Stochastic matching in simple settings

Someone

Abstract

Motivated by applications like kidney exchange, recommendation system and online dating, we study a matching problem with query-commit process. In this setting, a centralized system allocates agents into pairs. If a pair of agents accept their assignments, they are matched and leave the market. Otherwise, they will stay in the market. Most previous work focus on giving worst case guarantee on different settings. However, surve

1 Introduction

2 The setting

Our problem can be modeled in an undirected graph $G = (V, E)$, $V = \{v_1, v_2, \ldots, v_n\}$ denotes the set of agents. $e_{ij} \in E$ denotes an edge between v_i and v_j. The goal of the matching is to maximize cardinality. Each time, the system prob a pair of agents v_i and v_j with an edge e_{ij} connecting them. (1) If v_i and v_j accept to match with each other with p probability, both of the two agents are removed from the market. (2) If v_i and v_j do not accept with $1-p$ probability, the system knows that they cannot match and removes e_{ij} from E.

3 ESD algorithm

To maximize the number of matched agents, we apply edge serial dictatorship algorithm (ESD). The outline is described In Algorithm 1.

Algorithm 1 is quite straightforward. The system just picks edges one by one in some order (denoted by S) and try to match the current edge. Our goal is to maximize the expected size of the output set A.

In Algorithm 1, The generating of array S is not mentioned. Even though we find the S that can maximize the expected size of A, the current matching policy may not be optimal. However, ESD has the following advantages that motivates us to find a good policy based on it.

The most important advantage is that given S the expected size of A can be solved. It is not natural for other policies for optimizing stochastic matching. The expected cardinality of the matching can be solved as follows.

Algorithm 1 ESD algorithm

Require: A sorted array of E, denoted by $S = s_1, s_2, \ldots, s_m$ and the set of vertices V.

Ensure: A set of disjoint edges A, such that each two agents connected by an edge in A agree to match.

$A = \emptyset$
for do $i = 1, 2, \ldots, m$
  Let v_x and v_y be the two agents connected by s_i
  if then $v_x \in V$ and $v_y \in V$
    Prob v_x and v_y whether they agree to match with each other.
    if B **then**oth v_x and v_y accept to match
      Put s_i into A
      Remove v_x and v_y from V
    end if
  end if
end for
Outputs A as the set of matched edges.

$$E(S) = 2\sum_{i=1}^{m} p * \prod_{j=1}^{i-1} (1-p)^{\delta(s_i, s_j)} \tag{1}$$

$\delta(s_i, s_j)$ is an indicator, which equals to 1 if s_i and s_j are disjoint, otherwise equals to 0.