UNIVERSIDAD DE GRANADA

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GRADO EN FÍSICA

TRABAJO FIN DE GRADO

Herramientas para cálculos perturbativos en renormalización de Hamiltonianos

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Resumen

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Abstract

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Contents

1 Introduction

It's in our interest to formulate a theory compatible with special relativity and quantum mechanics, this culminates in quantum field theories.

we should expect the most general theories to be scale invariant,

In the field of theoretical physics, divergences are a common occurrence,

In the context of theoretical physics renormalization is an important procedure

Tipical renormalization procedures apply to Lagrangian dynamics,

2 Theoretical background

To describe the relativistic interactions of effective particles in the framework of the renormalization group procedure for effective particles, making use of quantum theory in the front form of Hamiltonian dynamics.

2.1 Canonical Hamiltonian

2.1.1 Classical mechanics

The canonical Hamiltonian is a function of the canonical coordinates q_i, the canonical momenta p_i, and time t. The Hamiltonian is a function that describes the total energy of the system, and is obtained from applying the Legendre transformation to the Lagrangian of the system, this is defined as,

$$H(q,p,t) = \sum_i p_i \dot{q}_i - L(q,\dot{q},t), \tag{2.1}$$

where H is the Hamiltonian, L is the Lagrangian, q_i are the canonical coordinates, $\dot{q}_i$ are the canonical velocities, and p_i are the canonical momenta. The canonical momenta are defined as,

$$p_i = \frac{\partial L}{\partial \dot{q}_i}, \tag{2.2}$$

The canonical Hamiltonian governs the time evolution of the system via the Hamilton equations of motion, which are given by,

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}. \tag{2.3}$$

2.1.2 Field theory

In the case of field theories, a Hamiltonian density can be defined such that integrating over space coordinates the Hamiltonian is obtained.

$$H = \int d^3x \mathcal{H}, \tag{2.4}$$

for simplicity we will call the Hamiltonian density $\mathcal{H}$, as Hamiltonian.

For a classical field $\phi(x)$, with the Lagrangian density $\mathcal{L}(\phi, \partial_\mu \phi)$, the Hamiltonian density is defined as,

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L}, \tag{2.5}$$

where π is the canonical momentum, defined as,

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}. \tag{2.6}$$

2.1.3 Quantum field theory

In quantum mechanics, or quantum field theory, the Hamiltonian becomes an operator acting on a Hilbert space.

Solving the system means obtaining the spectrum of the theory, this is finding the eigenenergies of the system. In general this is a non trivial process.

In field theory calculations the fundamental quantities to consider are the energy-momenta, angular momentum, and boosts, due to its symmetries under the Poincaré group.

The Poincaré group contains the Lorentz group and the translations, the generators of the Poincaré group are the four-momentum operator P^μ and the angular momentum $M^{\mu\nu}$, which satisfy the following commutation relations,

$$[P^\mu, P^\nu] = 0, \tag{2.7}$$

$$[M^{\mu\nu}, P^\rho] = i\left(g^{\nu\rho}P^\mu - g^{\mu\rho}P^\nu\right), \tag{2.8}$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i\left(g^{\nu\rho}M^{\mu\sigma} - g^{\mu\rho}M^{\nu\sigma} + g^{\sigma\mu}M^{\nu\rho} - g^{\sigma\nu}M^{\mu\rho}\right). \tag{2.9}$$

2.2 Fock space

In order to describe a variable number of particles in our system, the use of the Fock space is needed. Intoduced by V.A. Fock in 1932 [2], the Fock space is a sum of a set of Hilbert spaces, each one corresponding to a different number of particles in the system.

The Fock space is defined as the direct sum of tensor products of the single particle Hilbert space $\mathbb{H}$,

$$\mathbb{F}_\nu = \bigoplus_{n=0}^{\infty} S_\nu \mathbb{H}^{\otimes n} = \mathbb{C} \oplus \mathbb{H} \oplus S_\nu(\mathbb{H}\otimes\mathbb{H}) \oplus S_\nu(\mathbb{H}\otimes\mathbb{H}\otimes\mathbb{H}) \oplus \cdots, \tag{2.10}$$

where S_ν is the symmetrization operator depending on whether the particles described are bosonic or fermionic, it symmetrizes or antisymmetric the tensors, and $\mathbb{C}$ is the complex scalar, corresponding to the states with no particles.

This way a general state in the Fock space can be expressed as,

$$|\Psi\rangle_\nu = |\Psi_0\rangle_\nu \oplus |\Psi_1\rangle_\nu \oplus |\Psi_2\rangle_\nu \oplus \cdots = a\,|0\rangle_\nu + \sum_{i=1} a_i\,|\psi_i\rangle_\nu + \sum_{i,j=1} a_{ij}\,|\psi_i\psi_j\rangle_\nu + \cdots, \tag{2.11}$$

where $|\Psi_0\rangle_\nu$ is the vacuum state, $|\Psi_1\rangle_\nu$ is the one particle state, $|\Psi_2\rangle_\nu$ is the two particle state, and so on. The coefficients a_i are the amplitudes of the states, in general complex numbers.

In the case of QCD, a geneal state in the Fock space involves a superposition of all possible multiparticle states, build from quarks, antiquarks, and gluons, with the correct quantum numbers. This way a general state in the Fock space can be expressed as,

$$|\Psi\rangle = c_0 |0\rangle + c_1 |q\bar{q}\rangle + c_2 |q\bar{q}g\rangle + c_3 |qqq\rangle + c_4 |gg\rangle + \cdots, \tag{2.12}$$

2.3 Front form of Hamiltonian Dynamics

The front form of dynamics introduced by Dirac (1949) [1] offers a couple advantages to the

The quantization hypersurface considered is,

$$x^+ = t + z = x^0 + x^3 = 0, \tag{2.13}$$

then the rest of coordinates will be defined as

$$x^- = x^0 - x^3, \quad x^\perp = (x^1, x^2), \tag{2.14}$$

In this set of coordinates, the fundamental quantities

The Hamiltonian in FF quantization is obtained from the Lagrangian density using the Legendre transformation, similar to the procedure explained in the section 2.1, but with respect to the new coordinates.

2.4 Counterterms

The counterterms is an additional term added to the initial or bare Hamiltonian $\mathcal{H}_0$ to deal with the divergences due to loops, produced during the process. This way, ensure that the effective Hamiltonian $\mathcal{H}_t$ remains finite at all values of t.

2.5 RGPEP

The renormalization group procedure for effective particles (RGPEP), as it's name indicate is a renormalization group procedure applied to the Hamiltonian formulation.

By considering a series of unitary transformations, the RGPEP is able to construct a series of effective Hamiltonians $\mathcal{H}_s$, and the corresponding effective particles, by the use of effective particle operators (namely creation and annihilation operators) that differs from the canonical ones by the unitary transformation $\mathcal{U}_s$,

$$a_s = \mathcal{U}_s a_0 \mathcal{U}_s^\dagger, \tag{2.15}$$

labeled by the parameter s, the renormalization group parameter. This parameter has dimension of length. Physically s has the interpretation of the characteristic size of the effective particles.

Due to dimensional and notational reasons that will be explained latter, it's convenient to consider the scale parameter $t = s^4$ instead of s.

The effective Hamiltonian is related to the regulated canonical one with counterterms by the condition,

$$\mathcal{H}_t(a_t) = \mathcal{H}_0(a_0) \tag{2.16}$$

Then $s = 0$ correspond to the point-like or bare particles, and recovering the original

combining with the equation (2.15), and considering the parameter t instead of s, the condition becomes,

$$\mathcal{H}_t(a_0) = \mathcal{U}_t^{\dagger}\mathcal{H}_0(a_0)\mathcal{U}_t, \tag{2.17}$$

differentiating with respect of t,

$$\mathcal{H}_t'(a_0) \equiv \frac{d}{dt}\mathcal{H}_t(a_0) = \left[-\mathcal{U}_t^{\dagger}\mathcal{U}_t', \mathcal{H}_t(a_0)\right] = \left[\mathcal{G}_t(a_0), \mathcal{H}_t(a_0)\right], \tag{2.18}$$

where $\mathcal{G}_t$ is the RGPEP generator, this generator is defined as

We will consider the generator from Ref. [3]

$$\mathcal{G}_t = \left[\mathcal{H}_f, \mathcal{H}_{Pt}\right], \tag{2.19}$$

where $\mathcal{H}_f$, the free part of $\mathcal{H}_t$, and $\mathcal{H}_{Pt}$ is defined as function of the interacting term.

$$= \left[\left[\mathcal{H}_f, \mathcal{H}_{Pt}\right], \mathcal{H}_t\right] \tag{2.20}$$

where $\mathcal{H}_t$ is the Hamiltonian interested in solving.

The free Hamiltonian $\mathcal{H}_f$ is the part of $\mathcal{H}_0(a_0)$ that does not depend on the coupling constants,

2.6 Diagram representation

From the expression of the Hamiltonian, different terms can be separated and correlate to a diagram representation of the process, similar to the Feynman diagrams.

3 Cases

3.1 Scalar case

4 Code implementation

5 Diagrams obtained

5.1 Scalar

5.2 QED

5.3 QCD

6 Conclusions and future work

References

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