P8104 Homework Assignment 9

Due Wed 11/19. 11:59pm

Problem 1

Cast a fair die and let $X = 0$ if 1,2, or 3 spots appear, let $X = 1$ if 4 or 5 spots appear, and let $X = 2$ if 6 spots appear. Do this two independent times, obtaining X_1 and X_2. Calculate $P(|X_1 - X_2| = 1)$.

Problem 2

Let X and Y be independent standard normal random variable

(a) Show that $\frac{X}{X+Y}$ has a Cauchy distribution.

(b) Find the pdf of $\frac{X}{|Y|}$ and identify what distribution it is.
Hint: Find the joint pdf of $U = \frac{X}{|Y|}$ and $V = |Y|$ first.

Problem 3

X_1 and X_2 are independent random variables following $N(0, \sigma^2)$.

(a) Find the joint distribution of $Y_1 = X_1^2 + X_2^2$ and $Y_2 = \frac{X_1}{\sqrt{Y_1}}$.

(b) Show that Y_1 and Y_2 are independent, and interpret this result geometrically.

Problem 4

A point is generated at random in the plane according to the following polar scheme: A radius R is chosen, where $R^2 \sim \chi^2(2)$. Independently, an angle θ is chosen, where $\theta \sim Unifrom(0, 2\pi)$. Find the joint distribution of $X = Rcos\theta$ and $Y = Rsin\theta$.

Problem 5

Let (X_1, X_2, X_3) have joint pdf

$$f_{X_1,X_2,X_3}(x_1, x_2, x_3) = \begin{cases} 48\, x_1 x_2 x_3, & 0 < x_1 < x_2 < x_3 < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Define the transformation

$$Y_1 = \frac{X_1}{X_2}, \qquad Y_2 = \frac{X_2}{X_3}, \qquad Y_3 = X_3.$$

(a) Find the joint pdf of Y_1, Y_2, and Y_3.

(b) Are Y_1, Y_2, and Y_3 mutually independent?