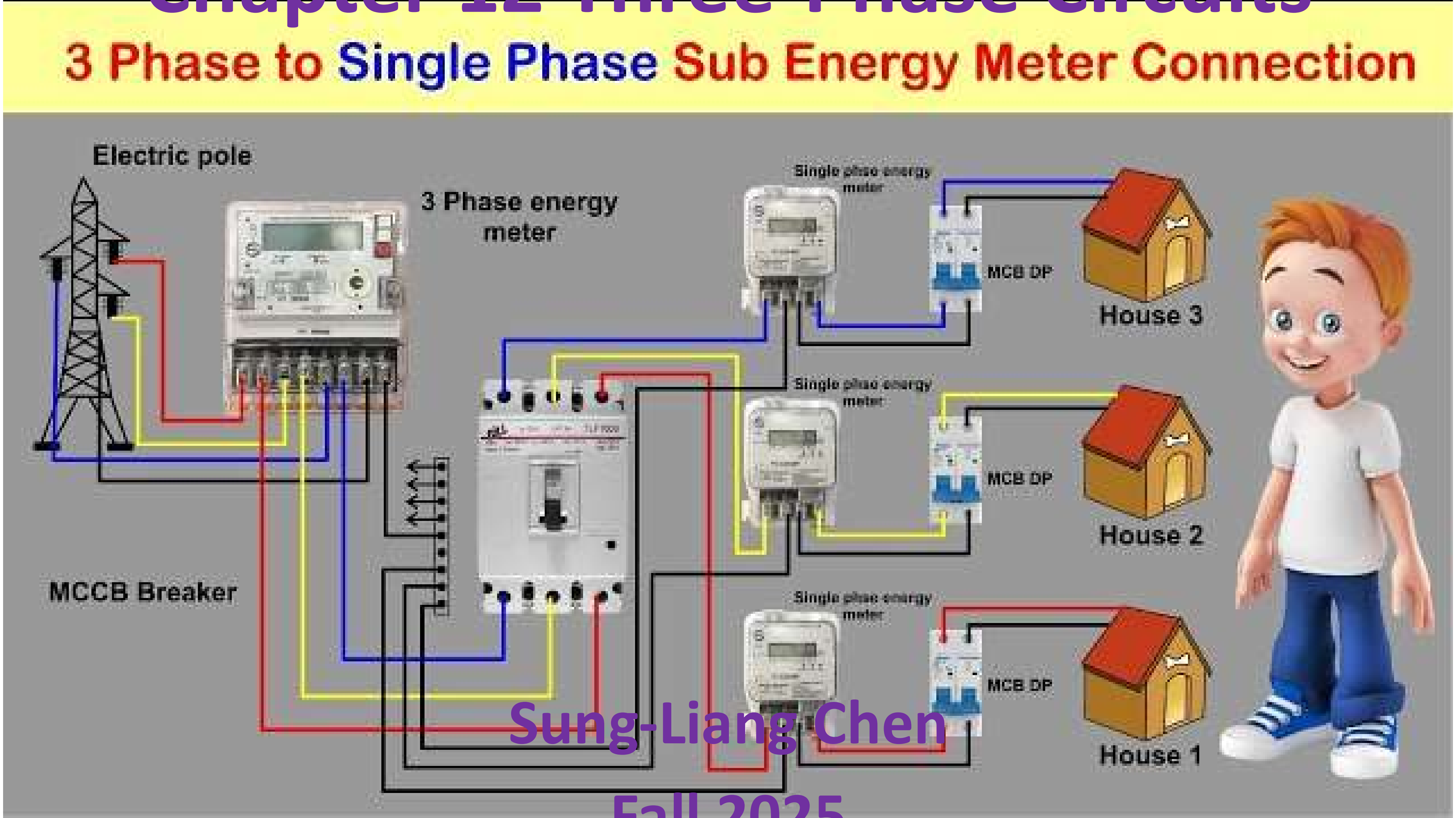
Chapter 12 Three-Phase Circuits



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Fall 2025

12.1 Introduction

So far, we have dealt with *single-phase* circuits. A single-phase ac power system consists of a generator connected through a pair of wires (a transmission line) to a load. Figure 12.1(a) depicts a single-phase two-wire system.

Figure 12.1(a) Single-phase two-wire system.

Circuits or systems in which the sources operate at the same frequency but different phases are known as *polyphase*. Figure 12.3 shows a three-phase four-wire system.

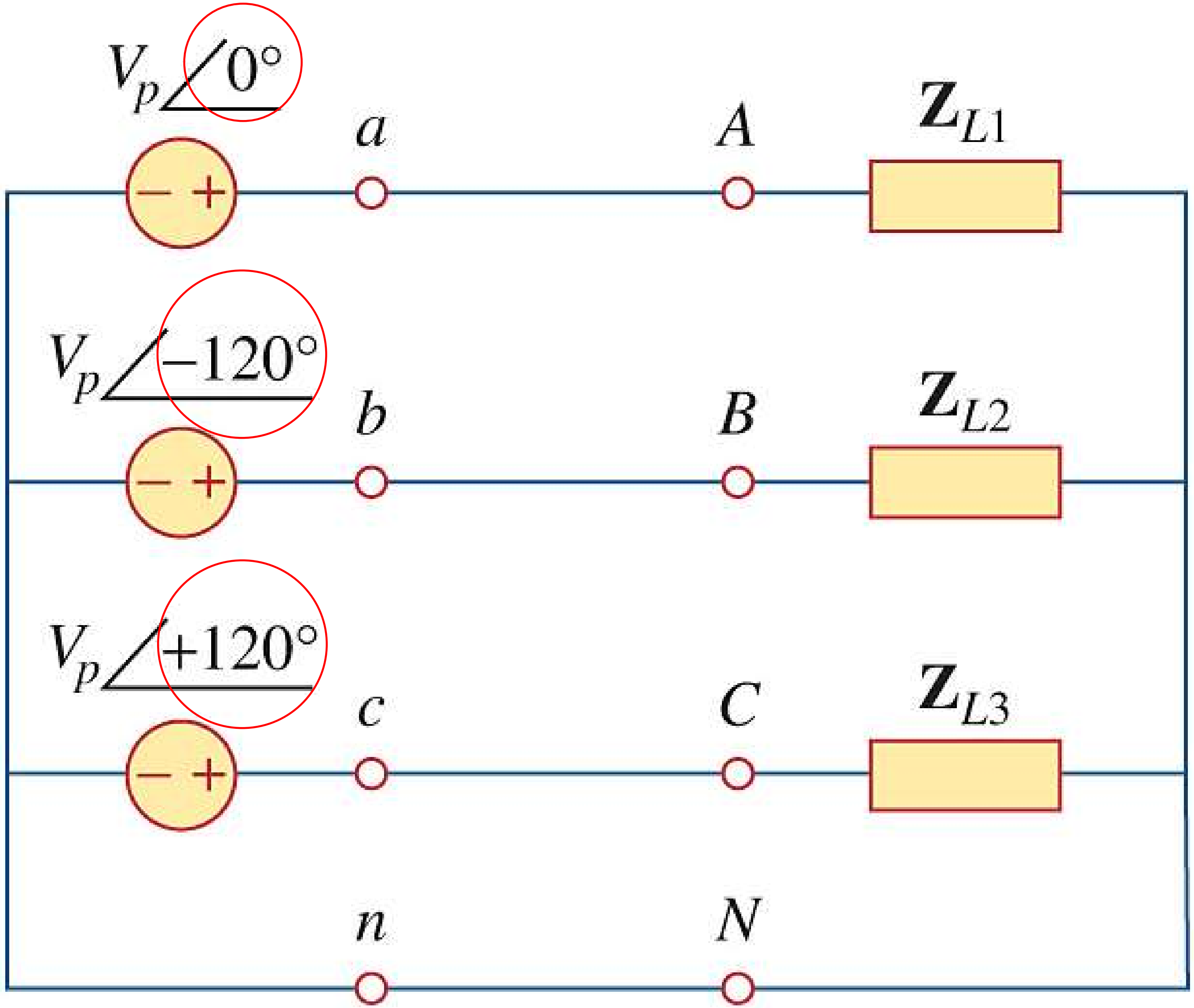


Figure 12.3 Three-phase four-wire system.

As distinct from a single-phase system, a three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°. In this chapter, we study three-phase systems.

12.2 Balanced Three-Phase Voltages

Three-phase voltages are often produced with a three-phase generator (or alternator) whose cross-sectional view is shown in Fig. 12.4. The generator basically consists of a rotating magnet (called the *rotor*) surrounded by a stationary winding (called the *stator*).

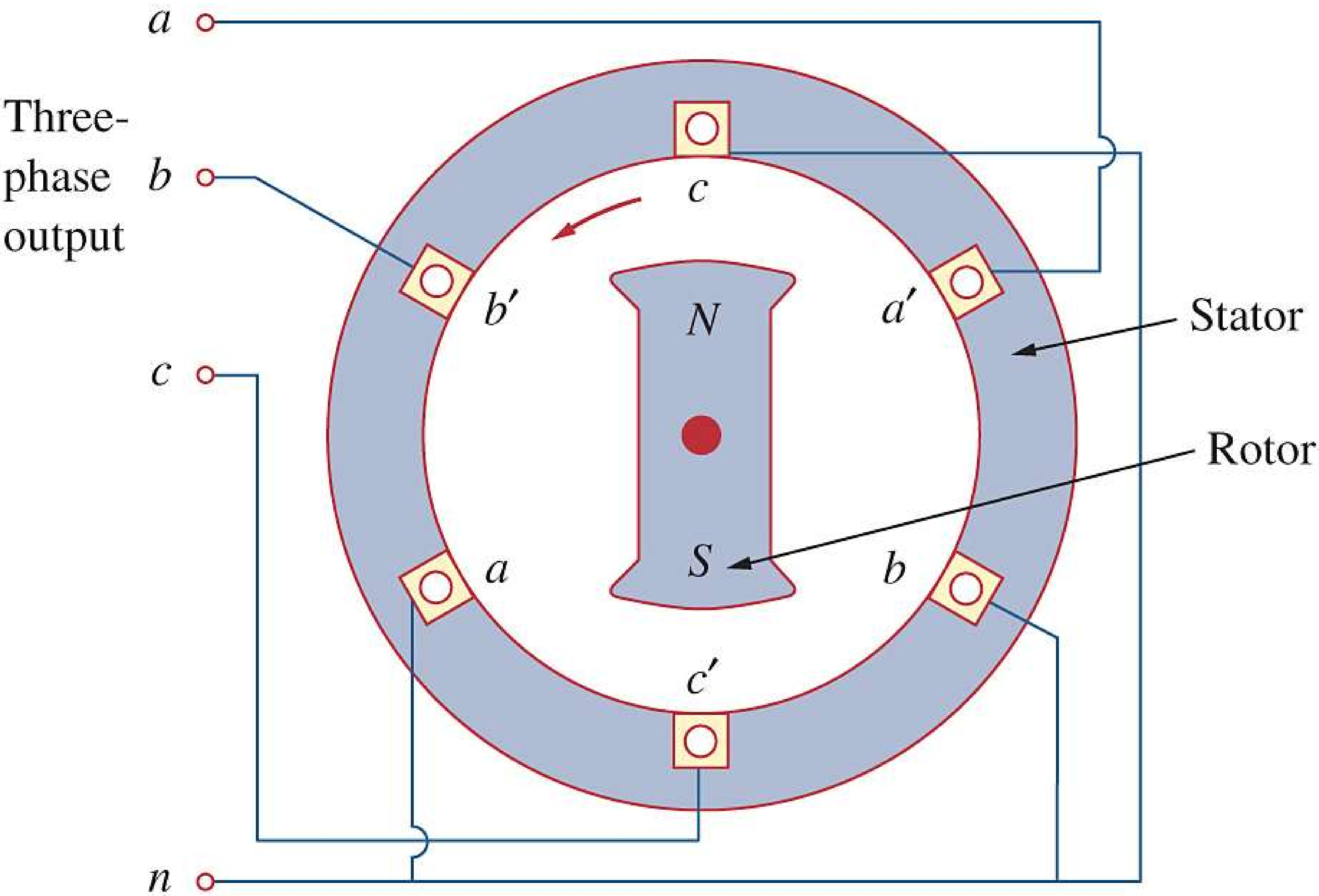


Figure 12.4 A three-phase generator.

As the rotator rotates, the flux through each coil varies sinusoidally with time, inducing a sinusoidal voltage. Because the coils are placed 120° apart, the induced voltages are out of phase by 120° (Fig. 12.5).

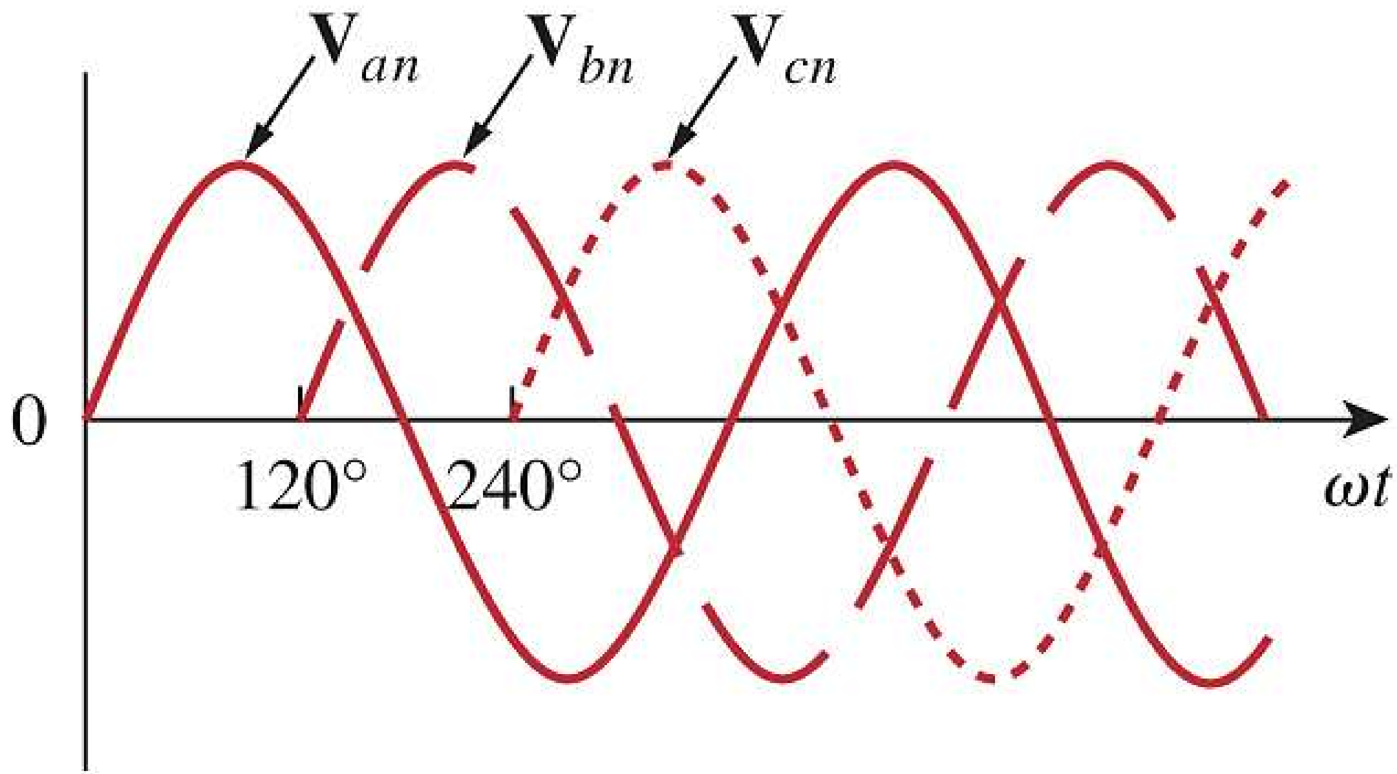


Figure 12.5 The generated voltages are 120° apart from each other.

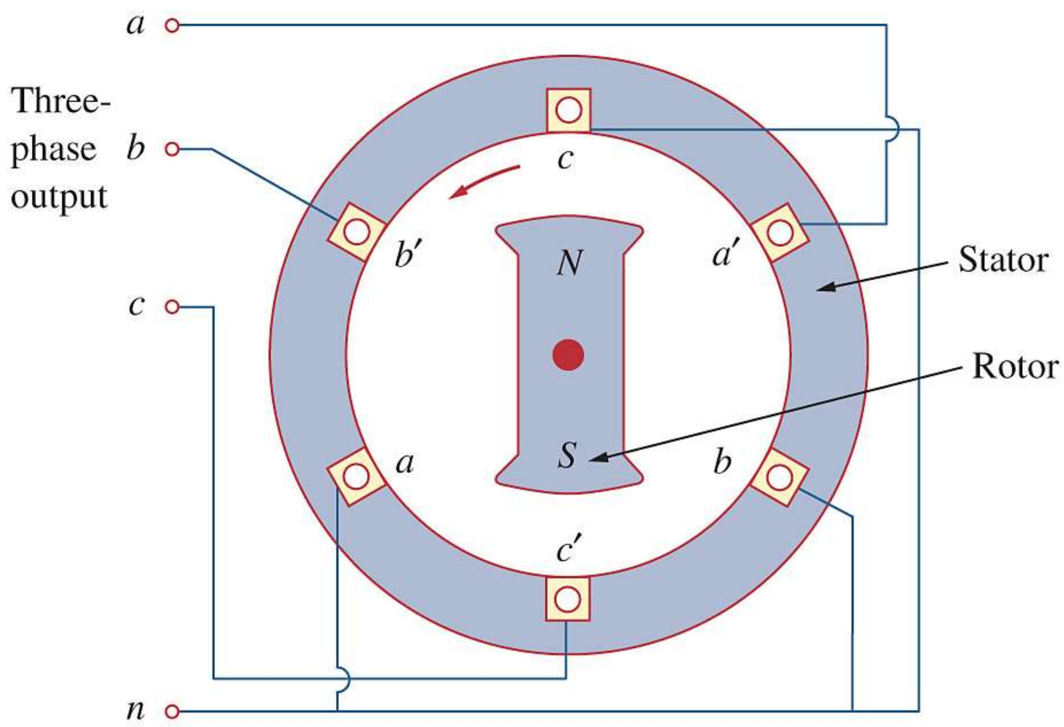


Figure 12.4 A three-phase generator.

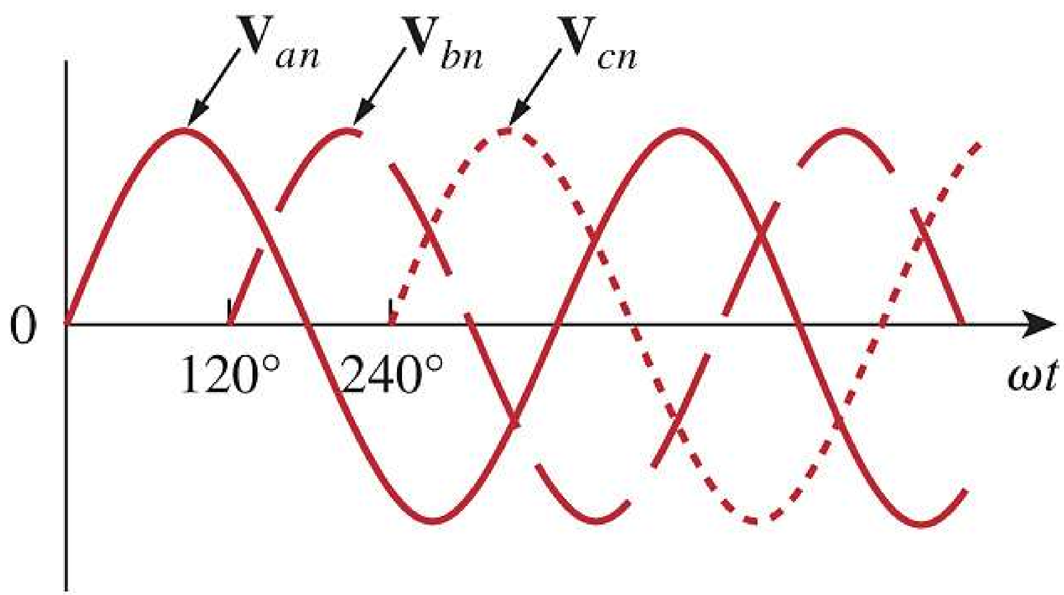
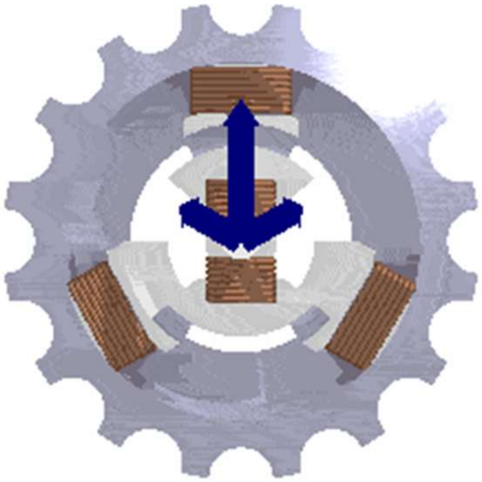
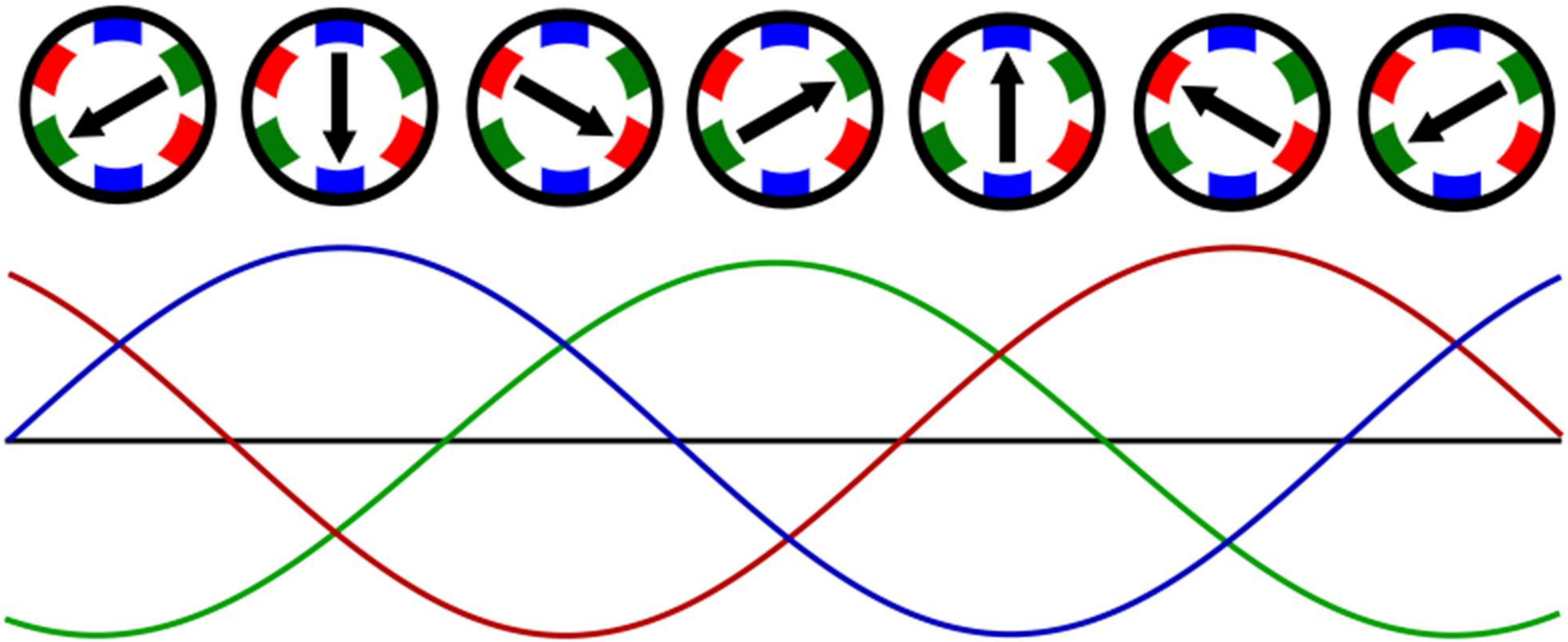
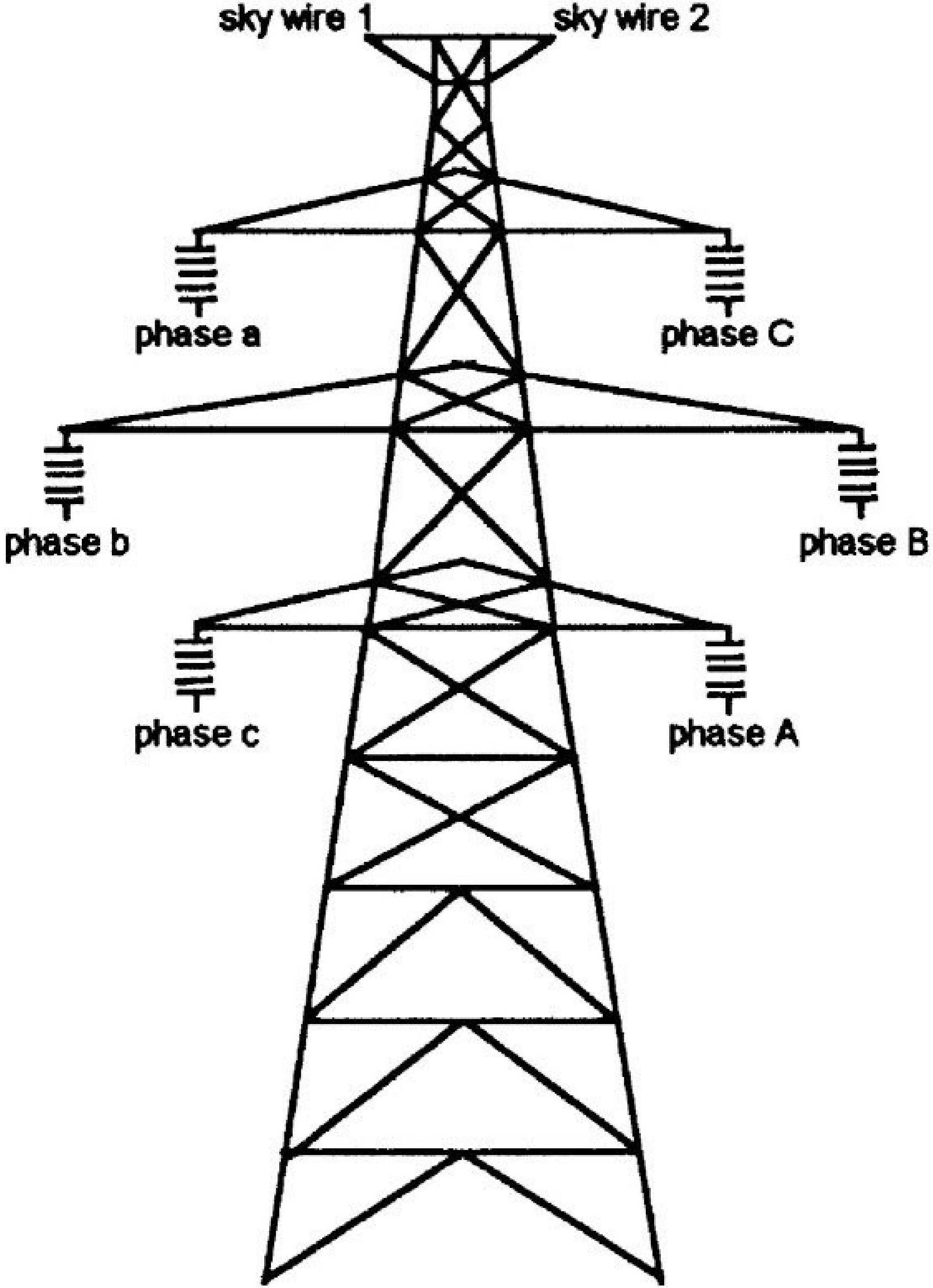


Figure 12.5 The generated voltages are 120° apart from each other.






sky wire 1
sky wire 2
phase a
phase C
phase b
phase B
phase c
phase A

A typical three-phase system consists of three voltage sources connected to loads by three or four wires. A three-phase system is equivalent to three single-phase systems. The voltage sources can be either wye- or delta-connected, as shown in Fig. 12.6.

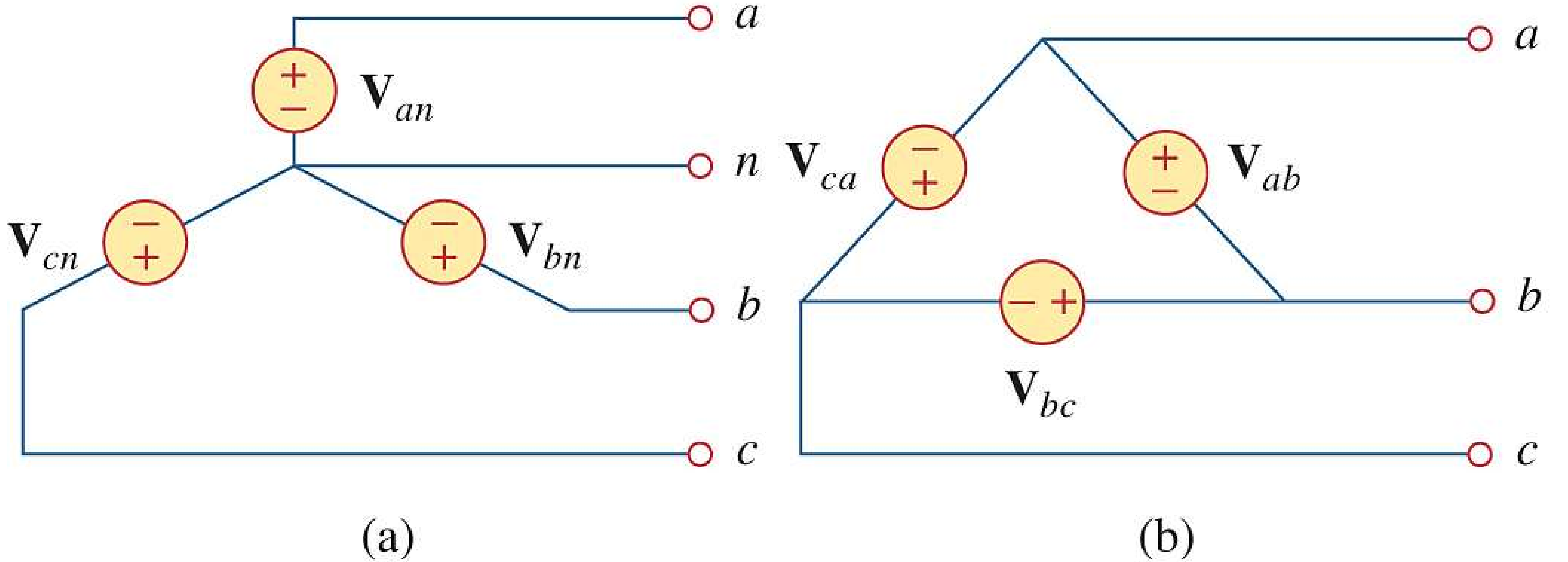
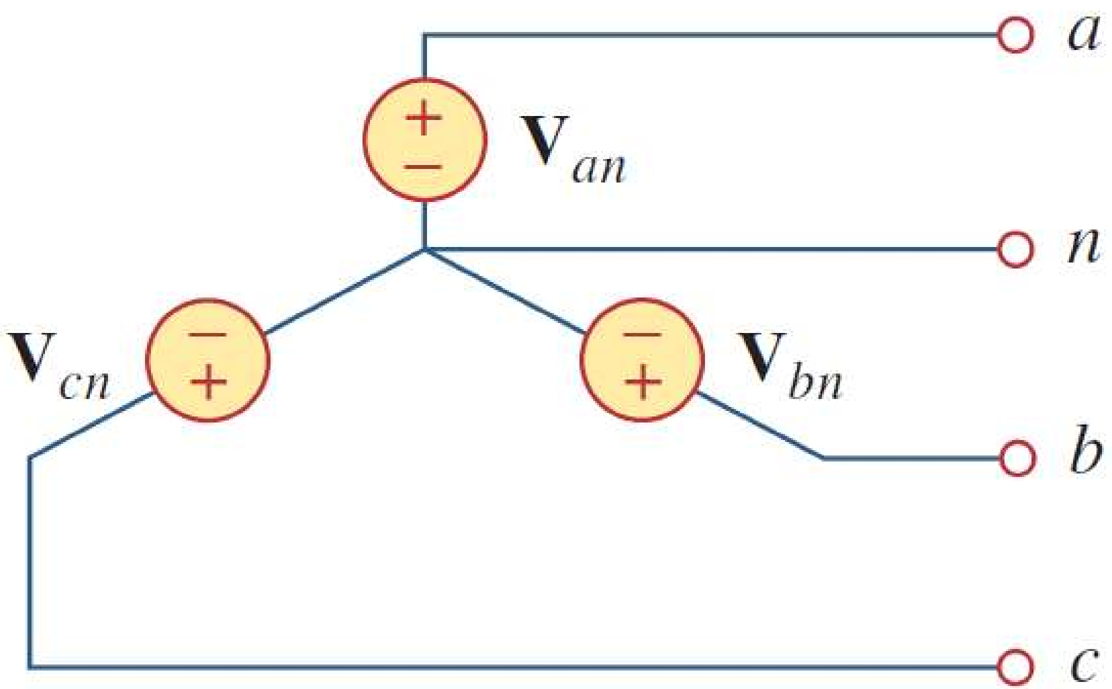


Figure 12.6 Three-phase voltage sources: (a) wye-connected, (b) delta-connected.

Let us consider the wye-connected voltages in Fig. 12.6(a). The voltages $\tilde{V}_{an}$, $\tilde{V}_{bn}$, and $\tilde{V}_{cn}$ are respectively between lines $a, b,$ and c, and the neutral line n. These voltages are called *phase voltages*.



If the voltages have the same amplitude and frequency and are out of phase with each other by 120°, the voltages are said to be *balanced*. This implies that (Fig. 12.7)

$$\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn} = 0$$

Balanced source:
Same amplitude and frequency
Out of phase with each other by 120°

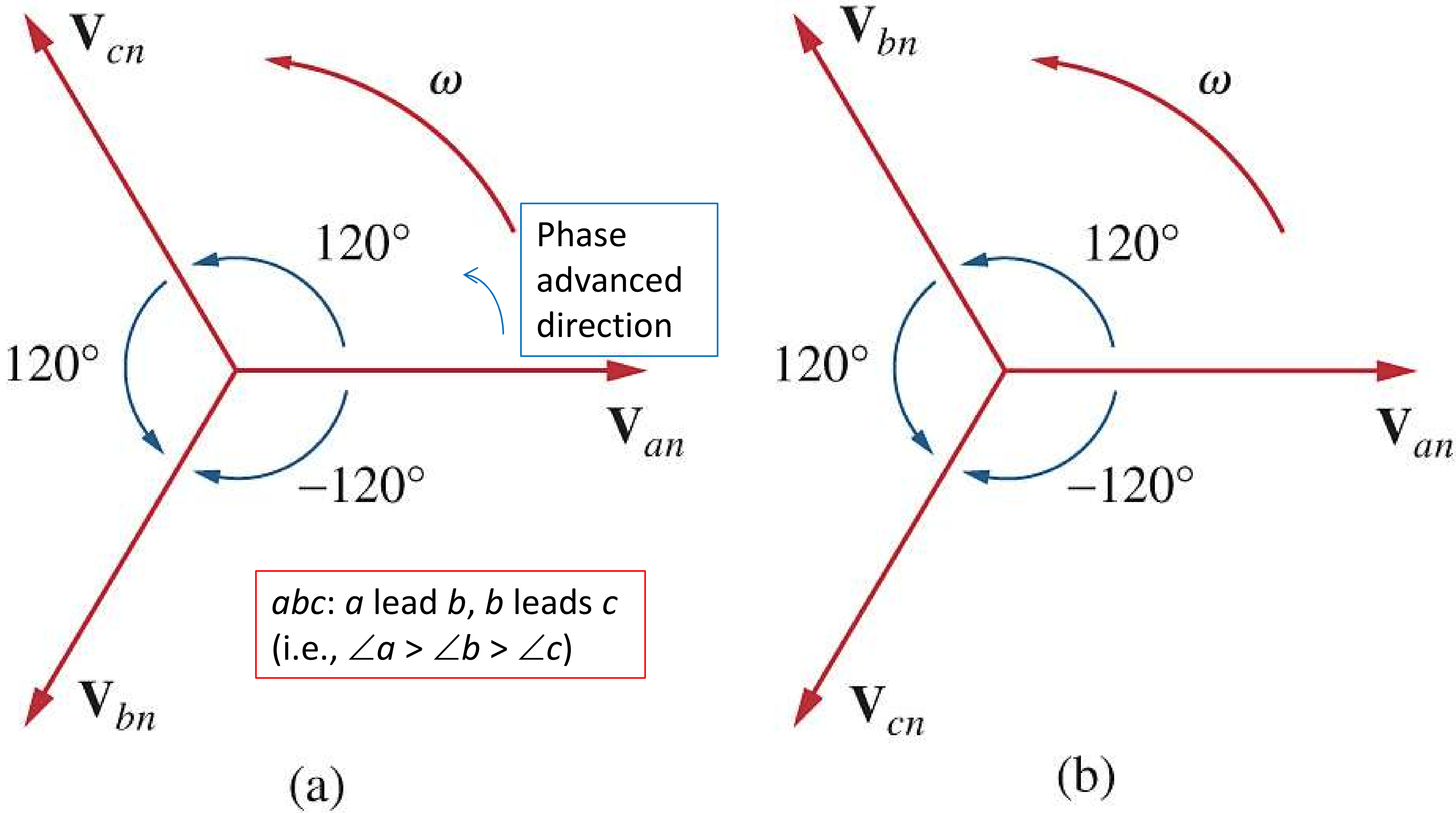


Figure 12.7 Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

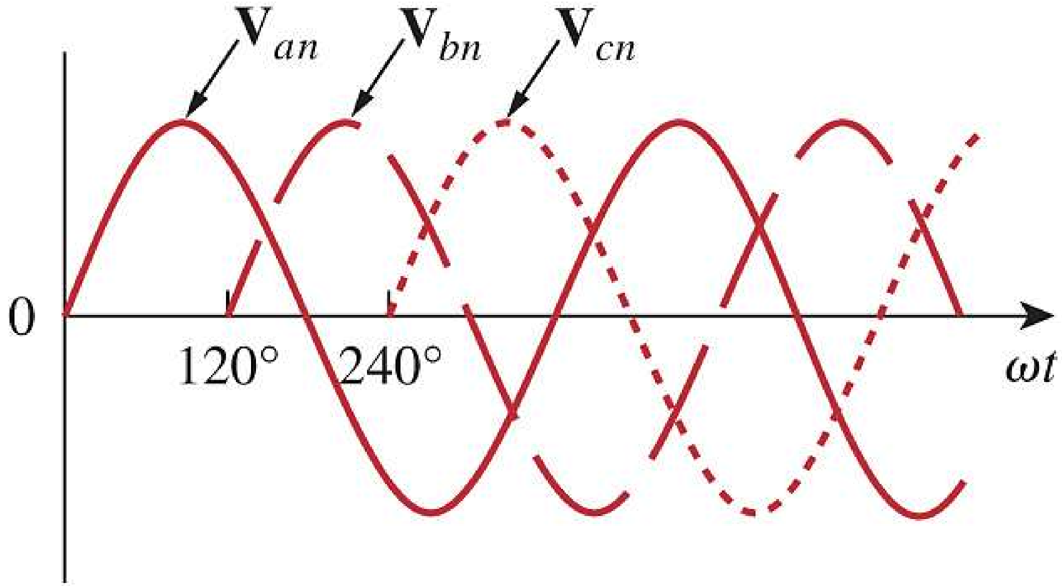


Figure 12.5 The generated voltages are 120° apart from each other.

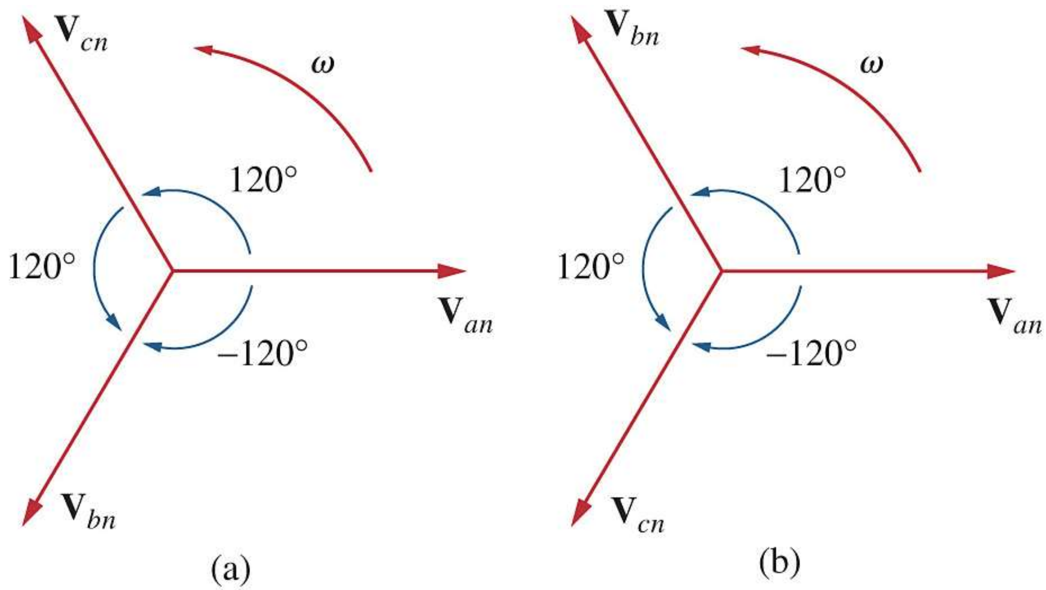


Figure 12.7 Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

In Fig. 12.7(a),

$$\tilde{V}_{an} = V_p \angle 0°$$

$$\tilde{V}_{bn} = V_p \angle -120°$$

$$\tilde{V}_{cn} = V_p \angle -240° = V_p \angle +120°$$

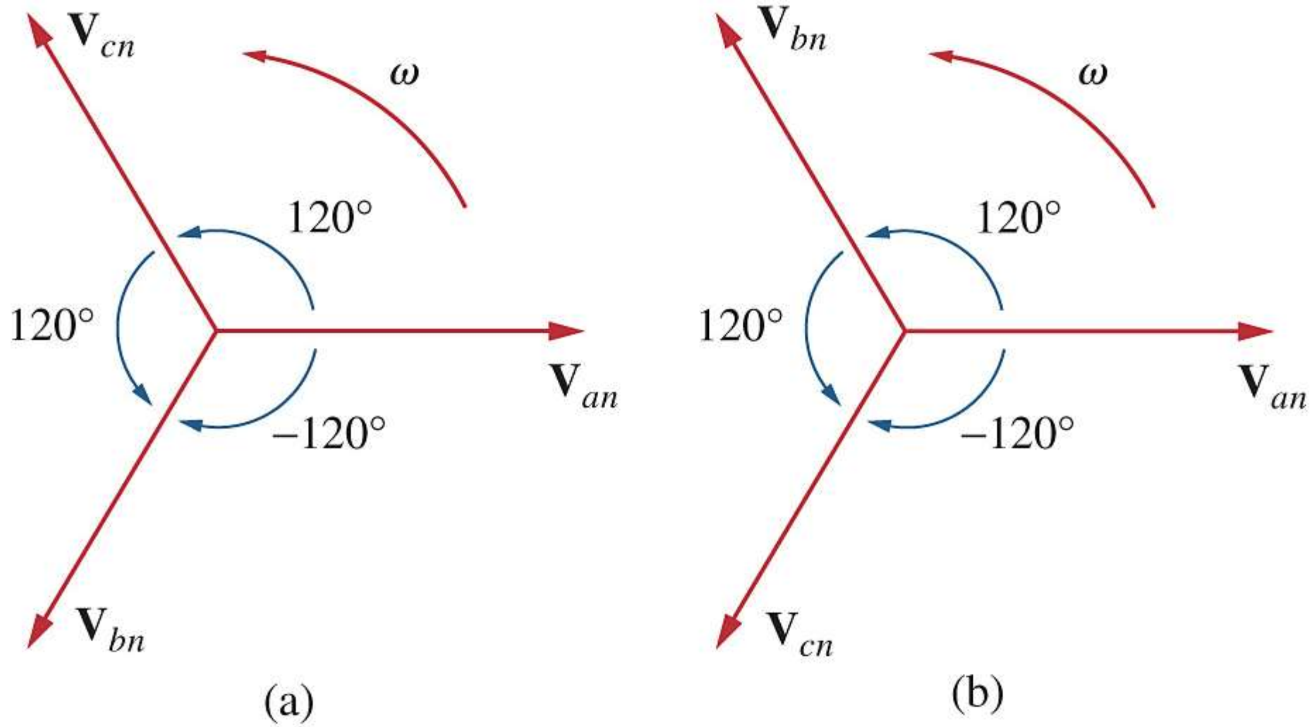


Figure 12.7 Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

where V_p is the effective or rms value of the phase voltages. This is known as the *abc sequence* or *positive sequence*.

In Fig. 12.7(b),

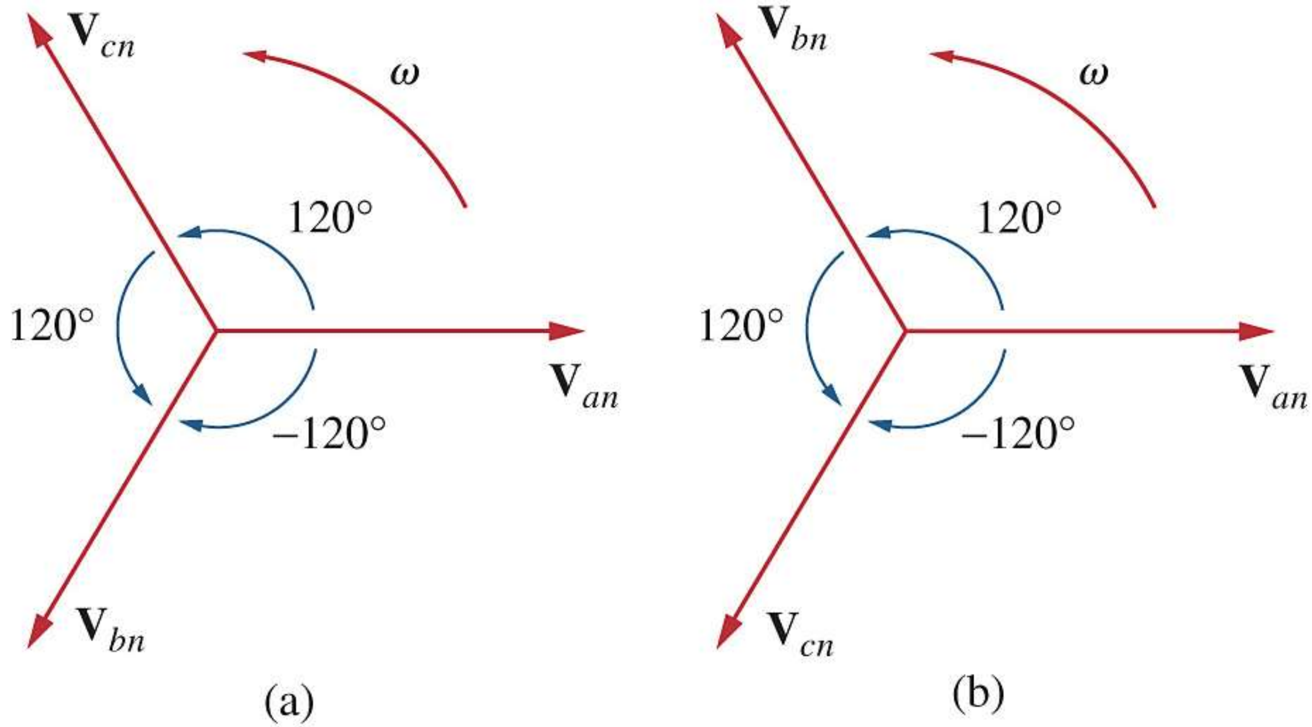


Figure 12.7 Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

$$\tilde{V}_{an} = V_p \angle 0°$$

$$\tilde{V}_{cn} = V_p \angle -120°$$

$$\tilde{V}_{bn} = V_p \angle -240° = V_p \angle +120°$$

This is called the *acb sequence* or *negative sequence*.

Like the source connections, a three-phase load can be either wye-connected or delta-connected, as shown in Fig. 12.8. The neutral line in Fig. 12.8(a) may or may not be there, depending on whether the system is four- or three-wire.

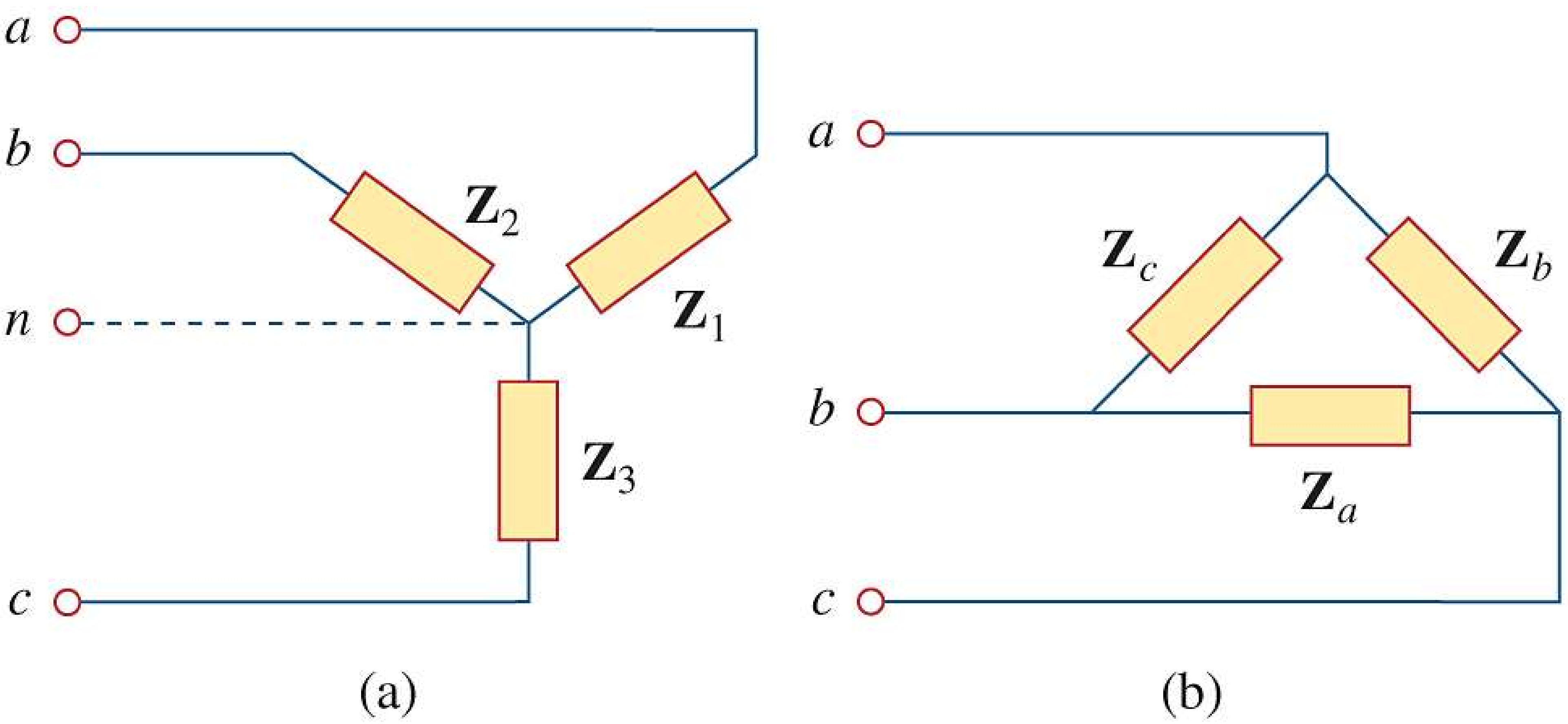


Figure 12.8 Two possible three-phase load configurations: (a) a wye-connected load, (b) a delta-connected load.

A wye- or delta-connected load is said to be balanced if the phase impedances are equal in magnitude and in phase.

For a balanced wye-connected load,

$$Z_1 = Z_2 = Z_3 = Z_Y$$

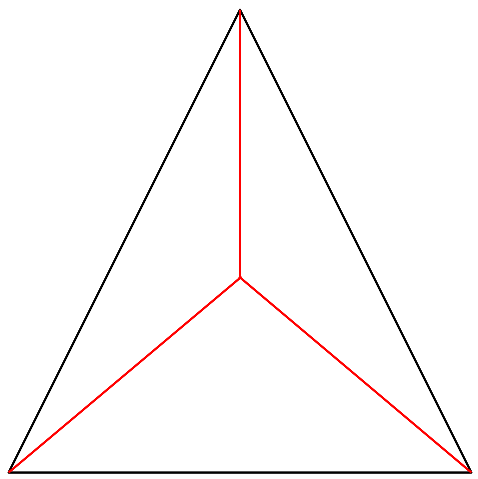
For a balanced delta-connected load,

$$Z_A = Z_B = Z_C = Z_\Delta$$

Balanced load:
Same amplitude and phase of impedances

A wye-connected load can be transformed into a delta-connected load, or vice versa.

$$Z_\Delta = 3Z_Y \text{ or } Z_Y = \frac{1}{3} Z_\Delta$$



Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

1. Y-Y connection (i.e., Y-connected source with a Y-connected load).
2. Y-Δ connection.
3. Δ-Δ connection.
4. Δ-Y connection.

$\boxed{\text{Source-Load}}$

12.3 Balanced Wye-Wye Connection

A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load. Figure 12.9 shows a balanced four-wire Y-Y system, which can be simplified to that shown in Fig. 12.10.

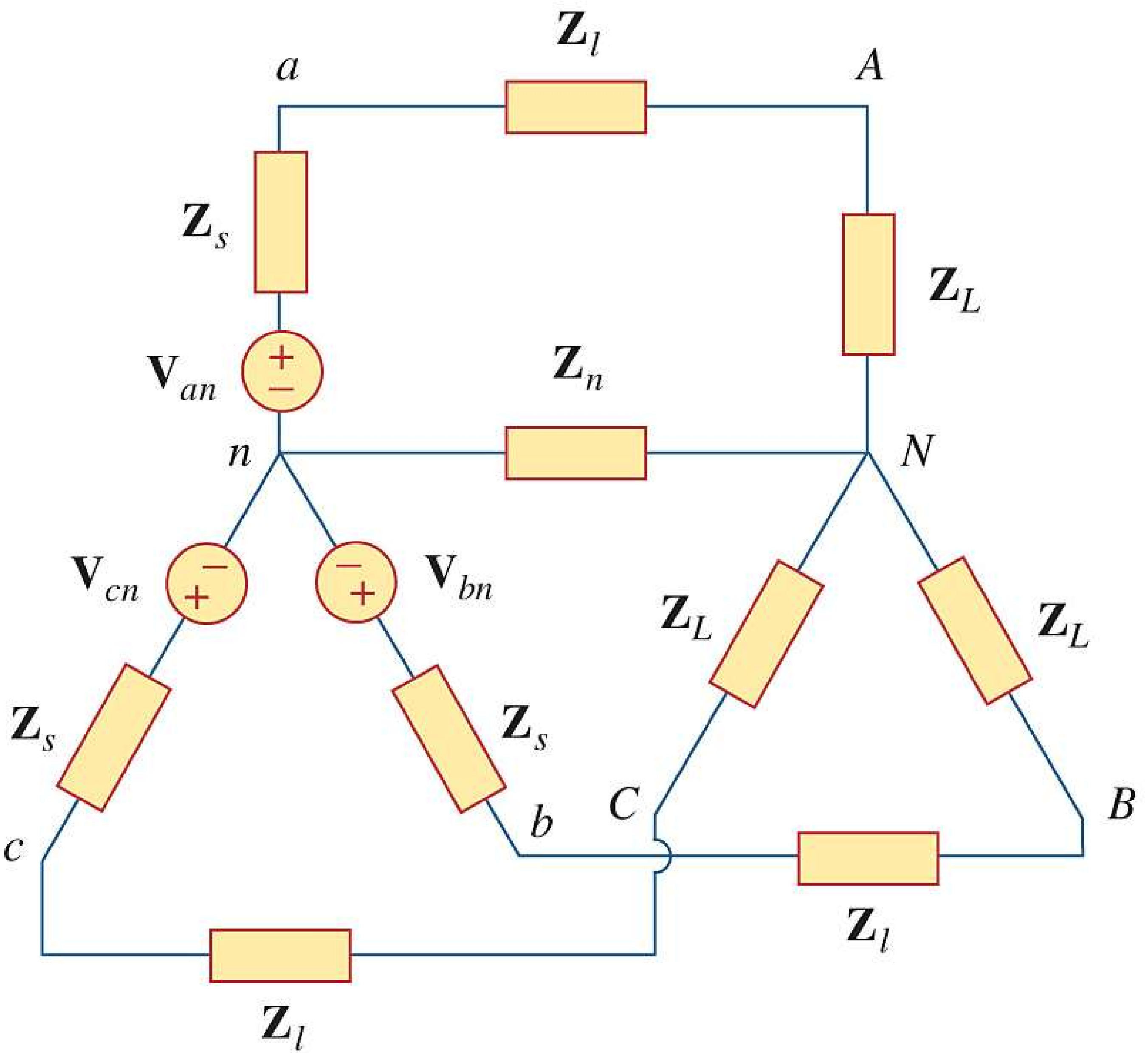


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

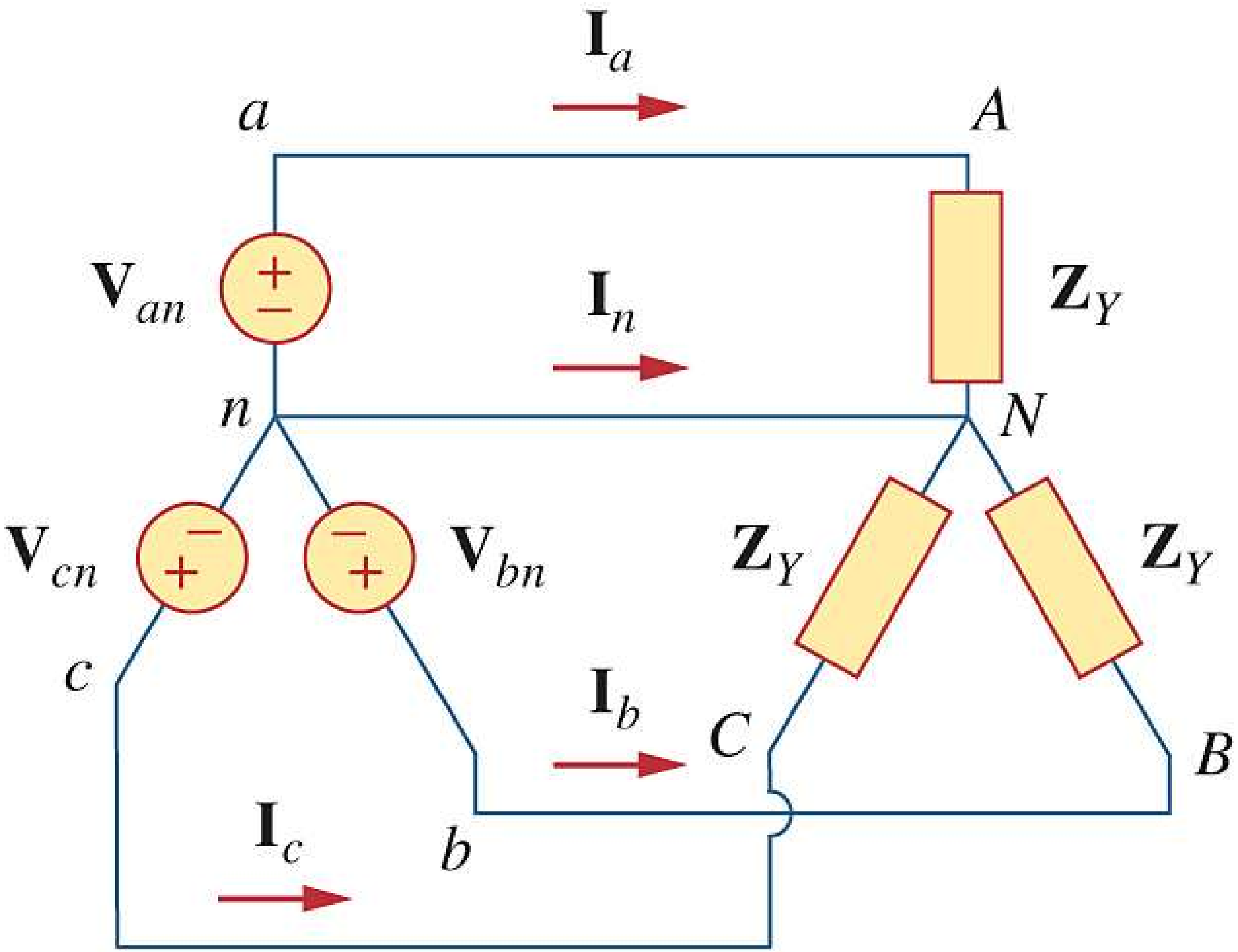


Figure 12.10 Balanced Y-Y connection.

Fig.12.9 → Fig. 12.10

If balanced, the system can be simplified.

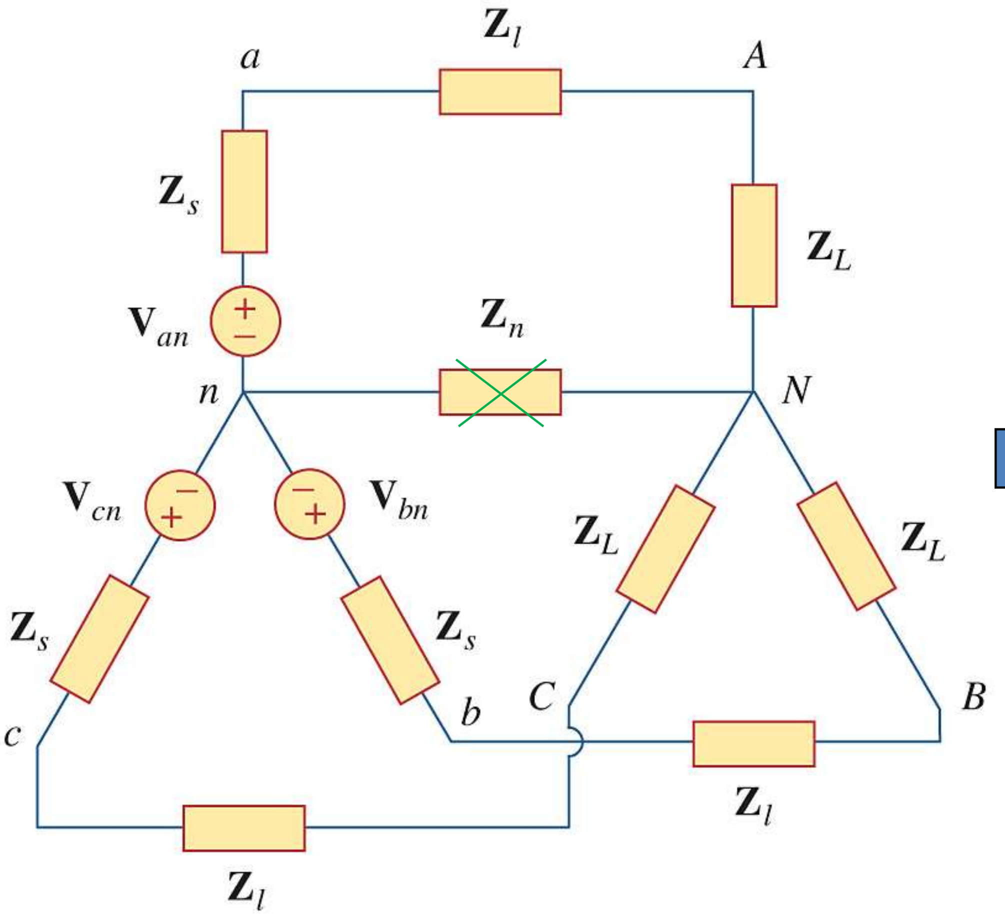


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

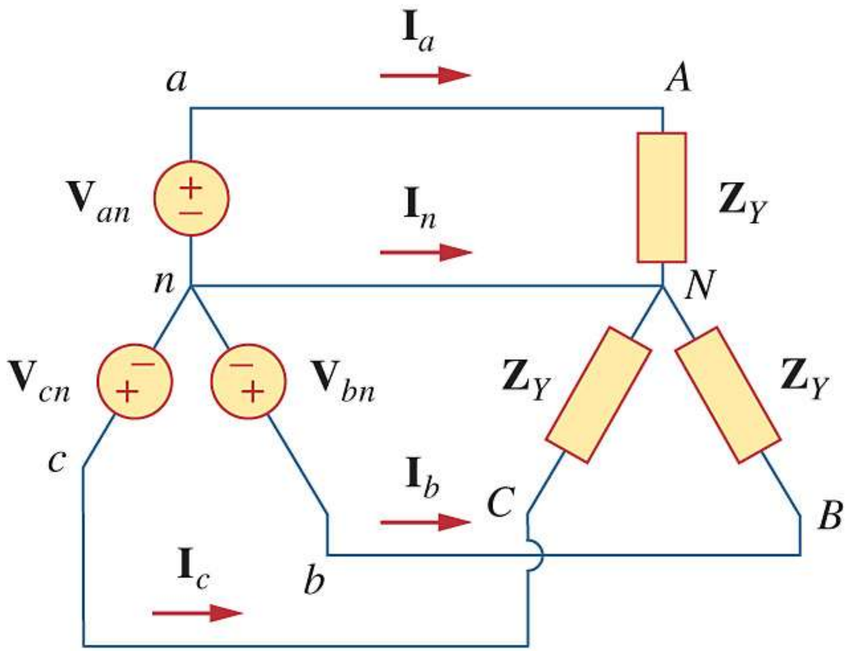


Figure 12.10 Balanced Y-Y connection.

$\boxed{Z_Y = Z_S + Z_l + Z_L}$

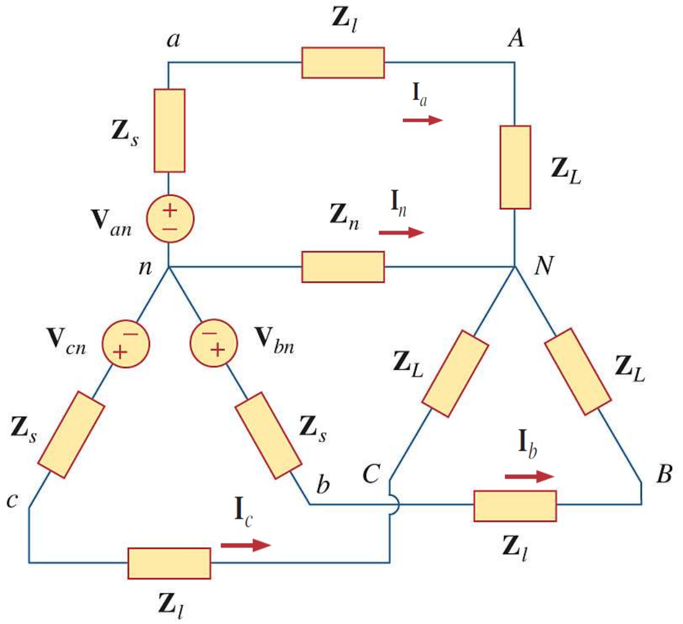


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

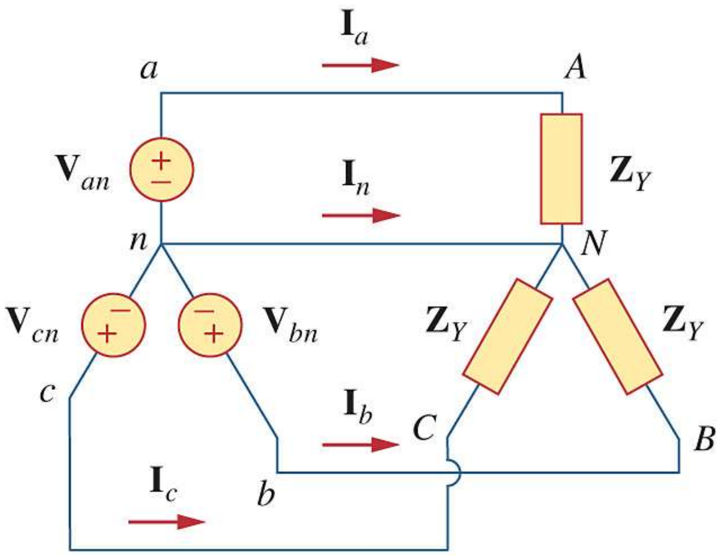


Figure 12.10 Balanced Y-Y connection.

$$\tilde{V}_{an} = \tilde{I}_a(Z_s + Z_l + Z_L) - \tilde{I}_n Z_n$$

$$\tilde{V}_{bn} = \tilde{I}_b(Z_s + Z_l + Z_L) - \tilde{I}_n Z_n$$

$$\tilde{V}_{cn} = \tilde{I}_c(Z_s + Z_l + Z_L) - \tilde{I}_n Z_n$$

$$\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn}$$

$$= (\tilde{I}_a + \tilde{I}_b + \tilde{I}_c)(Z_s + Z_l + Z_L) - 3\tilde{I}_n Z_n$$

$$\text{but } \tilde{I}_a + \tilde{I}_b + \tilde{I}_c + \tilde{I}_n = 0, \text{ i.e., } \tilde{I}_a + \tilde{I}_b + \tilde{I}_c = -\tilde{I}_n$$

$$\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn} = \underline{-\tilde{I}_n}(Z_s + Z_l + Z_L) - 3\tilde{I}_n Z_n$$

$$= -\tilde{I}_n(Z_s + Z_l + Z_L + 3Z_n) \underline{= 0}$$

Balanced source

$$\tilde{I}_n = 0$$

$$\tilde{V}_{nN} = \tilde{I}_n Z_n = 0$$

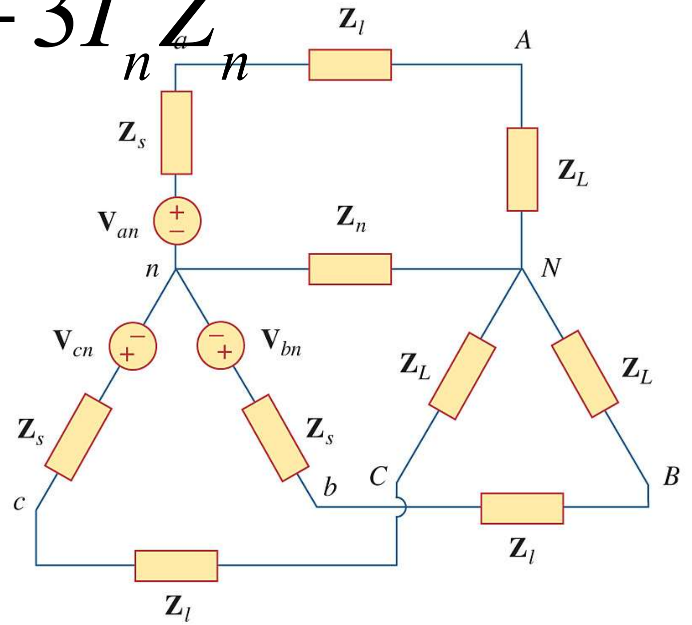


Figure 12.9: A balanced Y-Y system, showing the source, line, and load impedances.

so the neutral line can be replaced with an open circuit or a short circuit.

Consider Fig. 12.10. Assuming the positive sequence, the phase voltages (or line-to-neutral volatges) are

$$\tilde{V}_{an} = V_p \angle 0^\circ$$

$$\tilde{V}_{bn} = V_p \angle -120^\circ$$

$$\tilde{V}_{cn} = V_p \angle -240^\circ$$

Balanced source:
Same amplitude and frequency
Out of phase with each other by 120°

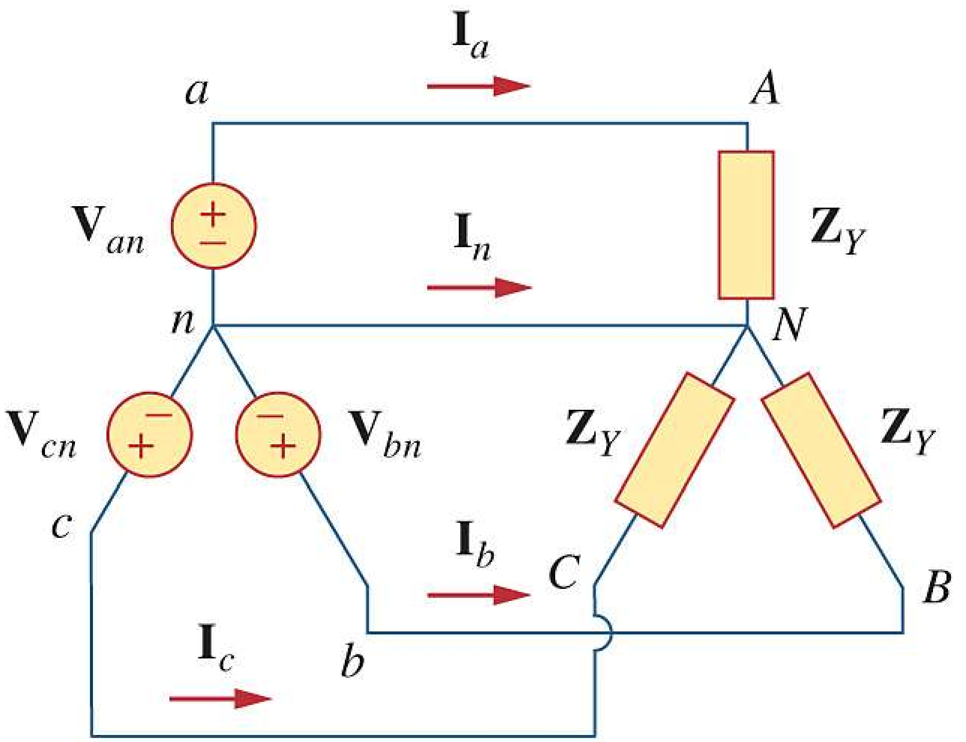


Figure 12.10 Balanced Y-Y connection.

The line voltages (or line-to-line volatges) are

$$\tilde{V}_{ab} = \tilde{V}_{an} - \tilde{V}_{bn} = \sqrt{3}V_p\angle 30°$$

$$\tilde{V}_{bc} = \tilde{V}_{bn} - \tilde{V}_{cn} = \sqrt{3}V_p\angle -90° = \tilde{V}_{ab}\angle -120°$$

$$\tilde{V}_{ca} = \tilde{V}_{cn} - \tilde{V}_{an} = \sqrt{3}V_p\angle -210° = \tilde{V}_{ab}\angle -240°$$

$$\tilde{V}_{ab} + \tilde{V}_{bc} + \tilde{V}_{ca} = 0$$

Line voltages are also balanced.

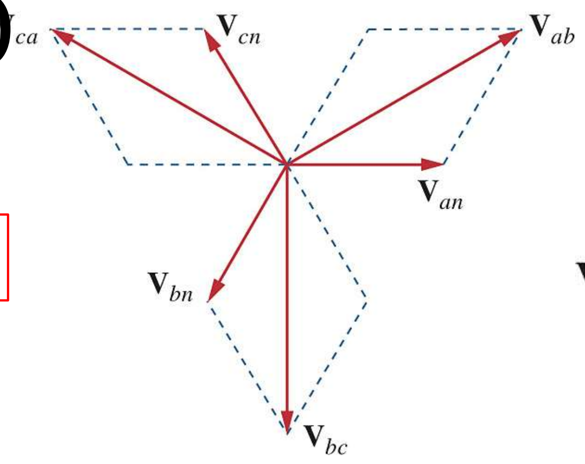


Figure 12.11(b) Phasor diagram illustrating the relationship between line voltages and phase voltages.

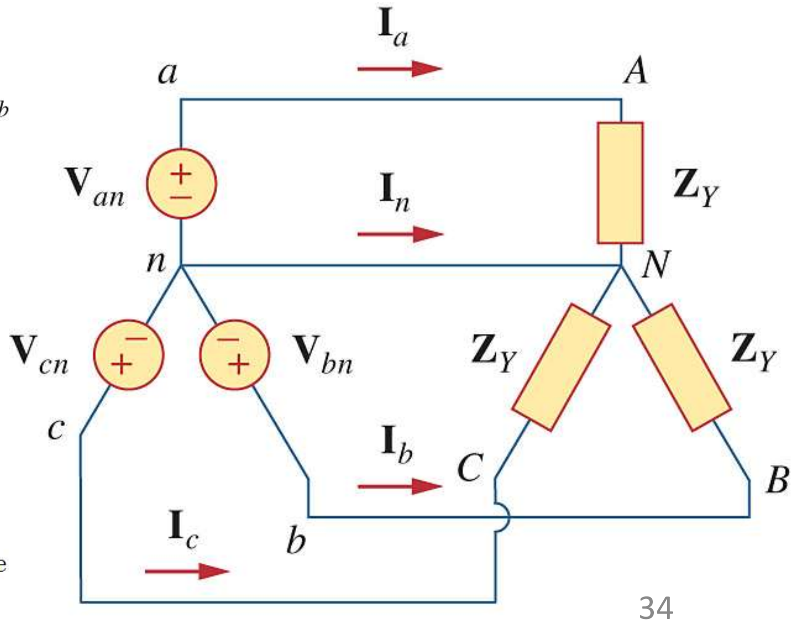


Figure 12.10 Balanced Y-Y connection.

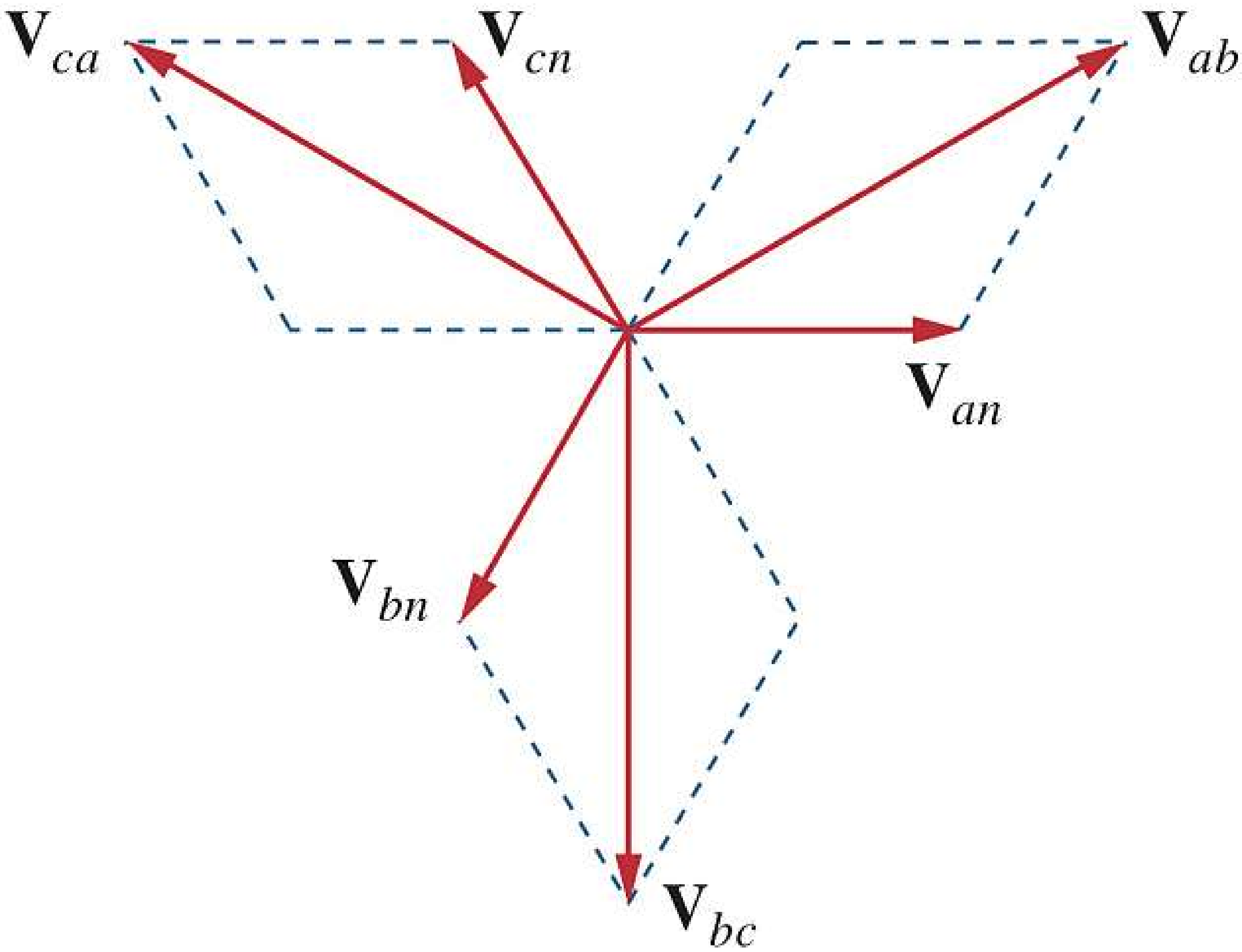


Figure 12.11(b) Phasor diagram illustrating the relationship between line voltages and phase voltages.

V_L

The magnitude of the line voltages is $\sqrt{3}$ times the magnitude of the phase volatges.

V_P

$$V_L = \sqrt{3} V_p$$

where

$$V_p = \left|\tilde{V}_{an}\right| = \left|\tilde{V}_{bn}\right| = \left|\tilde{V}_{cn}\right|$$

$$V_L = \left|\tilde{V}_{ab}\right| = \left|\tilde{V}_{bc}\right| = \left|\tilde{V}_{ca}\right|$$

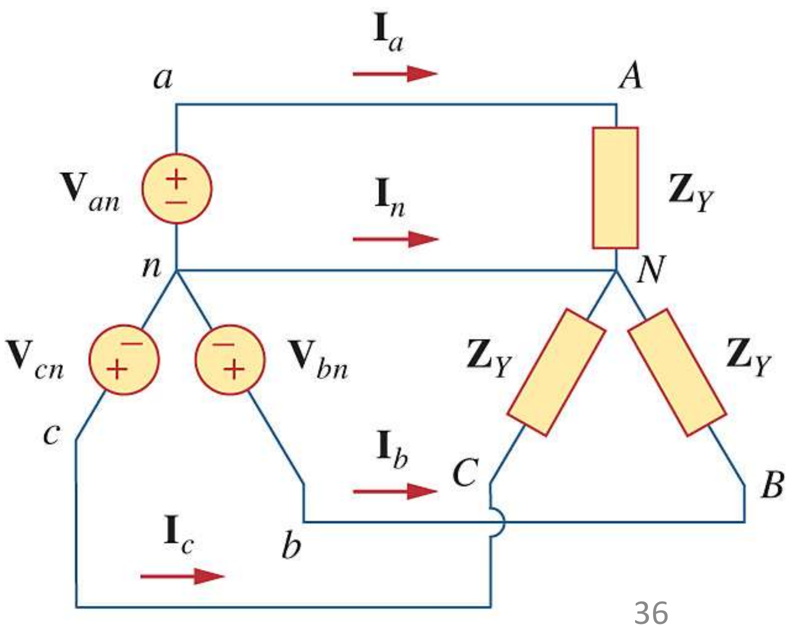


Figure 12.10 Balanced Y-Y connection.

The line voltages lead their corresponding phase volatges by 30°.

$$\tilde{V}_{ab} = \tilde{V}_{an}\angle 30°$$

$$\tilde{V}_{bc} = \tilde{V}_{bn}\angle 30°$$

$$\tilde{V}_{ca} = \tilde{V}_{cn}\angle 30°$$

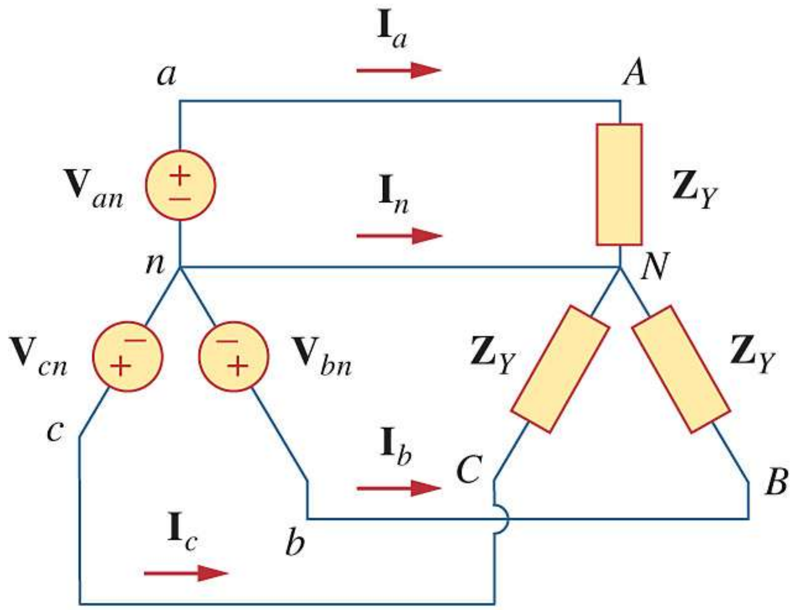


Figure 12.10 Balanced Y-Y connection.

The line currents are

$$\tilde{I}_a = \frac{\tilde{V}_{an}}{Z_Y}$$

$$\tilde{I}_b = \frac{\tilde{V}_{bn}}{Z_Y} = \tilde{I}_a \angle -120°$$

$$\tilde{I}_c = \frac{\tilde{V}_{cn}}{Z_Y} = \tilde{I}_a \angle -240°$$

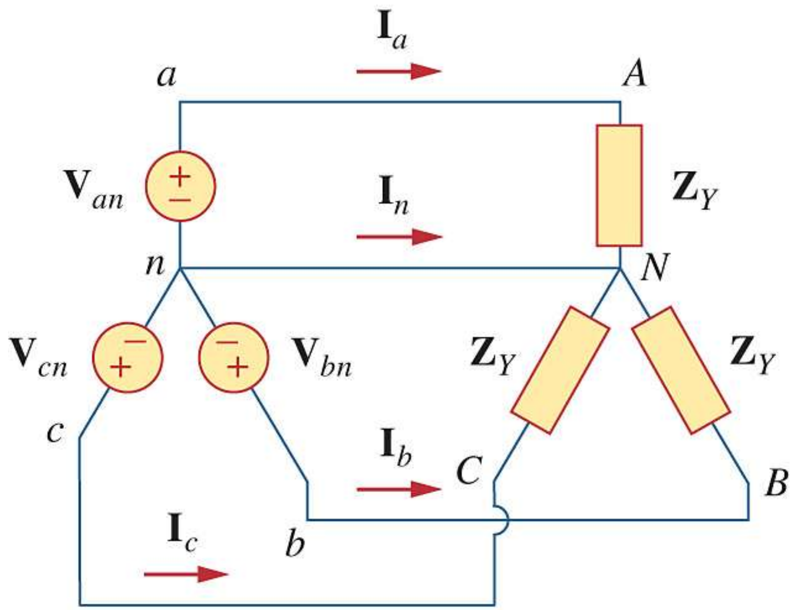


Figure 12.10 Balanced Y-Y connection.

Since $\tilde{I}_n = 0$, the neutral line can be removed without affecting the system. In fact, in long distance power transmission, conductors in multiples of three are used with the earth itself as the neutral ground. Power systems designed in this way are well ground at all critical points to ensure safety.

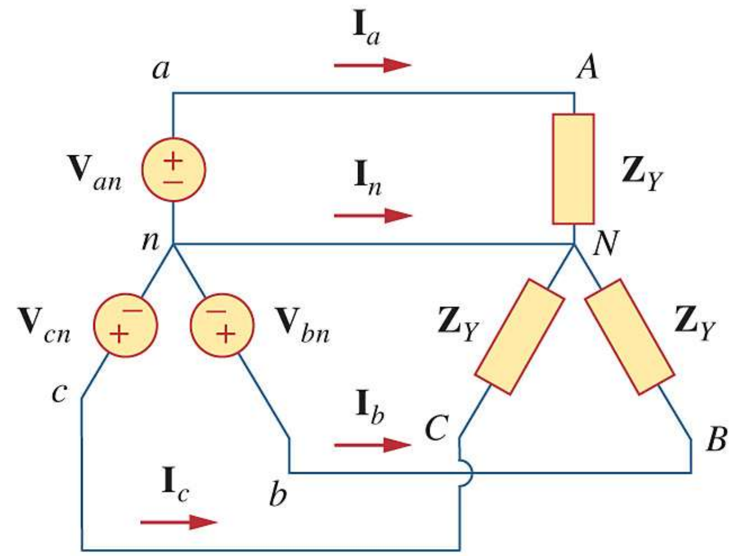


Figure 12.10 Balanced Y-Y connection.

While the $\boxed{\text{line current}}$ is the current in each line, the $\boxed{\text{phase current}}$ is the current in each phase of ~~the source~~ or load. In the Y-Y system, the line current is the same as the phase current.

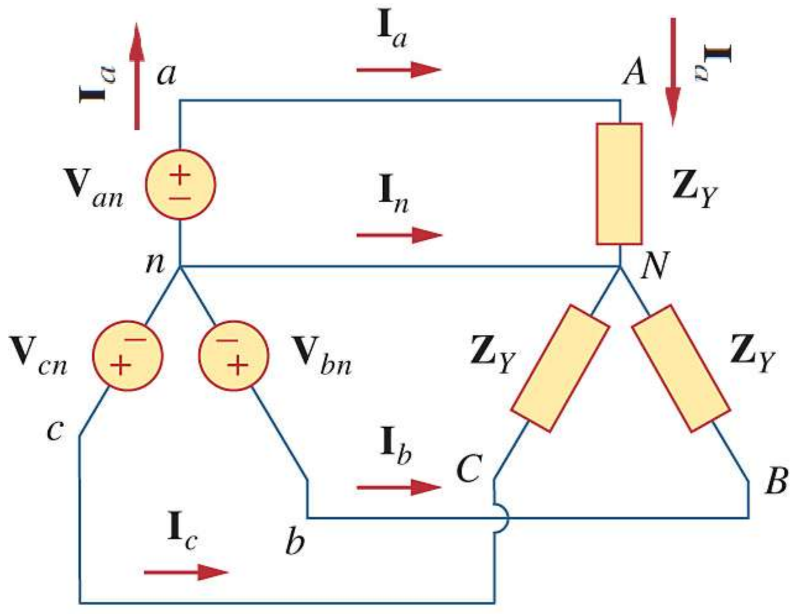


Figure 12.10 Balanced Y-Y connection.

An alternative way of analyzing a balanced Y-Y system is to do so on a per phase basis. We look at one phase, say phase *a*, and analyze the single-phase equivalent circuit in Fig. 12.12.

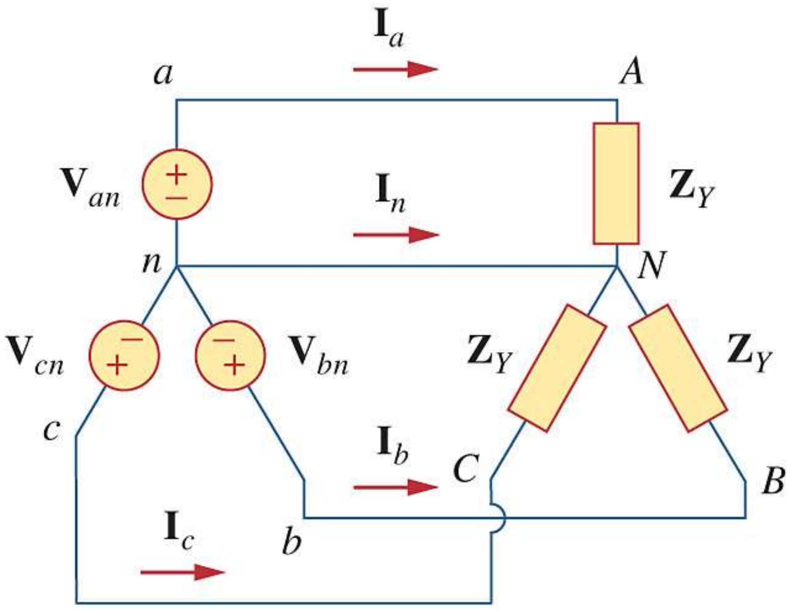


Figure 12.10 Balanced Y-Y connection.

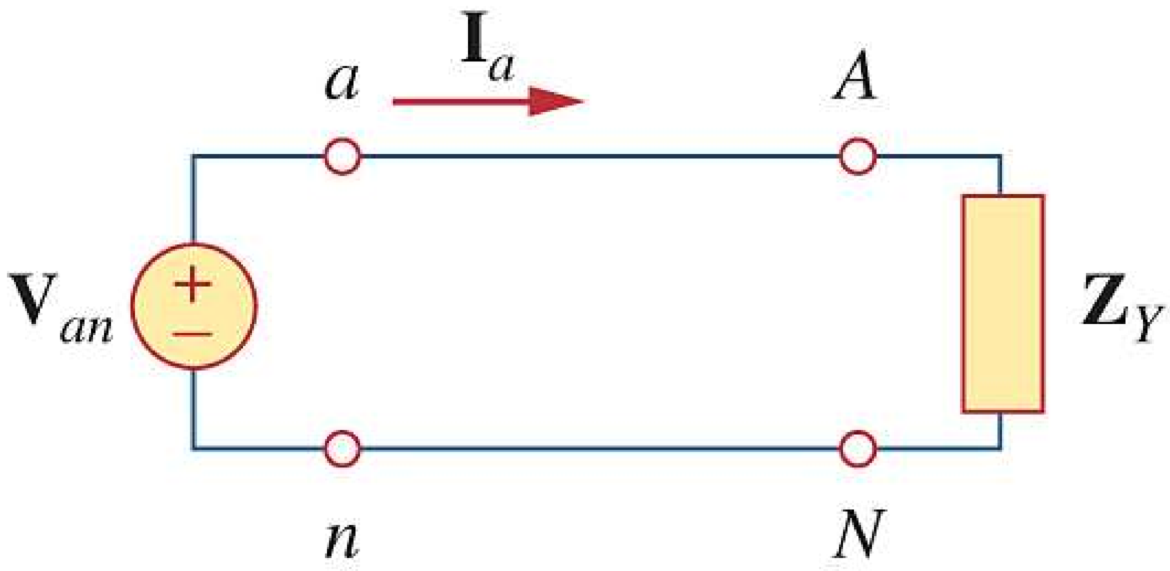


Figure 12.12 A single-phase equivalent circuit.

Summary of balanced Y-Y

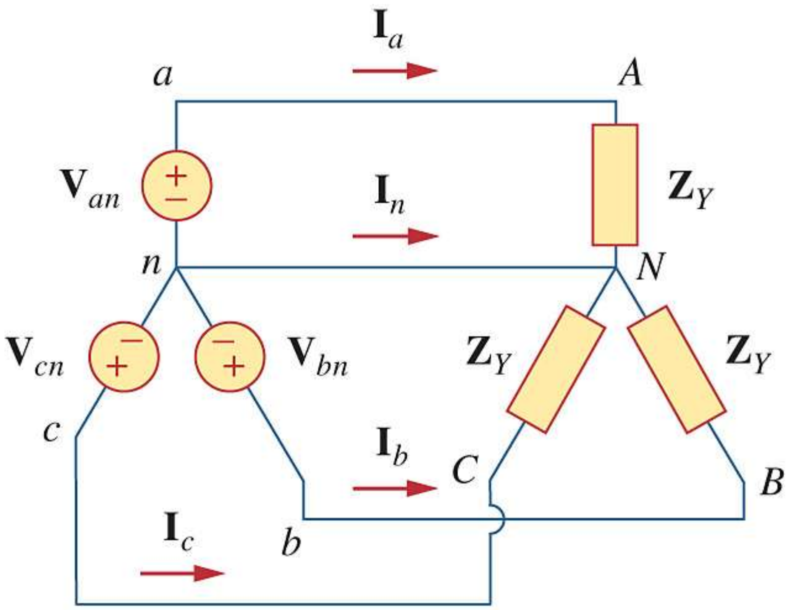


Figure 12.10 Balanced Y-Y connection.

If the source (phase voltages) are in *abc* sequence,

- Voltages: <u>Magnitude</u> <u>Phase</u>

$$V_L = \sqrt{3} V_p$$

$$\angle\tilde{V}_{ab} = \angle\tilde{V}_{an} + \angle 30^\circ$$

$$\angle\tilde{V}_{bc} = \angle\tilde{V}_{bn} + \angle 30^\circ$$

$$\angle\tilde{V}_{ca} = \angle\tilde{V}_{cn} + \angle 30^\circ$$

Line voltages and phase voltages are in *abc* sequence

- Currents:

Line currents and phase currents are same, and are in *abc* sequence

Practice Problem 12.2 A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3\ \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + j19\ \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7\ \Omega$ per phase. Assuming a positive sequence for the source voltages and that $\tilde{V}_{an} = 120\angle 30°$ V, find the line voltages and the line currents.

Solution :

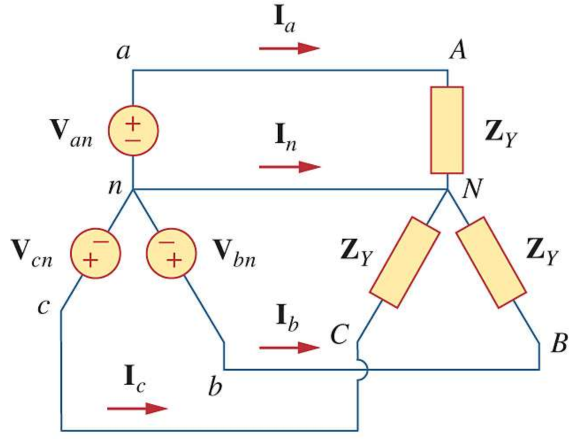


Figure 12.10 Balanced Y-Y connection.

$$Z_Y = Z_s + Z_l + Z_L = (0.4 + j0.3) + (24 + j19)$$

$$+(0.6 + j0.7) = 25 + j20\ (\Omega)$$

$$\tilde{I}_a = \frac{\tilde{V}_{an}}{Z_Y} = \frac{120\angle 30^\circ}{25 + j20} \approx \frac{120\angle 30^\circ}{32.0156\angle 38.66^\circ}$$

$$\approx 3.7482\angle -8.66^\circ\ (\mathrm{A})$$

$$\tilde{I}_b = \tilde{I}_a \angle -120^\circ = 3.7482\angle -128.66^\circ\ (\mathrm{A})$$

$$\tilde{I}_c = \tilde{I}_a \angle -240^\circ = 3.7482\angle -248.66^\circ\ (\mathrm{A})$$

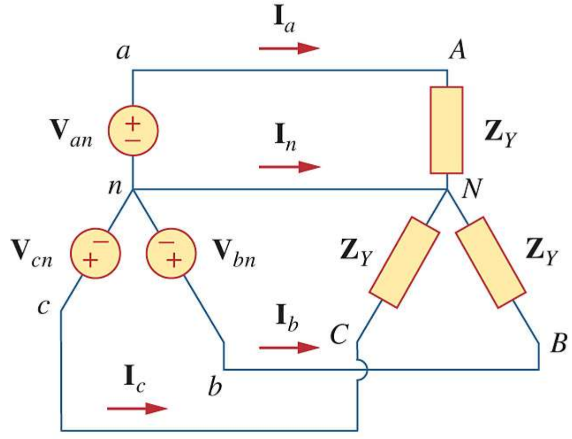


Figure 12.10 Balanced Y-Y connection.

The "ideal" line voltages are

$$\boxed{\tilde{V}_{ab} = \tilde{V}_{an}\sqrt{3}\angle 30°} = 120\angle 30° \times \sqrt{3}\angle 30°$$

$$\approx 207.8461\angle 60° \text{ (V)}$$

$$\tilde{V}_{bc} = \tilde{V}_{ab}\angle -120° = 207.8461\angle -60° \text{ (V)}$$

$$\tilde{V}_{ca} = \tilde{V}_{ab}\angle -240° = 207.8461\angle -180° \text{ (V)}$$

Note that for a balanced system, the other two voltages/currents can be quickly obtained by their **phase relation**.

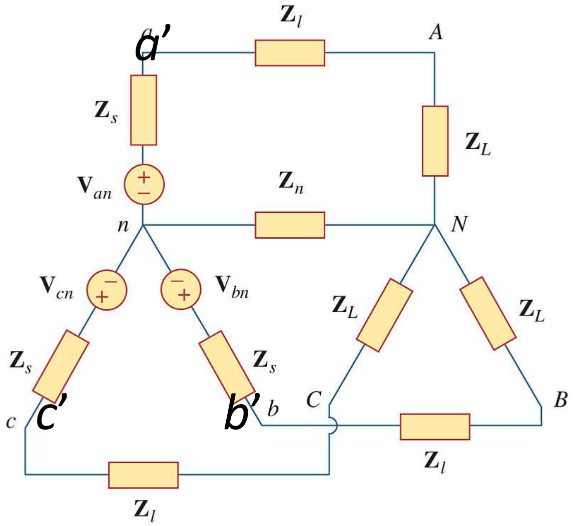


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

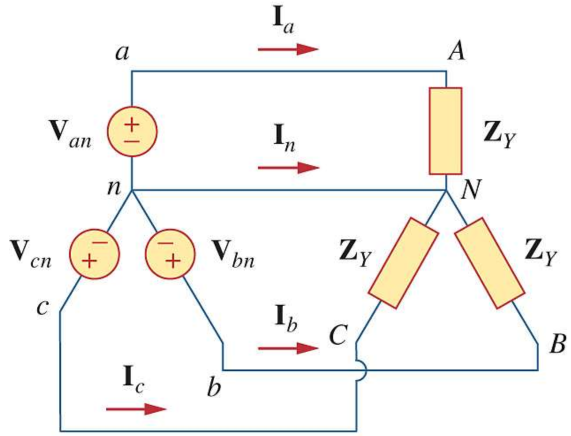


Figure 12.10 Balanced Y-Y connection.

The line voltages seen from the generator terminals are

$$\tilde{V}_{a'b'} = \tilde{I}_a(Z_l + Z_L) - \tilde{I}_b(Z_l + Z_L)$$

$$= \tilde{I}_a \sqrt{3}\angle 30°(Z_l + Z_L)$$

$$= \frac{\tilde{V}_{an}}{Z_Y}\sqrt{3}\angle 30°(Z_l + Z_L) = \tilde{V}_{ab}\left(\frac{Z_l + Z_L}{Z_Y}\right)$$

Line voltage V_{ab}

$$= 207.8461\angle 60^\circ \times \left(\frac{24.6 + j19.7}{25 + j20} \right)$$

$$\approx 207.8461\angle 60^\circ \times \frac{31.5159\angle 38.69^\circ}{32.0156\angle 38.66^\circ}$$

$$\approx 204.6020\angle 60.03^\circ \text{ (V)}$$

$$\tilde{V}_{b'c'} = 204.6020\angle -59.97^\circ \text{ (V)}$$

$$\tilde{V}_{c'a'} = 204.6020\angle -179.97^\circ \text{ (V)}$$

The line voltages seen from the load terminals are

$\boxed{V_{AN}-V_{BN}}$

$\boxed{I_a - I_b = I_a \sqrt{3}\angle 30^\circ = V_{an}/Z_Y \sqrt{3}\angle 30^\circ = V_{ab}/Z_Y}$

$$\tilde{V}_{AB} = \tilde{I}_a Z_L - \tilde{I}_b Z_L = \tilde{V}_{ab}\frac{Z_L}{Z_Y}$$

$$= 207.8461\angle 60^\circ \times \left(\frac{24+19}{25+j20}\right)$$

$$\approx 207.8461\angle 60^\circ \times \frac{30.6105\angle 38.37^\circ}{32.0156\angle 38.66^\circ}$$

$$\approx 198.7242\angle 59.71^\circ \text{ (V)}$$

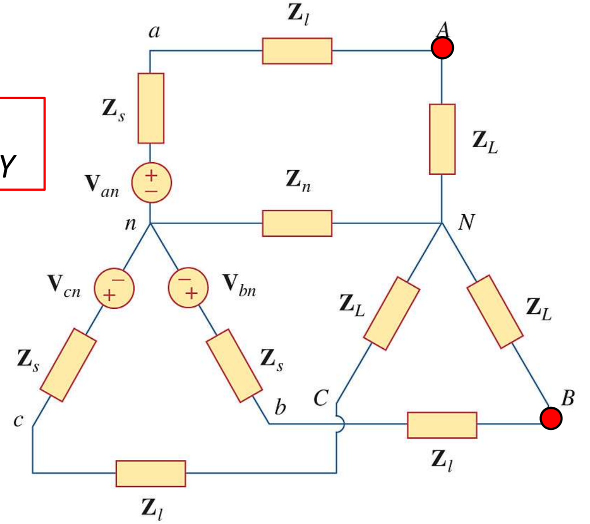


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

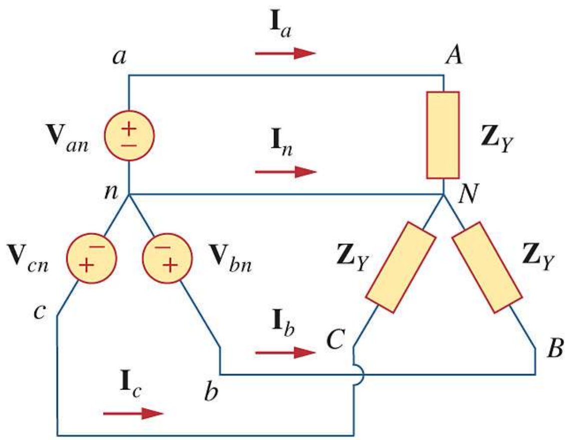


Figure 12.10 Balanced Y-Y connection.

$$\tilde{V}_{BC} \approx 198.7242\angle -60.29^\circ \text{ (V)}$$

$$\tilde{V}_{CA} \approx 198.7242\angle -180.29^\circ \text{ (V)}$$

The line voltage drops are

$$\tilde{I}_a Z_l = 3.7482\angle -8.66^\circ \times (0.6 + j0.7)$$

$$\approx 3.4558\angle 40.74^\circ \text{ (V)}$$

$$\tilde{I}_b Z_l = 3.4558\angle -79.26^\circ \text{ (V)}$$

$$\tilde{I}_b Z_l = 3.4558\angle -199.26^\circ \text{ (V)}$$

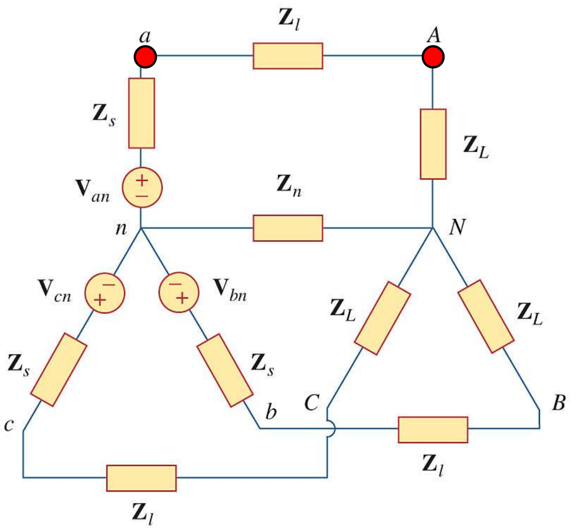


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

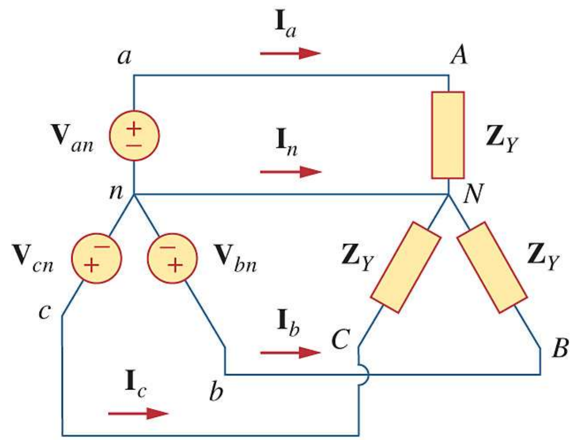


Figure 12.10 Balanced Y-Y connection.

12.4 Balanced Wye-Delta Connection

A Balanced Y-Δ system consists of a balanced Y-connected source feeding a balanced Δ-connected load, as shown in Fig. 12.14.

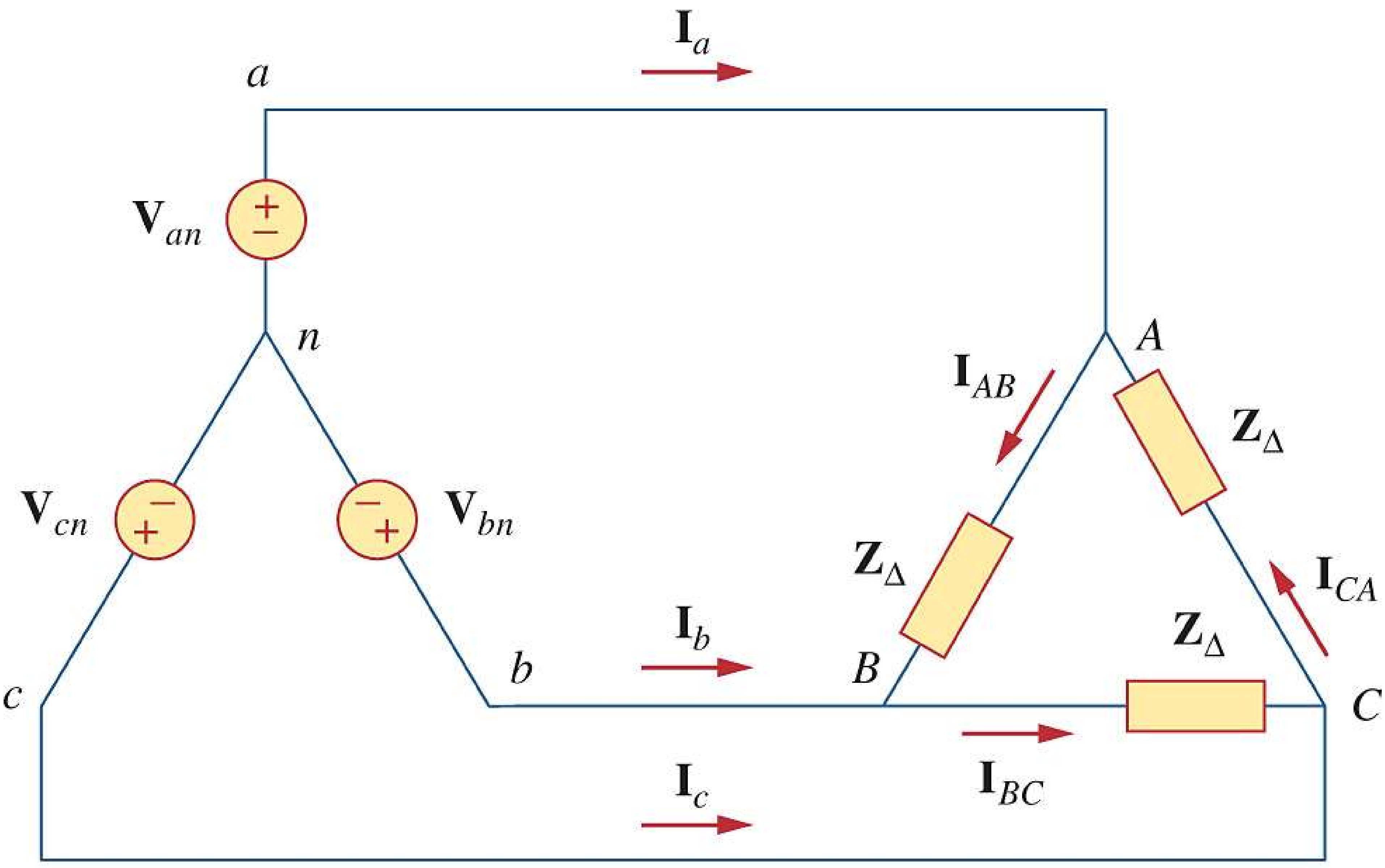


Figure 12.14 Balanced wye-delta connection.

Assume that

$$\tilde{V}_{an} = V_p\angle 0°$$

$$\tilde{V}_{bn} = V_p\angle -120°$$

$$\tilde{V}_{cn} = V_p\angle -240°$$

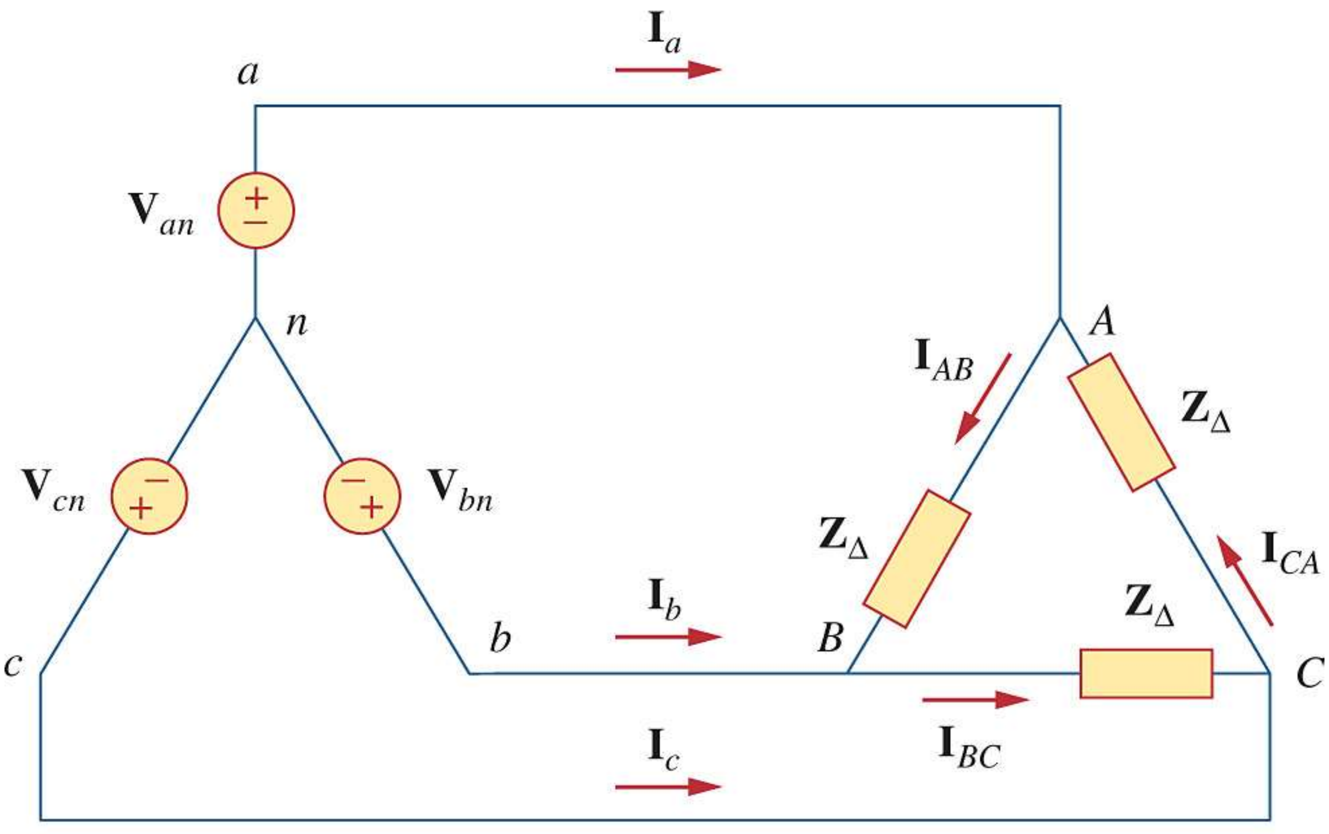


Figure 12.14 Balanced wye-delta connection.

The line voltages are

$$\tilde{V}_{ab} = \sqrt{3}V_p\angle 30° = \tilde{V}_{an}\sqrt{3}\angle 30° = \tilde{V}_{AB}$$

$$\tilde{V}_{bc} = \tilde{V}_{ab}\angle -120° = \tilde{V}_{BC}$$

$$\tilde{V}_{ca} = \tilde{V}_{ab}\angle -240° = \tilde{V}_{CA}$$

Same as Y-Y case

The phase currents

$$\tilde{I}_{AB} = \frac{\tilde{V}_{AB}}{Z_{\Delta}}$$

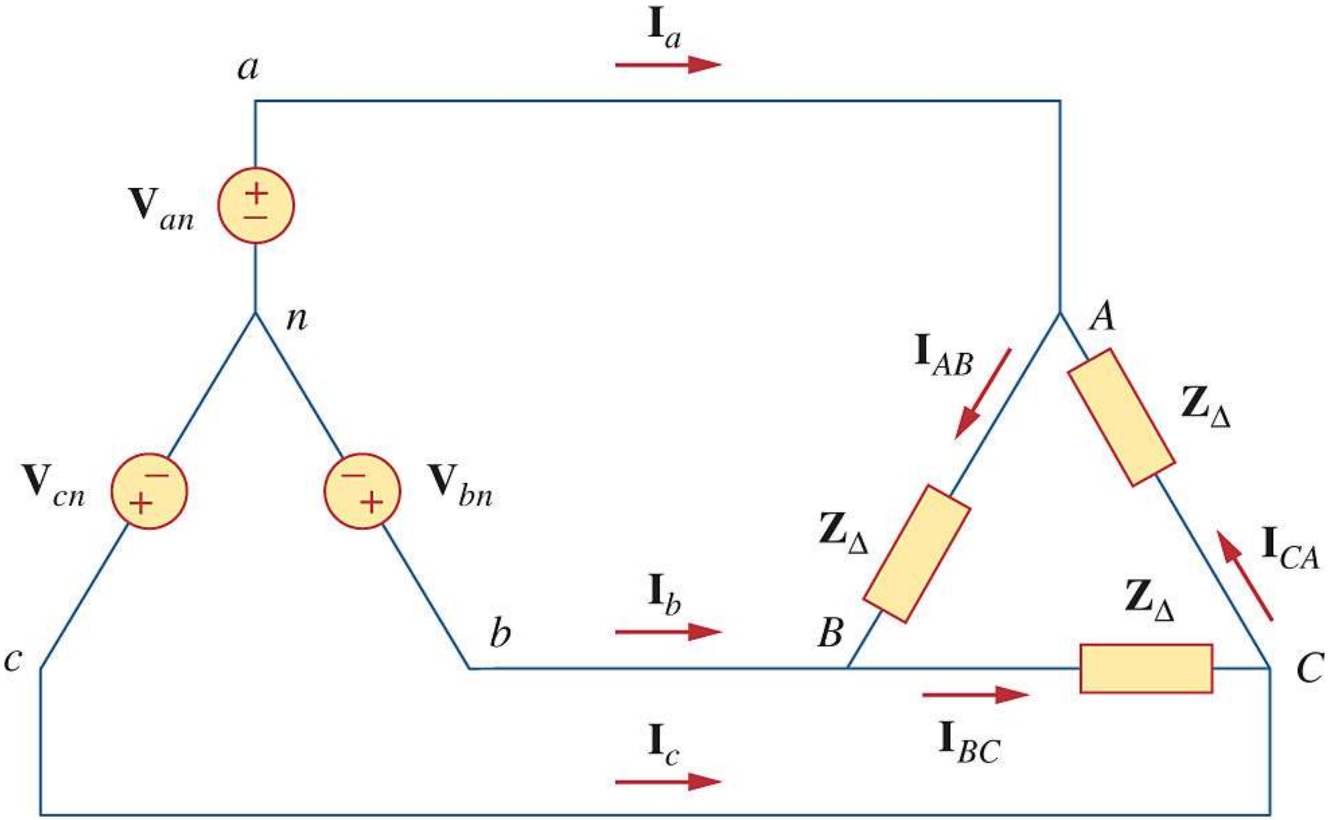


Figure 12.14 Balanced wye-delta connection.

$$\tilde{I}_{BC} = \frac{\tilde{V}_{BC}}{Z_{\Delta}} = \tilde{I}_{AB}\angle -120°$$

$$\tilde{I}_{CA} = \frac{\tilde{V}_{CA}}{Z_{\Delta}} = \tilde{I}_{AB}\angle -240°$$

The line currents are

$$\tilde{I}_a = \tilde{I}_{AB} - \tilde{I}_{CA} = \tilde{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$\tilde{I}_b = \tilde{I}_{BC} - \tilde{I}_{AB} = \tilde{I}_{BC}\sqrt{3}\angle -30^\circ = \tilde{I}_a\angle -120^\circ$$

$$\tilde{I}_c = \tilde{I}_{CA} - \tilde{I}_{BC} = \tilde{I}_{CA}\sqrt{3}\angle -30^\circ = \tilde{I}_a\angle -240^\circ$$

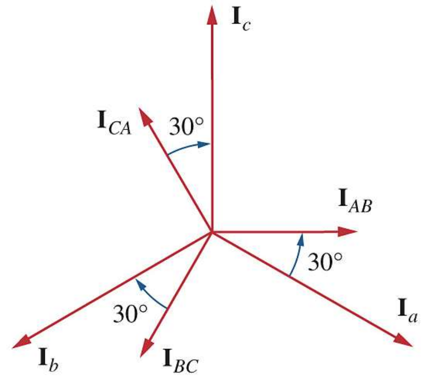


Figure 12.15 Phasor diagram illustrating the relationship between phase and line currents.

showing that

$$I_L = \sqrt{3} I_p$$

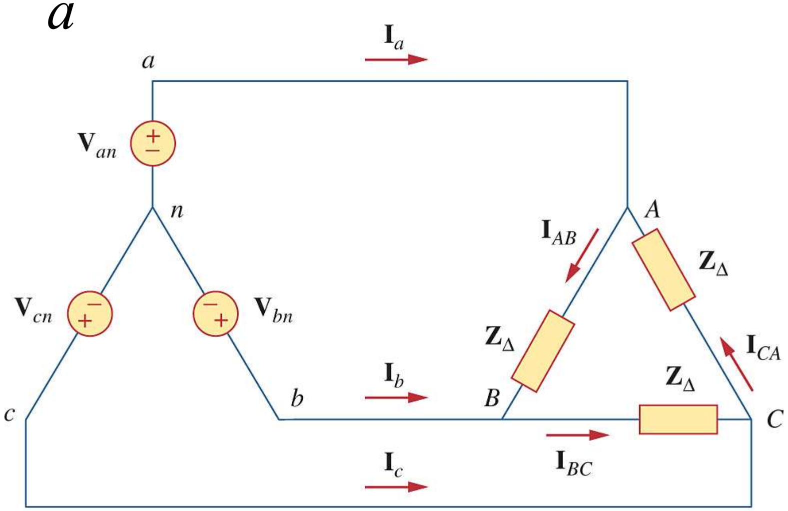


Figure 12.14 Balanced wye-delta connection.

where

$$I_L = |\tilde{I}_a| = |\tilde{I}_b| = |\tilde{I}_c|,\ I_p = |\tilde{I}_{AB}| = |\tilde{I}_{BC}| = |\tilde{I}_{CA}|$$

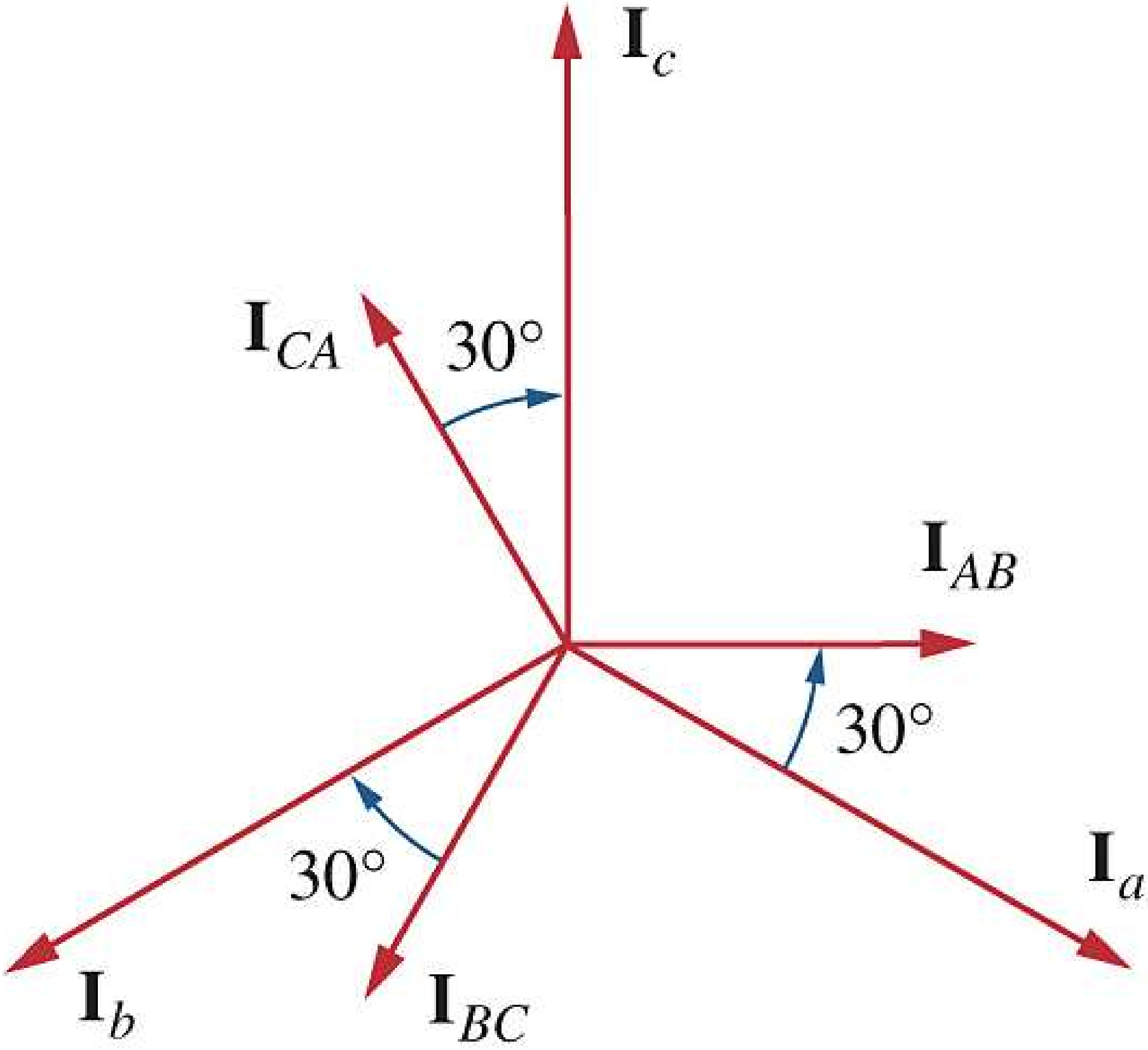


Figure 12.15 Phasor diagram illustrating the relationship between phase and line currents.

Summary of balanced Y-Δ

If the source (phase voltages) are in *abc* sequence,

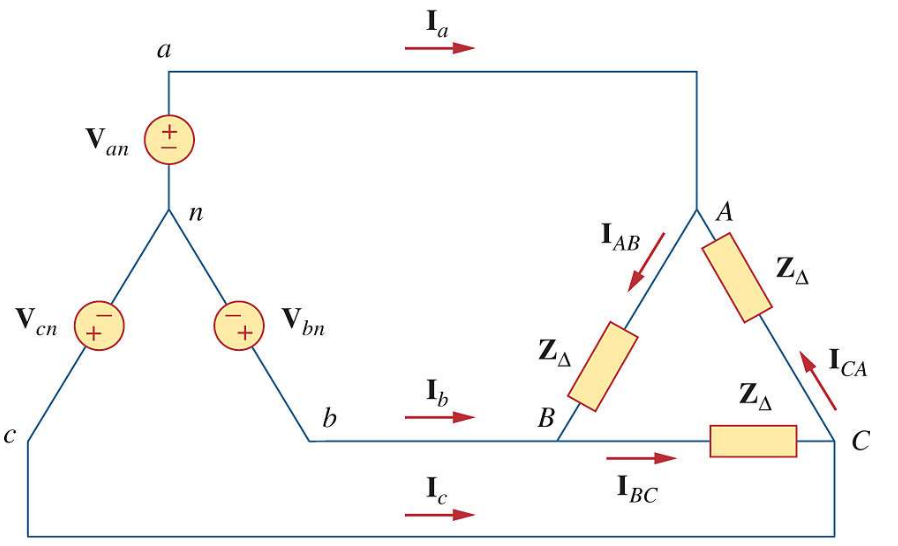


Figure 12.14 Balanced wye-delta connection.

- Voltages: <u>Magnitude</u> <u>Phase</u>

$$V_L = \sqrt{3}V_p \qquad \angle\tilde{V}_{ab} = \angle\tilde{V}_{an} + \angle 30^\circ$$

$$\angle\tilde{V}_{bc} = \angle\tilde{V}_{bn} + \angle 30^\circ$$

$$\angle\tilde{V}_{ca} = \angle\tilde{V}_{cn} + \angle 30^\circ$$

Line voltages and phase voltages are in *abc* sequence

- Currents: <u>Magnitude</u> <u>Phase</u>

$$I_L = \sqrt{3}I_p \qquad \tilde{I}_a = \tilde{I}_{AB}\sqrt{3}\angle -30^\circ$$

……

Line currents and phase currents are in *abc* sequence

Practice Problem 12.3 One line voltage of a balanced Y-connected source is $\tilde{V}_{AB} = 240\angle -20°$ V. If the source is connected to a Δ-connected load of $20\angle 40°$ Ω, find the phase and line currents. Assume the *abc* sequence.

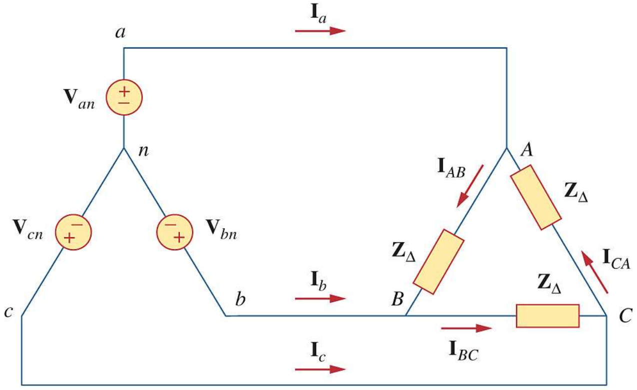


Figure 12.14 Balanced wye-delta connection.

Solution :

$$\tilde{I}_{AB} = \frac{\tilde{V}_{AB}}{Z_\Delta} = \frac{240\angle -20°}{20\angle 40°} = 12\angle -60° \text{ (A)}$$

$$\tilde{I}_{BC} = \tilde{I}_{AB}\angle -120° = 12\angle -180° \text{ (A)}$$

$$\tilde{I}_{CA} = \tilde{I}_{AB}\angle +120° = 12\angle 60° \text{ (A)}$$

$$\tilde{I}_{a} = \tilde{I}_{AB}\sqrt{3}\angle -30° \approx 20.7846\angle -90° \text{ (A)}$$

$$\tilde{I}_{b} = \tilde{I}_{a}\angle +240° = 20.7846\angle 150° \text{ (A)}$$

$$\tilde{I}_{c} = \tilde{I}_{a}\angle +120° = 20.7846\angle 30° \text{ (A)}$$

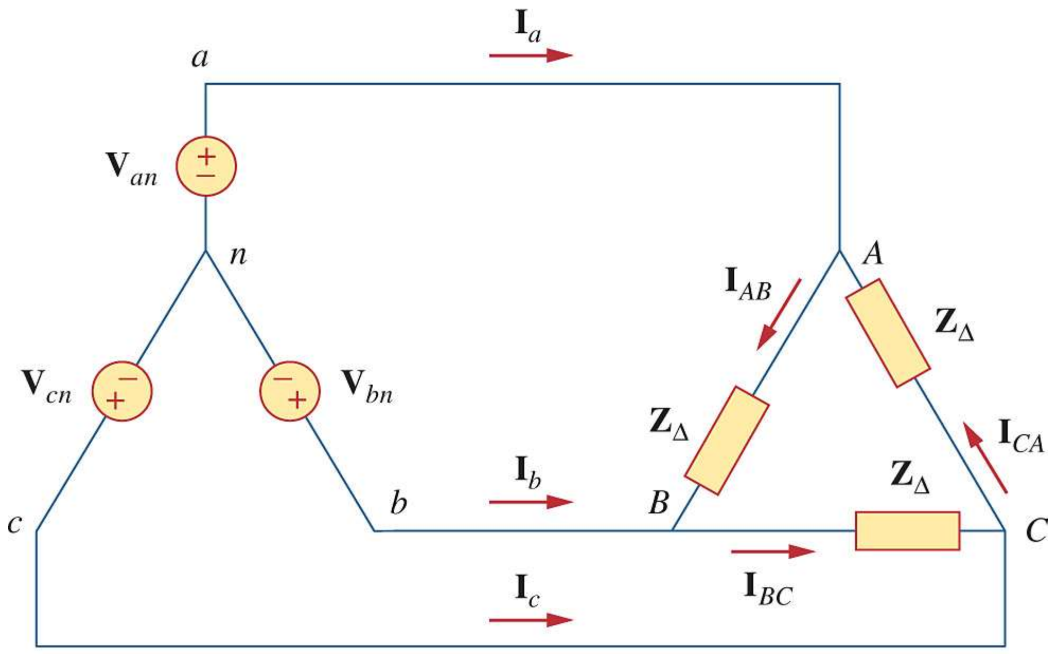


Figure 12.14 Balanced wye-delta connection.

12.5 Balanced Delta-Delta Connection

A balanced Δ-Δ system is one in which both the balanced source and balanced load are Δ-connected, as shown in Fig. 12.17.

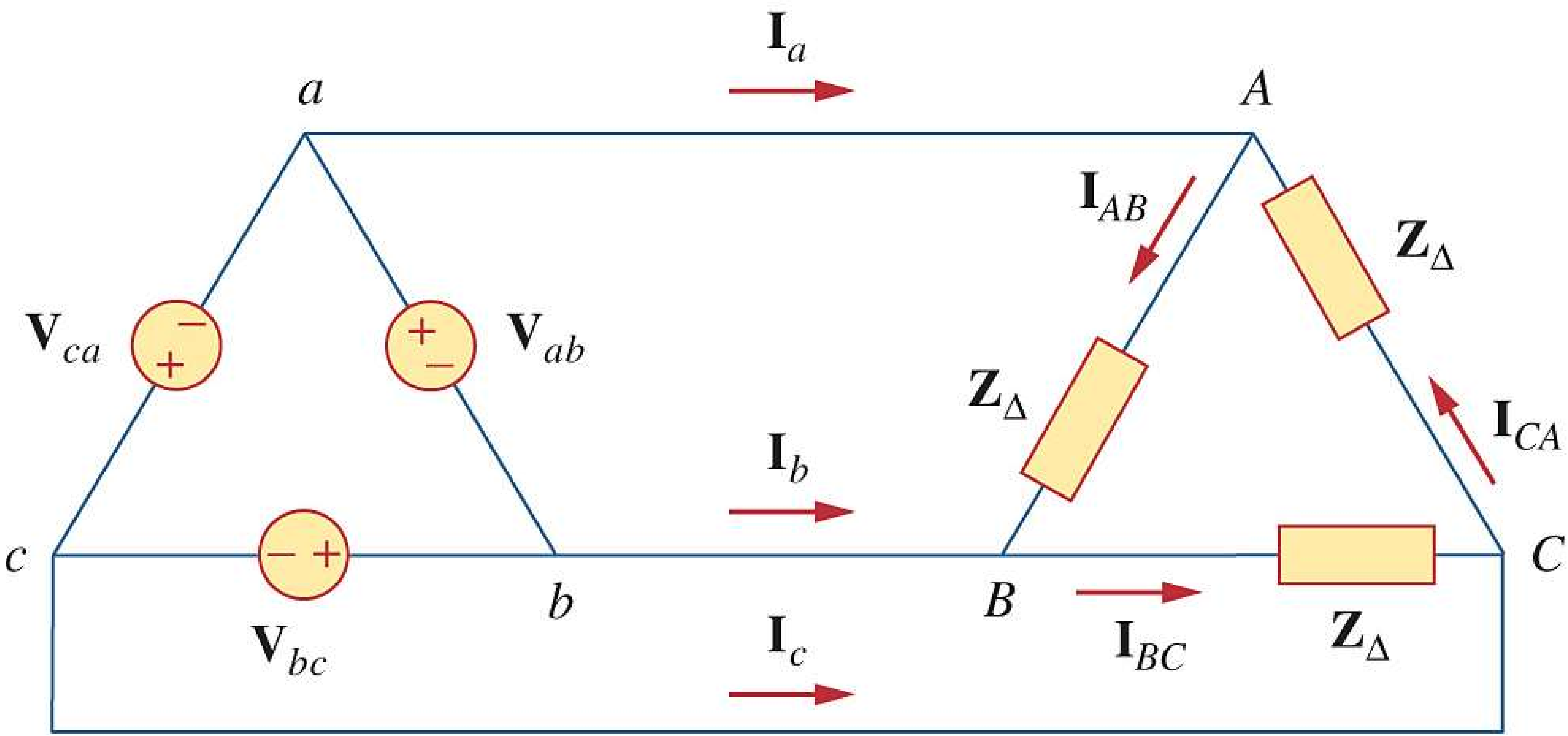


Figure 12.17 A balanced delta-delta connection.

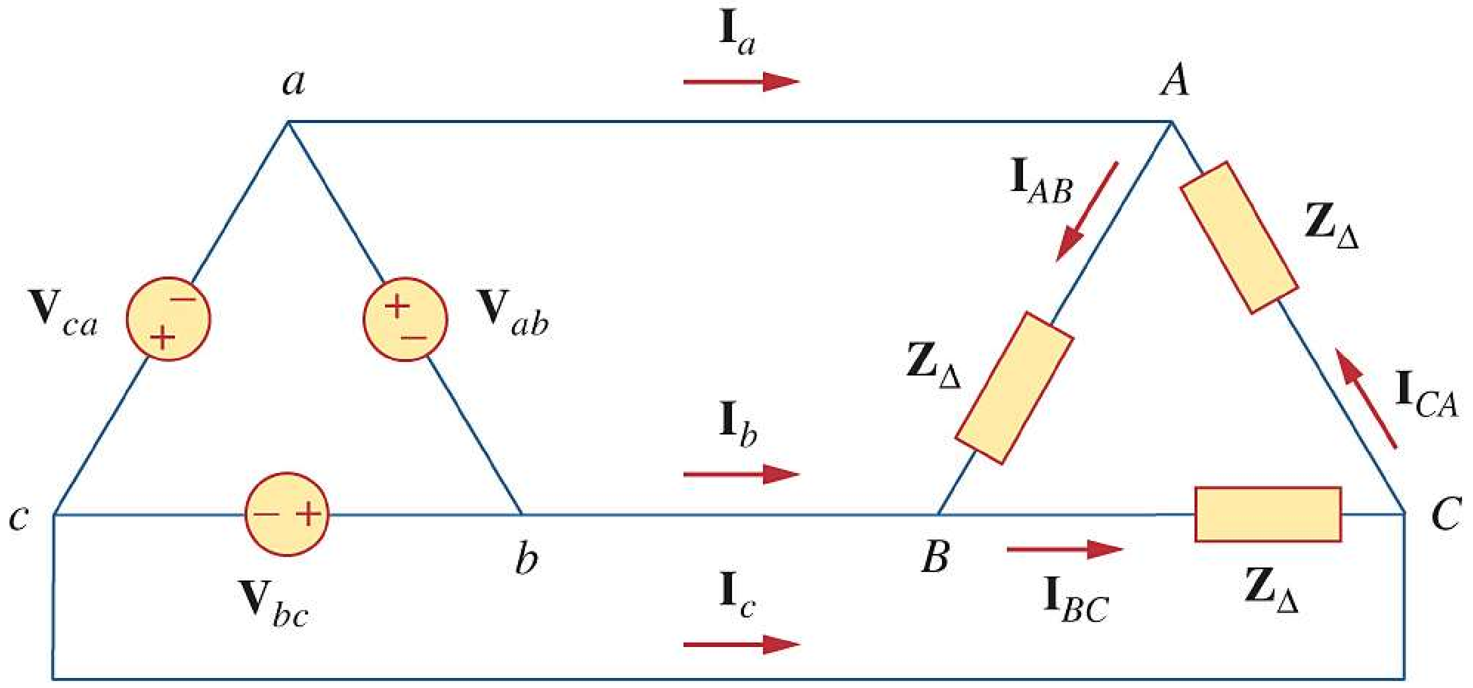


Figure 12.17 A balanced delta-delta connection.

Assume that

$$\tilde{V}_{ab} = V_p \angle 0^\circ$$

$$\tilde{V}_{bc} = V_p \angle -120^\circ$$

$$\tilde{V}_{ca} = V_p \angle +120^\circ$$

The line voltages are

$$\tilde{V}_{ab} = \tilde{V}_{AB}, \tilde{V}_{bc} = \tilde{V}_{BC}, \tilde{V}_{ca} = \tilde{V}_{CA}$$

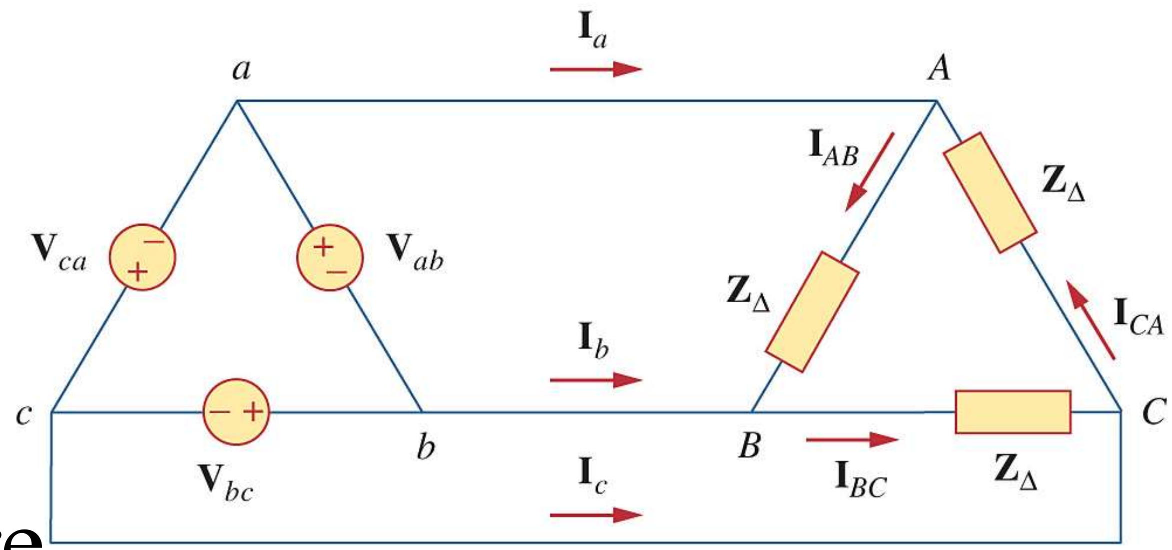


Figure 12.17 A balanced delta-delta connection.

The phase currents are

$$\tilde{I}_{AB} = \frac{\tilde{V}_{AB}}{Z_\Delta}$$

$$\tilde{I}_{BC} = \frac{\tilde{V}_{BC}}{Z_\Delta} = \tilde{I}_{AB}\angle -120^\circ$$

$$\tilde{I}_{CA} = \frac{\tilde{V}_{CA}}{Z_\Delta} = \tilde{I}_{AB}\angle +120^\circ$$

The line currents are

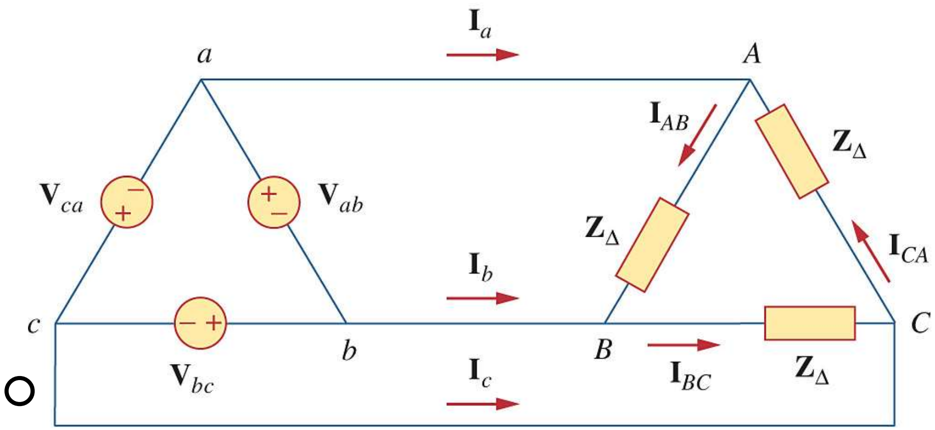


Figure 12.17 A balanced delta-delta connection.

$$\tilde{I}_a = \tilde{I}_{AB} - \tilde{I}_{CA} = \tilde{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$\tilde{I}_b = \tilde{I}_{BC} - \tilde{I}_{AB} = \tilde{I}_{BC}\sqrt{3}\angle -30^\circ = \tilde{I}_a\angle -120^\circ$$

$$\tilde{I}_c = \tilde{I}_{CA} - \tilde{I}_{BC} = \tilde{I}_{CA}\sqrt{3}\angle -30^\circ = \tilde{I}_a\angle -240^\circ$$

showing that

$$I_L = \sqrt{3}I_p$$

where

$$I_L = \left|\tilde{I}_a\right| = \left|\tilde{I}_b\right| = \left|\tilde{I}_c\right|, I_p = \left|\tilde{I}_{AB}\right| = \left|\tilde{I}_{BC}\right| = \left|\tilde{I}_{CA}\right|$$

Summary of balanced Δ-Δ

If the source (phase voltages) are in *abc* sequence,

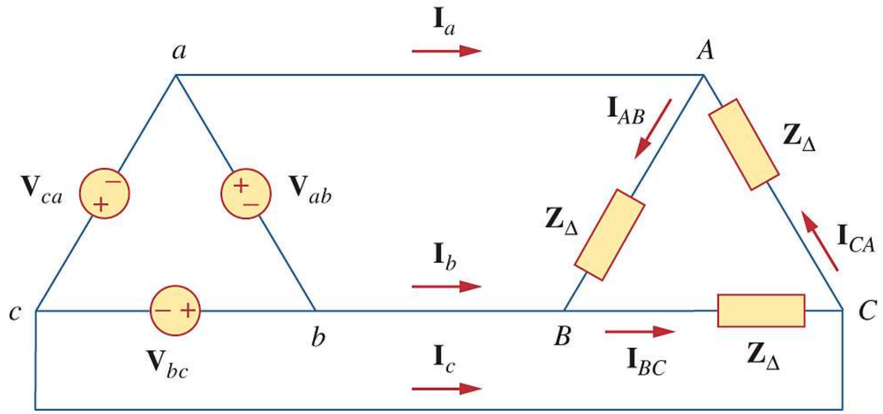


Figure 12.17 A balanced delta-delta connection.

- Voltages:
  Line voltages and phase voltages are same, and are in *abc* sequence

- Currents: <u>Magnitude</u> <u>Phase</u>

$$I_L = \sqrt{3} I_p \qquad \tilde{I}_a = \tilde{I}_{AB} \sqrt{3} \angle -30^\circ$$

……

  Line currents and phase currents are in *abc* sequence

12.6 Balanced Delta-Wye Connection

A balanced Δ-Y system consists of a balanced Δ-connected source feeding a balanced Y-connected load, as shown in Fig. 12.18.

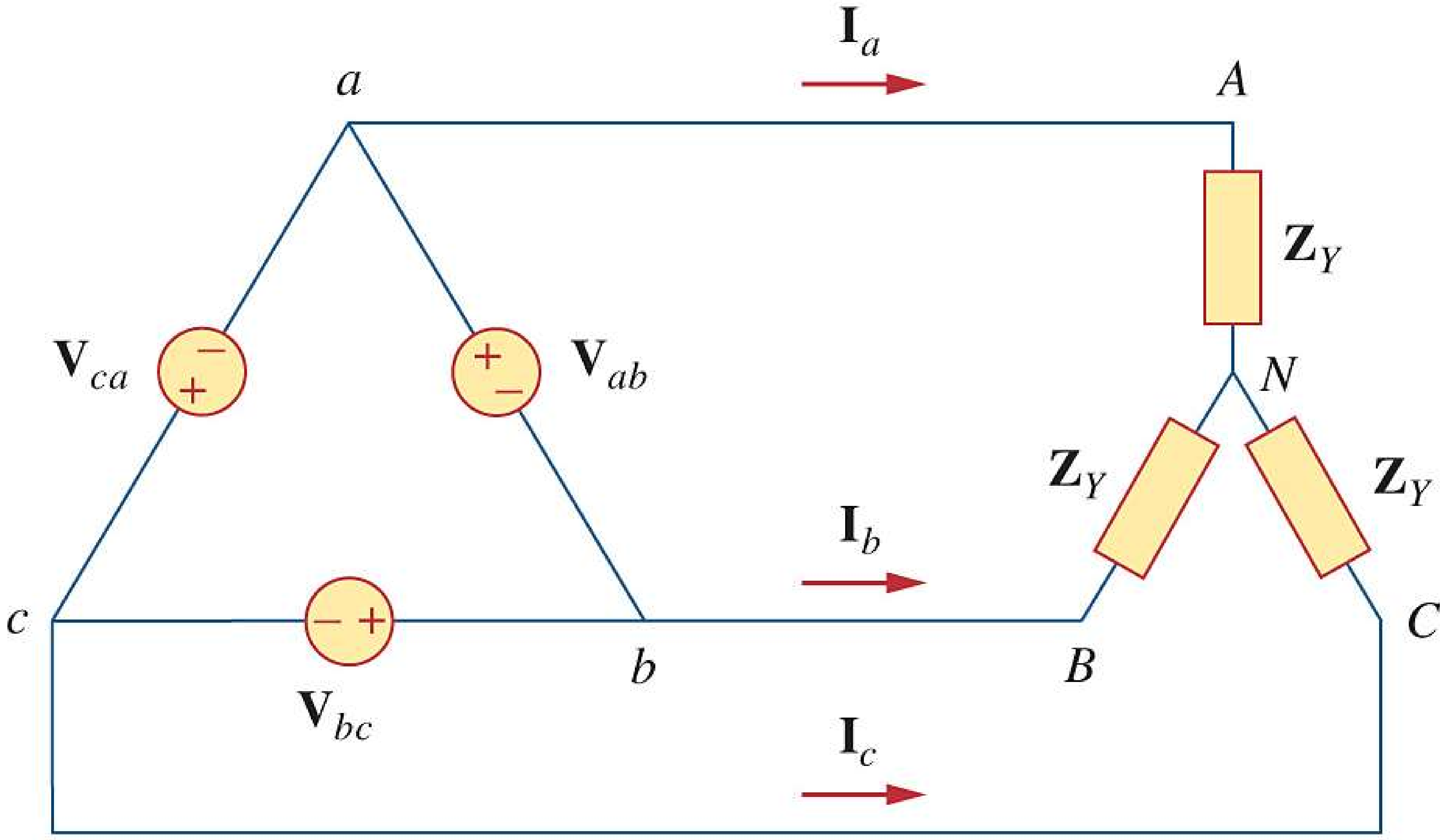


Figure 12.18 A balanced delta-wye connection.

Assume that

$$\tilde{V}_{ab} = V_p \angle 0°$$

$$\tilde{V}_{bc} = V_p \angle -120°$$

$$\tilde{V}_{ca} = V_p \angle +120°$$

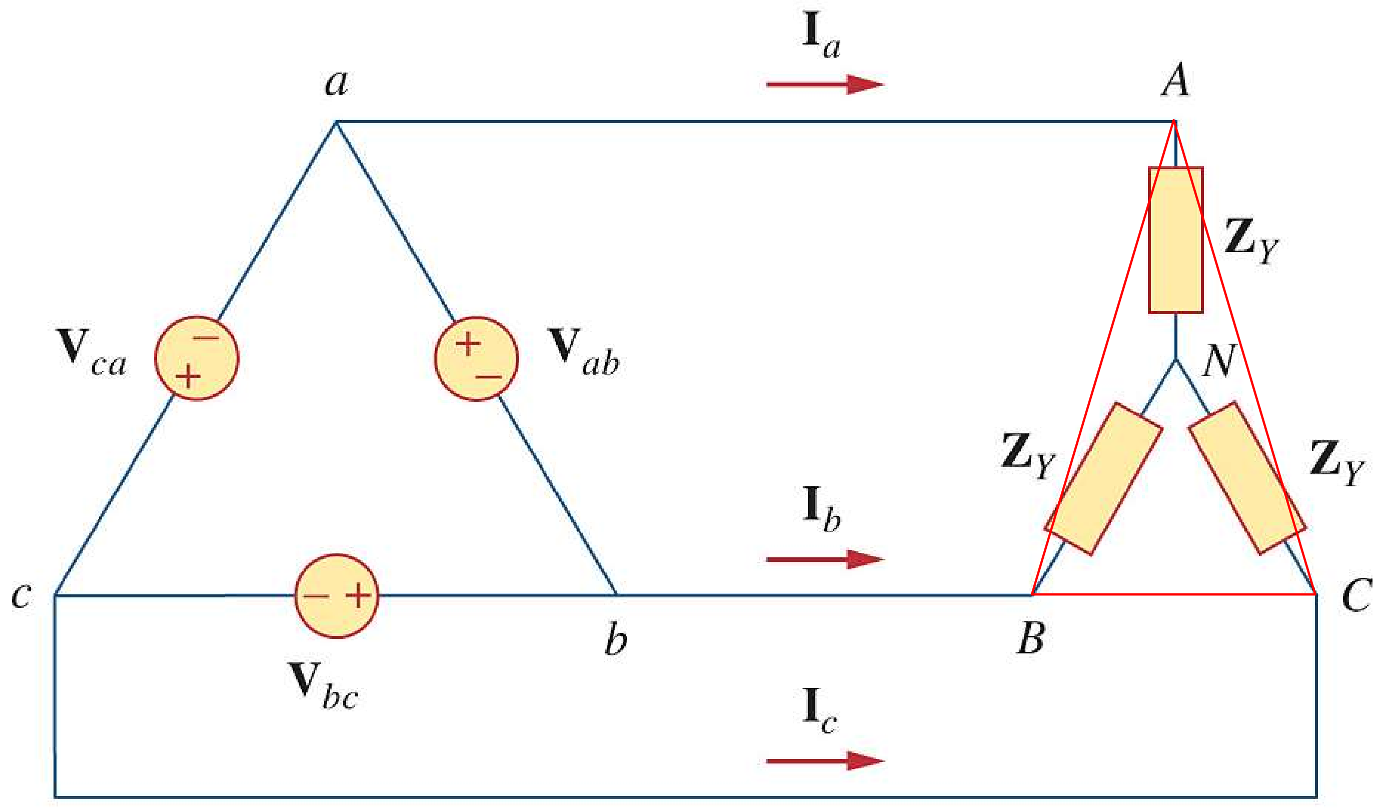


Figure 12.18 A balanced delta-wye connection.

Replace the Y-connected load with its equivalent Δ-connected load and recall that for a Δ-connected load,

$$\tilde{I}_a = \tilde{I}_{AB} \sqrt{3} \angle -30°$$

$$\tilde{I}_a = \tilde{I}_{AB}\sqrt{3}\angle -30° = \frac{\tilde{V}_{AB}}{Z_\Delta}\sqrt{3}\angle -30°$$

$Z_\Delta = 3Z_Y$
$Z_Y = Z_\Delta/3$

$$= \frac{\tilde{V}_{ab}}{3Z_Y}\sqrt{3}\angle -30° = \frac{\tilde{V}_{ab}}{\sqrt{3}Z_Y}\angle -30°$$

$$\tilde{I}_b = \frac{\tilde{V}_{bc}}{\sqrt{3}Z_Y}\angle -30° = \tilde{I}_a\angle -120°$$

$$\tilde{I}_c = \frac{\tilde{V}_{ca}}{\sqrt{3}Z_Y}\angle -30° = \tilde{I}_a\angle +120°$$

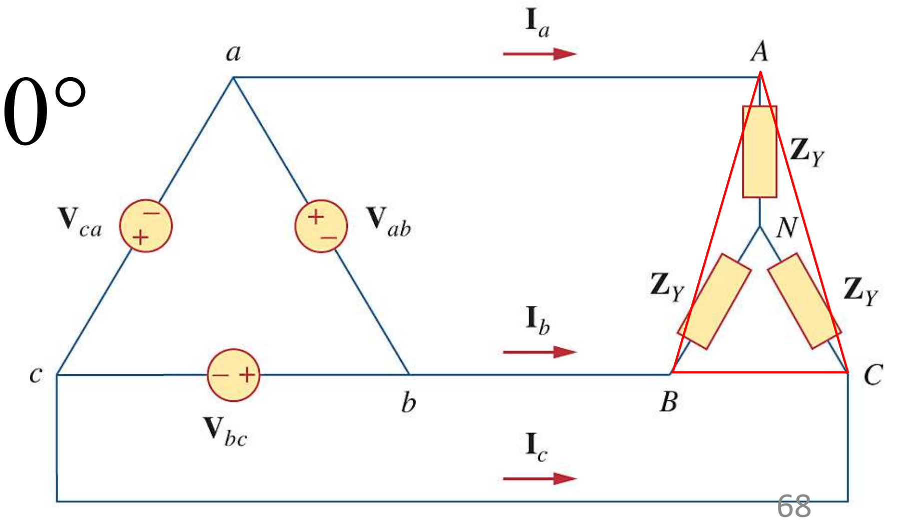


Figure 12.18 A balanced delta-wye connection.

Summary of balanced Δ-Y

If the source (phase voltages) are in *abc* sequence,

- Voltages:
  Line voltages and phase voltages are same, and are in *abc* sequence

- Currents:
  Line currents and phase currents are same, and are in *abc* sequence

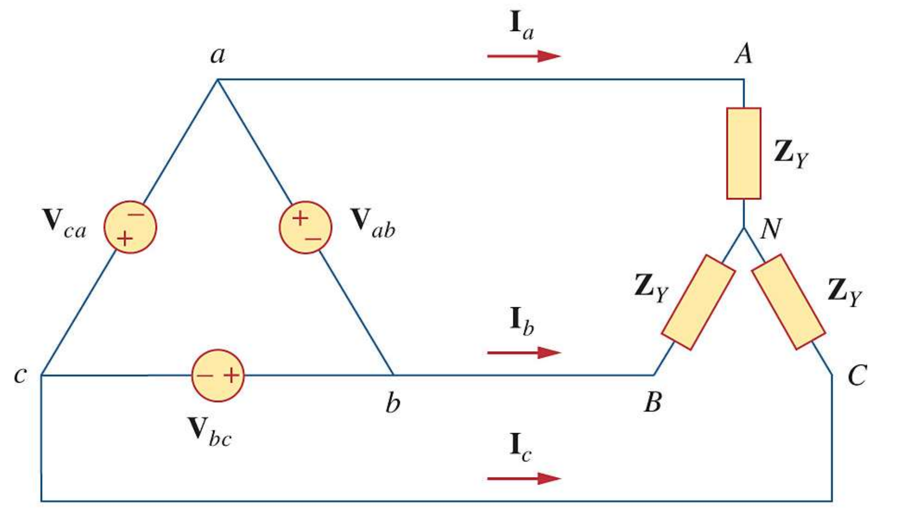


Figure 12.18 A balanced delta-wye connection.

12.7 Power in a Balanced System

We begin by examining the instantaneous power absorbed by the load. For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos \omega t$$

$$v_{BN} = \sqrt{2}V_p \cos(\omega t - 120°)$$

$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120°)$$

V_p is the rms value

➔ Peak value $\sqrt{2}V_p$

If $Z_Y = Z\angle\theta$, the phase currents are

$$i_a = \sqrt{2} I_p \cos(\omega t - \theta)$$

$$i_b = \sqrt{2} I_p \cos(\omega t - \theta - 120°)$$

$$i_c = \sqrt{2} I_p \cos(\omega t - \theta + 120°)$$

I* = *V*/*Z

➔ $\angle I = \angle V - \angle Z$

The total instantaneous power is

$$p = p_a + p_b + p_c = v_{AN} i_a + v_{BN} i_b + v_{CN} i_c$$

$$
\begin{aligned}
&= 2V_p I_p [\cos\omega t \cos(\omega t - \theta) \\
&+ \cos(\omega t - 120°)\cos(\omega t - \theta - 120°) \\
&+ \cos(\omega t + 120°)\cos(\omega t - \theta + 120°)] \\
&= 2V_p I_p \{\frac{\cos\theta + \cos(2\omega t - \theta)}{2} \\
&+ \frac{\cos\theta + \cos(2\omega t - \theta - 240°)}{2} \\
&+ \frac{\cos\theta + \cos(2\omega t - \theta + 240°)}{2}] \\
&= 3V_p I_p \cos\theta
\end{aligned}
$$

$$p(t) = 3V_pI_p\cos(\theta)$$

Thus the total instantaneous power in a balanced three-phase system is constant – it does not change with time as the instantaneous power of each phase does. This result is true whether the load is Y- or Δ-connected. This is one important reason for using a three-phase system to generate and distribute power.

The advantage over a single-phase AC system: **constant $p(t)$**

Since the total instantaneous power is independent of time, the average power per phase P_p for the Y- or Δ-connected load is $p/3$, or

$$P_p = V_p I_p \cos\theta$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin\theta$$

Recall:
$Z_Y = Z\angle\theta$
θ is the power factor angle

The apparent power per phase is

$$\left|S_p\right| = V_p I_p$$

The complex power per phase is

$$S_p = P_p + jQ_p = \tilde{V}_p \tilde{I}_p^*$$

The total average power is

$$P = P_a + P_b + P_c = 3P_p$$

By AC power conservation

$$= 3V_p I_p \cos\theta$$

and Y-connected source

For a Y-connected load, $I_L = I_p$ but $V_L = \sqrt{3}V_p$, whereas for a Δ-connected load, $I_L = \sqrt{3}I_p$ but $V_L = V_p$. Thus,

and Δ-connected source

$$P = 3V_p I_p \cos\theta = \sqrt{3}V_L I_L \cos\theta$$

V_p: phase voltage
I_p: phase current

V_L: line voltage
I_L: line current

The total reactive power is

$$Q = 3Q_p = 3V_p I_p \sin\theta = \sqrt{3}V_L I_L \sin\theta$$

The total complex power is

$$S = 3S_p = 3\tilde{V}_p \tilde{I}_p^* = 3I_p^2 Z_p = \frac{3V_p^2}{Z_p^*}$$

where Z_p is the load impedance per phase.

$$S = P + jQ = 3V_p I_p \cos\theta + j3V_p I_p \sin\theta$$

$$= \sqrt{3}V_L I_L \cos\theta + j\sqrt{3}V_L I_L \sin\theta$$

Remember that V_p, I_p, V_L, and I_L are all rms values and that θ is the angle of the load impedance or the angle between the phase voltage and the phase current.

Recall:

$Z_Y = Z\angle\theta$ $\theta = \angle \mathbf{V_p} - \angle \mathbf{I_p}$

θ is the power factor angle

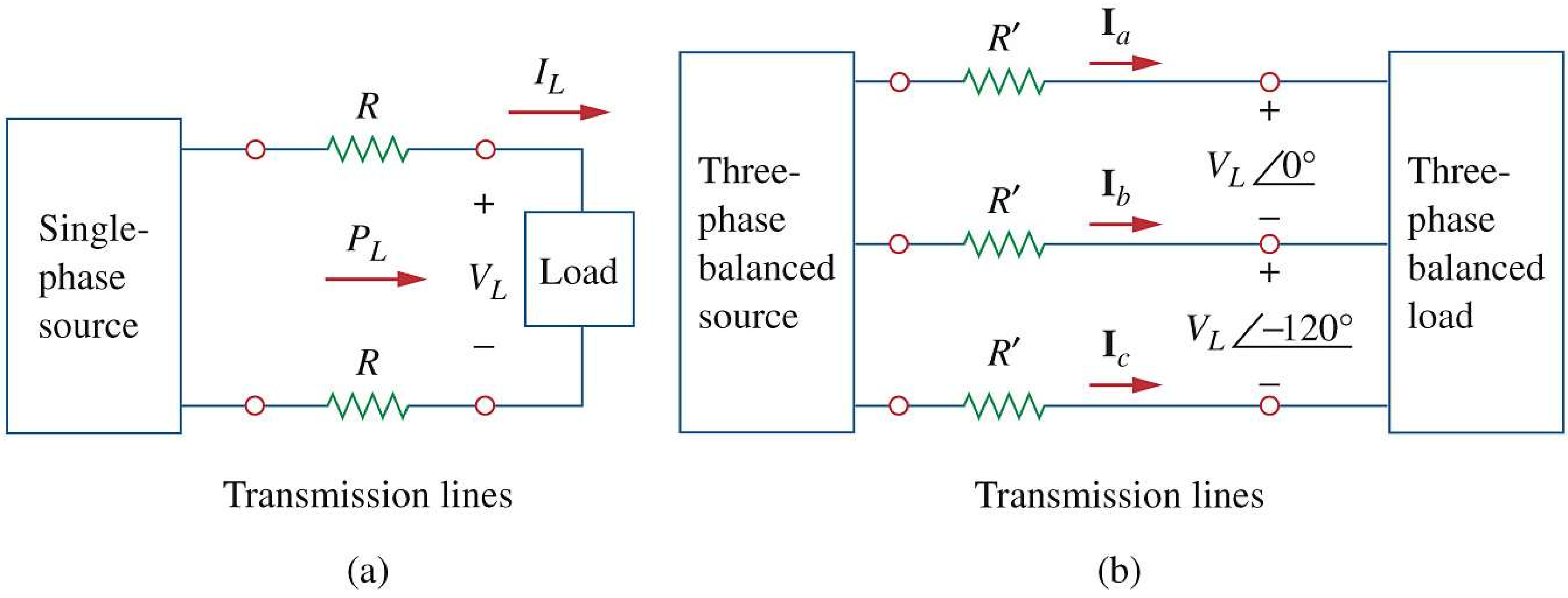


Figure 12.21 Comparing the power loss in (a) single-phase system, and (b) a three-phase system.

Conditions in the comparison:

1. Same line voltages V_L in the two cases
2. To deliver the same total power P_L
3. The same power loss P_{loss} is tolerated *during power transmission*

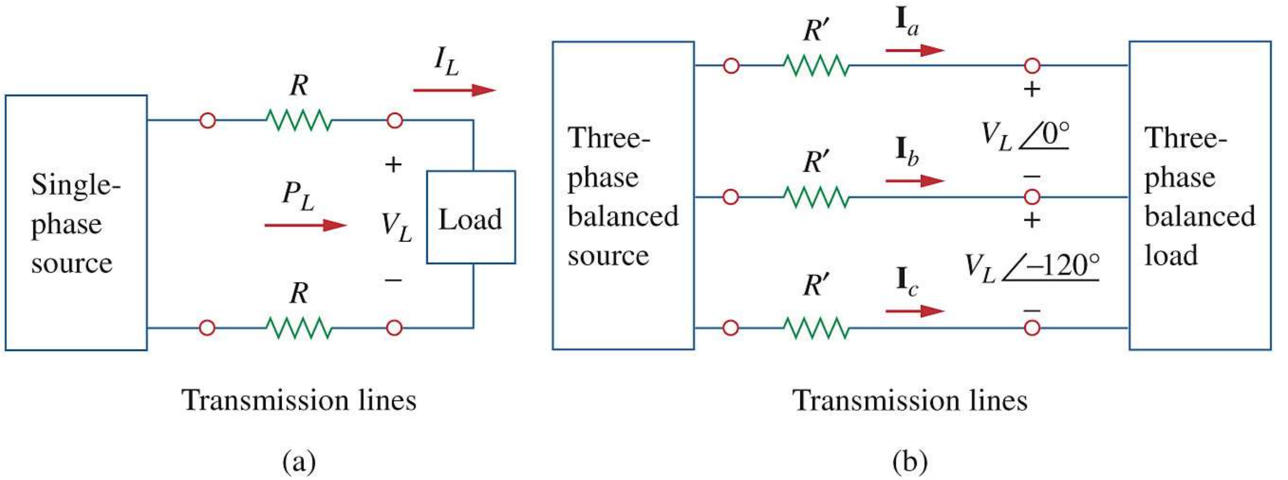
Consider Fig. 12.21. For the single-phase two-wire system, $P_L = V_L I_L$, the power loss in the two wires is

2 wires

$$P_{loss} = 2I_L^2 R = 2\left(\frac{P_L}{V_L}\right)^2 R$$

R: the total resistance of one transmission line

For the three-phase three-wire system, $P_L = \sqrt{3} V_L I'_L$, the power loss in the three wires is



’ means case 2, not derivative

I_L': line current
R': line resistance

Figure 12.21 Comparing the power loss in (a) single-phase system, and (b) a three-phase system.

3 wires

$$P'_{loss} = 3I_L'^2 R' = 3\left(\frac{P_L}{\sqrt{3}V_L}\right)^2 R' = \left(\frac{P_L}{V_L}\right)^2 R'$$

The equations show that for the same total power delivered P_L and the same line voltage V_L,

$$\frac{P_{loss}}{P'_{loss}} = \frac{2R}{R'} = \frac{2\rho l/(\pi r^2)}{\rho l/(\pi r'^2)} = \frac{2r'^2}{r^2}$$

ρ: resistivity
l: length of transmission line
r: radius of the transmission line

If the same power loss is tolerated in both systems, then $r^2 = 2r'^2$, so

Two wires

$$\frac{\text{Material for single-phase}}{\text{Material for three-phase}} = \frac{2(\pi r^2 l)}{3(\pi r'^2 l)}$$

Three wires

$$= \frac{2r^2}{3r'^2} = \frac{4}{3}$$

This is the second major advantage of three-phase systems for power distribution.

The advantage over a single-phase AC system: **Less materials for transmission lines (economic)**

Summary of the advantages of 3-ϕ system

1. The total instantaneous power in a balanced three-phase system is constant.

2. The material (transmission line) required to deliver the same power and to tolerate the same loss is 25% less.

Example 12.8 Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig. 12.22(a). Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence,

Line voltage

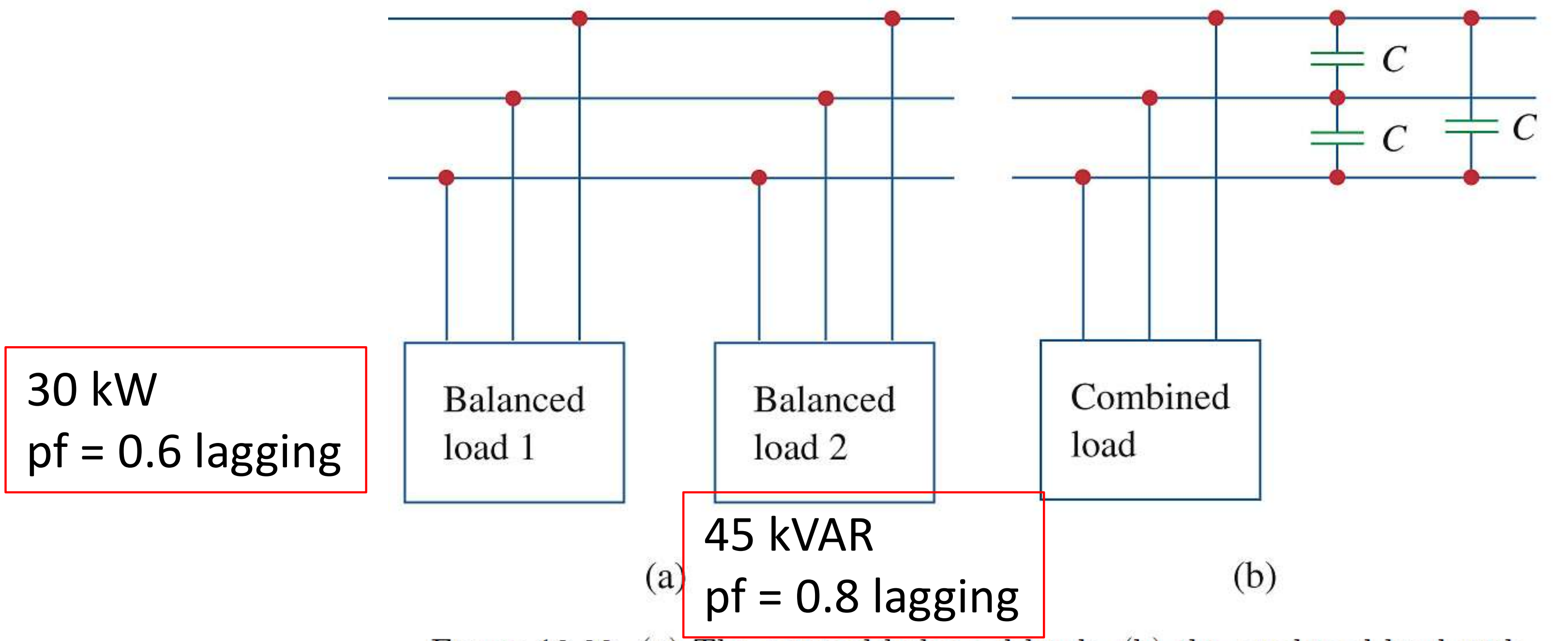


Figure 12.22 (a) The original balanced loads, (b) the combined load with improved power factor.

determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line current, and (c) the kVAR rating of the three capacitors Δ-connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.

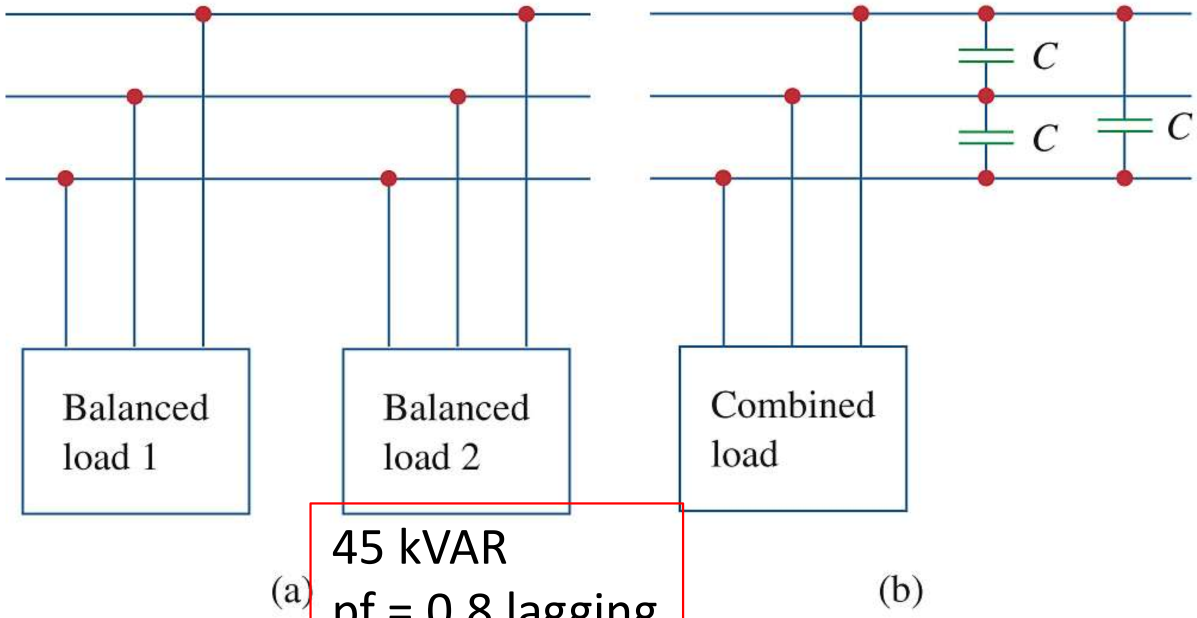
$V_L = 240\text{kV}$

(a) S, P, Q

(b) I_L

(c) Q_C

30 kW
pf = 0.6 lagging



45 kVAR
pf = 0.8 lagging

Figure 12.22 (a) The original balanced loads, (b) the combined load with improved power factor.

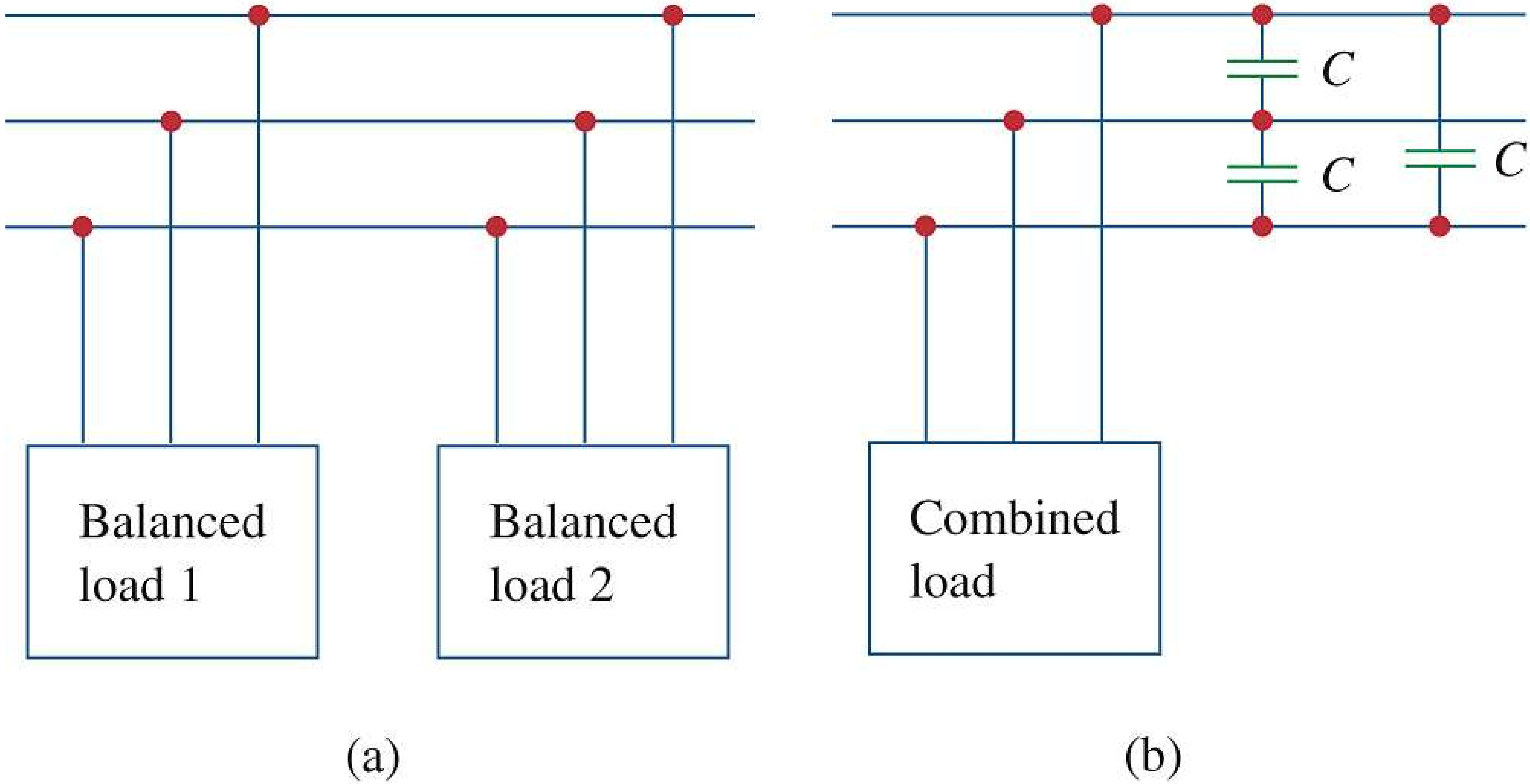


Figure 12.22 (a) The original balanced loads, (b) the combined load with improved power factor.

Solution :

V_L = 240kV

(a) S, P, Q

(b) I_L

(c) Q_C

$$\text{(a)}\ \cos\theta_1 = 0.6 \Rightarrow \sin\theta_1 = 0.8$$

$$Q_1 = P_1 \tan\theta_1 = 30\times\frac{0.8}{0.6} = 40\ \text{(kVAR)}$$

$$S_1 = P_1 + jQ_1 = 30 + j40\ \text{(kVA)}$$

$$\cos\theta_2 = 0.8 \Rightarrow \sin\theta_2 = 0.6$$

$$P_2 = Q_2 / \tan\theta_2 = 45/(0.6/0.8) = 60\ \text{(kW)}$$

$$S_2 = P_2 + jQ_2 = 60 + j45\ \text{(kVA)}$$

$$S = S_1 + S_2 = (30 + j40) + (60 + j45)$$

$$= 90 + j85 \text{ (kVA)}$$

$$P = 90 \text{ kW}, Q = 85 \text{ kVA}$$

V_L = 240kV
(a) S, P, Q
(b) I_L
(c) Q_C

$$\text{(b) } \theta = \tan^{-1}\left(\frac{85}{90}\right) \approx 43.36°$$

$$P = \sqrt{3} V_L I_L \cos\theta \Rightarrow I_L = \frac{P}{\sqrt{3} V_L \cos\theta}$$

$$I_L = \frac{90 \times 10^3}{\sqrt{3} \times 240 \times 10^3 \times \cos 43.36°} \approx 0.2978 \text{ (A)}$$

V_L = 240kV

(a) S, P, Q

(b) I_L

(c) Q_C

$$\text{(c) } \theta_{old} = \tan^{-1}\left(\frac{85}{90}\right) \approx 43.36°$$

$$\theta_{new} = \cos^{-1} 0.9 \approx 25.84°$$

$$|Q_C| = P(\tan\theta_{old} - \tan\theta_{new})$$

$$= 90 \times (\tan 43.36° - \tan 25.84°)$$

$$\approx 41.40 \text{ (kVAR)}$$

This reactive power is for the three capacitors. For each capacitor,

$$|Q'_C| = |Q_C| / 3 \approx 13.80 \text{ (kVAR)}$$

$$C = \frac{|Q'_C|}{\omega V_L^2} = \frac{13.80 \times 10^3}{2\pi \times 60 \times (240 \times 10^3)^2}$$

$$\approx 6.3551 \times 10^{-10} \text{ (F)}$$

$$\approx 635.51 \text{ pF}$$

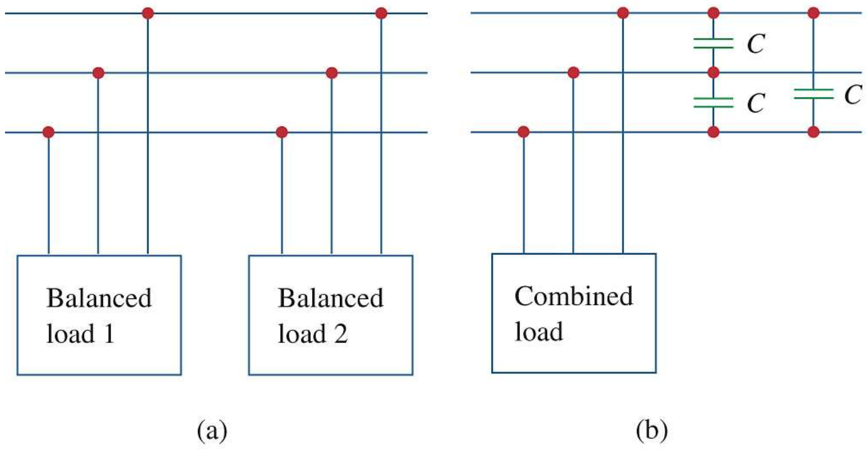


Figure 12.22 (a) The original balanced loads, (b) the combined load with improved power factor.

$|Q_C| = |I^2 X| = |V^2/X^*| = \omega C V^2$
for the purely capacitive case