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EEL 4750 / EEE 5502 (Fall 2019) – Exam #02

ExamID: 010001

Date: Oct. 28, 2019

Question	# of Points Possible	# of Points Obtained	Grader
# 1	22	~~17~~ 20	IK
# 2	21	21	MS
# 3	19	15.5	JBH
# 4	19	19	Y.B
# 5	19	13	MS
Total	100	~~85.5~~ 88.5	

For full credit when sketching: remember to label axes and make locations and amplitudes clear.

Before starting the exam, read and sign the following agreement.

By signing this agreement, I agree to solve the problems of this exam while adhering to the policies and guidelines of the University of Florida and EEL 4750 / EEE 5502 and without additional external help. The guidelines include, but are not limited to,

- The University of Florida honor pledge: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."
- Only one 8.5 by 11 inch cheat sheet (double-sided) may be used
- No calculators or computers may be used
- No textbooks or additional notes may be used
- No collaboration is allowed
- No cheating is allowed

Zexi Liu

Student

10.28 2019

Date

Full Name: ______________________________ **ExamID: 010001**

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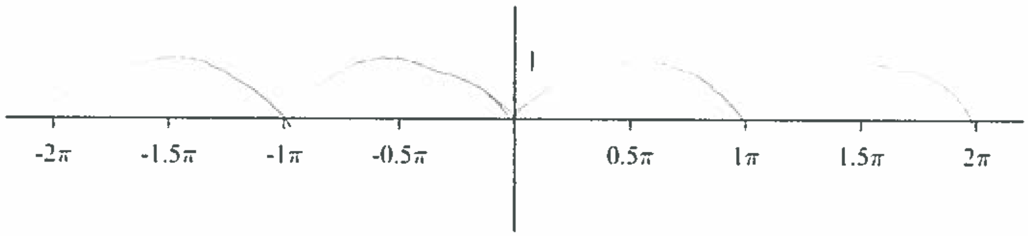
Question #1: Consider the following frequency response $H(\omega)$ and impulse response $g[n]$

$$H(\omega) = \sin(\omega)\, e^{-j(\omega/2+\pi/2)}$$

(a) *(8 pts)* Sketch the magnitude response of $H(\omega)$ (i.e., $|H(\omega)|$).

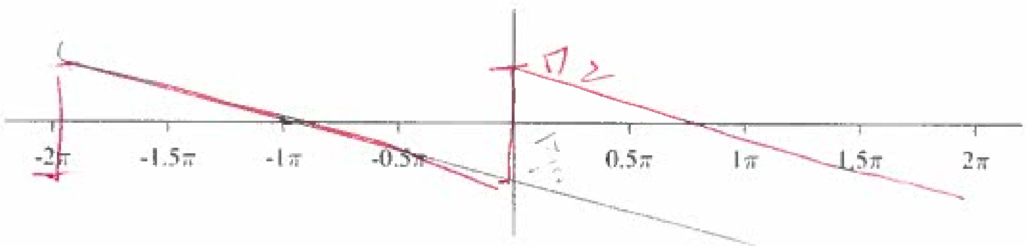


$H(w) = \sin(w)\left[\cos\left(\frac{w}{2}+\frac{\pi}{2}\right) - j\sin\left(\frac{w}{2}+\frac{\pi}{2}\right)\right]$

$= \sin(w)\left[-\sin\left(\frac{w}{2}\right) - j\cos\left(\frac{w}{2}\right)\right]$

$|H(w)| = |\sin w|$

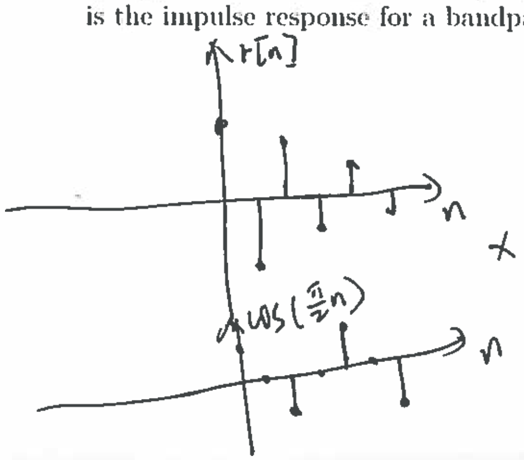
(b) *(8 pts)* Sketch the phase response of $H(\omega)$ (i.e., $\angle H(\omega)$) for $-2\pi < \omega < 2\pi$.

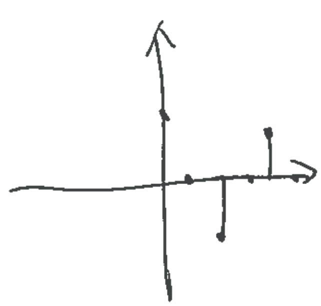


-2

$\angle H(w) = \text{atan}\,\frac{\cos\frac{w}{2}}{\sin\frac{w}{2}} = \text{atan}\,\frac{1}{\tan\frac{w}{2}}$

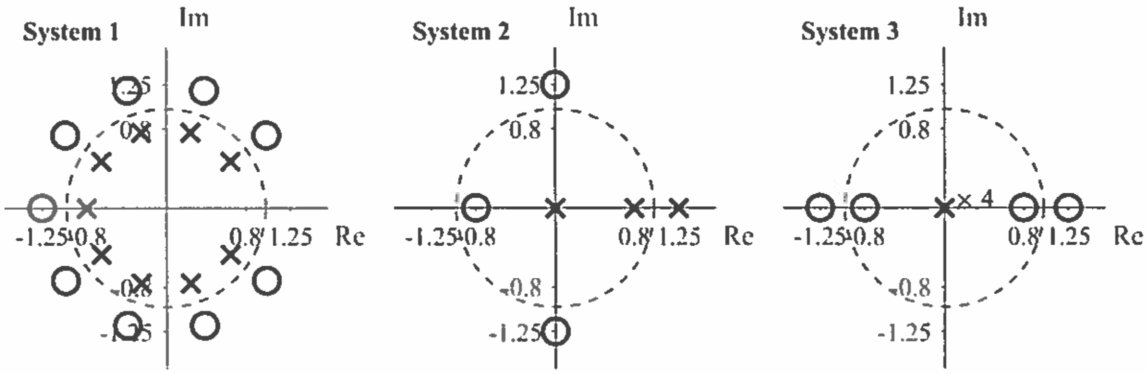
(c) *(6 pts)* **(True or False)** If $r[n]$ is the impulse response for a highpass filter, then $r[n]\cos((\pi/2)n)$ is the impulse response for a bandpass filter. **Explain why.**





Yes. it's a bandpass fil
we can see this from
the impulse response of
$r[n]\cos(\frac{\pi}{2}n)$

Question #2: Consider the following pole-zero plots, representing causal LTI systems.



(a) *(4 pts)* **For each system:** Would you describe the system as a low pass filter, bandpass filter, high pass filter, all-pass filter, or none-of-the-above?

System 1 : all-pass filter.
System 2 : low-pass filter.
System 3 : band-pass filter.

(b) *(4 pts)* **For each system:** Is the system stable?

1: Stable
2: not stable
3: stable

(c) *(4 pts)* **For each system:** Is the system an FIR or IIR filter?

1: IIR filter.
2: IIR filter
3: FIR filter.

(d) *(4 pts)* **For each system:** Is the filter linear phase?

1: not linear phase
2: not linear phase.
3: linear phase.

(e) *(5 pts)* Write the z-domain transfer function $H(z)$ for **System 2**. Assume a gain of 1.

$$H(z) = \frac{(z+0.8)(z-1.25j)(z+1.25j)}{z(z-0.8)(z-1.25)}$$

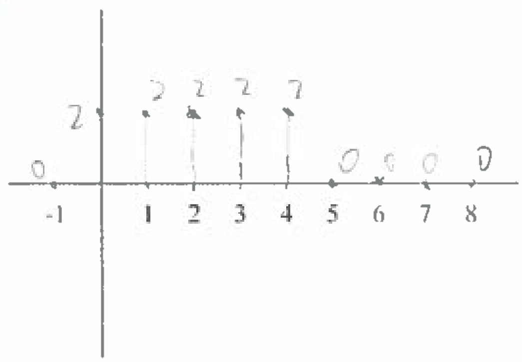
Question #3: Consider the desired frequency response

$$H_{da}(\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi k) \quad , \quad H_{db}(\omega) = \frac{1}{2 + \cos((5/2)\omega - \pi)}$$

(a) *(10 pts)* Approximate $H_{da}(\omega)$ with a length $N = 5$ windowing method. Use a rectangular window. **Force the resulting filter to be causal.** Sketch the time-domain filter coefficients $h_a[n]$ (for $-1 \leq n \leq 8$) with these requirements.

$$h_a[n] = 2\sum_{k=-\infty}^{\infty} \delta[n-2k]$$

$$w[n] = u[n] - u[n-5].$$



-3 5

(b) *(9 pts)* Approximate $H_{db}(\omega)$ with a length $N =$ 5 frequency sampling method. **Force the resulting filter to be causal and linear phase.** Compute the time-domain filter coefficients $h_b[n]$ with these requirements.

$N = 5.$

$$h[n] = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{(N-1)/2} H[k] \cos\left(\frac{2\pi}{N}\left(n - \frac{N-1}{2}\right)k\right)\right]$$

$$w = \frac{2\pi k}{N} = \frac{2\pi k}{5}$$

$k=0.\ H[0] = 1.$

$k=1\ H[1] = \frac{1}{3}$

$k=2\ H[2] = 1$

$$h[n] = \frac{1}{5}\left[1 + 2\cdot\frac{1}{3}\cos\left(\frac{2\pi}{5}(n-2)\right) + 2\cdot 1\cdot\cos\left(\frac{4\pi}{5}(n-2\right.\right.$$

$$= \frac{1}{5}\left[1 + \frac{2}{3}\cos\left(\frac{2\pi}{5}(n-2)\right) + 2\cos\left(\frac{4\pi}{5}(n-2)\right)\right]$$

$e^{\ln x} = x$. $e^{-\ln x} = x^{-1}$

Question #4: Consider the desired filter transfer function

$$H_{da}(s) = 4s^2 \quad , \quad H_{db}(s) = \frac{2}{s} + \frac{1}{s + \ln(2)}$$

(a) *(9 pts)* Approximate $H_{da}(s)$ as a discrete-time filter with the approximation of the differential operator with sampling period $T_s = 2$. **Force the filter to be causal.** Determine the resulting impulse response $h_a[n]$. **Also,** is this a low pass, high pass, band pass, or all-pass filter?

$$s = \frac{1}{T_s}(1 - z^{-1}) = \frac{1}{2}(1 - z^{-1})$$

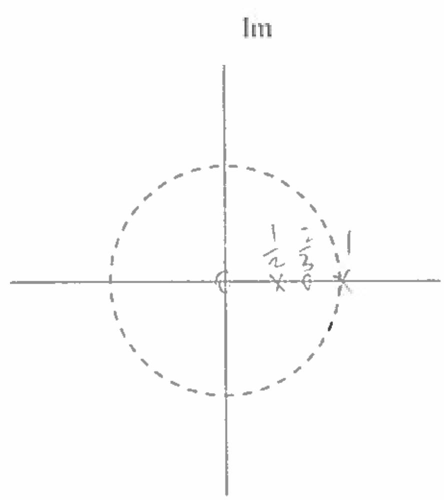
$$H(z) = (1 - z^{-1})^2$$

$$= 1 - 2z^{-1} + z^{-2}.$$

$$h_a[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$$

It is an high pass filter.

(b) *(10 pts)* Approximate $H_{db}(s)$ as a discrete-time IIR filter with the impulse invariance method and a sampling period $T_s = 1$. Determine the resulting pole-zero plot for the filter.



$$H(s) = \frac{A}{s - \alpha}.$$

$$H(z) = \frac{2}{1 - z^{-1}} + \frac{1}{1 - (e^{-\ln 2})^{1} z^{-1}}$$

$$= \frac{2}{1 - z^{-1}} + \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$= \frac{2 - z^{-1} + 1 - z^{-1}}{(1 - z^{-1})(1 - \frac{1}{2}z^{-1})}$$

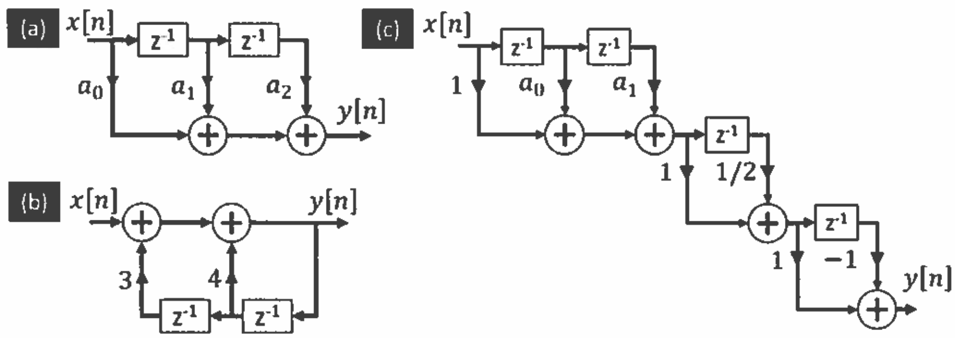
$$= \frac{3 - 2z^{-1}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$$

$$= \frac{3z^2 - 2z}{z^2 - \frac{3}{2}z + \frac{1}{2}}.$$

zeros: $z = 0, \frac{2}{3}$

poles: $z = 1, \frac{1}{2}$

Question #5: Consider the IIR all-pole form (left), FIR cascade form (center), and IIR direct form I (right) implementations below.



(a) *(6 pts)* Determine the weights a_0, a_1, a_2 for the FIR direct form in (a) with transfer function

$$H(z) = (1 + 2z^{-1})^2$$

$H(z) = 1 + 4z^{-1} + 4z^{-2}$

$a_0 = 1. \quad a_1 = 4 \quad a_2 = 4$

(b) *(6 pts)* Write the difference equation for the recursive, IIR direct form in (b).

$y[n] = x[n] + 4y[n-1] + 3y[n-2]$

(c) *(7 pts)* Choose a_0 and a_1 in the FIR cascade form in (c) to give the filter linear phase.

$H(z) = (1 + a_0 z^{-1} + a_1 z^{-2})(1 + \frac{1}{2}z^{-1})(1 - z^{-1})$

$= \frac{z^2 + a_0 z + a_1}{z^2} \cdot \frac{z + \frac{1}{2}}{z} \cdot \frac{z-1}{z}$

$= \frac{(z^2 + a_0 z + a_1)(z + \frac{1}{2})(z-1)}{z^4}$ $\quad -6$

poles and zeros are symmetric around the unit circle

$\frac{-a_0 \pm \sqrt{a_0^2 - 4a_1}}{2} = 1$ or $=$

zeros: $z = -\frac{1}{2}, 1, \frac{-a_0 \pm \sqrt{a_0^2 - 4a_1}}{2}$

$\therefore$ ~~$a_1 = \frac{1}{2}$~~

$a_1 = -\frac{3}{2}$