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Joint estimation and identification for stochastic systems with unknown inputs

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Abstract: Motivated by tracking a manoeuvring target in electronic counter environments, the authors present the problem of joint estimation and identification of a class of discrete-time stochastic systems with unknown inputs in both the plant and sensors. Based on the expectation-maximum criterion, the joint optimisation scheme of state estimation, parameter identification and iteration terminate decision were derived. A numerical example of tracking a manoeuvring target accompanied range gate pull-off is utilised to verify the proposed scheme.

1 Introduction

It is well recognised that the systematic modelling errors, for example caused by parameter variations, inaccurate parameter identification and unknown external disturbances, can be represented as unknown inputs (UI) to the nominal model. For example in chemical processes or network-based control systems, the effects of time delays and packet loss are usually represented by the UI to the nominal model [1–3]. Another example is the fault-tolerant diagnosis and control, where faults or even failures can be represented by the UI to the fault-free model [4–6]. Much attention has been paid on adaptive or robust filters for stochastic dynamical systems in the presence of UI. In our opinions, these filters belong to one of the following four categories.

The first category is the unknown input observer (UIO), whose estimate error is designed to be decoupled with UI [7, 8]. The necessary and sufficient conditions for the existence of the observer were derived [9]. The joint estimation of system state and inputs with UI was presented based on a reduced-order observer [10]. The minimum variance unbiased filter was proposed for the simultaneous input and state estimation of discrete-time linear systems and the stability conditions were given [11]. Furthermore, a reduced-order disturbance decoupled observer was proposed based on a parametric approach for linear systems with UI [12]. An adaptive sliding mode observer for parameter identification was presented under the assumption that the time derivatives of some outputs should be measurable [13]. Based on the gain switches, a high-gain observer was proposed to balance the conflicting performance indexes about error convergence and the robustness to model uncertainty [14].

The second category is the so-called robust filter especially applied in the field of fault detection and isolation. In general, the most widely considered performance measures are H_2 and H_∞ indices, representing the averaged estimation error and the peak estimation error, respectively [15–17]. In the case of singular systems with norm-bounded uncertainties, a robust H_2 filter was proposed via the linear matrix inequality (LMI) optimisation [16]. A delay-dependent H_∞ filtering design was established for a class of discrete-time switched systems with bounded time-varying delays [18]. The problem of finite horizon H_∞ filtering and/or smoothing for linear discrete time-varying descriptor systems with UI was presented and the derived H_∞ filter and smoother were proposed based on the innovation analysis technology [19]. A parameter-dependent robust mixed H_2/H_∞ filtering problem for discrete-time switched polytopic systems was presented and both the full-order and reduced-order filters were obtained in a unified framework via the LMI optimisation [20].

The third category is multiple model (MM) estimation [21] successfully applied in target tracking and fault diagnosis, where the UI is modelled as a randomly switching parameter obeying a known Markov chain. The corresponding estimators include the generalised pseudo–Bayesian estimator, the interacting multiple model estimation (IMM) and variable-structure MM estimation [22, 23]. However, the MM estimators require that the 'model dictionary' should be available and their computation burden increases significantly as the model number increases.

The fourth category is the minimum upper bound filter (MUBF) [24, 25], where a statistically-constrained UI was considered, representing an arbitrary combination of a class of unmodelled dynamics, random UI with unknown covariance matrix and deterministic UI. The corresponding filter design was transformed into the online scalar convex optimisation through constructing and minimising the upper bounds of covariance matrices of the state prediction error, residual and estimation error.

Up to date, none of the above-mentioned approaches consider the problem of state estimation for stochastic systems with UIs appeared in both dynamic model and measurement model, though such situation exists in general. For example, in tracking a manoeuvring target, the target may abruptly manoeuvre accompanied by its electronic counter-measure (ECM) [26, 27]. The existing solution is based on the IMM estimator, which has to establish the multiple dynamic models (because of all possible manoeuvre modes) and multiple measurement models (because of possible ECM modes). Such solution has the following two defects in engineering applications. One is that it is hard to guarantee the required model completeness by the fact the target is non-cooperative or even hostile. The other is that even the modes about possible manoeuvres and ECM ways were given beforehand, the computation complexity of the MM estimator would be very high because each combination of a dynamic model and a measurement model needs running a dedicated filter. In fact, the unknown manoeuvres and the ECM can be represented as two kinds of time-varying UIs in dynamic model and measurement model, respectively. It is highly demanded to develop the novel filter to deal with UIs to both the plant and sensors.

In this paper, we formulate the problem of joint estimation and identification for the stochastic systems coupling with UIs in both dynamic model and measurement model, respectively. The solution is obtained via iterative optimisation the state estimation and the parameter identification. An example of tracking a manoeuvring target accompanied by range gate pull-of (RGPO) is given to verify our proposed method.

The rest of the paper is organised as follows. Section 2 formulates the problem, where the stochastic systems model with two kinds of UIs is presented. Section 3 proposes the proposed joint estimation and identification based on the iterative optimisation strategy. The method simulation about target tracking and the concluding remarks are given in Sections 4 and 5, respectively.

Throughout this paper, the superscripts '-1' and 'T' represent the inverse and transpose operations of a matrix, respectively; $N(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{P})$ represents the Gaussian probability function of $\boldsymbol{x}$ with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{P}$; E is the mathematical expectation and 'Tr' denotes the matrix trace. The superscripts '$\wedge$' and '$\sim$', represent the corresponding estimate and the estimation error, respectively. For example, $\hat{\boldsymbol{x}}$ denotes the estimate of variable $\boldsymbol{x}$ and its estimation error is $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$. The notation $\otimes$ refers to the Kronecher product. $\mathbf{I}$ and $\mathbf{0}$ denote the identity matrix and the zero matrix with proper dimensions, respectively. For a vector $\boldsymbol{x}$, define $\boldsymbol{C}(\boldsymbol{x}) = \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}$ and $\boldsymbol{D}(\boldsymbol{x}, \boldsymbol{P}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{x}$.

2 Problem formulation

In modern target tracking, to destroy the track lock, a target manoeuvres always accompanied by the ECM. Over the past four decades, many important progresses have been made in tracking manoeuvring targets in the presence of clutters and missed detections [28, 29]. Recently, some concerns have been made on the important issue of tracking hostile targets in the presence of ECM [30–32]. Taking a particular type ECM called RGPO as an example, as shown in the upper subfigure of Fig. 1, the hostile pilot starts a constant-velocity (CV) motion, manoeuvres with constant acceleration at the time interval [10,20), switches to a CV motion, manoeuvres with circular motion at the time interval [50,70), and returns to a CV motion. The modelling error to the nominal CV model reflects such manoeuvres. Here both manoeuvre types and the time instants of the start and end of the manoeuvre are unknown. Meanwhile, the RGPO

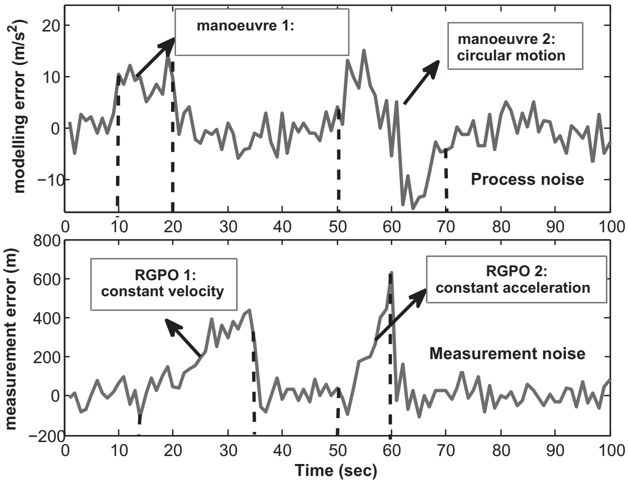

Fig. 1 *Illustration of tracking a manoeuvring target in the presence of RGPO*

products the false measurements to pull the radar-tracking gate step-by-step far away from the true target. As shown in lower subfigure of Fig. 1, the pilot implements the RGPO self-protection two times within time intervals [15,35) and [50,60), the measurement error in relative-range reflects such RGPOs.

Of all the algorithms considered for the benchmark problem of tracking a manoeuvring target accompanied by the ECM, the interacting multiple model probability data association (IMMPDA) [26] and the interacting multiple model multiple hypothesis tracking (IMMMHT) [27] were the best solutions, where the IMM estimator dealt with target manoeuvre, and the ECM including RGPO was handled via comparing the RGPO-originated false echo with the target-originated echo based on signal intensity and position information. In more serious case, the RGPO echo has much stronger intensity than the target echo and hence the target echo is neglected in the stage of signal processing. In other words, for our filter design in the stage of data processing, the obtained measurements are all RGPO-originated. In such situation, the IMM estimator will still work if the RGPO-free case and the RGPO-originated case can be modelled. As pointed out in [30], these algorithms were manoeuvre-specific and ECM-specific [26, 27, 33], namely, they require the model-completeness, that is, all possible modes about manoeuvres and ECM modes should be known beforehand. However, the completeness requirement is hardly satisfied by the fact that the possible manoeuvre and ECM are diverse, still being developed but seldom made public, especially for non-cooperative or even hostile targets [34–36].

Motivated by the above consideration, we present a new discrete-time linear stochastic system with UIs in both dynamic model and measurement model

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{F}_k\boldsymbol{x}_k + \boldsymbol{\Gamma}_k\boldsymbol{q}_k + \boldsymbol{M}_k\boldsymbol{a}_k \\ \boldsymbol{y}_{k+1} = \boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1} + \boldsymbol{N}_{k+1}\boldsymbol{b}_{k+1} \end{cases} \tag{1}$$

where $\boldsymbol{x} \in R^n$, $\boldsymbol{y} \in R^m$ represent the system state and measurement, respectively. The matrices $\boldsymbol{F}$, $\boldsymbol{\Gamma}$, $\boldsymbol{H}$, $\boldsymbol{M}$ and $\boldsymbol{N}$ are known with proper dimensions. The process noise $\boldsymbol{q}_k \in R^p$ and the measurement noise $\boldsymbol{v}_{k+1} \in R^m$ are zero-mean white Gaussian noises with known covariances $\boldsymbol{Q}_k > 0$ and $\boldsymbol{R}_{k+1} > 0$, respectively. The initial state $\boldsymbol{x}_0$ is Gaussian distributed with known mean $\bar{\boldsymbol{x}}_0$ and associated covariance $\boldsymbol{\Sigma}_0$. Here $\boldsymbol{q}$, $\boldsymbol{v}$

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and $\boldsymbol{x}_0$ are assumed to be independent; $\boldsymbol{M}_k$ and $\boldsymbol{N}_k$ are of full column-rank; $\boldsymbol{a}_k \in R^d$ and $\boldsymbol{b}_k \in R^r$ are UIs, which may depict actuator and sensor faults in fault diagnosis, or manoeuvres and man-made jams in target tracking. Our objective is to perform joint estimation and identification of $\boldsymbol{x}_k$, $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$ based on the measurement set $\{\boldsymbol{y}_i\}_{i=1}^k$.

Remark 1: Since the $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$ are the generalised UIs, it is difficult for the UIO to find the decoupling conditions. The H_2 or H_∞ filters face the design conservativeness because of the inequality derivation and can hardly output the needed parameter identification. The MM algorithms handle the UIs via establishing possible model set, and hence require more *a priori* information of UIs. The MUBF is constrained to deal with the UIs only appeared in the dynamic model. In brief, all of the above filters are NOT applicable. In this paper, we consider this problem as a joint state estimation and parameter identification and its solution is obtained via iterative optimisation, since it is in essence that state estimation and parameter identification affect each other.

3 Joint estimation and identification based on iterative optimisation

The UI identification mistake deteriorates the state estimate while the state estimate error increases the identification risk. It is thus necessary to develop an approach that performs state estimation and UI identification simultaneously. To do that, we propose the identification and estimation as a joint optimisation problem and then use the iterative optimisation strategy to resolve it. The expectation-maximum (EM) is one of the most popular iterative optimisation algorithms [37–40], and hence the following derivation follows the idea of the EM strategy.

Definition 1: Let $\boldsymbol{Z}_{k-l}^k = \{\boldsymbol{z}_{k-l}, \ldots, \boldsymbol{z}_k\}$ with $\boldsymbol{z}_i = \{\boldsymbol{x}_i, \boldsymbol{y}_i\}$, $\boldsymbol{Y}_{k-l}^k = \{\boldsymbol{y}_{k-l}, \ldots, \boldsymbol{y}_k\}$, $\boldsymbol{X}_{k-l}^k = \{\boldsymbol{x}_{k-l}, \ldots, \boldsymbol{x}_k\}$, and $\boldsymbol{\rho}_{k-l}^k = \{\boldsymbol{a}_{k-l}^k, \boldsymbol{b}_{k-l}^k\}$. Define the conditional mean and its cross-covariance by $\hat{\boldsymbol{x}}_{i|m:k} = E[\boldsymbol{x}_i|\boldsymbol{y}_m, \ldots, \boldsymbol{y}_k]$, and $\boldsymbol{P}_{i,j|m:k} = \mathrm{cov}(\boldsymbol{x}_i, \boldsymbol{x}_j|\boldsymbol{y}_m, \ldots, \boldsymbol{y}_k)$, respectively. In the following part, we call $\hat{\boldsymbol{x}}_{i|m:i}$, and $\hat{\boldsymbol{x}}_{i|i+1:k}$ as the forward and backward state estimates, respectively, and $\boldsymbol{P}_{i,i|m:i}$, $\boldsymbol{P}_{i,i|i+1:k}$ are the corresponding state estimation covariance, respectively.

Definition 2: Define the complete-data log-likelihood function at time interval $[k-l, k]$ by $L_{k-l}^k = \log p(\boldsymbol{Z}_{k-l}^k|\boldsymbol{\rho}_{k-l}^k, \boldsymbol{Y}_1^{k-l-1})$, let $G_{k-l}^k = E(L_{k-l}^k|\boldsymbol{Y}_{k-l}^k, \hat{\boldsymbol{\rho}}_{k-l}^k(r))$ be the conditional expectation of the complete-data log-likelihood function. $\hat{\boldsymbol{x}}_{k-l-1} = E[\boldsymbol{x}_{k-l-1}|\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{k-l-1}]$ is the conditional mean, and $\boldsymbol{P}_{k-l-1} = \mathrm{cov}(\boldsymbol{x}_{k-l-1}, \boldsymbol{x}_{k-l-1}|\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{k-l-1})$ is the corresponding covariance matrix.

The complete-data log-likelihood function L_{k-l}^k can be expressed as

$$\begin{aligned} L_{k-l}^k &= \log p(\boldsymbol{x}_{k-l}, \ldots, \boldsymbol{x}_k, \boldsymbol{y}_{k-l}, \ldots, \boldsymbol{y}_k|\boldsymbol{\rho}_{k-l}^k, \boldsymbol{Y}_1^{k-l-1}) \\ &= \log p(\boldsymbol{x}_{k-l-1}|\boldsymbol{Y}_1^{k-l-1}) + \sum_{i=k-l}^{k} \log p(\boldsymbol{x}_i|\boldsymbol{x}_{i-1}, \boldsymbol{\rho}_{k-l}^k) \\ &\quad + \sum_{i=k-l}^{k} \log p(\boldsymbol{y}_i|\boldsymbol{x}_i, \boldsymbol{\rho}_{k-l}^k) \end{aligned} \tag{2}$$

Theorem 1 (likelihood function)*:* The complete-data log-likelihood function L_{k-l}^k is given by

$$L_{k-l}^k = L_{0,k-l}^k + L_{1,k-l}^k + L_{2,k-l}^k + L_{3,k-l}^k \tag{3}$$

with

$$L_{0,k-l}^k = -\frac{n + l(m+n)}{2}\log(2\pi) - \frac{1}{2}\sum_{i=k-l}^{k}(\log|\boldsymbol{Q}_i| + \log|\boldsymbol{R}_i|) \tag{4}$$

$$L_{1,k-l}^k = -\frac{1}{2}\log|\boldsymbol{P}_{k-l-1}| - \frac{1}{2}D(\boldsymbol{x}_{k-l-1} - \hat{\boldsymbol{x}}_{k-l-1}, \boldsymbol{P}_{k-l-1}) \tag{5}$$

$$L_{2,k-l}^k = -\frac{1}{2}\sum_{i=k-l}^{k} D(\boldsymbol{x}_i - \boldsymbol{F}_i\boldsymbol{x}_{i-1} - \boldsymbol{M}_{i-1}\boldsymbol{a}_{i-1}, \boldsymbol{Q}_i) \tag{6}$$

$$L_{3,k-l}^k = -\frac{1}{2}\sum_{i=k-l}^{k} D(\boldsymbol{y}_i - \boldsymbol{H}_i\boldsymbol{x}_i - \boldsymbol{N}_i\boldsymbol{b}_i, \boldsymbol{R}_i) \tag{7}$$

Proof: See Appendix 1 □

Theorem 2 (state estimation)*:* The conditional expectation G_{k-l}^k is given by

$$G_{k-l}^k = L_{0,k-l}^k + G_{1,k-l}^k + G_{2,k-l}^k + G_{3,k-l}^k \tag{8}$$

with

$$G_{1,k-l}^k = -\frac{1}{2}\log|\boldsymbol{P}_{k-l-1}| - \frac{n}{2} \tag{9}$$

(see (10) and (11))

Proof: See Appendix 2 □

In order to obtain the conditional expectation of likelihood function G_{k-l}^k, it requires evaluating the conditional expectation of the state $\hat{\boldsymbol{x}}_{i|k-l:k}$ and its covariance $\boldsymbol{P}_{i,i|k-l:k}$. For a linear dynamic system, this can be carried out by using

$$\begin{aligned} G_{2,k-l}^k = -\frac{1}{2}Tr\Bigg\{\sum_{i=k-l}^{k} \boldsymbol{Q}_i^{-1} \times \{&C(\hat{\boldsymbol{x}}_{i|k-l:k} - \boldsymbol{F}_i\hat{\boldsymbol{x}}_{i-1|k-l:k} - \boldsymbol{M}_{i-1}\hat{\boldsymbol{a}}_{i-1}) - (\boldsymbol{P}_{i,i|k-l:k} - \boldsymbol{F}_i\boldsymbol{P}_{i,i-1|k-l:k}) \\ &+ (\boldsymbol{P}_{i,i-1|k-l:k}\boldsymbol{F}_i^{\mathrm{T}} - \boldsymbol{F}_i\boldsymbol{P}_{i-1,i-1|k-l:k}\boldsymbol{F}_i^{\mathrm{T}})\}\Bigg\} \end{aligned} \tag{10}$$

$$G_{3,k-l}^k = -\frac{1}{2}Tr\left\{\sum_{i=k-l}^{k} R_i^{-1} \times \{-(H_iP_{i,i|k-l:k}H_i^{\mathrm{T}}) + C(y_i - H_i\hat{x}_{i|k-l:k} - N_i\hat{b}_i)\}\right\} \tag{11}$$

a fixed interval smoother [41]. The smoother that combines the forward and backward filtered outputs can be formulated as follows:

State estimation

$$\hat{\boldsymbol{x}}_{i|k-l:k} = \boldsymbol{P}_{i,i|k-l:k}[(\boldsymbol{P}_{i,i|k-l:i})^{-1}\hat{\boldsymbol{x}}_{i|k-l:i} + (\boldsymbol{P}_{i,i|i+1:k})^{-1}\hat{\boldsymbol{x}}_{i|i+1:k}] \tag{12}$$

$$\boldsymbol{P}_{i,i|k-l:k} = [(\boldsymbol{P}_{i,i|k-l:i})^{-1} + (\boldsymbol{P}_{i,i|i+1:k})^{-1}]^{-1} \tag{13}$$

where the forward estimate $\hat{\boldsymbol{x}}_{i|k-l:i}$, $\boldsymbol{P}_{i,i|k-l:i}$ and the backward estimate $\hat{\boldsymbol{x}}_{i|i+1:k}$, $\boldsymbol{P}_{i,i|i+1:k}$ can be obtained by a standard Kalman filter.

Theorem 3 (parameter identification): $\hat{\boldsymbol{\rho}}^k_{k-l}(r+1)$ in (19) is given by

Dynamic model UI $\hat{\boldsymbol{a}}^k_{k-l}$

$$\hat{\boldsymbol{a}}^k_{k-l}(r+1) = \boldsymbol{A}^k_{k-l}\left[\sum_{i=k-l}^{k} \boldsymbol{Q}_i^{-1}(\hat{\boldsymbol{x}}_{i|k-l:k} - \boldsymbol{F}_i\hat{\boldsymbol{x}}_{i-1|k-l:k})\right] \tag{14}$$

with

$$\boldsymbol{A}^k_{k-l} = \left[\left(\sum_{i=k-l}^{k}\boldsymbol{Q}_i^{-1}\boldsymbol{M}_i\right)^{\mathrm{T}}\left(\sum_{i=k-l}^{k}\boldsymbol{Q}_i^{-1}\boldsymbol{M}_i\right)\right]^{-1} \times \left(\sum_{i=k-l}^{k}\boldsymbol{Q}_i^{-1}\boldsymbol{M}_i\right)^{\mathrm{T}} \tag{15}$$

Measurement model UI $\hat{\boldsymbol{b}}^k_{k-l}$

$$\hat{\boldsymbol{b}}^k_{k-l}(r+1) = \boldsymbol{B}^k_{k-l}\left[\sum_{i=k-l}^{k}\boldsymbol{R}_i^{-1}(\boldsymbol{y}_i - \boldsymbol{H}_i\hat{\boldsymbol{x}}_{i|k-l:k})\right] \tag{16}$$

with

$$\boldsymbol{B}^k_{k-l} = \left[\left(\sum_{i=k-l}^{k}\boldsymbol{R}_i^{-1}\boldsymbol{N}_i\right)^{\mathrm{T}}\left(\sum_{i=k-l}^{k}\boldsymbol{R}_i^{-1}\boldsymbol{N}_i\right)\right]^{-1} \times \left(\sum_{i=k-l}^{k}\boldsymbol{R}_i^{-1}\boldsymbol{N}_i\right)^{\mathrm{T}} \tag{17}$$

Proof: See Appendix 3 □

The joint estimation and identification consists of the following two iterative steps:

E-Step

$$G^k_{k-l} = E(L^k_{k-l}|\boldsymbol{Y}^k_{k-l}, \hat{\boldsymbol{\rho}}^k_{k-l}(r)) \tag{18}$$

M-Step

$$\hat{\boldsymbol{\rho}}^k_{k-l}(r+1) = \arg\max_{\boldsymbol{\rho}^k_{k-l}} G^k_{k-l} \tag{19}$$

where r corresponds to the rth iteration. The *E-Step* involves the calculation of the conditional expectation using the current estimate of the parameters $\hat{\boldsymbol{\rho}}^k_{k-l}(r)$ and the measurements. The *M-Step* provides an updated parameter estimate through maximising the likelihood function.

The iteration will be terminated if the values of likelihood functions at two consecutive iterations (defined by Theorem 1) are close enough or the number of iterations reaches the upper bound, that is

$$\frac{|L(r+1) - L(r)|}{|L(r+1)|} < \delta_L \text{ or } r \ge r_{\max} \tag{20}$$

where $0 < \delta_L \le 1$ is the iterative terminated threshold and $r_{\max}$ is the upper bound.

After the iteration terminated, the state estimate $\hat{\boldsymbol{x}}_{i|k-l:k}$ and its covariance $\boldsymbol{P}_{i,i|k-l:k}$ are obtained by the last iterative state estimate (given by Theorem 2). In the case that the UIs are zeroes, their estimates $\hat{\boldsymbol{a}}^k_{k-l}(r)$ and $\hat{\boldsymbol{b}}^k_{k-l}(r)$ will

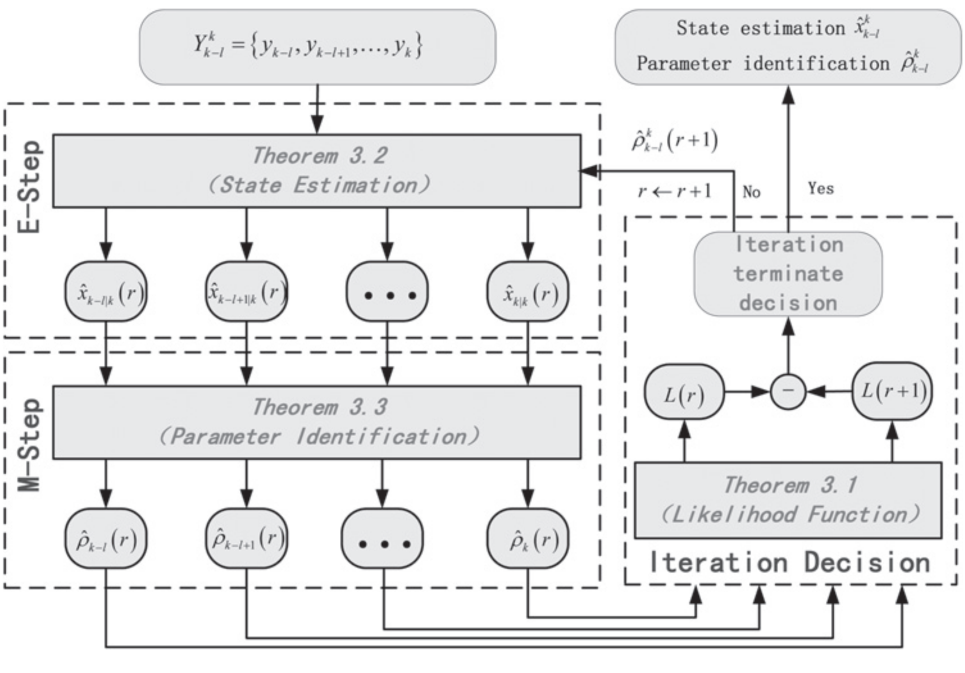

Fig. 2 *Function flow of the proposed method*

Table 1 Recursive computation of the proposed method

Initialisation: ($r = 0, k - l = 1$) Given a set of measurements $\boldsymbol{Y}_{k-l}^{k}$, initialise the parameter $\hat{\rho}_{k-l}^{k}(r)$ to suitable values.
Iterate the joint estimation and identification algorithm: for $r = 1, 2, \ldots$
1. E-Step
State estimation: use a fixed interval smoother to obtain the state estimate $\hat{\boldsymbol{x}}_{i\|k-l:k}$, and its covariance $\boldsymbol{P}_{i,i\|k-l:k}$ based on all the measurements $\boldsymbol{Y}_{k-l}^{k}$ and the rth parameter $\hat{\rho}_{k-l}^{k}(r)$ by (12)–(13).
Conditional expectation: compute the conditional expectation E_{k-l}^{k} using the state estimate and the parameter $\hat{\rho}_{k-l}^{k}(r)$ by (8)–(11).
2. M-Step
Parameter identification: obtain the updated parameters $\hat{\rho}_{k-l}^{k}(r+1)$ by (14)–(21).
3. Termination: If the values of likelihood functions $L(r)$ and $L(r+1)$ are close enough or the number of iteration reaches to the maximum number of iterations by (20), then the iteration terminates, else set $r = r + 1$, and go to step 1.
Recursion: ($r = 0, k = k + 1$), set the initialisation $\hat{\rho}_{k-l}^{k}(0)$ equal to the last ultimate value, and go to iterate loop.

not always be zeroes due to the effect of process noises and measurement noises, but will stay within the neighbourhood of zeroes. Hence, we output the estimates through multiplying the iterated estimates by a threshold function as follows

$$\hat{\boldsymbol{a}}_{k-l}^{k} = \hat{\boldsymbol{a}}_{k-l}^{k}(r)\boldsymbol{I}\{\|\hat{\boldsymbol{a}}_{k-l}^{k}(r)\| \le \delta_a\} \tag{21}$$

$$\hat{\boldsymbol{b}}_{k-l}^{k} = \hat{\boldsymbol{b}}_{k-l}^{k}(r)\boldsymbol{I}\{\|\hat{\boldsymbol{b}}_{k-l}^{k}(r)\| \le \delta_b\} \tag{22}$$

where the positive small number δ_a and δ_b are decision thresholds, and $\boldsymbol{I}\{\bullet\}$ is the indicator function, which will be one if the event $\{\bullet\}$ is true and zero otherwise.

The derived algorithm is shown in Fig. 2 and summarised as Table 1.

Remark 2: As shown in Fig. 2, the rth state estimate $\hat{\boldsymbol{x}}_{k-l}^{k}(r)$ is used to identify the UIs parameter $\hat{\boldsymbol{\rho}}_{k-l}^{k}(r)$, and then the identified UIs parameter $\hat{\boldsymbol{\rho}}_{k-l}^{k}(r)$ is uitlised to recalculate the $(r+1)$th state estimate $\hat{\boldsymbol{x}}_{k-l}^{k}(r+1)$. Owing to such iterative optimisation, the closed-loop of state estimation and parameter identification is established. In the view of feedback control, such closed loop is much helpful to improve the performance of the joint estimation and identification under multiple UIs.

Remark 3: The appealing property of EM is that the likelihood function monotonously increases with respect to the iteration number before reaching the local optimisation value. The convergence of the EM algorithm is discussed based on the Zangwill's global convergence theorem [42]. Considering the joint state estimation and mode identification of jump Markov linear systems, three EM schemes were presented and their convergence was given [43]. However, as stated in [44], there is no general convergence theorem for the EM algorithm: the convergence of the EM depends on the likelihood function, the condition expectation function and also the starting point. It is worth mentioning that our proposed EM scheme identifies the parameters (also called UIs) in the continues space while all the other EM schemes deal with parameters in the discrete set. Hence, the convergence condition of our proposed EM scheme is hardly derived from the existing results. This important but open problem will be considered in our future work.

4 Simulation results

A numerical simulation example is presented about tracking a manoeuvring target in the presence of the RGPO. Consider the target's initial position is $(\xi_0, \eta_0) = (70, 70)$ in km and the initial velocity is $(\dot{\xi}_0, \dot{\eta}_0) = (0, -0.5)$ in km/s. The target's positions are sampled at every $T_s = 1$ s. The target manoeuvres at $k_0 = 21$ and last 20 time steps with a constant acceleration motion $(\ddot{\xi}_0, \ddot{\eta}_0) = (-50, -50)$ in m/s^2. This manoeuvre is accompanied by RGPO with a constant pull-off acceleration $u(k) = 50\,\text{m/s}^2$. In other words, the systematic error because of the RGPO is $r_k^{\text{op}} = \frac{1}{2}u(k)(k-k_0)^2$. Finally, the CV motion remains in the last 20 time steps. The covariance of measurement noise $R_k = \text{diag}\{\sigma_r^2, \sigma_\theta^2, \sigma_v^2\}$, where the range standard deviation $\sigma\text{r} = 100\,\text{m}$, azimuth standard deviation $\sigma_\theta = 1 \times 10^{-3}$ rad and the range rate standard deviation $\sigma_v = 5\,\text{m/s}$.

The parameters of our proposed method are as follows: the system state $\boldsymbol{x}_k = [\xi_k, \dot{\xi}_k, \eta_k, \dot{\eta}_k]^{\text{T}}$, $\boldsymbol{F}_k = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{\Gamma}_k = \mathbf{I}_2 \otimes \begin{bmatrix} T_s^2/2 \\ T_s \end{bmatrix}$, and $\boldsymbol{Q}_k = 3\mathbf{I}_2\,\text{m/s}^2$. The initial dynamic model UI $\boldsymbol{a}_k = [\ddot{\xi}_k, \ddot{\eta}_k]^{\text{T}}$ and the initial measurement model UI $b_k = r_k^{\text{op}}$ are set to zeros. $\boldsymbol{M}_k = \mathbf{I}_2 \otimes \begin{bmatrix} T_s^2/2 \\ T_s \end{bmatrix}$, and

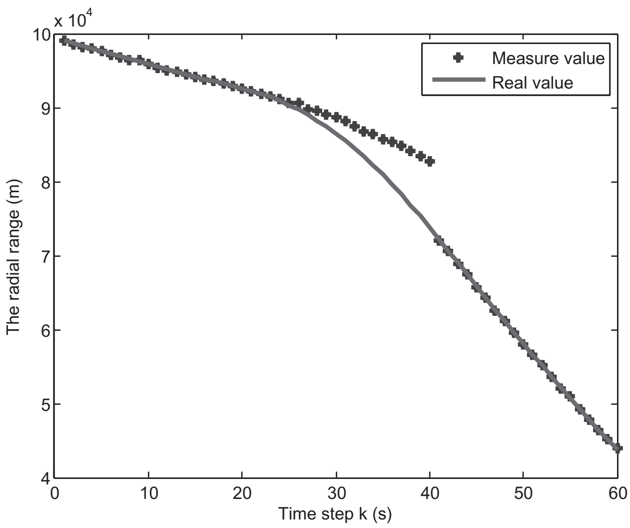

Fig. 3 *Measurement of target's radial range*

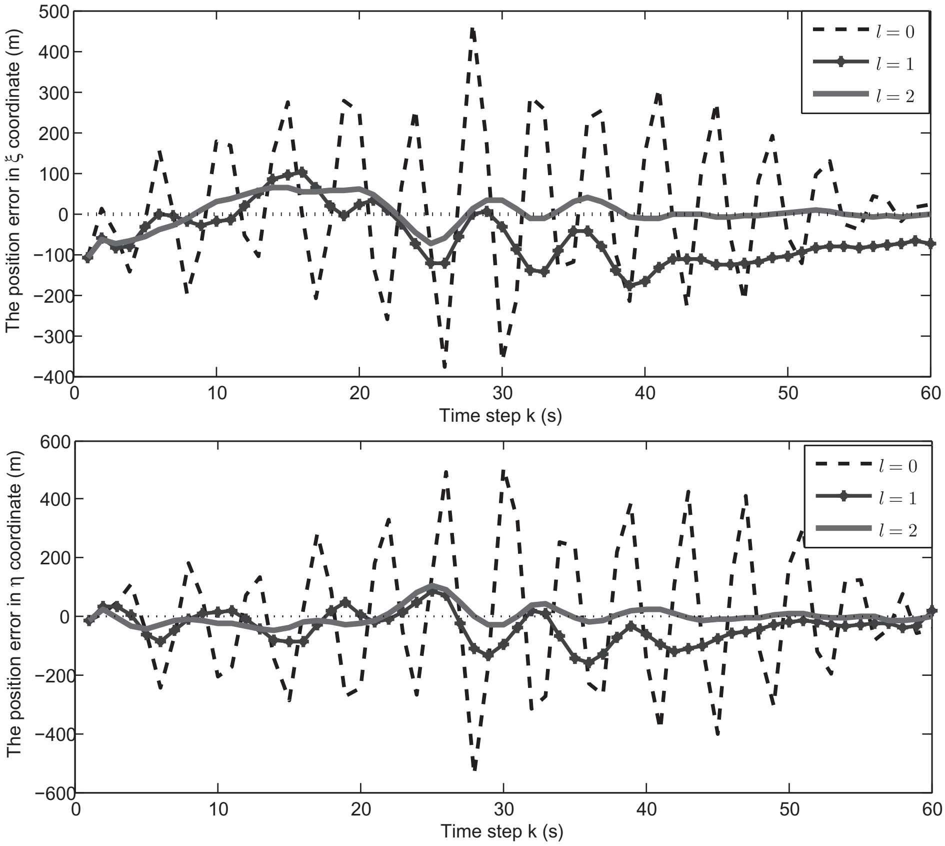

Fig. 4 *Position estimation errors with $l = 0, 1, 2$*

$\boldsymbol{N}_k = [1 \quad 0 \quad 0]^{\mathrm{T}}$. The iterative terminated threshold $\delta_L = 10^{-3}$, the maximum number of iterations $r_{\max} = 15$, and the detection thresholds $\delta_a = 5\,\mathrm{m/s^2}$, $\delta_b = 300\,\mathrm{m}$. The initial state covariance $\boldsymbol{\Sigma}_0 = \mathbf{I}_2 \otimes \mathrm{diag}(10^4, 25)$. The situation of above target tracking is non-linear, that is

$$\boldsymbol{y}_k = h(\boldsymbol{x}_k) = \left[\sqrt{\xi_k^2 + \eta_k^2} \quad \tan^{-1}\left(\frac{\eta_k}{\xi_k}\right) \quad \frac{(\xi_k\dot{\xi}_k + \eta_k\dot{\eta}_k)}{\sqrt{\xi_k^2 + \eta_k^2}}\right]^{\mathrm{T}}$$

and hence the measurement matrix $\boldsymbol{H}_k$ in (1) is obtained via the linearisation as $\boldsymbol{H}_k = \left.\frac{\partial h(\boldsymbol{x}_k)}{\partial \boldsymbol{x}_k}\right|_{\boldsymbol{x}_k = \hat{\boldsymbol{x}}_k(r)}$ in the rth iteration. Here, the parameter of window length $l = 0, 1, 2$ is used to analyse the estimate performance.

We also compare our proposed method with the IMM [45]. In the IMM, the target dynamic is covered by two models with CV and CA model, and the sensor is represented by two models, that is, RGPO with the constant pull-off acceleration (RW) and RGPO-free (RF). Therefore there are four kinds of model combination, that is, CV + RF, CV + RW, CA + RF and CA + RW. The parameters of IMM are as follows: the system state $\boldsymbol{x}_k = [\xi_k, \dot{\xi}_k, \ddot{\xi}_k, \eta_k, \dot{\eta}_k, \ddot{\eta}_k^{\mathrm{T}}]$, and the RGPO state $\boldsymbol{x}_k^{\mathrm{op}} = [r_k^{\mathrm{op}}, v_k^{\mathrm{op}}, a_k^{\mathrm{op}}]^{\mathrm{T}}$. The state transition matrices (see equations at the bottom of the page)

and $Q_k^{\mathrm{cv}} = \boldsymbol{0}_2\mathrm{m^2/s^4}$, $Q_k^{\mathrm{ca}} = 25 \times \mathbf{I}_2\,\mathrm{m^2/s^4}$, $Q_k^{\mathrm{op}} = 25\,\mathrm{m^2/s^4}$. The probability transition matrix $[\boldsymbol{p}_{ij}] = \left[\begin{smallmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{smallmatrix}\right] \otimes \left[\begin{smallmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{smallmatrix}\right]$, and the initial probability $\boldsymbol{u}_0 = [0.81 \quad 0.09 \quad 0.09 \quad 0.01]$. The measurement matrix

$$h(\boldsymbol{x}_k)^{\mathrm{RF}} = \begin{bmatrix} \sqrt{\xi_k^2 + \eta_k^2} \\ \tan^{-1}(\eta_k/\xi_k) \\ (\xi_k\dot{\xi}_k + \eta_k\dot{\eta}_k)/\sqrt{\xi_k^2 + \eta_k^2} \end{bmatrix}$$

$$h(\boldsymbol{x}_k)^{\mathrm{RW}} = \begin{bmatrix} \sqrt{\xi_k^2 + \eta_k^2} + r_k^{\mathrm{op}} \\ \tan^{-1}(\eta_k/\xi_k) \\ (\xi_k\dot{\xi}_k + \eta_k\dot{\eta}_k)/\sqrt{\xi_k^2 + \eta_k^2} \end{bmatrix}$$

The scenario of the target's radial range as shown in Fig. 3, at time interval [21, 40], the measurement of radial range has the system biases because of the RGPO with constant pull-off acceleration.

$$\boldsymbol{F}_k^{\mathrm{cv}} = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & T_{\mathrm{s}} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{F}_k^{\mathrm{ca}} = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & T_{\mathrm{s}} & T_{\mathrm{s}}^2/2 \\ 0 & 1 & T_{\mathrm{s}} \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{F}_k^{\mathrm{op}} = \begin{bmatrix} 1 & T_{\mathrm{s}} & T_{\mathrm{s}}^2/2 \\ 0 & 1 & T_{\mathrm{s}} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\Gamma}_k^{\mathrm{cv}} = \mathbf{I}_2 \otimes \begin{bmatrix} T_{\mathrm{s}}^2/2 \\ T_{\mathrm{s}} \\ 0 \end{bmatrix}, \quad \boldsymbol{\Gamma}_k^{\mathrm{ca}} = \mathbf{I}_2 \otimes \begin{bmatrix} T_{\mathrm{s}}^2/2 \\ T_{\mathrm{s}} \\ 1 \end{bmatrix}, \quad \boldsymbol{\Gamma}_k^{\mathrm{op}} = \begin{bmatrix} T_{\mathrm{s}}^2/2 \\ T_{\mathrm{s}} \\ 1 \end{bmatrix},$$

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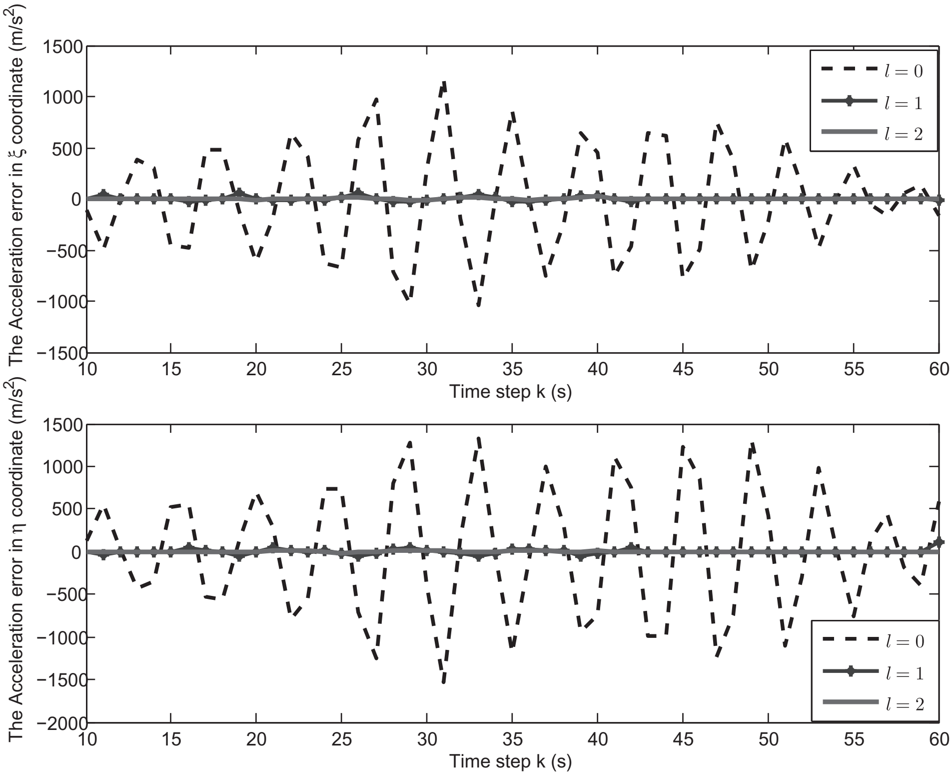

Fig. 5 *Acceleration identification errors with $l = 0, 1, 2$*

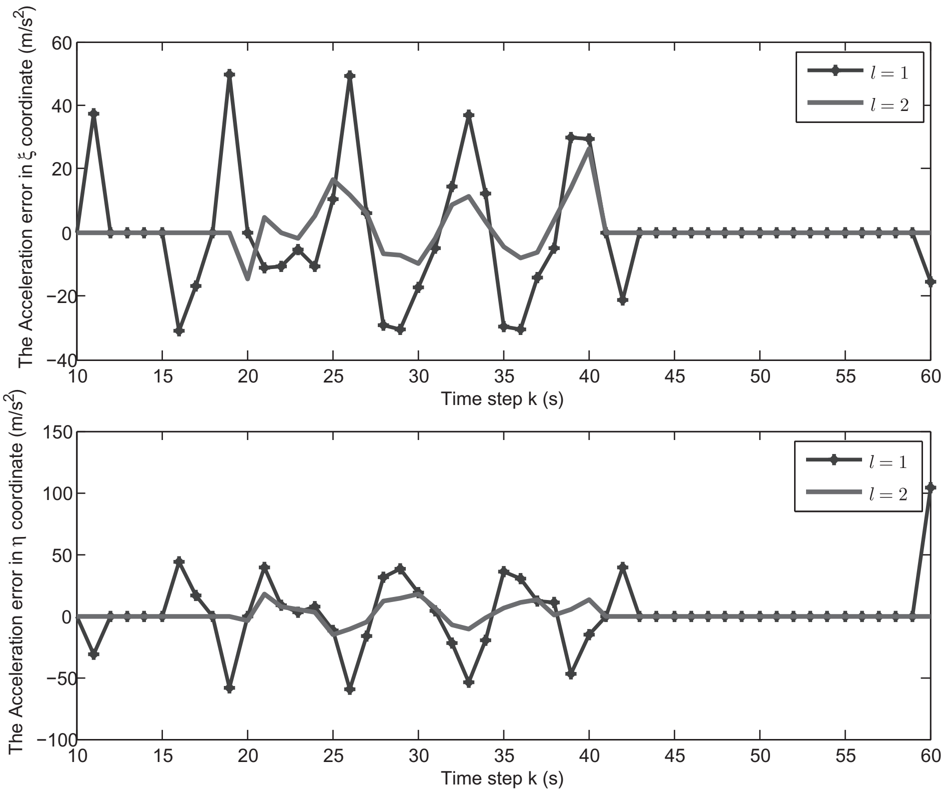

Fig. 6 *Acceleration identification errors with $l = 1, 2$*

The joint estimation and identification results with the different window length $l = 0, 1, 2$ are shown in Figs. 4–7, respectively. The proposed method successfully maintain good state estimate performance with and without the RGPO and manoeuvres, and the accuracy is improved as the window length l increases. The reason is that more measurements will be utilised in the joint state estimation and parameter identification if the window length l becomes larger. As shown in Fig. 8, as the window length l increases, the computational complexity decreases rapidly, and then

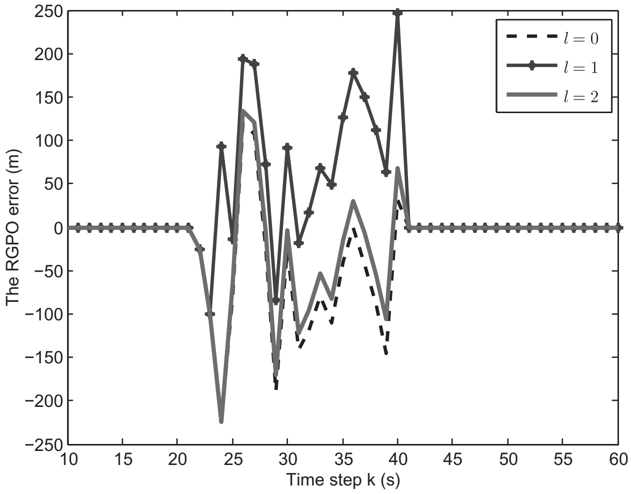

Fig. 7 *RGPO identification error $l = 0, 1, 2$*

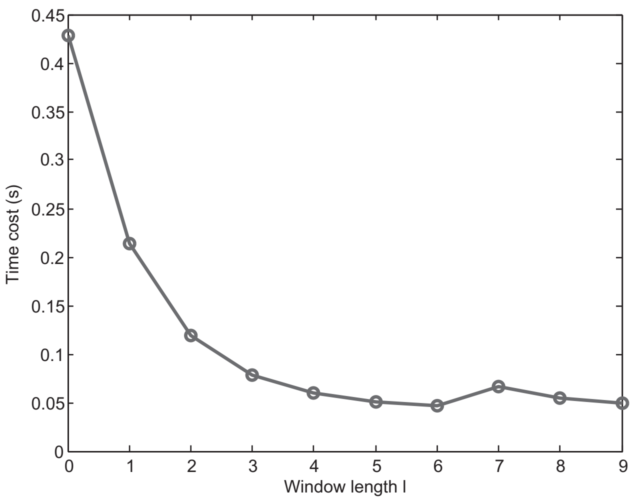

Fig. 8 *Computational complexity for different l*

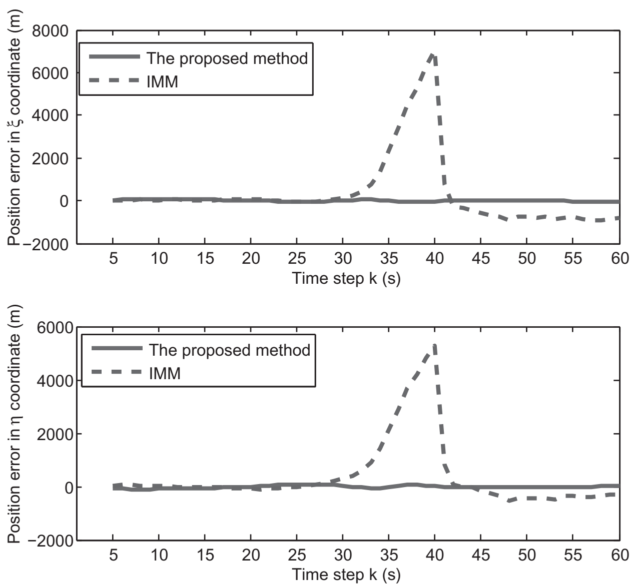

Fig. 9 *Position estimation errors with $l = 2$*

remain stable with the minimum point at $l = 6$. Meanwhile, both the manoeuvre and the RGPO are successfully estimated.

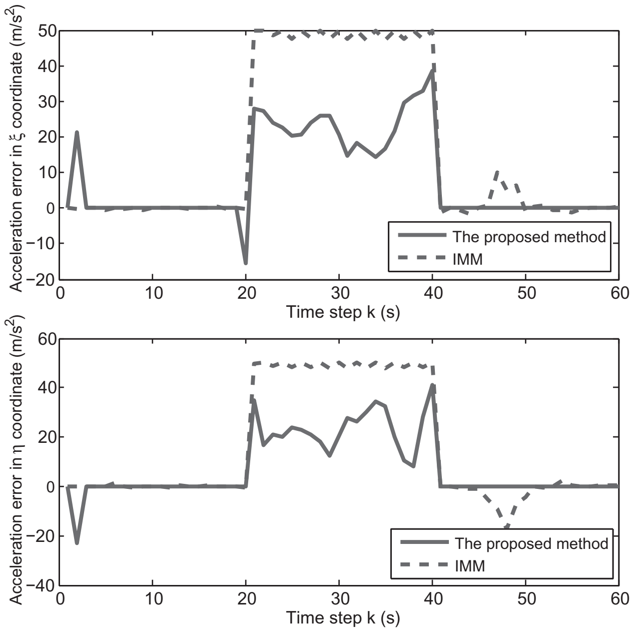

Fig. 10 *Acceleration identification errors with $l = 2$*

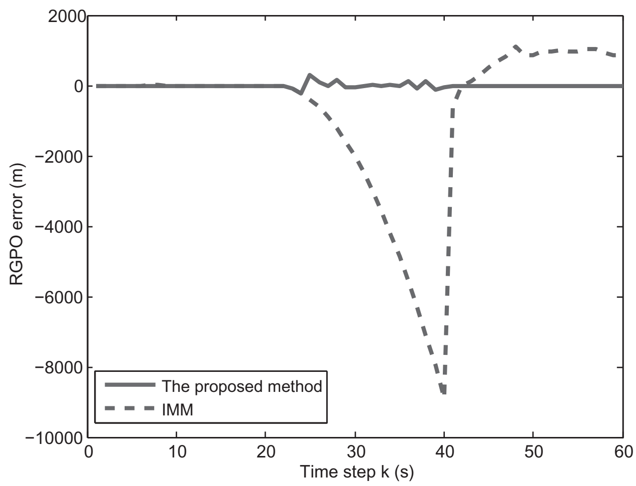

Fig. 11 *RGPO identification error with $l = 2$*

Our proposed method and IMM are compared in Figs. 9–11. The proposed method and the IMM have similar estimation accuracy at time interval $1 \leq k \leq 20$ when there is neither manoeuvre UI nor RGPO UI. During the time interval $21 \leq k \leq 40$ when the target manoeuvre and RGPO take place simultaneously. It is observed that the proposed method is better than IMM. It is mainly because our proposed method is derived based on the ideas of the joint optimisation and has the closed loop in iterative data processing.

5 Conclusion

By the fact that the existing filters are not suitable for the stochastic systems coupling with UIs in both dynamic model and measurement model, the joint estimation and

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identification problem is proposed. The solution is obtained via iterative optimising the estimated state and the identified parameters. The simulation about tracking a manoeuvring target under RGPO shows that the proposed method has good performances in both state estimate and UIs parameter identification.

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7 References

1 Kurz, H., Goedecke, W.: 'Digital parameter-adaptive control of processes with unknown dead time', *Automatica*, 1981, **17**, (1), pp. 245–252

2 Henson, M.A., Seborg, D.E.: 'Time delay compensation for nonlinear processes', *Ind. Eng. Chem. Res.*, 1994, **33**, (6), pp. 1493–1500

3 Shi, Y., Fang, H.: 'Kalman filter based identification for systems with randomly missing measurements in a network environment', *Int. J. Control*, 2010, **83**, (3), pp. 538–551

4 Chilin, D., Liu, J.F., Munoz de la Pena, D., Christofides, P.D., Davis, J.F.: 'Detection, isolation and handling of actuator faults in distributed model predictive control systems', *J. Process Control*, 2010, **20**, (9), pp. 1059–1075

5 Zhao, H., Zhong, M., Zhang, M.: 'H_∞ fault detection for linear discrete time-varying systems with delayed state', *IET Control Theory Appl.*, 2010, **4**, (11), pp. 2303–2314

6 Liu, W., Wang, H.I.: 'Robust estimation and fault detection and isolation algorithms for stochastic linear Hybrid systems with unknown fault input', *IET Control Theory Appl.*, 2011, **5**, (12), pp. 1353–1368

7 Wang, S.H., Wang, E., Dorato, P.: 'Observing the states of systems with unmeasurable disturbance', *IEEE Trans. Autom. Control*, 1975, **20**, (5), pp. 716–717

8 Bhattacharyya, S.: 'Observer design for linear systems with unknown inputs', *IEEE Trans. Autom. Control*, 1978, **23**, (3), pp. 483–484

9 Darouach, M., Zasadzinski, M., Xu, S.J.: 'Full-order observer for linear systems with unknown inputs', *IEEE Trans. Autom. Control*, 1994, **39**, (3), pp. 606–609

10 Trinh, H., Ha, Q.P.: 'State and input simultaneous estimation for a class of time delay systems with uncertainties', *IEEE Trans. Circuits Syst. II: Express Briefs*, 2007, **54**, (6), pp. 527–531

11 Fang, H., Shi, Y., Yi, J.: 'On stable simultaneous input and state estimation for discrete-time linear systems', *Int. J. Adapt. Control Signal Process.*, 2011, **25**, (8), pp. 671–686

12 Trinh, H., Tran, T.D., Fernando, T.: 'Disturbance decoupled observers for systems with unknown inputs', *IEEE Trans. Autom. Control*, 2008, **53**, (10), pp. 2397–2402

13 Yan, X.G., Edwards, C.: 'Adaptive sliding-mode-observer-based fault reconstruction for non-linear systems with parametric uncertainties', *IEEE Trans. Ind. Electron.*, 2008, **55**, (11), pp. 4029–4036

14 Ahrens, J.H., Khalil, H.K.: 'High-Gain observers in the presence of measurement noise: a Switched-Gain approach', *Automatica*, 2009, **45**, (4), pp. 936–943

15 Wang, H.Q., Dai, Q.H., Zhang, C.H., Liu, X.D.: 'An innovation approach to H_∞ fixed-lag smoothing for descriptor systems', *IEEE Trans. Circuits Syst. II: Express Briefs*, 2006, **53**, (12), pp. 1451–1455

16 Boulkroune, B., Darouach, M., Zasadzinski, M.: 'Moving horizon state estimation for linear Discrete-Time singular systems', *IET Control Theory Appl.*, 2010, **4**, (3), pp. 339–350

17 Darouach, M.: 'H_∞ unbiased filtering for linear descriptor systems via LMI', *IEEE Trans. Autom. Control*, 2009, **54**, (8), pp. 1966–1972

18 Mahmoud, M.S.: 'Delay-Dependent H_∞ filtering of a class of switched discrete-time state delay systems', *Signal Process*, 2008, **88**, (11), pp. 2709–2719

19 Zhao H, Zhang, C., Xing, G.: 'H_∞ filtering and smoothing for linear discrete time-varying descriptor systems with unknown inputs', *IET Control Theory Appl.*, 2012, **6**, (5), pp. 734–743

20 Qiu, J., Feng, G., Yang, J.: 'Robust mixed H_2/H_∞ filtering design for discrete-time switched polytopic linear systems', *IET Control Theory Appl.*, 2008, **2**, (5), pp. 420–430

21 Bar-Shalom, Y., Li, X.R., Kirubarajan, T.: 'Estimation with application to tracking and navigation: theory algorithms and software' (New York, Wiley, 2001)

22 Mazor, E., Averbuch, A., Bar-Shalom, Y., Dayan J.: 'Interacting multiple model methods in target tracking: a survey', *IEEE Trans. Aerosp. Electron. Syst.*, 1998, **34**, (1), pp. 103–123

23 Li, X.R., Jilkov, V.P.: 'Survey of maneuvering target tracking. Part, V. Multiple-model methods', *IEEE Trans. Aerosp. Electron. Syst.*, 2005, **41**, (4), pp. 1255–1321

24 Liang, Y., Zhang, L., Zhou, D.H., Pan, Q.: 'Estimation of systems with statistically-constrained inputs', *Appl. Math. Comput.*, 2010, **217**, (6), pp. 2644–2656

25 Liang, Y., Zhou, D.H., Zhang, L., Pan, Q.: 'Adaptive filtering for stochastic system with generalised disturbance inputs', *IEEE Signal Process. Lett.*, 2008, **15**, pp. 645–648

26 Kirubarajan, T., Bar-Shalom, Y., Blair, W.D., Watson, G.A.: 'IMM-PDAF for radar management and tracking benchmark with ECM', *IEEE Trans. Aerosp. Electron. Syst.*, 1998, **34**, (4), pp. 1115–1134

27 Blair, W.D., Watson, G.A., Kirubarajan, T., Bar-Shalom, Y.: 'Benchmark for radar allocation and tracking in ECM', *IEEE Trans. Aerosp. Electron. Syst.*, 1998, **34**, (4), pp. 1097–1114

28 Bar-Shalom, Y., Li, X.R.: 'Multitarget-multisensor tracking: principles and techniques' (Storrs, CT, YBS Publishing, 1995)

29 Blackman, S.S., Popoli, R. F.: 'Design and analysis of modern tracking systems' (Norwood, MA, Artech House, 1999)

30 Li, X. R., Slocumb, B., West, P.D: 'Tracking in the presence of range deception ECM and clutter by decomposition and fusion'. Proc. SPIE Signal and Data Processing of Small Targets, 1999, pp. 198–211

31 Blair, W.D., Watson, G.A., Hoffman, S. A.: 'Benchmark problem for beam pointing control of phased array radar against maneuvering target'. Proc. 1994 American Control Conf., Baltimore, MD, June 1994, pp. 2071–2075

32 Blair, W. D., Watson, G.A., Hoffman, S. A.: 'Benchmark for radar resource allocation and tracking targets in the presence of ECM'. Proc. 1995 American Control Conf., Seattle, WA, June 1995, pp. 2601–2605

33 Slocumb, B.J., West, P.D., Shirey, T.N., Kamen, E.W.: 'Tracking a maneuvering target in the presence of false returns and ECM using a variable state dimension Kalman filter'. Proc. 1995 American Control Conf., Seattle, WA, June 1995, pp. 2611–2615

34 Lothes, R.N., Szymanski, M.B., Wiley, R. G.: 'Radar vulnerability to jamming' (Norwood, MA, Artech House, 1990)

35 Chrzanowski, E.: 'Active radar electronic counter-measures' (Norwood, MA, Artech House, 1990)

36 West, P.D., Slocumb, B.J.: 'ECM modeling for assessment of target tracking algorithms'. Proc. 29th Southeastern Symp. System Theory, Cookeville, TN, March 1997, pp. 500–504

37 Dempster, A.P., Laird, N.M., Rubin, D.B.: 'Maximum likelihood from incomplete data'. *R. Stat. Soc.*, 1977, **39**, (1), pp. 1–38

38 Roweis, S., Ghahramani, Z.: 'Learning non-linear dynamical systems using the expectation-maximisation algorithm Kalman filtering and neural networks' (New York, Dover, 2001)

39 Zia, A., Kirubarajan, T., Reilly, J.P., Yee, D., Punithakumar, K., Shirani, S.: 'An EM algorithm for non-linear state estimation with model uncertainties', *IEEE Trans. Signal Process.*, 2008, **56**, (3), pp. 921–936

40 Li, Z.H., Chen, S.Y., Leung, H., Bosse, E.: 'Joint data association, registration, and fusion using EM-KF', *IEEE Trans. Aerosp. Electron. Syst.*, 2010, **46**, (2), pp. 496–507

41 Rauch, H.: 'Solutions to the linear smoothing problem', *IEEE Trans. Autom. Control*, 1963, **8**, (4), pp. 371–372

42 Wu, C.F.J.: 'On the convergence properties of the EM algorithm', *Ann. Stat.*, 1983, **11**, (1), pp. 95–103

43 Logothetis, A., Krishnamurthy, K.: 'Expectation maximisation algorithms for MAP estimation of jump Markov linear systems', *IEEE Trans. Signal Process.*, 1999, **47**, (8), pp. 2139–2156

44 Gupta, M.R, Chen, Y.: 'Theory and use of the EM algorithm', *Found. Trends Signal Process.*, 2011, **4**, (3), PP. 223–296

45 Daeipour, E., Bar-Shalom, Y.: 'IMM tracking of maneuvering targets in the presence of glint', *IEEE Trans. Aerosp. Electron. Syst.*, 1998, **34**, (3), pp. 996–1002

8 Appendix 1

The complete-data log-likelihood function is given by

$$L^k_{k-l} = \log p(\boldsymbol{x}_{k-l-1}|\boldsymbol{Y}^{k-l-1}_1) + \sum_{i=k-l}^{k} \log p(\boldsymbol{x}_i|\boldsymbol{x}_{i-1}, \boldsymbol{\rho}^k_{k-l}) + \sum_{i=k-l}^{k} \log p(\boldsymbol{y}_i|x_i, \boldsymbol{\rho}^k_{k-l}) \quad (23)$$

The PDF state estimation $\boldsymbol{x}_{k-l-1}$, state prediction $\boldsymbol{x}_i$, and measurement prediction $\boldsymbol{y}_i$ conditioned by $\boldsymbol{\rho}^k_{k-l}$ are all Gaussian. That is

$$\boldsymbol{p}(\boldsymbol{x}_{k-l-1}|, \boldsymbol{\rho}^k_{k-l}) = \boldsymbol{N}(\hat{\boldsymbol{x}}_{k-l-1}, \boldsymbol{P}_{k-l-1}) \quad (24)$$

$$\boldsymbol{p}(\boldsymbol{x}_i|\boldsymbol{x}_{i-1}, \rho^k_{k-l}) = \boldsymbol{N}(\boldsymbol{F}_i\boldsymbol{x}_{i-1} + \boldsymbol{M}_{i-1}\boldsymbol{a}_{i-1}, \boldsymbol{Q}_i) \quad (25)$$

$$\boldsymbol{p}(\boldsymbol{y}_i|\boldsymbol{x}_i, \boldsymbol{\rho}^k_{k-l}) = \boldsymbol{N}(\boldsymbol{H}_i\boldsymbol{x}_i + \boldsymbol{N}_i\boldsymbol{b}_i, \boldsymbol{R}_i) \quad (26)$$

By taking (24)–(26) into (23), we obtain (3)–(7).

9 Appendix 2

For $G^k_{1,k-l}$

$$\begin{aligned} G^k_{1,k-l} = E[L^k_{1,k-l}] &= -\frac{1}{2}\log|\boldsymbol{P}_{k-l-1}| \\ &\quad - \frac{1}{2}Tr\{(\boldsymbol{P}_{k-l-1})^{-1}E[C(\boldsymbol{x}_{k-l-1} - \hat{\boldsymbol{x}}_{k-l-1})|\boldsymbol{\rho}^k_{k-l}]\} \\ &= -\frac{1}{2}\log|\boldsymbol{P}_{k-l-1}| - \frac{n}{2} \end{aligned} \quad (27)$$

For $G^k_{2,k-l}$

$$G^k_{2,k-l} = E[L^k_{2,k-l}] = -\frac{1}{2}\mathrm{Tr}(\boldsymbol{\xi}_{Bk}) \quad (28)$$

(see (29))

Through putting (29) into (28), we obtain (10).

For $G^k_{3,k-l}$

$$G^k_{3,k-l} = E[L^k_{3,k-l}] = -\frac{1}{2}Tr(\boldsymbol{\xi}_{Ck}) \quad (30)$$

$$\begin{aligned} \boldsymbol{\xi}_{Ck} &= \sum_{i=k-l}^{k} \boldsymbol{R}_i^{-1}E[C(\boldsymbol{y}_i - \boldsymbol{H}_i\boldsymbol{x}_i - \boldsymbol{N}_i\boldsymbol{b}_i)|\boldsymbol{Y}^k_{k-l}, \boldsymbol{\rho}^k_{k-l}] \\ &= \sum_{i=k-l}^{k} \boldsymbol{R}_i^{-1}E[C(\boldsymbol{y}_i - \boldsymbol{H}_i(\hat{\boldsymbol{x}}_i + \tilde{\boldsymbol{x}}_i) - \boldsymbol{N}_i\boldsymbol{b}_i)|\boldsymbol{Y}^k_{k-l}, \boldsymbol{\rho}^k_{k-l}] \\ &= \sum_{i=k-l}^{k} \boldsymbol{R}_i^{-1}[C(\boldsymbol{y}_i - \boldsymbol{H}_i\hat{\boldsymbol{x}}_{i|k-l:k} - \boldsymbol{N}_i\boldsymbol{b}_i) \\ &\quad + \boldsymbol{H}_i\boldsymbol{P}_{i,i|k-l:k}\boldsymbol{H}_i^{\mathrm{T}}] \end{aligned} \quad (31)$$

Through putting (31) into (30), we obtain (11).

10 Appendix 3

Take the corresponding partial derivatives of the expected log-likelihood with respect to the UIs $\{a_i\}^k_{i=k-l}$, $\{b_i\}^k_{i=k-l}$, respectively. By the fact that these partial derivations should be zeroes at the optimal point of state estimate and parameter identification, we have

$$\begin{aligned} \frac{\partial G^k_{k-l}}{\partial \boldsymbol{a}_{i-1}} &= \sum_{i=k-l}^{k} \boldsymbol{Q}_i^{-1}(\hat{\boldsymbol{x}}_{i|k-l:k} - \boldsymbol{F}_i\hat{\boldsymbol{x}}_{i-1|k-l:k} - M_{i-1}\boldsymbol{a}_{i-1}) \\ &= 0 \end{aligned} \quad (32)$$

$$\begin{aligned} \frac{\partial G^k_{k-l}}{\partial \boldsymbol{b}_i} &= \sum_{i=k-l}^{k} \boldsymbol{R}_i^{-1}(\boldsymbol{y}_i - \boldsymbol{H}_i\hat{\boldsymbol{x}}_{i|k-l:k} - \boldsymbol{N}_i\boldsymbol{b}_i) \\ &= 0 \end{aligned} \quad (33)$$

From (33)–(32), we obtain (14)–(16).

$$\begin{aligned} \boldsymbol{\xi}_{Bk} &= \sum_{i=k-l}^{k} \boldsymbol{Q}_i^{-1}E[C(\boldsymbol{x}_i - \boldsymbol{F}_i\boldsymbol{x}_{i-1} - \boldsymbol{M}_{i-1}\boldsymbol{a}_{i-1})|\boldsymbol{Y}^k_{k-l}, \boldsymbol{\rho}^k_{k-l}] \\ &= \sum_{i=k-l}^{k} \boldsymbol{Q}_i^{-1}E\{C((\hat{\boldsymbol{x}}_i + \tilde{\boldsymbol{x}}_i) - \boldsymbol{F}_i(\hat{\boldsymbol{x}}_{i-1} + \tilde{\boldsymbol{x}}_{i-1}) - \boldsymbol{M}_{i-1}\boldsymbol{a}_{i-1})|\boldsymbol{Y}^k_{k-l}, \boldsymbol{\rho}^k_{k-l}\} \\ &= \sum_{i=k-l}^{k} \boldsymbol{Q}_i^{-1} \times E[C(\hat{\boldsymbol{x}}_i - \boldsymbol{F}_i\hat{\boldsymbol{x}}_{i-1} - \boldsymbol{M}_{i-1}\boldsymbol{a}_{i-1})|\boldsymbol{Y}^k_{k-l}, \boldsymbol{\rho}^k_{k-l}] + \sum_{i=k-l}^{k} \boldsymbol{Q}_i^{-1}E[C(\tilde{\boldsymbol{x}}_i - \boldsymbol{F}_i\tilde{x}_{i-1})|\boldsymbol{Y}^k_{k-l}, \boldsymbol{\rho}^k_{k-l}] \\ &= \sum_{i=k-l}^{k} C(\hat{\boldsymbol{x}}_{i|k-l:k} - \boldsymbol{F}_i\hat{\boldsymbol{x}}_{i-1|k-l:k} - \boldsymbol{M}_{i-1}\boldsymbol{a}_{i-1}) + \sum_{i=k-l}^{k} (\boldsymbol{P}_{i,i|k-l:k} - \boldsymbol{P}_{i,i-1|k-l:k}\boldsymbol{F}_i^{\mathrm{T}} - \boldsymbol{F}_i\boldsymbol{P}_{i,i-1|k-l:k} + \boldsymbol{F}_i\boldsymbol{P}_{i-1,i-1|k-l:k}\boldsymbol{F}_i^{\mathrm{T}}) \end{aligned} \quad (29)$$

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