

Full Name: Zexi Lin 55314399  
EEL 4750 / EEE 5502 (Fall 2019) – Exam #03

ExamID: 010001  
Date: Dec. 4, 2019

| Question | # of Points Possible | # of Points Obtained | Grader |
|----------|----------------------|----------------------|--------|
| # 1      | 20                   | 20                   | IK     |
| # 2      | 18                   | 18                   | MS     |
| # 3      | 20                   | 16                   | JBH    |
| # 4      | 22                   | 18                   | Y.B    |
| # 5      | 20                   | 18                   | JBH    |
| Total    | 100                  | <del>90</del> 90     | Y.B    |

**For full credit when sketching:** remember to label axes and make locations and amplitudes clear.

**Before starting the exam, read and sign the following agreement.**

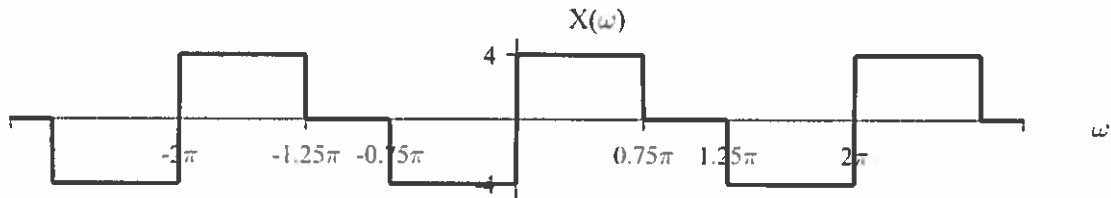
By signing this agreement, I agree to solve the problems of this exam while adhering to the policies and guidelines of the University of Florida and EEL 4750 / EEE 5502 and without additional external help. The guidelines include, but are not limited to,

- The University of Florida honor pledge: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."
- Only one 8.5 by 11 inch cheat sheet (double-sided) may be used
- No calculators or computers may be used
- No textbooks or additional notes may be used
- No collaboration is allowed
- No cheating is allowed

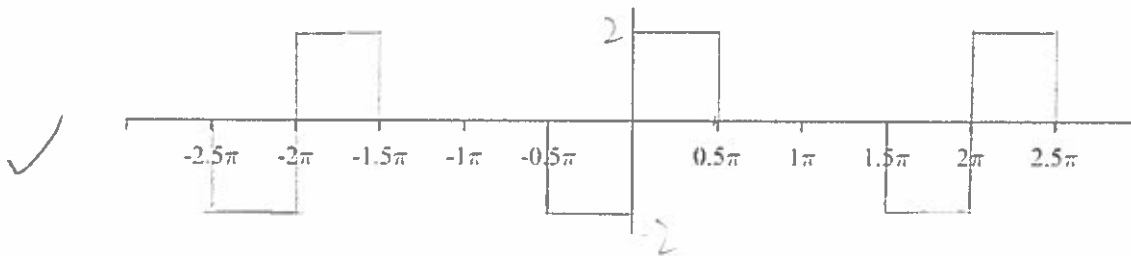
Zexi Lin  
Student

12.4.2019.  
Date

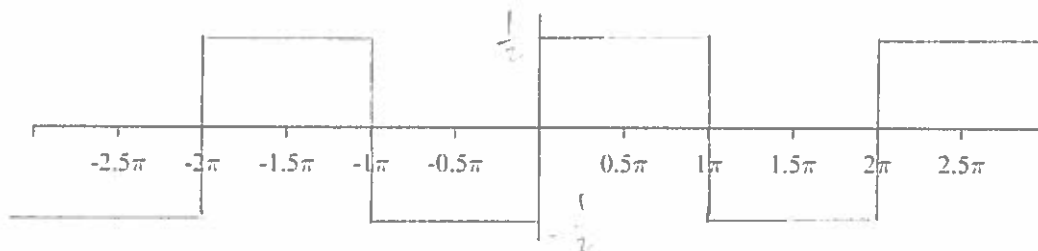
**Question #1:** Consider the DTFT of the signal  $x[n]$  (i.e.,  $X(\omega)$ ) shown below.



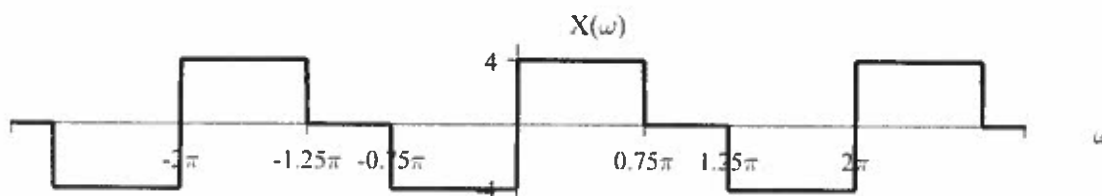
- (a) (10 pts) Sketch the DTFT (from  $\omega = -3\pi$  to  $\omega = 3\pi$ ) of  $x[n]$  after downsampling by 2 (with no anti-aliasing filter). Remember to label important locations / values.



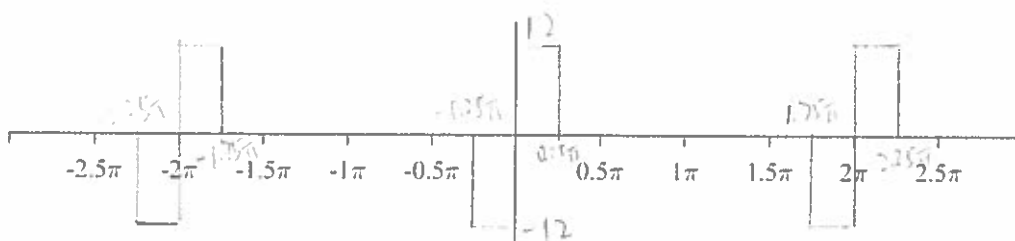
- (b) (10 pts) Sketch the DTFT (from  $\omega = -3\pi$  to  $\omega = 3\pi$ ) of  $x[n]$  after downsampling by 8 (with an anti-aliasing filter). Remember to label important locations / values.



**Question #2:** Consider the DTFT of the signal  $x[n]$  (i.e.,  $X(\omega)$ ) shown below.



- (a) (10 pts) Sketch the DTFT (from  $\omega = -3\pi$  to  $\omega = 3\pi$ ) of  $x[n]$  after upsampling by 3 (with an interpolation filter). Remember to label important locations / values.



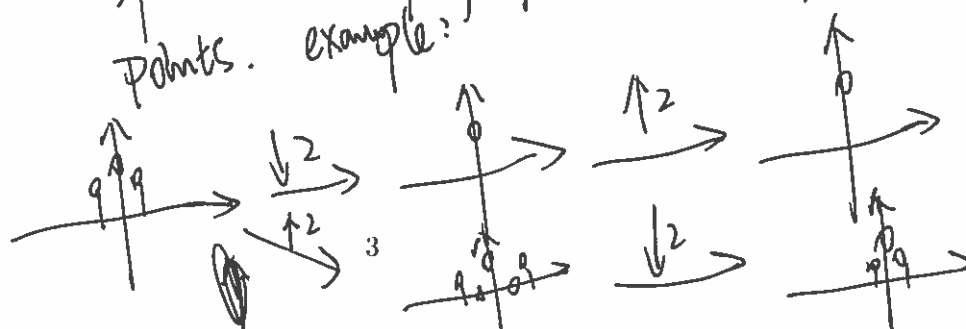
10

- (b) (8 pts) (True or False) Upsampling by  $M$  followed by downsampling by  $N$  is equivalent to downsampling by  $N$  followed by upsampling by  $M$ . Justify why.

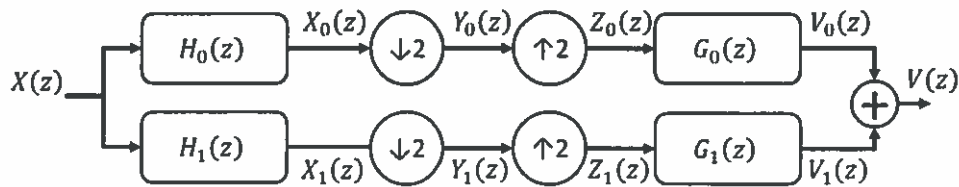
False.

Because downsampling may cause ~~aliasing~~ aliasing and we may loss some information.

If we downsampling first. we may loss some points. example:



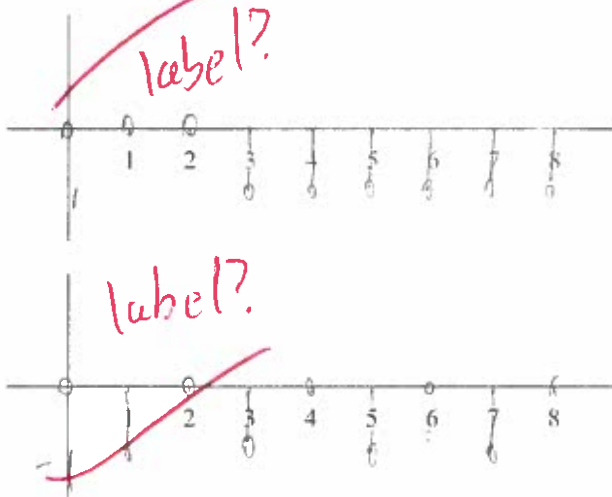
Question #3: Consider a 2-channel filter bank shown below.



Let the filters and input be defined by the impulse responses

$$h_0[n] = g_0[-n] = \delta[n-2], \quad h_1[n] = g_1[-n] = \delta[n-5], \quad x[n] = (-1)^n u[n]$$

(a) (12 pts) Sketch  $y_1[n]$  and  $v_1[n]$  (inverse z-transforms of  $Y_1(z)$  and  $V_1(z)$ ) for  $0 \leq n \leq 8$ .



$$\begin{aligned} X(z) &= \frac{1}{1+z^{-1}} & H_1(z) &= z^{-5} \\ Y_1(z) &= H_1(z)X(z) = \frac{z^{-5}}{1+z^{-1}} \\ Y_1[n] &= (-1)^{n-5} u[n-5] \\ g_1[n] &= \delta[n+5] \\ G_1(z) &= z^5 \end{aligned}$$

(b) (8 pts) (True or False) If the frequency responses  $H_0(\omega)$  and  $H_1(\omega)$  are periodic with a period of  $\pi$  (i.e.,  $H_0(\omega) = H_0(\omega - \pi)$  and  $H_1(\omega) = H_1(\omega - \pi)$ ), then the filter bank always satisfies the alias canceling conditions. Justify why.

False ~~True~~. Because it satisfies ~~Orthogonal Filter Bank~~.

$$\begin{cases} \cancel{G_0(z)G_0(z^{-1}) + G_0(z)G_1(z^{-1}) = 2} \\ \cancel{G_1(z)G_1(z^{-1}) + G_1(z)G_0(z^{-1}) = 2} \\ \cancel{G_0(z)G_1(z^{-1}) + G_1(z)G_0(z^{-1}) = 0} \end{cases} \quad \text{Because}$$

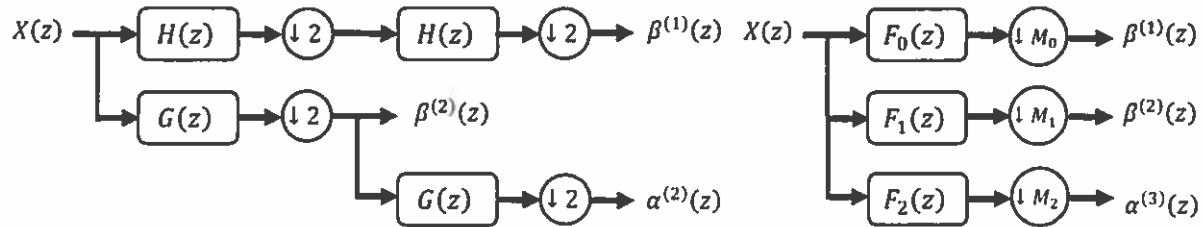
$$\begin{aligned} \omega &\rightarrow \omega - \pi \\ z &\rightarrow z^{-1} \end{aligned}$$

$$\begin{aligned} & (H_0(\omega)G_0(\omega) + G_1(\omega)G_0(\omega)) \neq 0 \\ & (G_0(\omega)G_0(\omega) + G_0(\omega)G_1(\omega)) \neq 0 \end{aligned}$$

Not satisfies alias canceling ~~why?~~

Be more specific?

**Question #4:** Consider the following non-standard wavelet bank and filter bank.



Let  $H(z)$  and  $G(z)$  be defined by the transfer functions:

$$H(z) = 1 - z^{-1}, \quad G(z) = 1 + z^{-1}$$

- (a) (10 pts) Use the Noble identities to simplify the reconstruction bank (left) and represent it as a filter bank (right). Determine  $M_0$ ,  $M_1$ ,  $M_2$ ,  $F_0(z)$ ,  $F_1(z)$ ,  $F_2(z)$ . Fully simplify.

$$\downarrow 2 \rightarrow H(z) = H(z^2) \rightarrow \downarrow 2.$$

$$\downarrow 2 \rightarrow G(z) = G(z^2) \rightarrow \downarrow 2.$$

$$M_0 = 4, \quad M_1 = 2, \quad M_2 = 4.$$

$$F_0(z) = H(z)H(z^2) = 1 - z^{-1} - z^{-2} + z^{-3} \\ F_1(z) = G(z) = 1 + z^{-1} \\ F_2(z) = G(z)G(z^2) = 1 + z^{-1} + z^{-2} + z^{-3}$$

- (b) (12 pts) Let the input signal be defined by

$$X(z) = -2z^{-2}.$$

Compute  $\beta^{(1)}(z)$ ,  $\beta^{(2)}(z)$ , and  $\alpha^{(2)}(z)$ .

4

$$F_0(z)X(z) = -2z^{-2}(1 - z^{-1} - z^{-2} + z^{-3}) \\ = -2z^{-2} + 2z^{-3} + 2z^{-4} - 2z^{-5}$$

$$\beta^{(1)}(z) = 2(-z^{-1} + z^{-2})$$

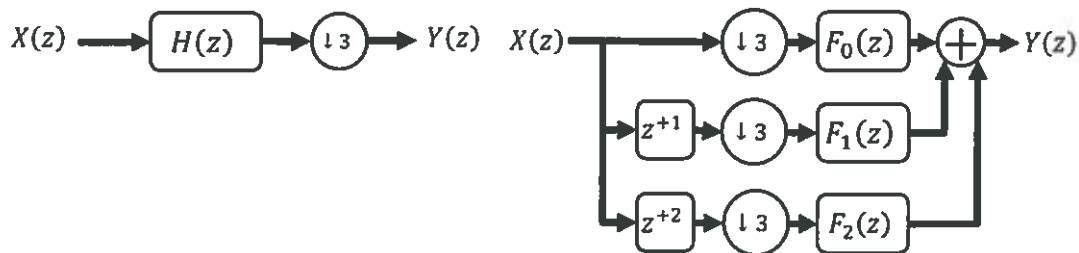
$$F_1(z)X(z) = -2z^{-2}(1 + z^{-1}) = -2(-z^{-2} - z^{-3})$$

$$\beta^{(2)}(z) = -2z^{-1}$$

$$F_2(z)X(z) = -2z^{-2}(1 + z^{-1} + z^{-2} + z^{-3}) = 2(-z^{-2} - z^{-3} - z^{-4} - z^{-5})$$

$$\alpha^{(2)}(z) = -2(z^{-1} + z^{-2})$$

**Question #5:** Consider a filter and its polyphase implementation, as shown below.



Let  $h[n]$ ,  $f_0[n]$ ,  $f_1[n]$ , and  $f_2[n]$  be the inverse z-transforms of  $H(z)$ ,  $F_0(z)$ ,  $F_1(z)$ , and  $F_2(z)$ .

(a) (12 pts) Let  $h[n] = \delta[n] + \sum_{k=-\infty}^{\infty} 4\delta[n-3k-2]$ . Mathematically express  $f_0[n]$ ,  $f_1[n]$ , and  $f_2[n]$ .

$$f_0[n] = h[3n] = \delta[n] + \sum_{k=-\infty}^{\infty} 4\delta[n-3k-2]$$

$$f_1[n] = h[3n+1] = \delta[n+1] + \sum_{k=-\infty}^{\infty} 4\delta[n-3k-1]$$

$$f_2[n] = h[3n+2] = \delta[n+2] + \sum_{k=-\infty}^{\infty} 4\delta[n-3k]$$

*Simplify*

(b) (8 pts) (True or False) If  $H(z)$  represents an FIR filter, then the filters represented by  $F_0(z)$ ,  $F_1(z)$ , and  $F_2(z)$  will be  $1/3$  the length of  $H(z)$  in time. Justify why.

False. Because

$$\begin{cases} f_0[n] = h[3n] \\ f_1[n] = h[3n+1] \\ f_2[n] = h[3n+2] \end{cases}$$

So, they will be  $\frac{1}{3}$  the length of  $H(z)$  in time