P8104 Homework Assignment 2

Due Wed 9/24. 11:59pm

Problem 1

Let X be the sum of the faces when rolling two fair six-sided dice.

(a) List all possible values of X and determine the probability mass function (pmf) $p_X(x)$.

(b) Compute:

- $P(X \in \{7, 11\})$
- $P(X \in \{2, 3, 12\})$

(c) Determine the cumulative distribution function (cdf) $F_X(x)$.

Problem 2

Let X be a discrete random variable with pmf:

$$p_X(x) = \begin{cases} cx & x = 1, 2, \ldots, 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) What's the support of X .

(b) Find the constant c.

(c) Compute $P(2 < X \leq 4)$.

(d) Sketch the cdf $F_X(x)$.

Problem 3

Let X be the number of tails before the first head in independent flips of a coin with head probability p.

(a) Write the pmf of X and justify it.

(b) Compute $P(X \text{ is even})$.

(c) What is the support of X?

Problem 4

Let $X \sim \text{Uniform}(0, 1)$, i.e., a real number chosen at random between 0 and 1.

(a) Write the cdf $F_X(x)$ and pdf $f_X(x)$.

(b) Verify that $f_X(x) = \frac{d}{dx} F_X(x)$.

(c) Let $A = (0.3, 0.7]$. Compute $P(X \in A)$ using both the cdf and the pdf.

(d) State whether X is discrete, continuous, or neither, and explain why.

Problem 5

Let $X \sim \text{Binomial}(n = 4, p)$, i.e., the number of heads in 4 independent coin flips where the probability of heads is p. Define a new random variable: $Y = |X - 2|$.

(a) List all possible values of X and write out its pmf $p_X(x)$.

(b) List all possible values of Y and determine its support.

(c) Express each y value in terms of the corresponding x values from the binomial distribution.

(d) Is the transformation one-to-one? Derive the pmf of Y.

(e) Verify that $\sum_y p_Y(y) = 1$.