

P8104 Homework Assignment 7

Due Wed 11/5. 11:59pm

Problem 1

Suppose we toss a pair of fair, four-sided dice, in which one of the dice is RED and the other is BLACK. Let

- X : the outcome on the RED die = $\{1, 2, 3, 4\}$
 - Y : the outcome on the BLACK die = $\{1, 2, 3, 4\}$
- (a) What is the probability that X takes on a particular value x , and Y takes on a particular value y , i.e., what is $P(X = x, Y = y)$?
- (b) What is the expectation of X ?

Problem 2

A pdf is defined by

$$f(x) = \begin{cases} c(x + 2y), & 0 < x < 2 \quad \text{and} \quad 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Calculate c .
- (b) Find the marginal distribution of X .
- (c) Find the joint cdf of X and Y .

Problem 3

Let the joint pdf of X, Y be given by

$$f(x, y) = \begin{cases} \frac{2}{81}x^2y, & 0 \leq x \leq c, \quad 0 \leq y \leq c \\ 0, & \text{otherwise.} \end{cases}$$

- (a) What value of c makes $f_{XY}(x, y)$ a valid joint pdf?
- (b) Find $P(X > 3Y)$.

Problem 4

Let the joint pdf of X, Y be given by

$$f(x, y) \begin{cases} 6x_1x_2, & 0 \leq x_1 \leq x_2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Let $Y_1 = 2X_1^2X_2^3 + 3X_2$. Find $E(Y_1)$.

(b) Let $Y_2 = X_1/X_2^2$. Find $E(Y_2)$.

Problem 5

Let the joint pdf of X, Y be given by

$$f(x, y) \begin{cases} cxy, & 0 \leq x \leq 1, \quad 0 \leq y \leq 1, \quad 0 \leq x + y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Calculate c .

(b) Obtain the conditional pdf of X given $Y = y$.

(c) Calculate $E(XY)$.

(d) Find the joint cdf $F_{X,Y}(x, y)$.