UNIVERSIDAD DE GRANADA

Facultad de Ciencias

GRADO EN FÍSICA

TRABAJO FIN DE GRADO

Herramientas para cálculos perturbativos en renormalización de Hamiltonianos

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Resumen

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Abstract

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Contents

1 Introduction

It's in our interest to formulate a theory compatible with special relativity and quantum mechanics, this culminates in quantum field theories.

we should expect the most general theories to be scale invariant,

In the context of theoretical physics renormalization is an important procedure

Tipical renormalization procedures apply to Lagrangian dynamics,

2 Theoretical background

To describe the relativistic interactions of effective particles in the framework of the renormalization group procedure for effective particles, making use of quantum theory in the front form of Hamiltonian dynamics.

2.1 Canonical Hamiltonian

This is the Hamiltonian that describes the theory, it dictates the evolution and interactions of the elements in the theory. In

In the case of field theories, a Hamiltonian density can be defined such that integrating over space coordinates the Hamiltonian is obtained.

$$H = \int d^3x \mathcal{H} \tag{2.1}$$

for simplicity we will call the Hamiltonian density $\mathcal{H}$, as Hamiltonian.

Solving the system means diagonalizing the Hamiltonian matrix, and finding the eigenvalues, or in this context eigenenergies of the system. In general this is a non trivial process.

In field theory calculations the fundamental quantities to consider are the energy-momentums, angular momentums, and boosts, due to it's relation or conservation under the Poincaré group, this is a set of transformation.

It's expression in point form of dynamics, this is expressing the set of dynamical variables refering to the physical condition of the system at some instant of time, we could say this is the "usual way" of expressing the dynamical variables, are,

2.2 Fock space

In order to describe a varible number of particles, the theory needs to be build in the Fock space, this is of the form,

2.3 Front form of Hamiltonian Dynamics

The front form of dynamics introduced by Dirac (1949) [1] offers a couple advantages to the

The quantization hypersurface considered is,

$$x^+ = t + z = x^0 + x^3 = 0, \tag{2.2}$$

then the rest of coordinates will be defined as

$$x^- = x^0 - x^3, \quad x^\perp = \{x^1, x^2\}, \tag{2.3}$$

In this set of coordinates, the fundamental quantities

2.4 RGPEP

The solution

RGPEP introduces effective particle operators (namely creation and annihilation operators) that differs from the canonical ones by a unitary transformation

$$a_s = \mathcal{U}_s a_0 \mathcal{U}_s^\dagger, \tag{2.4}$$

labeled by the parameter s, the renormalization group parameter. This parameter has dimension of length. Physically s has the interpretation of the characteristic size of the effective particles.

Then $s = 0$ correspond to the point-like or bare particles

Due to dimensional and notational reasons, it's convenient to use the scale parameter $t = s^4$ instead of s.

The effective Hamiltonian is related to the regulated canonical one with counterterms by the condition,

$$\mathcal{H}_t(a_t) = \mathcal{H}_0(a_0) \tag{2.5}$$

combining with the equation (2.4), and considering the parameter t instead of s, the condition becomes,

$$\mathcal{H}_t(a_0) = \mathcal{U}_t^\dagger \mathcal{H}_0(a_0) \mathcal{U}_t, \tag{2.6}$$

differentiating with respect of t,

$$\mathcal{H}_t'(a_0) \equiv \frac{d}{dt}\mathcal{H}_t(a_0) = \left[-\mathcal{U}_t^\dagger \mathcal{U}_t', \mathcal{H}_t(a_0)\right] = \left[\mathcal{G}_t(a_0), \mathcal{H}_t(a_0)\right], \tag{2.7}$$

where $\mathcal{G}_t$ is the RGPEP generator, this generator is defined as

We will consider the generator from Ref. [2]

$$\mathcal{G}_t = \left[\mathcal{H}_f, \mathcal{H}_{Pt}\right], \tag{2.8}$$

where $\mathcal{H}_f$, the free part of $\mathcal{H}_t$, and $\mathcal{H}_{Pt}$ is defined as function of the interacting term.

$$= \left[\left[\mathcal{H}_f, \mathcal{H}_{Pt}\right], \mathcal{H}_t\right] \tag{2.9}$$

where $\mathcal{H}_t$ is the Hamiltonian interested in solving,

2.5 Counterterms

The counterterms is an additional term added to the initial or bare Hamiltonian $\mathcal{H}_0$ to deal with the divergences due to loops, produced during the process. This way, ensure that the effective Hamiltonian $\mathcal{H}_t$ remains finite at all values of t.

2.6 Diagram representation

From the expression of the Hamiltonian, different terms can be separated and correlate to a diagram representation of the process, similar to the Feymann diagrams.

3 Cases

3.1 Scalar case

4 Code implementation

5 Diagrams obtained

5.1 Scalar

5.2 QED

5.3 QCD

6 Conclusions and future work

References

[1] P. A. M. Dirac. Forms of Relativistic Dynamics. *Reviews of Modern Physics*, 21(3):392–399, July 1949.

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