# P8104 Homework Assignment 5

Due Wed 10/22. 11:59pm

## Problem 1

Let the number of chocolate chips in a certain type of cookie have a Poisson distribution. We want the probability that a randomly chosen cookie has at least two chocolate chips to be greater than 0.99. Find the smallest value of the mean of the distribution that ensures this probability.

## Problem 2

A shipment of 1,000 items is delivered to a factory. Suppose 5% of the items are defective. The factory inspects a random sample of 10 items without replacement and records the number of defectives as $X$. If $X \geq 2$, the factory decides to return the entire shipment. What is the probability that the factory will return a shipment?

## Problem 3

Of all customers purchasing automatic garage-door openers, 75% purchase a chain-driven model, and the remaining 25% buy a shaft-driven model. Let $X$ be the number among the next 15 buyers who select the chain-driven model. Assume that the preferences of individual customers are independent of each other.

(a) Calculate the expectation and variance of $X$.

(b) If the store currently has 12 chain-driven and 6 shaft-driven models, what is the probability that the requests of these 15 buyers can all be met from the existing stock?

# Problem 4

You play a game where you toss a fair coin until the first head appears. If the first head shows up on the $i$-th toss, you receive $2^i$ dollars. For example, if the first head is on toss 2, you win $2^2 = 4$ dollars. Denote the reward you can get as $X$.

(a) Find the probability mass function of $X$.

(b) What is the expected value of the reward you can get?

# Problem 5

Suppose $X \sim \text{Binomial}(n, p)$, and its cdf is denoted as $F_X$. Suppose $Y \sim \text{NegativeBinomial}(r, p)$, and its cdf is denoted as $F_Y$. In this problem, $Y$ counts the number of failures before the $r$-th success (not the total number of trials). Show that

$$F_X(r-1) = 1 - F_Y(n-r)$$

*Hint: Start with the definition of each cdf. Then interpret their practical meanings.*