上 海 交 通 大 学 试 卷

（2022~ 2023~1 Academic Year/Fall Semester）

Class No.________________ _ Name in English or Pinyin: _____________________

Student ID No._______________ Name in Hanzi(if applicable): _____________________

VE215 Introduction to Circuits

Mid-term Exam

9th November 08:00 – 09:40

The exam paper has 14 pages in total.

Pledge of Honor

The University of Michigan –Shanghai Jiao Tong University Joint Institute trusts its students to participate in examinations in an honorable and respectful manner, following a spirit of fairness and equality. Cheating, seeking unfair advantage, and disturbing the safe and harmonious environment of examinations are contrary to the ethical principles of students of the Joint Institute. The letter and spirit of the Honor Code shall guide the behavior of students, faculty and all members of the Joint Institute. Therefore, I hereby declare that

(i) I will neither give nor receive unauthorized aid during the present examination, nor will I conceal any violations of the Honor Code by others or myself.

(ii) I confirm that I have read and understood the rules and procedures for the examination set out by SJTU. I will follow them to the best of my ability.

(iii) I understand that violating the rules and procedures for examinations or the Honor Code will lead to administrative and/or academic sanctions.

Date:

Signature:

Please enter grades here:

Exercises No. 题号	Points 得分	Grader's Signature 流水批阅人签名
1		
2		
3		
4		
5		
Total 总分		

Instructions

You have **100 minutes** to complete this midterm. Please write your answers in this booklet. Remember to write neatly and clearly, so your answers can be fully understood.

• You **may not use** your electronic devices other than your calculator.

• **One double-sided** cheating-sheet can be used.

• **Chapter 1-8** are covered in this exam.

• There are **5 questions** in total.

• Please **write your partial steps** and that will be counted.

• Please **manage your time properly**. If you encounter some questions and feel hard, please move on and go back after finishing all the rest.

• **No tolerance to cheating!** Any intentional violations of the SJTU exam rule of a vile nature will be assigned an "F" as course grade.

Fingers Crossed!

1. Discrete Problems [28 points]

1.1 Please choose **one right** description. [5 points]

A. In a linear circuit, we can apply a superposition method to both independent and dependent sources.

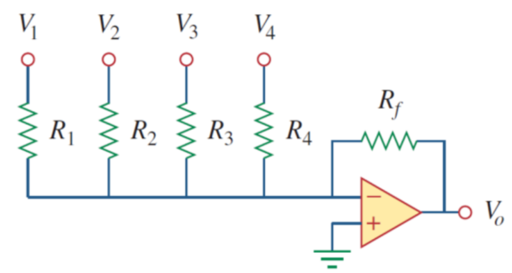
B. In a linear circuit, a source transformation can be applied to dependent sources.

C. Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source and a resistor.

D. Maximum power efficiency can be achieved when the load resistance R_L equals the Thevenin resistance (R_{TH}) as seen from the load.

E. No right answer.

1.2 A circuit below shows 4 bits DAC and an output voltage equation. Please choose **one not right** description. [5 points]



$$-V_O = V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

A. V_1 is called a most significant bit (MSB)

B. V_4 is called a least significant bit (LSB)

C. Resolution of the DAC is 0.125 V.

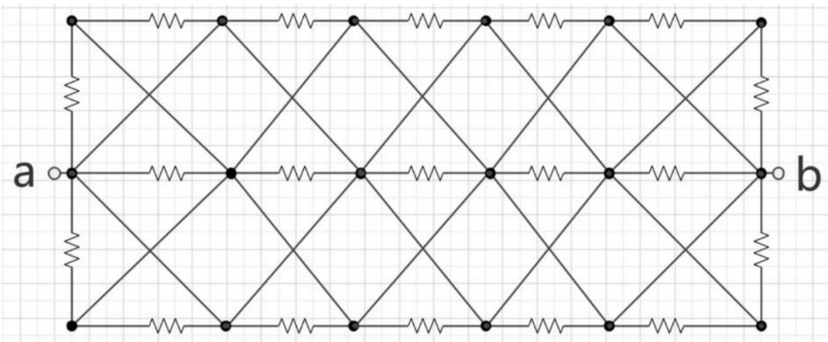
D. With inputs from [0000] – [1111], we get output voltage ($-V_O$) from 0 to 1.875.

E. It is impossible to change LSB of the circuit above regardless of value R_4 and R_f.

1.3 We have a 40 μH inductor with the terminal voltage of $V_L(t) = e^{-t}$ V. Please calculate the expression of power supplied by it when t ≥ 0 and i(0) = 0A. [6 points]

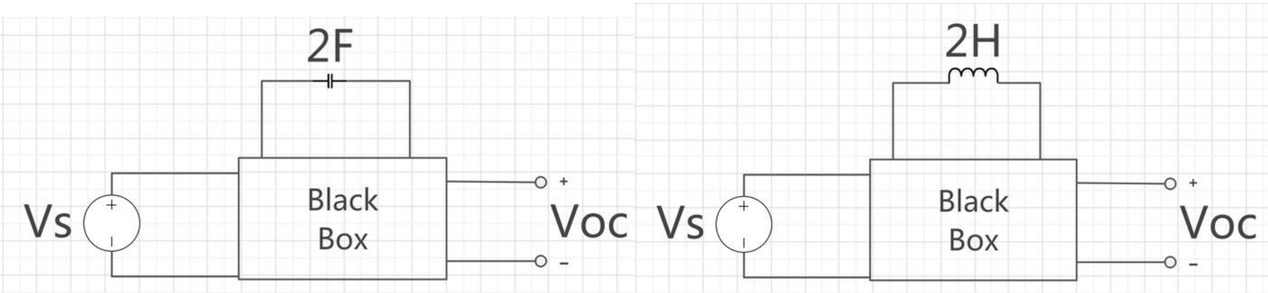
$P_L(t)$ (supplied) = _____________________________ W, t ≥ 0.

1.4 Suppose all the resistors in the figure below have the resistance of 15Ω. Please calculate the equivalent resistance between terminals a and b. Note that the deep dark dots mean that the wires are connected. Otherwise, taking X-shape wires as example, they are not connected at the cross in the figure. [6 points]



R_{ab} = _____________________________ Ω.

1.5 Below shows a black box which is consisted of purely resistors. With a 2F capacitor connected in parallel, Voc is found to be $\frac{1}{4}+\frac{1}{8}e^{-t/6}$ [V], t ≥ 0. V_s is a DC constant voltage source. If we replace the capacitor with a 2H inductor, please calculate the expression of V_{oc} in terms of t. Suppose no energy on both capacitor and inductor initially. [6 points]



$V_{oc}(t)$ = ___________________________ V, t ≥ 0.

1.1 A: superposition is not applicable for dependent source.

B: correct

C: Norton theorem instead of Thevenin theorem

D. max power instead of max power efficiency

$$\varepsilon = \frac{P_R}{P_{source}} = \frac{U_R I}{U_s I} = \frac{U_R}{U_s} = \frac{R}{R+R_{Th}}$$

$\varepsilon \uparrow$ as $R\uparrow$, ε max as R as large as possible

1.2 answer: E

obviously, changing the value of LSB, i.e. V_4, is not related to the circuit elements. You just need to alter the input voltage.

1.3 $i_L(t) = \frac{1}{L}\int_0^t v_L(t)\,dt + C$

$= -25000\, e^{-t} + 25000$

By passive sign convention,

$P_{absorbed} = V_L(t)\cdot I_L(t) = -25000\, e^{-2t} + 25000\, e^{-t}$

Hence, $P_{supplied} = -P_{absorbed} = 25000\, e^{-2t} - 25000\, e^{-t}$

Or, considering qualitatively, I increases so inductor is absorbing energy, i.e. power supplied is negative.

1.4 Observing on each resistor, you will find each is connected to both terminals directly by wire, so 19 resistors are all in parallel,

$$R_{ab} = \frac{1}{\frac{1}{15}\times 19} = \frac{15}{19}\,\Omega$$

1.5 We know that: C final state: open
L final state: short

Here we need a deduced result:
given no initial energy
C initial state: short
L initial state: open

proof: note that C feature: U
L feature: I

no energy $\Rightarrow$ $V_C = 0$
$I_L = 0$

while $V=0$ means short
$I=0$ means open

ends!

So now given the property, it's easy to derive that

$V_{oc(L)}(0) = V_{oc(C)}(\infty) = \frac{3}{8}$

$V_{oc(L)}(\infty) = V_{oc(C)}(0) = \frac{1}{4}$

$\tau_C = RC = 6 \Rightarrow R_{Th} = 3\,\Omega$

$\tau_L = \frac{L}{R} = \frac{2}{3}$

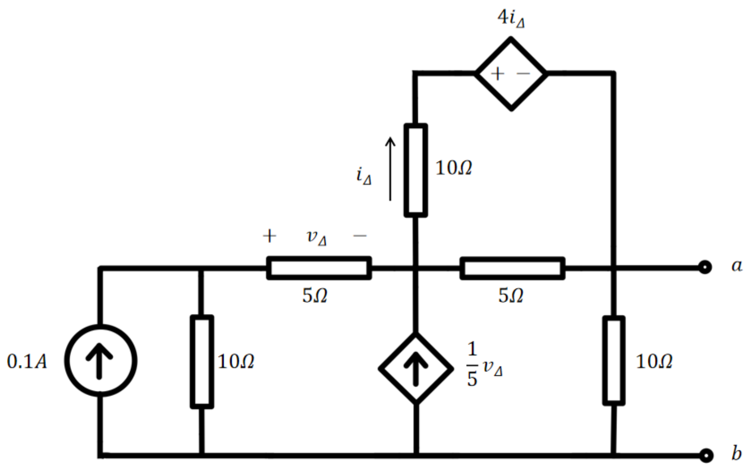
$\therefore V_{oc(L)}(t) = \frac{3}{8} + \left(\frac{1}{4} - \frac{3}{8}\right)e^{-\frac{t}{\tau}}$

$= \frac{3}{8} - \frac{1}{8}e^{-\frac{3t}{2}}$

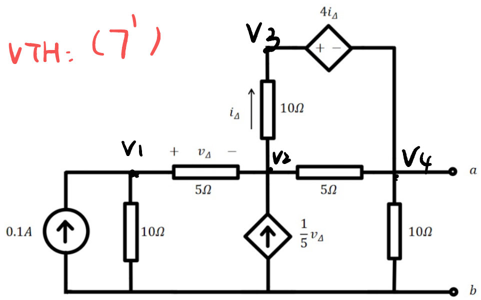
2. Below shows a circuit with an open terminal between a and b. Please answer the following questions. [16 points]

(1) Find the Thevenin's equivalent voltage V_{TH} and the Thevenin's equivalent resistance R_{TH} between terminal a and b.

(2) If we add a load resistance between a and b terminal, please find the value of the load resister R_{load} which gives the maximum power and find the maximum power P_{max}.



VTH: (7')



$$-0.1 + \frac{V_1}{10} + \frac{V_1-V_2}{5} = 0$$

$$\frac{V_2-V_1}{5} + \frac{V_2-V_3}{10} + \frac{V_2-V_4}{5} - \frac{1}{5}(V_1-V_2) = 0$$

$$\frac{V_4}{10} + \frac{V_4-V_2}{5} + \frac{V_3-V_2}{10} = 0$$

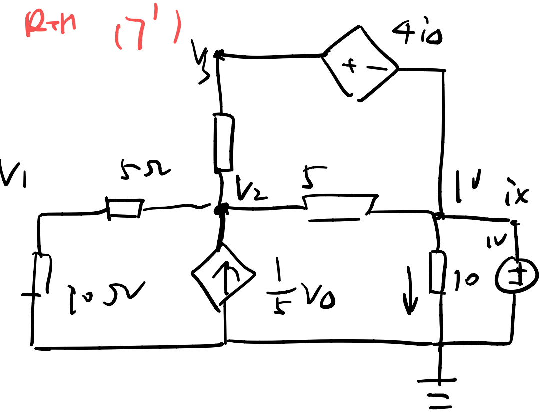
$$V_3 - V_4 = 4 i_\Delta$$

$$i_\Delta = \frac{V_2-V_3}{10}$$

(4')

$$\Rightarrow V_1 = \frac{123}{161}V, \quad V_2 = \frac{104}{161}V, \quad V_3 = \frac{12}{23}V, \quad \boxed{V_4 = \frac{76}{161}V = V_{Th}}$$ (3')

Rth (7')



$$V_1 + 2V_1 - 2V_2 = 0$$

$$\frac{V_1}{10} + \frac{V_1-V_2}{5} = 0$$

$$\frac{V_2-V_1}{5} + \frac{V_2-V_3}{10} + \frac{V_2-1}{5} - \frac{1}{5}(V_1-V_2) = 0$$

$$V_3 - 1 = 4 \cdot \frac{V_2-V_3}{10}$$

(4')

$$\Rightarrow V_1 = \frac{38}{85}V, \quad V_2 = \frac{57}{85}V, \quad V_3 = \frac{77}{85}V.$$

$$\Rightarrow i_x = \frac{1}{10} + \frac{1-\frac{57}{85}}{5} + \frac{\frac{77}{85}-\frac{57}{85}}{10} = \frac{161}{850}A$$

$$\boxed{R_{TH} = \frac{V_x}{i_x} = \frac{850}{161}\ \Omega}$$ (3')

Pmax (2')

$$P_{max} = \boxed{\frac{V_{TH}^2}{4R_{TH}}} = \boxed{0.01055\ W}$$

(1') (1')

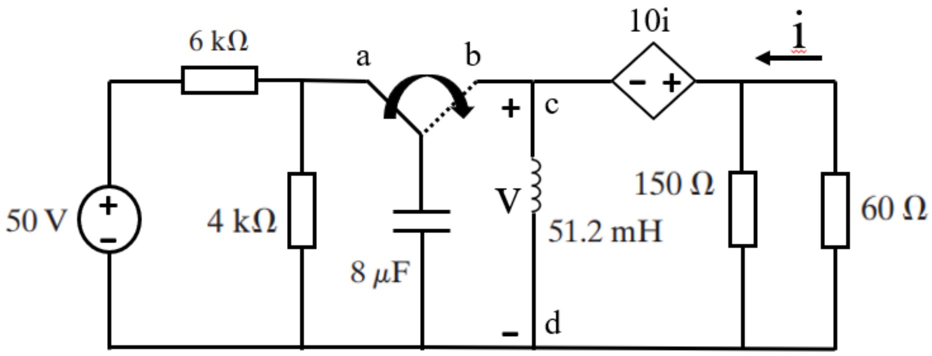
when $R_{Load} = R_{TH}$

3. Below shows a 2nd order ODE circuit. The switch in the circuit has been in position a for a long time. At t = 0 the switch moves instantaneously to position b. Please answer the following questions. [16 points]

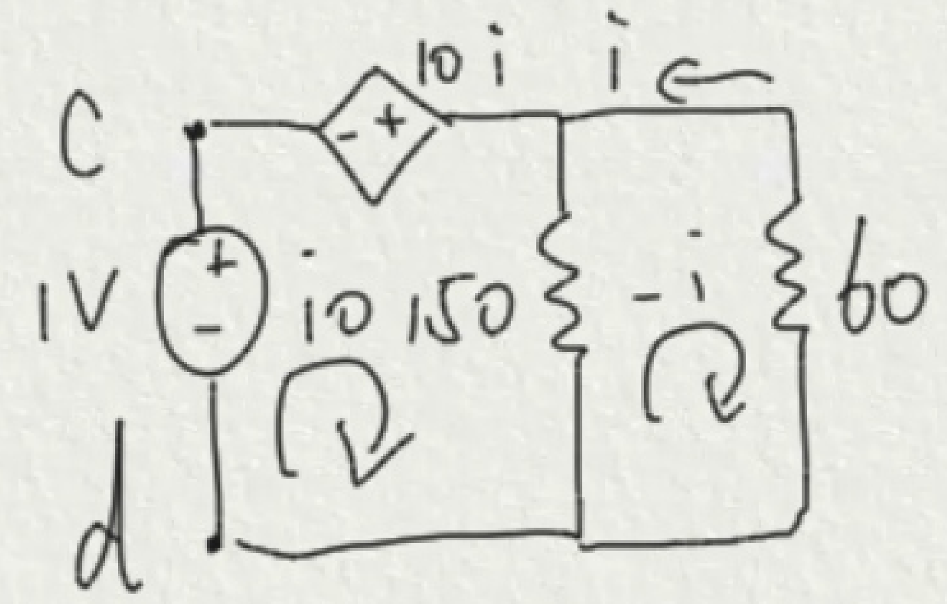
(1) Find the Thevenin equivalent resistance R_{eq} between port c and d when the circuit comes to the steady state (t > 0).

(2) Find $v(0^+)$ and $\frac{dv(0^+)}{dt}$.

(3) Find $v(t)$ for $t > 0$.



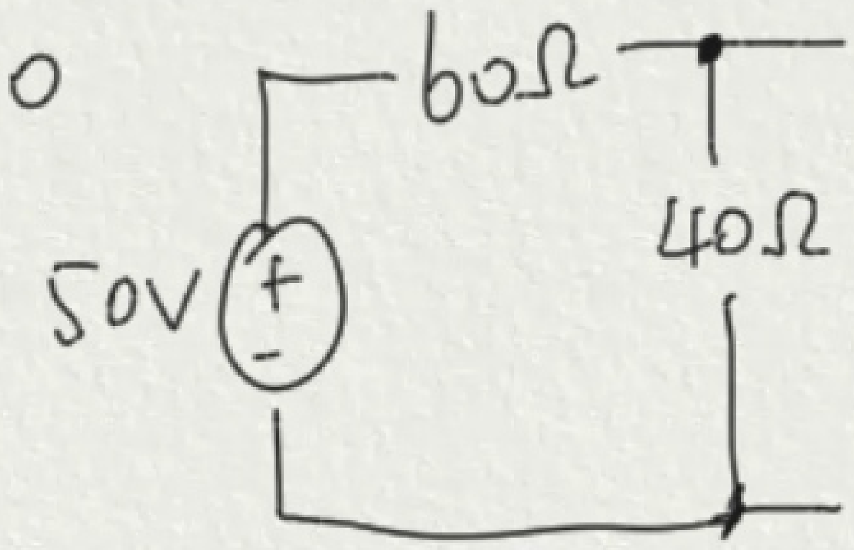
1)



$$\begin{cases} -1-10i+150(i_0+i)=0 \\ 150(-i-i_0)-60i=0 \end{cases}$$

$$\Rightarrow i_0 = \frac{1}{50}A \qquad R_{eq} = 50\Omega \quad 4'$$

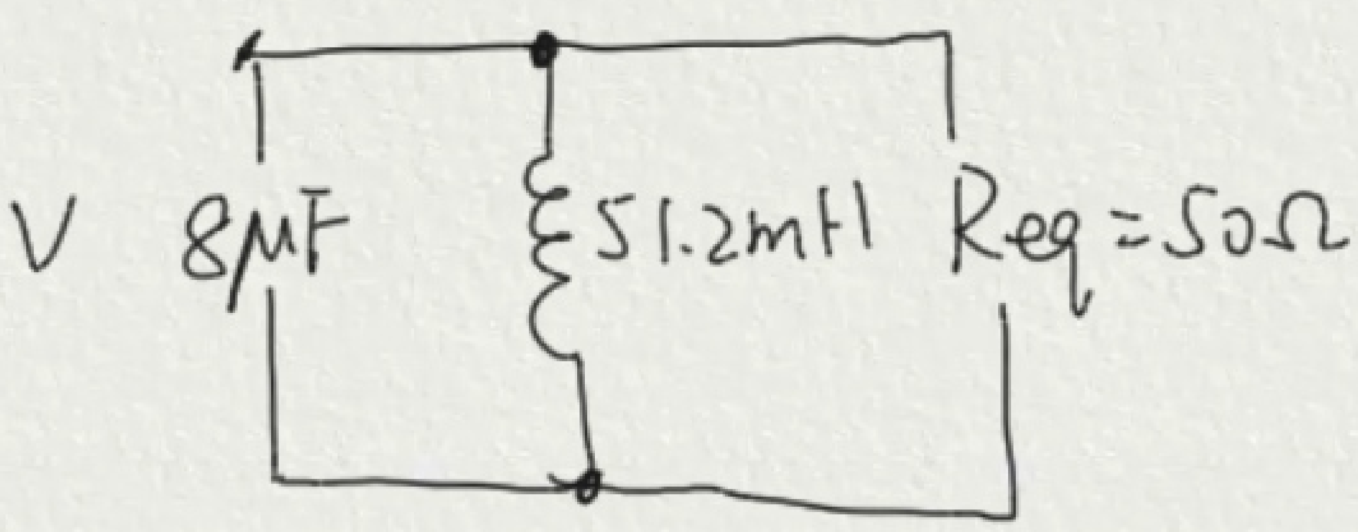
2). $t<0$



$$V(0^+) = V(0^-) = 50 \times \frac{40}{60+40}$$

$$= 20V \quad 3'$$

$t=0$



KCl. $C\frac{dv}{dt} + i_L + \frac{V}{R_{eq}} = 0$

$$C\frac{dv(0^+)}{dt} + i_L(0^+) + \frac{V(0^+)}{R_{eq}} = 0$$

$$\frac{dv(0^+)}{dt} = \frac{-V(0^+)}{R_{eq}C} = -50000\ V/s \quad 3'$$

3) $V_L = L\frac{di}{dt}$

$$C\frac{d^2V}{dt^2} + \frac{V}{L} + \frac{1}{R}\frac{dV}{dt} = 0 \Rightarrow V'' + \frac{1}{RC}V' + \frac{1}{LC}V = 0 \quad 3'$$

$$V'' + 2500V' + 2441406.25 = 0 \Rightarrow S = -1250 \pm 937.5i$$

$$V(t) = e^{-1250t}(A\cos 937.5t + B\sin 937.5t)$$

$$V(0) = A = 20$$

$$\frac{dV(0)}{dt} = -1250A + 937.5B = -50000$$

$$B = -\frac{80}{3}$$

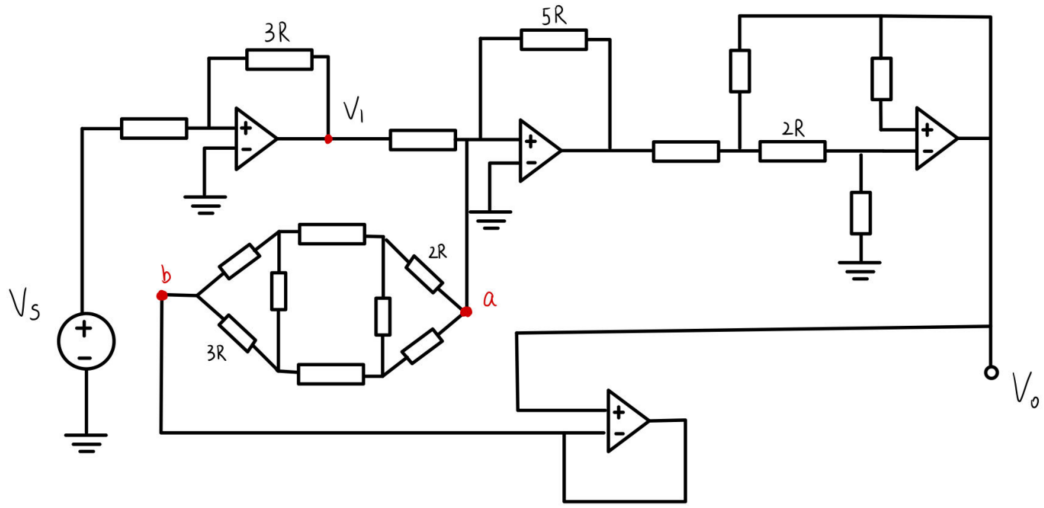
$$V(t) = e^{-1250t}\left(20\cos 937.5t - \frac{80}{3}\sin 937.5t\right) \quad 3'$$

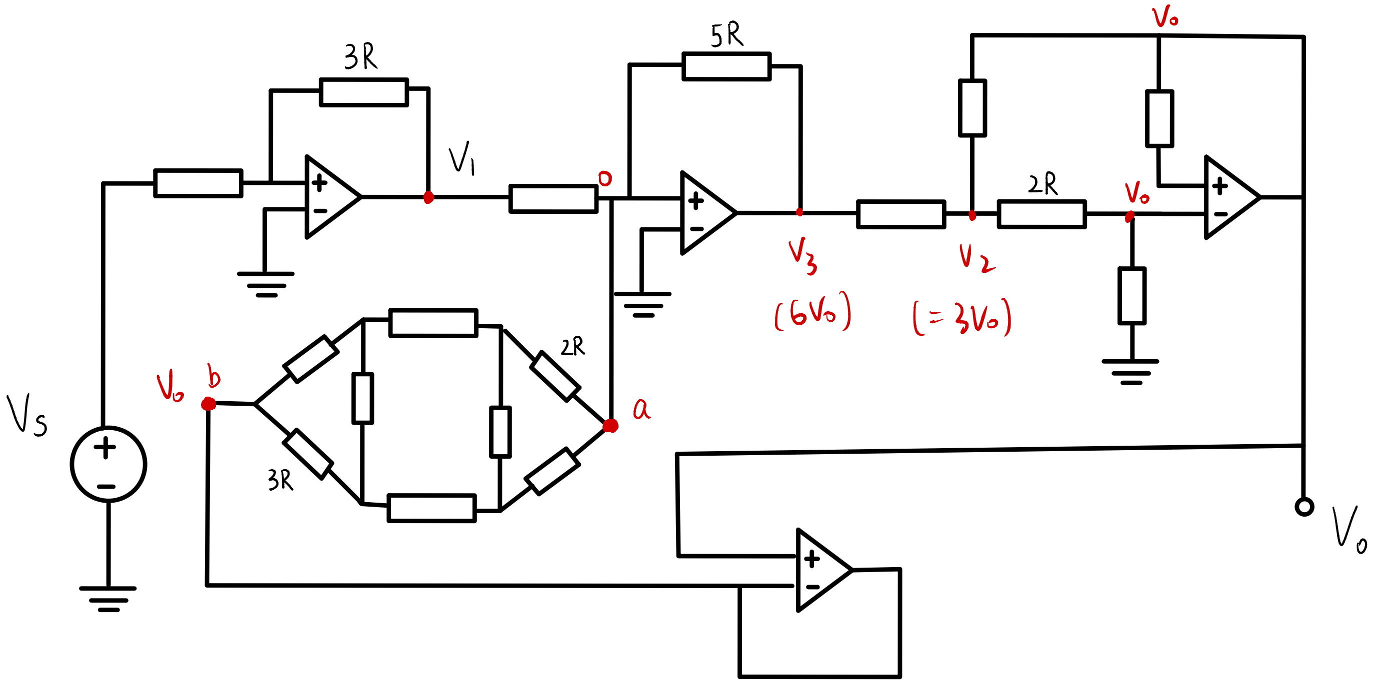
4. Suppose all resistors without labels have resistance of R. Please answer the following questions. [16 points]

(1) V_1 in terms of V_s and R.

(2) The equivalent resistance R_{ab} between terminals a and b.

(3) The overall gain $G = \frac{V_o}{V_s}$.

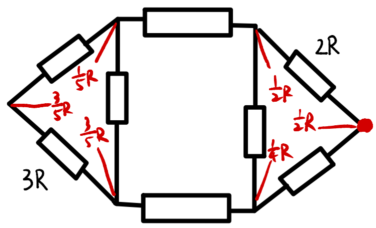




(1) $\frac{V_1}{3R} + \frac{V_s}{R} = 0$

$V_1 = -3V_s$ (4%)

(2)



← (2%)

$R_{ab} = \frac{3}{5}R + \left(\frac{1}{5}R + R + \frac{1}{2}R\right) \| \left(\frac{3}{5}R + R + \frac{1}{4}R\right) + \frac{1}{2}R$

$= \frac{3}{5}R + \frac{17}{10}R \| \frac{37}{20}R + \frac{1}{2}R$

$= \frac{141}{71}R = 1.986R$ (2%)

(3)
$$\begin{cases} V_2 = 3V_0 & (2\%) \\ \frac{3V_s}{R} + \frac{-V_3}{5R} - \frac{V_0}{\frac{141}{71}R} = 0 & (2\%) \\ \frac{V_2 - V_3}{R} + \frac{V_2 - V_0}{R} + \frac{V_2}{3R} = 0 & (2\%) \end{cases}$$

$V_3 = 6V_0$

$3V_s - \frac{6}{5}V_0 - \frac{71}{141}V_0 = 0$ $\frac{V_0}{V_s} = \frac{2115}{1201} = 1.76$ (2%)

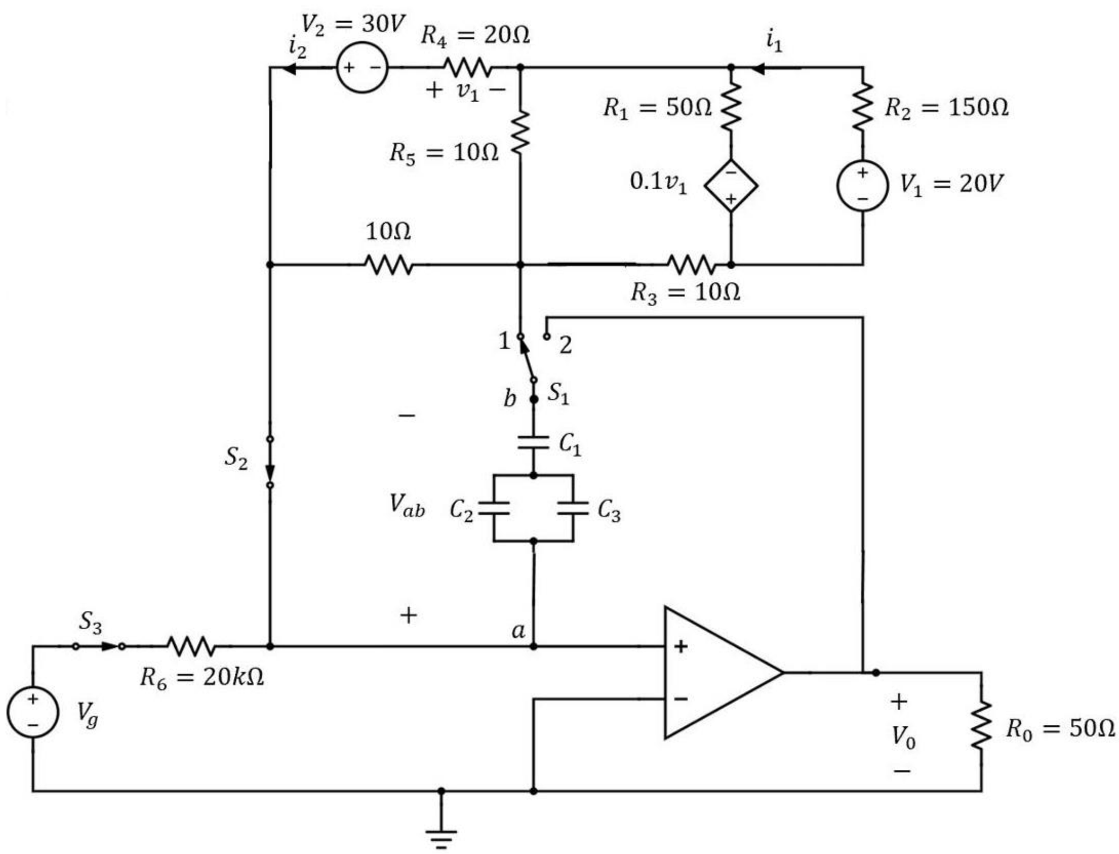
5. Consider the following circuit below.

When $t < 0s$, the switch S_1 is connected to position 1, with switch S_2 and S_3 closed, which lasts a long time. $C_1 = C_2 = C_3 = 100\ \mu F$ and $V_g(t) = 10r(t) - 20r(t-1) + 20r(t-3) - 10r(t-4)$ where unit of V_g is [V] and t is [s].

At $t = 0s$, S_1 is connected to position 2 and S_2 is open instantaneously.

At $t = 3.6s$, S_1 is reconnected to position 1, S_3 is open and S_2 is closed instantaneously.

Assume that during the process of turning on and off the switches, V_{ab} remains to be constant. [24 points]

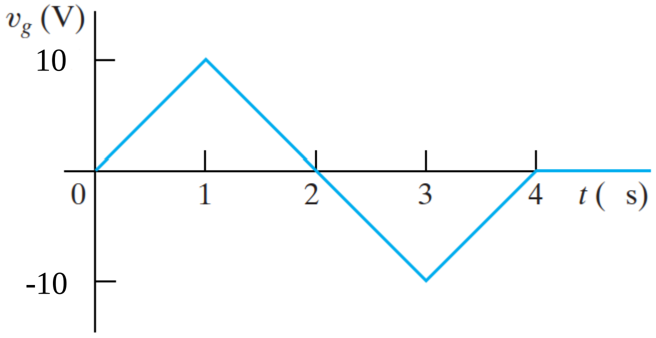


(1) Plot the curve that shows the relationship between V_g and t.

(2) Calculate V_{ab} at $t = 0s$ and $t = 3.6s$. Derive the function of $V_{ab}(t)$ $(t \geq 0s)$. Hint: Consider the situations $0 \leq t \leq 1s; 1 < t \leq 3s; 3 < t \leq 3.6s; t > 3.6s$.

(3) Calculate the instantaneous power of R_0 at $t = 2.5s$.

6’ (1)



6’
(0-1s 2’;1-3s 2’; >3s 2’)

13’ (2) $V_{ab}(0) = V_{Th} = \frac{3550}{437}V = 8.124V\ (V_{Th}\ 1')$, $\frac{dV_{ab}}{dt} = \frac{V_g}{R_6C} = 0.75V_g = \begin{cases} 7.5t\ (0 \le t \le 1s) \\ -7.5(t-2)\ (1 < t \le 3s) \\ 7.5(t-4)\ (3 < t \le 3.6s) \end{cases}$

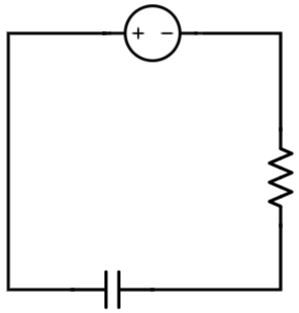
$\therefore V_{Th} = 8.124V, R_{Th} = \frac{140}{19}\Omega = 7.368\Omega, C_{eq} = \frac{200}{3}\mu F.$ (R_{Th} 2’; C_{eq} 1’)

$\therefore V_{ab}(3.6) = 8.724V(V_{ab}(3.6)\ 1'), V_f = V_{Th} = 8.124V$

$\therefore V_{ab}(t) = V_f + (V_0 - V_f)e^{-\frac{t-3.6}{R_{Th}C_{eq}}} = 8.124 + 0.6e^{-\frac{14250}{7}(t-3.6)}(t \ge 3.6s)$

$\therefore V_{ab}(t) = \begin{cases} 3.75t^2 + 8.124(0 \le t \le 1s) \\ -3.75(t-2)^2 + 15.624\ (1 < t \le 3s) \\ 3.75(t-4)^2 + 8.124\ (3 < t \le 3.6s) \\ 8.124 + 0.6e^{-\frac{14250}{7}(t-3.6)}(t > 3.6s) \end{cases}$ (4*2’, each equation 1’, each answer 1’)

$V_{Th} = 8.124V$



$R_{Th} = \frac{140}{19}\Omega$

$C_{eq} = \frac{200}{3}\mu F$

5’ (3) $\therefore V_0\ (2.5) = -14.6865V, P = \frac{V_0^2}{R} = \frac{14.6865^2}{50} = 4.314W$ (V_0 2’; P 2’; provide equation 1’)