

XDSY118019 期末考试第一部分

冯伊然 22302010009

问题描述

1. 给定参数 a, b, u_0, N , 计算坐标数组 $[u_0, \dots, u_N]$
2. 从1.中得到的坐标数组, 绘制散点图
3. 对于从多个参数 a 得到的坐标数组, 以 a 为横坐标、 x 为纵坐标绘制散点图
4. 分析3.中得到的orbit diagram, 寻找使henon_map收敛为周期数列的 a 值

解答思路

将问题拆分成几个模块:

1. henon_map函数
给定参数 a, b , 返回一个二元函数 $f(x, y)$, 它是 a 和 b 对应的henon map函数。
2. 迭代并生成坐标数组 $[(x_0, y_0), \dots, (x_N, y_N)]$ 的函数trajectory()
给定函数 f 、初值 $u_0 = (x_0, y_0)$ 和迭代次数 N 。初始化列表 $point_list = [u_0]$ 。每次计算下一个 (x, y) 并加入列表, 最后返回这个列表。
3. 根据坐标数组绘制散点图的函数plot_trajectory()
用matplotlib.pyplot.scatter()。
值得注意的是, 这里应该把整个数组传递给scatter(), 而不是用一个for循环一个一个点地传递, 那样会导致性能非常低。
4. 使用这些函数完成问题中的工作。具体地:
 - ① 输出轨迹 $[u_0, \dots, u_N]$: 直接打印trajectory()的返回值即可
 - ② 用轨迹绘制散点图: 直接对trajectory()的返回值调用plot_trajectory()
 - ③ 绘制orbit diagram: 取出trajectory()返回值中的 x 部分, 与 a 进行plt.scatter()
 - ④ 寻找使henon_map收敛为周期数列的 a 值:
在③中对不同的 a 值进行尝试, 并且去掉迭代中前一半的点(只关注最终是否进入循环, 前面的点会扰乱我们的分析),
如果观察到图像上只有离散的点, 对应的 a 值即使henon_map收敛为周期数列; 否则, 若图像看起来像是连续的线, 则说明没有成为周期。

结论

我找到了以下使得henon map收敛为周期数列的 a 的值 ($b=0.3, u_0=(0,0)$): $-0.12 \sim 1.05$
 $1.23 \sim 1.25$
 $1.300 \sim 1.304$
 $1.4219 \sim 1.4221$

代码使用方法

使用者只需要关注主函数部分。

要求的4部分中, 每部分均用1个注释开头, 标明代码对应哪部分要求。

第3、4部分展示了以上找到的收敛的henon_map，对应的取值在a_list中。可以修改a_list来观察a取其他值时的orbit diagram。

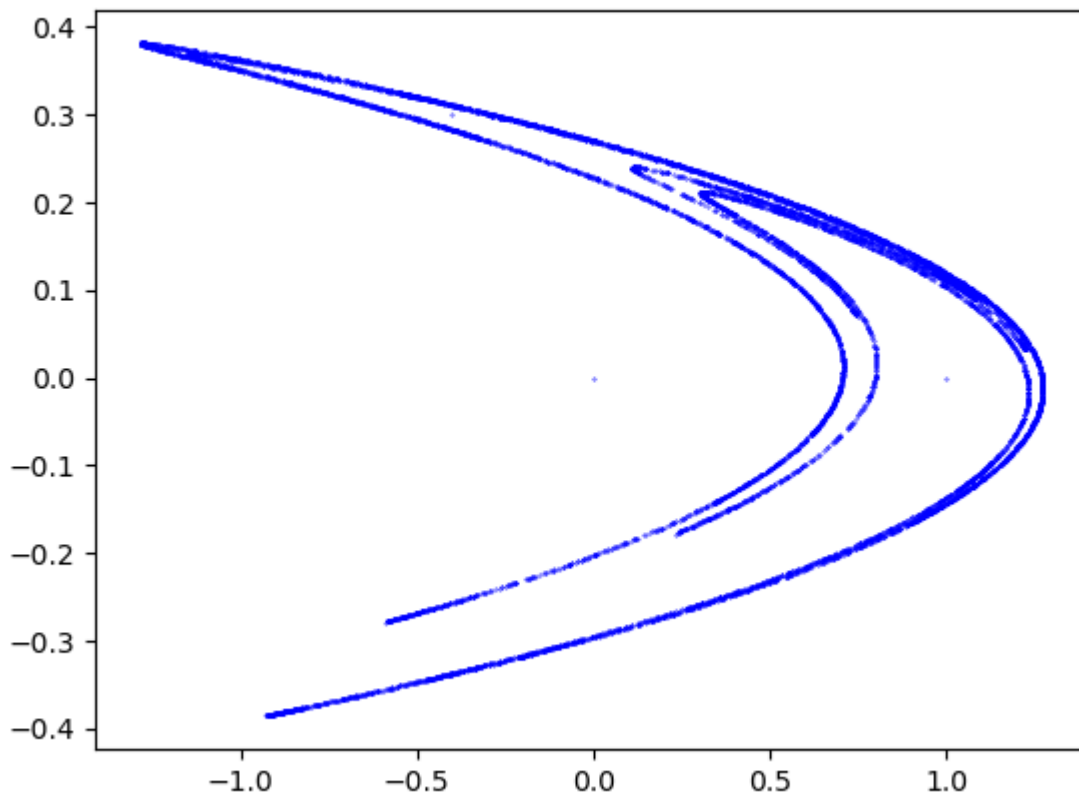
结果图

1. 输出轨迹

这里N=10000, 结果极长，只展示一小部分。

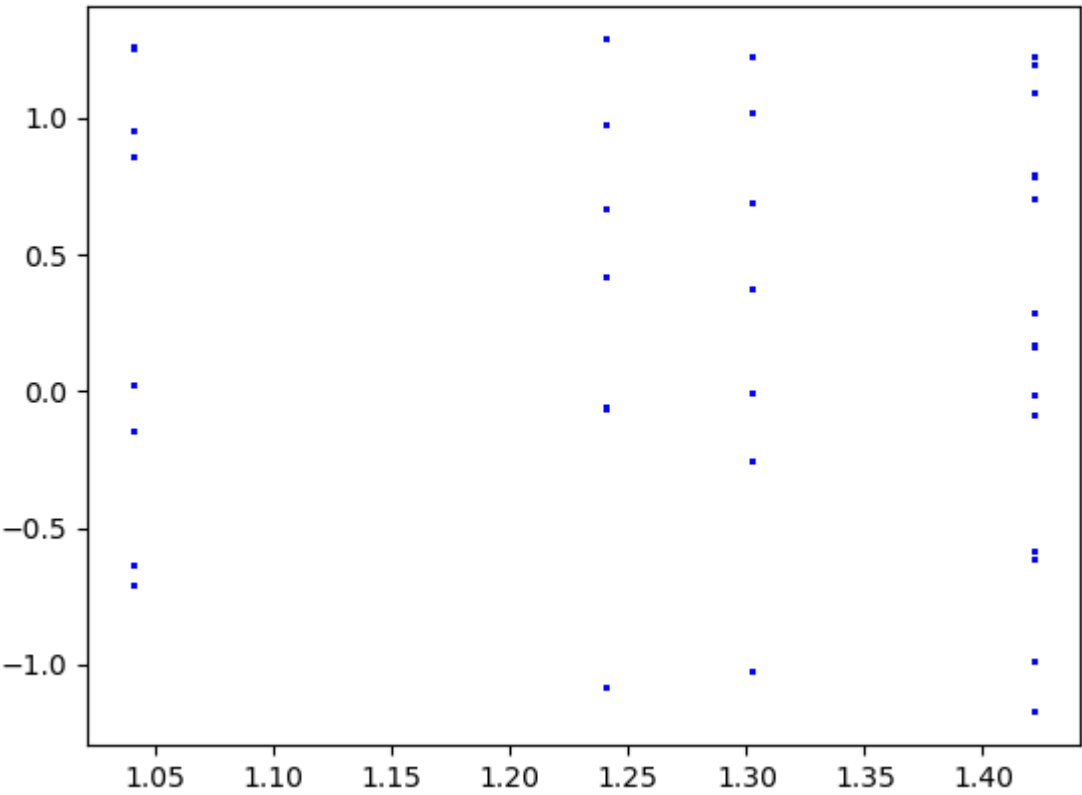
```
15942152023238, -0.33036481791539674), (0.7992261894207722, -0.18478264560697139), (-0.07905014820543849, 0.23976785682623164), (1.2310193605224091, -0.023715044461631547), (-1.1452871768350332, 0.3693058081567227), (-0.46704999623514215, -0.34358615305059993), (0.35102386837294525, -0.14011499887054263), (0.6873801424949479, 0.10530716051188357), (0.4438191160969588, 0.20621404274848437), (0.9304484718101642, 0.13314573482908762), (-0.07888236734233026, 0.27913454154304923), (1.2704231425145067, -0.023664710202699075), (-1.283296555637073, 0.381126942754352), (-0.9242227460544328, -0.3849688966961122), (-0.5808316547502674, -0.27726682381632983), (0.2504216005597628, -0.17424949642508022), (0.737955134337241, 0.07512648016792885), (0.31271758775535524, 0.2213865403011723), (1.084477347330328, 0.09381527632660656), (-0.5527122490429207, 0.32534320041990983), (0.8976560380809928, -0.1658136747128762), (-0.29391458249744695, 0.26929681142429784), (1.1483567168977897, -0.08817437474923408), (-0.9343867836912123, 0.345070150693369), (0.12219688891780522, -0.2803160351073637), (0.6987790533669697, 0.036659066675341564), (0.3530500350811281, 0.2096337160100909), (1.0351316578309908, 0.10591501052433842), (-0.394181581371711, 0.31053949734929726), (1.0930087562636701, -0.11825446744115133), (-0.7907898652178281, 0.32790262687910104), (0.45241457157537807, -0.23723695956534843), (0.4762125180314256, 0.1357243714726134), (0.8182346642108502, 0.14286375540942767), (0.2055526034066877, 0.24547039926325506), (1.1863177773890812, 0.06166578102200631), (-0.9086240355071109, 0.35589533321672434), (0.2000586401550058, -0.27258721065213326), (0.6713799460469285, 0.060017592046501735), (0.42896614731093446, 0.20141398381407855), (0.9437972460597777, 0.12868984419328033), (-0.11836469414474837, 0.2831391738179333), (1.2635248926696614, -0.03550940824342451), (-1.2706026243987167, 0.3790574678009884), (-0.8811459729794804, -0.381180787319615), (-0.46816630329675213, -0.2643437918938441), (0.42880464554659076, -0.14044980909802563), (0.6021273153517022, 0.128641393663977723), (0.6210611682142698, 0.18063819460551064), (0.6406344300763678, 0.18631835046428094), (0.6117408882652994, 0.19219032902291033), (0.6682726488970459, 0.1835222664795898), (0.5582985999101658, 0.20048179466911376), (0.7641055373428017, 0.16748957997304975), (0.3500893988959453, 0.2292316612028405), (1.0576440390957864, 0.10502681966878359), (-0.46102845914000556, 0.3172932117287359), (1.019727075536925, -0.13830853774200166), (-0.5940891697583268, 0.3059181226610775), (0.8117994043872843, -0.17822675092749804), (-0.108852337646721, 0.24353982131618526), (1.229300150994432, -0.030255699922940162), (-1.1459961056518463, 0.36879004529832954), (-0.4695510788599226, -0.34377183169555386), (0.3475866638252203, -0.4086532365797677), (0.6900188391333871, 0.1042675999147566), (0.43769120221217456, 0.20700565174001614), (0.9388026278485021, 0.13130736066365237), (-0.10258316301370193, 0.28164078835455064), (1.2669081608870965, -0.030774948904110578), (-1.2778537522753655, 0.3800724482661289), (-0.9060018488197947, 0.3833561256826096), (-0.5325312157734503, -0.2718005546459384), (0.3311741512716529, -0.1597593647320351), (0.6866937894092654, 0.00935224538149587), (0.4391845488029368, 0.2060081368227796), (0.9359718515903787, 0.13175536224088102), (-0.09470526751644937, 0.2807915554771136), (1.2682348327036064, -0.028411580254934808), (-1.2801990074907768, 0.3804704498110819), (-0.9148028484814359, 0.384095702247233), (-0.553621392092283, -0.27420085454443077), (0.296703841360493, -0.1660864176276849), (0.710671451030135, 0.0800111524081479), (0.3819442448277332, 0.21320014353090405), (1.0808661749110657, 0.11458327344831995), (-0.31063456551221413, 0.302680852473197), (1.1675984858659727, -0.09319036965366424), (-1.001791893528781, 0.3502795457597918), (-0.05473997929308494, 0.38053732505863434), (0.6952676234751554, -0.01642199378792548), (0.3068221106581806, 0.2085802870425466), (1.0767845564183203, 0.00204663219745042), (-0.5312043401199477, 0.32203652660254066), (0.9279860955782193, -0.1593613020590428), (0.3649827730573554, 0.27839582867349577), (1.091894434193404, -0.10949483191720662), (-0.7786338987488166, 0.32750953025802116), (0.47879048266402596, -0.2335001662644496), (0.44547337356863714, 0.1436371447994778), (0.8658120076174063, 0.13364201207059115), (0.0841504052231427, 0.2597436022852186), (1.2408276742965584, 0.02524782195660428), (1.1616490796558672, 0.374948302889675), (0.514251715824851, 0.3484047238967601), (0.2812685181879087, 0.15427551470474551), (0.7349677142421838, 0.0013055545637262), (0.32813198088640824, 0.2204903142726551), (1.0697514628369817, 0.0984399594259921), (-0.5036758697126449, 0.32029543885109753), (0.9657603044279928, -0.15110276991379346), (-0.45687291276618247, 0.28972809132839783), (0.9975020895411598, -0.13706187382985474), (-0.5300764599244268, 0.29925062686234793), (0.9058771521499306, -0.15902293797372803), (-0.307881718679504, 0.27176314564497916), (1.1390553186911158, -0.09236451560385119), (-0.9087910221581389, 0.3417166595607347), (0.18545508882340578, -0.2726373066474417), (0.6792116673938621, 0.05563652664702173), (0.40977664187349133, 0.20376350021815862), (0.9686798455029968, 0.1229329925620474), (-0.19074390775514613, 0.2906039536508099), (1.239667419969138, -0.057223172326543836), (-1.208786093049377, 0.3719002259900741), (-0.6734668771809529, -0.3626125827914813), (0.002406728835564753, -0.20204006313028586), (0.797951827588551, 0.0007220186506694259), (0.10930405183799749, 0.23938554827656527), (1.2226592222990802, 0.032791215551399244), (-1.0600625876311478, 0.36679776666872405), (-0.20642799890461877, -0.31801877628934433), (0.6223236974061843, -0.06192839967138563), (0.39587010209458817, 0.18669710924585528), (0.9672987164205237, 0.11876103062837645), (-0.1911724988759333, 0.2901896149261571), (1.239023920869101, -0.057351749662779984), (-1.206604136742956, 0.371071762607303), (-0.6665437836665697, -0.3619812410228868), (0.01602589721474751, -0.1999631350999709), (0.7996773037658764, 0.004807769164424253), (0.10953046294285783, 0.2399031911297629), (1.2231074998922955, 0.032859138882857346), (-1.061529599270367, 0.36693224996768864), (-0.21065087816206746, -0.31845887978111), (0.6194178105632588, -0.06319526344862024), (0.39965494289120607, 0.18582534316897764), (0.9622116404406551, 0.11989648286736182)]
```

2. 绘制a=1.4, b=0.3, u0=(0, 0)时的轨迹



3. 绘制orbit diagram

注意，这里省略了前一半的点，以免收敛到周期前的点干扰分析。



4. 绘制Henon map收敛到周期轨道时的轨迹

注意，这里没有省略前一半点，而是显示完整轨迹，所以有许多比较淡的点，他们是收敛到周期前的。

