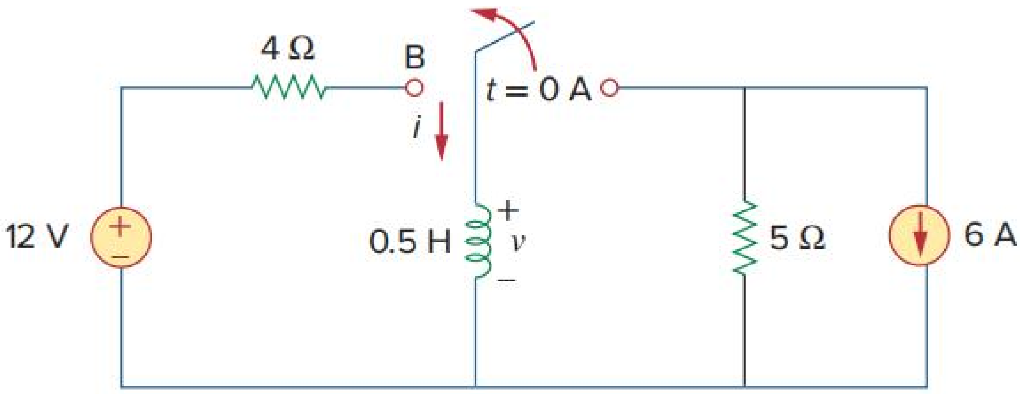
Homework 3 Rubric

In order to get full marks, you shall write all the intermediate steps of calculation or proof unless otherwise indicated. This assignment covers content from chapters 6 to 8.

1. 20%

The switch in Fig. 6.86 has been in position A for a long time. At $t = 0$, the switch moves from position A to B. The switch is a make-before-break type so that there is no interruption in the inductor current. Find:

(a) $i(t)$ for $t > 0$,
(b) v *just after* the switch has been moved to position B,
(c) $v(t)$ long after the switch is in position B.



(a) When the switch is in position A,

$i = -6 = i(0)$ (2)

When the switch is in position B,

$i(\infty) = 12/4 = 3,$ (2) $\tau = L/R = 1/8$ (2)

$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$ (4)

$i(t) = \mathbf{(3 - 9e^{-8t})\ A}$ (2)

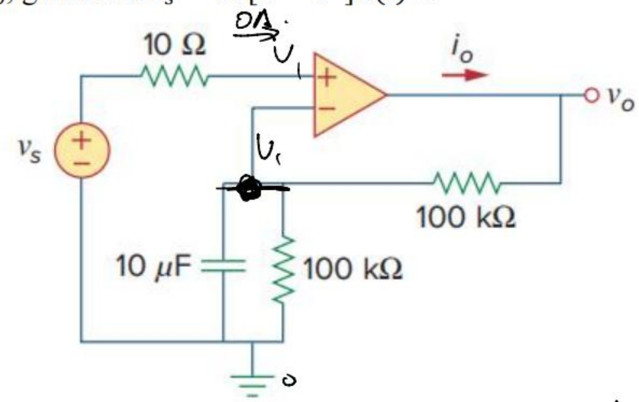
(b) $-12 + 4i(0) + v = 0$, i.e. (4) $v = 12 - 4i(0) = \mathbf{36\ V}$ (2)

(c) At steady state, the inductor becomes a short circuit so that

$v = \mathbf{0\ V}$ (2)

2. 20%

Find v_o and i_o, given that $v_s = 10[1 - e^{-t}]u(t)$ V.



$v_1 = V_s = 10(1-e^{-t})u(t)$ (2) When $t>0$, $\frac{dv_s}{dt} = 10e^{-t}$. (2)

$\frac{v_1}{100k} + \frac{v_1 - v_o}{100k} + C\frac{dv_s}{dt} = 0$. (10)

$\Rightarrow V_o = 2V_s + \frac{dV_s}{dt} = 20(1-e^{-t}) + 10e^{-t} = 20 - 10e^{-t}$ (2)

When $t<0$, $V_o = 0$ $\Rightarrow$ $V_o = (20 - 10e^{-t})u(t)$ V. (1)

$i_o(t) = \frac{V_o - V_1}{100k} = \frac{10}{100k} = 10^{-4}$ (2)

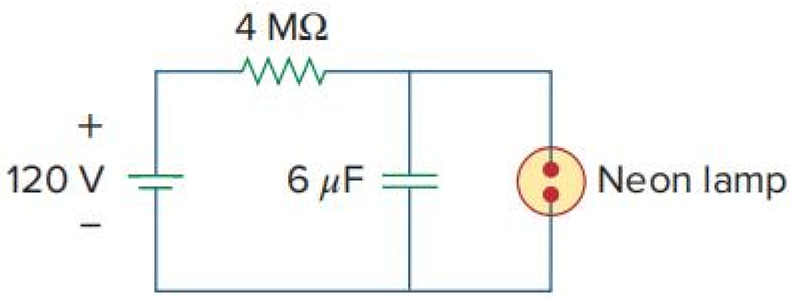
When $t<0$, $i_o = 0$ $\Rightarrow$ $i_o = 0.1u(t)$ mA. (1)

3. 20%

A simple relaxation oscillator circuit is shown in Fig. 7.145. The neon lamp fires when its voltage reaches 75 V and turns off when its voltage drops to 30 V. Its resistance is 120 Ω when on and infinitely high when off.

(a) For how long is the lamp on each time the capacitor discharges?

(b) What is the time interval between light flashes?



(a) The light is on from 75 volts until 30 volts. During that time we essentially have a 120-ohm resistor in parallel with a 6-μF capacitor.

(2)

$v(0) = 75$, $v(\infty) = 0$, $\tau = 120\text{x}6\text{x}10^{-6} = 0.72$ ms (2)

(4)

$v(t_1) = 75\,e^{-t_1/\tau} = 30$ which leads to $t_1 = -0.72\ln(0.4)$ ms = **659.7 μs** of lamp on time.
(2)

(b) $\tau = RC = (4\times10^6)(6\times10^{-6}) = 24$ s (2)

Since $v(t) = v(\infty) + \left[v(0) - v(\infty)\right] e^{-t/\tau}$

$$v(t_1) - v(\infty) = \left[v(0) - v(\infty)\right] e^{-t_1/\tau} \qquad (1) \quad (2)$$

$$v(t_2) - v(\infty) = \left[v(0) - v(\infty)\right] e^{-t_2/\tau} \qquad (2) \quad (2)$$

Dividing (1) by (2),

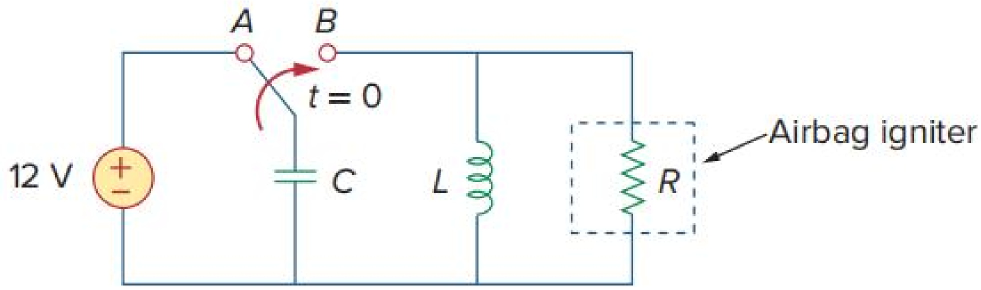
$$\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)} = e^{(t_2 - t_1)/\tau}$$

$$t_0 = t_2 - t_1 = \tau \ln\left(\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)}\right) \quad (2)$$

$$t_0 = 24\ln\left(\frac{75-120}{30-120}\right) = 24\ln(2) = \mathbf{16.636\ s} \quad (2)$$

4. 20%

An automobile airbag igniter is modeled by the circuit in Fig. 8.122. Determine the time it takes the voltage across the igniter to reach its first peak after switching from A to B. Let $R = 3\ \Omega$, $C = 1/30$ F, and $L = 60$ mH.



The voltage across the igniter is $v_R = v_C$ since the circuit is a parallel RLC type.

$$v_C(0) = 12, \text{ and } i_L(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2\text{x}3\text{x}1/30) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{60\text{x}10^{-3}\text{x}1/30} = 22.36$$

$\alpha < \omega_o$ produces an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm j21.794 \quad (5)$$

$$v_C(t) = e^{-5t}(A\cos 21.794t + B\sin 21.794t) \quad (4) \qquad (1)$$

$$v_C(0) = 12 = A \quad (2)$$

$$dv_C/dt = -5[(A\cos 21.794t + B\sin 21.794t)e^{-5t}]$$
$$+ 21.794[(-A\sin 21.794t + B\cos 21.794t)e^{-5t}] \quad (2) \qquad (2)$$

$$dv_C(0)/dt = -5A + 21.794B$$

But, $\quad dv_C(0)/dt = -[v_C(0) + Ri_L(0)]/(RC) = -(12+0)/(1/10) = -120$

Hence, $\quad -120 = -5A + 21.794B$, leads to $B\ (5\text{x}12 - 120)/21.794 = -2.753$ (2)

At the peak value, $\quad dv_C(t_o)/dt = 0$, i.e.,

$$0 = A + B\tan 21.794t_o + (A21.794/5)\tan 21.794t_o - 21.794B/5 \quad (3)$$

$$(B + A21.794/5)\tan 21.794t_o = (21.794B/5) - A$$

$$\tan 21.794t_o = [(21.794B/5) - A]/(B + A21.794/5) = -24/49.55 = -0.484$$

$21.794t_0 = \arctan(-0.484)$

The general solution is $n\pi - 0.451$. We need the first positive time $t_0 > 0$. This occurs for $n = 1$:

$21.794t_0 = \pi - 0.451 = 3.1416 - 0.451 = 2.6906$

$$t_0 = \frac{2.6906}{21.794} \approx 0.12345 \text{ s} \quad (2)$$

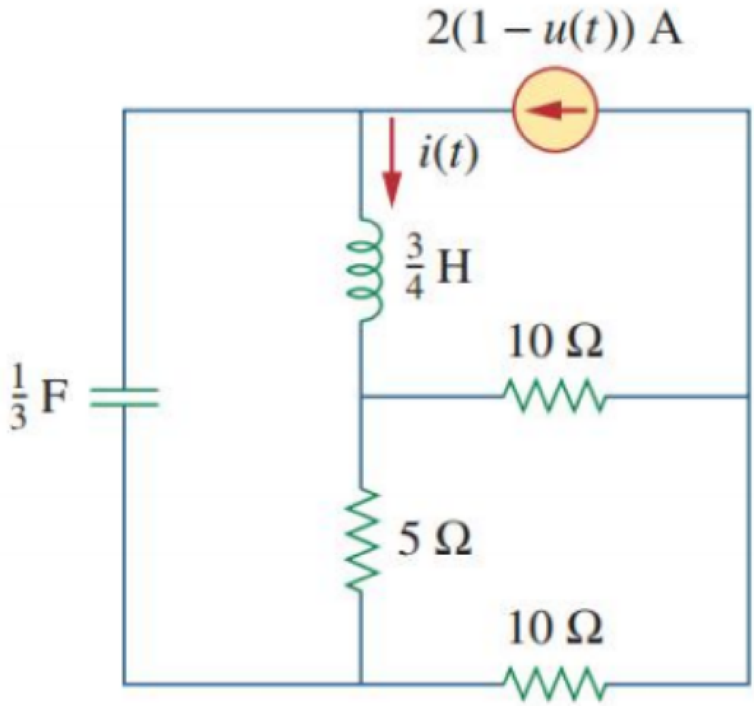
The time it takes for the voltage to reach its first peak (which is actually a valley/minimum in this case) is **123.5 ms**.

5. 20%

(a) When t = 0, what is the energy stored in the capacitor?

(b) Calculate i(t) for t > 0.

(c) Draw the dual of this circuit.



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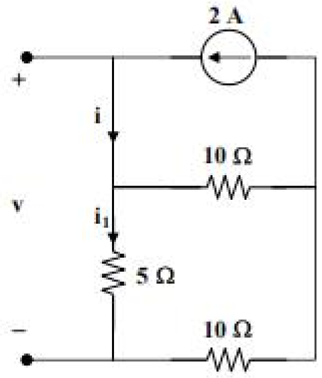
At $t = 0^{-}$, the equivalent circuit is as shown.



$i(0) = 2A, \quad i_1(0) = 10(2)/(10 + 15) = 0.8\ A$

$v(0) = 5i_1(0) = 4V$ (3)

For $t > 0$, we have a source-free series RLC circuit.

$R = 5||(10 + 10) = 4$ ohms (2)

$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/3)(3/4)} = 2$

$\alpha = R/(2L) = (4)/(2x(3/4)) = 8/3$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -4.431, -0.903$ (3)

$i(t) = [Ae^{-4.431t} + Be^{-0.903t}]$ (2)

$$i(0) = A + B = 2 \qquad (1)$$

$di(0)/dt = (1/L)[-Ri(0) + v(0)] = (4/3)(-4x2 + 4) = -16/3 = -5.333$

$$\text{Hence, } -5.333 = -4.431A - 0.903B \qquad (2)$$

From (1) and (2), $A = 1$ and $B = 1$. (2)

$$i(t) = [e^{-4.431t} + e^{-0.903t}]\ A \qquad (2)$$

(b) 1. $I = \frac{2}{10+15} \times 10 = \frac{4}{5}A$

$V = IR = \frac{4}{5} \times 5 = 4V.$ (3)

$W = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{1}{3} \times 4^2 = \frac{8}{3}J.$