# ECE2150J 2025FA Assignment 6

**Due Date: 23:59 December 20th**

In order to get full marks, you shall write all the intermediate steps of calculation or proof unless otherwise indicated.

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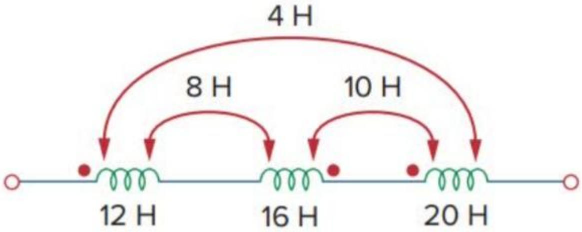
**Exercise 6.1** (25%)

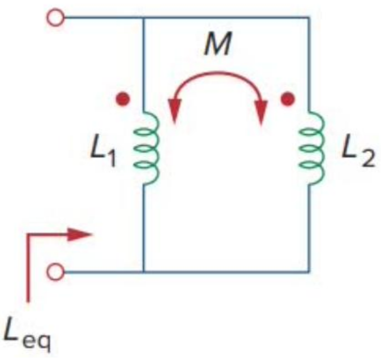
(a) Find the total inductance for the three coupled coils.

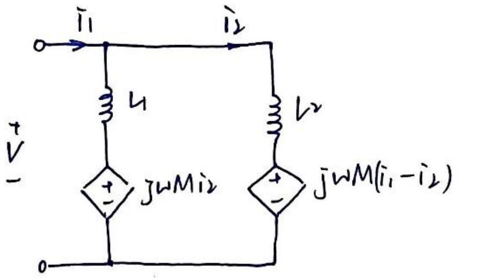



$$L = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{13}$$

$$= 12 + 16 + 20 - 2\cdot 8 - 2\cdot 10 + 2\cdot 4 = 20H$$

(b) Find the equivalent inductance $L_{eq}$.




$$V = j\omega L_1 (i_1 - i_2) + j\omega M i_2$$

$$V = j\omega L_2 i_2 + j\omega M (i_1 - i_2)$$

$$i_2 = \frac{L_1 - M}{L_1 + L_2 - 2M} i_1$$

$$V = j\omega M i_1 + j\omega (L_2 - M) \frac{L_1 - M}{L_1 + L_2 - 2M} i_1 = j\omega \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} i_1$$

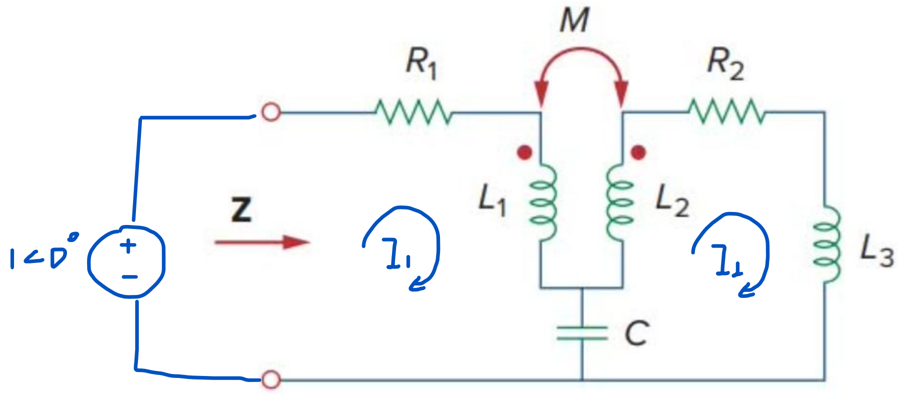
$V = j\omega L_{eq} i_1$, so $L_{eq} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

# ECE2150J 2025FA Assignment 6

GLOBAL COLLEGE
SHANGHAI JIAO TONG UNIVERSITY

**Exercise 6.2** (25%)

$R1 = R2 = 5\text{k}\Omega, M = 20\text{ H}, L1 = 10\text{H}, L_2 = 5\text{H}, L_3 = 10\text{H}, C = 0.1\text{ F}$. Suppose frequency of source is $\omega = 10\text{k}$ rad/s. Find the equivalent impedance Z. All currents flow clockwise.




$Z_{L1} = j\omega L_1 = j\,100\text{ k}\Omega$

$Z_{L2} = j\omega L_2 = j\,50\text{ k}\Omega$

$Z_M = j\omega M = j\,200\text{ k}\Omega$

$Z_C = \frac{1}{j\omega C} = -j\,10^{-3}\ \Omega$

Apply test voltage $V = 1\angle 0^\circ$ V

$V = (R_1 + Z_{L1} + Z_C) I_1 - Z_C I_2 - Z_M I_2$

$0 = (Z_C + Z_{L2} + R_2 + Z_{L3}) I_2 - Z_C I_1 - Z_M I_1$

$1 = (5\times10^3 + j(10^5 - 10^{-3})) I_1 + j(10^{-3} - 2\times10^5) I_2$

$0 = (5\times10^3 + j(1.5\times10^5 - 10^{-3})) I_2 + j(10^{-3} - 2\times10^5) I_1$

$I_2 = (1.3319 + j0.0444) I_1$

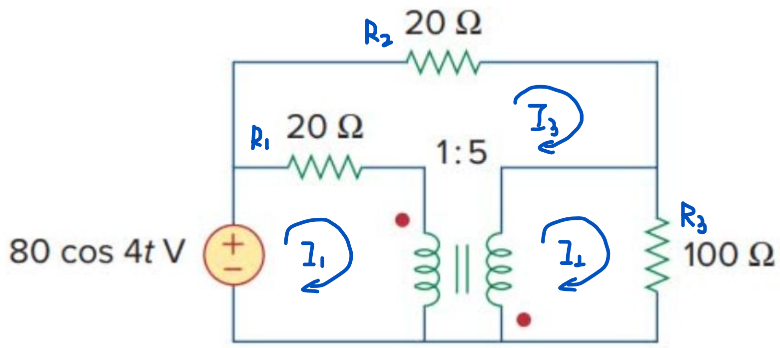
$1 = (13879.0 - j166370.7) I_1$

$I_1 = 5.99\times10^{-6} \angle 85.23^\circ$ A

Therefore, $Z = \frac{V}{I_1} = 1.67\times10^5 \angle -85.23^\circ\ \Omega$

**Exercise 6.3** (25%)

Determine the average power absorbed by each resistor in this circuit. All currents flow clockwise.




$$\begin{cases} 80\angle 0^\circ = 20(I_1 - I_3) + V_1 \\ V_2 = 100 I_2 \\ 20 I_3 + V_2 - V_1 + 20(I_3 - I_1) = 0 \end{cases}$$

ideal transformer : $V_2 = -5V_1$

$I_1 - I_3 = -5(I_2 - I_3)$

$V_1 = -20 I_2$

$40 I_3 - 20 I_1 + 120 I_2 = 0$ $\quad$ $80 = 20 I_1 - 20 I_3 - 20 I_2$

$11 I_2 - 4 I_3 = 0$ $\quad$ $4 = 5 I_3 - 6 I_2$

$I_3 = 1.419$ A

$I_2 = 0.516$ A

$I_1 = 5.935$ A

Therefore, $P_{R_1} = \frac{1}{2}(I_1 - I_3)^2 R_1 = 203.94$ W

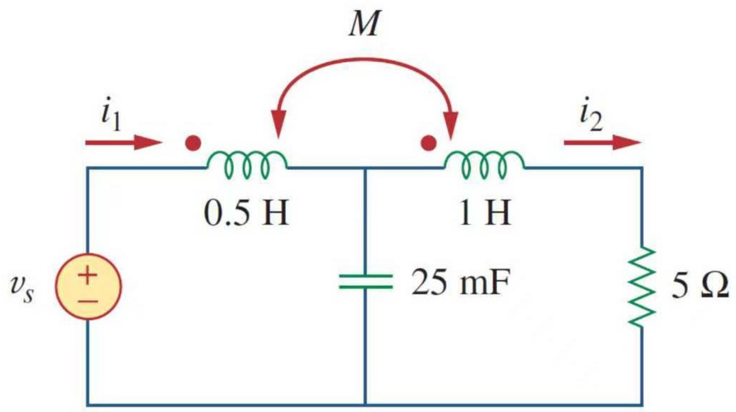
$P_{R_2} = \frac{1}{2} I_3^2 R_2 = 20.14$ W

$P_{R_3} = \frac{1}{2} I_2^2 R_3 = 13.31$ W

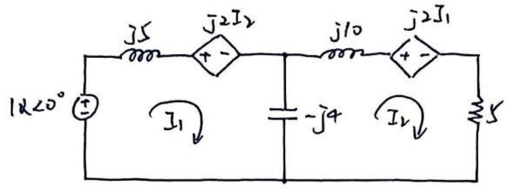
**Exercise 6.4** (25%)

$M = 0.2$ H, $v_s = 12 \cos 10t$ V.

Find $i_1$ and $i_2$. Calculate the energy stored in the coupled coils at $t = 15$ms.




$V_s = 12\angle 0°$ V    $\omega = 10$ rad/s




$$12 - j5I_1 - j2I_2 - (-j4)(I_1 - I_2) = 0$$

$$j10I_2 + j2I_1 + 5I_2 + (-j4)(I_2 - I_1) = 0$$

$$j1I_1 + j6I_2 = 12$$

$$j6I_1 + (j6+5)I_2 = 0$$

$\Rightarrow I_1 = 3.082\angle 40.73°$    $I_1 = 3.082 \cos(10t + 40.73°)$ A

$I_2 = 2.367\angle -99.46°$    $I_2 = 2.367 \cos(10t - 99.46°)$ A

At $t = 15$ ms:

$10 \times 15 \times 10^{-3} = 0.15$ rad $= 8.594°$

$I_1 = 2.008$ A

$I_2 = -0.036$ A

$E_1 = \frac{1}{2}L_1I_1^2 = 1.008$ J

$E_2 = \frac{1}{2}L_2I_2^2 = 6.4 \times 10^{-4}$ J

$E_{coupling} = MI_1I_2 = -7.22 \times 10^{-3}$ J

Therefore, $E_{total} = E_1 + E_2 + E_{coupling} = 1.001$ J