The Coming Oil Crisis

Team # 321

February 7, 2005

Abstract

In this paper, we study oil, a typical vital nonrenewable resource, to model the depletion of nonrenewable resources.

First, based on the demand-supply theory, we establish a differential equation system including oil demand $D(t)$, oil supply $S(t)$, and oil price $P(t)$, and thereby get the explicit formulas of the three variables, respectively. Considering the intrinsic law of nonrenewable resources, i.e. the value of nonrenewable resources increases with the elapse of time, we suitably modify the above-mentioned model to reflect the worldwide oil demand dependent on time. The modified demand function $D(t)$ can be written as follows:

$$D(t) = a_1 + a_2 \cos(a_3 t + a_4) + a_5 \exp(a_6 t).$$

Using the modified model to fit the worldwide oil demand data from 1970 to 2003, we find the goodness of fit is very satisfactory. From this model and available related data, we conclude that the worldwide endowment of oil will be used up in 2032 without any effective measure. Then we take the economic, demographic, political, and environmental factors into account, and discuss the influences of these factors on oil demand.

To meet the needs of contemporary human beings without compromising the capacity of future generations to meet their needs, we establish the criterion of the rational oil allocation between generations, and construct the optimal oil allocation model under this criterion. To explain it clearly, an illustration and the corresponding optimal oil allocation scheme are provided. As for the disasters accompanied by oil exploitation, we provide a strategy for oil exploitation to reduce the possibility of disasters in short term. In the end, according to marginal utility replacement rules, we also study the trade-off between oil and its alternatives.

Owing to the fact that the model building in this paper is on basis of demand-supply theory and intrinsic law of nonrenewable resources, our model can be applied to general nonrenewable resources.

Contents

1 Introduction

Natural resources can be classified into two categories: nonrenewable resources and renewable resources. Natural resources such as oil, copper or iron that once used cannot renew itself, at least not in this geological age and are called nonrenewable resources. Other resources such as fish or trees can renew themselves if not overused and so are called renewable resources.

In today's world, human produce, distribute and consume large quantities of oil. Oil is used as a major power source to fuel our factories and various modes of transportation, and in many everyday-products, such as plastics, nylon, paints, tires, cosmetics, and detergents.

With the development of science and technology, nowadays we can produce more products than we did, but at the cost of consuming a huge amount of oil. Whenever we ponder the future of the human enterprise, questions about oil come up. The Earth's oil reserves are finite, so we must choose how best to use it.

2 Problem restatement

This problem wants us to select a vital nonrenewable resource, and find out appropriate worldwide historical data on its endowment, discovery, annual consumption, and price.

Task 1 asks us to use the data we obtain to design a model to predict the depletion or degradation of the commodity over a long horizon.

Task 2 requires us to modify our model to account for future economic, demographic, political and environmental factors, and explain its limitation.

Task 3 wants us to create a practical policy which sustains the usage of the resource for a long period of time while avoiding rapid exhaustion of the resource.

Task 4 asks us to develop a "security" policy to protect the resource against misuse.

Task 5 requires us to develop policies to control any short-or long-term "environmental effects".

Task 6 wants us to compare this resource with any other alternatives for its purpose, and develop a research policy to advance these alternatives.

3 Task 1 Modeling the Depletion of Oil

Under the following assumptions, we will have a rather pessimistic situation, where no restriction is made to protect oil, and obviously oil will be in the state of total exhaustion in the fastest way.

3.1 Assumptions:

1. Oil refinement capacity is enough.
2. All the undiscovered oil is available when necessary.

In other words, when there is a demand, there is a supply, until the day all the oil on the earth is completely used up.

3.2 Notations:

$U(t)$: Oil undiscovered at year t .

$R(t)$: Oil discovered but has not been used (reserves) at year t.

$D(t)$: Worldwide oil demand at year t.(thousand barrels)

$S(t)$: Worldwide oil supply at year t.

$p(t)$: Oil price at year t.

p_0: The equilibrium price of oil.

3.3 Modeling:

From the above definition of notations, we know $U(t)+R(t)$ denotes the total remaining oil on the earth at year t, and $\sum_{i=t}^{n} D(i)$ is the total demand from year t to year n.

$$\sum_{i=t}^{n} D(i) \leq U(t)+R(t) < \sum_{i=t}^{n+1} D(i) \tag{1}$$

Based on the above inequalities we can say that oil will be depleted between year n to year $n+1$.

The data we can find are:

1. The estimation of undiscovered oil worldwide in 1997 is 180 billion barrels, that is to say, $U(1997) = 180$ (billion barrels). (see [4])
2. The worldwide oil reserve in 1997 is 1018.5 billion barrels, viz. $R(1997) = 1018.5$ (billion barrels). (see [3])
3. The worldwide oil demands from 1980 to 2003, $D(i)(i = 1980, \cdots, 2003)$ are shown in the table below. (see [3])

Table 1: World-wide oil demand, 1970 ∽ 2003 (thousands of barrels/day)

1970	46,808	1980	63,108	1990	66,443	2000	76,954
1971	49,416	1981	60,944	1991	67,061	2001	78,105
1972	53,094	1982	59,543	1992	67,273	2002	78,439
1973	57,237	1983	58,779	1993	67,372	2003	79,813
1974	56,677	1984	59,822	1994	68,679		
1975	56,198	1985	60,087	1995	69,955		
1976	59,673	1986	61,825	1996	71,522		
1977	61,826	1987	63,104	1997	73,292		
1978	64,158	1988	64,963	1998	73,932		
1979	65,220	1989	66,092	1999	75,826		

If we have some information of future oil demand, namely $D(i)(i = 2004, 2005, \ldots)$, then (1) changes to

$$\sum_{i=1997}^{n} D(i) \le U(1997) + R(1997) < \sum_{i=1997}^{n+1} D(i).$$

Then n, the year when oil is used up it can be easily calculated.

In order to predict future oil demand, we now consider the following ordinary differential equation system according to 'supply-demand' principles:

$$\begin{cases} \frac{dS}{dt} = a\tilde{P} & (1.1) \\ \frac{d\tilde{P}}{dt} = -b(S - D) & (1.2) \\ \frac{dD}{dt} = -c\tilde{P} & (1.3) \end{cases}$$

where $\tilde{P} = P(t) - P_0$, and $a > 0, b > 0, c > 0$(constant).

Now we give some explanations of the system. Eq.(1.1) means that if oil piece is greater than its equilibrium price, the output will increase accordingly, and vice versa. Eq.(1.2) shows that if oil supply exceeds its demand, the price will decline. Eq.(1.2) indicates when oil price is up or down, the demand of oil will expand or shrink correspondingly.

After careful calculation, we get the solution of the above ordinary differential equation system:

$$\begin{cases} \tilde{P}(t) = \sqrt{\tilde{c_1}^2 + \tilde{c_2}^2} \times \sin[\sqrt{(ba + c)} \times t + \varphi] & (2.1) \\ S(t) = S_0 - \frac{a\sqrt{\tilde{c_1}^2+\tilde{c_2}^2} \times \cos[\sqrt{b(a+c)} \times t + \varphi]}{\sqrt{b(a+c)}} & (2.2) \\ D(t) = D_0 + \frac{c\sqrt{\tilde{c_1}^2+\tilde{c_2}^2}}{\sqrt{b(a+c)}} \cos[\sqrt{b(a + c)} \times t + \varphi] & (2.3) \end{cases}$$

where $\varphi = \arctan \frac{\tilde{c_1}}{\tilde{c_2}}$, and $S_0, D_0, \tilde{c_1}, \tilde{c_2}$ are parameters to be determined.

Here, we place our interests on Eq.(2.3). It implies that oil demand is in a periodical form. However, with the time passing by, the population on the earth is expanding in an exponential way, and the demand of oil is accordingly increasing in a similar way. Therefore we should modify Eq.(2.3) to reflect the intrinsic increasing tendency. We consider adding a term of exponential function $k_1 \exp(k_2(t - t_0))(k_1, k_2, t_0$are constants) to the right side of Eq.(2.3). And after some simplification, we can easily get the following equation:

$$D(t) = a_1 + a_2 \cos(a_3 t + a_4) + a_5 \exp(a_6 t) \tag{2}$$

Using Eq.(2) to fit the data in Table 1, we can get the fitting curve of Figure 1, from which we can easily find the goodness of fit is quite satisfactory. And the function we get after fitting is:

$$D(t) = 365 * (31950 + 556.7 \cos(1.605t - 3159.659) + (1.239 * 10^{-16}) * \exp(0.02366t))$$

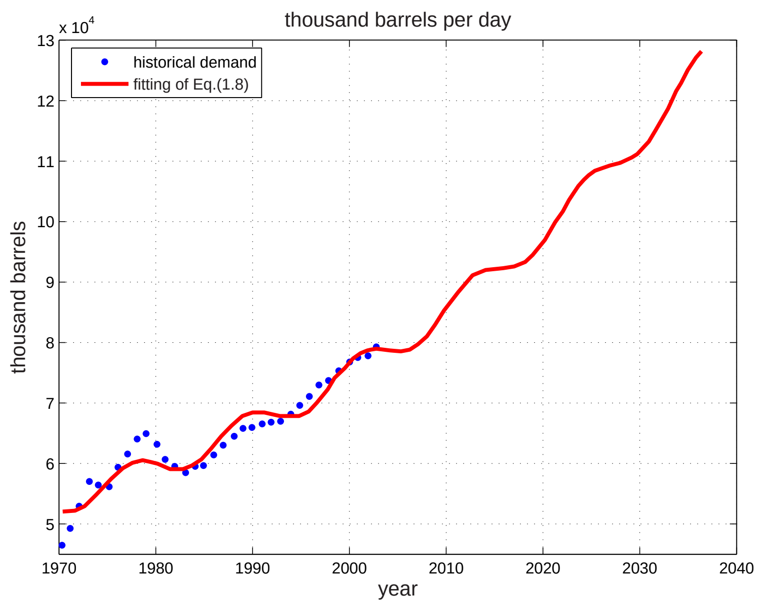

Figure 1: The fitting curve by Eq.(2)

Noticing that with the passing of time, the third term $(a_5 \exp(a_6 t))$ on the right side of Eq.(2) will play a more important role and the second term$(a_2 \cos(a_3 t + a_4))$ can be neglected, thus for the sake of convenience, we reduce Eq.(2) to:

$$D(t) = a_1 + a_5 \exp(a_6 t) \tag{3}$$

Hence we choose exponential fitting to predict future demand, and for the purpose of comparison, we also choose linear fitting and a invariable demand case in which future demand is assumed the same as that of 2003. The data for fitting is world-wide oil demand $D(i)(i = 1970, \dots, 2003)$ in Table 1 , and the fitting results are given as follows:

Exponential fitting:

$$D(t) = 365 * (29820 + 2.265 * 10^{-15} * \exp(0.02223 * t))(t \geq 2004)$$

Linear fitting:

$$D(t) = 365 * (771.2 * t + (-1.467e + 006))(t \geq 2004)$$

The predicted demand is shown in Figure 2. (For convenience, annual demand is in the form of average daily demand.)

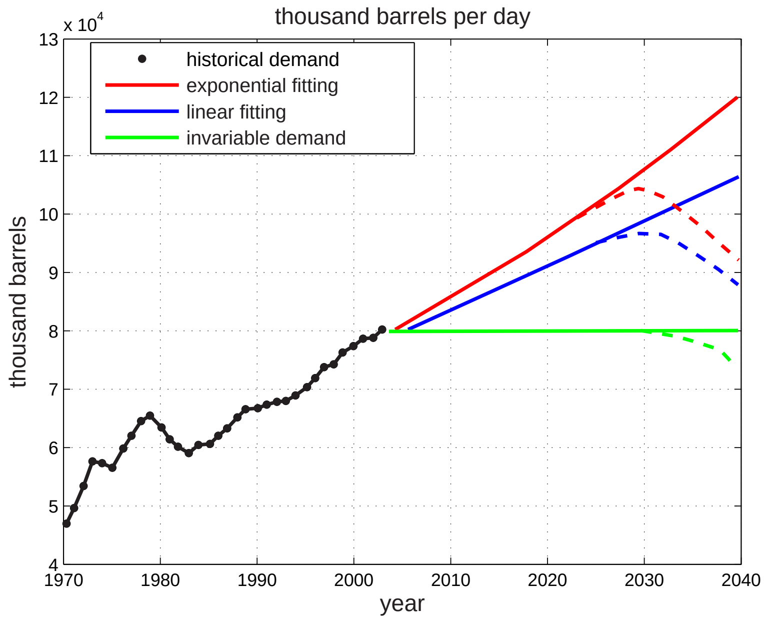

Figure 2: Estimation of future oil demand

Remark: With the increase of oil demand, its price will accordingly rise. As is shown by the broken lines in figure 2, this will lead to the decline of oil demand, and we will discuss this phenomenon in detail later.

3.4 Sensitivity Analysis:

Based on the above assumption, $U(t)$ is the undiscovered oil on earth at year t, so it is difficult to get a precise value. What we can acquire is merely an estimation, which may contain a

Table 2: the relationship between the fluctuation of $U(t)$ and the year of oil exhaustion

The year of oil exhaustion(n)		Fluctuation of $u(t)$(t=2000)		
The way of fitting	Exponential	-10%	0%	+10%
	Linear	2032	2032	2032
	Invariable	2033	2033	2034
	demand case	2036	2037	2037

certain degree of deviation. We now investigate whether the deviation will bring about a serious distortion on the determination on n.

We give $U(t)(t = 1997)$ a fluctuation of $\pm 10\%$, and study the corresponding change of n. The conclusion is shown in Table 2.

From table 2, we can learn that the fluctuation of $U(t)$ does not have a strong influence on the estimated year of oil's exhaustion.

4 Task 2 The Influence of Economic, Demographic, Political and Environmental Factors on the Oil

4.1 Assumptions

1. We assume that the annual demand of oil could reflect the annual consumption of oil.
2. When we consider the effect of one factor, the other factors are neglected, in other words, we do not take the interaction of arbitrary two different factors into account.
3. When considering the future consumption of oil, we ignore the intrinsic fluctuation, because its influence is really small. That means we only consider the tendency of the future tendency.

In task 1, we've assumed that oil demand is in an exponential form, but in reality, many factors such as economy, population, politics and environment, will influence oil consumption. Thus, in the following discussion, we modify the model provided in Task 1 to include the above-mentioned factors and thereby make our results more approximate to the reality.

4.2 The Influence of Economic Factors

We use GDP as the measure of economy. First we investigate the relationship between demand and supply. Based on the data of worldwide oil demand and supply between 1970 and 2003, we perform a correlation analysis between oil demand and oil supply, and get the correlation coefficient $r = 0.9911$. Therefore, we justifiably deem that demand is almost linearly dependent on supply. So when we consider the influence of oil on demand, caused by economic, we take the demand as a dependent variable. And the data we obtain about the recent world total GDP is shown in Table 3, while the data about the total world oil consumption in recent years is shown in Table 4.

Table 3: Recent World Total GDP($ 108),1995-2003

1995	1996	1997	1998	1999	2000	2001	2002	2003
27.134	28.247	29.433	30.257	31.377	32.85	33.64	34.6487	36

Table 4: Recent Total World Oil Consumption(10^3 thousands of barrels/day),1995-2003

1995	1996	1997	1998	1999	2000	2001	2002	2003
69955	71522	73292	73932	75826	76954	78105	78439	79813

We know from the table that the GDP value and the worldwide oil consumption both increase as time elapses. Hence we make a correlation analysis for these two lists of data, and the correlation coefficient turns out to be 0.9930. That is to say, the GDP value and oil consumption are of strong positive linear dependence.

In order to find out the intrinsic relationship between them, we make a linear regression for these data. The regression equation is given as follows:

$$y = 1183 \times x + 38140 \tag{4}$$

Where x: global GDP value

y: worldwide oil consumption

And *R-square.*(square of regression coefficient) is 0.9861. And the further statistical significance test shows that there indeed exists a linear dependence between the global GDP value and worldwide oil consumption.

Next, using Eq.(4), we can predict the cumulative oil demand as the global GDP grows at the rate of 10%, 5%, 3%, 1% respectively. For the convenience, we take the year 2001 as the starting point, and we get that the total remaining oil on earth at that time is $1.1178 \times 10^{+12}$(barrels). And then we calculate the time of the oil exhaustion under different cases of economic growth. Oil depletion time is shown in the following figure:

The horizontal line denotes the total remaining oil at year 2001. The x-axis coordinates of the intersection points of the horizontal line and the curves denote oil exhaustion time at the GDP growth rate of 10%, 5%, 3%, 1% , respectively.

We find that the faster the GDP growth rate is, the larger the oil consumption will be, making the advent of the exhaustion time to come sooner. When the GDP growth rate is 10%, oil will be depleted at 2020; when the GDP growth rate is 5%, oil will be used up at

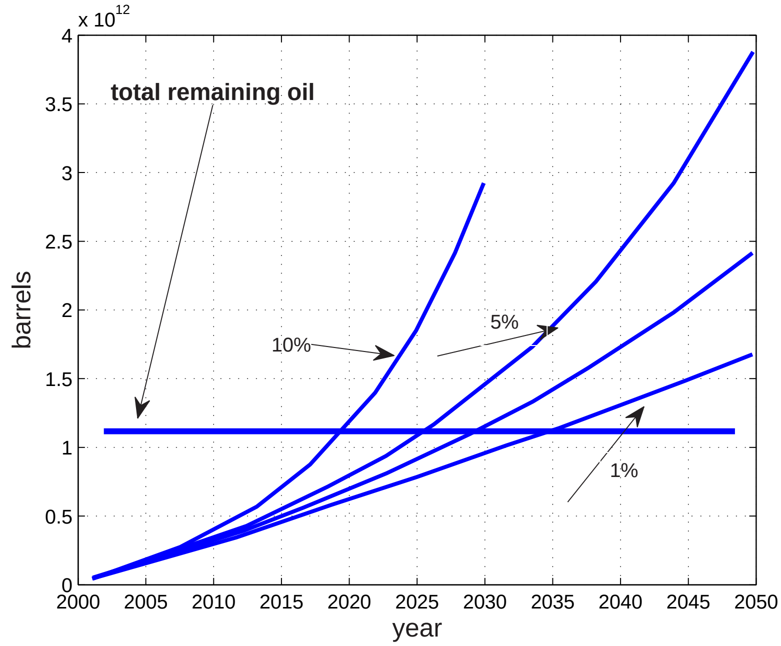

Figure 3: the curves of cumulative consumption under different rates of GDP growth

2026; When the GDP growth rate is 3%, oil will be used out at 2029,and finally, When the GDP growth rate is 1%, oil will be used out at 2035.

As for a specific nation, its GDP growth rate can be controlled through some economic approaches, so as to delay the advent of oil exhaustion.

4.3 The Demographic Influence on Oil Consumption

First, we resort to ***Logistic*** model to predict the change of world population in years to come. The model is as follows:

$$x(t) = \frac{k}{1 + (\frac{k}{x_0} - 1)e^{-(t-t_0)r}} \tag{5}$$

Where t: denotes time;

t_0: denotes initial time;

x_0: denotes the population at initial time;

k: is the environment capacity, namely the maximum number of population the earth can accommodate;

r: is the intrinsic growth rate of population.

We use the data of the population in recent decades to fit the above equation, and get the formula of the change of population with time:

$$x(t) = \frac{100}{1 + (\frac{100}{44.585} - 1)e^{-0.0327 \times (t-1980)}} \tag{6}$$

Now, using Eq. (6) we can the prediction of future population, which is shown in the following table:

Table 5: The population estimated by the ***Logistic model***(0.1billion)

1980	1990	2000	2010	2020	2030	3040	2050	2060
44.585	52.736	60.744	68.212	74.849	80.495	85.126	88.811	91.560

Similarly, we try to obtain the relationship between oil consumption and total population as we do in the section of studying GDP's influence. The correlation coefficient is 0.9877. Obviously, there exists a strong positive linear dependence between oil consumption and total population. Theoretically, the larger the population is, the greater the oil demand will be, which coincides with the above result.

Naturally, we make a linear regression for oil demand and population, and get the relationship between them as follows:

$$y = 1443 \times x - 11170 \tag{7}$$

Where y: oil consumption;.

x: total population.

From (6) we can get the future population, and from (7) we may obtain oil demand in the future. Hence, the time of oil exhaustion can be estimated under such a growth of population.

From the above figure we can clearly see that oil will be in the state of exhaustion if population increases in the way of (6). And the intersection point of the horizontal line and the curve is the corresponding exhaustion time under a specific growth rate of population, which is not difficult to estimate.

4.4 The Political Influence on Oil Consumption

Here, we mainly discuss the influences caused by wars.

In Task 1, we have fitted the function of demand dependent on time as follows:

$$y(t) = 7.95 \times 10^{-6} \times e^{0.01149t}$$

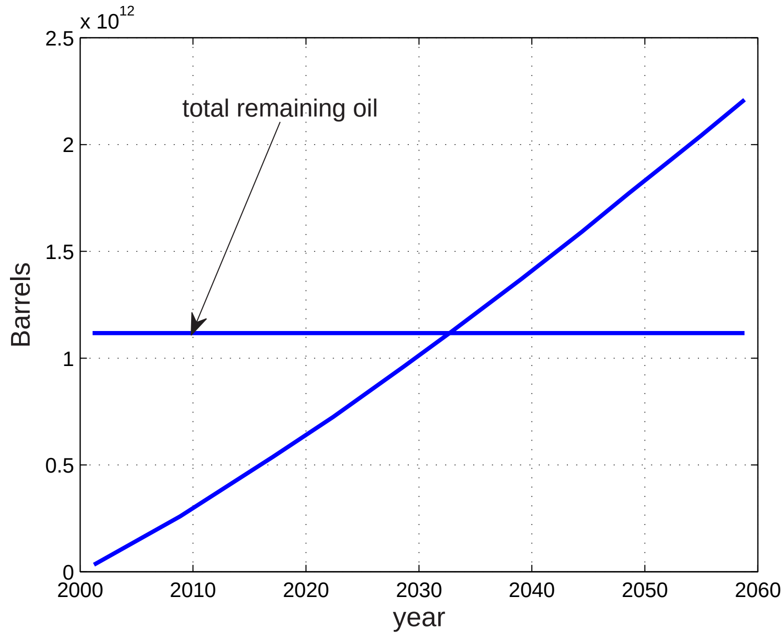

Figure 4: the curve of cumulative oil consumption with the growth of population

from which we can estimate the oil demand at any given time. Let rate denote the growth rate of oil consumption, thus

$$rate = \frac{y(t+1)}{y(t)} - 1 = \frac{7.95 \times 10^{-6} \times e^{0.01149(t+1)}}{7.95 \times 10^{-6} \times e^{0.01149t}} - 1 = e^{0.01149} - 1 = 1.56\%$$

From the above equation, we find that the annual growth rate of oil consumption remains at the same level of 1.56% if oil consumption increases in an exponential form, and we call it ***average growth rate***.

Now, we investigate the growth rate of oil consumption during the past decades, and the results are shown in the figure below.

From the figure above, we can obviously see that the growth rate declines sharply in the year 1974 1980 and 1990, which coincides with the outbreak of the fourth Middle East War (1973), the Iran-Iraq War (1980) and the Gulf War (1990). And these three wars all broke out in the Middle East, which is the center of oil production. So the wars strongly impacted the oil price, and consequently impacted the oil demand.

Define

$$\Delta rate = rate(t) - \overline{rate} \tag{8}$$

Where $rate(t)$: the growth rate of oil consumption at year t

$\overline{rate}$: the average growth rate of oil consumption

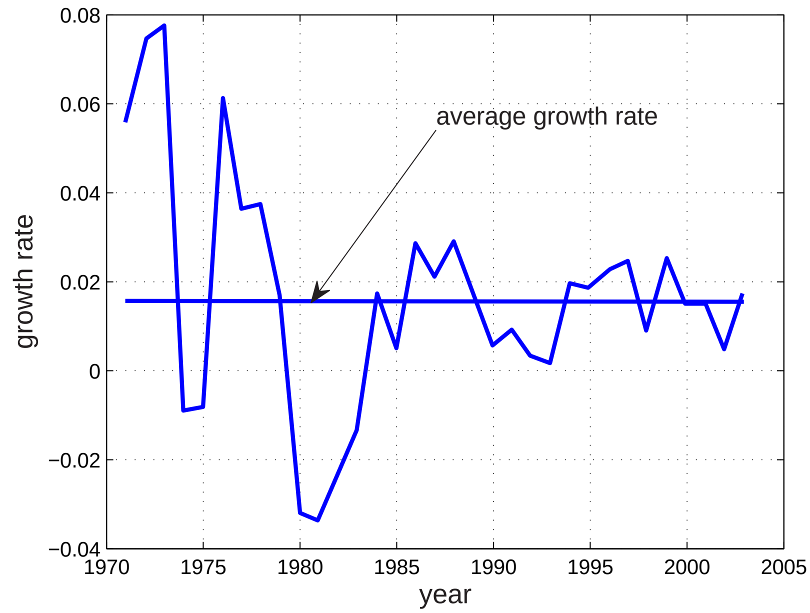

Figure 5: the curve of oil consumption growth rate and mean growth rate

So $\Delta rate$ indicates the deviation of oil assumption off its mean level. We calculate the $\Delta rate$ in year 1974, 1980 and 1990, respectively, and the three numbers may reflect the affects on oil demand caused by these three wars, respectively. The results are in the table below.

Table 6: Δ rate in year 1974,1980 and 1990

1974	1980	1990
-0.02533	-0.048	-0.0103

We reach the conclusion that wars will make the growth rate of oil consumption well bellow its average level.

As for other political influences, we can use the similar method, and the key is to choose the crucial factor to analysis.

4.5 The Environmental Influence on Oil Consumption

The use of oil inevitably leads to environment pollution. To protect the environment against excessive pollution, the government would adopt certain measures to limit the use of oil. So the environmental factors will also influence the oil demand.

We take the discharge amount of carbon dioxide generated by oil consumption as the scale to measure the environment pollution, and study the relation between the discharge amount of carbon dioxide and oil consumption. The data of discharge amount of carbon dioxide is shown in the table below.

Table 7: World Carbon Dioxide Emissions From the Consumption of Oil (million metric tons of carbon dioxide)

1993	1994	1995	1996	1997	1998	1999	2000	2001
9220	9284	9388	9586	9691	9766	9939	10138	10292

First, we also make a correlation analysis for the discharge amount of carbon dioxide and oil consumption, and the correlation coefficient is 0.9937. That is to say, the two variables are of strong positive linear dependence. Then we make a linear regression, and get the relationship between carbon dioxide emission and oil consumption as follows:

$$y = 10.09 \times x - 25320 \tag{9}$$

Where y: oil consumption;

x: discharge amount of carbon dioxide caused by oil consumption.

From Eq (9) we can work out the oil consumption at a controlled annual CO_2 emission growth rate. We simulate the future oil consumption under different annual CO_2 emission growth rate of 1%, 3%... The corresponding oil exhaustion time is shown in the figure below.

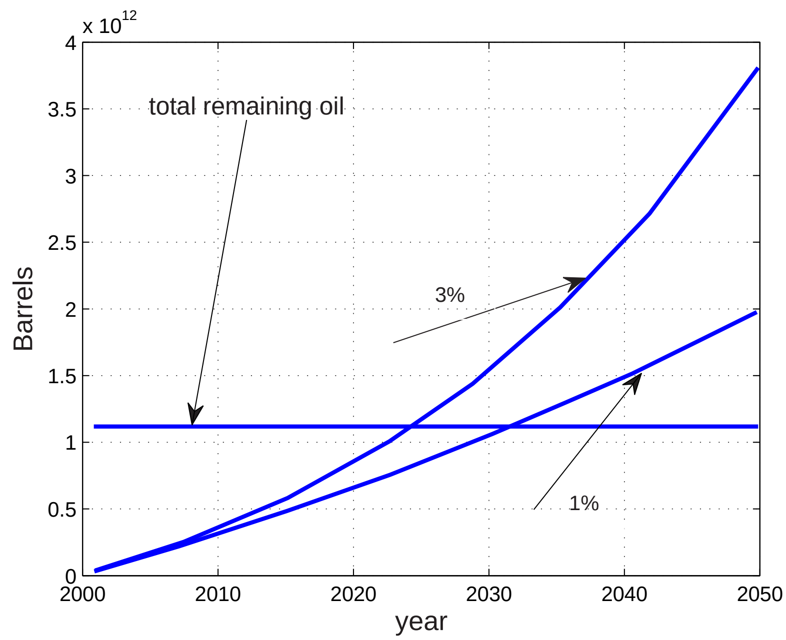

Figure 6: the curve of cumulative oil consumption under different annual CO_2 emission growth rates

Based on the analysis above, if we want to keep the annual CO_2 emission growth rate at a given level, we can give the corresponding annual oil consumption, and then fulfill the purpose of protecting the environment.

4.6 Limitations

The above models are based on the assumption that all the other factors are invariable when modeling for a specific factor. But this cannot be true in reality, because one factor may be interacted with others. Thus, the interactions should be taken into account in further study.

5 Task 3 The Sustainable Use of Oil

In order to prevent the excessive consumption of nonrenewable resource from rapid depletion, and to take into account of our offspring's interests, we must develop a policy to control the consumption of nonrenewable resource to give a rational consumption allocation between generations.

5.1 Assumptions

1. We assume that annual demand of oil can truly reflect oil consumption.
2. Oil consumption in year t can not be far less than that in year $t-1$.
3. The sustainable use of oil means that we must take the needs of our offspring into account. That is to say, we should provide a rational consumption allocation between generations.
4. We assume that one generation consists of n years.

5.2 The Criterion of Rational Consumption Allocation of Oil Between Generations.

Based on Task 1, we know that $U(t)+R(t)$ denotes the total remaining oil on earth at year t, and obviously, it holds that:

$$U(t)+R(t)=m_1+m_2 \tag{10}$$

Where m_1: the amount of oil for contemporary man;

m_2: the amount of oil left for offspring.

In order to give a rational allocation, we define:

$$\eta=\frac{m_2}{m_1}\times 100\%(0\leq\eta\leq\infty) \tag{11}$$

η is called **the degree of rational consumption allocation of oil.**

A large value of η indicates a high degree of rational consumption allocation, and vice versa. But if the value of η is too high, the amount of oil for contemporary human beings is too small to meet the needs. So we may give an alert analysis and management by choosing a suitable critical value of η, say η'. Having known the value of η', we can obtain the optimal allocation in the n years of one generation.

5.3 Modeling the Rational Consumption Allocation

We hope that future oil demand will not undergo a sharp decline, and expect oil to be allocated among generations fairly. Meanwhile, we wish that the resource is utilized in the most efficient way.

As is assumed, one generation consists of n years, and now we model at the interval of n years, i.e. one generation. Based on the assumptions, we can construct the following optimization model:

$$\max \sum_{i=1}^{n} c_i \times d_i$$

$$s.t. = \begin{cases} \frac{r}{\sum_{i=1}^{n} d_i} \geq \eta' \\ d_i \geq a \times d_{i-1}, i = 1, 2, \cdots, n \\ d_i \geq 0, i = 1, 2, \cdots, n \end{cases} \tag{12}$$

Where c_i: the utilization rate of oil at year i;

η': the degree of rational consumption allocation of oil between generations.

d_i: oil consumption at year i, and d_0 indicates oil consumption at initial time.

r: the total remaining oil in the first year of one generation.

α: a given percentage such that the oil consumption at a given year must not be less than α times the consumption in the previous year. And α is close to 1.

The objective here is to obtain the maximum total utilization rate of oil in n years. The first constraint condition assures a high rate of rational oil allocation between generations, while $d_i \geq \alpha \times d_{i-1}, i = 1, 2, \cdots, n$ makes sure that the oil consumption at year i is not less than α times the consumption at year $i - 1(i = 1, \ldots, n)$. The purpose is to guarantee a smooth oil demand change.

When η', c_i, r, α is given, we can obtain the optimal consumption allocation of oil between n years by solving the linear programming (12). As for the estimation of c_i here, we believe that the utilization rate should increase as time passes by, but is always smaller than 1, nevertheless. Thus,c_i can be regarded as in the following form:

$$c_i = 1 - a_1 e^{a_2 t} \tag{13}$$

where a_1 and a_2 are constants and can be determined by fitting historical data.

The result we get here may not increase in an exponential way. After determining the annual consumption, we can adjust it to include the factors such as economy, demography, politics and so on, until it is optimal.

Next, we give an illustration to show the consumption under optimal allocation after year 2004. For instance, we set $\alpha = 1$, $\eta' = 1.67$, $r = 1.0 \times 10^{12}$ and year 2004 to be the base time, i.e.d_0 is the oil consumption at 2004. Thus we can give the prediction of the oil consumption after 2005, which is shown in the following figure.

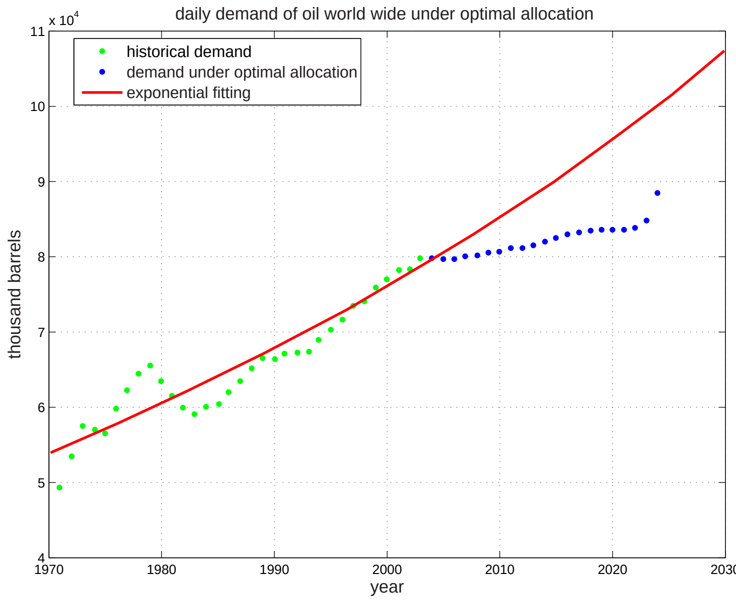

Figure 7: Oil Consumption Under Optimal Allocation

From the above figure, we can find that annual oil consumption under optimal allocation is far less than that in an exponential way. That is to say, the optimal consumption varies smoothly. But in the late phase of the prediction, consumption fluctuates sharply. This is due to the reason that we may have chosen an inappropriate η'-value. However, the choosing of η'-value is rather difficult because it should incorporate many factors such as population, price, specific economic environment, etc. Fortunately, the data at the prophase can clearly reflect the trend of oil consumption.

5.4 The Policy

To guarantee a sufficient amount of oil for our offspring to sustain their development, we must keep a rational allocation of oil. This can be done through the following measurements:

1. We may levy a relatively heavy tax on oil compared with other resources.
2. We should encourage the development of alternatives for oil.

'Security' Policy for Oil We believe that the problem of oil security arises mainly due to the different utilization rates among countries. For example, if a country with low oil utilization rate is assigned a redundancy of oil, whereas a country with high utilization rate is assigned an insufficient amount of oil, this phenomenon will lead to a great waste. Based on this idea, we can establish one model to find out the optimal distribution of oil among the countries with different utilization rates.

5.5 Assumptions:

1. The utilization rates among countries are different.
2. The low utilization rate is the primary source to result in the misuse and waste of oil.
3. The annual oil consumption of the whole world is according to optimal oil allocation model in Task 3
4. We do not take the trade barrier into account, and assume that reallocation of oil between countries is feasible.

5.6 Modeling:

Suppose there are n main oil-consuming countries in the world. Given the year t, we can establish the linear programming as follows:

$$\max \sum_{i=1}^{n} l_i(t) \times x_i(t)$$

$$s.t. \begin{cases} \sum_{i=1}^{n} x_i(t) = d(t) \\ x_i(t) \geq \alpha_i(t) \times x_i(t-1), i = 1, 2, \cdots, n \\ x_i(t) \geq 0, i = 1, 2, \cdots, n \end{cases} \tag{14}$$

Where $l_i(t)$: the oil utilization rate of country i at year t;

$x_i(t)$: the oil consumption of country i at year t;

$d(t)$: the world-wide oil consumption allocation at year t, which can be obtained using the model in Task 3.

$\alpha_i(t)$: the minimum ratio of $x_i(t)$ to $x_i(t-1)$.in percentage.

Remark: $\alpha_i(t)$, as a parameter, is close to 1, and it needs to be evaluated according to the different situations and the rationality of oil consumption of countries.

The first constraint condition means the sum of oil consumption of different countries at year t should equal total oil consumption at year t under optimal oil allocation.

The constraint condition $x_i(t) \geq \alpha_i(t) \times x_i(t-1), i = 1, 2..., n$ means the oil consumption of country i at year t is not less than a certain degree of the previous year's consumption. This degree is different among countries. From this model, we can see that the country with higher oil utilization rate has relative privilege to get the oil supply. The optimal solution of the linear programming represents the optimal oil distribution among countries at year t. Thus, we can meet the needs of every country with the minimal waste of oil.

5.7 Limitations of the model

The model we have designed above is for the sake of the interest of the entire world, but in reality, countries would more likely consider their own interests, leading to the result of tight trade barriers among countries, and consequently making it impossible to get the optimal distribution.

5.8 Conclusion

For a country with large oil demand but low utilization rate, we should fulfill its consume level while cut down the extra oil demand, while for a country with small oil demand but high utilization rate, we should meet its demand to the maximum extent, in order to prevent oil waste.

5.9 The Policy

To assure a good utilization of oil, and reduce unnecessary waste, we give the following measurements:

1. We may levy a relatively heavy tax on oil to countries with low utilization rate.
2. We can set a limit on the annual oil consumption for countries with low utilization rate

6 Task 5 Oil Exploitation vs. Natural Disasters

Oil exploitation is a tough task because the nature is very vulnerable. And if our activities go against the intrinsic laws of nature, we will be punished by it. Oilfield occupies a large range of area, destroys the vegetation in the vicinity, changes the components of the soil, and deteriorates the environment near by, and hence the animals will lose their habitat. With the exploitation of the field, it will influence the groundwater and cause desertification. An obvious instance is oil spill, which often leads to the pollution of nearby water area, and further destroys the entire water area eco-system.

Next, we mainly consider the effects of oil exploitation on natural disasters. We hope that the susceptibilities to disasters to be as low as possible while the demand for oil is satisfied.

6.1 Assumptions

1. The amount of oil exploited entirely turns into consumption.
2. The total amount of oil exploited can meet the need of economic development.

6.2 Modeling the short-term effects

We mainly consider the short-term effects, and assume that the total number of oilfields on the earth is n. According to the first assumption, oil exploitation is at the same level of oil consumption. In order to satisfy the sustainable development of the economy, we must try to keep the total output of all oilfields to be the same as the world-wide oil consumption under optimal allocation in Task 3. Thus, we have the following equation:

$$\sum_{i=1}^{n} x_i(t) = d(t) \tag{15}$$

Where $x_i(t)$: the output of the ith oilfield at year t;

$d(t)$: the world wide oil consumption under optimal oil allocation in Task 3.

For a specific oilfield i,we believe that its susceptibility to disasters has something to do with its output at a given year and the ratio of its cumulative output to its initial oil reserve.

Naturally, the more the oil is exploited, the greater the likelihood of disasters will be. We believe that they are of linear independence. And of course, a new oilfield and an old one will have different effects on the environment. This difference is shown by the ratio of the cumulative output to the initial oil reserve. Here we introduce a penalty function $e^{-\alpha(1-\lambda_i(t))}$,and get the following equation:

$$p_i(t) = k_i \times x_i(t) \times e^{-\alpha(1-\lambda_i(t))}, \tag{16}$$

Where k_i: proportion coefficient, which is determined by the position and exploitation method of the oilfield. Here, a small value of k_i represents a prior position and an advanced exploitation method.

p_i: the susceptibility to disasters.

$\lambda_i(t)$: the ith oilfield's ratio of its cumulative output until year t to its initial oil reserve.

We hope to minimize the total susceptibilities of different oilfields under the condition that the worldwide oil demands can be satisfied. That is:

$$\min \sum_{i=1}^{n} p_i(t)$$
$$s.t. \sum_{i=1}^{n} x_i(t) = d(t)$$

The solution to this optimization problem generates the outputs of every oilfield that can assure the least possibility to disasters. Obviously, we find out those oilfields with small value of k_i will have larger outputs and vice versa.

We also increase the value of n tentatively, i.e. to increase the number of oilfields, and find that the total susceptibilities will decrease. This is mainly due to the fact that during the prophase of exploitation,$\exp^{-\alpha(1-\lambda_i(t))}$ plays an important role, leading to the exploitation of new oilfields, and this will decrease the likelihood of disasters.

6.3 The Policy

Our policy is to increase the output of old oilfields with small k_i-value (those with a prior position and a priority of exploitation), and reduce the output of those with large k_i-value (those with an inferior position and a backward exploitation method). Also, if possible, we should exploit as many new oilfields as possible and decrease the exploitation of old oilfields, so as to control the susceptibilities to disaster.

6.4 Long-term Effects

As for the long-term effects, for a given oilfield, the average annual output should be minimal. So it allows us to carry out an intensive exploitation in one year and then a mild one in another. By doing so, there is no necessity for us to increase the number of new oilfields in a short time, and the benefit is that it allows us to have enough time to search for and construct new oil fields.

7 Task 6 The Development of The Alternatives for Oil

With the development of human, oil is being used ceaselessly. And we have estimated that if oil consumption increases in an exponential way, it will be used out in about thirty years. Even if we control the use of it, the time can only be prolonged by four to five years. So, at present stage, we urgently need an alternative for oil. For the sake of sustainable development, we must gradually accelerate the consumption of oil's alternative at the anaphase of the depletion of oil. The question is how should we apply the alternative in order to keep the economy stable during the transition period.

7.1 Assumptions

1. We only consider one kind of alternative.
2. Oil and its alternative have precisely the same function as energy resource .
3. The criterion of the measurement of oil and its alternative is their contributions to GDP.
4. We assume that the quantity of oil to produce unit of energy will not change as time elapses.

7.2 Analysis

We assume that the cost for oil to produce unit energy is c_1, and that of its alternative is c_2 m such that $c_1 \leq c_2$. This is on the basis the fact that the cost for oil to produce unit energy is lowest compared with any other resources (See [5]). Because of this, the consumption of oil is far greater than those of its alternative.

But we must realize the fact that the total amount of remaining oil on the earth is declining, so the price of oil will correspondingly increase on the whole, leading to the rise of oil cost. On the other hand, with the advancement of the technology for the alternative, their prince will fall, and as a result makes a low cost possible. The general tendency can be found in the following figure:

From the trend above we get the conclusion that with the rising of the cost for unit oil, the consumption of it will become less and less, but its unit alternative will have low cost, and then the demand of them will according increase, until the day when oil is completely replaced by them. Now, the question we are faced with is at what a speed should oil be replaced.

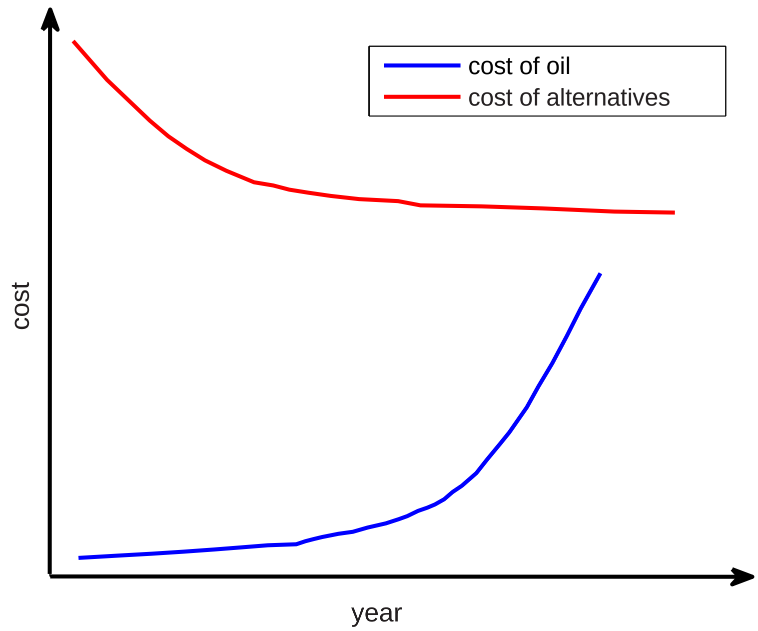

Figure 8: The Trend of Cost for Oil and Its Alternative to Produce unit of energy

7.3 Modeling

From the above model for optimal oil distribution between countries, we have obtained the conclusion that the growth of consumption in future years will increase slowly. Hence, what our research is interested in is the time when oil demand begins to decline, i.e. the transition time of oil and its alternative.

Let GDP value to be G, x to be the consumption of oil, y to be the consumption of the alternative, and t to be time. We can see that G is a function dependent on x and y, so

$$G = G(x, y) \tag{17}$$

From (17), we have:

$$\frac{dG}{dt} = \frac{\partial G}{\partial x} \times \frac{dx}{dt} + \frac{\partial G}{\partial y} \times \frac{dy}{dt} \tag{18}$$

During the anaphase of oil consumption, we want to keep it at a low level in order to assure a smooth transition. We think that an exponential decline will seem to be reasonable, so we let:

$$x(t) = x(t_0) \times e^{b \times (t - t_0)} (t > t_0, b < 0) \tag{19}$$

then

$$\frac{dx}{dt} = x(t_0) \times b \times e^{b\times(t-t_0)} \tag{20}$$

Where b :the decline rate of oil consumption;

$x(t)$:oil consumption at year t.

t_0:the time when oil demand begins to decline.

Substitute (20) into (18),and then we get:

$$\frac{dy}{dt} = \frac{\frac{dG}{dt} + x(t_0) \times b \times e^{b\times(t-t_0)} \times \frac{\partial G}{\partial x}}{\frac{\partial G}{\partial y}} \tag{21}$$

$\frac{dy}{dt}$ denotes replacement rate, $\frac{\partial G}{\partial x}$ means the contribution rate of oil to GDP,and $\frac{\partial G}{\partial y}$ means the contribution rate of the alternative to GDP. After we have known the value of b, t_0, $\frac{dG}{dt}$, $\frac{\partial G}{\partial x}$ and $\frac{\partial G}{\partial y}$, the replacement ratio of the alternative — $\frac{dy}{dt}$, can be calculated, and with the information of $\frac{dy}{dt}$, we can work out the consumption of the alternative which will guarantee a stable economy. This consumption works as a guidance when we exploit the alternative. Next, we choose the year 2010 as t_0 and then simulate to study the consumption of oil and the alternative. The result is shown in the following figure:

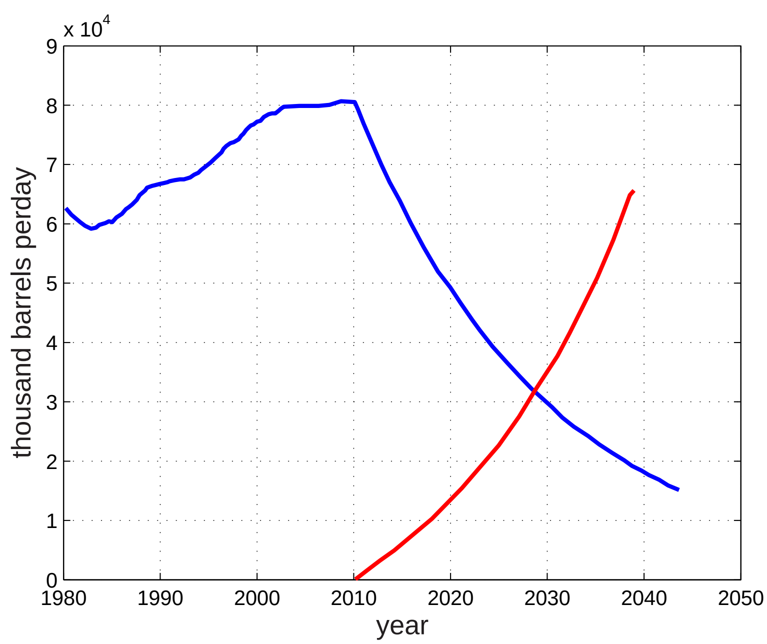

Figure 9: The Curves of the Daily Consumption of Oil and Its Alternative During the Transition Time

Thus, we can determine the quantity of the alternative to be exploited which will assure the sustainable development of the economy.

7.4 Improvements of the Model

As for the case where more than one alternative is available, we can also solve the problem using the method above. But the degree of exploitation difficulty is involved. To this end, we've searched for some information, and have drafted the following report. In this report, we've specifically introduced some alternatives for oil, and hope this will pave the way for future work.

Alternative Energy and Potential Oil Substitutes

Oil, a kind of nonrenewable resource, will be used up in the near future.

As it is pointed by C.J. Campbell —– " The coming oil crisis will be just that because the transition will not be easy, but I sometimes think that the world needs a change in direction in any case. From the ashes of the oil crisis may arise a better and more sustainable planet. It must at least become more sustainable as mankind lives out his allotted life span in the fossil record. Whether or not it is better depends on how well we manage the transition."

Transition to an entirely renewable sustainable energy resource economy with resulting changes in lifestyles is inevitable. Will it be done with intelligence and foresight or will it be done by harsh natural forces? This is one of the main challenges, which lie before us.

The crisis is imminent. Those who anticipate can do well from the economic and political discontinuity; those who react can survive; but those who continue to live in the past will suffer. And we don't have long to prepare.

In this report, alternative energy and related technologies will be talked about.

Alternative energy:

Alternative energy refers to energy sources, which are not based on the burning of fossil fuels or the splitting of atoms. The renewed interest in this field of study comes from the undesirable effects of pollution (as witnessed today) both from burning fossil fuels and from nuclear waste byproducts. Fortunately there are many means of harnessing energy, which have less damaging impacts on our environment. Here are some possible alternatives:

Solar energy:

Solar energy is one the most resourceful sources of energy for the future. One of the reasons for this is that the total energy we receive each year from the sun is around 35,000 times the total energy used by man. However, about 1/3 of this energy is either absorbed by the outer atmosphere or reflected back into space (a process called albedo).

Using solar energy provide a ticket for the environmental lobby. Solar energy is presently being used on a smaller scale in furnaces for homes and to heat up swimming pools. On a larger scale use, solar energy could be used to run cars, power plants, and space ships. But the restrictions are: A huge number of solar panels, which are not cheap, and of course, less sun means less power, so it's not fit in countries where sunshine is not a constant.

Wind power:

Wind power is another alternative energy source that could be used without producing by-products that are harmful to nature. And human have a long history of using wind power in the form of windmill. Like solar power, harnessing the wind is highly dependent upon weather and location. The average wind velocity of Earth is around 9 m/sec. And the power that could be produced when a windmill is facing the wind of 10 mi/hr. is around 50 watts.

Geothermal energy:

Geothermal energy is an alternative energy source, although it is not resourceful enough to replace more than a minor amount of the future's energy needs. Geothermal energy is obtained from the internal heat of the planet and can be used to generate steam to run a steam turbine. This in turn generates electricity, which is a very useful form of energy.

Because of the costs required in upkeep and the shortage of potential sites, geothermal energy systems are more inefficient than other alternative energy sources.

Hydroelectricity:

Hydroelectricity comes from the damming of rivers and utilizing the potential energy stored in the water. As the water stored behind a dam is released at high pressure, its kinetic energy is transferred onto turbine blades and used to generate electricity. This system has enormous costs up front, but has relatively low maintenance costs and provides power quite cheaply. In the United States approximately 180,000 MW of hydroelectric power potential is available, and about a third of that is currently being harnessed.

Tide:

Similar to the more conventional hydroelectric dams, the tidal process utilizes the natural motion of the tides to fill reservoirs, which are then slowly discharged through electricity-producing turbines. The former USSR produced 300 MW in its Lumkara plant using this method.

Oil substitutes:

Biodiesel is made entirely from vegetable oil, it does not contain any sulfur, aromatic hydrocarbons, metals or crude oil residues. The absence of sulfur means a reduction in the formation of acid rain by sulfate emissions which generate sulfuric acid in our atmosphere. The reduced sulfur in the blend will also decrease the levels of corrosive sulfuric acid accumulating in the engine crankcase oil over time.

The lack of toxic and carcinogenic aromatics (benzene, toluene and xylene) in Biodiesel means the fuel mixture combustion gases will have reduced impact on human health and the environment. The high cetane rating of Biodiesel (ranges from 49 to 62) is another measure of the additive's ability to improve combustion efficiency. Unlike other "clean fuels" such as compressed natural gas (CNG), Biodiesel and other biofuels are produced from renewable agricultural crops that assimilate carbon dioxide from the atmosphere to become plants and vegetable oil. The carbon dioxide released this year from burning vegetable oil Biodiesels, in effect, will be recaptured next year by crops growing in fields to produce more vegetable oil starting material. But unfortunately it haven't been putted into wide use.

Gas hydrate is an ice-like crystalline solid, its building blocks consist of a gas molecule surrounded by a cage of water molecules. Thus, it is similar to ice, except that the crystalline structure is stabilized by the guest gas molecule within the cage of water molecules. Many gases have molecular sizes suitable to form hydrate, including such naturally occurring gases as carbon dioxide, hydrogen sulfide, and several low-carbon-number hydrocarbons, but most marine gas hydrates that have been analyzed are methane hydrates. They occur in the pore spaces of sediments, and may form cements, nodes or layers. **Gas Hydrate** is found in sub-oceanic sediments in the polar regions (shallow water) and in continental slope sediments (deep water), where pressure and temperature conditions combine to make it stable.

Gas hydrate is an important topic for study for three reasons:

It contains a great volume of methane, which indicates a potential as a future energy resource

It may function as a source or sink for atmospheric methane, which may influence global climate

It can affect sediment strength, which can initiate landslides on the slope and rise

Other things may be used as a substitute of oil. For example, some countries confronted with the oil crisis use **ethanol** as a substitute of oil. Although these substitutes may provide a possible outlet to the oil crisis, whether they can solve the energy problem is really unknown.

Conclusion:

Considering the concurrent problems of population size and stabilization, the adjustment of economies and lifestyles, the challenge of conversion to alternative energy resources is clearly exigent and formidable. A realistic appraisal of the future encourages people to properly prepare for the coming events. Delay in dealing with the issues will surely result in unpleasant surprises. Let us get on with the task of moving orderly into the post-petroleum paradigm.

References

[1] Data of world population
http://www.cpirc.org

[2] Chris Nelder, Oil Crises DelayCA World Price Forecast,2004,
http://www.betterworld.com

[3] Data of oil demand, supply, reserves, and emission of CO_2
http://www.eia.doe.gov

[4] C.J Campbell, The Coming Oil Crisis, Published by MULTI-SCIENCE PUBLISHING CO.LTD. 1997

[5] Raymond Vernon, The Oil Crisis, Published by George J. McLeod Limited, Toronto

[6] Atmospheric CO_2 concentrations (ppmv),
http://serc.carleton.edu/files/introgeo/interactive/examples/mlco2.doc

[7] http://www.cc.utah.edu

[8] Alternative resources
http://curtrosengren.typepad.com

[9] Alternative resources
http://www.hubbertpeak.com

[10] Alternative resources
http://www.cytoculture.com

[11] Alternative resources
http://www.epa.gov/

[12] Gas Hydrate web sites,
http://woodshole.er.usgs.gov/project-pages/hydrates/external.html