

P8104 Homework Assignment 4

Due Wed 10/8. 11:59pm

Problem 1

Let $X \sim \text{Laplace}(0, 1)$ with pdf:

$$f_X(x) = \frac{1}{2}e^{-|x|}, \quad x \in \mathbb{R}$$

- (a) Show that $f_X(x)$ is a valid pdf.
- (b) Derive the cdf $F_X(x)$.
- (c) Use symmetry to argue what the mean of X are.
- (d) Compute $\text{Var}(X)$.

Problem 2

Let X be a continuous random variable with pdf:

$$f_X(x) = x^2 \left(2x + \frac{3}{2}\right), \quad 0 < x \leq 1$$

Let $Y = \frac{2}{X} + 3$

- (a) Express $\text{Var}(Y)$ in terms of the variance of a function of X .
- (b) Compute $E\left[\frac{1}{X}\right]$ and $E\left[\frac{1}{X^2}\right]$.
- (c) Compute $\text{Var}(Y)$.

Problem 3

Let X be a continuous random variable with probability density function:

$$f(x) = \begin{cases} \lambda^2 x e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda > 0$ is a constant.

- (a) Calculate the mean $E(X)$
- (b) Calculate the variance $\text{Var}(X)$

Problem 4

Let X be the outcome of rolling a fair 4-sided die (values 1, 2, 3, 4).

- (a) Write the moment generating function (mgf) $M_X(t)$.
- (b) Use $M_X(t)$ to compute $E(X)$ and $\text{Var}(X)$.
- (c) Compare your answers with direct calculation using the pmf.

Problem 5

Let X_1, X_2, \dots, X_n be independent and identically distributed exponential random variables with rate λ , i.e., $X_i \sim \text{Exp}(\lambda)$.

- (a) Show that if X and Y are independent, then $M_{X+Y}(t) = M_X(t)M_Y(t)$.
- (b) Define $S_n = \sum_{j=1}^n X_j$. Calculate the mgf of S_n using the result from part (a).
- (c) Show that if $Y = aX + b$, then $M_Y(t) = e^{bt}M_X(at)$.
- (d) Define Y_n as the standardized version of S_n , and calculate the mgf of Y_n using the result from part (c).