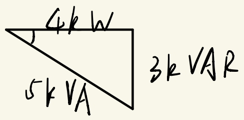
Three loads are connected in parallel across a 300 V(rms) line, as shown in Fig. P10.28. Load 1 absorbs 3 kW at unity power factor; Load 2 absorbs 5 kVA at 0.8 leading; Load 3 absorbs 5 kW and delivers 6 kvars.

a) Find the impedance that is equivalent to the three parallel loads.

b) Find the power factor of the equivalent load as seen from the line's input terminals.

a) $S_1 = 3kW$

$|S_2| = 5k\ VA \Rightarrow S_2 = 4k - j3k$



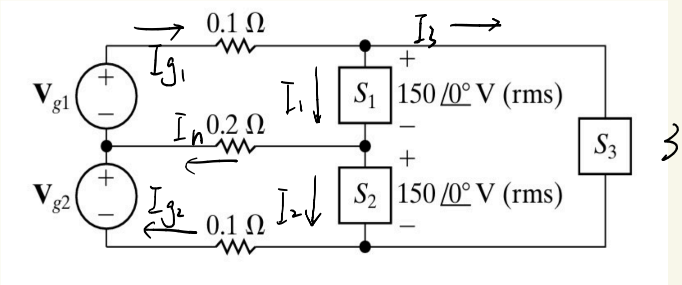
$S_3 = 5k - j6k$

$S = S_1 + S_2 + S_3 = 12k - j9k$

$300\tilde{I}_{rms}^{*} = (12 - j9) \times 10^3 \qquad \tilde{I}_{rms} = 40 + j30\ A$

$$Z = \frac{300}{40 + j30} = 4.8 - j3.6 = 6\angle -36.87°$$

(b) $pf = \cos(-36.87°) = 0.80$ leading.



$300\angle 0^\circ$

$S_1 = (6 + j3)k \quad S_2 = (7.5 - j4.5)k \quad S_3 = (12 + j9)k$

$\tilde{V}_1 = 150\angle 0^\circ \Rightarrow \tilde{I}_1 = \left(\frac{S_1}{\tilde{V}_1}\right)^* = \frac{(6 - j3)k}{150} = 40 - j20 \text{ A}$

$\tilde{I}_2 = \left(\frac{S_2}{\tilde{V}_2}\right)^* = \frac{(7.5 + j4.5)k}{150} = 50 + j30 \text{ A}$

$\tilde{I}_3 = \frac{(12 - j9)k}{300} = 40 - j30 \text{ A}$

$\tilde{I}_{g_1} = \tilde{I}_1 + \tilde{I}_3 = 80 - j50 \text{ A}$

$\tilde{I}_n = \tilde{I}_1 - \tilde{I}_2 = -10 - j50 \text{ A}$

$\tilde{I}_{g_2} = \tilde{I}_2 + \tilde{I}_3 = 90 + j0 \text{ A}$

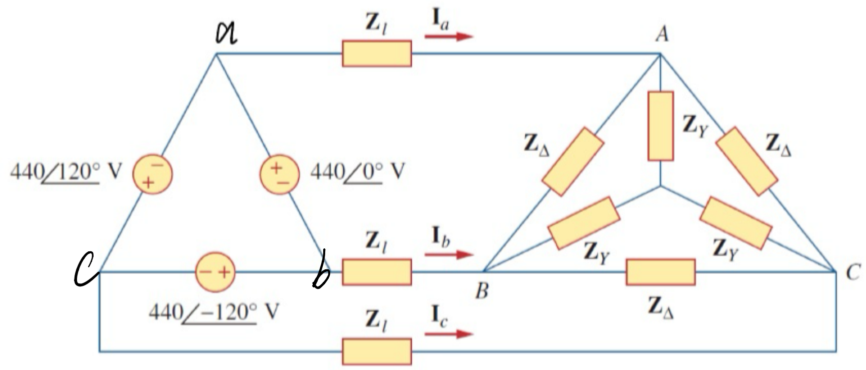
$\tilde{V}_{g_1} = 0.1 I_{g_1} + 150 + 0.2 \tilde{I}_n = 156 - j15 \text{ V (rms)}$

$\tilde{V}_{g_2} = -0.2 \tilde{I}_n + 150 + 0.1 \tilde{I}_{g_2} = 161 + j10 \text{ V (rms)}$

$S_{g_1} = -[(156 - j15)(80 + j50)] = -13230 - j6600 \text{ VA}$

$S_{g_2} = -[(161 + j10)(90 + j0)] = -14490 - j900 \text{ VA}$

Find the line currents in the three-phase network. Take $Z_\Delta = 12 - j15\,\Omega$, $Z_Y = 4 + j6\,\Omega$, $Z_L = 2\,\Omega$.



Convert Δ-connected source to Y-connected.

$$\tilde{V}_{an} = \frac{\tilde{V}_{ab}}{\sqrt{3}} \angle -30^\circ = \frac{440}{\sqrt{3}} \angle -30^\circ = 254 \angle -30^\circ$$

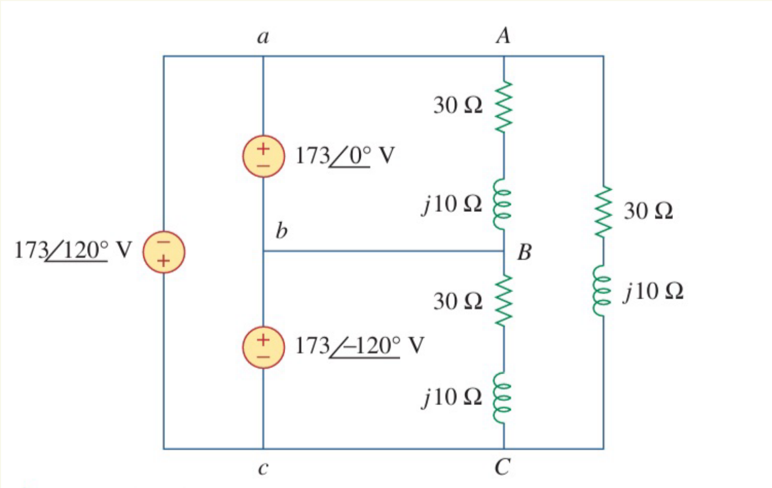
Convert Δ-connected load to Y-connected load.

$$Z = Z_Y \parallel \frac{Z_\Delta}{3} = (4+j6) \parallel (4-j5) = 5.723 - j0.2153$$

$$\tilde{I}_a = \frac{\tilde{V}_{an}}{Z_l + Z} = \frac{254\angle -30^\circ}{7.723 - j0.2153} = 32.88 \angle -28.4^\circ \text{ A}$$

$$\tilde{I}_b = \tilde{I}_a \angle -120^\circ = 32.88 \angle -148.4^\circ \text{A}$$

$$\tilde{I}_c = I_a \angle 120^\circ = 32.88 \angle 91.6^\circ \text{ A}$$



$Z_\Delta = 30 + j10\,\Omega$

Phase Current $\tilde{I}_{AB} = \dfrac{\tilde{V}_{ab}}{Z_\Delta} = \dfrac{173\angle 0^\circ}{31.62\angle 18.43^\circ} = 5.47\angle -18.43^\circ\,A$

$\tilde{I}_{BC} = \tilde{I}_{AB}\angle -120^\circ = 5.47\angle -138.43^\circ\,A$

$\tilde{I}_{CA} = \tilde{I}_{AB}\angle 120^\circ = 5.47\angle 101.57^\circ\,A$

Line Current $\tilde{I}_a = \tilde{I}_{AB} - \tilde{I}_{CA} = \tilde{I}_{AB}\sqrt{3}\angle -30^\circ = 9.474\angle -48.43^\circ\,A$

$\tilde{I}_b = \tilde{I}_a\angle -120^\circ = 9.474\angle -168.43^\circ\,A$

$\tilde{I}_c = \tilde{I}_a\angle 120^\circ = 9.474\angle 71.57^\circ\,A$