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TRABAJO FIN DE GRADO

Herramientas para cálculos perturbativos en renormalización de Hamiltonianos

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Resumen

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Abstract

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1 Introduction

When trying to build a theory that describes the particles and their interactions, it's fundamental that the theory is compatible with the 2 pillars of modern physics, the special relativity and quantum mechanics. The special relativity is a theory that's able to describe the behavior of particles at high energies, and quantum mechanics is a theory that describes the behavior of particles at small scales. The combination of these two theories is the basis of the quantum field theory, building a theory that describes the particles as excitations of a field.

Other important aspect of the theory is that it should be able to describe the different phenomena that occur at different scales. At low energies, the dominant physics involves, bound states, and the interactions between particles are weak. While at high energies, the dominant physics involves scattering processes, and the interactions between particles are strong. The theory should be able to describe the transition between these two regimes.

In the context of theoretical physics, the renormalization group procedure is a powerful tool to study the behavior of physical systems at different scales. In the case of quantum field theories, renormalization allows to study the system at different energy scales by introducing a scale parameter, and by changing this parameter, the "resolution" of the system is changed, allowing to focus from the smallest details (the short distance behavior) to the largest ones (the grand scale behavior).

The typical renormalization procedures tend to apply to Lagrangian dynamics, but in the framework of RGPEP, the renormalization group procedure is applied to Hamiltonian dynamics, the benefit of this approach is the ability obtain directly the solutions of the system (the spectrum of the theory, and the eigenstates of the Hamiltonian). RGPEP introduces a effective Hamiltonian, that describes the system at a given scale. This effective Hamiltonian is the solution to a differential equation, that describes the evolution of the effective Hamiltonian with respect to the scale parameter.

In general, the exact solution of the Hamiltonian is a non-trivial task, and a perturbative method is used to obtain the solution, by identifying each order in the Hamiltonian expansion with a series of diagrams, the next order in the expansion can be obtained by some operations on the diagrams of previous orders. In the past, these diagrams were obtained by hand, but the number of diagrams increases exponentially with each order, making the process tedious and error-prone. The goal of this thesis is to develop a code that automates the process of obtaining the diagrams, and to obtain the diagrams for a given order in the expansion.

This thesis is organized as follows. Section 2 describes the theoretical background needed to understand the renormalization group procedure for effective particles, and the Hamiltonian dynamics. Section 3 describes the different cases studied in this thesis, mainly the gluon case, where the simplicity similar of the scalar case is preserved, but the some of the peculiarities of the QCD theory are present. The section 4 describes the connection between the diagrams and the objects are defined in the code, as well as the steps taken to obtain higher order diagrams, that are presented in the section 5. Finally, the section 6 presents the conclusions, future work and the improvement that can be done to the code.

2 Theoretical background

To describe the relativistic interactions of effective particles in the framework of the renormalization group procedure for effective particles, and making use of quantum theory in the front form of Hamiltonian dynamics. It's important to understand each of the concepts involved in the process.

2.1 Front form of Hamiltonian Dynamics

The front form of dynamics introduced by Dirac (1949) [1] offers a couple advantages to the

The quantization hypersurface considered is,

$$x^+ = t + z = x^0 + x^3 = 0, \tag{2.1}$$

then the rest of coordinates will be defined as

$$x^- = x^0 - x^3, \quad x^\perp = (x^1, x^2), \tag{2.2}$$

In this set of coordinates, the fundamental quantities

The Hamiltonian in FF quantization is obtained from the Lagrangian density using the Legendre transformation, similar to the procedure explained in the section 2.2, but with respect to the new coordinates.

2.2 Canonical Hamiltonian

2.2.1 Scalar fields

2.2.2 Gluon fields

The Lagrangian density for the gluon fields is given by,

$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}F^{\mu\nu}F_{\mu\nu} \tag{2.3}$$

where $F^{\mu\nu}$ is the field strength tensor, defined as,

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu], \tag{2.4}$$

and $A^\mu = A^{a\mu}t^a$, t^a are the generators of the gauge group, and g is the coupling constant. Verifying the following relations,

$$[t^a, t^b] = if^{abc}t^c, \quad \mathrm{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}, \tag{2.5}$$

We will be working in the gauge $A^+ = 0$, in this gauge the Lagrange equations constrain the component A^- to become,

$$A^- = \frac{1}{\partial^+} 2\partial^\perp A^\perp - \frac{2}{\partial^{+2}} ig \left[\partial^+ A^\perp, A^\perp\right], \tag{2.6}$$

this way the only degree of freedom left is the transverse component $A^\perp$.

This way the associated energy-momentum tensor reads,

$$\mathcal{T}^{\mu\nu} = -F^{a\mu\alpha}\partial^\nu A^a_\alpha + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}. \tag{2.7}$$

The Hamiltonian density in FF quantization is obtained from integrating the component $\mathcal{T}^{+-}$ of the energy-momentum tensor, over the hyperplane $x^+ = 0$.

2.3 Fock space

In order to describe a variable number of particles in our system, the use of the Fock space is needed. Intoduced by V.A. Fock in 1932 [2], the Fock space is a sum of a set of Hilbert spaces, each one corresponding to a different number of particles in the system.

The Fock space is defined as the direct sum of tensor products of the single particle Hilbert space $\mathbb{H}$,

$$\mathbb{F}_\nu = \bigoplus_{n=0}^{\infty} S_\nu \mathbb{H}^{\otimes n} = \mathbb{C} \oplus \mathbb{H} \oplus S_\nu(\mathbb{H} \otimes \mathbb{H}) \oplus S_\nu(\mathbb{H} \otimes \mathbb{H} \otimes \mathbb{H}) \oplus \cdots, \tag{2.8}$$

where S_ν is the symmetrization operator depending on whether the particles described are bosonic or fermionic, it symmetrizes or antisymmetric the tensors, and $\mathbb{C}$ is the complex scalar, corresponding to the states with no particles.

This way a general state in the Fock space can be expressed as,

$$|\Psi\rangle_\nu = |\Psi_0\rangle_\nu \oplus |\Psi_1\rangle_\nu \oplus |\Psi_2\rangle_\nu \oplus \cdots = a\,|0\rangle_\nu + \sum_{i=1} a_i\,|\psi_i\rangle_\nu + \sum_{i,j=1} a_{ij}\,|\psi_i\psi_j\rangle_\nu + \cdots, \tag{2.9}$$

where $|\Psi_0\rangle_\nu$ is the vacuum state, $|\Psi_1\rangle_\nu$ is the one particle state, $|\Psi_2\rangle_\nu$ is the two particle state, and so on. The coefficients a_i are the amplitudes of the states, in general complex numbers.

In the case of QCD, a geneal state in the Fock space involves a superposition of all possible multiparticle states, build from quarks, antiquarks, and gluons, with the correct quantum numbers. This way a general state in the Fock space can be expressed as, quarkonium

$$|\Psi\rangle = c_1\,|q\bar{q}\rangle + c_2\,|q\bar{q}g\rangle + c_3\,|qqq\rangle + c_4\,|gg\rangle + \cdots, \tag{2.10}$$

2.4 Counterterms

The counterterms is an additional term added to the initial or bare Hamiltonian $\mathcal{H}_0$ to deal with the divergences due to loops, produced during the process. This way, ensure that the effective Hamiltonian $\mathcal{H}_t$ remains finite at all values of t.

2.5 RGPEP

The renormalization group procedure for effective particles (RGPEP), as it's name indicate is a renormalization group procedure applied to the Hamiltonian formulation.

By considering a series of unitary transformations, the RGPEP is able to construct a series of effective Hamiltonians $\mathcal{H}_s$, and the corresponding effective particles, by the use of effective particle operators (namely creation and annihilation operators) that differs from the canonical ones by the unitary transformation $\mathcal{U}_s$,

$$a_s = \mathcal{U}_s a_0 \mathcal{U}_s^\dagger, \tag{2.11}$$

labeled by the parameter s, the renormalization group parameter. This parameter has dimension of length. Physically s has the interpretation of the characteristic size of the effective particles.

Due to dimensional and notational reasons that will be explained latter, it's convenient to consider the scale parameter $t = s^4$ instead of s.

The effective Hamiltonian is related to the regulated canonical one with counterterms by the condition,

$$\mathcal{H}_t(a_t) = \mathcal{H}_0(a_0) \tag{2.12}$$

Then $s = 0$ correspond to the point-like or bare particles, and recovering the original

combining with the equation (2.11), and considering the parameter t instead of s, the condition becomes,

$$\mathcal{H}_t(a_0) = \mathcal{U}_t^\dagger \mathcal{H}_0(a_0) \mathcal{U}_t, \tag{2.13}$$

differentiating with respect of t,

$$\mathcal{H}_t'(a_0) \equiv \frac{d}{dt}\mathcal{H}_t(a_0) = \left[-\mathcal{U}_t^\dagger \mathcal{U}_t', \mathcal{H}_t(a_0)\right] = \left[\mathcal{G}_t(a_0), \mathcal{H}_t(a_0)\right], \tag{2.14}$$

where $\mathcal{G}_t$ is the RGPEP generator, this generator is defined as

We will consider the generator from Ref. [3]

$$\mathcal{G}_t = \left[\mathcal{H}_f, \mathcal{H}_{Pt}\right], \tag{2.15}$$

where $\mathcal{H}_f$, the free part of $\mathcal{H}_t$, and $\mathcal{H}_{Pt}$ is defined as function of the interacting term.

$$= \left[\left[\mathcal{H}_f, \mathcal{H}_{Pt} \right], \mathcal{H}_t \right] \tag{2.16}$$

where $\mathcal{H}_t$ is the Hamiltonian interested in solving.

The free Hamiltonian $\mathcal{H}_f$ is the part of $\mathcal{H}_0(a_0)$ that does not depend on the coupling constants,

2.6 Diagram representation

From the expression of the Hamiltonian, different terms can be separated and correlate to a diagram representation of the process, similar to the Feynman diagrams.

3 Cases

3.1 Scalar case

4 Code implementation

5 Diagrams obtained

5.1 Scalar

5.2 QED

5.3 QCD

6 Conclusions and future work

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