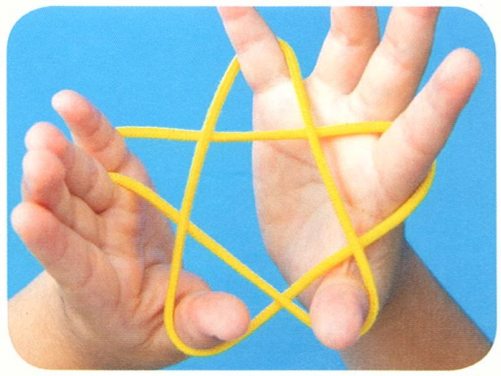
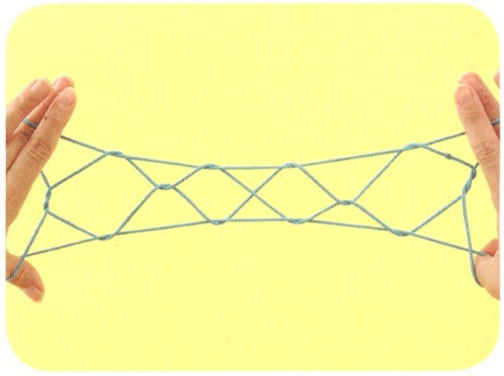
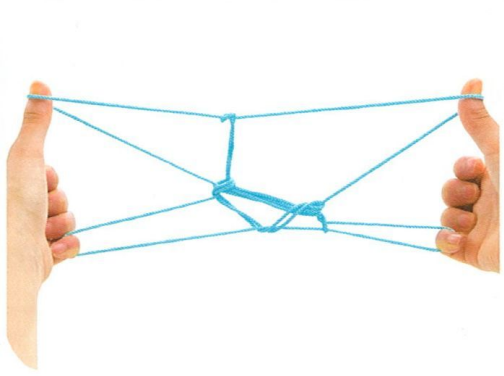
A Computational Approach to String Figures

Yulong Liu

2023-11-29

String Figures

- Designs formed from a loop of string
- Commonly known as a children's game

People have also been playing with the string throughout history.

- Entertainment during polar nights in the Arctic region
- Storytelling and illustrating scenes from myths and legends

---

Noguchi, T. (2020). Ayatori Daizenshu. Shufunotomosha.

A Computational Approach

To make a string figure:

- Start with an initial position (opening)
- Apply a sequence of moves
- Each move transforms a string figure to another

String figures computations

- Represent string figures: simple, precise
- Apply moves directly to the representations

Motivation

- Precise language of describing string figures
- Computer simulations & animations

Representation: Diagrams

Fingers are named $L1, \cdots, L5$ and $R1, \cdots, R5$ from thumb to pinky

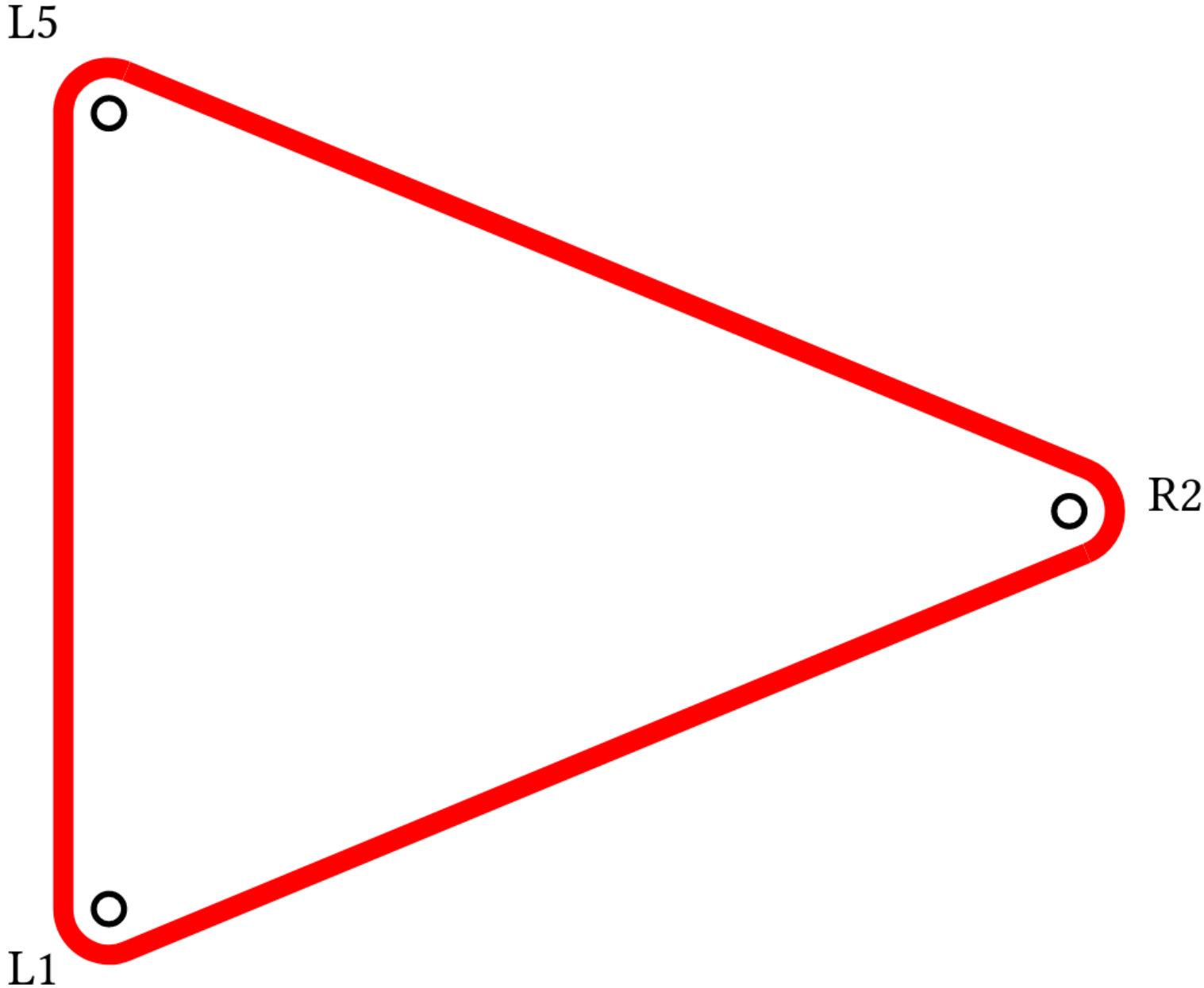
- Ordered from nearest to furthest

$$L1 < L2 < L3 < L4 < L5$$
$$R1 < R2 < R3 < R4 < R5$$

String segments are named by finger $F \in \{L1, \cdots, L5, R1, \cdots, R5\}$

- Fn is the near string, Ff is the far string
- Lp and Rp are palmar strings

Representation: Diagrams



Representation: Linear Sequences

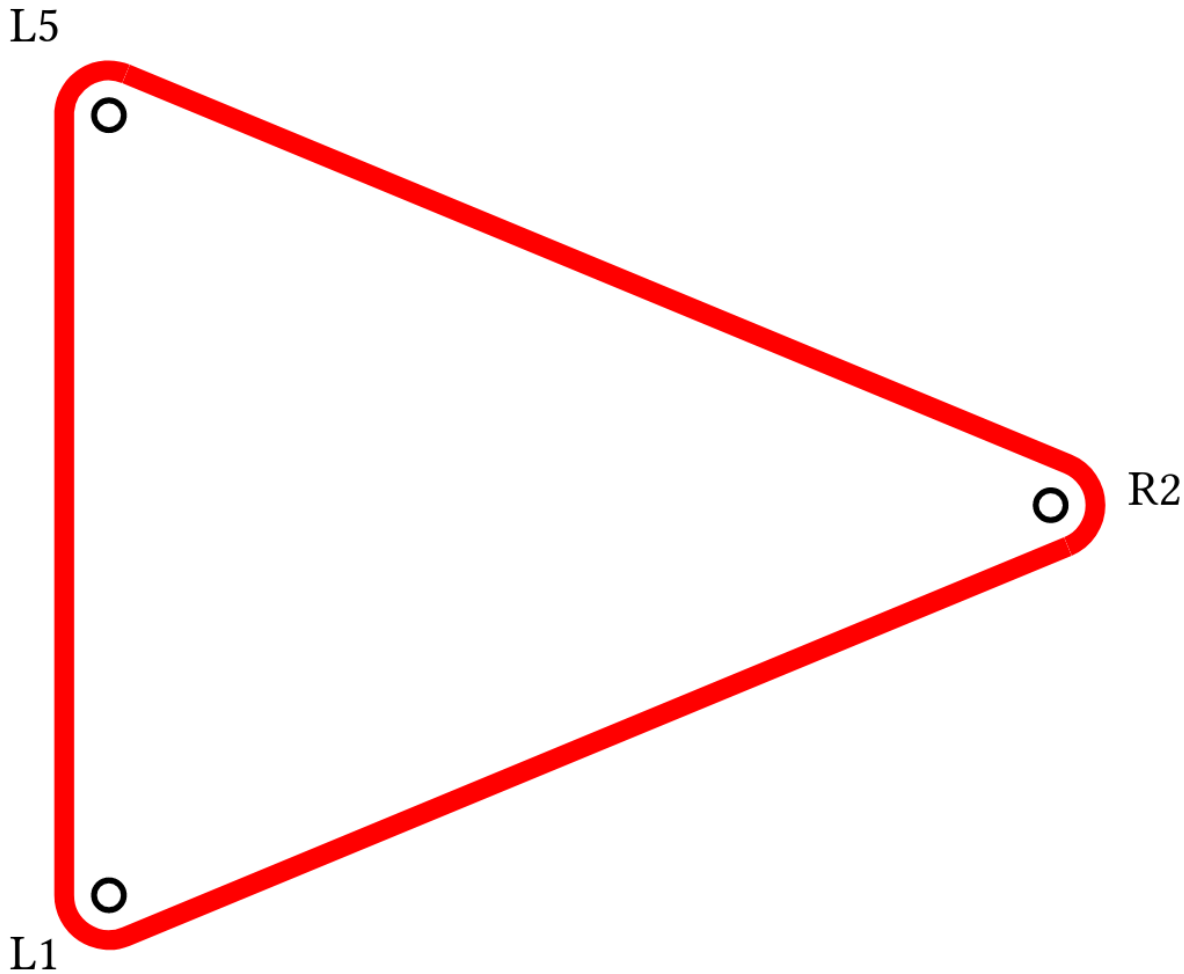
Two components

- Fingers that hold the string
- Crossings between two segments

Diagram $\longrightarrow$ linear sequence

- Start with left nearest finger and travel clockwise
- Visit fingers and crossings

Representation: Linear Sequences



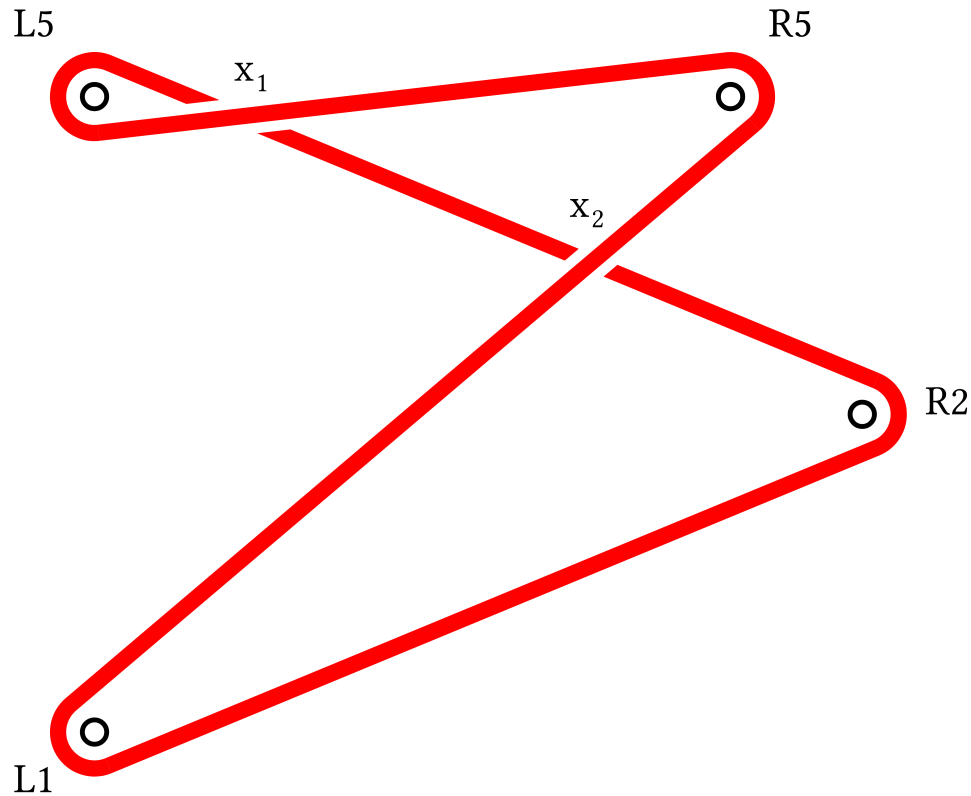
$L1 : L5 : R2$

Linear Sequences with Crossings

Diagram $\longrightarrow$ linear sequence

- Name each crossing as x_i for some i
- Visit overcrossing $\implies$ write $x_i(o)$
- Visit undercrossing $\implies$ write $x_i(u)$

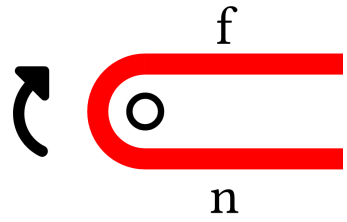
Linear Sequences with Crossings



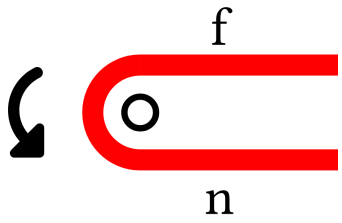
$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

Identifying String Segments from Linear Sequences

Consider a left-hand finger L_i in the sequence

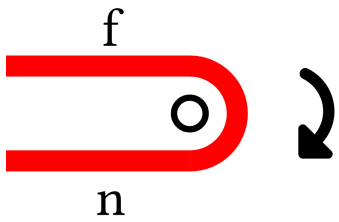
- Traverse clockwise 

$\Longrightarrow \ldots : [n]\, L_i\, [f] : \ldots$

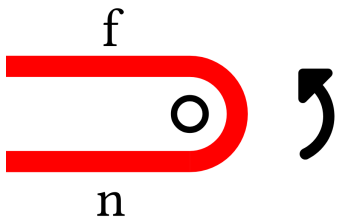
- Traverse counterclockwise 

$\Longrightarrow \ldots : [f]\, L_i\, [n] : \ldots$

Similarly for finger R_i on the right hand

- 

$\Longrightarrow \ldots : [f]\, R_i\, [n] : \ldots$

- 

$\Longrightarrow \ldots : [n]\, R_i\, [f] : \ldots$

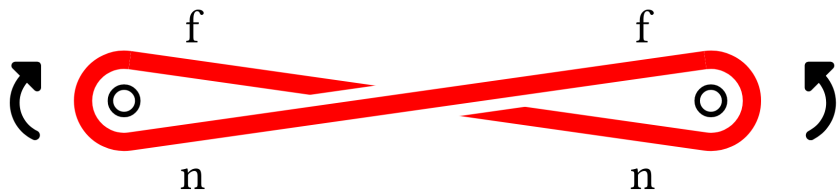
Identifying String Segments : Opposite Hand

Consider $\ldots : L_i : \ldots : R_j : \ldots$

- Even number of crossings between L_i and $R_j \implies$ orientation persists

$$[n]L_i[f] : [f]R_j[n]$$

- Odd number of crossings between L_i and $R_j \implies$ orientation reverses

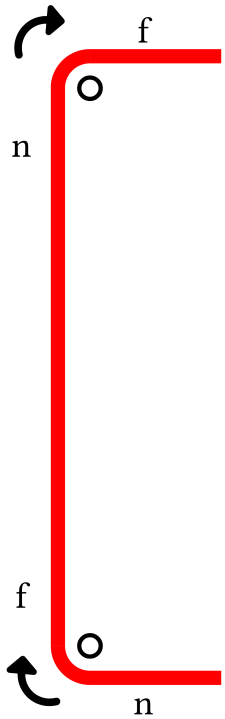


$$[n]L_i[f] : x_1(u) : [n]R_j[f] : x_1(o)$$

Identifying String Segments : Same Hand

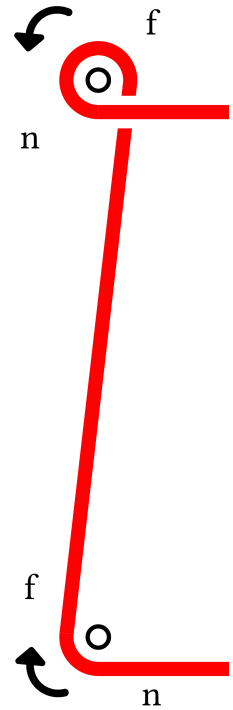
Consider $\ldots : L_i : \ldots : L_j : \ldots$

Even $\Longrightarrow$ orientation persists



$\ldots:[n]L_i[f]:[n]L_j[f]:\ldots$

Odd $\Longrightarrow$ orientation reverses



$\ldots:[n]L_i[f]:x_1(u):[f]L_j[n]:x_1(o):\ldots$

Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

By convention, the first finger in the linear sequence is clockwise

$$L1\text{: } x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

By convention, the first finger in the linear sequence is clockwise

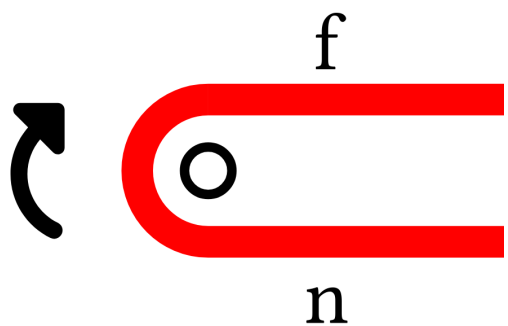
$$\overset{\curvearrowright}{L1} : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

By convention, the first finger in the linear sequence is clockwise

$$[n] \overset{\curvearrowright}{L1} [f] : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$



Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

By convention, the first finger in the linear sequence is clockwise

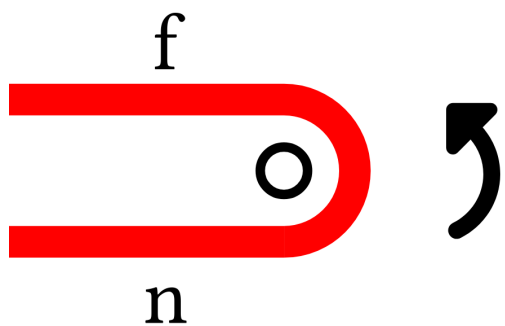
$$[n] \overset{\curvearrowright}{L1} [f] : x_2(o) : \overset{\curvearrowleft}{R5} : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

By convention, the first finger in the linear sequence is clockwise

$$[n] \overset{\curvearrowright}{L1} [f] : x_2(o) : [n] \overset{\curvearrowleft}{R5} [f] : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$



Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

By convention, the first finger in the linear sequence is clockwise

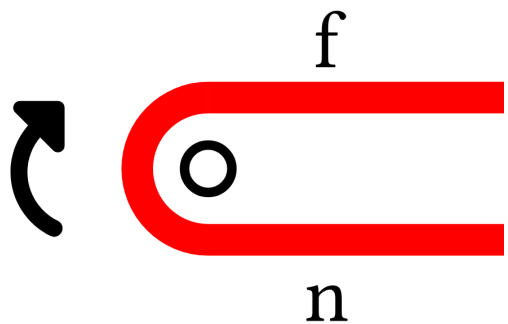
$$[n]L1[f] : x_2(o) : [n]\ \overset{\curvearrowleft}{R5}\ [f] : x_1(o) : \overset{\curvearrowright}{L5} : x_1(u) : x_2(u) : R2$$

Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

By convention, the first finger in the linear sequence is clockwise

$$[n]L1[f] : x_2(o) : [n]\ \overset{\curvearrowleft}{R5}\ [f] : x_1(o) : [n]\ \overset{\curvearrowright}{L5}\ [f] : x_1(u) : x_2(u) : R2$$



Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

By convention, the first finger in the linear sequence is clockwise

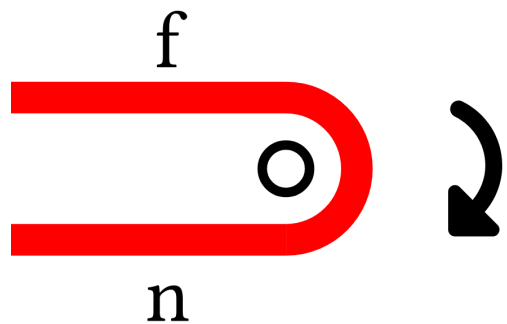
$$[n]L1[f] : x_2(o) : [n]R5[f] : x_1(o) : [n] \overset{\frown}{L5} [f] : x_1(u) : x_2(u) : \overset{\frown}{R2}$$

Identifying String Segments : Example

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

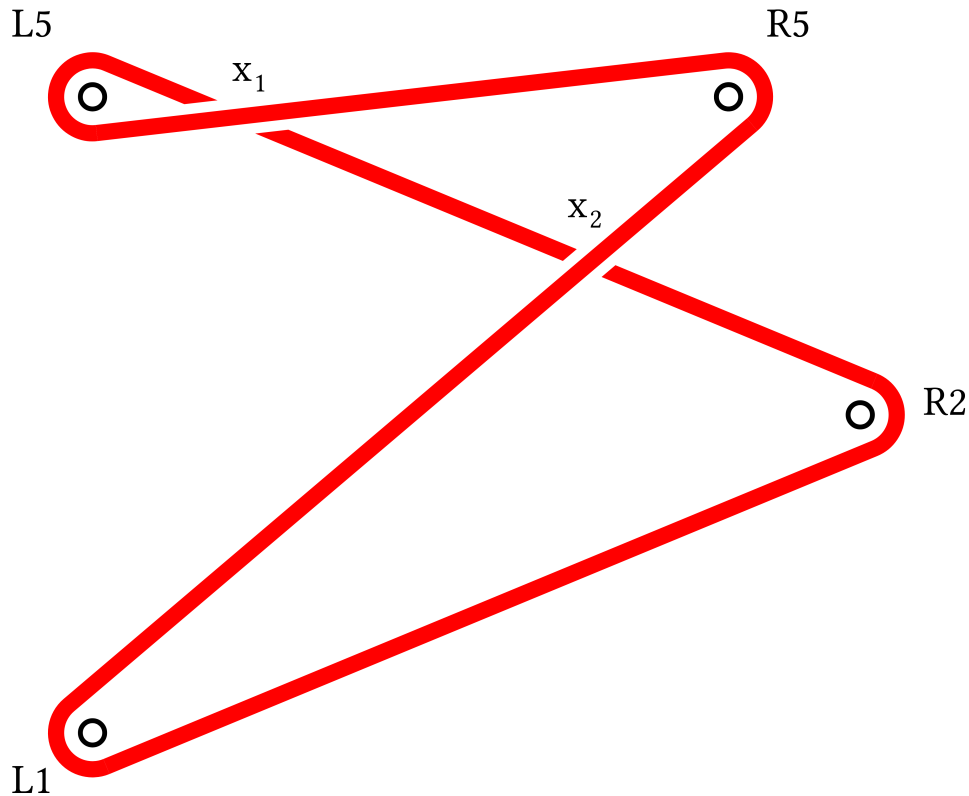
By convention, the first finger in the linear sequence is clockwise

$$[n]L1[f] : x_2(o) : [n]R5[f] : x_1(o) : [n]\ \widetilde{L5}\ [f] : x_1(u) : x_2(u) : [f]\ \widetilde{R2}\ [n]$$



Identifying String Segments : Example

$$[n]L1[f] : x_2(o) : [n]R5[f] : x_1(o) : [n]L5[f] : x_1(u) : x_2(u) : [f]R2[n]$$

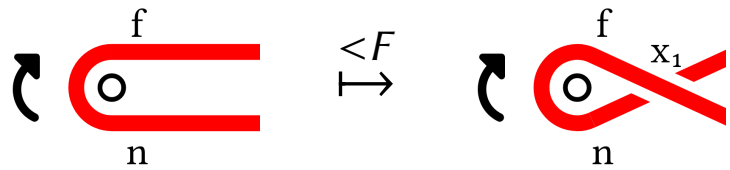


Moves: Twist

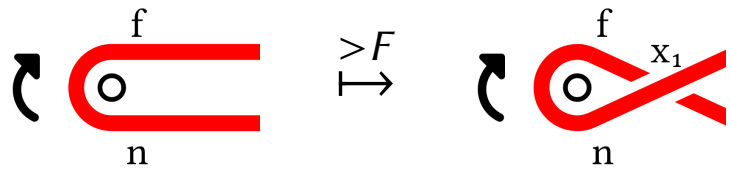
Two variations: twist towards and twist away

- Twist the loop on finger F *towards* player: $< F$
- Twist the loop on finger F *away* from player: $> F$

Consider $\ldots : [n]F[f] : \ldots$

- 

$$\ldots : [n]F[f] : \ldots \overset{<F}{\mapsto} \ldots : x_1(u) : F : x_1(o) : \ldots$$

- 

$$\ldots : [n]F[f] : \ldots \overset{>F}{\mapsto} \ldots : x_1(o) : F : x_1(u) : \ldots$$

- $\ldots : [f]F[n] : \ldots \overset{<F}{\mapsto} \ldots : x(o) : F : x(u) : \ldots$
- $\ldots : [f]F[n] : \ldots \overset{>F}{\mapsto} \ldots : x(u) : F : x(o) : \ldots$

Moves: Pick

Finger F picks a string segment s

- Written as $F(s)$

Four variations:

- F passes *over/under* all intermediate segments
- F picks s from *above/below*

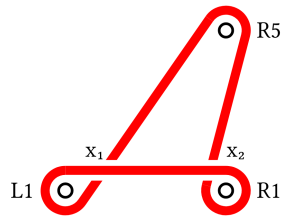
Examples

- "$R5$ passes *over* all intermediate segments and picks Lp from *above*" is denoted as $\overleftarrow{R5}(\overline{Lp})$
- "$R1$ passes *over* all intermediate segments and picks $R5n$ from *below*" is denoted as $\overrightarrow{R1}(\underline{R5n})$
- "$R4$ passes *below* all intermediate segments and picks $L1n$ from *below*" is denoted as $\underleftarrow{R4}(\underline{L1n})$

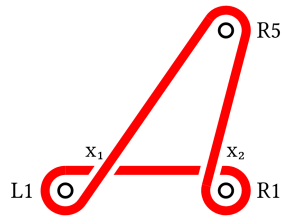
Pick: Examples

Starting with 

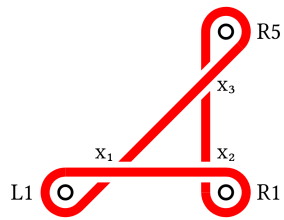
$[n]L1[f] : [f]R1[n]$

$\underleftarrow{R5}(\underline{R1n})$ gives 

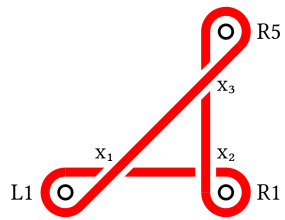
$L1 : x_1(o) : x_2(o) : R1 : x_2(u) : R5 : x_1(u)$

$\overleftarrow{R5}(\underline{R1n})$ gives 

$L1 : x_1(u) : x_2(u) : R1 : x_2(o) : R5 : x_1(o)$

$\underleftarrow{R5}(\overline{R1n})$ gives 

$L1 : x_1(o) : x_2(o) : R1 : x_2(u) : x_3(u) : R5 : x_3(o) : x_1(u)$

$\overleftarrow{R5}(\overline{R1n})$ gives 

$L1 : x_1(u) : x_2(u) : R1 : x_2(o) : x_3(u) : R5 : x_3(o) : x_1(o)$

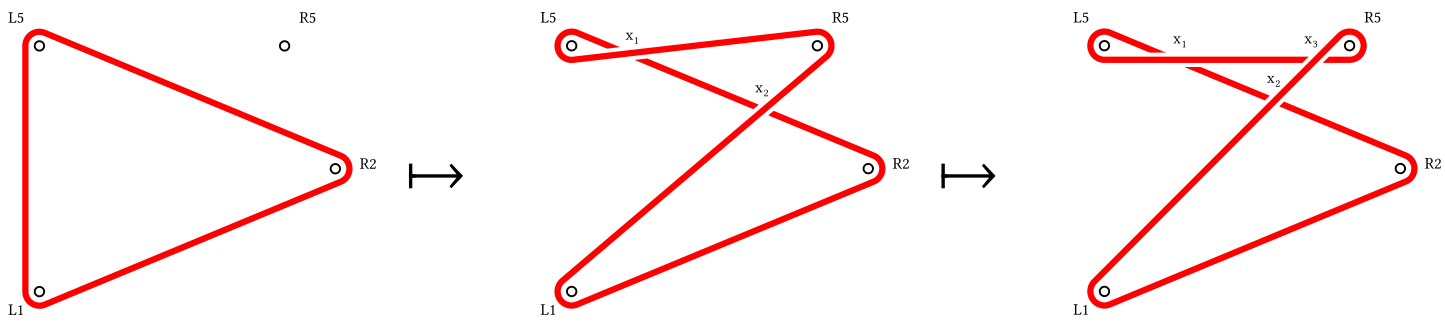
Observations

- A pair of crossings for each intermediate string
- $\overleftarrow{F}(s)$ and $\underleftarrow{F}(s)$ differ by crossing parity
- $F(\overline{s})$ and $F(\underline{s})$ differ by a twist

Pick: Construction

General steps for applying $F(s)$ to a string figure

- Identify intermediate segments
- Insert a pair of crossings for each intermediate segment
- Insert F at s with crossings
- Add twist if pick from above



Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

- Identify Lp

$$[n]L1[f] : [n]L5[f] : [f]R2[n]$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

- Identify Lp

$$[n]\,L1\,[Lp]\,L5\,[f] : [f]\,R2\,[n]$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

- Identify Lp

$$[n]\,L1\,[Lp]\,L5\,[f] : [f]\,R2\,[n]$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

- Identify Lp

$$[n]\,L1\,[Lp]\,L5\,[f] : [f]\,R2\,[n]$$

- Only the segment between $L5$ and $R2$ is intermediate

$$\underline{R2n = L1n} < L1 < Lp = L5n < \underline{L5f = R2f} < R5$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : [f] : [f] : R2$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : x_1(u) : x_2(u) : R2$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : x_1(u) : x_2(u) : R2$$

- Insert $R5$ at Lp with x_1 and x_2

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : x_1(u) : x_2(u) : R2$$

- Insert $R5$ at Lp with x_1 and x_2

$$\overset{\frown}{L1}\ [Lp]\ \overset{\frown}{L5}\colon x_1(u) : x_2(u) : \overset{\frown}{R2}$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : x_1(u) : x_2(u) : R2$$

- Insert $R5$ at Lp with x_1 and x_2

$$\overset{\curvearrowright}{L1}: x_2(o) : \overset{\curvearrowleft}{R5}: x_1(o) : \overset{\curvearrowright}{L5}: x_1(u) : x_2(u) : \overset{\curvearrowright}{R2}$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : x_1(u) : x_2(u) : R2$$

- Insert $R5$ at Lp with x_1 and x_2

$$\overset{\frown}{L1}: x_2(o) : \overset{\curvearrowleft}{R5}: x_1(o) : \overset{\frown}{L5}: x_1(u) : x_2(u) : \overset{\frown}{R2}$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : x_1(u) : x_2(u) : R2$$

- Insert $R5$ at Lp with x_1 and x_2

$$\overset{\curvearrowright}{L1}: x_2(o) : \overset{\curvearrowleft}{R5}: x_1(o) : \overset{\curvearrowright}{L5}: x_1(u) : x_2(u) : \overset{\curvearrowright}{R2}$$

$$L1 : x_2(o) : R5 : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : x_1(u) : x_2(u) : R2$$

- Insert $R5$ at Lp with x_1 and x_2

$$\overset{\curvearrowright}{L1}: x_2(o) : \overset{\curvearrowleft}{R5}: x_1(o) : \overset{\curvearrowright}{L5}: x_1(u) : x_2(u) : \overset{\curvearrowright}{R2}$$

$$L1 : x_2(o) : x_3(o) : R5 : x_3(u) : x_1(o) : L5 : x_1(u) : x_2(u) : R2$$

Pick: Construction Example

$$L1 : L5 : R2 \xmapsto{\overleftarrow{R5}(\overline{Lp})} ???$$

Found $L5f = R2f$ as an intermediate segment

- Insert crossings x_1 and x_2 at intermediate segment

$$L1 : L5 : x_1(u) : x_2(u) : R2$$

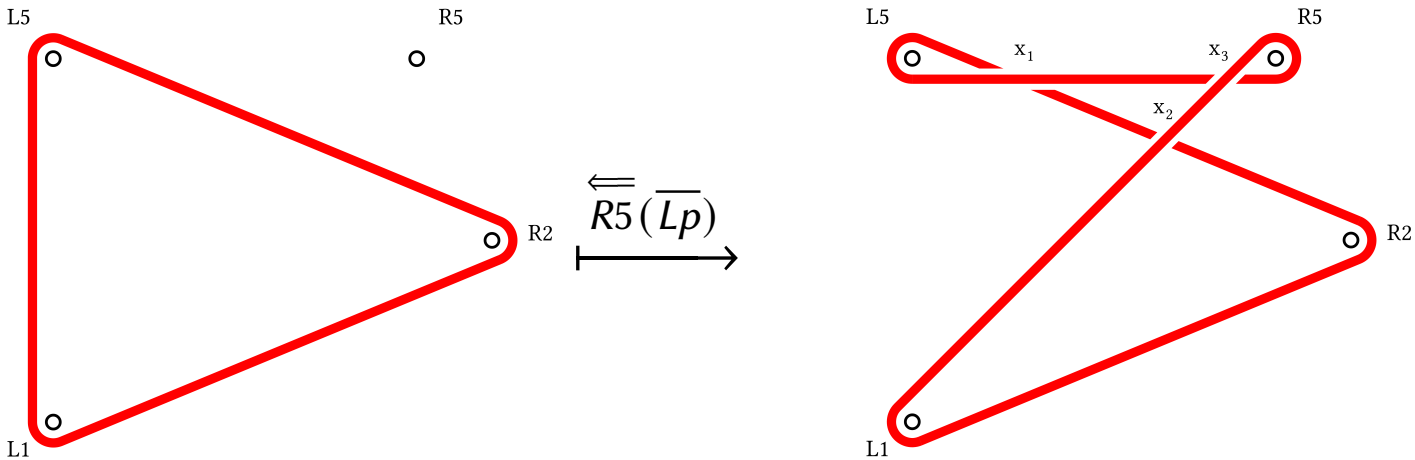
- Insert $R5$ at Lp with x_1 and x_2

$$\overset{\frown}{L1}: x_2(o) : \overset{\curvearrowleft}{R5}: x_1(o) : \overset{\frown}{L5}: x_1(u) : x_2(u) : \overset{\frown}{R2}$$

$$\overset{\frown}{L1}: x_2(o) : x_3(o) : \overset{\frown}{R5}: x_3(u) : x_1(o) : \overset{\frown}{L5}: x_1(u) : x_2(u) : \overset{\frown}{R2}$$

Pick: Construction Example

$$L1{:}L5{:}R2 \xrightarrow{\overleftarrow{R5}\,(\overline{Lp})} \overset{\frown}{L1}{:}x_2(o){:}x_3(o){:}\overset{\frown}{R5}{:}x_3(u){:}x_1(o){:}\overset{\frown}{L5}{:}x_1(u){:}x_2(u){:}\overset{\frown}{R2}$$



Summary

What we covered

- Representing string figures as linear sequences
- Applying twist and pick to linear sequences

Going deeper

- More moves
- Drawing diagrams from linear sequences

Thank you!