

Problem 1:

$$\begin{aligned}
 (1) \quad V_{\text{rms}} &= \sqrt{\frac{1}{T} \int_0^T [1 + 2 \sin(100t + 30^\circ)]^2 dt} = \sqrt{\frac{1}{T} \int_0^T [1 + 4 \sin(100t + 30^\circ) + 4 \cdot \frac{1 - \cos(100t + 60^\circ)}{2}] dt} \\
 &= \sqrt{\frac{1}{T} \int_0^T [3 + 4 \sin(100t + 30^\circ) - 2 \cos(200t + 60^\circ)] dt} = \sqrt{\frac{1}{T} (3T + 4 \cdot \left[\frac{-\cos(100t + 30^\circ)}{100} \right]_0^T - 2 \cdot \left[\frac{\sin(200t + 60^\circ)}{200} \right]_0^T)} \\
 &= \sqrt{3} V.
 \end{aligned}$$

$$(2) \quad I_0 = \begin{cases} I_s & t \in [0, 0.5T] \\ -\frac{4I_s}{T} \cdot t + 2I_s & t \in [0.5T, \frac{3}{4}T] \\ \frac{4I_s}{T} \cdot t - 4I_s & t \in [\frac{3}{4}T, T] \end{cases}$$

$$\text{Then } I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T I_0^2 dt} \approx$$

$$\begin{aligned}
 \text{Here } \int_0^T I_0^2 dt &= \int_0^{0.5T} I_s^2 dt + \int_{0.5T}^{0.75T} (-\frac{4t}{T} + 2)^2 I_s^2 dt + \int_{0.75T}^T (\frac{4t}{T} - 4)^2 I_s^2 dt \\
 &= I_s^2 \cdot \int_0^{0.5T} 1 dt + \int_{0.5T}^{0.75T} (\frac{16}{T^2} t^2 - \frac{16}{T} t + 16) dt + \int_{0.75T}^T (\frac{16}{T^2} t^2 - \frac{32}{T} t + 16) dt \\
 &= \frac{2}{3} T \cdot I_s^2.
 \end{aligned}$$

$$So \quad I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T I_0^2 dt} = \sqrt{\frac{2}{3} I_s^2} = \frac{\sqrt{6}}{3} I_s.$$

Problem 2:

$$I = \frac{\tilde{V}_{TH}}{\Sigma_{eq} + jZ_L} = \frac{\tilde{V}_{TH}}{(R_{eq} + R_L) + j(X_{eq} + X_L)}$$

$$\text{Then } P_L = \frac{1}{2} I^2 R_L = \frac{1}{2} \frac{\tilde{V}_{TH}^2 \cdot R_L}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}$$

To find the maximum, we need to find its derivative.

$$\frac{\partial P_L}{\partial R_L} = 0 \Rightarrow \frac{(X_{eq} + X_L)^2 + R_L^2 - R_L^2}{\dots} = 0 \Rightarrow \cancel{R_L^2} \Rightarrow R_{eq}^2 = R_L^2$$

$$\frac{\partial P_L}{\partial X_L} = 0 \Rightarrow \cancel{R_L} (X_{eq} + X_L) = 0 \Rightarrow X_{eq} = -X_L$$

$$(1) \text{ For } R_{eq} > 0, \quad Z_L = R_{eq} - jX_{eq}$$

$$(2) \text{ For } R_{eq} < 0, \quad Z_L = -R_{eq} - jX_{eq}.$$