期末考试试题：第二部分

- 课程： 跨入科学研究之门（XDSY118019）
- 该部分试题将于第11次课开始时在Github发布，见https://github.com/liuyxpp/XDSY118019-exam
- 答案提交截止时间: 2025.11.27, 21:05
- 答案提交方式： 以Pull Request的形式（**请在标题备注学号、姓名**）将所有相关材料提交到GitHub repo： https://github.com/liuyxpp/XDSY118019-exam

试题解答要求

1. 提交源代码文件：将所有解答相关的代码（如有）及答案（包括图片）写入一个Markdown（或LaTeX或Typst）文件中并提交。
2. 提交PDF文件：将上述文件渲染为一个PDF文件并提交。

试题（共4题）

1. （30分）Python编程题。请在当前目录下找到源代码文件 `matrix_stats.py` ，并据此回答下列问题：
    i. 修改代码，使函数增加返回'sum'（表示所有元素的和）；
    ii. 请利用该函数，计算如下矩阵的统计量，并写出结果：
        - $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$
        - $B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
    iii. 请设计至少5个测试用例，尽可能包含各种可能得输入情况，比如正常输入、异常输入、边界输入等，并给出测试结果。根据测试结果，分析该函数的功能是否正确。如果找到bug，试着修复它。将测试用例替换 `__main__` 代码块中的 `pass` 。
2. （20分）Matlab作图。请用 `surf` 函数渲染出如下莫比乌斯带（Möbius strip）的三维图像，其中 $R = 3, r = 0.7$:

$$
\begin{aligned}
x &= (R + u\cos\frac{v}{2})\cos v \\
y &= (R + u\cos\frac{v}{2})\sin v \\
z &= u\sin\frac{v}{2}
\end{aligned}
$$

其中$u \in [-r, r],\ v \in [0, 2\pi]$。

3. （20分）利用Mathematica

   i. 求如下无穷级数的和：

$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{\pi}{n}\right)}{n^3}$$

   ii. 求如下定积分的值：

$$\int_0^{\infty} \frac{\sin x}{x\left(e^x + 1\right)} dx$$

4. （30分）用LaTeX或Markdown或Typst写出如下文本内容（要求渲染后的显示效果与如下文本一致）：

Linear Least Squares

Linear least squares (LLS) is the least squares approximation of linear functions to data. It is a set of formulations for solving statistical problems involved in linear regression, including variants for ordinary (unweighted), weighted, and generalized (correlated) residuals. Numerical methods for linear least squares include inverting the matrix of the normal equations and orthogonal decomposition methods.

Basic Formulation

Consider the linear equation

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

where $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{b} \in \mathbb{R}^m$ are given and $x \in \mathbb{R}^n$ is variable to be computed. When $m > n$, it is generally the case that Eq. (1) has no solution. For example, there is no value of x that satisfies

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

because the first two rows require that $x = (1, 1)$, but then the third row is not satisfied. Thus, for $m > n$, the goal of solving Eq. (1) exactly is typically replaced by finding the value of x that minimizes some error. There are many ways that the error can be defined, but one of the most common is to define it as $||\mathbf{Ax} - \mathbf{b}||^2$. This produces a minimization problem, called a least squares problem

$$\text{minimize}_{x \in \mathbb{R}^n} ||\mathbf{Ax} - \mathbf{b}||^2$$

The solution to the least squares problem is computed by solving the *normal equation*

$$\mathbf{A}^T\mathbf{Ax} = \mathbf{A}^T\mathbf{b}$$

where $\mathbf{A}^T$ denotes the transpose of the matrix $\mathbf{A}$.