$$\begin{cases} \tilde{I}_1 = \frac{\tilde{V}_1}{j\omega L_b} + \frac{\tilde{V}_1 - \tilde{V}_2}{j\omega L_a} \\ \tilde{I}_2 = \frac{\tilde{V}_1 - \tilde{V}_2}{j\omega L_a} - \frac{\tilde{V}_2}{j\omega L_c} \end{cases}$$

$$\begin{aligned} \tilde{I}_1 &= \left(\frac{1}{j\omega L_b} + \frac{1}{j\omega L_a} \right) \tilde{V}_1 - \frac{1}{j\omega L_a} \tilde{V}_2 \\ \tilde{I}_2 &= \frac{1}{j\omega L_a} \tilde{V}_1 - \left(\frac{1}{j\omega L_a} + \frac{1}{j\omega L_c} \right) \tilde{V}_2 \end{aligned}$$

$$\Rightarrow \begin{cases} \frac{1}{L_a} + \frac{1}{L_b} = \frac{1}{L^2 - M^2} \\ \frac{1}{L_a} = \frac{-M}{L^2 - M^2} \\ \frac{1}{L_a} + \frac{1}{L_c} = \frac{1}{L^2 - M^2} \end{cases}$$

$$\Rightarrow \begin{cases} L_a = \frac{M^2 - L^2}{M} \\ L_b = \frac{L^2 - M^2}{L + M} = L - M \\ L_c = \frac{L^2 - M^2}{L + M} = L - M \end{cases}$$



$$\begin{cases} (4+3j)I_1 - 8jI_2 = 100\sqrt{2} \\ -8jI_1 + (5+18j)I_2 = 0 \end{cases}$$

$$\begin{cases} I_1 = 28.709 \cos(\omega t + 3.501^\circ) \\ I_2 = 12.294 \cos(\omega t + 19.026^\circ) \end{cases}$$

Then Instant $P = \frac{C V_c^2}{2} + \frac{1}{2} (I_1 - I_2)^2 L_1 + \frac{1}{2} I_2^2 L_2$
 $+ M(I_1 - I_2)I_2$

$$\begin{aligned} = & \frac{1}{2} \times \frac{1}{3\omega} \times (86.126)^2 \cos^2(\omega t - 86.499^\circ) + \\ & \frac{1}{2} \times \frac{6}{\omega} \times 17.181^2 \cos^2(\omega t - 7.539^\circ) + \\ & \frac{1}{2} \times \frac{8}{\omega} \times 12.294^2 \cos^2(\omega t + 19.026^\circ) + \\ & - \frac{2}{\omega} \times 17.181 \times 12.294 \times \cos(\omega t - 7.539^\circ) \\ & \times \cos(\omega t + 19.026^\circ) \end{aligned}$$

$$\begin{aligned} 2) \quad W_{AVG} = & \frac{1}{4} \times \frac{1}{3\omega} \times (86.126)^2 + \frac{1}{4} \times \frac{6}{\omega} \times 17.181^2 \\ & + \frac{1}{4} \times \frac{8}{\omega} \times 12.294^2 - \frac{1}{2} \times \frac{2}{\omega} \times 17.181 \times 12.294 \\ & \times \cos(26.565^\circ) \end{aligned}$$

$$\approx 1174$$

