VE215 2025SU Assignment 4

Due Date: 23:59, June.26th, 2025
In order to get full marks, you shall write all the intermediate steps of calculation or proof, unless otherwise indicated. **Please box your answers.**

Exercise 4.1(15%)
For the following 1st order circuit, find the value of the output voltage V_0 for all t.

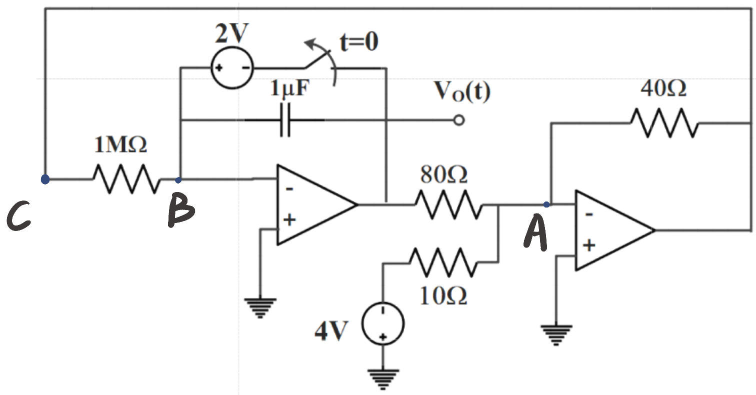


Figure 1: Exercise4.1

When $t<0$, 2 V voltage source connects in the circuit.

$$V_o(0^-) - 0 = -2$$

$$\Rightarrow V_o(0^-) = -2 \text{ V}.$$

At $t>0$: KCL(A): $\frac{V_o - 0}{80} + \frac{-4-0}{10} = \frac{0 - V_c}{40}$ ①

KCL(B): $\frac{V_c - 0}{10^6} = -10^{-6} \cdot \frac{dV_o}{dt}$ ②

① $\Rightarrow$ $V_o = 32 - 2V_c$

② $\Rightarrow$ $V_c = -\frac{dV_o}{dt}$

$$V_o - 2\frac{dV_o}{dt} = 32$$

And $V_o(0^+) = V_o(0^-) = -2$

$$\Rightarrow V_o(t) = \begin{cases} 32 - 34e^{t/2} & t>0 \\ -2, & t \le 0 \end{cases}$$

Exercise 4.2(25%)
In the circuit below, suppose both resistors have the same resistance of R and all the inductors have the same inductance of L. The power supply provides a voltage equal to R at $t < 0$ and suddenly turns off at $t = 0$.

(a) (15%)Suppose $R = L$, please calculate the mathematical expression of $I_x(t)$.

(b) (10%)Could we select the appropriate R and L to make the circuit working in under-damped condition? (R and L may not equal) Please prove your opinion.

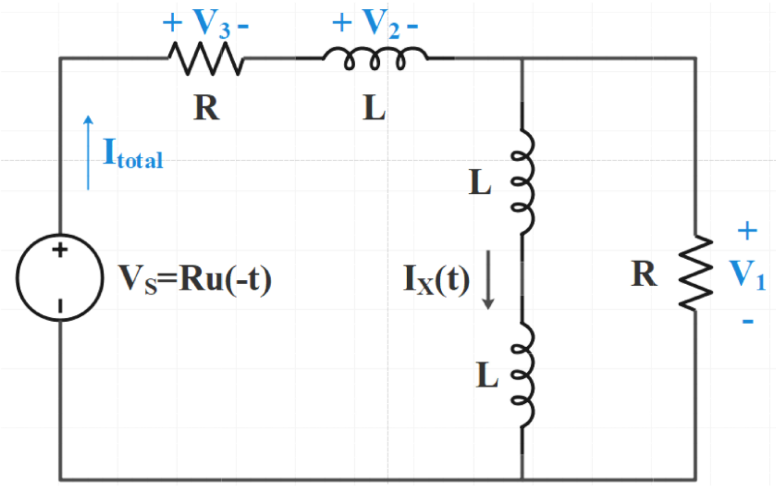


Figure 2: Exercise4.2

Solution:

(1) For the initial value, we have

$$\mathbf{I_X(0)} = \frac{\mathbf{V_S(0^-)}}{\mathbf{R}} = \mathbf{1A} \text{ (1 mark)}$$

According to the voltage-current relationship of inductor, we have

$$\mathbf{V_1(t) = 2L\frac{dI_X(t)}{dt}} \text{ (2 marks)}$$

Thus, at initial, we have

$$\mathbf{\frac{dI_X(0)}{dt}} = \mathbf{\frac{V_1(0)}{2L}} = \mathbf{0V} \text{ (1 mark)}$$

For the current $\mathbf{I_{total}(t)}$, according to KCL, we have

$$\mathbf{I_{total}(t) = I_x(t) + \frac{V_1(t)}{R} = I_x(t) + \frac{2L}{R}\frac{dI_X(t)}{dt}} \text{ (1 mark)}$$

For the voltage $\mathbf{V_2(t)}$ and $\mathbf{V_3(t)}$, we have

$$\mathbf{V_2(t) = L\frac{dI_{total}(t)}{dt} = L\frac{dI_X(t)}{dt} + \frac{2L^2}{R}\frac{d^2I_X(t)}{dt^2}} \text{ (1 mark)}$$

$$\mathbf{V_3(t) = I_{total}(t)R = I_x(t)R + 2L\frac{dI_X(t)}{dt}} \text{ (1 mark)}$$

According to KVL, we have

$$\mathbf{V_1(t) + V_2(t) + V_3(t) = 0} \text{ (1 mark)}$$

Thus, we can obtain the differential equation:

$$\mathbf{\frac{2L^2}{R}\frac{d^2I_X(t)}{dt^2} + 5L\frac{dI_X(t)}{dt} + I_x(t)R = 0} \text{ (2 marks)}$$

Since $\mathbf{R = L}$, we have

$$\mathbf{2\frac{d^2I_X(t)}{dt^2} + 5\frac{dI_X(t)}{dt} + I_x(t) = 0}$$

The two characteristic roots are

$$\mathbf{s_1 = -0.21 \text{ and } s_2 = -2.28} \text{ (2 marks, result)}$$

The general solution of $I_X(t)$ is

$$\mathbf{I_X(t) = C_1e^{-0.21t} + C_2e^{-2.28t}}$$

By derivate it, we have (1 mark, method)

$$\mathbf{\frac{dI_X(t)}{dt} = -0.21C_1e^{-0.21t} - 2.28C_2e^{-2.28t}}$$

Combine the two equations with initial values, we have

$$\mathbf{C_1 + C_2 = 1}$$

$$\mathbf{-0.21C_1 - 2.28C_2 = 0}$$

Thus, we have

$$\mathbf{C_1 = 1.10 \text{ and } C_2 = -0.10} \text{ (2 marks, result)}$$

Thus,

$$\mathbf{I_X(t) = 1.1e^{-0.21t} - 0.1e^{-2.28t}} \text{ A}$$

(2) Consider the general differential equation

$$\mathbf{\frac{2L^2}{R}\frac{d^2I_X(t)}{dt^2} + 5L\frac{dI_X(t)}{dt} + I_x(t)R = 0}$$

Suppose we want the circuit working in under-damped condition, we need $\mathbf{\Delta} < 0$. (2 marks)

However, for the equation, we have

$$\mathbf{\Delta} = (5L)^2 - 4 \times \frac{2L^2}{R} \times R = 17L^2 \;>\; 0 \text{ (2 marks)}$$

It's impossible to make this circuit work in under-damped condition. (1 mark)

VE215 2025SU Assignment 4

Exercise 4.3(25%)For the op-amp circuit shown below, the switch is connected to the branch connected with a 3Ω resister and a $24V$ independent voltage source at $t < 0$, and it is switched to the branch connected with a 8Ω resister and a $20V$ independent voltage source at $t \geq 0$.

(a) (10%)Find $v(t)$ for $t < 0$.

(b) (15%)Find $v(t)$ for $t > 0$.

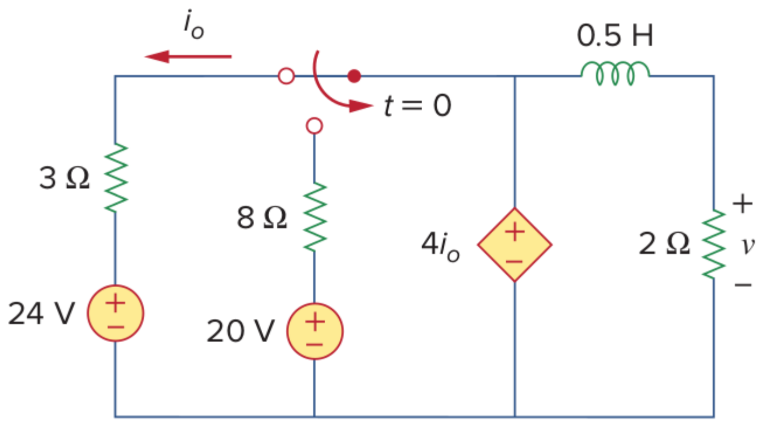
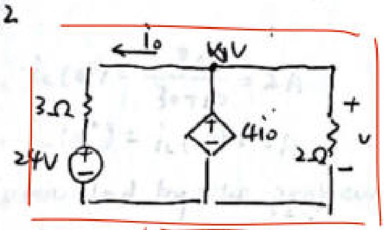


Figure 3: Exercise4.3

Exercise 4.2

(a) $t<0$,



→ draw equivalent circuit 4''
or state that L is short circuited,

$$\begin{cases} V_o = 4i_o \\ \dfrac{v-24}{3} = i_o \end{cases} \Rightarrow \begin{cases} \boxed{i_o = 24\,A} \\ \boxed{V = 96\,V} \end{cases}$$

3''

3''

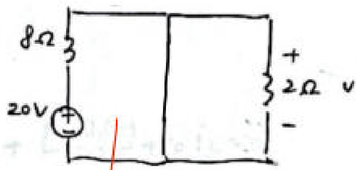
Then $v(t) = 96V$ $(t<0)$

(b) When $t<0$, $i_L(0^-) = \frac{v}{2} = 48$ A.

When $t>0$, $i_L(0^+) = i_L(0^-) = 48$A.

When $t=\infty$,

$i_o = 0A \rightarrow i_L(\infty) = 0$ A.



State that $\oplus\, 4i_o$ is short circuit. 4''

$$\boxed{\begin{aligned} &0.5\frac{di(t)}{dt} + 2i(t) = 0 \\ \Rightarrow\ &i(t) = ce^{-4t} \end{aligned}}$$

5'' (or use formula)

Because $\boxed{i(0^+) = 48\,A,\ \text{i.e.,}\ c = 48,}$ — 2'' (initial condition)

So $\boxed{i(t) = 48e^{-4t}}$ A. — i(t), 1''

$v(t) = i(t)\cdot R = \boxed{96e^{-4t}\,V}$ $(t>0)$ — answer, 3''

Exercise 4.4(15%)For the circuit shown below, please:

(a) (5%)Draw the equivalent cirrcuit at $t < 0$ and find $v(0^+)$ and $i(0^+)$

(b) (5%)Draw the equivalent cirrcuit at $t > 0$ and find $\frac{dv(0^+)}{dt}$ and $\frac{di(0^+)}{dt}$.

(c) (5%)Draw the equivalent cirrcuit at $t = \infty$ and find $v(\infty)$ and $i(\infty)$.

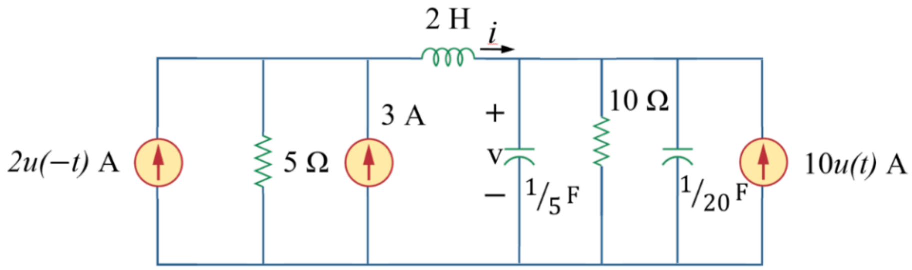
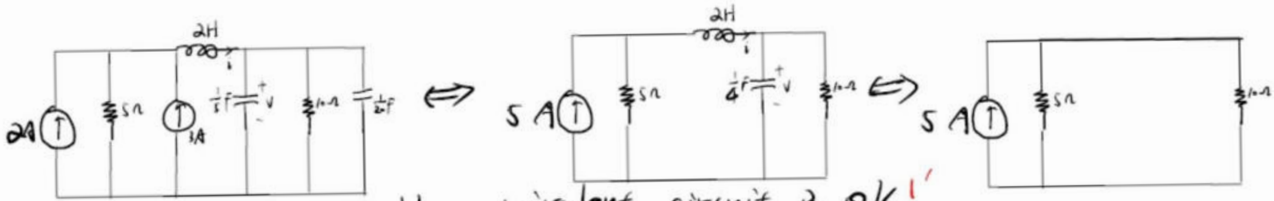


Figure 4: Exercise4.4

(a)

$t<0$

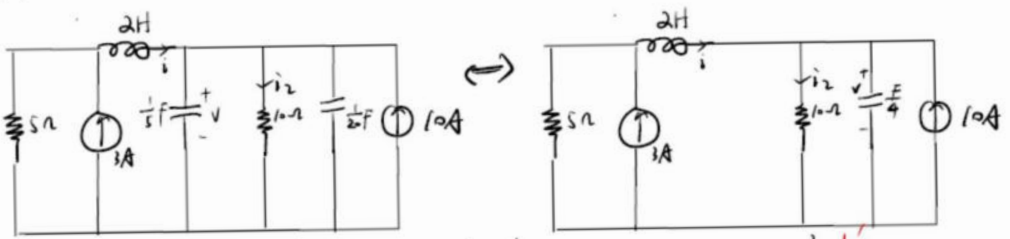


Any reasonable equivalent circuit is ok 1'

$i(0^-) = \frac{25}{15} = \frac{5}{3}A$ 1' $V(0^-) = 10\cdot\frac{5}{3} = \frac{50}{3}V$. 1'

$= i(0^+)$ $= V(0^+)$

(b) $t>0$



Any reasonable equivalent circuit is ok 1'

Since $\begin{cases} V(0^-) = V(0^+) = \frac{50}{3}V, \\ i(0^-) = i(0^+) = \frac{5}{3}A \end{cases}$ using KCL and KVL, we have:

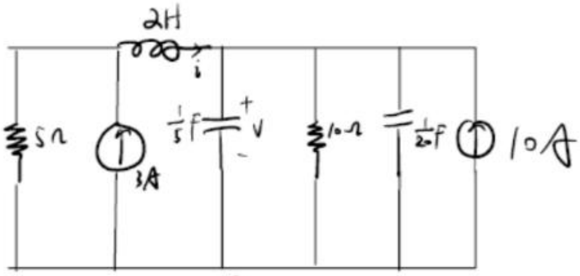
$$\frac{5}{3} + 10 = (C_1 + C_2)\frac{dV(0^+)}{dt} + \frac{\frac{50}{3}}{10}$$

$$\Rightarrow \frac{dV(0^+)}{dt} = 40 \quad 1'$$

$$-15 + 5\cdot\frac{5}{3} + 2\frac{di}{dt} + \frac{50}{3} = 0$$

$$\Rightarrow \frac{di(0^+)}{dt} = -5 \quad 1'$$

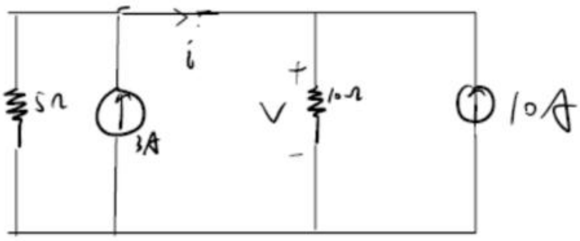
c) $t \to \infty$



$$i(\infty) = -\frac{17}{3}\ A \quad 1'$$

$$V(\infty) = \frac{130}{3}\ V \quad 1'$$

⇕



Any reasonable equivalent circuit is ok 1'

Exercise 4.5(20%)The input current source of the following circuit is $2(1-u(t))A$.

(a) (5%)Construct the dual of the circuit below.

(b) (15%)Find $i(t)$ for $t > 0$.

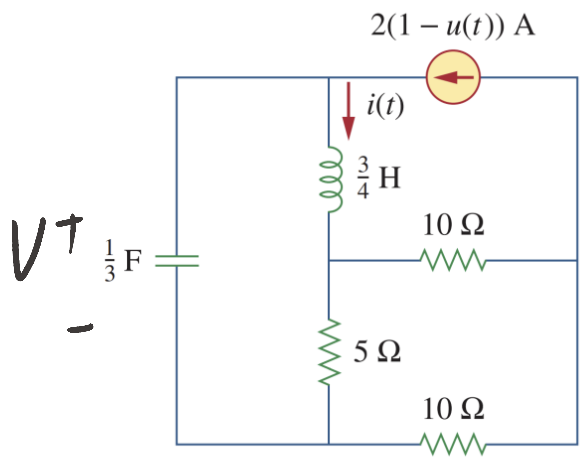
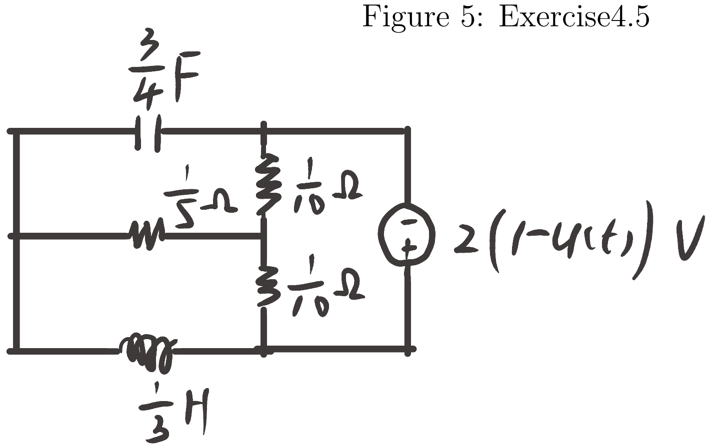
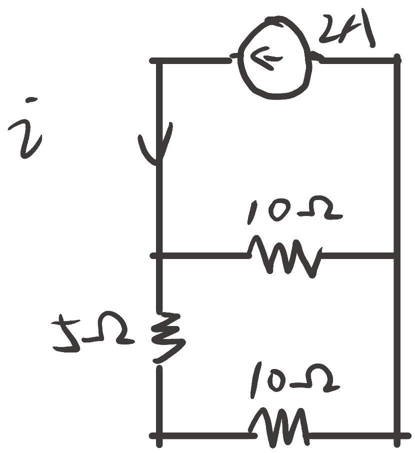


Figure 5: Exercise4.5

(a)



(b) When $t<0$, equivalent circuit:



$i(t) = 2A$

$v(t) = \left(2 \cdot \frac{10}{10+15}\right) \cdot 5$

$= 4V$

Therefore $i(0^+) = i(0^-) = 2A$

$$V(0^+) = V(0^-) = 4\ V$$

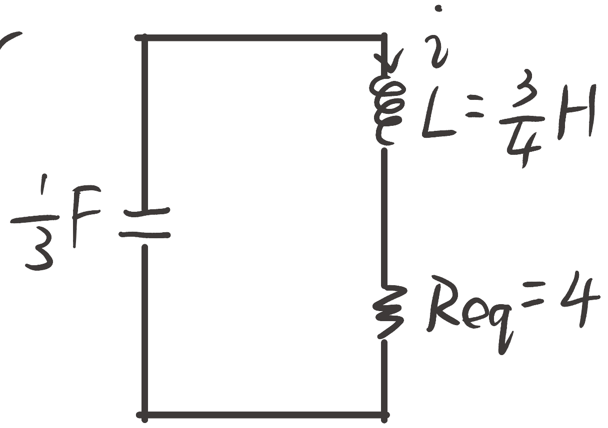
At $t=0$: KVL: $\frac{3}{4}\frac{di}{dt} + 4i - V = 0$

KCL: $i + \frac{1}{3}\frac{dV}{dt} = 0$

$$\Rightarrow \frac{3}{4}i'' + 4i' + 3i = 0$$

$$\frac{di(0^+)}{dt} = \frac{4}{3}\left(V(0) - 4i(0)\right) = -\frac{16}{3}$$

Or



$$\alpha = \frac{R}{2L} = \frac{8}{3}$$

$$w_0 = \frac{1}{\sqrt{LC}} = 2$$

$$S_1 = \frac{-8+2\sqrt{7}}{3} = -0.903$$

$$S_2 = \frac{-8-2\sqrt{7}}{3} = -4.431$$

$$i(t) = Ae^{-4.431t} + Be^{-0.903t}$$

$i(0)$ $i'(0)$ $\Rightarrow \begin{cases} A=1 \\ B=1 \end{cases}$ $\Rightarrow i(t) = e^{-4.431t} + e^{-0.903t}$