Homework 2 - Human Population Genetics

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Q1: ~~$p_1 = 0.3$, $p_2 = 0.2$ $\Rightarrow$ $p_3 = 0.5$~~

$p_1 = 0.3 \Rightarrow q_1 = 0.7$

$p_2 = 0.2$, $q_2 = 0.3 \Rightarrow r_2 = 0.5$

$P(A_1B_1) = p_1 p_2 = 0.06$ $\quad P(A_1B_2) = p_1 q_2 = 0.09$

$P(A_1B_3) = p_1 r_2 = 0.15$ $\quad P(A_2B_1) = q_1 p_2 = 0.14$

$P(A_2B_2) = q_1 q_2 = 0.21$ $\quad P(A_2B_3) = q_1 r_2 = 0.35$

Q2: Let $p = P_A$ and $q = P_B$, we have $P_a = 1-p$, $P_b = 1-q$

$P_AP_B + P_AP_b + P_aP_B + P_aP_b = pq + p(1-q) + (1-p)q + (1-p)(1-q)$

$= pq + (p - pq) + (q - pq) + (1 - p - q + pq) = 1$ Proved.

Q3: (a) $D = g_{AB} - P_AP_B = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$g_{aB} = P_aP_B - D = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$

$g_{Ab} = P_AP_b - D = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$

$g_{ab} = P_aP_b + D = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$

(b) If mating is at random and two loci are on different chromosomes, $\theta = 50\%$, ~~$D_1 = \theta \cdot D = \frac{1}{24}$~~

$D_1 = (1-\theta) \cdot D = \frac{1}{24}$

$g^*_{AB} = P_AP_B + D_1 = \frac{1}{4} + \frac{1}{24} = \frac{7}{24}$

$g^*_{Ab} = P_AP_b - D_1 = \frac{1}{4} - \frac{1}{24} = \frac{5}{24}$

$g^*_{aB} = P_aP_B - D_1 = \frac{1}{4} - \frac{1}{24} = \frac{5}{24}$

$g^*_{ab} = P_aP_b + D_1 = \frac{1}{4} + \frac{1}{24} = \frac{7}{24}$

Q3 (c) $\theta = 30\%$ $\quad D_1 = (1-\theta)\cdot D = 70\% \cdot \frac{1}{12} = \frac{7}{120}$

$$g^*_{AB} = p_A p_B + D_1 = \frac{1}{4} + \frac{7}{120} = \frac{37}{120}$$
$$g^*_{Ab} = p_A p_b - D_1 = \frac{1}{4} - \frac{7}{120} = \frac{23}{120}$$
$$g^*_{aB} = p_a p_B - D_1 = \frac{1}{4} - \frac{7}{120} = \frac{23}{120}$$
$$g^*_{ab} = p_a p_b + D_1 = \frac{1}{4} + \frac{7}{120} = \frac{37}{120}$$

(d) When $\theta > 0$, $\lim_{n\to\infty} D_n = 0$

$$g^*_{AB} = g^*_{Ab} = g^*_{aB} = g^*_{ab} = \frac{1}{4}$$

Q4 (a) $\quad a = \frac{14-6}{2} = 4 \quad d = 12 - \frac{14+6}{2} = 2$

(i) $q = 0.1 \quad p = 0.9 \Rightarrow \alpha = a + d(q-p) = 2.4$

effect of A_1: $\alpha_1 = q\alpha = 0.24$

effect of A_2: $\alpha_2 = -p\alpha = -2.16$

ii) $q = 0.4 \quad p = 0.6 \Rightarrow \alpha = a + d(q-p) = 3.6$

effect of A_1: $\alpha_1 = q\alpha = 1.44$

effect of A_2 = $\alpha_2 = -p\alpha = -2.16$

(b) (i) $q = 0.1 \Rightarrow p = 0.9$ $\quad$ (ii) $q = 0.4 \Rightarrow p = 0.6$

Breeding value:

	(i)	(ii)
Breeding value:	$A_{11} = \alpha_1 + \alpha_1 = 0.48$	$A_{11} = \alpha_1 + \alpha_1 = 2.88$
	$A_{12} = \alpha_1 + \alpha_2 = -1.92$	$A_{12} = \alpha_1 + \alpha_2 = -0.72$
	$A_{22} = \alpha_2 + \alpha_2 = -4.32$	$A_{22} = \alpha_2 + \alpha_2 = -4.32$
(c) Dominance deviations:	$D_{11} = -2q^2 d = -0.04$	$D_{11} = -2q^2 d = -0.64$
	$D_{12} = 2pqd = 0.36$	$D_{12} = 2pqd = 0.96$
	$D_{22} = -2p^2 d = -3.24$	$D_{22} = -2p^2 d = -1.44$

Q5. Since first cousins share ~~a~~ a pair of grandparents, two of the first cousin will get 1 allel each from 4 allels of their 2 ~~shared~~ shared grandparents.

So the chance 2 first cousin share one allel is $\frac{1}{4}$.

~~A~~ And, they ~~share~~ can't share 2 allels.

In all, $r_0 = \frac{3}{4}$, $r_1 = \frac{1}{4}$, $r_2 = 0$.

$$\mathrm{cov}(X_p, Y_p) = \frac{r_1}{2} V_A + r_2 (V_A + V_D) = \frac{1}{8} V_A$$

$$\Rightarrow h^2 = \frac{V_A}{V_P} = \frac{8\,\mathrm{Cov}(X_p, Y_p)}{\mathrm{Cov}(X_p, Y_p)/b} = 8b \text{. proved.}$$