Full Name: ______________________________

EEL 4750 / EEE 5502 (Fall 2019) – Code #02 **Due Date:** **Sept. 20, 2019**

Question #1: *(1 pts)* How many hours did you spend on this homework?

Question #2: *(4 pts)* *Correlation*

Consider the expression for correlation (also known as cross-correlation) between $x[n]$ and $y[n]$,

$$c[n] = x[-n] * y[n] = \sum_{m=-\infty}^{\infty} x[m]y[n+m] \tag{1}$$

Let us discuss some properties of the correlation function.

(a) **Bounds of Convolution with Causal Signals.** Show that if $x[n] = 0$ for $n < 0$ and $n > N_x - 1$ (i.e., casual with length N_x) and $y[n] = 0$ for $n < 0$ and $n > N_y - 1$ (i.e., causal with length N_y), then $x[n] * y[n]$ must satisfy

$$x[n] * y[n] = 0 \quad \text{for} \quad n < 0 \quad \text{and} \quad n > N_x + N_y - 2\ .$$

That is, the result is causal with length $N_x + N_y - 1$. Illustrations and correct intuition will be acceptable. *[Side note: This is a common DSP interview question.]*

(b) **Bounds of Convolution with Non-causal Signals.** Show that if $x[n] = 0$ for $n < 0$ and $n > N_x - 1$ (i.e., casual with length N_x) and $y[n] = 0$ for $n < 0$ and $n > N_y - 1$ (i.e., causal with length N_y), then $c[n]$ must satisfy

$$c[n] = 0 \quad \text{for} \quad n < -(N_x - 1) \quad \text{and} \quad n > N_y - 1\ .$$

That is, the result is non-causal with length $N_x + N_y - 1$. Illustrations and correct intuition will be acceptable.

(c) **(EEE 5502 Only) Maximum of Auto-Correlation.** Let $y[n] = x[n]$. Under this condition, the correlation is known as the auto-correlation,

$$c[n] = \sum_{m=-\infty}^{\infty} x[m]x[n+m] \tag{2}$$

For this condition, show that $c[n]$ is largest when $n = 0$.

Hint: Use the Cauchy-Schwarz inequality, defined by

$$\left| \sum_{m=-\infty}^{\infty} x[m]y[m] \right|^2 \leq \sum_{m=-\infty}^{\infty} |x[m]|^2 \sum_{k=-\infty}^{\infty} |y[k]|^2 \tag{3}$$

(d) **(EEE 5502 Only) Maximum of Cross-Correlation.** Let $y[n] = x[n - n_0]$. For this condition, show that $c[n]$ is largest when $n = n_0$.

Question #3: *(3 pts)* *Convolution*

Create the following signals in MATLAB:

```
>> x1 = sin(pi/10*(0:19));
>> x2 = 2*(mod((0:19)/10,1)-0.5);
>> x3 = cos(pi/2*(0:19));
>> x4 = (-1).^(0:19);
```

And concatenate each of these four signals:

```
>> z = [x1 x2 x3 x4];
```

(a) Plot `z` using `stem(z)`

(b) Create three systems with impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ (or `h1` and `h2` and `h3` in MATLAB) that are also 20 samples in length. Define these impulse responses as

$$h_1[n] = \delta[n]$$
$$h_2[n] = \delta[n-5]$$
$$h_3[n] = \delta[n-19]$$

Perform three convolutions in MATLAB between $h_1[n]$, $h_2[n]$, and $h_3[n]$ and $x_1[n]$ by running

```
>> y1 = conv(x1, h1)
>> y2 = conv(x1, h2)
>> y3 = conv(x1, h3)
```

Plot `y1`,`y2`,`y3` using `stem`.

(c) The output signals now have a length of $N = 39$ instead of $N = 20$. Based on the previous results, why is this necessary?

Question #4: *(4 pts)* *The Auto-Correlation*

(a) Perform a correlation of `x1` with itself, i.e., an auto-correlation. Perform the auto-correlation with the following commands:

```
>> a1 = conv(fliplr(x1), x1);
>> lag = -(20-1):(20-1);
>> stem(lag, a1)
>> xlabel('Lag [samples]')
>> ylabel('Amplitude')
```

As shown in Question #1, the auto-correlation is always maximum at $n = 0$. The x-axis for a correlation is referred to as the "lag" or "delay."

(b) Repeat this process for $x_2[n]$, $x_3[n]$, $x_3[n]$, and $x_4[n]$. Plot the auto-correlations (like above) for all four signals.

(c) Perform four correlations between $z[n]$ and $x_1[n]$; $z[n]$ and $x_2[n]$; $z[n]$ and $x_3[n]$; and $z[n]$ and $x_4[n]$. Hence you can plot the result `z1` with

```
>> z1 = % the correlation, determine yourself based prior
   results
>> n = % the x-axis, determine yourself based on Question #2
>> stem(n, z1)
>> xlabel('Lag [samples]')
>> ylabel('Amplitude')
```

Plot the correlations for all four signals.

(d) Based on your results, how can we use this result to determine the location of our four signals $x_1[n]$, $x_2[n]$, $x_3[n]$, $x_4[n]$ buried in a large signal $z[n]$?

Question #5: *(5 pts)* *Finding a Hidden Signal*

In radar, communications, and other systems, we need to find received signals that are similar to what we transmitted. In communication systems, these signals correspond to messages. In radar, these signals tell us about the surrounding environment. Yet, in communications, we may not know when the signal will arrive. In radar systems, the signal's location in time tells us how far away a target is. Hence, we need to search across time to find the signals of interest. This is common accomplished through the cross-correlation.

In this question, we will use cross-correlation to locate the position of a hidden signal in noisy data. Included with the zip file is a p-file (an obfuscated m-file) function

```
>> [message, code] = get_message(ufid);
```

This function provides a message with a hidden code. Your objective is to determine the location of the *first sample* of the code with cross-correlation. Submit your .m file used to achieve this. Also, plot your resulting correlation coefficient and display the first sample location.

Note: This works well for small data sets (like the example here), but this is extremely slow for larger datasets (like audio recordings). In this next coding assignment, we will see how to speed this up with the discrete Fourier transform.