CS 7641 CSE/ISYE 6740 Homework 4 Report

GTID:903070716
Liu Yujia

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Programming

In this problem, you will implement algorithm to analyze the behavior of *SP500* index over a period of time. For each week, we measure the price movement relative to the previous week and denote it using a binary variable (+1 indicates up and 1 indicates down). The price movements from week 1 (the week of January 5) to week 39 (the week of September 28) are plotted below.

Consider a Hidden Markov Model in which x_t denotes the economic state (good or bad) of week t and y_t denotes the price movement (up or down) of the *SP500* index. We assume that $x_{(t+1)} = x_t$ with probability 0.8, and $P_{(Y_t|X_t)}(y_t = +1|x_t = \text{good}) = P_{(Y_t|X_t)}(y_t = -1|x_t = \text{bad}) = q$. In addition, assume that $P_{(X_1)}(x_1 = \text{bad}) = 0.8$. Load the `sp500.mat`, implement the algorithm, briefly describe how you implement this and report the following :

Report

We use the forward-backward algorithm to solve the problem.
We denote $\alpha_i(t) = P(O_1, \ldots, O_t, S_t = i|\theta)$, then we have

$$\alpha_i(1) = \pi_i b_i(O_1)$$

$$\alpha_j(t+1) = b_j(O_{t+1}) \sum_{i=1}^{N} \alpha_i(t)\alpha_{ij}$$

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Then we denote $\beta_i(t) = P(O_{t+1}, \ldots, O_T|S_t = i, \theta)$, then we have

$$\beta_i(T) = 1$$

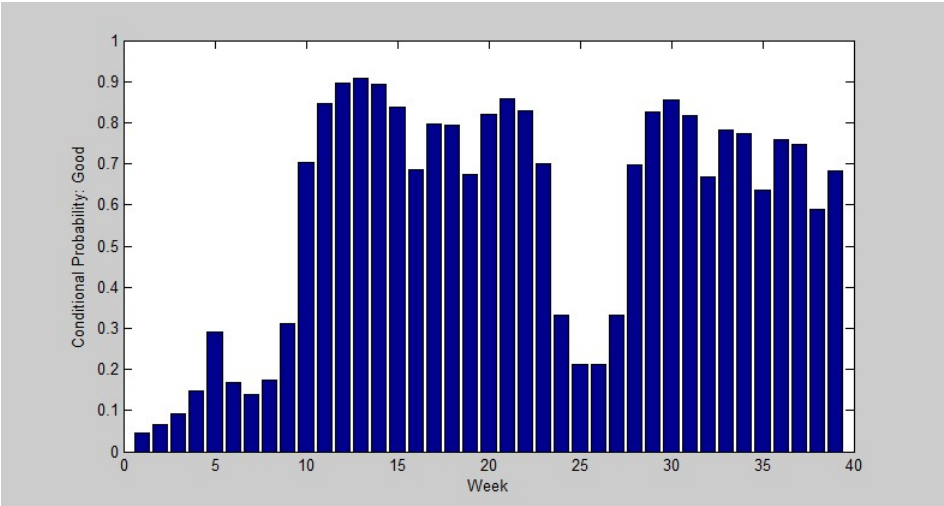
$$\beta_i(t) = \sum_{j=1}^{N} \beta_j(t+1)a_{ij}b_j(O_{t+1})$$

Finally, the conditional probability can be calculated as the normalized product of α and β.

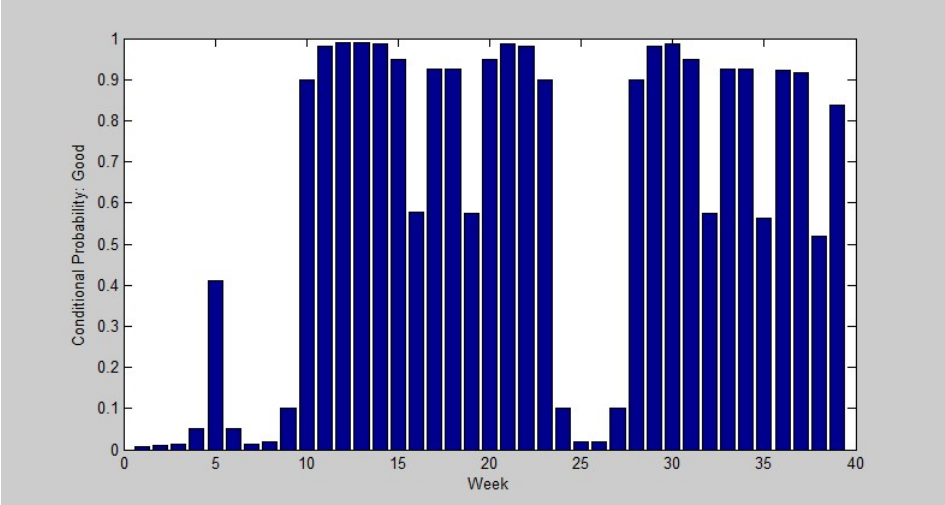
$$\gamma_i(t) = P(S_t = i|O, \theta) = \frac{\alpha_i(t)\beta_i(t)}{\sum_{j=1}^{N} \alpha_j(t)\beta_j(t)}$$

(a) Assuming $q = 0.7$, plot $P_{(X_t|Y)}(x_t = \textbf{good}|y)$ for $t = 1, 2, ..., 39$. What is the probability that the economy is in a good state in the week of week 39. [15 pts]

The $P(X_t|Y)(x_t = \text{good}|y)$ for $q = 0.7$ is plotted below. And the probability that the economy is in a good state in the week of week 39 is 0.6830.



(b) Repeat (a) for $q = 0.9$, and compare the result to that of (a). Explain your comparison in one or two sentences. [15 pts]



The $P(X_t|Y)(x_t = \text{good}|y)$ for $q = 0.9$ is plotted above. And the probability that the economy is in a good state in the week of week 39 is 0.8379.

When q increases from 0.7 to 0.9, which means that the probability of a price up given a good economy state or a price down given a bad economy state is increased. In other words, the economy state and the price are more correlated. Therefore, the probability tends to stay around at two ends on the y axis.