An EM Algorithm for Multipath State Estimation in OTHR Target Tracking

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***Abstract*—Multi-path tracks of a single target exist in sky-wave over-the-horizon radar (OTHR) surveillance, due to the availability of multiple signal propagation paths through the ionosphere. Different from the traditional multipath data association and estimation methods for OTHR target tracking, a novel scheme for joint multipath data association and state estimation (JMAE) is developed based on the expectation-maximization (EM) framework. The proposed scheme has the iterative optimization of identification (including data association and ionospheric mode identification) and estimation (including path-conditional state estimation and multipath track fusion). Due to the mechanism of iterative optimization, the closed loop between identification and estimation is established, which is desirable to deal with the coupling issue of identification risk and estimation errors. The simulation shows that JMAE is superior to the well-known multipath probabilistic data association (MPDA).**

***Index Terms*—Multipath, joint identification and estimation, expectation-maximization, target tracking, OTHR.**

I. Introduction

HIGH-FREQUENCY over-the-horizon radar (OTHR) systems that exploit skywave propagation, i.e., reflection and refraction of radar signals from the ionosphere, provide wide-area surveillance capabilities to detect and track targets beyond the line-of-sight horizon [1], [2]. The capability of OTHR systems to cover a surveillance area beyond the range of conventional line-of-sight radars makes them uniquely important in a number of applications [2].

A significant problem in OTHR is the phenomenon of multipath propagation, whereby radar signals scattered from the same target arrive at the receiver via different propagation paths. The difficulty in tracking a target for OTHR arises from the uncertain origin of the measurements and the uncertainty in the presence of multipath propagation, since it is often impossible to select a radar operating frequency that results in single-mode propagation to the region of interest, so that multipath propagation is unavoidable [3]. Much attention has been paid on the multipath data association and state estimation for OTHR. In our opinions, these methods belong to one of the following two categories [4].

The first category is track-based where multi-path tracks are obtained in slant or radar coordinates based on probabilistic data association (PDA) [5]–[7] or Viterbi data association (VDA) [8]–[11], and further fused in ground coordinates via coordinate registration transformation, which requires knowledge of the ionospheric propagation conditions. The most well-known fusion solution is the multi-hypothesis multipath track fusion [12]–[14] through establishing propagation path-dependent track-to-target association hypotheses, calculating the hypothesis probabilities, and further obtaining the fused track in the minimum mean squared error sense. This category is computationally effective but has two main drawbacks. One is the fusion risk by the fact that multipath tracks are always not accurate and even missed or intermittent due to the low detection probability of each mode, and hence the track fusion is not always reliable. The other is the fusion approximation that the errors among multipath tracks are assumed independent of each other, deviating from the fact that the multipath tracks contain the common process noises, and/or one measurement may be utilized in updating several multipath tracks.

An alternative category is measurement-based where track prediction and measurement-target-mode association are implemented in slant coordinates and the track updating fulfilled in ground coordinates. One well-known scheme is multi-path probabilistic data association (MPDA) [3], [15], which combines multi-path measurements for state updating. Another scheme is multipath Viterbi data association (MVDA) [16], which chooses the optimal hypothesis (measurement and propagation associations) to maximize the likelihood function. Measurement-based methods have the advantage over track-based methods in circumventing the problems posed by error dependence among multipath tracks, and utilizing multipath detections of the target simultaneously to improve the tracking performance. Recently, there have been some developments on OTHR multitarget tracking such as multiple detection MHT (MD-MHT) [17] and multiple detection JPDA (MD-JPDA) [18], which also belong to the measurement-based category.

In fact, the OTHR multipath data association and state estimation problem involves both identification and estimation. Identification[1] in this sense includes data association and propagation mode, while estimation includes path-conditional state

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[1]Here, the term 'identification' refers to the hypothesis decision of choosing the best one among a discrete set of candidates, instead of the conventional meaning of determining the most likely propagation mode of a multi-path track in OTHR target tracking.

estimation and multipath track fusion. Essentially, the identification and estimation are highly coupled and affect each other. Namely, the identification mistake of multipath data association deteriorates state estimation and track fusion while the estimate error triggers the identification risk. However, both track-based methods and measurement-based methods combine identification with estimation, i.e., all the methods mentioned above are open-loop[2]. It is highly demanded to develop the joint optimization of identification and estimation. Considering the case of multiple observations of a single target in clutter, Pulford and Logothetis [19] presented the expectation maximization data association (EMDA) for fixed-interval Kalman smoothing conditioned on the MAP estimation of measurement-to-mode association sequence in the expectation-maximization (EM) framework. However, simulations were not presented for EMDA so its performance is not known.

In this paper, a novel scheme for joint multipath data association and state estimation (JMAE) based on the EM framework is developed. The JMAE carries out the identification of ionospheric mode and measurement association in the *E-Step*, where the pseudo-measurement and *a posterior* probabilities of each propagation mode are derived. Meanwhile, the JMAE updates the state estimation in the *M-Step*, where path-conditional state estimates and multipath track fusion are implemented. More importantly, due to the iterative optimization, the closed loop between identification and estimation is established, which is desirable in dealing with the coupling of identification and estimation.

The rest of this paper is organized as follows: Section II formulates the problem of OTHR multipath target tracking. In Section III, the JMAE algorithm is discussed and developed. The performance of JMAE is compared with MPDA in Section IV, and the conclusion is presented in Section V.

Throughout this paper, the superscripts "-1" and "T" represent the inverse and transpose operations of a matrix, respectively; $N(x;\mu,P)$ represents the Gaussian probability function of x with mean μ and covariance P; E is mathematical expectation. The notation $\otimes$ refers to the kronecker product. I and 0 denote the identity matrix and the zero matrix with proper dimensions, respectively. $I\{\bullet\}$ denotes the indicator function, which equals one if the event $\{\bullet\}$ is true, or zero otherwise. For a vector x, define $\mathcal{D}(x,P) = x^T P^{-1} x$, where P is a positive-definite matrix.

II. Problem Formulation

Consider a single target in the clutter environment. The target state in ground coordinates at time instant k is defined by $x_k = [R_k, \dot{R}_k, \vartheta_k, \dot{\vartheta}_k]^T$, which consists of ground range R, range rate $\dot{R}$, bearing ϑ and bearing rate $\dot{\vartheta}$. The discrete-time state equation is

$$x_{k+1} = f(x_k) + v_k \tag{1}$$

[2]Here, the term 'open-loop' refers to there is no feedback between estimation and identification, that is, the updated state estimate is not feedback to re-identify the ionospheric mode and measurement association. From the perspective of control theory, an "open-loop" scheme is always weak in the simultaneous optimization of coupling estimation and identification, especially under severe uncertainties. On the contrary, the term "closed loop" means the existence of feedback between estimation and identification.

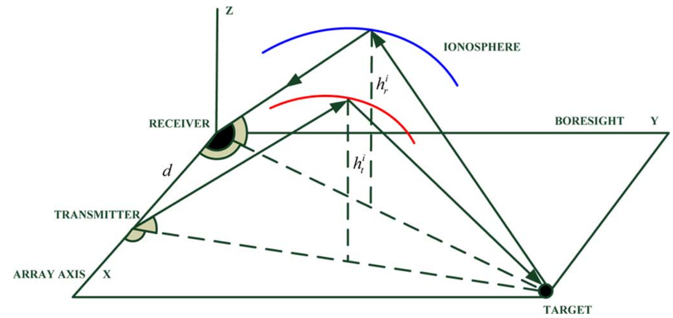


Fig. 1. Radar signal propagation via the ith propagation mode.

where the state transition function $f(\bullet)$ is given; v_k is a zero-mean white Gaussian noise vector with the known covariance Q_k; k is time instant.

In [3], the multipath propagation is shown in Fig. 1. The virtual ionospheric heights on transmission and reception via the ith propagation mode are denoted by h_t^i and h_r^i. The target measurement via the ith propagation mode is detected with the detection probability P_d^i, and the corresponding measurement of the detected target measurement at time instant k is

$$y_k^i = h^i(x_k) + w_k^i, \qquad i = 1, 2, \ldots, t \tag{2}$$

where $y_k = [r_k, \dot{r}_k, \theta_k]^T$ consists of slant range r_k, rang rate $\dot{r}_k$ and azimuth θ_k; t is the number of possible ionospheric propagation modes; the measurement function $h^i(\bullet)$ is assumed known in contrast to the approach in [15]; the measurement noise w_{k+1}^i is a zero-mean white Gaussian noise vector with known covariances $R_{k+1}^i > 0$. The initial state x_0 is Gaussian distributed with known mean $\bar{x}_0$ and associated covariance Σ_0. Here v_k, w_k^i and x_0 are mutually independent.

The standard uniform and Poisson models are used to represent the clutter. The probability density function (pdf) of clutter measurements in a region $G(k)$ with the corresponding volume $V_G(k)$ is assumed to be uniform distributed [3]:

$$p_c(y_k) = \begin{cases} V_G(k)^{-1} & \text{if } y_k \in G(k) \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The number of clutter points in a region $G(k)$ with the corresponding volume $V_G(k)$ is assumed to be Poisson distributed [3] with point mass function $\mu_c(\bullet)$:

$$\mu_c(n) = \frac{(\lambda V_G(k))^n e^{-\lambda V_G(k)}}{n!}, \quad n = 0, 1, 2, \ldots \tag{4}$$

where λ is the spatial density of the clutter.

The measurement set in radar or slant coordinates at time instant k is defined by

$$Y_k = \{y_k(1), y_k(2), \ldots, y_k(M_k)\} \tag{5}$$

where M_k is the number of measurements at time instant k.

The *validation region* for each filter is established in order to reduce the association hypothesis events that described in [3]. Denote the volumes of the gates and its corresponding number of measurements at time instant k via ith propagation mode by V_k^i and m_k^i, respectively.

Definition 2.1: Define the measurement-to-target association and the target measurement-to-mode association at time instant k by $a_k \in \{-1, 0, \ldots, M_k\}$ and $b_k \in \{1, 2, \ldots, t\}$, respectively:

- "$a_k = -1$": the target measurement doesn't exist and all measurements are due to clutter;
- "$a_k = 0$": the target exists, but all measurements are due to clutter;
- "$a_k = n, n > 0$": the target exists, and the nth measurement is originated from the interested target;
- "$b_k = i$": the detected target measurement is via the ith propagation mode.

Definition 2.2: Define the target existence at time instant k by E_k and $\bar{E}_k$, respectively [3]:

1) E_k: the target exists/is observable via any propagation modes;
2) $\bar{E}_k$: the target does not exist or is failed to be detected via any a propagation mode.

Assumption 2.1: Target existence is modelled as a two-state Markov chain with known transition probabilities [3]:

$$\Delta_0 \triangleq P(E_k|E_{k-1}) \qquad \Delta_1 \triangleq P(E_k|\bar{E}_{k-1}) \tag{6}$$

Up to present, all above considerations are the same as that in [3]. Here we present a new assumption about propagation modes as follows:

Assumption 2.2: The random process $\{b_k\}$ obeys a discrete-time t-state Markov chain with known transition probabilities:

$$p_b(i,j) \triangleq p\{b_{k+1} = j | b_k = i\} \quad \forall i, j \in \{1, \ldots, t\} \tag{7}$$

Remark 2.1: Different from the conventional assumption that the propagation modes should be independent of each other, we present a Markov chain model about propagation modes by the consideration at three aspects: (a) Markov-switching representation has been shown effective in dealing with abrupt mode changes of hybrid systems [20] in many fields, such as speech recognition [21], fault diagnosis [22], and maneuvering target tracking [23]; (b) Markov-switching representation explores the temporal correlation of mode evolvement in statistics and the existing independence assumption is just the special case of (7) with Markov transition matrix being the identity matrix; (c) The Markov parameters have physical meaning, for example, the mean sojourn time of a Markov chain can be easily determined from the diagonal transition probabilities [23], or be identified online [24]. In other words, application-specific information is permitted to be introduced for performance improvement.

The object of this paper is to obtain the optimal estimation $\hat{x}_k$ of target state x_k, given the measurement set $\{Y_1, \ldots, Y_k\}$.

It is worth mentioning that the measurement association a_k and propagation modes b_k should be identified accurately, i.e., jointly estimating continuous state and discrete association/mode information is required.

III. Joint Multipath Data Association and State Estimation

Notation 3.1: Denote the sequence of measurements $Y_{k-l}^k = \{Y_{k-l}, \ldots, Y_k\}$, the sequence of states $X_{k-l}^k = \{X_{k-l}, \ldots, X_k\}$, the sequence of measurement-to-target associations $A_{k-l}^k = \{a_{k-l}, \ldots, a_k\}$ and the sequence of measurement-to-mode associations $B_{k-l}^k = \{b_{k-l}, \ldots, b_k\}$, respectively. Let $\rho_{k-l}^k = \{\rho_{k-l}, \ldots, \rho_k\}$ with $\rho_i = \{a_i, b_i\}$.

Definition 3.1: Define $\hat{x}_{i|m:k} = E(x_i|y_m, \ldots, y_k)$ as the conditional mean, and $\Sigma_{i,j|m:k} = \mathrm{cov}(x_i, x_j|y_m, \ldots, y_k)$ as the corresponding cross-covariance. In the following part, we call $\hat{x}_{i|m:i}$, and $\hat{x}_{i|i+1:k}$ as the forward and backward state estimates, respectively; $\Sigma_{i,i|m:i}$ and $\Sigma_{i,i|i+1:k}$ are the corresponding state estimation covariances, respectively.

Definition 3.2: Define the complete-data log-likelihood function L_{k-l}^k and its corresponding conditional expectation, also called Q-function $\mathcal{Q}_{k-l}^k(r)$ at time interval $[k-l, k]$ by

$$L_{k-l}^k = \log p\left(X_{k-l}^k, Y_{k-l}^k, \rho_{k-l}^k | Y_1^{k-l-1}\right) \tag{8}$$

$$\mathcal{Q}_{k-l}^k(r) = E\left(L_{k-l}^k | Y_{k-l}^k, \hat{X}_{k-l}^k(r)\right) \tag{9}$$

where $\hat{X}_{k-l}^k(r)$ is the state estimate of X_{k-l}^k at the rth iteration.

Definition 3.3: Define the joint probability density function $\gamma_j^{(r)}(n,\tau)$, the forward function $\alpha_j^{(r)}(n,\tau)$, the backward function $\beta_j^{(r)}(n,\tau)$, and the observation merit function $\varepsilon_j^{(r)}(n,\tau)$ at time instant j and rth iteration by:

$$\gamma_j^{(r)}(n,\tau) \triangleq p\left(a_j = n, b_j = \tau | Y_{k-l}^k, \hat{X}_{k-l}^k(r)\right) \tag{10}$$

$$\alpha_j^{(r)}(n,\tau) \triangleq p(x_j, y_j, a_j = n, b_j = \tau) \tag{11}$$

$$\beta_j^{(r)}(n,\tau) \triangleq p\left(X_{j+1}^k, Y_{j+1}^k | a_j = n, b_j = \tau, x_j, y_j\right) \tag{12}$$

$$\varepsilon_j^{(r)}(n,\tau) \triangleq p\left(x_j, y_j | \hat{x}_{j-1}(r), y_{j-1}, a_j = n, b_j = \tau\right) \tag{13}$$

The EM algorithm has been widely used in the engineering and statistical literature as an iterative optimization procedure for computing maximum likelihood (or MAP) parameter estimates of *incomplete data problem* [25]–[31]. The well-known probabilistic multi-hypothesis tracker (PMHT) [27], [32], [33], regarding the data association as the missing data and based on the EM framework to obtain the state estimation in MAP sense. While, the proposed JMAE addresses the problem of optimal target tracking in a multipath environment. Specifically, to obtain the target state estimate $\hat{X}_{k-l}^k$, the measurements Y_{k-l}^k, data association A_{k-l}^k and propagation mode B_{k-l}^k should be given. However, data association A_{k-l}^k and propagation mode B_{k-l}^k is missing. In other words, the OTHR target tracking can be treated as the problem of estimation in the presence of incomplete data, and hence be solved in the EM framework.

The JMAE consists of the following two iterative steps:

E-Step

$$\mathcal{Q}_{k-l}^k(r) = E\left(L_{k-l}^k | Y_{k-l}^k, \hat{X}_{k-l}^k(r)\right) \tag{14}$$

M-Step

$$\hat{X}_{k-l}^k(r+1) = \arg\max_{X_{k-l}^k} \mathcal{Q}_{k-l}^k(r) \tag{15}$$

where r corresponds to the rth iteration. The *E-Step* involves the calculation of the conditional expectation with respect to ρ_{k-l}^k using the current state estimate $\hat{X}_{k-l}^k(r)$ and measurements Y_{k-l}^k. The *M-Step* provides an updated state estimate $\hat{X}_{k-l}^k(r+1)$ through maximizing Q-function $\mathcal{Q}_{k-l}^k(r)$.

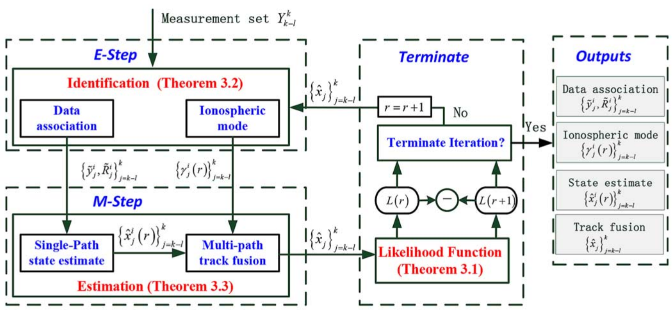


Fig. 2. Function flow of the proposed JMAE.

A fusion framework for joint multipath data association and estimation is proposed as shown in Fig. 2, including the following three modules, respectively.

- *The likelihood function* L_{k-l}^k *(Theorems 3.1)*. The JMAE based on EM algorithm consists of two steps, namely, getting Q-function $\mathcal{Q}_{k-l}^k$ in *E-Step* and maximizing it in *M-Step*. The primary is to get the likelihood function.
- *The identification of joint data association and propagation modes (Theorems 3.2)*. Given the rth iterative state estimate $\hat{X}_{k-l}^k(r)$ and measurements Y_{k-l}^k, the joint probability density of data association $\hat{A}_{k-l}^k$ and propagation modes $\hat{B}_{k-l}^k$ is calculated in *E-Step*. After obtaining the joint probability density, data association and propagation modes are identified via computing marginal distribution density, and further obtained the posterior probability of each mode and its corresponding pseudo-measurement.
- *The estimation of path-conditional state and multipah track fusion (Theorems 3.3)*. Given the identified data association $\hat{A}_{k-l}^k$ and propagation modes $\hat{B}_{k-l}^k$, the path-conditional track $\hat{X}_{k-l:k}^i$ for each propagation mode is estimated in *M-Step*, and then fuse those path-conditional tracks to obtain the state estimate $\hat{X}_{k-l}^k$.

Remark 3.1: There are several differences between EMDA and the proposed JMAE although both of them are based on the EM framework.

1) The first difference is on modeling: the EMDA treats the target state as missing data whereas the JMAE treats data association and propagation mode as missing data. The EMDA regards each measurement-target-mode association as one association hypothesis and searches the best one among all possible hypotheses, while the JMAE splits the measurement-target-mode association into measurement-target and measurement-mode associations to calculate the posteriori probability of each mode and its corresponding pseudo-measurement.
2) The second difference is on implementation: the EMDA implements the state estimation in *E-Step* by using the Kalman Smoother (KS) and identifies multipath data association in *M-Step* via the Viterbi algorithm; On the contrary, the JMAE implements the identification of multipath data association in *E-Step* via the hidden Markov smoother (HMMS) and obtains the state estimate in *M-Step* by using the KS.
3) The third difference is on processing structure: the EMDA consists of two closed loop iterative modules that are multipath data association and state estimate, while the JMAE consists of four closed loop iterative modules that are data association, propagation mode, path-conditional state estimation and multipath track fusion.
4) The fourth difference is on computational cost: the EMDA deals with the OTHR multipath data association and state estimation in series while the JMAE in parallel. Hence, the proposed JMAE will be more computation-effective than the EMDA, and the computational comparison is given as follows. Each iteration of EMDA requires $O(\sum_{j=k-l}^{k}(\prod_{i=1}^{t} m_j^i)^2) + O(p^3 l)$ computational cost, and $O(\sum_{j=k-l}^{k}(\sum_{i=1}^{t} m_j^i)^2) + O(p^3 l)$ for our proposed JMAE, where p is the dimension of the state.

Remark 3.2: As shown in Fig. 2, the rth state estimate $\hat{X}_{k-l}^k(r)$ is used to identify the parameter $\hat{\rho}_{k-l}^k(r)$, and then the identified parameter $\hat{\rho}_{k-l}^k(r)$ is utilized to recalculate the $r+1$th state estimate $\hat{X}_{k-l}^k(r+1)$. Due to such a iterative optimization, the closed loop of estimation and identification is established, which is helpful in improving the performance of estimation and identification under multiple uncertainties, and avoids an exhaustive search through a larger solution space.

Remark 3.3: The proposed framework is hybrid of measurement-based and track-based ideas. Namely, it is via single mode measurement-based methods in the identification stage, which combines multipath measurements to produce a pseudo measurement and *a posterior* probability of each ionosphere mode. Whereas it is track-based in the estimation stage, which uses the each mode state estimate and its corresponding probability to obtain the fused state. It has the advantages of making full use of the multipath measurements to overcome the problem of low detection probability via measurement-based methods, while inheriting the practicable computational cost of the track-based methods.

Theorem 3.1 (Likelihood Function): The log-likelihood function L_{k-l}^k defined in (8) has the following calculation expression:

$$L_{k-l}^k = L_{0,k-l}^k + L_{1,k-l}^k + L_{2,k-l}^k + L_{3,k-l}^k + L_{4,k-l}^k \quad (16)$$

with

$$\begin{aligned} L_{0,k-l}^k = & -\frac{1}{2}\sum_{j=k-l}^{k}\log(|2\pi Q_j|) - \sum_{j=k-l}^{k}\log(V_j^i)m_j^i \\ & + \sum_{j=k-l}^{k}\left(\log\left(V_j^i\right) - \frac{1}{2}\log\left(|2\pi R_j^i|\right)\right) \\ & \times I\left\{a_j^i \neq -1, 0\right\} \end{aligned} \quad (17)$$

$$\begin{aligned} L_{1,k-l}^k = & -\frac{1}{2}\log(|2\pi\Sigma_{s,s|1:s}|) \\ & -\frac{1}{2}\mathcal{D}\left(x_s - \hat{x}_{s|1:s}, \Sigma_{s,s|1:s}\right) \end{aligned} \quad (18)$$

$$L_{2,k-l}^k = -\frac{1}{2}\sum_{j=k-l}^{k}\mathcal{D}(x_j - f(x_{j-1}), Q_j) \quad (19)$$

$$L^k_{3,k-l} = -\frac{1}{2}\sum_{j=k-l}^{k} \mathcal{D}\left(y_j - h^i(x_j), R^i_j\right) \tag{20}$$

$$L^k_{4,k-l} = \sum_{j=k-l}^{k} \log(p(a_j|b_j)p_b(u,v)) + \log(p(b_s)) \tag{21}$$

where $s = k - l - 1$.

Proof: See Appendix A

The Q-function $\mathcal{Q}^k_{k-l}(r)$ defined in (9) is given by

$$\mathcal{Q}^k_{k-l}(r) = \mathcal{Q}^k_{0,k-l} + L^k_{1,k-l} + L^k_{2,k-l} + \mathcal{Q}^k_{3,k-l} + \mathcal{Q}^k_{4,k-l} \tag{22}$$

with

$$\begin{aligned}\mathcal{Q}^k_{0,k-l} = &-\frac{1}{2}\sum_{j=k-l}^{k}\log(|2\pi Q_j|) - \sum_{j=k-l}^{k}\sum_{i=1}^{t}\log\left(V^i_j\right) m^i_j P_{\mathcal{A}}\\ &+ \sum_{j=k-l}^{k}\sum_{i=1}^{t}\left(\log\left(V^i_j\right) - \frac{1}{2}\log\left(|2\pi R^i_j|\right)\right) P_{\mathcal{B}}\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{Q}^k_{3,k-l} = &-\frac{1}{2}\sum_{j=k-l}^{k}\sum_{i=1}^{t}\sum_{n=1}^{m^i_j}\mathcal{D}\left(y_j(n) - h^i(x_j), R^i_j\right)\\ &\times \gamma^{(r)}_j(n,i)\end{aligned} \tag{24}$$

$$\begin{aligned}\mathcal{Q}^k_{4,k-l} = &\sum_{j=k-l}^{k}\sum_{i=1}^{t}\sum_{n=1}^{m^i_j}\log(p(a_j = n|b_j = i))P_{\mathcal{C}}\\ &+ \sum_{j=k-l}^{k}\sum_{u=1}^{t}\sum_{v=1}^{t}\log(p_b(u,v))P_{\mathcal{D}}\\ &+ \sum_{i=1}^{t}\log(p(b_{k-l-1} = i))P_{\mathcal{E}}\end{aligned} \tag{25}$$

where the variables of $P_{\mathcal{A}}$, $P_{\mathcal{B}}$, $P_{\mathcal{C}}$, $P_{\mathcal{D}}$, $P_{\mathcal{E}}$ are denoted by

$$P_{\mathcal{A}} = p\left(b_j = i|Y^k_{k-l}, \hat{X}^k_{k-l}(r)\right) \tag{26}$$

$$P_{\mathcal{B}} = 1 - p\left(a^i_j = -1, 0|Y^k_{k-l}, \hat{X}^k_{k-l}(r)\right) \tag{27}$$

$$P_{\mathcal{C}} = p\left(a_j = n|b_j = i, Y^k_{k-l}, \hat{X}^k_{k-l}(r)\right) \tag{28}$$

$$P_{\mathcal{D}} = p\left(b_{j-1} = u, b_j = v|Y^k_{k-l}, \hat{X}^k_{k-l}(r)\right) \tag{29}$$

$$P_{\mathcal{E}} = p\left(b_{k-l-1} = i|Y^k_{k-l}, \hat{X}^k_{k-l}(r)\right) \tag{30}$$

In order to obtain the Q-function $\mathcal{Q}^k_{k-l}$, the key is to evaluate the conditional expectation of the joint probability density function $\gamma^{(r)}_j(n,i)$, which can be obtained by a HMMS.

Theorem 3.2 (E-Step): The joint probability density function $\gamma^{(r)}_j(n,i)$ defined by (5) is given as follows:

$$\gamma^{(r)}_j(n,i) = \frac{\alpha^{(r)}_j(n,i)\beta^{(r)}_j(n,i)}{\sum_{\tau=1}^{t}\sum_{d=-1}^{m^\tau_j}\alpha^{(r)}_j(d,\tau)\beta^{(r)}_j(d,\tau)} \tag{31}$$

with

$$\begin{aligned}\alpha^{(r)}_{j+1}(n,\tau) = &\varepsilon^{(r)}_{j+1}(n,\tau)p(a_{j+1} = n|b_{j+1} = \tau)\\ &\times\sum_{i=1}^{t}\sum_{d=-1}^{m^i_j}\alpha^{(r)}_j(d,i)p_b(i,\tau)\end{aligned} \tag{32}$$

$$\begin{aligned}\beta^{(r)}_j(n,\tau) = &p(a_j = n|b_j = \tau)\\ &\times\sum_{i=1}^{t}\sum_{d=-1}^{m^i_j}\varepsilon^{(r)}_{j+1}(d,i)\beta^{(r)}_{j+1}(d,i)p_b(\tau,i)\end{aligned} \tag{33}$$

$$\alpha^{(r)}_1(n,\tau) = \varepsilon^{(r)}_1(n,\tau)p(a_1 = n|b_1 = \tau)p(b_1 = \tau) \tag{34}$$

$$\beta^{(r)}_k(n,\tau) = 1 \tag{35}$$

Proof: See Appendix B

The *posteriori* probability $\gamma^{(r)}_j(i)$ of mode i is as follows:

$$\gamma^{(r)}_j(i) \triangleq p\left(b_j = i|Y^k_{k-l}, \hat{X}^k_{k-l}(r)\right) = \sum_{n=-1}^{m^i_j}\gamma^{(r)}_j(n,i) \tag{36}$$

The pseudo-measurement in the MAP sense $\tilde{y}^i_j(r)$ of mode i and its corresponding covariance $\tilde{R}^i_j(r)$ are given by:

$$\tilde{y}^i_j(r) = \frac{\sum_{n=1}^{m^i_j}\gamma^{(r)}_j(n,i)y_j(n)}{1 - \gamma^{(r)}_j(-1,i) - \gamma^{(r)}_j(0,i)} \tag{37}$$

$$\left(\tilde{R}^i_j(r)\right)^{-1} = \left(R^i_j\right)^{-1}\left(1 - \gamma^{(r)}_j(-1,i) - \gamma^{(r)}_j(0,i)\right) \tag{38}$$

Through putting the (36)–(38) into (22), followed by some simple algebraic manipulations, the Q-function $\mathcal{Q}^k_{k-l}(r)$ is rewritten as follows:

$$\begin{aligned}\mathcal{Q}^k_{k-l}(r) = &-\frac{1}{2}\sum_{j=k-l}^{k}\sum_{i=1}^{t}\mathcal{D}\left(\tilde{y}^i_j(r) - h^i(x_j), \tilde{R}^i_j(r)\right)\gamma^{(r)}_j(i)\\ &-\frac{1}{2}\sum_{j=k-l}^{k}\mathcal{D}(x_j - f(x_{j-1}), Q_j)\\ &-\frac{1}{2}\mathcal{D}(x_s - \hat{x}_{s|1:s}, \Sigma_{s,s|1:s}) + \mathcal{C}\end{aligned} \tag{39}$$

where $\mathcal{C}$ is an additive term not involving X^k_{k-l}.

Furthermore, on equating the derivative of the right hand of (39) with respect to X^k_{k-1} to zero, we directly obtain the MAP state estimate of $\hat{X}^k_{k-l}(r+1)$ that should satisfy (40),

$$\begin{aligned}&\sum_{i=1}^{t}\left(H^i(x_j)\right)^T\left(\tilde{R}^i_j(r)\right)^{-1}\left(\tilde{y}^i_j(r) - h^i(x_j)\right)\gamma^{(r)}_j\\ &\quad\times(i) - Q^{-1}_j(x_j - f(x_{j-1}))\big|_{x_j=\hat{x}_{j|k-l:k}(r+1)} = 0\end{aligned} \tag{40}$$

where H^i is the Jacobian matrix of h^i.

The computation-intensive numerical optimization, such as gradient descent, can be utilized to solve a symmetric block tridiagonal nonlinear system of equations in (40). Here we present an alternative approximate solution but computation-effective.

Theorem 3.3 (M-Step): An approximate solution for state estimation is given as follows:

$$\hat{x}_{j|k-l:k}(r+1) = \frac{\sum_{i=1}^{t} \gamma_j^{(r)}(i) \left(\Sigma^i_{j,j|k-l:k}\right)^{-1} \hat{x}^i_{j|k-l:k}}{\sum_{i=1}^{t} \gamma_j^{(r)}(i) \left(\Sigma^i_{j,j|k-l:k}\right)^{-1}} \tag{41}$$

$$\Sigma^{-1}_{j,j|k-l:k}(r+1) = \sum_{i=1}^{t} \gamma_j^{(r)}(i) \left(\Sigma^i_{j,j|k-l:k}\right)^{-1} \tag{42}$$

where $\hat{x}^i_{j|k-l:k}$ and $\Sigma^i_{j,j|k-l:k}$ is the state smoothing at time instant j of mode i. This can be carried out by using a fixed interval smoother [34], which insists of the forward and backward filtered outputs as follows:

$$\hat{x}^i_{j|k-l:k} = \Sigma^i_{j,j|k-l:k}\left[\left(\Sigma^i_{j,j|k-l:j}\right)^{-1}\hat{x}^i_{j|k-l:j} + \left(\Sigma^i_{j,j|j+1:k}\right)^{-1}\hat{x}^i_{j|j+1:k}\right] \tag{43}$$

$$\Sigma^i_{j,j|k-l:k} = \left[\left(\Sigma^i_{j,j|k-l:j}\right)^{-1} + \left(\Sigma^i_{j,j|j+1:k}\right)^{-1}\right]^{-1} \tag{44}$$

where the forward estimate $\hat{x}^i_{j|k-l:j}$, $\Sigma^i_{j,j|k-l:j}$ and the backward estimate $\hat{x}^i_{j|j+1:k}$, $\Sigma^i_{j,j|j+1:k}$ can be obtained by a linear filter, and the linearization can be used as in the EKF framework for nonlinear case.

: See Appendix C

The iteration will be terminated if the values of likelihood functions at two consecutive iterations are close enough or the number of iterations reaches the upper bound, i.e.,

$$\frac{\left|L^k_{k-l}(r+1) - L^k_{k-l}(r)\right|}{\left|L^k_{k-l}(r+1)\right|} < \delta_L \text{ or } r \geq r_{\max} \tag{45}$$

where $0 < \delta_L \ll 1$ is the iterative terminated threshold, and $r_{\max}$ is the upper bound of iterative number.

The appealing property of EM is that the likelihood function is monotonically nondecreasing with the iteration number before reaching the local optimization value. The proof of convergence of the proposed method is similar to that in [35] based on the Zangwill's global convergence theorem [36].

The derived JMAE is summarized as Table I.

TABLE I
THE JMAE ALGORITHM

Step 1: Initialization. ($r = 0$) Given the measurement set Y^k_{k-l}, initialize the state estimate $\hat{X}^k_{k-l}(0)$ to suitable values.
Step 2: Iteration. For $r = 1, 2, \ldots$.
Step 3a: *E-Step*. (1) Using the current state estimate $\hat{X}^k_{k-l}(r)$ and measurement set Y^k_{k-l}, compute the joint probability density function $\gamma_j^{(r)}(n,i)$ via a hidden Markov smoother by (31)-(35). (2) Calculate the posteriori mode probability $\gamma_j^{(r)}(i)$ by (36), and the pseudo-measurement $\tilde{y}^i_j(r)$ and its corresponding covariance $\tilde{R}^i_j(r)$ of each mode i by (37)-(38).
Step 3b: *M-Step*. (3) Compute the state estimation $\hat{x}^i_{j\|k-l:k}$ of each mode i using a fixed interval smoother by (43)-(44). (4) Combined the state estimation $\hat{x}^i_{j\|k-l:k}$ and the posteriori model probability $\gamma_j^{(r)}(i)$, we can obtain the state fusion $\hat{x}_{j\|k-l:k}$ by (41)-(42).
Step 3c: *Termination*. (5) If the values of likelihood functions $L^k_{k-l}(r)$ and $L^k_{k-l}(r+1)$ are close enough or the number of iteration reaches to the maximum number of iterations by (45), then the iteration terminates, else set $r = r + 1$, and go to Step 3a.
Step 4: Recursion. ($r = 0, k = k + 1$), initialize $\hat{x}_{j\|k-l:k}(r)$ to the last ultimate value, and go to iterate loop.

TABLE II
THE PERFORMANCE OF JMAF FOR DIFFERENT SCENARIO

	R(km)	$\dot{R}$(km/s)	b(rad)	$\dot{b}$(rad/s)	$Time$(s)
$P_d = 1$, $\lambda V = 0$	0.65	$8.9e-4$	$1.0e-4$	$7.5e-7$	1.9
$P_d = 1$, $\lambda V = 400$	1.41	$6.9e-4$	$8.2e-4$	$1.5e-6$	19.2
$P_d = 0.4$, $\lambda V = 0$	0.87	$6.6e-4$	$3.8e-4$	$7.1e-7$	7.8
$P_d = 0.4$, $\lambda V = 400$	2.2	$7.4e-4$	$7.9e-4$	$1.4e-6$	23.4

IV. SIMULATION

Considering the same simulation environment for a simple non-maneuvering tracking using a two-layer ionospheric model as [3], the target state equation is assumed to be linear since the target is far away from the receiver, and the detailed scenario parameters refer to Table III in Appendix. We compare the proposed JMAE with the MPDA. The filter initialization of the JMAE requires an initial ground target state and covariance to be specified, as well as the initial target confidence. The stochastic initialization strategies, including emEM [37] and RndEM [38] algorithms, share the common idea of trying different initial values of parameters and choosing the one that yields the largest local maximum. In this paper, we present the simulation of JMAE with different initialization choices.

The target tracking trajectory, OTHR multipath detections and clutter are shown in Fig. 3. Fig. 4 shows the procedure of the proposed JMAE at the fifth time step, the EE propagation mode (the uppermost ellipsoid region) has the maximum mode posteriori probability by 0.786, the FF propagation mode has the secondary by 0.152, the EF and FE propagation mode has the minimum by 0.0285 and 0.0329. The reason is that the target measurement and less clutter fall into the validation region of EE propagation mode, while too much clutter of the EF and FE propagation modes, which makes the identification of EF and FE propagation mode unreliable.

The comparison between the proposed JMAE and the MPDA is shown in Fig. 5, supporting that the JMAE outperforms MPDA in state estimation. It is mainly because the proposed JMAE is a joint estimation and identification solution and has the closed loop in iterative batch processing consists of

TABLE III
THE PARAMETERS SETTING OF THE SIMULATION SCENARIO

Category	**Parameters**	**Value**
Scenario	number of dwells	20
	time between dwells	20 seconds
	region size (range)	1000-1400km
	region size (azimuth)	0.069813-0.17453rad
	region size (range rate)	0.013889-0.22222km/s
	Target Initial State	(1100km,0.15km/s,0.10rad,8.72e-05rad/s)
	expected number of clutter	400 per dwell
	sensor noise covariances R^i	$diag(25km^2, 1e-6(km/s)^2, 9e-6(rad)^2)$
	detection probability P_d^i	0.4
Ionosphere	number of modes t	4
	ionosphere height(h_t, h_r)	(100km, 260km)
	TX-RX distance	100km
JMAE	state transition matrix F	$I_2 \otimes \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$
	measurement matrix h^i and its jacobian matrix H^i	see [3] and its correction [39]
	process noise covariance Q	$blockdiag\left(\begin{bmatrix} 7.8e-6 & 4.4e-6 \\ 4.4e-6 & 1.3e-6 \end{bmatrix}, \begin{bmatrix} 1.5e-12 & 1.1e-13 \\ 1.1e-13 & 1.1e-14 \end{bmatrix}\right)$
	mode transition matrix $p(u, v)$	$0.05\mathbf{1}_4 + 0.8I_4$
	initial state covariance $\Sigma_{0\mid 0}$	$diag(25, 1e-6, 9e-6, 4.5e-8)$
	target confidence transition matrix	$\begin{bmatrix} 0.75 & 0.25 \\ 0.05 & 0.95 \end{bmatrix}$
	gate probability P_g	0.971
	initial track confidence	0.2
	track existence	0.8
	track deletion	0.3
	window length l	5 dwells
	iterative terminated threshold δ_L	$1e-5$
	maximum number of iterations r_{max}	10

two smoothers (HMMS in the *E-Step* and KS in the *M-Step*), while the MPDA is a recursion algorithm with a filter.

Fig. 6 shows estimation performance with different values of the iteration number r. As the number of iteration increases, the estimation errors decrease, which verifies the convergence of the proposed JMAE. Meanwhile, the JMAE with $r = 0$ is an open-loop smoother, otherwise, it is a closed loop processing. It shows that the closed loop processing is benefit of dealing with the coupling of identification and estimation.

The estimation performance with different values of window length l is illustrated as Figs. 7 and 8. As the window length l increases, the estimation performance improves and the computational cost linearly increases with the regression confidence 97.8%.

The performance comparison of state filtering and one-step smoothing is shown in Fig. 9, and the smoothing is a litter bit better than filtering. The RMSE of each mode state estimate and the multipath track fusion based on 100 Monte Carlo runs are shown in Fig. 10. Depending on the probability of detection P_d and the clutter level λV, we consider four different scenarios illustrated in Table II. It shows that the proposed JMAE maintains good estimation performance at the cost of computation of a low detection probability P_d, and as the clutter density increases, the estimation performance decreases while still maintains steady tracking.

V. CONCLUSION

Considering target tracking of OTHR inevitably facing the problem of multiple propagation modes, a new joint multipath data association and state estimation is considered. The JMAE solution is obtained via iterative optimizing identification (including data association and ionosphere mode identification) and state estimation (including path-conditional state estimation and multipath track fusion) based on the EM framework. The simulation verifies the proposed JMAE. Along the result of this paper, several further researches could be done. One is how to extend the proposed EM scheme to the case of the uncertainty of ionosphere parameters. Another is to compare the proposed EM scheme with EMDA and further find out its rapid implementation.

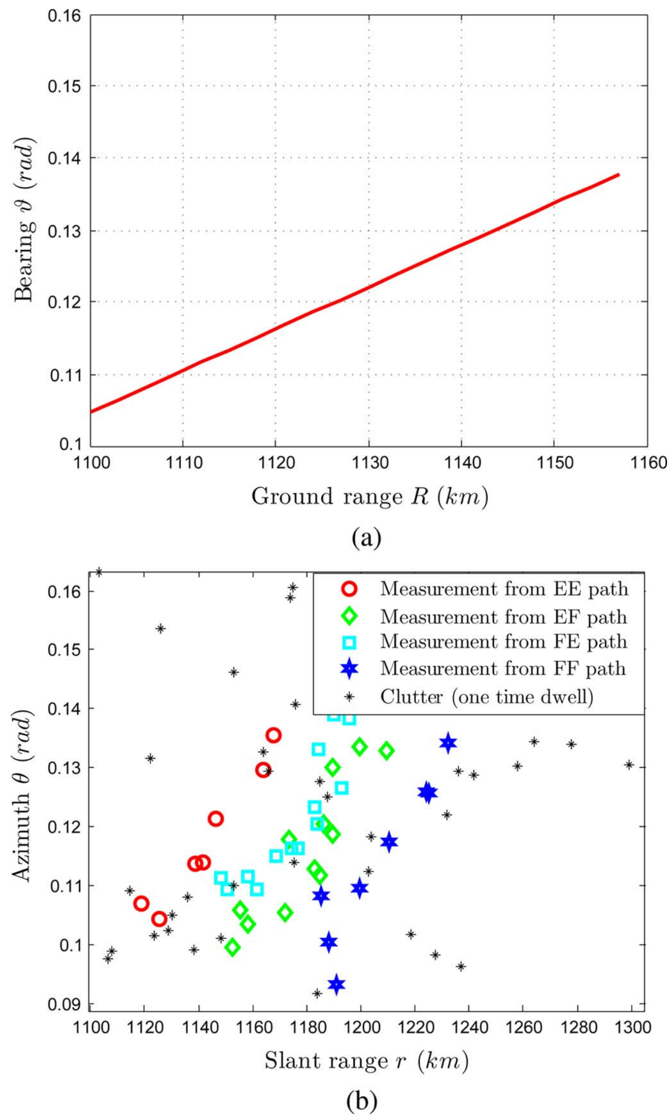


Fig. 3. The scenario of target trajectory and its corresponding OTHR multipath detection. (a) Target trajectory in ground coordinates. (b) OTHR multipath detection in slant coordinates.

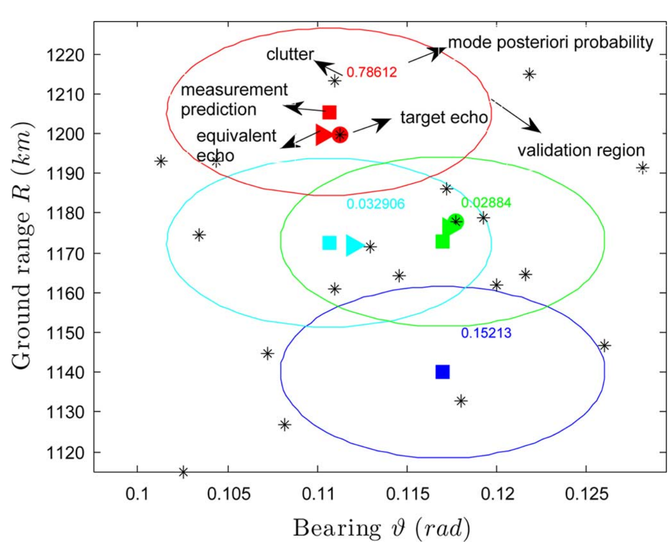


Fig. 4. The display of OTHR multipath target tracking process (The fifth time instant).

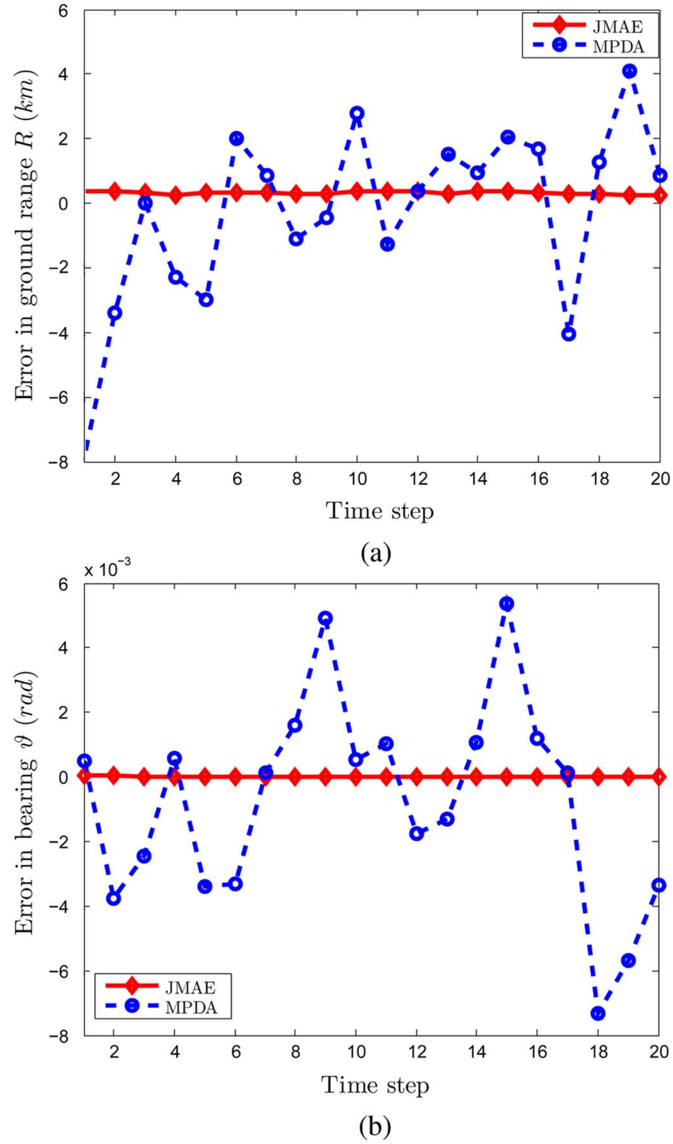
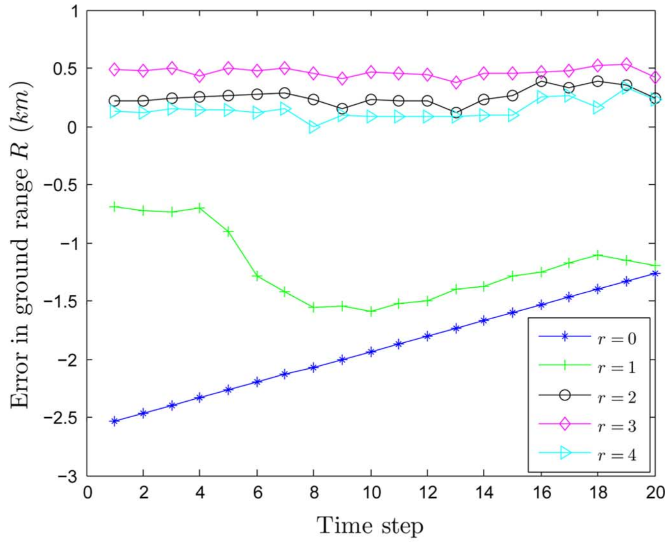


Fig. 5. The performance comparison of JMAE and MPDA. (a) Estimate error in ground range. (b) Estimate error in bearing.

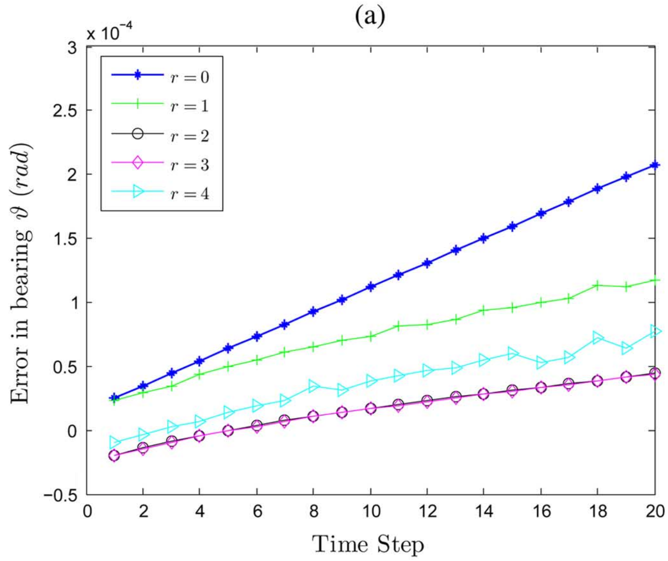
APPENDIX A

The complete-data log-likelihood function L_{k-l}^{k} can be expressed as

$$\begin{aligned} L_{k-l}^{k} &= \log p\left(X_{k-l}^{k}, Y_{k-l}^{k}, \rho_{k-l}^{k} | Y_1^{k-l-1}\right) \\ &= \sum_{j=k-l}^{k} \log p\left(y_j | x_j, a_j, b_j\right) + \sum_{j=k-l}^{k} \log p\left(x_j | x_{j-1}\right) \\ &\quad + \log p\left(x_{k-l-1} | Y_1^{k-l-1}\right) + \sum_{j=k-l}^{k} \log p(a_j | b_j) \\ &\quad + \sum_{j=k-l}^{k} \log p_b(u,v) + \log p(b_{k-l-1}) \end{aligned} \tag{46}$$



(a)



(b)

Fig. 6. The estimate error with different iteration $r = 0, 1, \ldots, 4$. (a) Estimate error in ground range. (b) Estimate error in bearing.

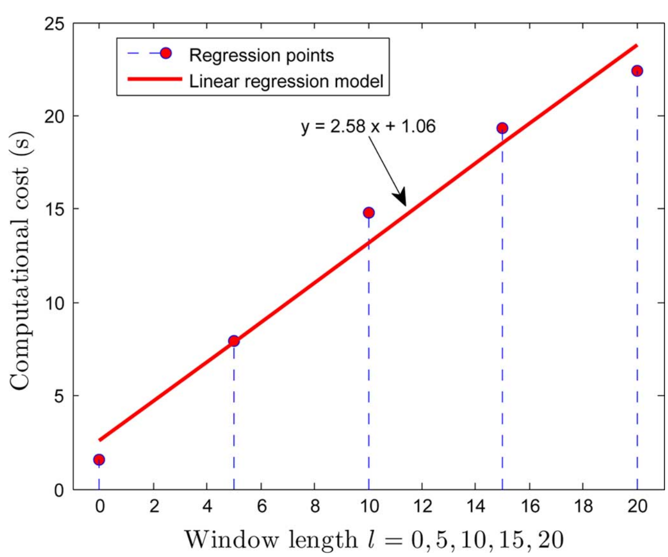
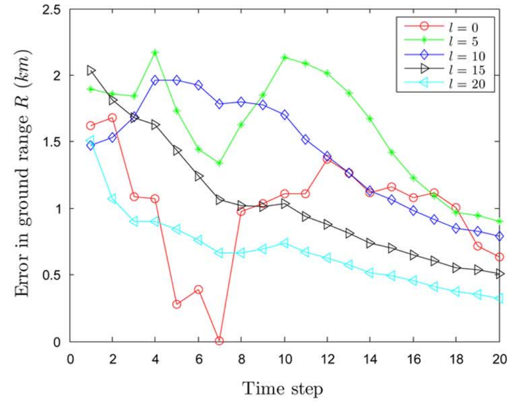
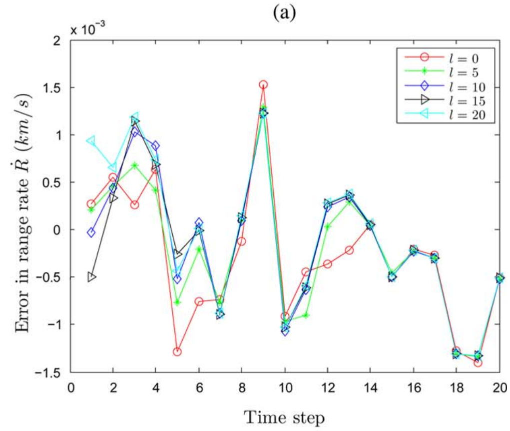


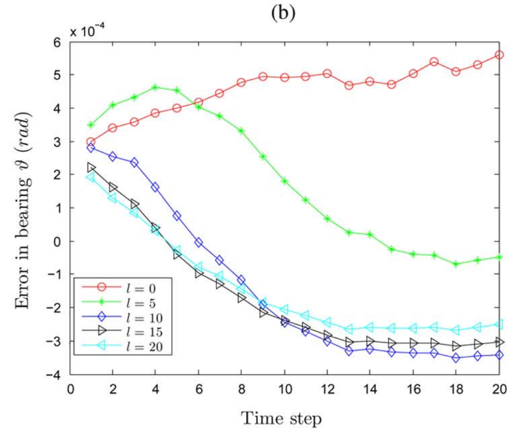
Fig. 7. Computational cost with different $l = 0, 5, 10, 15, 20$.



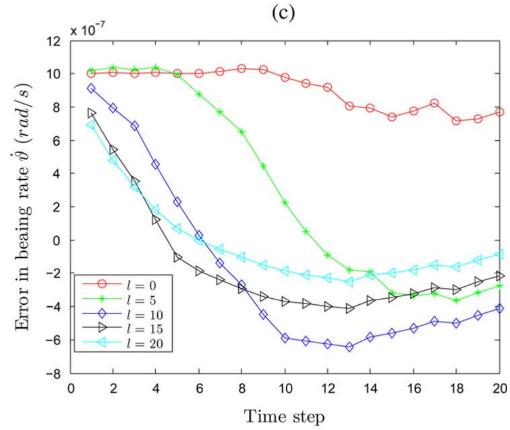
(a)



(b)



(c)



(d)

Fig. 8. The estimate error with different $l = 0, 5, 10, 15, 20$. (a) Estimate error in ground range. (b) Estimate error in range rate. (c) Estimate error in bearing. (d) Estimate error in bearing rate.

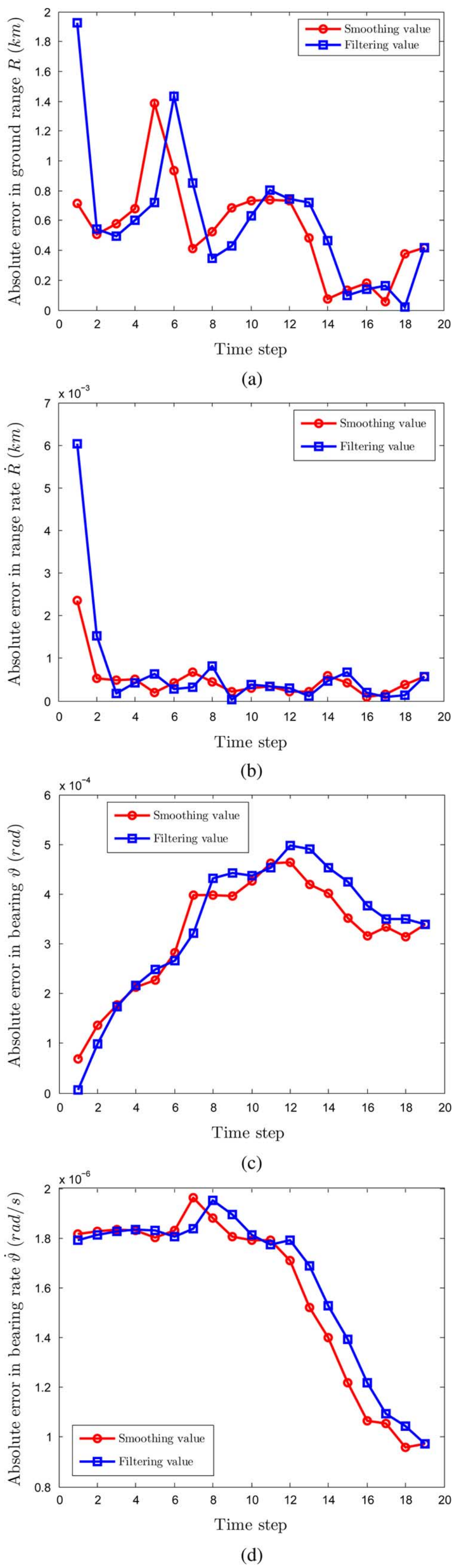


Fig. 9. The performance comparison of filtering and smoothing. (a) Estimate error in ground range. (b) Estimate error in range rate. (c) Estimate error in bearing. (d) Estimate error in bearing rate.

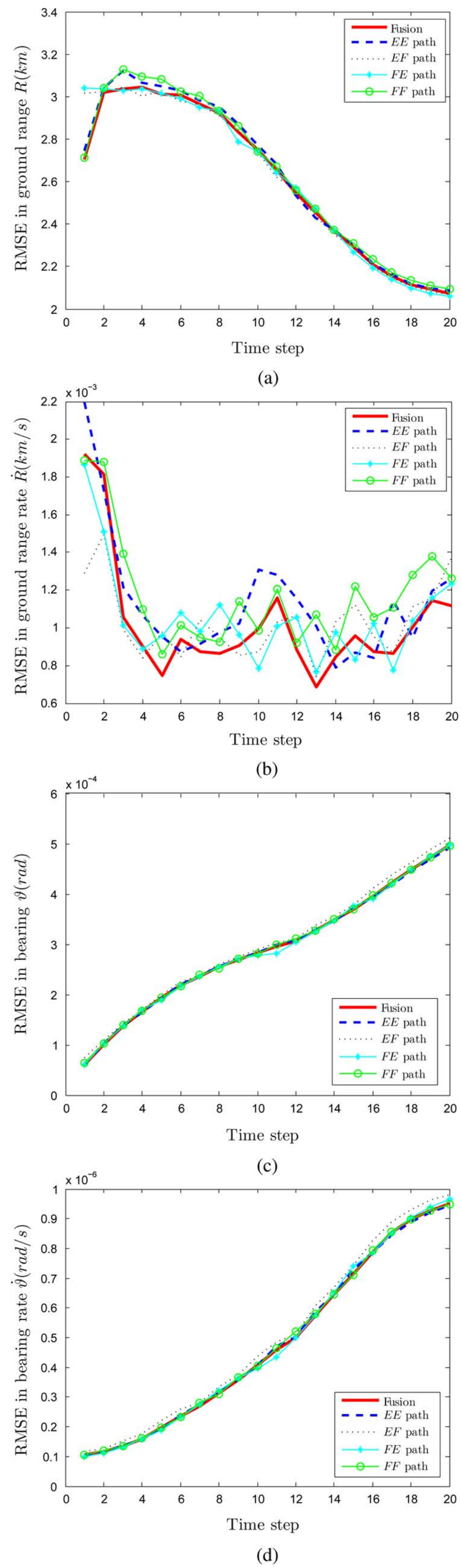


Fig. 10. RMSE of the proposed JMAE. (a) RMSE in ground range. (b) RMSE in ground range rate. (c) RMSE in bearing. (d) RMSE in bearing rate.

Consider the log-likelihood function L_{k-l}^{k} in (46), the PDF of x_{k-l-1}, x_j, and $y_j^i(n)$ are all Gaussian. That is,

$$p\left(x_{k-l-1}|Y_1^{k-l-1}\right) = N(x_{k-l-1}; \hat{x}_{k-l-1}, \Sigma_{k-l-1}) \tag{47}$$
$$p(x_j|x_{j-1}) = N(x_j; f(x_{j-1}), Q_j) \tag{48}$$
$$p(y_j|x_j, a_j = n, b_j = i) = \begin{cases} \left(V_j^i\right)^{-m_j^i+1} p\left(y_j^i(n)|x_j\right), & a_j \in \{1,\ldots,m_j^i\} \\ \left(V_j^i\right)^{-m_j^i}, & a_j \in \{-1,0\} \end{cases} \tag{49}$$

where

$$p\left(y_j^i(n)|x_j\right) = N\left(y_j(n); h^i(x_j), R_j^i\right) \tag{50}$$

The association of measurement-to-target at mode

$$p(a_j|b_j = i) = \begin{cases} \frac{P_E(j)P_d^iP_g\mu_c(m_j^i-1)}{\left(m_j^i\delta_j\right)}, & a_j \in \{1,\ldots,m_j^i\} \\ P_E(j)\left(1-P_d^iP_g\right)\mu_c\left(m_j^i\right)\delta_j^{-1}, & a_j = 0 \\ (1-P_E(j))\mu_c\left(m_j^i\right)\delta_j^{-1}, & a_j = -1 \end{cases} \tag{51}$$

where $\delta_j^{-1} \triangleq (1-P_E(j)P_d^iP_g)\mu_c(m_j^i) + P_E(j)P_d^iP_g\mu_c(m_j^i - 1)$ is a normalization constant, and put (47)–(51) into (46), we obtain (16)–(21).

Appendix B

According to the Bayesian rule and the Markov model, we have

$$\gamma_j^{(r)}(n,i) = \frac{\alpha_j^{(r)}(n,i)\beta_j^{(r)}(n,i)}{\sum_{b=1}^{t}\sum_{a=-1}^{m_j^b}\alpha_j^{(r)}(a,b)\beta_j^{(r)}(a,b)} \tag{52}$$

Based on the Bayesian total probability theorem, the forward function $\alpha_j^{(r)}(n,\tau)$ is given by
(See (53)),

$$\begin{aligned}\alpha_{j+1}^{(r)}(n,\tau) &= p(x_{j+1}, y_{j+1}|a_{j+1} = n, b_{j+1} = \tau) \\ &\quad \times p(a_{j+1} = n, b_{j+1} = \tau) \\ &= \sum_{i=1}^{t}\sum_{d=-1}^{m_j^b} p(x_{j+1}, y_{j+1}|x_j(r), y_j, a_{j+1} = n, b_{j+1} = \tau) \\ &\quad \times p(x_j(r), y_j, a_j = d, b_j = i) \\ &\quad \times p(a_{j+1} = n, b_{j+1} = \tau) \\ &= \varepsilon_{j+1}^{(r)}(n,\tau)p(a_{j+1} = n|b_{j+1} = \tau) \\ &\quad \times \sum_{i=1}^{t}\sum_{d=-1}^{m_j^i}\alpha_j^{(r)}(d,i)p_b(i,\tau)\end{aligned} \tag{53}$$

and the backward function $\beta_j^{(r)}(n,\tau)$ is given by (See (54)).

$$\begin{aligned}\beta_j^{(r)}(n,\tau) &= p\left(x_{j+1}, y_{j+1}, X_{j+2}^k, Y_{j+2}^k|a_j = n, b_j = \tau, x_j, y_j\right) \\ &= p(x_{j+1}, y_{j+1}|x_j, y_j)p\left(X_{j+2}^k, Y_{j+2}^k|x_{j+1}, y_{j+1}\right) \\ &= \sum_{i=1}^{t}\sum_{d=-1}^{m_j^i} p(x_{j+1}, y_{j+1}|x_j, y_j, a_{j+1} = d, b_{j+1} = i) \\ &\quad \times p\left(X_{j+2}^k, Y_{j+2}^k|x_{j+1}, y_{j+1}, a_{j+1} = d, b_{j+1} = i\right)\big) \\ &\quad \times p(a_j = n|b_j = \tau)p(b_j = \tau, b_{j+1} = i) \\ &= p(a_j = n|b_j = \tau) \\ &\quad \times \sum_{i=1}^{t}\sum_{d=-1}^{m_j^i}\varepsilon_{j+1}^{(r)}(d,i)\beta_{j+1}^{(r)}(d,i)p_b(\tau,i)\end{aligned} \tag{54}$$

where $\varepsilon_j^{(r)}(n,\tau)$ is defined by (13) with $p(x_j|\hat{x}_{j-1}(r))$ and $p(y_j|x_j, a_j = n, b_j = \tau)$ in (48) and (49), respectively.

As in [21], the initialization of $\alpha_j^{(r)}(n,\tau)$ and $\beta_j^{(r)}(n,\tau)$ are given as follows:

$$\alpha_1^{(r)}(n,\tau) = \varepsilon_1^{(r)}(n,\tau)p(a_1 = n|b_1 = \tau)p(b_1 = \tau) \tag{55}$$
$$\beta_k^{(r)}(n,\tau) = 1 \tag{56}$$

Appendix C

The Q-function $Q_{k-l}^k(r)$ can be written as follows:

$$\mathcal{Q}_{k-l}^k(r) = \sum_{i=1}^{t}\gamma_j^{(r)}(i)\mathcal{Q}_{i,k-l}^k(r) + \mathcal{C} \tag{57}$$

with

$$\begin{aligned}\mathcal{Q}_{i,k-l}^k(r) &= -\frac{1}{2}\sum_{j=k-l}^{k}\mathcal{D}\left(\tilde{y}_j^i(r) - h^i(x_j), \tilde{R}_j^i(r)\right) \\ &\quad -\frac{1}{2}\sum_{j=k-l}^{k}\mathcal{D}(x_j - f(x_{j-1}), Q_j) \\ &\quad -\frac{1}{2}\mathcal{D}(x_s - \hat{x}_{s|1:s}, \Sigma_{s,s|1:s})\end{aligned} \tag{58}$$

Consider the following Gaussian state space model:

$$x_j = f(x_{j-1}) + v_{j-1} \tag{59}$$
$$\tilde{y}_j^i = h^i(x_j) + \tilde{w}_j \tag{60}$$

The initial state x_{k-l-1}, v_j and $\tilde{w}_j$ are mutually independent white Gaussian noise processes. The averaged variables $\tilde{y}_j^i(r)$, $\tilde{R}_j^i(r)$ are given by (37) and (38). Then the MAP state estimate of $X_{k-l:k}^i$ via the ith propagation path on the rth iteration is computed by a fixed-interval smoother on the average state space model (59) and (60) as

$$X_{k-l:k}^i(r) = \left\{x_{k-l|k-l:k}^i, \ldots, x_{k|k-l:k}^i\right\} \tag{61}$$

where $x^i_{j|k-l:k} = E\{x_j|\tilde{y}^i_{k-l},\ldots,\tilde{y}^i_k\}$, which can be computed by (43) and (44). That is,

$$\hat{X}^i_{k-l:k}(r) = \arg\max_{X^i_{k-l:k}} \mathcal{Q}^k_{i,k-l}(r) \tag{62}$$

The (57) shows that the target state X^k_{k-l} is a mixture Gaussian model with each Gaussian component $X^i_{k-l:k}$ and its relative weights $\gamma^{(r)}_j(i)$. Then we obtain the multipath state fusion as the (41) and (42).

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