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Herramientas para cálculos perturbativos en renormalización de Hamiltonianos

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Resumen

Los processos de renormalización son fundamentales para la comprensión de la teorías a distintas escalas, definiendo teorías efectivas que describen el comportamiento de los sistemas para las distintas energías. Sin embargo, los cálculos son no triviales, y se emplean la teoría de perturbaciones para obtener los resultados, representando los distintos términos de la teoría como diagramas que describen el proceso, similar a los diagramas de Feynman. Pero el número de diagramas aumenta exponencialmente con el orden de la perturbación, haciendo que el proceso de cálculo sea tedioso y propenso a errores.

El objetivo de este trabajo es desarrollar un programa que automatice el proceso de obtención de los diagramas asociados a un proceso determinado, hasta un orden dado, partiendo de unos diagramas base, llamado diagramas canónicos dados por la teoría. Siendo el programa capaz de descartar los diagramas que no contribuyen al proceso, detectar loops y añadir contratérminos a los diagramas para cancelar divergencias.

Analizando los diagramas obtenidos de ordenes inferiores, y comparando con resultados conocidos, se ha comprobado la veracidad del programa. Esto permite obtener los diagramas de ordenes superiores, y estudiar el comportamiento de la teoría a dichos ordenes, en las que se pueden observar fenómenos único de teorías no abelianas.

Abstract

The renormalization procedure is fundamental to understand the behavior of the theories at different scales, defining effective theories that describe the behaviour of the systems for different energy levels. However, the calculations are non-trivial, and perturbation theory is used to obtain the results, representing the different terms of the theory as diagrams that describe the process, similar to the Feynman diagrams. But the number of diagrams increases exponentially with the order of the perturbation, making the calculation process tedious and prone to errors.

The aim of this work is to develop a program that automates the process of obtaining the diagrams associated with a given process, up to a given order, starting from a set of base diagrams, called canonical diagrams given by the theory. The program is able to discard the diagrams that do not contribute to the process, detect loops and add counterterms to the diagrams to cancel divergences.

By analysing the diagrams obtained from lower orders, and comparing with known results, the correctness of the program has been verified. This allows to obtain the diagrams of higher orders, and study the behavior of the theory at those orders, where unique phenomena of non-abelian theories can be observed.

Contents

1 Introduction

When trying to build a theory that describes the particles and their interactions, it's fundamental that the theory is compatible with the two pillars of modern physics: special relativity and quantum mechanics. Special relativity is a theory that is able to describe the behavior of particles at high energies, and quantum mechanics is a theory that describes the behavior of particles at small scales. Combining these two pillars gives rise to quantum field theory (QFT) – a formalism in which particles are described as excitations of underlying fields.

An important aspect of any theoretical approach is that it should be able to describe different phenomena that occur at different scales. In the context of Quantum Chromodynamics (QCD), at low energies (long distances), experiments see only bound states (hadrons), with the internal structure smoothed out. In contrast, high energies (short distances), one can probe inside hadrons – quarks and gluons behave almost free (the phenomenon of asymptotic freedom). The theory must interpolate between these regimes.

We will adopt the Hamiltonian formalism to describe the dynamics of the system, working on the operator space, where the Hamiltonian operator governs the time evolution of the states.

In the context of theoretical physics, the renormalization group procedure for effective particle (RGPEP) is a powerful tool to study the behavior of physical systems at different energy scales. In the case of quantum field theories, the renormalization group procedures introduce a scale parameter, which moderates the "resolution" of the system, allowing one to select the focus on a particular energy scale, from the smallest details (the short distance behavior) to the largest ones (the grand scale behavior).

RGPEP introduces an effective Hamiltonian, that describes the system at a given scale. This effective Hamiltonian is the solution to a differential equation, the RGPEP equation, that describes the evolution of the effective Hamiltonian with respect to the scale parameter.

In general, obtaining the exact solution of the RGPEP equation is a non-trivial task, and a perturbative expansion of the effective Hamiltonian is used to obtain the solution. Every interaction term in the Hamiltonian can be identified with a diagram, which, in term, results as a product of diagrams of lower order. These diagrams are usually analyzed by hand, but the number of diagrams increases exponentially with each order, making the process tedious and error-prone. The goal of this thesis is to develop a code that automates the process of obtaining the diagrams associated with a certain interaction for a given order.

This thesis is organized as follows. Section 2 describes the theoretical background needed to understand the renormalization group procedure for effective particles, and the Hamiltonian dynamics. Section 3 describes the case studied in this thesis, gluon interactions, where holding the simplicity of the scalar case, some peculiarities of the QCD theory are present. Section 4 describes how diagrams are defined in the code, and section 5 describes the steps taken to obtain higher order diagrams, as product of elementary ones. Finally, section 6 presents

the conclusions, possible future work and further improvement to the code.

2 Theoretical background

To understand the basis of the RGPEP, we need to describe the different concepts involved in the framework, as well as the theories that the process is applied to.

2.1 Fock space

In quantum field theory, the number of particles is not conserved and processes such as particle creation and annihilation occur. The formal description of such features operates on the Fock space [1]. The Fock space is a sum of different Hilbert spaces, each one corresponding to a different number of particles, thus allowing the description of quantum systems with a variable number of particles.

The Fock space is defined as the direct sum of tensor products of the single particle Hilbert space $\mathbb{H}$,

$$\mathbb{F} = \bigoplus_{n=0}^{\infty} \mathbb{H}^{\otimes n} = \mathbb{C} \oplus \mathbb{H} \oplus (\mathbb{H} \otimes \mathbb{H}) \oplus (\mathbb{H} \otimes \mathbb{H} \otimes \mathbb{H}) \oplus \cdots , \tag{2.1}$$

where $\mathbb{C}$ is the complex scalar, corresponding to the states with no particles, and each terms $\mathbb{H}^{\otimes n}$ represent the Hilbert space for n-particle states.

This way a general state in the Fock space can be expressed as,

$$|\Psi\rangle = |\Psi_0\rangle \oplus |\Psi_1\rangle \oplus |\Psi_2\rangle \oplus \cdots = c\,|0\rangle + \sum_{i=1} c_i\,|\psi_i\rangle + \sum_{i,j=1} c_{ij}\,|\psi_i\psi_j\rangle + \cdots , \tag{2.2}$$

where $|\Psi_0\rangle$ is the vacuum state, $|\Psi_1\rangle$ is the one particle state, $|\Psi_2\rangle$ is the two particle state, and so on. The coefficients c_i are the amplitudes of the states, in general complex numbers.

Fock space provides a natural framework for quantum field theories, where, physical states are expressed as superposition of all allowable multiparticle configurations consistent with color confinement and other quantum numbers. For instance, a quarkonium state (a bound state of quark and antiquark) in QCD is given by:

$$|\Psi\rangle = c_1\,|q\bar{q}\rangle + c_2\,|q\bar{q}g\rangle + c_3\,|qqq\rangle + c_4\,|gg\rangle + \cdots , \tag{2.3}$$

2.2 Hamiltonian dynamics

In quantum field theory, 2 equivalent formulations of the dynamics can be used, the Lagrangian and the Hamiltonian formulations. Although one often starts with a Lagrangian formulation and then switches to a Hamiltonian via a Legendre transform, here we adopt the Hamiltonian (canonical) formalism because RGPEP is implemented in Hamiltonian dynamics.

2.2.1 Noether's theorem

Noether's theorem [**?**] states that every continuous symmetry of the Lagrangian of a physical system corresponds to a conserved quantity. In the context of quantum field theory, this theorem is fundamental as it relates symmetries of the Lagrangian to conservation laws, which are crucial for understanding the dynamics of the system.

Considering a general continuous transformation of the fields $\phi_a(x)$, which in the infinitesimal form can be expressed as,

$$\phi_a(x) \to \phi'_a = \phi_a(x) + \delta\phi_a(x) = \phi_a(x) + \epsilon\mathcal{Q}_a(x), \tag{2.4}$$

where ϵ is an infinitesimal parameter, and $\mathcal{Q}_a(x)$ is some deformation of the field. This transformation is a symmetry if it leaves the equations of motion invariant, which means that the Lagrangian density $\mathcal{L}(\phi_a, \partial_\mu\phi_a)$ remains unchanged under the transformation, up to a 4-divergence term:

$$\mathcal{L}(\phi'_a, \partial_\mu\phi'_a) = \mathcal{L}(\phi_a, \partial_\mu\phi_a) + \epsilon\partial_\mu J^\mu. \tag{2.5}$$

The conserved current j^μ is defined as the Noether current associated with the symmetry transformation, which is given by,

$$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)}\mathcal{Q}_a - J^\mu, \quad \partial_\mu j^\mu = 0, \tag{2.6}$$

where Einstein summation is assumed.[1]

Applying Noether's theorem, the conserved charge Q associated with the symmetry is defined as the integral of the time component of the current over all space:

$$Q = \int d^3x\, j^0(x). \tag{2.7}$$

Applying Noether's theorem to translations, in infinitesimal form,

$$x^\mu \to x'^\mu = x^\mu + \epsilon\mathcal{A}^\mu, \tag{2.8}$$

where $\mathcal{A}^\mu$ is a vector that represents the infinitesimal translation, the transformation of the fields is given by,

$$\phi_a(x) \to \phi'_a(x') = \phi_a(x) + \epsilon\mathcal{A}^\mu\partial_\mu\phi_a(x). \tag{2.9}$$

The Lagrangian density transforms as,

$$\mathcal{L} \to \mathcal{L} + \epsilon\mathcal{A}^\nu\partial_\mu\left(\delta^\mu_\nu\mathcal{L}\right), \tag{2.10}$$

Comparing with equation (2.5), four different conserved currents can be identified,

[1] The Einstein summation convention is a notational convention in which repeated indices in a mathematical expression imply summation over those indices. For example, in the expression A_iB^i, it is understood that there is an implicit sum over the index i, i.e., $A_iB^i = \sum_i A_iB^i$.

$$T^{\mu}{}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)} \partial_\nu \phi_a - \delta^{\mu}{}_{\nu} \mathcal{L}, \tag{2.11}$$

this is the energy-momentum tensor of the field. The conserved charge associated with the time translation symmetry is the Hamiltonian H,

$$H = \int d^3x \, T^{00}(x) = \int d^3x \mathcal{H}, \tag{2.12}$$

and the conserved charges associated with the spatial translations,

$$P^i = \int d^3x \, T^{0i}(x), \tag{2.13}$$

which it is interpreted as the physical momentum that the fields carry.

2.2.2 Canonical Hamiltonian

Consider a general field theory and a Lagrangian density $\mathcal{L}(\phi_a(x), \partial_\mu \phi_a(x))$[2], where $\phi_a(x)$ is a particular field of the system.

To formulate the theory in the Hamiltonian framework, we begin by defining the canonical conjugate momenta [2],

$$\pi_a(x) = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi_a(x))}. \tag{2.14}$$

The Hamiltonian density, from equation (2.12),

$$\mathcal{H} = \pi_a(x) \partial_0 \phi_a(x) - \mathcal{L}(\phi_a(x), \partial_\mu \phi_a(x)), \tag{2.15}$$

and the momentum,

$$P^i = \int d^3x \, T^{0i}(x) = \int d^3x \, \pi_a(x) \partial^i \phi_a(x). \tag{2.16}$$

In quantum mechanics, or quantum field theory, the canonical variables $\phi_a(x)$ and $\pi_a(x)$ are promoted to operators acting on a Hilbert space. Quantization is achieved by imposing equal-time canonical commutation relations [3],

$$[\phi_a(x), \pi_b(y)] = i\delta_{ab}\delta^3(x-y), \quad [\phi_a(x), \phi_b(y)] = 0, \quad [\pi_a(x), \pi_b(y)] = 0. \tag{2.17}$$

The field operators are expanded in terms of creation, a_i and annihilation $a_i^\dagger$ operators, that act on the Fock space, creating and annihilating the mode i of the field. Mathematically, the creation and annihilation operators are the Fourier components of the field.

Any state component in the Fock space can be expressed as the action of a series of creation operators on the vacuum state, $|0\rangle$,

[2] where x indicate a 4-vector, with x^μ its components, and $\partial_\mu = \frac{\partial}{\partial x^\mu}$

$$|\Psi_n\rangle = \sum_{i_1,i_2,\cdots,i_n} \frac{c_{i_1 i_2 \cdots i_n}}{\sqrt{n!}} a^\dagger_{i_1} a^\dagger_{i_2} \cdots a^\dagger_{i_n} |0\rangle \,, \tag{2.18}$$

making the full state,

$$|\Psi\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} \sum_{i_1,i_2,\cdots,i_n} c_{i_1 i_2 \cdots i_n} a^\dagger_{i_1} a^\dagger_{i_2} \cdots a^\dagger_{i_n} |0\rangle \,. \tag{2.19}$$

The creation and annihilation operators satisfy some commutation or anticommutation relations, depending on the nature of the particles, bosons or fermions, according to (2.17).

For bosons, the creation and annihilation operators satisfy the following commutation relations,

$$\left[a_i, a^\dagger_j\right] = \delta_{ij}, \quad \left[a_i, a_j\right] = 0, \quad \left[a^\dagger_i, a^\dagger_j\right] = 0. \tag{2.20}$$

For fermions, the creation and annihilation operators satisfy the following anticommutation relations,

$$\{q_i, q^\dagger_j\} = \delta_{ij}, \quad \{q_i, q_j\} = 0, \quad \{q^\dagger_i, q^\dagger_j\} = 0. \tag{2.21}$$

This way, the creation and annihilation operators are the fundamental objects acting on the Fock space, and the Hamiltonian and the fields are expressed in terms of these operators.

When a combination of creation and annihilation operators is considered, normal-ordering is used when defining operators that contain both creation and annihilation operators and avoid the infinite vacuum contribution. The normal-ordering is defined as the process of rearranging the creation and annihilation operators in such a way that all the creation operators are to the left of all the annihilation operators.

The normal-ordering is denoted by the symbol : :, and it is defined as follows:

$$: a_i a^\dagger_j : = a^\dagger_j a_i. \tag{2.22}$$

2.2.3 Front form of Hamiltonian Dynamics

The quantization of relativistic systems is most commonly performed in the instant form (IF) of dynamics, where the ordinary time coordinate $x^0 = t$ serves as the evolution parameter, and spatial coordinates $\vec{x}$ define the hypersurface of equal time. However, alternative forms of dynamics are possible and were classified by Dirac in 1949 [4].

One such alternative is the front form (FF) of dynamics, also known as light-front quantization. It is defined by choosing a new set of coordinates where the evolution parameter is,

$$x^+ = \frac{1}{\sqrt{2}}(x^0 + x^3), \tag{2.23}$$

and the remaining coordinates are,

$$x^- = \frac{1}{\sqrt{2}}(x^0 - x^3), \quad x^\perp = (x^1, x^2). \tag{2.24}$$

In this framework, quantization is performed on surfaces of constant x^+, treating it as the "light-front time", while $x^-, x^\perp$ play the role of spatial coordinates. The corresponding momenta are defined as,

$$p^+ = \frac{1}{\sqrt{2}}(p^0 + p^3), \quad p^- = \frac{1}{\sqrt{2}}(p^0 - p^3), \quad p^\perp = (p^1, p^2). \tag{2.25}$$

The light-front Hamiltonian P^- is derived from the conserved charge related to the component x^+, following a similar procedure as before. It governs the evolution in x^+, analogous to the role of $H = P^0$ in instant-form quantization.

A key feature of FF quantization is the positivity condition $p^+ > 0$ for all physical particles. This kinematic constraint ensures that particle creation from the vacuum, which has total $P^+ = 0$, is forbidden due to momentum conservation. As a result, the vacuum in FF is trivial or "empty" [5]: it contains no virtual particles and cannot mix with multi-particle states. [3]

This property simplifies the structure of the theory, particularly in the context of the RGPEP, where a clean separation between the vacuum and the particle spectrum is advantageous. FF quantization avoids the complexities associated with vacuum fluctuations that are typical in instant-form dynamics.

2.3 RGPEP

The RGPEP, is a renormalization scheme applied within the Hamiltonian formalism of quantum field theory, developed in Refs. [6–8]. By considering a series of unitary transformations applied to the canonical or bare Hamiltonian, the RGPEP is able to construct a sucession of effective Hamiltonians $\mathcal{H}_s$, each describing dynamics in terms of effective particles at a resolution scale set by the parameter s. This is associated with the renormalization group scale $\lambda = 1/s$, where λ have dimension of energy, and physically has the interpretation of the characteristic energy scale of the theory, while s has dimension of length, and physically has the interpretation of the characteristic scale of the theory.

Effective particles, are defined by the use of effective particle operators that differs from the canonical ones by the unitary transformation $\mathcal{U}_s$,

$$a_s = \mathcal{U}_s a_0 \mathcal{U}_s^\dagger. \tag{2.26}$$

[3] Although gluons are supposed to be massless and therefore this condition does not hold for them, in practice, it is often assumed to have infinitesimally small mass (which is taken to zero at the end of the calculation).

Due to dimensional and notational reasons, it's convenient to consider the scale parameter $t = s^4$ instead.

Then $s = 0$ or equivalently $t = 0$, the theory describes point-like or bare particles, and recovering the original Hamiltonian $\mathcal{H}_0(a_0)$.

The effective Hamiltonian $\mathcal{H}_t$, written in terms of the effective particles operator a_s, is related to the regulated canonical Hamiltonian with counter-terms by the condition (section 2.5),

$$\mathcal{H}_t(a_t) = \mathcal{H}_0(a_0), \tag{2.27}$$

Using Eq. (2.26) and expressing all operators in terms of the original a_0, the effective Hamiltonian becomes,

$$\mathcal{H}_t(a_0) = \mathcal{U}_t^\dagger \mathcal{H}_0(a_0)\mathcal{U}_t, \tag{2.28}$$

differentiating with respect of t, one obtains the RGPEP differential equation,

$$\mathcal{H}_t'(a_0) \equiv \frac{d}{dt}\mathcal{H}_t(a_0) = \left[-\mathcal{U}_t^\dagger \mathcal{U}_t', \mathcal{H}_t(a_0)\right] = \left[\mathcal{G}_t(a_0), \mathcal{H}_t(a_0)\right], \tag{2.29}$$

where $\mathcal{G}_t$ is the generator of the RGPEP transformation.

Considering the generator from Ref. [8],

$$\mathcal{G}_t = \left[\mathcal{H}_f, \mathcal{H}_{Pt}\right], \tag{2.30}$$

where $\mathcal{H}_f$, the free part of $\mathcal{H}_t$ and the part of $\mathcal{H}_0(a_0)$ that does not depend on the coupling constant g, while $\mathcal{H}_{Pt}$ is defined as function of the interacting term.

The resulting RGPEP equation have the form,

$$\mathcal{H}_t' = \left[\left[\mathcal{H}_f, \mathcal{H}_{Pt}\right], \mathcal{H}_t\right]. \tag{2.31}$$

In general, the solution of the RGPEP equation is a non-trivial task, and a perturbative expansion of the effective Hamiltonian is used to obtain them. Expressing $\mathcal{H}_t$ as a power series of the coupling constant g,

$$\mathcal{H}_t = \sum_{n=0}^{\infty} g^n \mathcal{H}_{tn} = \mathcal{H}_0 + g\mathcal{H}_{t1} + g^2\mathcal{H}_{t2} + g^3\mathcal{H}_{t3} + g^4\mathcal{H}_{t4} + \cdots. \tag{2.32}$$

Substituting into the RGPEP equation, and collecting terms order by order in g, the following differential equations are obtained,

$$\mathcal{H}_0' = 0, \tag{2.33}$$

$$g\mathcal{H}_{t1}' = \left[\left[\mathcal{H}_0, g\mathcal{H}_{Pt1}\right], \mathcal{H}_0\right], \tag{2.34}$$

$$g^2\mathcal{H}_{t2}' = \left[\left[\mathcal{H}_0, g^2\mathcal{H}_{Pt2}\right], \mathcal{H}_0\right] + \left[\left[\mathcal{H}_0, g\mathcal{H}_{Pt1}\right], g\mathcal{H}_{t1}\right], \tag{2.35}$$

$$g^3\mathcal{H}_{t3}' = \left[\left[\mathcal{H}_0, g^3\mathcal{H}_{Pt3}\right], \mathcal{H}_0\right] + \left[\left[\mathcal{H}_0, g^2\mathcal{H}_{Pt2}\right], g\mathcal{H}_{t1}\right] + \left[\left[\mathcal{H}_0, g\mathcal{H}_{Pt1}\right], g^2\mathcal{H}_{t2}\right]. \tag{2.36}$$

$$\vdots$$

The order 0 term is solvable from an initial condition, and the solution of the equation of order 1 will be an exponential of the parameter t. The 2nd order solution can be obtained from 1th order, and the 3rd order from the previous orders, and so on. At this point, the use of diagrams becomes crucial.

2.4 Order by order solutions.

The solution of the differential equations (2.34) to (2.36) and so on, without considering the multiplicative factors, and relative phases between the terms, can be expressed as,

$$\mathcal{H}_{t1} = \mathcal{H}_{01} \tag{2.37}$$
$$\mathcal{H}_{t2} = \mathcal{G}_{02} + \mathcal{H}_{01}\mathcal{H}_{01} \tag{2.38}$$
$$\mathcal{H}_{t3} = \mathcal{G}_{03} + \mathcal{H}_{01}\mathcal{H}_{01}\mathcal{H}_{01} + (\mathcal{G}_{02}\mathcal{H}_{01} + \mathcal{H}_{01}\mathcal{G}_{02}) \tag{2.39}$$
$$\mathcal{H}_{t4} = \mathcal{G}_{04} + \mathcal{H}_{01}\mathcal{H}_{01}\mathcal{H}_{01}\mathcal{H}_{01} + (\mathcal{G}_{02}\mathcal{H}_{01}\mathcal{H}_{01} + \mathcal{H}_{01}\mathcal{G}_{02}\mathcal{H}_{01} + \mathcal{H}_{01}\mathcal{H}_{01}\mathcal{G}_{02}) + (\mathcal{G}_{03}\mathcal{H}_{01} + \mathcal{H}_{01}\mathcal{G}_{03} + \mathcal{G}_{02}\mathcal{G}_{02})\,. \tag{2.40}$$
$$\vdots$$

This is shows how the different solutions for each order are obtained.

For instance, the diagrams for the first order is simply the canonical diagrams from $\mathcal{H}_{01}$. As for the second order, it contains the canonical diagrams from the second order $\mathcal{G}_{02}$, and the product of the first order diagrams. The third order contains the canonical diagrams from the third order $\mathcal{G}_{03}$, and the product of the first order, and second order canonical diagrams, as well as the product of exclusive first order diagrams. This way, we could rewrite the solution for each order in terms of the solution to the previous order,

$$\mathcal{H}_{t1} = \mathcal{H}_{01} = \mathcal{G}_{01}, \tag{2.41}$$
$$\mathcal{H}_{t2} = \mathcal{G}_{02} + \mathcal{H}_{t1}\mathcal{G}_{01}, \tag{2.42}$$
$$\mathcal{H}_{t3} = \mathcal{G}_{03} + \mathcal{H}_{t1}\mathcal{G}_{02} + \mathcal{H}_{t2}\mathcal{G}_{01}, \tag{2.43}$$
$$\mathcal{H}_{t4} = \mathcal{G}_{04} + \mathcal{H}_{t2}\mathcal{G}_{02} + \mathcal{H}_{t1}\mathcal{G}_{03} + \mathcal{H}_{t3}\mathcal{G}_{01} \tag{2.44}$$
$$\vdots$$

This is how different orders are related to each other, and how an iterative process can be used to obtain the solution for each order.

2.5 Regularization and Counterterms

In QCD, the bare Hamiltonian is ill-defined due to the presence of elements that contain ultraviolet (UV) divergences and infrared (IR) divergences [9, 10]. The UV divergences are produced in processes involving large momentum transfers,

whereas the IR divergences occur in processes with 'soft' particles carrying small longitudinal momentum fractions, $x_{p/P} = p^+/P^+$, yielding zeros in the denominator.

To deal with these divergences a regulating factor r is introduced in every interacting term. These factors make the interacting terms rapidly tend to zero, if the change in the transverse momentum of any gluon exceeds a certain cutoff parameter Δ, or if the change in longitudinal momentum of any gluon is greater than a cutoff parameter δ [11].

The particle operators is multiplied by the regulating factor,

$$r_{\Delta\delta}(k^\perp, x) = r_\Delta(k^\perp) r_\delta(x)\theta(x). \tag{2.45}$$

The transverse regulator factor will be of the form,

$$r_\Delta(z) = \exp\left(-\frac{z}{\Delta^2}\right), \tag{2.46}$$

with $\mathcal{M}$ the invariant mass of the system, ensuring that UV processes are suppressed.

The longitudinal regulator factor must verify a similar condition, preventing terms of the form $1/x$ or $1/x^2$ to blow up as x approaches 0. The exact form of the longitudinal regulator factor is not important for our purposes, as long as it verifies the condition, since it will be removed once we take the limit $\Delta \to \infty$ and $\delta \to 0$.

It's convenient to consider an abbreviated notation for the regulating factor, using the symbol $\tilde{r}_{P,p} = \tilde{r}_{\Delta\delta}(P, p)$, with,

$$\tilde{r}_{\Delta\delta}(P, p) = r_{\Delta\delta}(p^\perp - x_{p/P}P^\perp) r_{\Delta\delta}\left[P^\perp - p^\perp - \left(1 - x_{p/P}\right)P^\perp, 1 - x_{p/P}\right]. \tag{2.47}$$

The counterterms is an additional term added to the initial or bare Hamiltonian $\mathcal{H}_0$ to deal with the divergences due to loops, produced during the process. This way, ensure that the effective Hamiltonian $\mathcal{H}_t$ remains finite at all values of t.

The counterterms are defined in a way, such that the coefficients of products of creation and annihilation operators in the effective theory for gluons of size s become independent of the regularization parameter Δ when the regularization in dynamics of gluons of size zero is being removed. The rest of the unknown parts of the counterterms are adjusted to respect the symmetries of the theory, and must match the predictions of the theory with the experimental results.

The distinction between regularization and counterterms may be confusing, but the regularization is a procedure to expose and parameterize the divergences that arise in the theory, while the counterterms are the terms added to the Hamiltonian to cancel the regulated divergences, so that after the regulators are removed, the effective Hamiltonian remains finite and regulator independent. Making the regulation an essential first step in the renormalization process.

3 Case study: Gluons self-interactions

3.1 Quantum Chromodynamics (QCD)

QCD is the quantum field theory that describes the interactions of quarks and gluons, the fundamental constituents of hadrons. QCD is a non-abelian gauge theory based on the SU(3) gauge group, which describes the strong force, describing the interactions between quarks and gluons.

The Lagrangian for QCD is given by,

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^{a,\mu\nu}F^{a}_{\mu\nu} + \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi, \tag{3.1}$$

where $F^{a,\mu\nu}$ is the field strength tensor for the gluon fields,

$$F^{a,\mu\nu} = \partial^{\mu}A^{a,\nu} - \partial^{\nu}A^{a,\mu} + gf^{abc}A^{b,\mu}A^{c,\nu}, \tag{3.2}$$

$A^{a,\mu}$ are the gluon fields, ψ is the quark field, m is the quark mass, and D_{μ} is the covariant derivative defined as,

$$D_{\mu} = \partial_{\mu} - igA^{a}_{\mu}t^{a}, \tag{3.3}$$

where t^{a} are the generators of the SU(3) gauge group, and g is the coupling constant of the strong interaction.

The structure constants f^{abc} encodes the non-abelian nature of the theory, and are present explicitly in the commutation relations of the generators of the gauge group,

$$[t^{a}, t^{b}] = if^{abc}t^{c}, \tag{3.4}$$

as well as in the Hamiltonian, in the interacting terms involving the gluon fields.

3.2 Canonical Hamiltonian for the gluon fields

For our purposes, we will focus on the gluon fields. The Lagrangian density for the gluon fields is given by the first term of the QCD Lagrangian (3.1),

$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}F^{\mu\nu}F_{\mu\nu}, \tag{3.5}$$

We will be working in the gauge $A^{+} = 0$, where the Lagrange equations constrain the component A^{-} to become

$$A^{-} = \frac{1}{\partial^{+}}2\partial^{\perp}A^{\perp} - \frac{2}{\partial^{+2}}ig\left[\partial^{+}A^{\perp}, A^{\perp}\right]. \tag{3.6}$$

In this way, the only degree of freedom left is the transverse component $A^{\perp}$.

As for the associated energy-momentum tensor,

$$\mathcal{T}^{\mu\nu} = -F^{a\mu\alpha}\partial^{\nu}A^{a}_{\alpha} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}. \tag{3.7}$$

The Hamiltonian in FF quantization is obtained from integrating the component $\mathcal{T}^{+-}$ of the energy-momentum tensor, over the hyperplane $x^+ = 0$.

By working in the gauge $A^+ = 0$, the Hamiltonian density can be expressed as the sum of four terms, as denoted in [7]

$$\mathcal{T}^{+-} = \mathcal{H}_{A^2} + \mathcal{H}_{A^3} + \mathcal{H}_{A^4} + \mathcal{H}_{[\partial AA]^2}, \tag{3.8}$$

with each of the terms,

$$\mathcal{H}_{A^2} = -\frac{1}{2} A^{\perp a} (\partial^\perp)^2 A^{\perp a}, \tag{3.9}$$

$$\mathcal{H}_{A^3} = g i \partial_\alpha A^a_\beta \left[A^\alpha, A^\beta\right]^a, \tag{3.10}$$

$$\mathcal{H}_{A^4} = -\frac{1}{4} g^2 \left[A_\alpha, A_\beta\right]^a \left[A^\alpha, A^\beta\right]^a, \tag{3.11}$$

$$\mathcal{H}_{[\partial AA]^2} = -\frac{1}{2} g^2 \left[i\partial^+ A^\perp, A^\perp\right]^a \frac{1}{(i\partial^+)^2} \left[i\partial^+ A^\perp, A^\perp\right]^a. \tag{3.12}$$

Replacing A^μ with the operator $\hat{A}^\mu(x)$, defined by its Fourier composition on the plane $x^+ = 0$,

$$\hat{A}^\mu(x) = \sum_{\sigma c} \int [k] \left[t^c \epsilon^\mu_{k\sigma} a_{k\sigma c} e^{-ikx} + t^c \epsilon^{\mu *}_{k\sigma} a^\dagger_{k\sigma c} e^{ikx} \right]_{x^+=0}, \tag{3.13}$$

where $[k] = \theta(k^+) dk^+ d^2k^\perp / (16\pi^3 k^+)$, $\epsilon^\mu_{k\sigma}$ are the polarization vectors, and $a^\dagger_{k\sigma c}$, $a_{k\sigma c}$ are the creation and annihilation operators (particle operators), respectively.

Substituting this expression into each term of the Hamiltonian densities, integrating over space coordinates and taking into account the completeness and orthonormality of the polarization vectors, we obtain the following expression for the different terms of the Hamiltonian [9],

$$H_{A^2} = \sum_{\sigma c} \int [k] \frac{k^{\perp 2}}{k^+} a^\dagger_{k\sigma c} a_{k\sigma c}, \tag{3.14}$$

$$H_{A^3} = \sum_{123} \int [123] \delta(p^+ - p) \tilde{r}_{\Delta\delta}(3,1) g \left[Y_{123} a^\dagger_1 a^\dagger_2 a_3 + Y^*_{123} a^\dagger_3 a_2 a_1 \right], \tag{3.15}$$

$$H_{A^4} = \sum_{1234} \int [1234] \delta(p^+ - p) \frac{g^2}{4} \left[\left(\Xi_{A^4 1234} a^\dagger_1 a^\dagger_2 a^\dagger_3 a_4 + h.c. \right) + X_{A^4 1234} a^\dagger_1 a^\dagger_2 a_3 a_4 \right], \tag{3.16}$$

$$H_{[\partial AA]^2} = \sum_{1234} \int [1234] \delta(p^+ - p) g^2 \left[\left(\Xi_{[\partial AA]^2 1234} a^\dagger_1 a^\dagger_2 a^\dagger_3 a_4 + h.c. \right) + X_{[\partial AA]^2 1234} a^\dagger_1 a^\dagger_2 a_3 a_4 \right]. \tag{3.17}$$

Here the notation [123] indicates the integration over the momenta of the particles 1 to 3, $[123] = [k_1][k_2][k_3]$. The explicit expression for the functions $\Xi, Y, X, ...$,

can be found in appendix A, for the purpose of this thesis, we are only interested in the dependence of the functions on the structure constant f^{abc}.

The terms H_{A^2}, H_{A^3}, H_{A^4} and $H_{[\partial AA]^2}$ represent the different interaction terms of the gluon fields: H_{A^2} corresponds to the kinetic term of the gluon fields, H_{A^3} corresponds to the three-gluon vertex and H_{A^4} together with $H_{[\partial AA]^2}$ corresponds to four-gluon vertices.

In the context of RGPEP, the four-gluon vertices can be treated as a combination of two time ordered three-gluon vertices, by introducing an internal gluon propagator, or as a special case of instantaneous interaction, as in the term $H_{[\partial AA]^2}$, effectively reproducing the four-gluon interaction at that energy scale.

As for the expansion in terms of the coupling constant g, the term H_{A^2} is an order zero term, since it does not contain the coupling constant g. The term H_{A^3} is an order first order term, since it contains g to the first power, as well as a single structure constant f^{abc}. While the term H_{A^4} is a second order term, since it contains g^2 to the second power, and two structure constants, but since it can be expressed as a product of two first order terms, it will not be considered as a new canonical term, rather as a result of the perturbative expansion.

The term $H_{[\partial AA]^2}$ is a second order term, for the same reasons as H_{A^4}, but it is a special case that can't be considered as a product of two first order terms, and thus will be considered as a new canonical term.

As we can see, in the case of QCD, there are only canonical diagrams of order 1 and 2, then referring to the framework laid out in section 2.4, from order 3 and onwards, the diagrams only depend explicitly on the diagrams of previous order, as well as the order 1, and the order 2 canonical diagrams.

For example, in Eq. (2.44), the solution at order 4, $\mathcal{H}_{t4}$ consists of $\mathcal{H}_{t3}$ and the canonical diagrams, thus eliminating the term with $\mathcal{G}_{04}$ and $\mathcal{H}_{t1}\mathcal{G}_{03}$, since they are not present in the QCD theory. This simplifies the number of diagrams that can be obtained at higher orders, and making the diagrams have the form,

$$\mathcal{H}_{t1} = \mathcal{H}_{01} = \mathcal{G}_{01}, \tag{3.18}$$
$$\mathcal{H}_{t2} = \mathcal{G}_{02} + \mathcal{H}_{t1}\mathcal{G}_{01}, \tag{3.19}$$
$$\mathcal{H}_{t3} = \mathcal{H}_{t1}\mathcal{G}_{02} + \mathcal{H}_{t2}\mathcal{G}_{01}, \tag{3.20}$$
$$\mathcal{H}_{t4} = \mathcal{H}_{t2}\mathcal{G}_{02} + \mathcal{H}_{t3}\mathcal{G}_{01} \tag{3.21}$$
$$\mathcal{H}_{t5} = \mathcal{H}_{t3}\mathcal{G}_{02} + \mathcal{H}_{t4}\mathcal{G}_{01} \tag{3.22}$$
$$\vdots$$
$$\mathcal{H}_{tn} = \mathcal{H}_{t(n-1)}\mathcal{G}_{02} + \mathcal{H}_{t(n-2)}\mathcal{G}_{01} \quad \text{for } n \geq 3. \tag{3.23}$$

Obtaining a simpler recursive relation to obtain the diagrams of higher order, that can be implemented in a program to obtain the diagrams of higher order.

4 Code implementation

The code is implemented in Python, but the principles of the method can be written in any programming language. The program is designed to be modular, and applicable to other any theory, these require changing the canonical diagrams. These modifications are beyond the of this work, and they are not presented here explicitly.

4.1 Definition of diagrams

The diagrams are defined by 2 arrays,

- Points: arrays of dimension $N \times 2$, where N is the number of points in the diagram, each point is defined by its coordinates (x, y).
- Paths: arrays of dimension $M \times N' \times 2$, where M is the number of different types of particles to consider in the theory, N' is the number of paths for each types of particle in the diagram, and 2 indicate the points to connect.

This way of defining the diagrams is analogous to the way of defining the undirected graphs in the graph theory, where the points are the vertices and the paths are the edges.

In the case of gluon interactions, although only 1 type of particle is present, the instantaneous interactions have to be considered. This is done by defining this interaction as a new type of virtual particle in the program.

4.2 Order by order procedure

To calculate the diagram representing a certain order, the code follows the procedure described in Section 2.4. Having the canonical diagrams as an input, the program aims to obtain all possible diagrams of such order that contribute to a certain effective interaction, and discards those diagrams, with different structure.

The program would follow a recursive procedure, where the diagrams of order n are obtained from the diagrams of order $n-1$ and $n-2$, in a procedure that can be outlines as follows,

1. Start with the canonical diagrams of order 1 and 2.
2. For each order n, starting from 3, do the following,
   (a) Generate all possible combinations of diagrams of order $n-1$ with order 1, and $n-2$ with order 2.
   (b) For each combination, check if it is a valid diagram for the process being considered.
   (c) If it is a valid diagram, add it to the list of diagrams for order n.
3. After generating all the diagrams of order n, check for equivalent diagrams, and add their contributions to the list of diagrams.

4. If the order is greater than 2, check for loops, and add the counterterms to the list of diagrams.

5. Repeat the process for the next order, until the desired order is reached.

The step 3 is fundamental in order to reduce the number of diagrams that needs to be used to calculate the next order. This procedure is the most time-consuming process of the program, needing a search algorithm to find the equivalent diagrams, meaning that as the order increases, the number of diagrams increases exponentially, and the time to find the equivalent diagrams increases exponentially too.

Although the process of finding the equivalent diagrams is time-consuming, it is fundamental to reduce the global computational time. Since the number of diagrams tend to decrease by 1 to 2 orders of magnitude, depending on the order and the number of particles in the process. This way reduces the time needed to calculate the diagrams of the next order, so at the grand scale, the time needed to calculate the diagrams of till a certain order is reduced by performing the search algorithm in between the orders.

4.3 Applied to gluons self-interactions

Although in this thesis the focus is on the gluons and its self-interactions, and considering only one type of particle, there are tests that can be done to check the validity of the program for multiple types of particles. Thanks to the instantaneous interactions, the program can be adapted to consider this type of interactions as a new type of particles, and thus check for bugs or other issues that may arise when considering multiple types of particles.

This approach comes with its own problems. By considering an instantaneous process as a new type of particles, this process need to respect the time evolution of the system, meaning that in the eyes of the program, the instantaneous interaction is no longer instantaneous, but rather a process that happens in a certain time interval.

Nonetheless, this is a valid approach to consider the instantaneous interactions, since no possible diagrams will be lost, rather equivalent diagrams will be obtained, without the program being aware of it, and it rests on the user to identify these equivalences.

For lower orders, this approach works well, since the number of diagrams is manageable, but at higher orders, this is a problem that needs to be addressed in the future, and a general fix to the problem will be implemented in the program.

4.4 Diagram representation

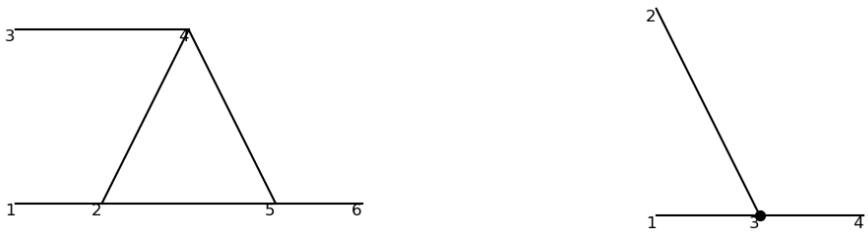
Every term in the Hamiltonian can be identified with a diagram, depending on the structure of the creation and annihilation operators present in it. Each diagram will be composed of different elements,

- External legs, representation of the incoming and outgoing particles or the annihilation and creation operators, respectively. They determine the type of process that is being considered.
- Internal legs, represent the virtual particles that are exchanged during the process, or the intermediate creation and annihilation operators that are eliminated in the normal ordering process.These particles are not observed in the final state.
- Vertices, points where interactions between particles occur, the number of vertices in a diagram indicates the order at which the diagram is contributing.
- Loops, closed paths in the diagrams, formed by the internal legs. It will indicate the presence of divergences in the process.
- Counterterms, additional terms added to the Hamiltonian to deal with the divergences produced during the process. These will be represented with a dot in the diagram.

Notice that the diagrams are similar to the Feynman diagrams [2], except for the presence of the counterterms. The main difference is the order that the processes occur, altering the order of the vertices in the diagram will produce a different process.

Other difference how the diagrams are read, in the context of RGPEP, the diagrams are read from the left to the right, meaning that the time evolution is from the left to the right.

As an example, consider the diagram in figure 1, this pair of diagrams are one of the contributions to the three-gluon vertex, with the counterterm associated with it.



(a) Diagram with a three-gluon loop. (b) Counterterm of the diagram

Figure 1: A example of a possible diagram, a third order contribution to the process of three-gluon vertex, 1 gluon incoming and 2 gluons outgoing. (Outputs of the programs.)

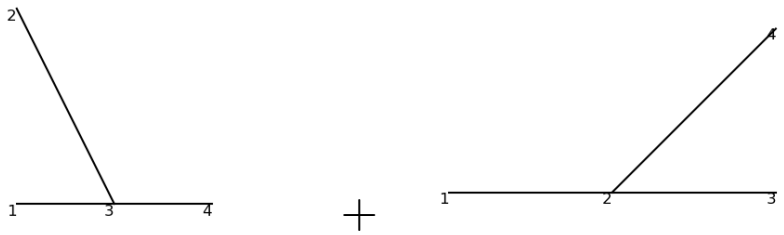
Following the previous definitions, the diagram represent a process where a gluon interacts in a third order process, forming a three-gluon loop, and 2 gluons are produced in the final state. Due to the presence of a loop, the diagram is divergent, and a counterterm diagram is needed to deal with the divergence. This diagram is represented with a dot at the position of the loop, with the same external legs as the original diagram.

5 Diagrams obtained

In this section, we will present the diagrams obtained from the program. At each order, we will consider the types of processes that is the most relevant.

5.1 Order 1

The first order diagrams correspond to the canonical diagrams of order 1, from equation (3.15), shown in figure 2. These diagrams represent the process of 1 gluon going to 2 gluons, and its hermitian conjugate of 2 gluons going to 1 gluon.



(a) (b)

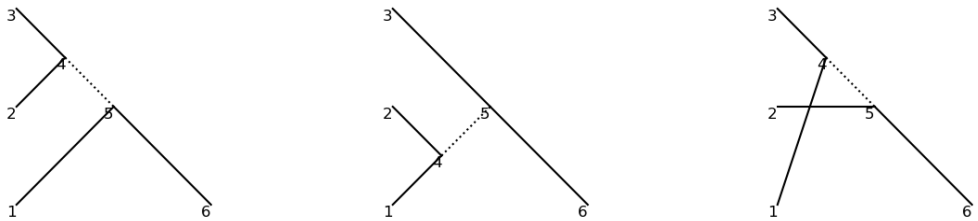
Figure 2: Canonical diagrams of order 1

5.2 Order 2

The second order diagrams correspond to the canonical diagrams of order 2, plus the diagrams obtained from the combination of the canonical diagrams of order 1 with itself.

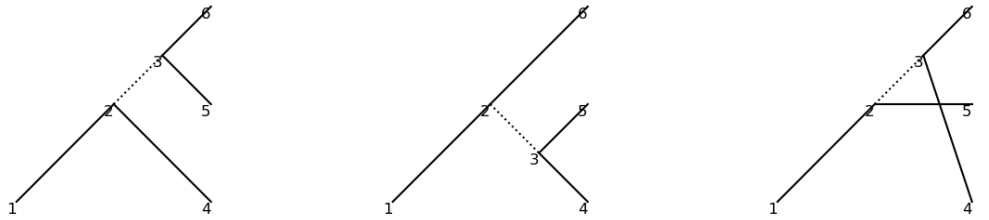
5.2.1 Canonical diagrams

The canonical diagrams of order 2 are obtained from the term $H_{[\partial AA]^2}$, from equation (3.17). The diagrams obtained are shown in figures 3, 4 and 5. Each of these diagrams corresponds to a different term in the Hamiltonian, contributing to different processes.



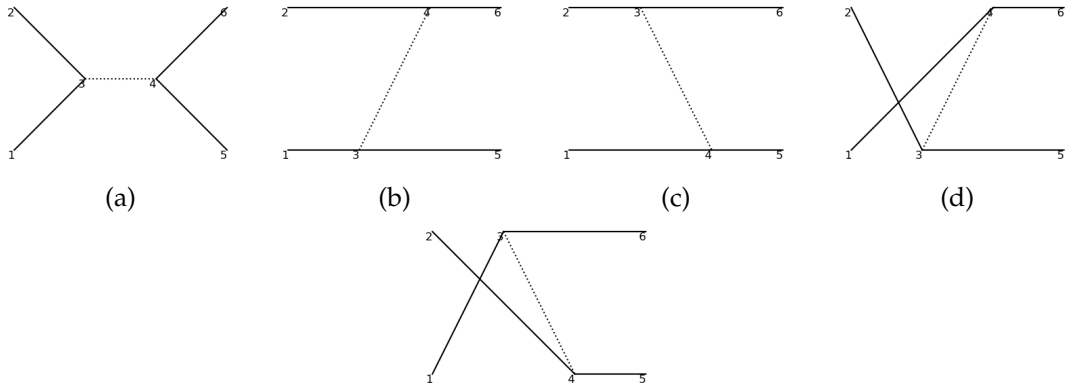
(a) (b) (c)

Figure 3: Canonical diagrams of order 2, for the term $\Xi_{[\partial AA]^2}$, where one gluon is annihilated, and 3 gluons are created.



(a) (b) (c)

Figure 4: Canonical diagrams of order 2, for the term hermitian conjugate of $\Xi_{[\partial AA]^2}$, where 3 gluons are annihilated, and 1 gluon is created.



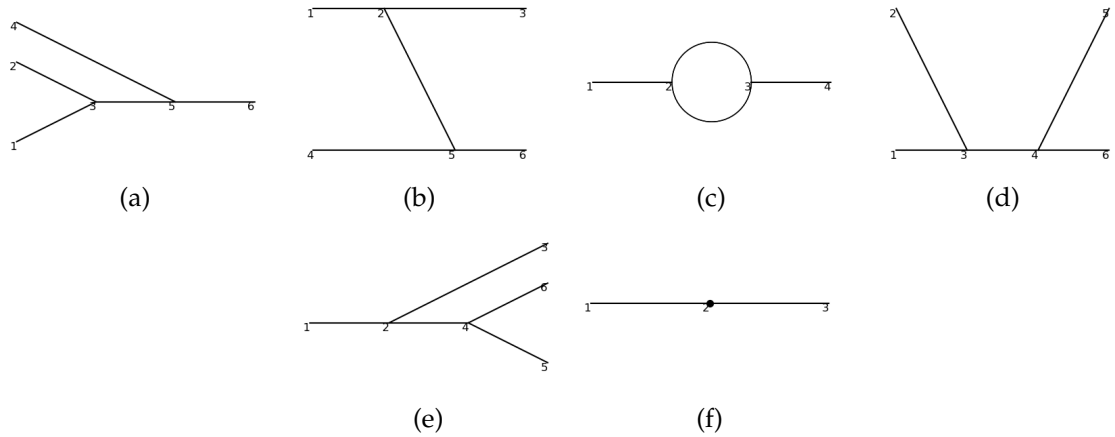
(a) (b) (c) (d)

(e)

Figure 5: Canonical diagrams of order 2, for the term $X_{[\partial AA]^2}$, where 2 gluons are annihilated, and 2 gluons are created.

5.2.2 Diagrams by combination of order 1

The second order diagrams can also be obtained from the combination of the canonical diagrams of order 1, shown in figure 2.



(a) (b) (c) (d)

(e) (f)

Figure 6: Diagrams of second order, obtained from the combination of the canonical diagrams of order 1, with the order 2 counterterms

The diagram 6a is the combination of 2 2a diagrams, where 1 one of the out-

going gluon from the first diagram is the incoming gluon of the second diagram. Similarly, the diagram 6e is the combination of 2 2b diagrams, the outgoing gluon of the first diagram is the one of the incoming gluon of the second diagram.

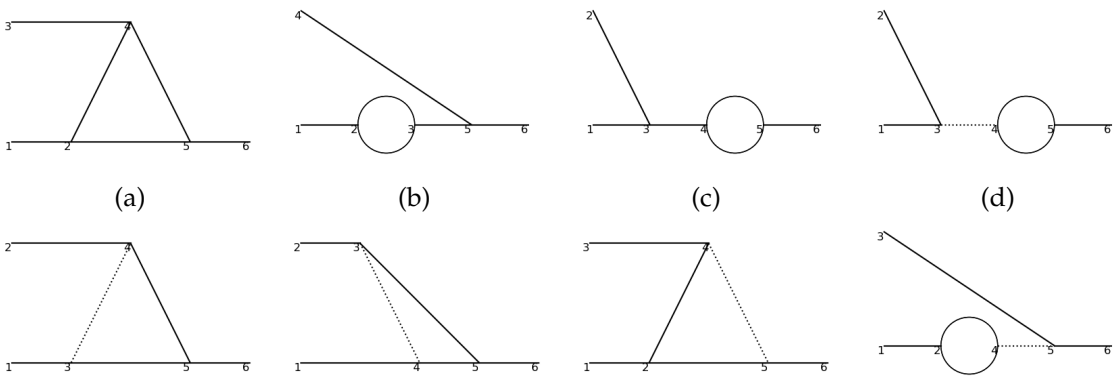
As for the diagrams 6b and 6c, both result from the combination of 2a and 2b diagrams, in that order, where the different diagrams are obtained depending on the number of connections between the incoming and outgoing gluons.

The diagram 6d is the combination of the 2b and 2a diagrams, where the outgoing gluon of the first diagram is the incoming gluon of the second diagram.

Finally, the diagram 6f is the counterterm associated with the second order diagrams, canceling the divergences that arises.

5.3 Three gluon vertex: 1 gluon to 2 gluons

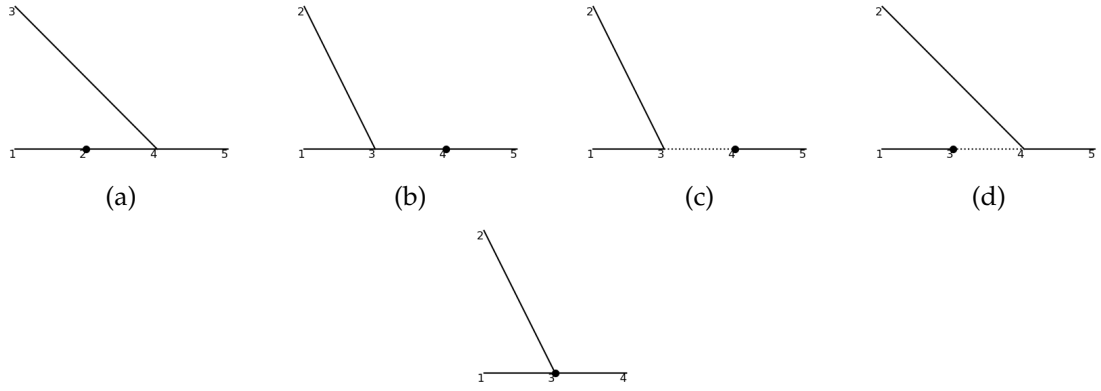
Considering the three-gluon vertex, the process of 1 gluon going to 2 gluons, up till order 3, the diagrams obtained from the program are shown in the figures 7 and 8.



(a) (b) (c) (d)

(e) (f) (g) (h)

Figure 7: Diagrams of third order, contributing to the three-gluon vertex.



(a) (b) (c) (d)

(e)

Figure 8: Third-order contributions containing counterterms [graphs (a) to (d)] and the third order counterterm [graph (e)].

In figure 7 we can see the diagrams obtained for the interaction of 1 gluon going to 2 gluons, with the corresponding counterterms in figure 8.

Depending on the types of particles in the process, we can deduce the origin of the different diagrams. With the once having dotted lines, and thus coming from the second order term canonical diagrams $H_{[\partial AA]^2}$, combined with a first order term canonical diagram H_{A^3}. While the diagrams without dotted lines come solely from the first order term canonical diagrams H_{A^3}

Referring to the problems mentioned in Section 4.3, the diagrams with dotted lines are instantaneous interactions, but the program considers them as a new type of particle, there are some artifacts that arise from this approach. For instance, the diagrams 7e and 7f are actually the same diagram, due to instant process being instantaneous. For any other particle, these 2 diagrams would be different, due to the importance of the order in the interactions. Hence, the program keeps them different, to avoid the lost of generality.

Other artifact of the program occur for the diagrams 8c and 8d, which are counterterms added to cancel the divergences in the canonical diagrams. These counterterms are already considered in the canonical diagrams, and thus not needed to be added again. But due to the way the program is implemented, it adds counterterms to all loop divergences in the diagrams.

Comparing with the 3rd order contribution diagrams in [9], the same diagrams for the three-gluon vertex are obtained, proving the validity of the program to reproduce the same results.

5.4 Self-energy: Gluon's effective mass

The self-energy describes the interaction of a gluon with its own field, leading to modifications of the propagator and the effective mass of the gluon, such that at the infrared limit, the gluon mimics a massive particle.

This way the gauge boson propagator in momentum space is of the form,

$$D(p^2) \sim \frac{1}{p^2 - m_t^2} \tag{5.1}$$

where m_t is the effective mass of the gluon at a particular scale, arising due to the presence of loop corrections.

To obtain the effective mass of the gluon, we need to consider one to one gluon interactions, or equivalently the propagation of a gluon. This way the effective mass can be obtained by considering the sum on the contributions of the diagrams at each order,

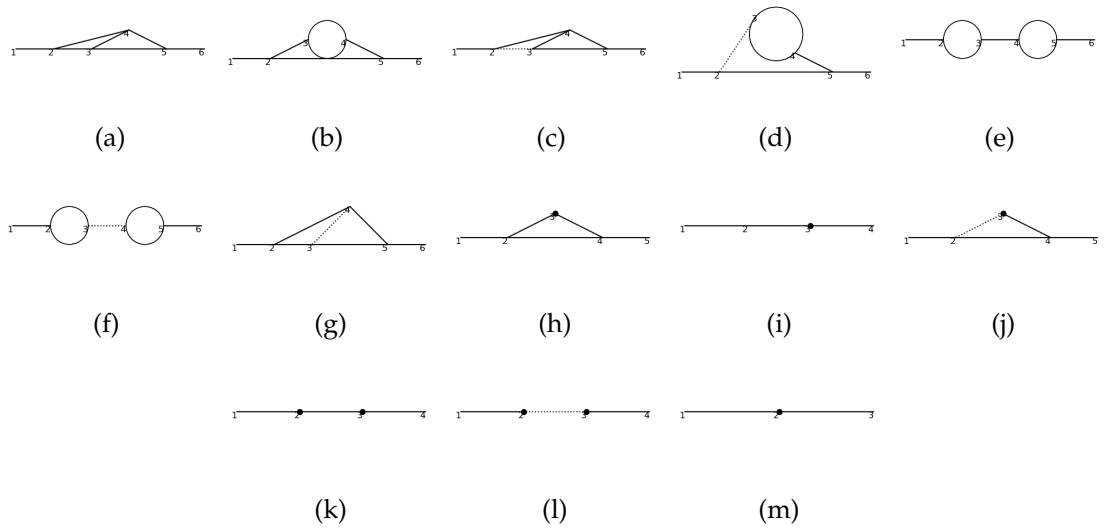
$$m_t^2 = g^2 m_{(2),t}^2 + g^4 m_{(4),t}^2 + g^6 m_{(6),t}^2 + \dots \tag{5.2}$$

with $m_{(n),t}^2$ being the effective mass correction at order n.

This way, it is in our best interest to obtain the diagrams at each order that contribute to the gluon's self-energy, and thus the effective mass.

At order 2, the diagram that contribute to the gluon's self-energy presented in figure 6c and 6f, the only diagrams from one gluon to one gluon.

At order 4, the diagrams that contribute to the gluon's self-energy are shown in figure



(a) (b) (c) (d) (e)

(f) (g) (h) (i) (j)

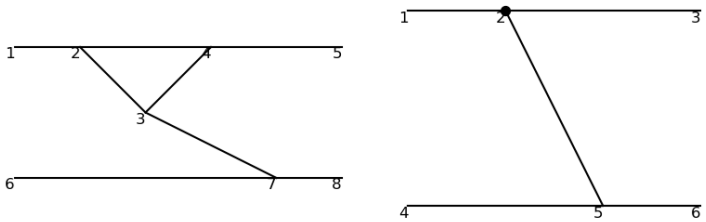
(k) (l) (m)

Figure 9: Diagrams of fourth order, contributing to the gluon's self-energy, with (h)-(l) diagrams with counterterms of order 2, and (m) the counterterm of order 4

5.4.1 Additional implications: Quark interactions

The implication of the gluon's self-energy extend to the quark interactions, since the gluon is the mediator of the strong force, and thus the effective mass of the gluon will impact the results of the quark interactions.

5.5 Bound states: Glueballs



(a) (b)

Figure 10: Diagrams of fourth order,

5.6 Program performance

By increasing the orders, the number of diagrams scale exponentially, and so does the computational time needed to calculate the diagrams. The program has been tested with orders up to 6, and the results are shown in table 1.

Order	Unique diagrams	Computational time (s)
2	16	0.150
3	76	0.198
4	612	1.166
5	5871	10.935
6	65000	157.121

Table 1: Number of diagrams obtained by the program at each order, and the Computational time taken to calculate the diagrams.

6 Conclusions and future work

In this thesis, we have presented a program that implements the RGPEP to obtain the diagrams of the gluons self-interactions, and the perturbative expansion of the Hamiltonian. The program is able to obtain the diagrams of higher order, starting from the canonical diagrams, and using the order by order procedure. The program is modular, and can be adapted to other theories, by changing the canonical diagrams and the types of particles considered. The program has been tested with the gluons self-interactions, and has been able to obtain the diagrams of higher order, with the corresponding counterterms to cancel the divergences produced in the process. The program has been able to reproduce the diagrams obtained in the literature, proving its validity to obtain the diagrams of higher order in the perturbative expansion of the Hamiltonian.

The program is able to obtain the diagrams of higher order, but there are still some issues that need to be addressed, such as the artifacts produced by the instantaneous interactions, and the time taken to calculate the diagrams at higher orders.

The program is still in development, and there are many improvements that can be made to optimize the process of obtaining the diagrams. Other main issue to address is the counting of the diagrams, as the program is able to obtain the diagrams, but it does not output the symmetry factor correctly, due to the problems with the factors associated to the canonical diagrams, and the counterterms, as well as the method used to find the diagrams that prioritize ensuring that no diagrams are lost, rather than ensuring that the symmetry factor is correct.

As future work, we plan to fix the issues mentioned above, and to implement and test the program with other theories, and more types of particles, particularly considering the quarks and antiquarks, in the general case of QCD.

7 Conclusiones y trabajo futuro

8 Acknowledgements

A Functions in the Hamiltonians

In this section, we present the functions that are present in the Hamiltonian from equations (3.15), (3.16), and (3.17).

$$Y_{123} = if^{c_1c_2c_3}\left[\varepsilon_1^*\varepsilon_2^*\cdot\varepsilon_3\kappa - \varepsilon_1^*\varepsilon_3\cdot\varepsilon_2^*\kappa\frac{1}{x^{2/3}} - \varepsilon_2^*\varepsilon_3\cdot\varepsilon_1^*\kappa\frac{1}{x^{1/3}}\right]. \tag{A.1}$$

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